# Дифференциальные уравнения листок 4

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Февраль 2019

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## Задача 1

**(a)** $\dot{x} = 0, \dot{y} = 0$

$$y = C_1, x = C_2$$

**(b)** $\dot{x} = 2, \dot{y} = 1$

$$y = t + C_1, x = 2t + C_2$$

**(c)** $\dot{x} = 1, \dot{y} = y$

$$x = t + C_1, y = C_2 e^t \to y(x) = C_2 e^{x - C_1}$$

**(d)** $\dot{x} = x, \dot{y} = y$

$$x = C_1 e^t, y = C_2 e^t \to y(x) = \frac{C_2}{C_1} x$$

**(e)** $\dot{x} = 2x, \dot{y} = y$

$$x = C_1 e^{2t}, y = C_2 e^t \to y(x) = C_2 \cdot \left( \sqrt{\frac{x}{C_1}} \right)$$

**(f)** $\dot{x} = x, \dot{y} = -y$

$$x = C_1 e^t, y = \frac{C_2}{e^t} \to y(x) = \frac{C_1 C_2}{x}$$

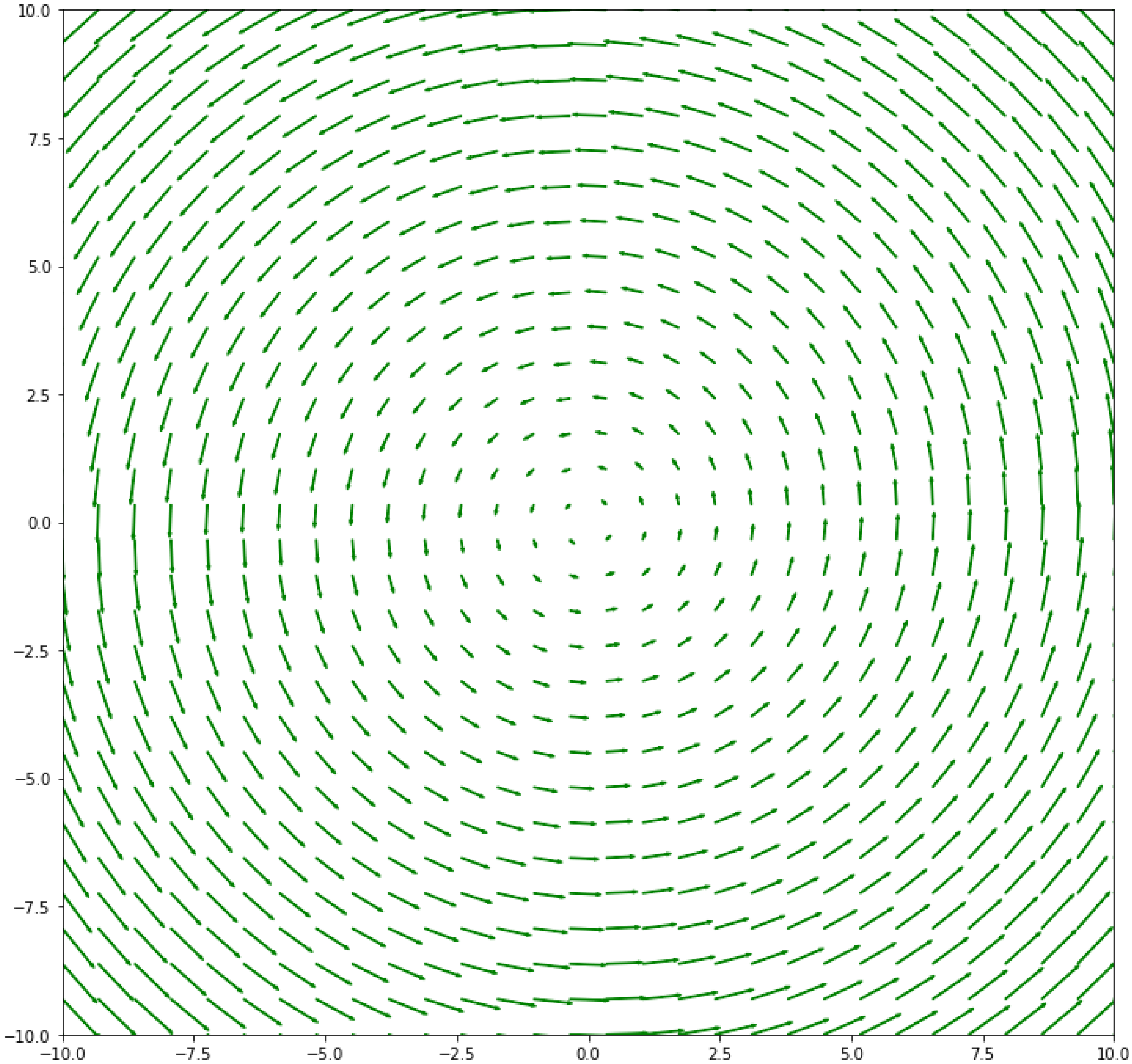
**(g)** $\dot{x} = x^2, \dot{y} = -y$

$$x = -\frac{1}{t + C_1}, y = \frac{C_2}{e^t} \to y(x) = C_2 e^{C_1 + \dfrac{1}{x}}$$

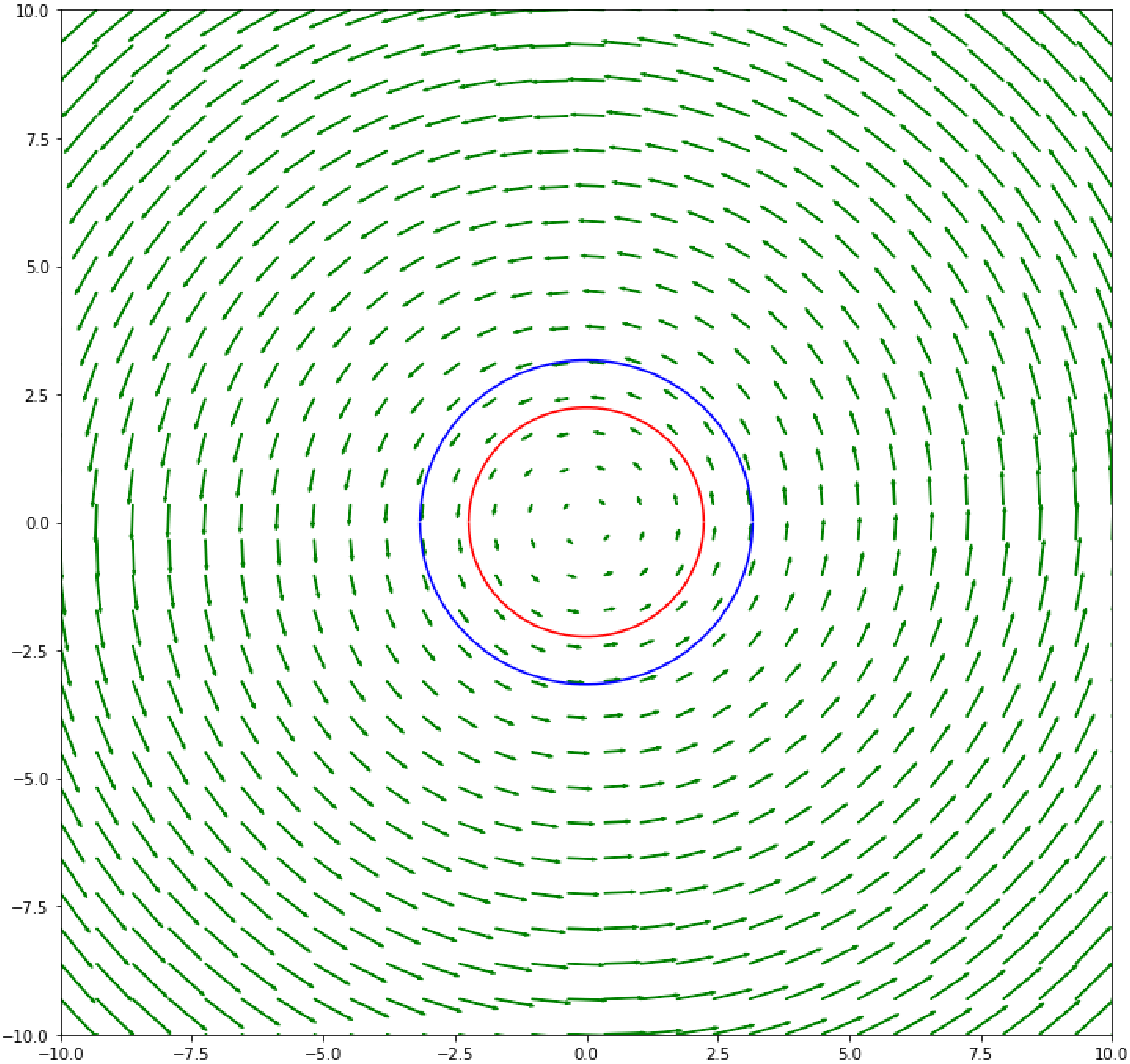
## Задача 2

$$\dot{x} = -y, \quad \dot{y} = x$$

**(a)** Поле направлений:



**(b)** Фазовые кривые:




**(c)** Настоящие фазовые кривые:

$x^2 + y^2 = C$

Проверим: $\dot{x}x + \dot{y}y = 0 \to [\dot{x} = -y, \dot{y} = x] \to -xy + xy = 0$. Верно.

## Задача 5

**(a)** $\dot{x} = x, \dot{y} = y$

$$\dot{x} = x, \dot{y} = y \to \frac{dx}{dt} = x, \frac{dy}{dt} = y \to \frac{dy}{dx} = \frac{y}{x} \to \frac{dy}{y} = \frac{dx}{x} \to \ln|y| = \ln|x| + C \to y = C_1|x|$$

**(b)** $\dot{x} = x, \dot{y} = -y$

$$\dot{x} = x, \dot{y} = -y \to \frac{dx}{dt} = x, \frac{dy}{dt} = -y \to \frac{dy}{dx} = -\frac{y}{x} \to \frac{dy}{y} = -\frac{dx}{x} \to \ln|y| = -\ln|x| + C \to y = \frac{C_1}{|x|}$$

**(c)** $\dot{x} = x^2, \dot{y} = -y$

$$\dot{x} = x^2, \dot{y} = -y \to \frac{dx}{dt} = x^2, \frac{dy}{dt} = -y \to \frac{dy}{dx} = \frac{-y}{x^2} \to \frac{dy}{y} = -\frac{dx}{x^2} \to \ln|y| = \frac{1}{x} + C \to y = e^{1/x}C_1$$

**(d)** $\dot{x} = -y, \dot{y} = x$

$$\dot{x} = -y, \dot{y} = x \to \frac{dx}{dt} = -y, \frac{dy}{dt} = x \to -ydy = xdx \to y^2 = C - x^2 \to y = \pm\sqrt{C - x^2}$$

## Задача 6

**(a)** $\ddot{x} = 1$

$$\dot{x} = y, \dot{y} = 1 \to \frac{dy}{dx} = \frac{1}{y} \to y^2/2 = x + C \to y = \pm\sqrt{2x + C}$$

**(b)** $\ddot{x} = x$

$$\dot{x} = y, \dot{y} = x \to \frac{dy}{dx} = \frac{x}{y} \to y^2 = x^2 + C \to y = \pm\sqrt{x^2 + C}$$

**(c)** $\ddot{x} = \dot{x}$

$$\dot{x} = y, \dot{y} = y \to \frac{dy}{dx} = 1 \to y = x + C$$