Proceedings of the 2001 IEEE
International Conference on Robotics & Automation
Seoul, Korea · May 21-26, 2001

Guaranteed 3-D mobile robot localization using an odometer, an automatic theodolite and indistinguishable landmarks

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Abstract

This paper deals with the determination of the full 3-D position and spatial orientation of a mobile robot from six azimuth and elevation angle measurements provided by an automatic theodolite. Stochastic approaches have already proved to be interesting for navigation algorithms, but they require a reliable initial estimation of the vehicle posture. Computing this initial posture without any a priori knowledge is not a straightforward mathematical exercise, especially if the landmarks are indistinguishable. In this paper, interval analysis is used to design a robust solution to this issue.

1 Introduction

The 3-D localization algorithm presented in this paper is intended for outdoor mobile robots equipped with an odometer and an automatic theodolite. Several of such devices use a minimum number of three non aligned beacons and two rotating laser fan-beams with different tilt angles [2, 9]. All these sensors allow to measure, at least when the robot is motionless, the full direction vector (both horizontal λ_k and vertical σ_k directions) to each beacon B_k, the position of which is assumed to be known.

For 2-D localization problems, different approaches have been proposed to estimate the mobile robot posture when it is stopped. In [3], the authors detail four triangulation methods to compute the position x, y and orientation ψ of the robot given the azimuth angles λ_k of three landmarks. Once the initial posture is determined, the mobile robot evolutions can be estimated by Extended Kalman Filtering as shown in [1, 7] for instance.

3-D navigation algorithms using an automatic theodolite have also been proposed, and they give very satisfying results, provided the initial estimation is close to the real 3-D posture of the mobile robot [2]. To our knowledge, this computation of the initial xyz position and spatial attitude has never been solved in a satisfying way: the iterative approach proposed in [6] for instance requires initial values not too far from the real ones, and if one excepts the situations where the theodolite is close to the plane defined by the three beacons (in which case 2-D triangulation methods described in [3] can be used), their initialization requires the intervention of a human operator or the use of an additional exteroceptive sensor.

What is needed is a robust three landmarks 3-D triangulation algorithm working autonomously (i.e. without any external help) whatever the posture of the mobile robot. The resolution of the problem is all the more difficult since the landmarks are indistinguishable and the azimuth and elevation measurements performed by the theodolite can correspond to one or two 3-D postures, depending on the position of the sensor relative to the beacons. We propose a method based on algorithm SIVIA (Set Inverter by Interval Analysis) presented by Jaulin and Walter [4]. With the assumption that the measurement errors are bounded, this approach yields a set of solutions, which is guaranteed to contain the actual position. A direct application of interval analysis to the full 6 DoF problem, although theoretically possible, would not be acceptable in practice because of its computational cost. We show how to transform the 6 DoF problem into a 3 DoF problem in section 2. In section 3, interval analysis and set insersion are used to compute a first set of admissible solutions to the localization problem: up to 12 postures can correspond to the theodolite measurements. In section 4, the method to eliminate the 'wrong' solutions is explained then tested in simulation and finally, section 5 gives some conclusions.

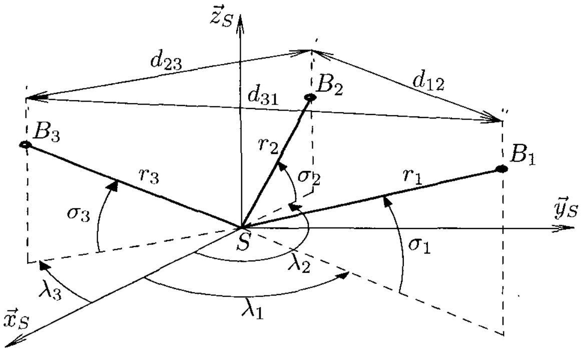

Figure 1: Variables involved in the computation of the initial 3-D posture

2 Principle of the 3-D localisation computation

In this section, we present the equations used to find the 3-D posture of a vehicle given the angular measurements of a theodolite.

Let us define the following frames:

- F_0: absolute reference frame,
- F_M: mobile robot frame,
- F_S: sensor frame,
- F_B: frame defined from 3 non-aligned beacons.

The homogeneous co-ordinates of beacon B_k in the sensor frame F_S are denoted ${}^S\mathbf{b}_k = [{}^Sx_k, {}^Sy_k, {}^Sz_k, 1]^t$ and the resulting theodolite measurements are given by (see figure 1):

$$\begin{array}{rcl} \lambda_k & = & \arctan2\left({}^Sy_k, {}^Sx_k\right) \\ \sigma_k & = & \arctan\left({}^Sz_k / \sqrt{{}^Sx_k^2 + {}^Sy_k^2}\right) \end{array} \quad (1)$$

Since the beacon co-ordinates are known in the reference frame, the homogeneous co-ordinates ${}^S\mathbf{b}_k$ are given by:

$${}^S\mathbf{b}_k = {}^S\mathbf{T}_M \cdot {}^M\mathbf{T}_0(\mathbf{x}) \cdot {}^0\mathbf{b}_k \quad (2)$$

where the homogeneous transformation matrix ${}^S\mathbf{T}_M$ is fixed once and for all after calibration of the sensor in the vehicle frame F_M and the homogeneous transformation matrix ${}^M\mathbf{T}_0$ depends on the 3-D posture $\mathbf{x}$ of the vehicle.

If one tries to solve the localization problem directly by writing system (1) with the position and attitude of the vehicle, a complex system of six nonlinear equations with six unknowns is obtained. But, as shown in [6], it is possible to reduce this system to a 3 unknowns problem by introducing the distances between the sensor and the beacons as new variables.

Let r_k be the distance from the origin of the sensor frame S to beacon B_k. The co-ordinates of B_k in the sensor frame are given by:

$$\begin{array}{rcl} {}^Sx_k & = & r_k \cos(\lambda_k)\cos(\sigma_k) \\ {}^Sy_k & = & r_k \sin(\lambda_k)\cos(\sigma_k) \\ {}^Sz_k & = & r_k \sin(\sigma_k) \end{array} \quad (3)$$

By writing the (known) distances d_{ij} between beacons B_i and B_j, we obtain three equations of the following form:

$$a_{ij} = r_i/r_j + r_j/r_i - d_{ij}^2/(r_i \cdot r_j) \quad (4)$$

where

$$\begin{array}{rl} a_{ij} = & 2(\cos(\sigma_i)\cos(\lambda_i)\cos(\sigma_j)\cos(\lambda_j) + \ldots \\ & \cos(\sigma_i)\sin(\lambda_i)\cos(\sigma_j)\sin(\lambda_j) + \sin(\sigma_i)\sin(\sigma_j)) \end{array} \quad (5)$$

Finding the values r_k provides the co-ordinates of the beacons in the sensor frame and thus the homogeneous transformation ${}^S\mathbf{T}_B$. Knowing ${}^0\mathbf{T}_B$ and ${}^M\mathbf{T}_S$, it is easy to obtain the homogeneous transformation matrix ${}^0\mathbf{T}_M$ and compute the 3-D position and spatial orientation of the vehicle in the reference frame:

$${}^0\mathbf{T}_M(\mathbf{x}) = {}^0\mathbf{T}_B \cdot {}^S\mathbf{T}_B^{-1} \cdot {}^S\mathbf{T}_M \quad (6)$$

As a consequence, the whole problem lies in finding the good association beacons-angles and in determining the solution $\tilde{\mathbf{r}}$ to the vector equation derived from (4):

$$\mathbf{a} = \mathbf{g}(\mathbf{r}) \quad (7)$$

where the output vector is $\mathbf{a} = [a_{12}, a_{23}, a_{31}]^t$ and the state vector $\mathbf{r} = [r_1, r_2, r_3]^t$.

3 Initial localization

3.1 Principle

Rather than trying to find the vehicle 3-D posture corresponding exactly to the theodolite measurements, we propose to take into account the inaccuracy of the sensor and use the bounds on the measurement errors in order to compute a set guaranteed to contain the real distances $\tilde{\mathbf{r}}$. These bounds are obtained by experimentation on the theodolite, and allow to define intervals $[\lambda_k] = \{\lambda \in \mathbb{R} | \underline{\lambda}_k \leq \lambda \leq \overline{\lambda}_k\} = [\underline{\lambda}_k, \overline{\lambda}_k]$ and $[\sigma_k] = \{\sigma \in \mathbb{R} | \underline{\sigma}_k \leq \sigma \leq \overline{\sigma}_k\} = [\underline{\sigma}_k, \overline{\sigma}_k]$ centred on the measured angles and guaranteed to contain the *real* azimuth and elevation angles. From these intervals and using equation (5), the following box is computed:

$$[\mathbf{a}_{123}] = [a_{12}] \times [a_{23}] \times [a_{31}] \tag{8}$$

and assuming that the distances sensor-beacon r_k belong to some initial box $[\mathbf{r}](0) = [r_1](0) \times [r_2](0) \times [r_3](0)$, we are looking for the set

$$\mathcal{R} = \{\mathbf{r} \in [\mathbf{r}](0) | \mathbf{g}(\mathbf{r}) \in [\mathbf{a}_{123}]\} \tag{9}$$

As aforesaid, this set inversion problem can be solved with SIVIA provided an inclusion function $[\mathbf{g}]$ of function $\mathbf{g}$ can be found, that it is to say an interval function such that $\forall \mathbf{r} \in [\mathbf{r}]$, $\mathbf{g}(\mathbf{r}) \in [\mathbf{g}]([\mathbf{r}])$.

For any function composed of elementary operators such as $+$, $-$, $*$, $/$, sin, cos, an inclusion function is simple to obtain by replacing these operators by their minimal inclusion function. For instance, considering the addition operation and intervals $[x]$ and $[y]$, we have:

$$[+]([x], [y]) = [\underline{x} + \underline{y}, \overline{x} + \overline{y}] \tag{10}$$

For a better understanding, the SIVIA principle is briefly summarized (for further details, the reader is referred to [4]). Considering box $[\mathbf{r}](k)$, the following cases can occur:

- $[\mathbf{g}]([\mathbf{r}](k)) \subset [\mathbf{a}_{123}]$: the box $[\mathbf{r}](k)$ is included in the solution set $\mathcal{R}$. Accordingly, it is stored in the subpaving $\hat{\mathcal{R}}$.
- $[\mathbf{g}]([\mathbf{r}](k)) \cap [\mathbf{a}_{123}] = \emptyset$: the box $[\mathbf{r}](\mathbf{k})$ is out of $\mathcal{R} = \mathbf{g}^{-1}([\mathbf{a}_{123}])$, and is consequently rejected.
- In any other case, a part of $[\mathbf{r}](k)$ may be included in set $\mathcal{R}$. If the width of box $[\mathbf{r}](k)$ is greater than a fixed threshold ϵ, the box is divided into two boxes of equal width which are stored in the stack containing all the boxes to be tested (and which initially contains the search box $[\mathbf{r}](0)$). Otherwise, $[\mathbf{r}](k)$ is stored in $\hat{\mathcal{R}}$.

All the boxes $[\mathbf{r}](k)$ are processed until the stack is empty. Finally, SIVIA yields a subpaving $\hat{\mathcal{R}}$ (i.e. an union of non overlapping boxes), enclosing the solution set $\mathcal{R}$. An example of application to static localization is given in [8]: the robot is equipped with an ultrasonic belt and moves in a 2-D mapped environment.

Actually, since the three beacons are unsigned, we have to consider the six possible associations between beacons B_1, B_2, B_3 and angular measurements (λ_a, σ_a), (λ_b, σ_b), (λ_c, σ_c). As a consequence, the whole set $\mathcal{R}(0)$ is given by

$$\begin{aligned}
&\mathcal{R}(0) = \ldots \\
&\{\mathbf{r} \in [\mathbf{r}](0) | \mathbf{g}(\mathbf{r}) \in [\mathbf{a}_{abc}]\} \cup \{\mathbf{r} \in [\mathbf{r}](0) | \mathbf{g}(\mathbf{r}) \in [\mathbf{a}_{acb}]\} \cup \\
&\{\mathbf{r} \in [\mathbf{r}](0) | \mathbf{g}(\mathbf{r}) \in [\mathbf{a}_{cba}]\} \cup \{\mathbf{r} \in [\mathbf{r}](0) | \mathbf{g}(\mathbf{r}) \in [\mathbf{a}_{bac}]\} \cup \\
&\{\mathbf{r} \in [\mathbf{r}](0) | \mathbf{g}(\mathbf{r}) \in [\mathbf{a}_{bca}]\} \cup \{\mathbf{r} \in [\mathbf{r}](0) | \mathbf{g}(\mathbf{r}) \in [\mathbf{a}_{cab}]\}
\end{aligned} \tag{11}$$

3.2 Example

The simulation we propose is based on the features of the LASERGUIDE designed for precise localization of civil-engineering machines [2]. The theodolite operates by continuous scanning of a pair of mutually inclined laser fan beams. The rotation of the beams is monitored internally by a single electronic angle encoder. When either of the fan beams strikes an electronic target, the instantaneous reading of the encoder is latched and recorded. At the completion of every full 360° sweep, the pairs of encoder readings corresponding to each target in the instruments field of view are used to compute its respective full spatial direction vector (λ_k, σ_k).

The inaccuracy of the sensor has been evaluated as follows:

- maximum azimuth error: $e_\lambda = 100$ arc seconds;
- maximum elevation error: $e_\sigma = 100$ arc seconds.

The minimum range of the system is 1 m and the maximum range 200 m. Accordingly the initial search box $[\mathbf{r}](0)$ is given by:

$$[\mathbf{r}](0) = [1 \text{ m}, 200 \text{ m}] \times [1 \text{ m}, 200 \text{ m}] \times [1 \text{ m}, 200 \text{ m}] \tag{12}$$

As shown on figure 2, the initial subpaving $\hat{\mathcal{R}}(0)$ has 8 connected components, which correspond to 8 different 3-D postures for the mobile robot (see figure 2), for which azimuth and elevation measurements are identical.

4 Set estimation for 3-D localization

Our purpose is to eliminate the 'wrong' connected components of initial $\hat{\mathcal{R}}(0)$ until $\hat{\mathcal{R}}$ is reduced to an unique connected set guaranteed to contain the real distances beacons-sensor. To this end, we define a set estimator which predicts, then updates the subpaving

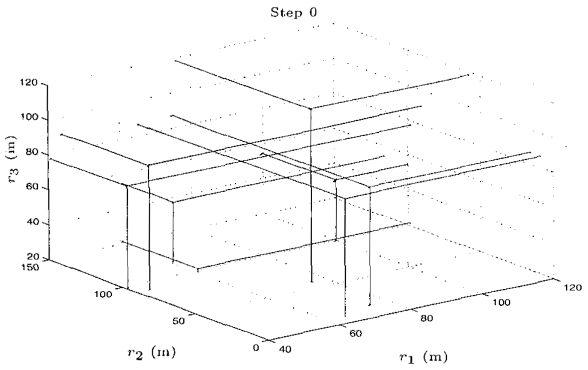

Figure 2: Initial subpaving $\hat{\mathcal{R}}(0)$ computed by SIVIA with a precision set to $\epsilon = 10$ cm (computation time: 13 s with a `Matlab` program running on a Pentium II 450 MHz PC)

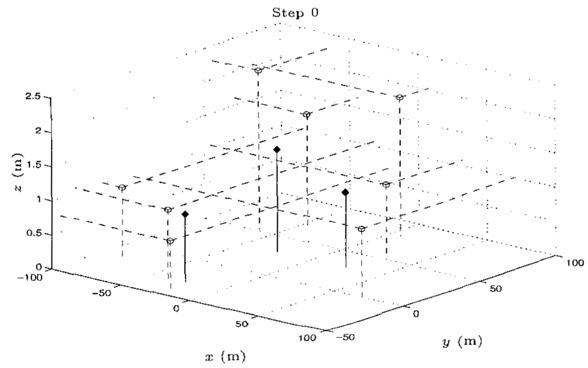

Figure 3: Corresponding 3-D positions (in white) and location of the beacons (in black)

$\hat{\mathcal{R}}$ at each displacement of the theodolite. Since the six angle measurements cannot generally be simultaneous, we need to stop the vehicle for each estimation step. Therefore, and contrary to the 2-D application presented in [5], the set observer we present cannot be used for vehicle tracking.

4.1 Prediction step

The distance between the different stops of the mobile robot is measured by an odometer and the result is an interval $[\delta]$ guaranteed to contain the real value. The prediction of the new beacon-sensor distances from this interval is based on the following assumptions:

- The rolling plane on which the vehicle moves is constant between two stops;

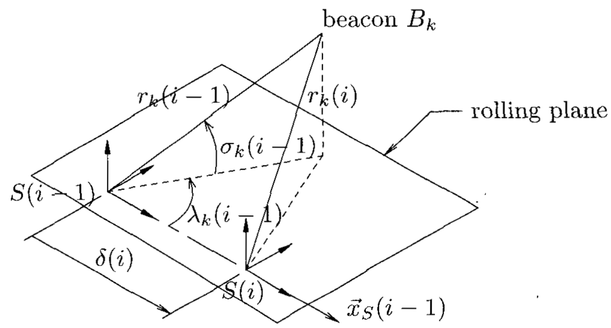

Figure 4: Evolution of distance r_k between two stops

- The rotation axis $\vec{z}_S$ of the sensor is perpendicular to the rolling plane;
- Between stops $i-1$ and i, the vehicle trajectory is rectilinear and directed by vector $\vec{x}_S(i-1)$ (see figure 4).

From the previous distances $\mathbf{r}(i-1)$, the travelled distance $\delta(i)$ and the measurements $(\lambda_k(i-1), \sigma_k(i-1))$ performed at stop $i-1$, the sensor-beacon distances are given by:

$$r_k(i) = \left(r_k(i-1)^2 + \delta(i)^2 - \ldots \right. \\ \left. 2\cos(\lambda_k(i-1))\cos(\sigma_k(i-1))\delta(i) r_k(i-1)\right)^{\frac{1}{2}} \tag{13}$$

which defines the following prediction equation:

$$\mathbf{r}(i) = \mathbf{f}_{Ai}(\mathbf{r}(i-1), \boldsymbol{\lambda}(i-1), \boldsymbol{\sigma}(i-1), \delta(i)) + \boldsymbol{\alpha}(i) \tag{14}$$

where index $Ai = 1, \ldots, 6$ indicates that the prediction function $\mathbf{f}$ depends on the way the measurements are associated with the beacons, and where $\boldsymbol{\alpha}$ is the model noise, which belongs to the known box $[\boldsymbol{\alpha}]$.
The initial subpaving $\hat{\mathcal{R}}(0)$ is divided into $N = n_1 + \ldots + n_6$ connected components $\hat{\mathcal{C}}_{Ai,j}$ corresponding to the six beacon-measurement associations Ai. As a consequence, we have:

$$\hat{\mathcal{R}}(0) = \hat{\mathcal{C}}_{A1,1} \cup \ldots \cup \hat{\mathcal{C}}_{A1,n_1} \cup \ldots \hat{\mathcal{C}}_{A6,1} \cup \ldots \cup \hat{\mathcal{C}}_{A6,n_6} \tag{15}$$

Writing an inclusion function $[\mathbf{f}_{Ai}]$ of the prediction function is quite simple and the resulting prediction step is processed as follows.

1. The mobile robot moves and stops after travelling a given distance;

2. For each connected component $\hat{\mathcal{C}}_{Ai,j}(i-1)$ of previous subpaving $\hat{\mathcal{R}}(i-1)$, the evolution of each box $[\mathbf{r}](i-1)$ is computed using the inclusion function $[\mathbf{f}_{Ai}]$ and intervals $[\delta](i), [\boldsymbol{\alpha}], [\lambda_k](i-1)$ and $[\sigma_k](i-1)$;

3. the different predicted boxes $[\mathbf{r}](i|i-1)$ of each connected component $\hat{\mathcal{C}}_{Ai,j}$ are gathered in a minimum sized box $[\mathcal{C}_{Ai,j}](i|i-1)$ and the resulting predicted subpaving $\hat{\mathcal{R}}(i|i-1)$ is the union of all these boxes:

$$\hat{\mathcal{R}}(i|i-1) = [\mathcal{C}_{A1,1}](i|i-1) \cup \ldots \cup [\mathcal{C}_{A6,n_6}](i|i-1) \tag{16}$$

4.2 Updating step

Once the mobile robot is stopped, a new set of measurements is performed by the theodolite, and box $[\boldsymbol{\alpha}](i)$ is computed. Set inversions are processed with SIVIA, but this time, the search sets are the predicted boxes $[\mathcal{C}_{Ai,j}](i|i-1)$.

After the updating, if only one component $\hat{\mathcal{C}}_{Ai,j}(i|i)$ is not empty, this means there is no more ambiguity on the mobile robot position and its 3-D posture is computed using distances r_1, r_2, r_3 obtained by a Newton-Raphson algorithm applied on equation (7) [6], which is initialized with values contained in $\hat{\mathcal{C}}_{Ai,j}](i|i)$. Otherwise, the mobile robot performs a new displacement, and the prediction-updating process is computed once more.

4.3 Example

The example of subsection 3.2 is considered. Since the initial set inversion yields a subpaving with 8 connected components, the mobile robot performs several one-metre-long displacements along axis $\vec{x}$ until subpaving $\hat{\mathcal{R}}(i)$ is reduced to an unique component $\hat{\mathcal{C}}_{Ai,j}(i|i)$. After one metre, we still have 4 possible positions, but step by step, the number of admissible 3-D postures diminishes and after four metres, the only component gives the real 3-D posture of the mobile robot (see figures 5 and 6).

5 Conclusion

For localizing outdoor vehicles, theodolites have the advantage, contrary to GPS systems, to be usable close to buildings or under bridges. On the other hand, the exploitation of the measures to compute the position and spatial orientation of the vehicle is less obvious.

As we have seen, experiments have already shown that by using an Extended Kalman Filter for instance, real-time positioning was possible with a theodolite [2]. Actually, the only theoretical difficulty lies in the determination of the initial posture of the mobile robot. Indeed, spatial triangulation is a complex nonlinear problem, with up to 12 possible solutions if the three considered landmarks are unsigned and until now, no method guaranteeing the convergence to a proper 3-D posture has been proposed.

By transforming the 6 DoF problem into a 3 DoF one, and by solving it through an algorithm based on set inversion by interval analysis, the method presented in this paper efficiently solves the problem. Of course, this method cannot be used in real time, but it gives a robust mean to initialize real-time positioning algorithms without any operator intervention.

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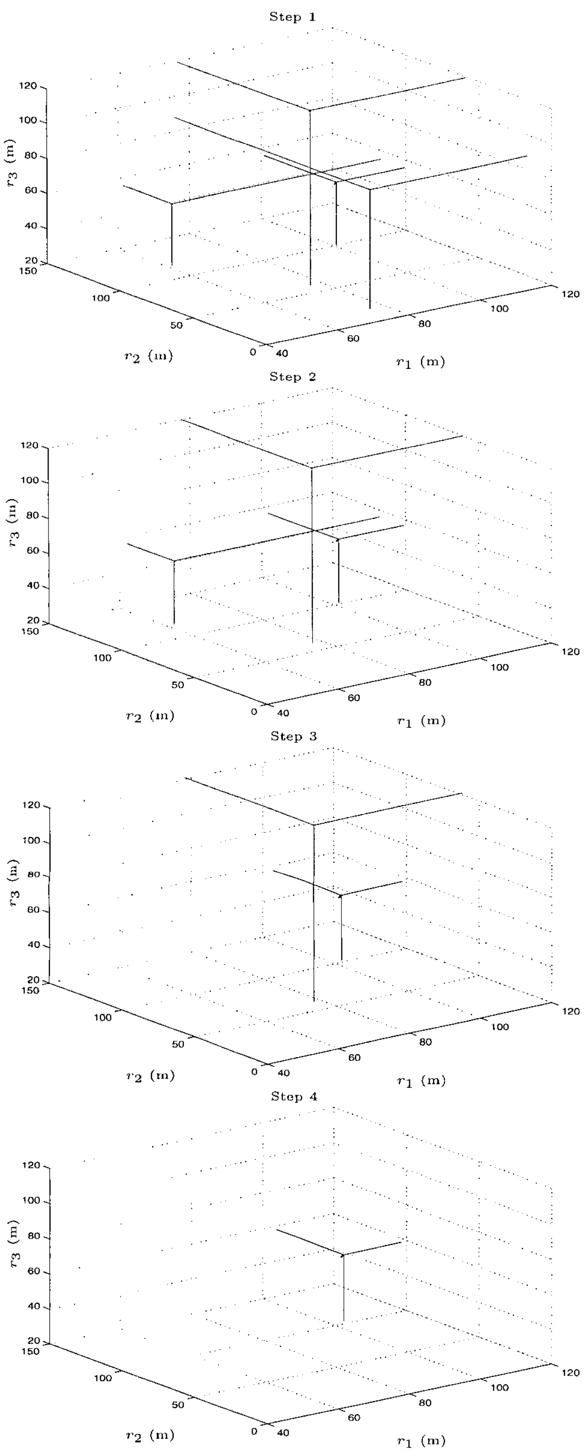

Figure 5: Subpaving computed by SESSYL for steps 1 to 4

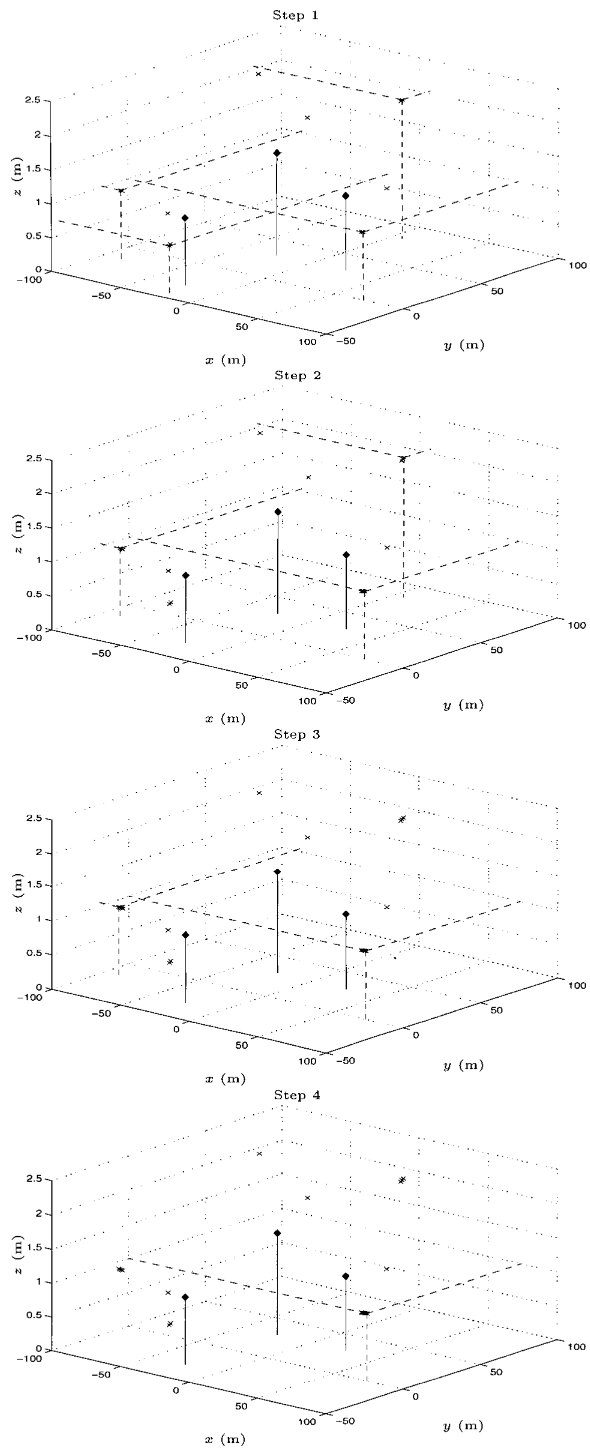

Figure 6: 3-D positions corresponding to the different connected components of subpavings $\hat{\mathcal{R}}$