

Дифференциальные уравнения листок 4

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Задача 1

(a) $\dot{x} = 0, \dot{y} = 0$

$$y = C_1, x = C_2$$

(b) $\dot{x} = 2, \dot{y} = 1$

$$y = t + C_1, x = 2t + C_2$$

(c) $\dot{x} = 1, \dot{y} = y$

$$x = t + C_1, y = C_2 e^t \rightarrow y(x) = C_2 e^{x-C_1}$$

(d) $\dot{x} = x, \dot{y} = y$

$$x = C_1 e^t, y = C_2 e^t \rightarrow y(x) = \frac{C_2}{C_1} x$$

(e) $\dot{x} = 2x, \dot{y} = y$

$$x = C_1 e^{2t}, y = C_2 e^t \rightarrow y(x) = C_2 \cdot \left(\sqrt{\frac{x}{C_1}} \right)$$

(f) $\dot{x} = x, \dot{y} = -y$

$$x = C_1 e^t, y = \frac{C_2}{e^t} \rightarrow y(x) = \frac{C_1 C_2}{x}$$

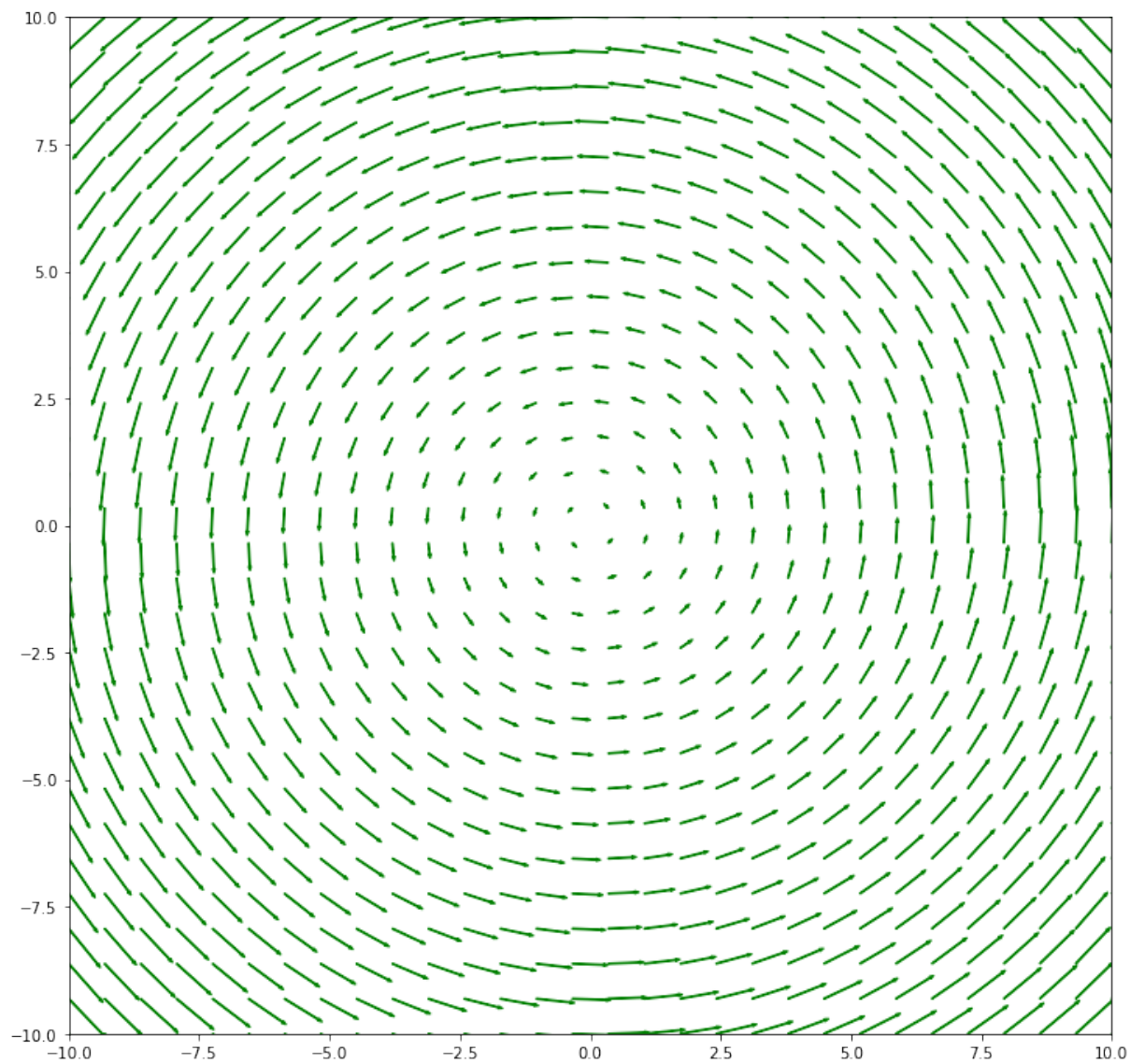
(g) $\dot{x} = x^2, \dot{y} = -y$

$$x = -\frac{1}{t + C_1}, y = \frac{C_2}{e^t} \rightarrow y(x) = C_2 e^{C_1 + \frac{1}{x}}$$

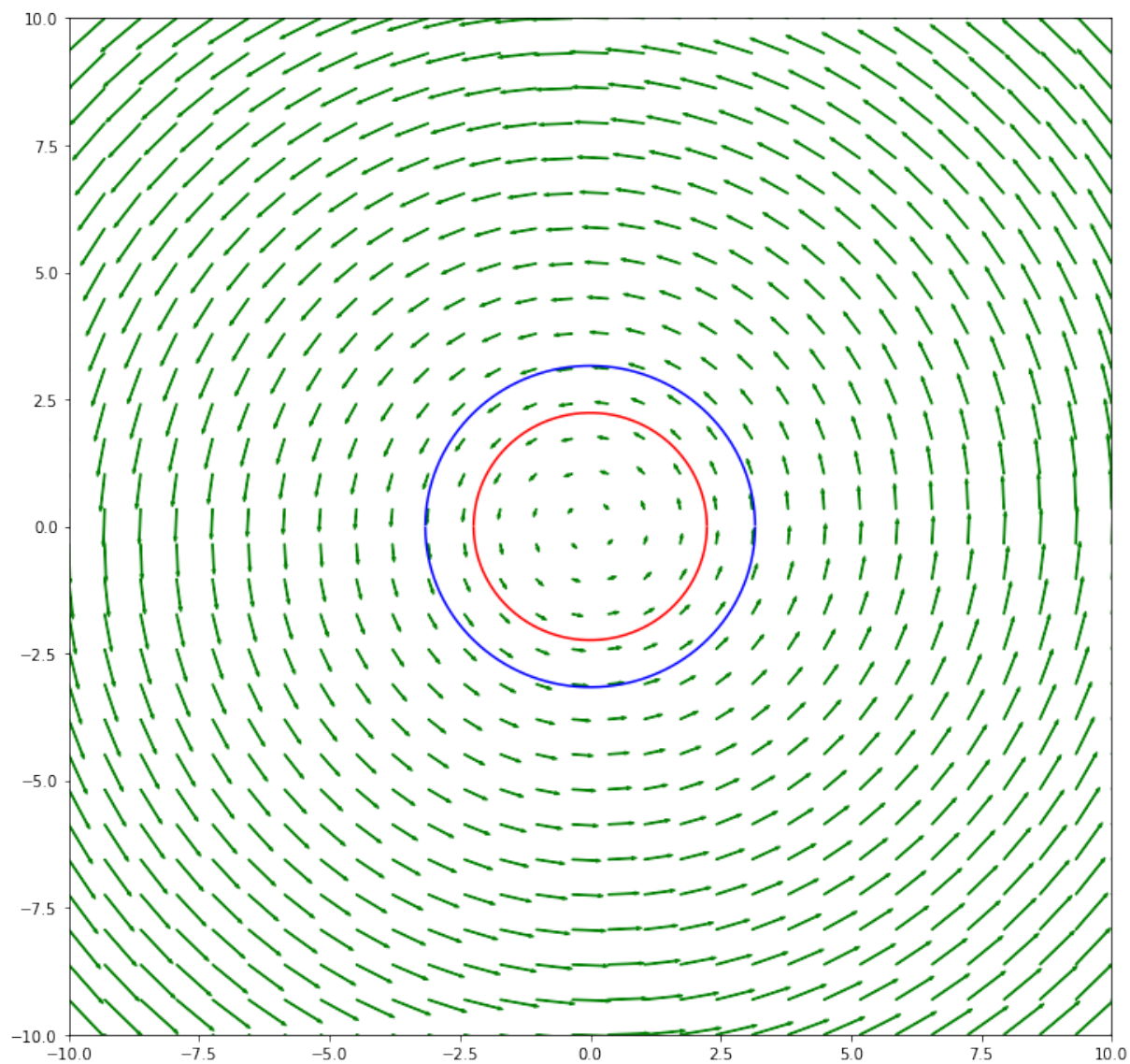
Задача 2

$$\dot{x} = -y, \quad \dot{y} = x$$

(a) Поле направлений:



(b) Фазовые кривые:



(c) Настоящие фазовые кривые:

$$x^2 + y^2 = C$$

Проверим: $\dot{x}x + \dot{y}y = 0 \rightarrow [\dot{x} = -y, \dot{y} = x] \rightarrow -xy + xy = 0$. Верно.

Задача 5

(a) $\dot{x} = x, \dot{y} = y$

$$\dot{x} = x, \dot{y} = y \rightarrow \frac{dx}{dt} = x, \frac{dy}{dt} = y \rightarrow \frac{dy}{dx} = \frac{y}{x} \rightarrow \frac{dy}{y} = \frac{dx}{x} \rightarrow \ln |y| = \ln |x| + C \rightarrow y = C_1 |x|$$

(b) $\dot{x} = x, \dot{y} = -y$

$$\dot{x} = x, \dot{y} = -y \rightarrow \frac{dx}{dt} = x, \frac{dy}{dt} = -y \rightarrow \frac{dy}{dx} = -\frac{y}{x} \rightarrow \frac{dy}{y} = -\frac{dx}{x} \rightarrow \ln |y| = -\ln |x| + C \rightarrow y = \frac{C_1}{|x|}$$

(c) $\dot{x} = x^2, \dot{y} = -y$

$$\dot{x} = x^2, \dot{y} = -y \rightarrow \frac{dx}{dt} = x^2, \frac{dy}{dt} = -y \rightarrow \frac{dy}{dx} = \frac{-y}{x^2} \rightarrow \frac{dy}{y} = -\frac{dx}{x^2} \rightarrow \ln |y| = \frac{1}{x} + C \rightarrow y = e^{1/x} C_1$$

(d) $\dot{x} = -y, \dot{y} = x$

$$\dot{x} = -y, \dot{y} = x \rightarrow \frac{dx}{dt} = -y, \frac{dy}{dt} = x \rightarrow -ydy = xdx \rightarrow y^2 = C - x^2 \rightarrow y = \pm\sqrt{C - x^2}$$

Задача 6

(a) $\dot{x} = 1$

$$\dot{x} = y, \dot{y} = 1 \rightarrow \frac{dy}{dx} = \frac{1}{y} \rightarrow y^2/2 = x + C \rightarrow y = \pm\sqrt{2x + C}$$

(b) $\dot{x} = x$

$$\dot{x} = y, \dot{y} = x \rightarrow \frac{dy}{dx} = \frac{x}{y} \rightarrow y^2 = x^2 + C \rightarrow y = \pm\sqrt{x^2 + C}$$

(c) $\dot{x} = \dot{y}$

$$\dot{x} = y, \dot{y} = y \rightarrow \frac{dy}{dx} = 1 \rightarrow y = x + C$$