p. 404 do livro

$$\overline{R}_1 = \begin{bmatrix} x_0 - x \\ y_0 - y \\ D - z \end{bmatrix} \qquad \overline{R}_2 = \overline{R}_1 - 2D\hat{z} = \begin{bmatrix} x_0 - x \\ y_0 - y \\ -D - z \end{bmatrix}$$

$\boxed{\text{eq. } 12.39}$

$$\overline{u} = \frac{C_m}{4\pi}\left\{ \overbrace{\frac{\overline{R}_1}{R_1^3}}^{\mathrm{I}} + \overbrace{(3-4\nu)\frac{\overline{R}_2}{R_2^3}}^{\mathrm{II}} \overbrace{- \frac{6z(z+D)\,\overline{R}_2}{R_2^5}}^{\mathrm{III}} \right.$$

$$\left. + \overbrace{\frac{2\hat{z}}{R_2^3}\left[(3-4\nu)(z+D) - z\right]}^{\mathrm{IV}} \right\} \quad V \propto \Delta p_f$$

I $\quad \dfrac{\overline{R}_1}{R_1^3} = \dfrac{1}{R_1^3}\begin{bmatrix} x_0 - x \\ y_0 - x \\ D - z \end{bmatrix} = \overline{\nabla} V_1 = \begin{bmatrix} \partial_x V_1 \\ \partial_y V_1 \\ \partial_z V_1 \end{bmatrix}, \quad V_1 = \dfrac{1}{R_1}$

derivadas em relação ao ponto de observação (x, y, z)

II

$$(3-4\nu)\frac{\bar{R}_2}{R_2^3} = (3-4\nu)\underbrace{\frac{1}{R_2^3}\begin{bmatrix} x_0-x \\ y_0-y \\ -D-z \end{bmatrix}}_{\bar{\nabla} V_2} \qquad \bar{\nabla} V_2 = \begin{bmatrix} \partial_x V_2 \\ \partial_y V_2 \\ \partial_z V_2 \end{bmatrix}$$

$$V_2 = \frac{1}{R_2}$$

DERIVADAS EM RELAÇÃO AO PONTO DE OBSERVAÇÃO (x, y, z)

III

$$-\frac{6z(z+D)}{R_2^5}\bar{R}_2 = 2z\,\frac{3(-D-z)}{R_2^5}\begin{bmatrix} x_0-x \\ y_0-y \\ -D-z \end{bmatrix}$$

IV

$$\frac{2\hat{z}}{R_2^3}\Big[(3-4\nu)(z+D) - z\Big] = -2(3-4\nu)\frac{1}{R^3}$$

$$= -2(3-4\nu)\frac{1}{R_2^3}\begin{bmatrix} 0 \\ 0 \\ -D-z \end{bmatrix} + \frac{2z}{R_2^3}\begin{bmatrix} 0 \\ 0 \\ -z \end{bmatrix} =$$

$$= -(3-4\nu)\begin{bmatrix} 0 \\ 0 \\ 2\partial_z V_2 \end{bmatrix} + 2z\begin{bmatrix} 0 \\ 0 \\ -1/R_2^3 \end{bmatrix}$$

III + IV

$$-(3-4\nu)\begin{bmatrix} 0 \\ 0 \\ 2\partial_z V_2 \end{bmatrix} + 2z\left\{\frac{3}{R_2^5}\begin{bmatrix} (x_0-x)(-D-z) \\ (y_0-y)(-D-z) \\ (-D-z)(-D-z) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1/R_2^3 \end{bmatrix}\right\}$$

...

IV

$$-(3-4\nu)\begin{bmatrix} 0 \\ 0 \\ -2\partial_z V_2 \end{bmatrix} + 2z \begin{bmatrix} \partial_{xz} V_2 \\ \partial_{yz} V_2 \\ \partial_{zz} V_2 \end{bmatrix}$$

$$\bar{u} = \frac{C_m}{4\pi}\left\{ \mathrm{I} + \mathrm{II} + \mathrm{III} + \mathrm{IV} \right\} V\alpha\Delta p_f =$$

$$= \frac{C_m}{4\pi}\left\{ \begin{bmatrix} \partial_x V_1 \\ \partial_y V_1 \\ \partial_z V_1 \end{bmatrix} + (3-4\nu)\begin{bmatrix} \partial_x V_2 \\ \partial_y V_2 \\ -\partial_z V_2 \end{bmatrix} + 2z\begin{bmatrix} \partial_{xz} V_2 \\ \partial_{yz} V_2 \\ \partial_{zz} V_2 \end{bmatrix} \right\} V\alpha\Delta p_f$$