Sharma

1

$$u = \frac{\partial \phi}{\partial x}$$

$$\phi = -\frac{mT}{4\pi R_1} d\Omega \quad \text{em que} \quad R_1 = \sqrt{x^2 + y^2 + (z-c)^2}$$

$$u = \frac{\partial \phi}{\partial x} = \frac{-mT}{4\pi} d\Omega \; \frac{\partial\, {}^1/R_1}{\partial x}$$

$$\frac{\partial\, {}^1/R_1}{\partial x} = \frac{\partial}{\partial x}\frac{1}{R_1} = \frac{\partial}{\partial x}\left(x^2 + y^2 + (z-c)^2\right)^{-1/2}$$

$$\frac{\partial}{\partial x}\frac{1}{R_1} = -\frac{1}{\cancel{2}}\left(x^2+y^2+(z-C)^2\right)^{-3/2} \cancel{2}x$$

$$\boxed{\frac{\partial}{\partial x}\frac{1}{R_1} = -\left(x^2+y^2+(z-c)^2\right)^{-3/2} x}$$

$$\frac{\partial}{\partial x} V_1 = -\left(x^2+y^2+(z-c)^2\right)^{-3/2} x$$

$$\frac{\partial}{\partial x} V_1 = \frac{-x}{\left(x^2+y^2+(z-c)^2\right)^{3/2}}$$

$$\frac{\partial}{\partial x} V_1 = \frac{-x}{R_1^{\,3}}$$

em que $V_1 = {}^1/R_1$

In Sharma we have

$$u = \frac{\partial \Phi}{\partial x} = \left(\frac{-mT}{4\pi}\, d\Omega\right) \frac{\partial V_1}{\partial x}$$

~~[illegible]~~ $V_1 = 1/R_1$.

constant is $-\frac{mT}{4\pi} d\Omega$

$$m = \frac{\alpha(1+\sigma)}{1-\sigma}$$

In equation 5 Sharma stated that

$$u = \frac{\partial \Phi}{\partial x} = \frac{A(1+\sigma)}{E} \frac{\partial V_1}{\partial x}$$

$$A = -\frac{\alpha E T d\Omega}{4\pi(1-\sigma)}$$

In next page I show that the constants.

$$\boxed{-\frac{mT}{4\pi} d\Omega = \frac{A(1+\sigma)}{E}}$$

constant in equation 4 shown

$$\frac{-mT}{4\pi}\, d\Omega \qquad (1)$$

Constant in equation 5 shown

$$\frac{A(1+\sigma)}{E} \qquad (2)$$

where $A = -\dfrac{\alpha E T d\Omega}{4\pi(1-\sigma)}. \qquad (3)$

How to obtain A

$$\underbrace{\frac{-mT}{4\pi}\, d\Omega}_{1} = \underbrace{\frac{A(1+\sigma)}{E}}_{2}$$

$$\frac{-mTE\, d\Omega}{4\pi(1+\sigma)} = A \qquad (3a)$$

Substituting A (3a) into equation (2) we have

$$\frac{A(1+\sigma)}{E} = \frac{-mT\cancel{E}d\Omega\,\cancel{(1+\sigma)}}{4\pi\cancel{(1+\sigma)}\,\cancel{E}}$$

This shows that ~~both~~ equation 1 and 2 is equal.

$$\boxed{\frac{A(1+\sigma)}{E} = \frac{-mT\, d\Omega}{4\pi}}$$

FIRST System

4

The x-component of the displacement in x is given by

$$u = \frac{\partial \phi}{\partial x} = \frac{A(1+G)}{E}\frac{\partial V_1}{\partial x} = \frac{A(1+G)}{E}\left(\frac{-x}{R_1^3}\right)$$

$$\boxed{u = \frac{A(1+G)}{E}\left(\frac{-x}{R_1^3}\right)}$$

The y-component of the displacement in y is given by

$$v = \frac{\partial \phi}{\partial y} = \frac{A(1+G)}{E}\frac{\partial V_1}{\partial y} = \frac{A(1+G)}{E}\left(\frac{-y}{R_1^3}\right)$$

$$\boxed{v = \frac{A(1+G)}{E}\left(\frac{-y}{R_1^3}\right)}$$

The z component of the displacement in the z direction is given by

$$w = \frac{\partial \phi}{\partial z} = \frac{A(1+G)}{E}\frac{\partial V_1}{\partial z} = \frac{A(1+G)}{E}\left(\frac{-(z-c)}{R_1^3}\right)$$

$$\boxed{w = \frac{A(1+G)}{E}\left(\frac{-(z-c)}{R_1^3}\right)}$$

STRESS PORT system (5)
equation (6) of Sharma.

$$\hat{xz}_1 = A \frac{\partial^2 V_1}{\partial x \partial z} \quad \text{where } V_1 = \frac{1}{R_1}$$

$$\hat{xz}_1 = A \frac{\partial}{\partial z} \frac{\partial V_1}{\partial x}$$

$$\hat{xz}_1 = A \frac{\partial}{\partial z} \frac{-x}{R_1^3} = A \frac{\partial}{\partial z} \frac{-x}{\left(x^2 + y^2 + (z-c)^2\right)^{3/2}}$$

$$\hat{xz}_1 = A \frac{\partial}{\partial z} \underbrace{-x\left[x^2 + y^2 + (z-c)^2\right]^{-3/2}}$$

$$\frac{\partial}{\partial z} \cdot \frac{-x}{R_1^3} = \frac{-(-x)(+\tfrac{3}{2})\left[x^2+y^2+(z-c)^2\right]^{+1/2} 2(z-c)}{R_1^6}$$

$$\frac{\partial}{\partial z}\left(\frac{-x}{R_1^3}\right) = \frac{3x}{\cancel{2}} \frac{1}{R_1^6} \frac{\cancel{2}(z-c)}{} R_1$$

$$\frac{\partial}{\partial z}\frac{\partial V_1}{\partial x} = \frac{3x(z-c) R_1}{R_1^6} = \frac{3x(z-c)}{R_1^5}$$

$$\hat{xz}_1 = A \cdot \frac{3x(z-c)}{R_1^5}$$

equacão (6a) no Sharma

STRESS FOR 7 System

$\hat{yz}_1 = A \frac{\partial^2 V_1}{\partial y \partial z} =$ where $V_1 \frac{1}{R_1}$

$\hat{yz}_1 = A \frac{\partial}{\partial z} \frac{\partial}{\partial y} V_1 = A \frac{\partial}{\partial z} \frac{\partial}{y} {}^1/R_1$

$\hat{yz}_1 = A \frac{\partial}{\partial z} \frac{-y}{R_1^3} = A \frac{\partial}{\partial z} \frac{-y}{(x^2+y^2+(z-c)^2)^{3/2}}$

$\hat{yz}_1 = A \frac{\partial}{\partial z} -y\left[x^2+y^2+(z-c)^2\right]^{-3/2}$

$= A \; -y\left(-\frac{3}{\cancel{2}}\right)\left[x_2+y^2+(z-c)^2\right]^{-5/2} \cancel{2}(z-c)$

$= A \frac{3y(z-c)}{R_1^5}$

$\hat{yz}_1 = A \frac{3y(z-c)}{R_1 5}$

Eq (6 b) in Sharma

STRESS FIRST System (7)

$$\hat{z}z_1 = A \frac{\partial^2}{\partial z^2} V_1 = A \frac{\partial}{\partial z} \underbrace{\frac{\partial}{\partial z} 1/R_1}_{\frac{-(z-c)}{R_1^3}}$$

$$\hat{z}z_1 = A \frac{\partial}{\partial z} \frac{-(z-c)}{R_1^3}$$

$$\hat{z}z_1 = A \frac{\partial}{\partial z} \frac{-(z-c)}{\left[x^2 + y^2 + (z-c)^2\right]^{3/2}}$$

$$\hat{z}z_1 = A \frac{\partial}{\partial z} -(z-c) \left[x^2 + y^2 + (z-c)^2\right]^{-3/2}$$

$$\hat{z}z_1 = A \left[\frac{-1}{R_1^3} + -(z-c)\left(-\frac{3}{\cancel{2}}\right)\left[x^2+y^2+(z-c)^2\right]^{-5/2} \cancel{2}(z-c) \right.$$

$$\boxed{\hat{z}z_1 = A \left[\frac{-1}{R_1^3} + 3\frac{(z-c)^2}{R^5} \right]}$$

Equação 6c em Sharma.

STRESS system 2

$$\boxed{\hat{xz}_2 = \psi_{21} = A\frac{\partial^2 V_2}{\partial x \partial z}}$$ (Equation 8a in Sharma)

$$V_2 = \frac{1}{R_2} \qquad R_2 = \left(x^2 + y^2 + (z \pm c)^2\right)^{1/2}$$

$$\boxed{\hat{yz}_2 = \psi_{22} = A\frac{\partial^2 V_2}{\partial y \partial x}}$$ (Equation 8b in Sharma)

Lets's calculate:

$$\frac{\partial V_2}{\partial x} = \frac{\partial}{\partial x}\frac{1}{(R_2)} = \frac{\partial}{\partial x}\left[x^2 + y^2 + (z+c)^2\right]^{-1/2}$$

$$= -\frac{1}{\cancel{2}}\left[x^2 + y^2 + (z+c)^2\right]^{-3/2} \cancel{2}x$$

$$\frac{\partial V_2}{\partial x} = \frac{-x}{\left[x^2 + y^2 + (z+c)^2\right]^{3/2}} = \frac{-x}{R_2^3}$$

$$\frac{\partial}{\partial z}\frac{\partial}{\partial x}V_2 = \frac{\partial}{\partial z}\frac{-x}{R_2^3} = \frac{\partial}{\partial z}\left(-x\left[x^2 + y^2 + (z+c)^2\right]^{-3/2}\right)$$

$$= -x\left(-\frac{3}{\cancel{2}}\right)R_2^{-5/2} \cdot \cancel{2}(z+c)$$

$$\boxed{\frac{\partial}{\partial z}\frac{\partial}{\partial x}V_2 = \frac{3x(z+c)}{R_2^5}}$$