Nagy

Sharma

(1)

$$R = \left((x_0-x)^2+(y_0-y)^2+(z_0-z)^2\right)^{1/2}$$

$$R_1 = \left((x-x_0)^2+(y-y_0)^2+(z-c)^2\right)^{1/2}$$

$$\frac{\partial 1/R}{\partial x} = -\frac{1}{\cancel{2}}\,\frac{\cancel{2}(x_0-x)(-1)}{R^3}$$

$$\boxed{\frac{\partial 1/R}{\partial x} = \frac{+(x_0-x)}{R^3}}$$

Nagy

$$\frac{\partial 1/R_1}{\partial x} = \frac{\partial V_1}{\partial x}$$

$$\frac{\partial V_1}{\partial x} = -\frac{1}{\cancel{2}}\,\frac{\cancel{2}(x-x_0)(+1)}{R_1^3}$$

$$\boxed{\frac{\partial 1/R_1}{\partial x} = \frac{\partial V_1}{\partial x} = \frac{-(x-x_0)}{R_1^3}}$$

Sharma

$$\boxed{\frac{\partial 1/R}{\partial y} = \frac{+(y_0-y)}{R^3}}$$

Nagy

$$\boxed{\frac{\partial 1/R_1}{\partial y} = \frac{\partial V_1}{\partial y} = \frac{-(y-y_0)}{R_1^3}}$$

$$\boxed{\frac{\partial 1/R}{\partial z} = \frac{+(z_0-z)}{R^3}}$$

Nagy

$$\boxed{\frac{\partial 1/R_1}{\partial z} = \frac{\partial V_1}{\partial z} = \frac{-(z-c)}{R_1^3}}$$

Shama.

Não Há uma Troca de sinais.

Second Derivative

$$\frac{\partial^2 1/R}{\partial x\partial z} = \frac{\partial}{\partial z}\left(\frac{(x_0-x)}{R^3}\right)$$

$$= (x_0-x)\cdot\left(\frac{-3}{\cancel{2}}\right)\frac{1}{R^5}\,\cancel{2}(z_0-z)(-1)$$

$$\boxed{\frac{\partial^2 1/R}{\partial x\partial z} = \frac{+3(x_0-x)(z_0-z)}{R^5}}$$

Nagy

$$\boxed{\frac{\partial^2 1/R}{\partial y\partial z} = \frac{3(y_0-y)(z_0-z)}{R^5}}$$

Nagy

São Idênticas

$$\frac{\partial^2 1/R_1}{\partial x\partial z} = \frac{\partial}{\partial z}\left(\frac{-(x-x_0)}{R_1^3}\right)$$

$$= -(x-x_0)\left(-\frac{3}{\cancel{2}}\right)\frac{1}{R_1^5}(\cancel{2})(z-c)$$

$$\frac{\partial^2 1/R_1}{\partial x\partial z} = +\frac{3(x-x_0)(z-c)}{R_1^5}$$

Sharma

$$\boxed{\frac{\partial^2 1/R_1}{\partial y\partial z} = \frac{3(y-y_0)(z-c)}{R^5}}$$

Nagy

$$R = \left((x_0-x)^2 + (y_0-y)^2 + (z_0-z)^2\right)^{1/2}$$

$$\frac{\partial^2 1/R}{\partial z \partial z} = \frac{\partial}{\partial z}\left[\frac{(z_0-z)}{R^3}\right]$$

$$= (z_0-z)\left(\frac{-3}{\cancel{2}}\right)\frac{1}{R^5}\cancel{2}(z_0-z)(-1)$$

$$+ (-1)\frac{1}{R^3}$$

$$\boxed{\frac{\partial^2 1/R}{\partial z^2} = +3\frac{(z_0-z)^2}{R^5} - \frac{1}{R^3}}$$

Nagy

Sharma ②

$$R_1 = \left[(x-x_0)^2 + (y-y_0)^2 + (z-c)^2\right]^{1/2}$$

$$\frac{\partial^2 1/R_1}{\partial z \partial z} = \frac{\partial}{\partial z}\left[\frac{-(z-c)}{R_1^3}\right]$$

$$= -(z-c)\left(\frac{-3}{\cancel{2}}\right)\frac{1}{R^5}\cancel{2}(z-c)(+1)$$

$$+ \frac{1}{R^3} - (1)$$

$$= \frac{3(z-c)^2}{R_1^5} - \frac{1}{R_1^3}$$

$$\boxed{\frac{\partial^2 1/R_1}{\partial z \partial z} = \frac{3(z-c)^2}{R_1^5} - \frac{1}{R_1^3}}$$

Sharma

São idênticas

Derivadas Terceiras

Nagy

$$\frac{\partial}{\partial x}\frac{\partial^2 1/R}{\partial z^2} =$$

$$\frac{\partial}{\partial x}\frac{\partial^2 1/R}{\partial z^2} = \frac{\partial}{\partial x}\left[\frac{3(z_0-z)^2}{R^5} - \frac{1}{R^3}\right]$$

$$= 3(z_0-z)^2\left(\frac{-5}{\cancel{2}}\right)\frac{1}{R^7}\cancel{2}(x_0-x)(-1) - \left(\frac{-3}{\cancel{2}}\right)\frac{\cancel{2}(x_0-x)(-1)}{R^5}$$

$$\frac{\partial \partial^2 1/R}{\partial x \partial z^2} = \frac{+15(z_0-z)^2(x_0-x)}{R^7} - \frac{3(x_0-x)}{R^5}$$

Nagy

Sharma

$$\frac{\partial^2 1/R_1}{\partial x \partial z^2} = \frac{\partial}{\partial x}\left[\frac{3(z-c)^2}{R_1^5} - \frac{1}{R_1^3}\right]$$

$$= 3(z-c)^2\left(\frac{-5}{\cancel{2}}\right)\frac{\cancel{2}(x-x_0)(1)}{R^7} - \left(\frac{-3}{\cancel{2}}\right)\frac{\cancel{2}(x-x_0)}{R^3}$$

$$\frac{\partial \partial^2 1/R_1}{\partial x \partial z^2} = \frac{-15(z-c)^2(x-x_0)}{R^7} + \frac{3(x-x_0)}{R^3}$$

Sharma

Nagy

$$R = \left[(x_0 - \cancel{x})^2 + (y_0 - y)^2 + (z_0 - z)^2\right]^{1/2}$$

$$\frac{\partial}{\partial y}\frac{\partial^2 1/R}{\partial z^2} = \frac{+15(z_0 - z)^2 (y_0 - y)}{R^7} - \frac{3(y_0 - y)}{R^5}$$

Nagy

$$15 z_0^3 - 30 z z_0^2 + 15 z^2 z_0 - 15 z z_0^2 + 30 z^2 z_0 - 15 z^3$$

$=$ São idênticas.

$$\frac{\partial^3 1/R}{\partial z \partial z^2} = \frac{\partial}{\partial z}\left[3\frac{(z_0 - z)^2}{R^5} - \frac{1}{R^3}\right]$$

$$= \frac{6(z_0 - z)(-1)}{R^5} + 3(z_0 - z)^2\left(\frac{-5}{\cancel{2}}\right)\frac{1}{R^7}\cancel{2}(z_0 - z)(-1) - \left(\frac{-3}{\cancel{2}}\right)\frac{1}{R^5}\cancel{2}(z_0 - z)(-1)$$

$$= \frac{-6(z_0 - z)}{R^5} + \frac{15(z_0 - z)^3}{R^7} \cancel{+}3\frac{(z_0 - z)}{R^5}$$

$$\frac{\partial^3 1/R}{\partial z^3} = \frac{-9(z_0 - z)}{R^5} + 15\frac{(z_0 - z)^3}{R^7}$$

Nagy

Sharma (3)

$$R_1 = \left((x - x_0)^2 + (y - y_0)^2 + (z - c)^2\right)^{1/2}$$

$$\frac{\partial}{\partial y}\frac{\partial^2 1/R_1}{\partial z^2} = \frac{-15(z - c)^2 (y - y_0)}{R_1^7} + \frac{3(y - y_0)}{R_1^5}$$

Sharma

$$-15 z^3 + 30 z^2 c - 15 z c^2 + 15 z^2 c - 30 z c^2 + 15 c^3$$

$$\frac{\partial^3 1/R_1}{\partial z \partial z^2} = \frac{\partial}{\partial z}\left[\frac{3(z - c)^2}{R_1^5} - \frac{1}{R_1^3}\right]$$

$$= \frac{6(z - c)}{R_1^5} + 3(z - c)^2\left(\frac{-5}{\cancel{2}}\right)\frac{1}{R_1^7}\cancel{2}(z - c) - \left(\frac{-3}{\cancel{2}}\right)\frac{1}{R_1^5}\cancel{2}(z - c)$$

$$= \frac{6(z - c)}{R_1^5} - \frac{15(z - c)^3}{R_1^7} + \frac{3(z - c)}{R_1^5}$$

$$\frac{\partial^3 1/R_1}{\partial z^3} = \frac{9(z - c)}{R_1^5} - \frac{15(z - c)^3}{R_1^7}$$

Sharma