

# Bitwise Operations

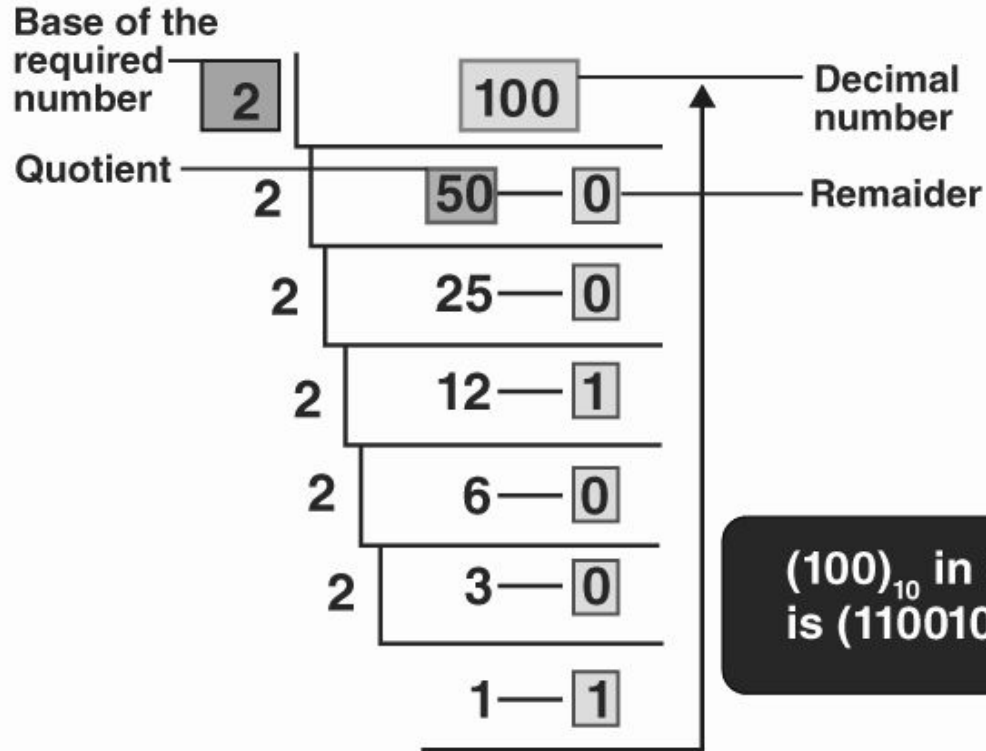
# What are Bits ?

# Bit Representation for unsigned?

# Why Bitwise Operations?

- Some questions require bit manipulations.
- It can be used to optimize solutions and simplify solutions.

# Bit Representation



$(100)_{10}$  in Binary  
is  $(1100100)_2$

# Bit Representation for signed?

# Bit Representation

## Signed Integer

$\begin{array}{c|c|c|c|c|c|c|c|} \text{+/-} & & \text{number} & & & & & \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$

# 2's Complement

A method to represent negative numbers

Most popular method

Allows adding negative numbers with the same logic gates as positive numbers

Main Idea:  $x + (-x) = 0$



# 2's Complement

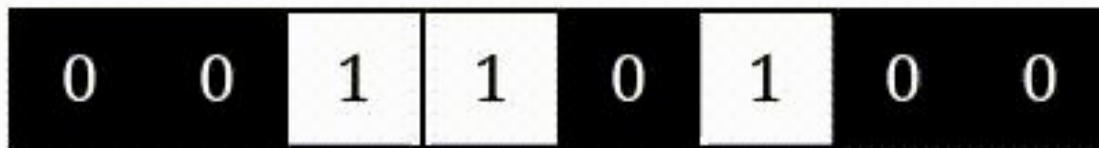
How to convert number to 2's complement

1. Convert the positive number to binary
2. Flip the bits (1 to 0 and 0 to 1)
3. Add 1

# Bit Operators

- NOT (~)
- AND (&)
- OR (|)
- XOR (^)
- Bit Shifts (<<,>>)

# NOT



NOT (~)

# AND

1	0	0	1	1	1	0	0
---	---	---	---	---	---	---	---

AND (&)

0	0	1	1	0	1	0	0
---	---	---	---	---	---	---	---

# Check yourself

$$3 \& 2 = ?$$

# OR

1	0	0	1	1	1	0	0
---	---	---	---	---	---	---	---

OR (|)

0	0	1	1	0	1	0	0
---	---	---	---	---	---	---	---

# Check yourself

$$7 \mid 8 = ?$$

# XOR

1	0	0	1	1	1	0	0
---	---	---	---	---	---	---	---

XOR (^)

0	0	1	1	0	1	0	0
---	---	---	---	---	---	---	---



# Check yourself

$$4 \wedge 9 = ?$$

# Left Shift



Left Shift (<<)

# Check yourself

$$1 \ll 3 = ?$$

# Right Shift



Right Shift (>>)

# Check yourself

$$16 \gg 3 = ?$$

- Each right shift operation reduces the number to its half.

# Bit Operators

A	B	$A \mid B$	$A \& B$	$A \wedge B$	$\sim A$
0	0	0	0	0	1
0	1	1	0	1	1
1	0	1	0	1	0
1	1	1	1	0	0

## Question #1

First let's solve it using normal approach.

Then let's solve it using bits (shifting operation)

### 338. Counting Bits

Hint ...

Easy



8.6K

408



Companies

Given an integer `n`, return an array `ans` of length `n + 1` such that for each `i` ( $0 \leq i \leq n$ ), `ans[i]` is the **number of 1's** in the binary representation of `i`.

#### Example 1:

**Input:** `n = 2`

**Output:** `[0,1,1]`

**Explanation:**

`0 --> 0`

`1 --> 1`

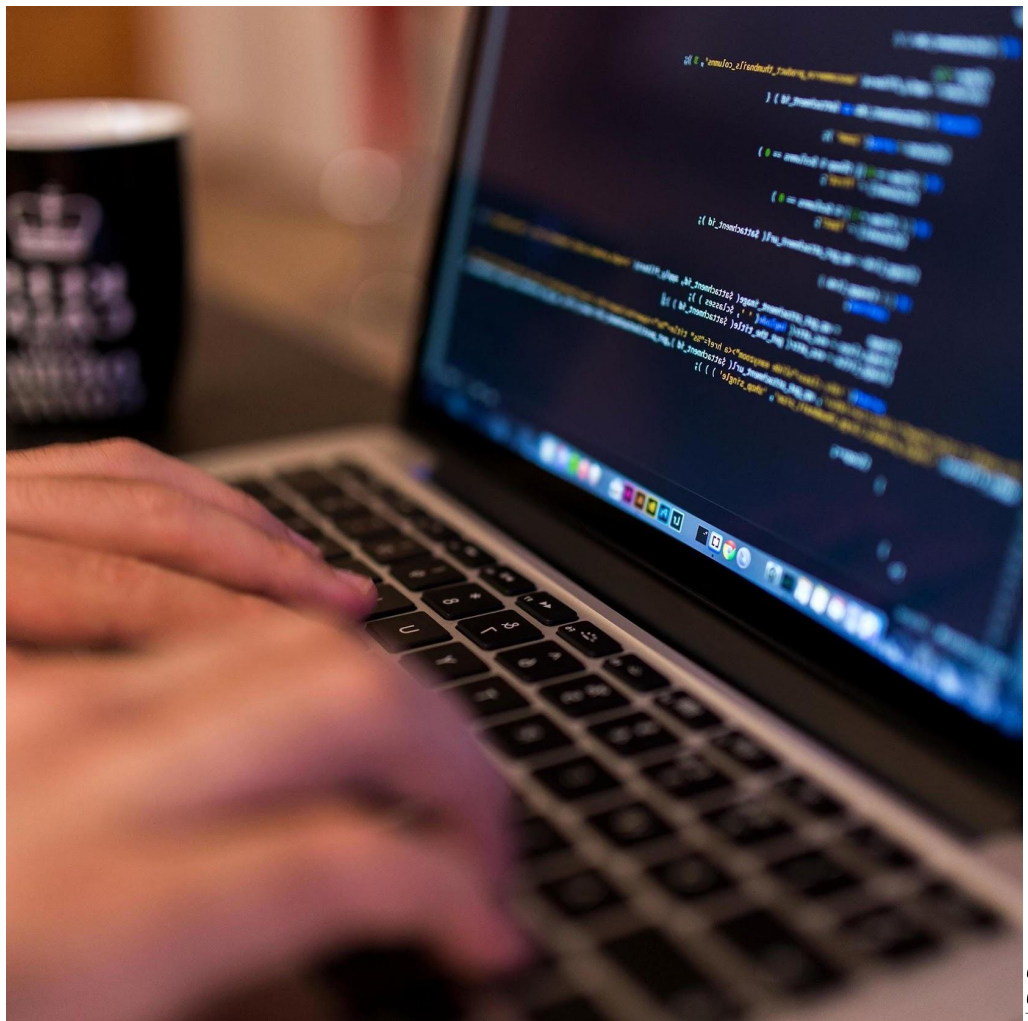
`2 --> 10`

# Bitwise operators properties

- Commutative
  - $x \wedge y = y \wedge x$
- Associative
  - $x \& (y \& z) = (x \& y) \& z$
- Does this property hold for all bit operations?



# Practice Time



## Question #2

First let's solve it  
using normal approach.

Then let's solve it  
using bits.

$[0,2] \Rightarrow 2$

### 268. Missing Number

Easy



7834



3002



Add to List



Share

Given an array `nums` containing `n` distinct numbers in the range `[0, n]`, return *the only number in the range that is missing from the array*.

#### Example 1:

**Input:** `nums = [3,0,1]`

**Output:** 2

**Explanation:** `n = 3` since there are 3 numbers, so all numbers are in the range `[0,3]`. 2 is the missing number in the range since it does not appear in `nums`.

#### Example 2:

**Input:** `nums = [0,1]`

**Output:** 2

**Explanation:** `n = 2` since there are 2 numbers, so all numbers are in the range `[0,2]`. 2 is the missing number in the range since it does not appear in `nums`.

#### Example 3:

**Input:** `nums = [9,6,4,2,3,5,7,0,1]`

**Output:** 8

**Explanation:** `n = 9` since there are 9 numbers, so all numbers are in the range `[0,9]`. 8 is the missing number in the range since it does not appear in `nums`.

# Bit masking

- Way of optimizing storage
- Store information in a single bit

# Test 5th Bit

num = 10001000**1**00111

1 = 0000000000000001

# Test 5th Bit

num = 10001000100111

1 = 000000000000001

1<<5 = 00000000100000

10001000100111  
& 00000000100000  
00000000100000 (!= 0)

# Implement

```
import sys

# kth bit from the right, 1 indexed
def kthBitTest(num : int, k : int) -> int:
    # TODO
    Mask = 1 << k
    Result = Num & num
    If result != 0:
        Return True
    Retur False
```

```
def test():
```

# Implement

```
import sys

# Set the Kth bit from the right, 1 indexed
def kthBitSet(num : int, k : int) -> int:
    # make a mask, the mask goes to 1<<k
    # we or it
    Mask = 1<<k
    New_num = num | mask

def test():
    assert kthBitSet(6, 1) == 7, 'Oops'
    assert kthBitSet(3, 4) == 11, 'Oops'
    print('Niceee')
```

# Implement

```
import sys

# turn off the kth bit from the right, 1 indexed

def turnOffKthBit(num : int, k : int) -> int:

def test():
    assert kthBitTest(6, 1) == 6, 'Oops'
    assert kthBitTest(6, 2) == 4, 'Oops'
    assert kthBitTest(3, 4) == 3, 'Oops'
    print('Niceee')

test()
```



Test  $k^{\text{th}}$  bit is set: `num & (1 << k) != 0` .

Set  $k^{\text{th}}$  bit: `num |= (1 << k)` .

Turn off  $k^{\text{th}}$  bit: `num &= ~(1 << k)` .

Toggle the  $k^{\text{th}}$  bit: `num ^= (1 << k)` .

# Bit masking

## 46. Permutations



Medium



15.1K



256



Companies

Given an array `nums` of distinct integers, return *all the possible permutations*. You can return the answer in **any order**.

### Example 1:

Input: `nums = [1,2,3]`

Output: `[[1,2,3], [1,3,2], [2,1,3], [2,3,1], [3,1,2], [3,2,1]]`

### Example 2:

Input: `nums = [0,1]`

Output: `[[0,1], [1,0]]`

### Example 3:

Input: `nums = [1]`

Output: `[[1]]`

## Question #3

- Use bit mask to keep track of used numbers

# Python Built-in Functions

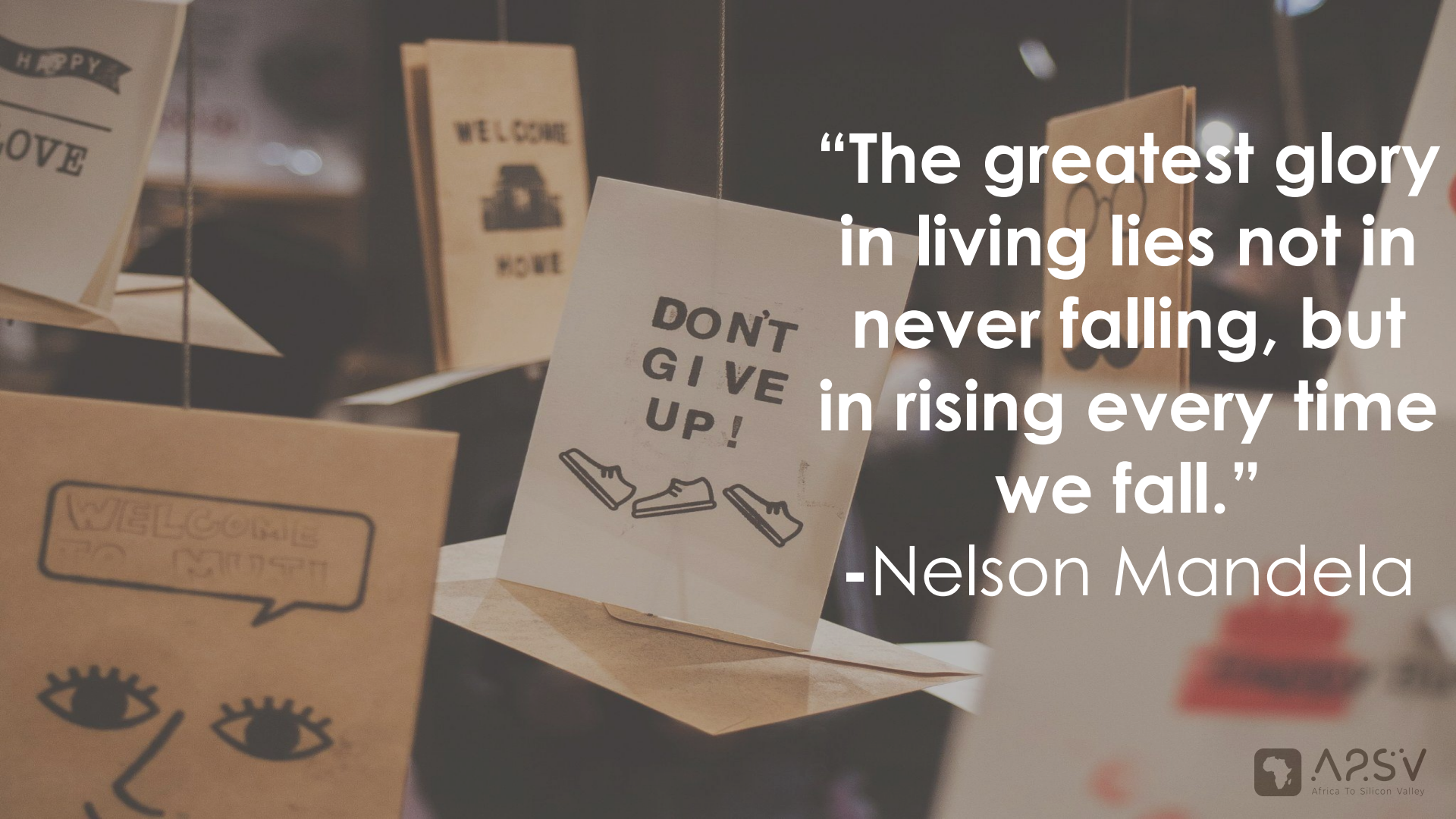
- `bin()`: This built-in function can be used to convert an integer to a binary string.
- `int()`: This built-in function can be used to convert a binary string to an integer.

# Python Built-in Functions

- `bit_length()`: This method can be called on an integer and returns the number of bits required to represent the integer in binary, excluding the sign bit.
- `bit_count()` : this method can be called on an integer and returns the number of set bits.

# Practice Problems

- [Single Number II](#)
- [Single Number III](#)
- [Number Complement](#)
- [Sum of Two Integers](#)
- [Add Binary](#)
- [Hamming Distance](#)
- [Counting Bits](#)
- [Subsets](#)
- [Count Words Obtained After Adding a Letter](#)

The background of the image is a collection of several paper cards hanging from thin strings. The cards are made of light-colored paper and feature various messages and drawings. One card in the foreground has the text "DON'T GIVE UP!" with three simple line drawings of sneakers below it. Another card to the left has a speech bubble saying "WELCOME TO MUSA" and a drawing of a face with large eyes. Other cards in the background have "HAPPY LOVE", "WELCOME HOME", and "WELCOME" written on them. The overall tone is warm and motivational.

**“The greatest glory  
in living lies not in  
never falling, but  
in rising every time  
we fall.”**

**-Nelson Mandela**