Stress System 2

Equation 8 in Shear

$$\hat{xz}_2 = \psi_{21} = A \frac{\partial^2 V_2}{\partial x \, \partial z}$$

$$\boxed{\hat{x}z_2 = A \left[\frac{3x(z+c)}{R_2^5} \right]}$$

if $z=0$

$$\hat{xz}_2 = A \frac{3xc}{(x^2+y^2+c^2)^{5/2}}$$

→ Equation 7a.

Stress in y system 2:

$$\hat{y}z_2 = \psi_{22} = A \frac{\partial^2 V_2}{\partial y \, \partial z}$$

$$\hat{y}z_2 = A \left[\frac{3y\,(z+c)}{R_2^5} \right]$$

If $z=0$

$$\hat{y}z_2 = \frac{A\,3yc}{(x^2+y^2+c^2)^{5/2}}$$

Equation 7b.

Shear Sey Tem 2

Let's verify the condition

$$\boxed{\hat{X}Z_1\Big|_{z=0} + \hat{X}Z_2\Big|_{z=0} = 0}$$

if $\Rightarrow z=0$

$\hat{X}Z_1$ if $z=0$ is. $\hat{X}Z_1\Big|_{z=0} = \dfrac{-A\,3xc}{R_1^5}$

$$\hat{X}Z_1\Big|_{z=0} = \frac{-A\,3xc}{[x^2+y^2+(-c)^2]^{5/2}}$$

$$\hat{X}Z_1\Big|_{z=0} = \frac{-A\,3xc}{[x^2+y^2+c^2]^{5/2}}$$

$\hat{X}Z_2$ if $z=0$ $= \dfrac{+A3c}{(x^2+y^2+c^2)^{52}}$

Thus. $\hat{X}Z_1 + \hat{X}Z_2 = 0$.

The same occurs to the condition

$\hat{y}Z_1\Big|_{z=0} + \hat{y}Z_2\Big|_{z=0} = 0$ if $z=0$.

$$\hat{y}Z_1\Big|_{z=0} = \frac{-A\,3yc}{R_1^5} = \frac{-A\,3yc}{(x^2+y^2+(-c)^2)^{5/2}} = \frac{-A\,3yc}{[x^2+y^2+c^2]^{5/2}}$$

$\hat{y}Z_2\Big|_{z=0} = \dfrac{A\,3yc}{(x^2+y^2+c^2)^{5/2}}$ Thus $\hat{y}Z_1 + \hat{y}Z_2 = 0$

Shaw Sy... 2 (11)

Who is the Shear $\hat{z}\tilde{z}_2$

we know that.

$$\hat{z}\hat{z}_1 - \hat{z}\tilde{z}_2 = 0. \quad \text{if } z=0$$

$$\hat{z}\hat{z}_1 = A\left[\frac{-1}{R_1^3} + \frac{3(z-c)^2}{R_1^5}\right] \quad \text{eq 6c in Shaw}$$

$$\hat{z}\hat{z}_1 = A\left[\frac{-1}{[x^2+y^2+(z-c)^2]^{3/2}} + \frac{3(z-c)^2}{[x^2+y^2+(z-c)^2]^{5/2}}\right]$$

if $z=0$ we have

$$\hat{z}\hat{z}_1\Big|_{z=0} = A\left[\frac{-1}{(x^2+y^2+c^2)^{3/2}} + \frac{3c^2}{[x^2\ y^2+c^2]^{5/2}}\right]$$

To satisfy $\hat{z}\hat{z}_1 + \hat{z}\tilde{z}_2 = 0$ in $z=0$.

$$\hat{z}\hat{z}_2\Big|_{z=0} = A\left[\frac{+1}{(x^2+y^2+c^2)^{3/2}} - \frac{3c^2}{(x^2+y^2+c^2)^{5/2}}\right]$$

equation 7c in Shaw

Shaw said that 7c is satisfy for

(7). $$\hat{z}\tilde{z}_2 = -A\frac{\partial^2 V_2}{\partial z^2}$$

Let's check equation. (12)

$$\tilde{z}z_2 = -A \frac{\partial^2 V_2}{\partial z^2}$$

if it is corect ~~as~~ this relationship

$$\tilde{z}z_1 + z\tilde{z}_2 = 0 \quad \text{if } z = 0$$

$$\tilde{z}z_2 = -A \frac{\partial^2}{\partial z \partial z} V_2 = -A \frac{\partial}{\partial z} \frac{\partial}{\partial z} \frac{1}{R_2}$$

$$\frac{\partial}{\partial z} \frac{1}{R_2} = \frac{\partial}{\partial z} \frac{1}{(x^2 + y^2 + (z+c)^2)^{1/2}}$$

$$\frac{\partial}{\partial z} \frac{1}{R_2} = -\frac{1}{\cancel{2}} (x^2 + y^2 + (z+c)^2)^{-3/2} \cancel{2}(z+c)$$

$$\frac{\partial}{\partial z} \frac{1}{R_2} = \frac{-(z+c)}{(x^2 + y^2 + (z+c)^2)^{3/2}} = \frac{-(z+c)}{R_2^3}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial}{\partial z} \frac{1}{R_2}\right) = \frac{\partial}{\partial z} \frac{-(z+c)}{(x^2 + y^2 + (z+c)^2)^{3/2}}$$

$$= \frac{\partial}{\partial z} \underline{-(z+c)\left[x^2 + y^2 + (z+c)^2\right]^{-3/2}}$$

$$= -1\left[x^2 + y^2 + (z+c)^2\right]^{-3/2} + \left[-(z+c)\right]\left(\frac{-3}{\cancel{2}}\right)\left[x^2 + y^2 + (z+c)^2\right]^{-5/2} \cancel{2}(z+c)$$

$$\frac{\partial}{\partial z} \frac{\partial}{\partial z} V_2 = \frac{-1}{R_2^3} + \frac{3(z+c)^2}{R_2^5}$$

According To eq 8c in Sharma

$$\hat{z}z_2 = -A \underbrace{\frac{\partial^2}{\partial z^2} V_2}$$

$$\tilde{z}z_2 = -A\left[\frac{-1}{R_2^3} + \frac{3(z+c)^2}{R_2^5}\right]$$

$$\hat{z}z_2 = A\left[\frac{1}{R_2^3} - \frac{3(z+c)^2}{R_2^5}\right]$$

if $\hat{z}z_2\Big|_{z=0}$ we have

$$\left[\hat{z}z_2\right]_{z=0} = A\left[\frac{1}{(x^2+y^2+c^2)^{3/2}} - \frac{3(c)^2}{(x^2+y^2+c^2)^{5/2}}\right]$$

~~checking if~~ verifying. if we Take

$$\underbrace{\left[\tilde{z}z_1\right]_{z=0}}_{\text{eq c sharma}} + \left[\tilde{z}z_2\right]_{z=0} = 0$$

$$A\left[\frac{-1}{R_1^3} + \frac{3(z-c)^2}{R_1^5}\right] + A\left[\frac{1}{R_2^3} - \frac{3(z+c)^2}{R_2^5}\right]$$

$$A\left[\frac{-1}{\left[x^2+y^2+(z-c)^2\right]^{3/2}} + \frac{3(z-c)^2}{\left[x^2+y^2+(z-c)^2\right]^{5/2}}\right] + A\left[\frac{1}{\left[x^2+y^2+(z+c)^2\right]^{3/2}} - \frac{3(z+c)^2}{\left[x^2+y^2+(z+c)^2\right]^{5/2}}\right]$$

if $z=0$

$$0 = A\left[\frac{-1}{(x^2+y^2+c^2)^{3/2}} + \frac{3c^2}{(x^2+y^2+c^2)^{5/2}}\right] + A\left[\frac{1}{(x^2+y^2+c^2)^{3/2}} - \frac{3(c)^2}{\left[x^2+y^2+c^2\right]^{5/2}}\right]$$

Equilibrium equation.

(14)

$$\frac{\partial}{\partial t}\Theta_r = 2(1+\sigma)\left\{\frac{\partial \psi_{r1}}{\partial x} + \frac{\partial \psi_{r2}}{\partial y} + \frac{\partial \psi_{r3}}{\partial z}\right\} \cdots$$

$$\Theta_r = \hat{x}x_r + \hat{y}y_r + \hat{z}z_r$$

equation (2) Strain.

ψ_{r1}, ψ_{r2} ψ_{r3} are harmonic functions

$$\boxed{\Theta_2 = \hat{x}x_2 + \hat{y}y_2 + \hat{z}z_2}$$

From equation 2 we have

$$\frac{\partial \Theta_R}{\partial z} = \frac{\partial \Theta_2}{\partial t} = 2(1+\sigma)\left\{\frac{\partial \psi_{21}}{\partial x} + \frac{\partial \psi_{22}}{\partial y} + \frac{\partial \psi_{23}}{\partial z}\right\}$$

$$\psi_{21} = A\frac{\partial^2 V_2}{\partial x \partial z}$$

$$\psi_{22} = \frac{A\,\partial^2 V_2}{\partial y \partial z}$$

$$\psi_{23} = -\frac{A\,\partial^2 V_2}{\partial z^2}$$

$$\frac{\partial \Theta_2}{\partial z} = 2(1+\sigma)A\left\{\frac{\partial}{\partial x}\frac{\partial^2 V_2}{\partial x \partial z} + \frac{\partial\, V_2}{\partial y \partial y \partial z} - \frac{\partial}{\partial z}\frac{\partial^2 V_2}{\partial z \partial z}\right\}$$

Equation (2) in Sharma can be rewritten as

if $R=2$

$$\frac{\partial \theta_2}{\partial z} = 2A(1+\sigma)\left\{\frac{\partial}{\partial z}\frac{\partial^2 V_2}{\partial x \partial x} + \frac{\partial}{\partial z}\frac{\partial^2 V_2}{\partial y \partial y} - \frac{\partial}{\partial z}\frac{\partial^2 V_2}{\partial z \partial z}\right\}$$

integrating both sides in z we have

$$\int \frac{\partial \theta_2}{\partial z} dz = 2A(1+\sigma)\left\{\int \frac{\partial}{\partial z}\frac{\partial^2}{\partial x^2}V_2 dz + \int \frac{\partial}{\partial z}\frac{\partial^2 V_2}{\partial y^2} dz - \int \frac{\partial}{\partial z}\frac{\partial^2 V_2 dz}{\partial z^2}\right\}$$

$$\theta_2 = 2A(1+\sigma)\left\{\underbrace{\frac{\partial^2}{\partial x^2}V_2 + \frac{\partial^2 V_2}{\partial y^2}} - \frac{\partial^2 V_2}{\partial z^2}\right\}$$

Laplace Equation

$$\frac{\partial^2 V_2}{\partial x^2} + \frac{\partial^2 V_2}{\partial y^2} + \frac{\partial^2 V_2}{\partial z^2} = 0$$

$$\boxed{\frac{\partial^2 V_2}{\partial x^2} + \frac{\partial^2 V_2}{\partial y^2} = -\frac{\partial^2 V_2}{\partial^2 z}}$$

$$\theta_2 = 2A(1+\sigma)\left\{-\frac{\partial^2 V_2}{\partial^2 z} - \frac{\partial^2 V_2}{\partial^2 z}\right\}$$

$$\theta_2 = 2A(1+\sigma)\left\{-2\left(\frac{\partial^2 V_2}{\partial^2 z}\right)\right\}$$

$$\theta_2 = -4A(1+\sigma)\frac{\partial^2 V_2}{\partial^2 z} \rightarrow \text{Equation 9 Sharma}$$

From Equations (3) and (1) in Shama we can calculate eq (10a)

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$\hat{xz}_2 = ?$

$$\hat{xz}_R = -\frac{1}{2(1+\sigma)} z \frac{\partial \theta_R}{\partial x} + \psi_{R1}$$

equação 1a

$$\hat{xz}_2 = -\frac{1}{2(1+\sigma)} z \frac{\partial}{\partial x}\theta_2 + \psi_{21}$$

$$\hat{xz}_2 = -\frac{1}{2(1+\sigma)} z \frac{\partial}{\partial x}\underbrace{\left(-4A(1+\sigma)\right)\frac{\partial^2 V_2}{\partial z^2}}_{\theta_2} + A\frac{\partial^2 V_2}{\partial x \partial z}.$$

$$\hat{xz}_2 = \frac{-1}{\cancel{2(1+\sigma)}} z\left[\cancel{-4}^{2}A\cancel{(1+\sigma)}\right]\frac{\partial^3 V_2}{\partial x \partial z^2} + A\frac{\partial^2 V_2}{\partial x \partial z}.$$

$$\hat{xz}_2 = 2A.z\frac{\partial^3 V_2}{\partial x \partial z^2} + A\frac{\partial^2 V_2}{\partial x \partial z}.$$

diferente da Timpone eq (16a) $\neq$ Equation 10a Shama