From Beltrami equations (1a)

$$\hat{X}Z_R = -\frac{1}{2(1+\sigma)} z \frac{\partial \Theta_R}{\partial x} + \psi_{R1}$$

$$\hat{Y}Z_R = -\frac{1}{2(1+\sigma)} z \frac{\partial \Theta_R}{\partial y} + \psi_{R2} \qquad (1)$$

$$\hat{Z}Z_R = -\frac{1}{2(1+\sigma)} z \frac{\partial \Theta_R}{\partial z} + \psi_{R3}$$

$$\boxed{\Theta_R = \hat{X}X_R + \hat{Y}Y_R + \hat{Z}Z_R}$$

Θ_R FIRST STRESS INVARIANT

ψ_{R1}, ψ_{R2} and ψ_{R3} are harmonic function

Equations of Equilibrium

$$\frac{\partial \Theta_R}{\partial z} = \frac{\partial}{\partial z}\hat{X}X_R + \frac{\partial}{\partial z}\hat{Y}Y_R + \frac{\partial}{\partial z}\hat{Z}Z_R$$

According eq. 2 of Sharma we have:

$$\frac{\partial \Theta_R}{\partial z} = 2(1+\sigma)\left\{\frac{\partial \psi_{R1}}{\partial x} + \frac{\partial \psi_{R2}}{\partial y} + \frac{\partial \psi_{R3}}{\partial z}\right.$$

→ Equation (2) Sharma
(2) Pamela Tempone

From the stress-strain relations (1b)

$$\left.\begin{aligned} \frac{\partial w_R}{\partial z} &= \frac{1}{E}\left[(1+\sigma)\hat{zz}_R - \sigma \cdot \Theta_r\right] \\ \frac{\partial v_r}{\partial z} + \frac{\partial w_R}{\partial y} &= \frac{2(1+\sigma)}{E}\widehat{yz_R} \\ \frac{\partial u_R}{\partial z} + \frac{\partial w_R}{\partial x} &= \frac{2(1+\sigma)}{E}\widehat{xz_R} \end{aligned}\right\} 3$$

In Sharma is stated that from equation (3) and (1) we obtained in equation 10. How to obtain $\hat{xz}_2$ $\hat{yz}_2$ and $\hat{zz}_2$. by using ~~xz~~ these quantities by using eq 3 and 1?

In equation 1a we have if $R=2$

$$\tilde{xz}_2 = -\frac{1}{2(1+\sigma)} z \frac{\partial \Theta_2}{\partial x} + \Psi_{21}.$$

(1C)

$$\frac{\partial \Theta_R}{\partial z} = 2(1+\vartheta)\left\{\frac{\partial \Psi_{R1}}{\partial x} + \frac{\partial \Psi_{R2}}{\partial y} + \frac{\partial \Psi_{R3}}{\partial z}\right\}$$

$R=2$

$$\frac{\partial \Theta_2}{\partial z} = 2(1+\vartheta)\left\{\frac{\partial \Psi_{21}}{\partial x} + \frac{\partial \Psi_{22}}{\partial y} + \frac{\partial \Psi_{23}}{\partial z}\right\}$$

How to calculate the first stress invariant. Θ_2

$$\cancel{\Theta_2 = -4A(1+\sigma)\frac{\partial^2 V_2}{\partial z^2}}$$

$$\cancel{\frac{\partial \Theta_2}{\partial z} = -4A(1+\sigma)\frac{\partial}{\partial z}\frac{\partial^2 V_2}{\partial z^2} =}$$

$$\cancel{\frac{\partial \Theta_2}{\partial z} = -4A(1+\sigma)\frac{\partial^3 V_2}{\partial z^3}}$$

$$\frac{\partial \Theta_2}{\partial z} = 2(1+\vartheta)\left\{\frac{A\partial}{\partial x}\overbrace{\frac{\partial^2 V_2}{\partial x\partial z}}^{\Psi_{21}} + \frac{A\partial}{\partial y}\overbrace{\frac{\partial^2 V_2}{\partial y\partial z}}^{\Psi_{22}} - \frac{A\partial}{\partial z}\overbrace{\frac{\partial^2 V_2}{\partial z^2}}^{\Psi_{23}}\right\}$$

$$\frac{\partial \Theta_2}{\partial z} = 2A(1+\vartheta)\left\{\frac{\partial}{\partial x}\frac{\partial^2 V_2}{\partial x\partial z} + \frac{\partial}{\partial y}\frac{\partial^2 V_2}{\partial y\partial z} - \frac{\partial}{\partial z}\frac{\partial^2 V_2}{\partial z^2}\right\}$$

1d

$$\theta_2 = \int \frac{\partial \theta_2}{\partial z}\, dz.$$

$$\frac{\partial \theta_2}{\partial z} = 2A(1+\delta)\left\{\frac{\partial}{\partial x}\frac{\partial^2 V_2}{\partial x \partial z} + \frac{\partial}{\partial y}\frac{\partial^2}{\partial y \partial z} - \frac{\partial^3 V_2}{\partial z^3}\right\}$$

$$\theta_2 = \int \frac{\partial \theta_2}{\partial z} = 2A(1+\delta)\left\{\frac{\partial^2}{\partial x^2}V_2 + \frac{\partial^2}{\partial y^2}V_2 - \frac{\partial^2 V_2}{\partial z^2}\right\}$$

$$\theta_2 = 2A(1+\delta)\frac{\partial^2 V_2}{\partial x^2} + 2A(1+\delta)\frac{\partial^2 V_2}{\partial y^2} - 2A(1+\delta)\frac{\partial^2 V_2}{\partial z^2}$$

~~Diferente da equação de~~ Sharma

$$\theta_2 = 2A(1+\delta)\underbrace{\left[\frac{\partial^2 V_2}{\partial x^2} + \frac{\partial^2 V_2}{\partial y^2}\right]}_{\text{Pela eq Laplace } = -\frac{\partial^2 V_2}{\partial z^2}} - 2A(1+\delta)\frac{\partial^2 V_2}{\partial z^2}$$

$$\theta_2 = -2A(1+\delta)\frac{\partial^2 V_2}{\partial z^2} - 2A(1+\delta)\frac{\partial^2 V_2}{\partial z^2}$$

$$\theta_2 = -4A(1+\delta)\frac{\partial^2 V_2}{\partial z^2}.$$

Igual a Eq 9 Sharma