Operador $\nabla_2 V_2$ → operação sistema 2 Mindlin and Cheng (1950) equação pag 931

(1)

$$\nabla_2 = \underbrace{(3-4\nu)\begin{bmatrix}\partial_x V_2\\ \partial_y V_2\\ \partial_z V_2\end{bmatrix}}_{I_1} + \underbrace{2\begin{bmatrix}\partial_x\\ \partial_y\\ \partial_z\end{bmatrix} z\,\partial_z V_2}_{I_2} - \underbrace{4(1-\nu)\hat{z}\,\nabla_z^2 V_2}_{I_3}$$

$$I_2 = 2\begin{bmatrix}\partial_x(z\,\partial_z V_2)\\ \partial_y(z\,\partial_z V_2)\\ \partial_z(z\,\partial_z V_2)\end{bmatrix} = 2\begin{bmatrix} z\,\partial_{xz} V_2\\ z\,\partial_{yz} V_2\\ \underbrace{\partial_z(z)}_{1}\partial_z V_2 + z\,\partial_{zz} V_2\end{bmatrix} = 2\begin{bmatrix} z\,\partial_{xz} V_2\\ z\,\partial_{yz} V_2\\ \partial_z V_2 + z\,\partial_{zz} V_2\end{bmatrix}$$

$$I_2 = 2z\begin{bmatrix}\partial_{xz} V_2\\ \partial_{yz} V_2\\ \partial_{zz} V_2\end{bmatrix} + 2\begin{bmatrix}0\\ 0\\ \partial_z V_2\end{bmatrix}$$

$$I_3 = \begin{bmatrix}0\\ 0\\ -8\,\partial_z V_2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ +8\nu\,\partial_z V_2\end{bmatrix}$$

leia pag 2

$$\nabla_2 = \begin{bmatrix}(3-4\nu)\,\partial_x V_2\\ (3-4\nu)\,\partial_y V_2\\ 3\partial_z V_2 - 4\nu\,\partial_z V_2\end{bmatrix} + 2z\begin{bmatrix}\partial_{xz} V_2\\ \partial_{yz} V_2\\ \partial_{zz} V_2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 2\partial_z V_2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -8\partial_z V_2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 8\nu\,\partial_z V_2\end{bmatrix}$$

$$\nabla_2 = \begin{bmatrix}(3-4\nu)\,\partial_x V_2\\ (3-4\nu)\,\partial_y V_2\\ \overbrace{3\partial_z V_2 + 2\partial_z V_2 - 8\partial_z V_2}^{-3\partial_z V_2}\ \overbrace{-4\nu\,\partial_z V_2 + 8\nu\,\partial_z V_2}^{+4\nu\,\partial_z V_2}\end{bmatrix}$$

$$+\ 2z\begin{bmatrix}\partial_{xz} V_2\\ \partial_{yz} V_2\\ \partial_{zz} V_2\end{bmatrix}$$

$$\nabla_2 = \begin{bmatrix}(3-4\nu)\,\partial_x V_2\\ (3-4\nu)\,\partial_y V_2\\ (3-4\nu)-\partial_z V_2\end{bmatrix} + 2z\begin{bmatrix}\partial_{xz} V_2\\ \partial_{yz} V_2\\ \partial_{zz} V_2\end{bmatrix}$$

$$\nabla_2 = (3-4\nu)\begin{bmatrix}\partial_x V_2\\ \partial_y V_2\\ -\partial_z V_2\end{bmatrix} + 2z\begin{bmatrix}\partial_{xz} V_2\\ \partial_{yz} V_2\\ \partial_{zz} V_2\end{bmatrix}$$

$$I_3 = -4(1-\nu)\,\hat{z}\,\nabla_z^2 V_2 = -4(1-\nu)\begin{bmatrix}0\\0\\1\end{bmatrix}\nabla^2\left(\frac{z}{R_2}\right)$$ ②

calculando $\nabla^2\left(\frac{z}{R_2}\right)$

$$\nabla^2\left(\frac{z}{R_2}\right) = \partial_{xx}\left(\frac{z}{R_2}\right) + \partial_{yy}\left(\frac{z}{R_2}\right) + \partial_{zz}\left(\frac{z}{R_2}\right)$$

$$= z\,\partial_{xx}\left(\frac{1}{R_2}\right) + z\,\partial_{yy}\left(\frac{1}{R_2}\right) + z\,\partial_{zz}\left(\frac{1}{R_2}\right) + 2\partial_z\left(\frac{1}{R_2}\right)$$

$$\nabla^2\left(\frac{z}{R_2}\right) = z\underbrace{\cancel{\left[\partial_{xx}\left(\frac{1}{R_2}\right) + \partial_{yy}\left(\frac{1}{R_2}\right) + \partial_{zz}\left(\frac{1}{R_2}\right)\right]}^{0}}_{\nabla^2 1/R_2 = 0} + 2\partial_z\left(\frac{1}{R_2}\right)$$

calculamos $\partial_{zz}\left(\frac{z}{R_2}\right)$

$$\partial_{zz}\left(\frac{z}{R_2}\right) = \partial_z\left[\partial_z\left(\frac{z}{R_2}\right)\right]$$

$$= \partial_z\left[\frac{1}{R_2} + z\partial_z\left(\frac{1}{R_2}\right)\right]$$

$$= \partial_z\left(\frac{1}{R_2}\right) + 1\,\partial_z\left(\frac{1}{R_2}\right) + z\,\partial_{zz}\left(\frac{1}{R_2}\right)$$

$$\partial_{zz}\left(\frac{z}{R_2}\right) = 2\,\partial_z\left(\frac{1}{R_2}\right) + z\,\partial_{zz}\left(\frac{1}{R_2}\right)$$

$$I_3 = -4(1-\nu)\begin{bmatrix}0\\0\\2\partial_z\left(\frac{1}{R_2}\right)\end{bmatrix}$$

$$I_3 = \begin{bmatrix}0\\0\\-8\,\partial_z\left(\frac{1}{R_2}\right)\end{bmatrix} + \begin{bmatrix}0\\0\\+8\nu\,\partial_z\left(\frac{1}{R_2}\right)\end{bmatrix}$$