Squared Euclidean Distance Matrix (SEDM)

(x_i, y_i, z_i)
ith observation point

Observation points matrix

$$P = \begin{bmatrix} x_0 & x_1 & \cdots & x_{N-1} \\ y_0 & y_1 & \cdots & y_{N-1} \\ z_0 & z_0 & \cdots & z_{N-1} \end{bmatrix}_{N \times 3}$$

x

y

z

Source points matrix

$$S = \begin{bmatrix} x'_0 & x_{1'} & \cdots & x'_{M-1} \\ y'_0 & y'_1 & \cdots & y'_{M-1} \\ z'_0 & z'_1 & \cdots & z'_{M-1} \end{bmatrix}_{3 \times M}$$
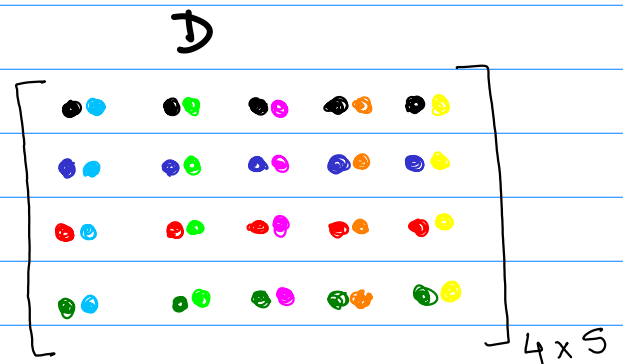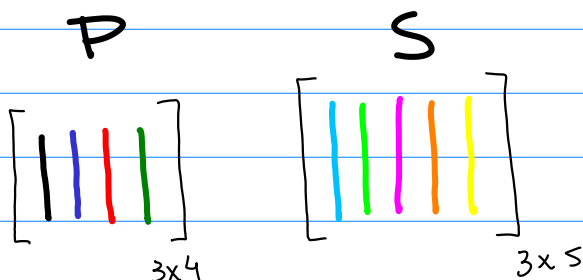
(x'_j, y'_j, z'_j)
jth source point

Squared Euclidean distance

$$d_{ij}^2 = (x_i - x'_j)^2 + (y_i - y'_j)^2 + (z_i - z'_j)^2$$

$$i = 0, \ldots, N-1 \quad j = 0, \ldots, M-1$$

Squared Euclidean Distance Matrix (SEDM)

$$D = \begin{bmatrix} d_{00}^2 & \cdots & d_{0m-1}^2 \\ \vdots & & \vdots \\ d_{N-1\,0}^2 & \cdots & d_{N-1\,m-1}^2 \end{bmatrix}_{N \times M}$$

P

S

D

3×4

3×5

4×5

$$d_{ij}^2 = P_i^T P_i + s_j^T s_j - 2 P_i^T s_j$$

$$D_1 = \begin{bmatrix} P_0^T P_0 & \cdots & P_0^T P_0 \\ & & \\ P_{N-1}^T P_{N-1} & \cdots & P_{N-1}^T P_{N-1} \end{bmatrix}_{N \times M}$$

$$P_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \qquad s_j = \begin{bmatrix} x'_j \\ y'_j \\ z'_j \end{bmatrix} \qquad D_2 = \begin{bmatrix} s_0^T s_0 & & s_{m-1}^T s_{m-1} \\ \vdots & & \vdots \\ s_0^T s_0 & & s_{m-1}^T s_{m-1} \end{bmatrix}_{N \times M}$$

$$\boxed{D = D_1 + D_2 + D_3}$$

$$D_3 = \begin{bmatrix} P_0^T s_0 & P_0^T s_{m-1} \\ & \\ P_{N-1}^T s_0 & P_{N-1}^T s_{m-1} \end{bmatrix}_{N \times M} = P^T S$$