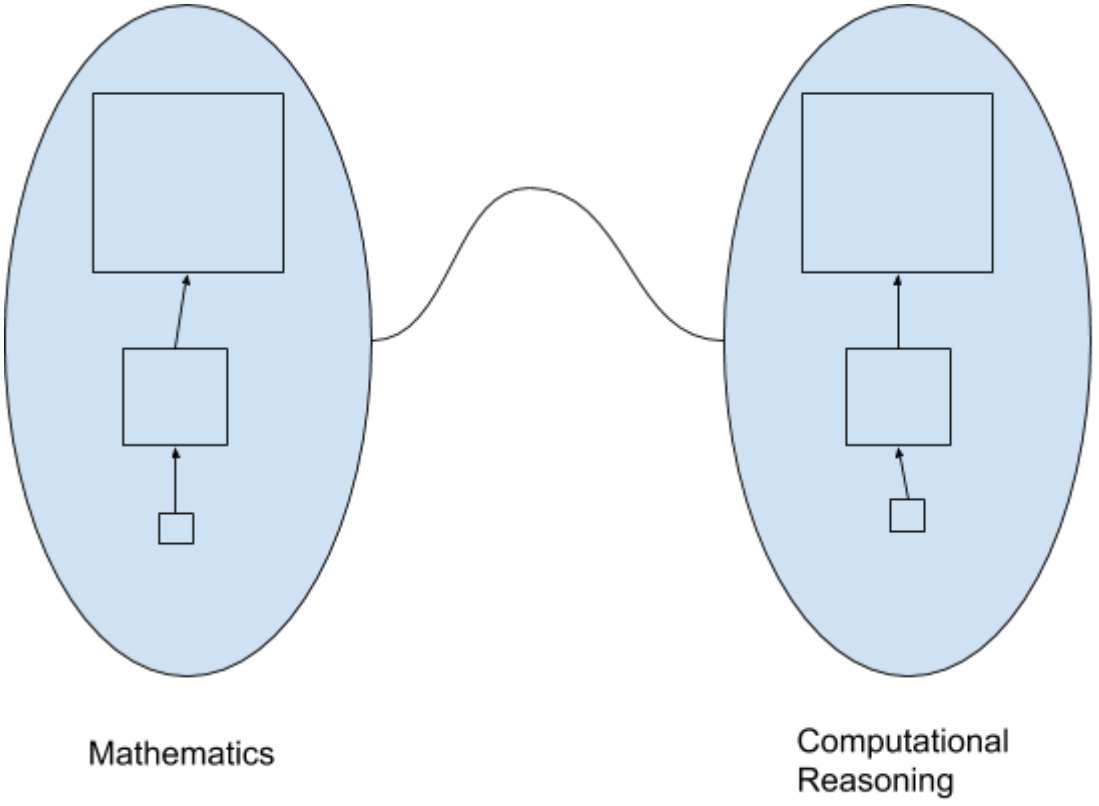
Basic Methods and Tools in Category Theory
Nature of Information Project 3 of Q4, 2018

Introduction

Category Theory is a branch of pure mathematics seeking to understand a formal part of relationship between different objects or structures. One could inquire if Category Theory could be used to understand human cognition and development of effective representation systems of somewhat complex concepts. Here one could make use of Category Theory straight away by observing the parallels between mathematics and computational reasoning



In both mathematics and computational reasoning everything starts with elementary observations, which are then generalized into some bigger theories and concepts. Mathematics provides us with a language of talking about generalizations. One of such languages is the language of Category Theory. One of the prominent problems in theory of computational reasoning is representation of rather complex concepts and processes. One could then apply language of Category Theory in developing tools to represent such concepts and processes.

Ideas and Results

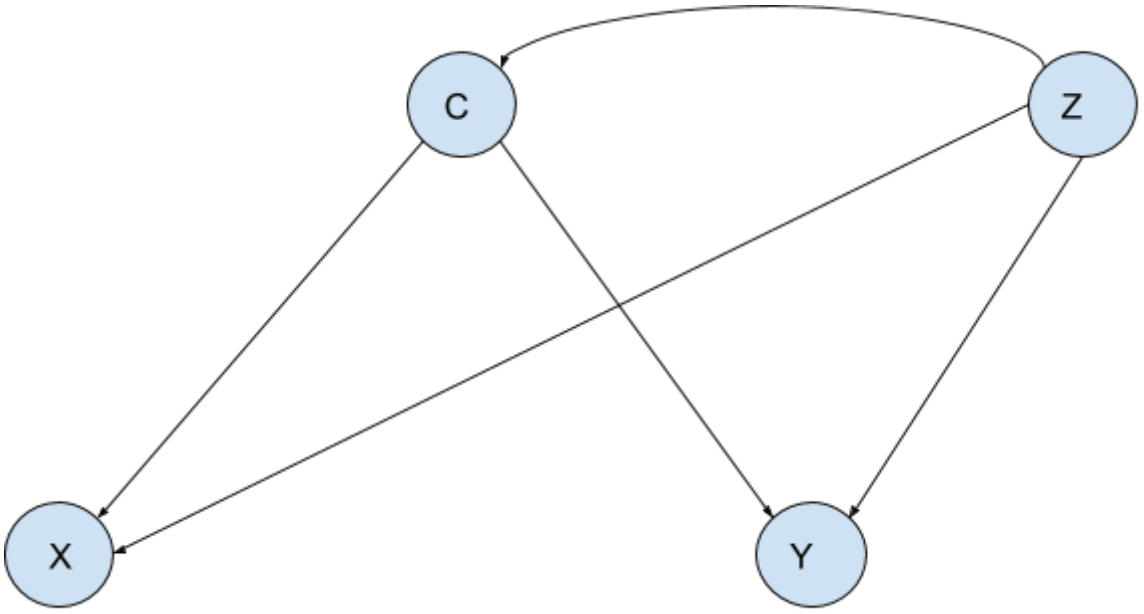
There are many important tools employed in Category Theory. We would like to outline three tools in this note:

- A notion of limit (colimit)
- An idea of going from syntax to semantics
- An idea of compositionality (how complex things can be assembled out of simpler parts)

Let’s briefly go through these ideas

Limit (colimit)

Given some diagram in some category, its limit and colimit somehow codifies information about this diagram. For example, consider a product of two objects X and Y. It is an object C through which X and Y could be accessed.



Q: What is an example of limits in computational reasoning?

Given separate instances of certain observations of some learning agent, the limit of this observation could be thought as a mechanism which explains these observations. Though this example is too general, in a sense that almost any scientific field has this kind of ‘limit’, it is might be a decent starting point.

Idea of Going from Syntax to Semantics

In the above idea, syntax refers to a set of rules, say grammar, while semantics refers to meaning, say meaning of a sentence. In mathematics, a famous example of transition from syntax to semantics is a notion of group representation. Within Category Theory itself an example could be Yoneda lemma.

Q: What could an example of a transition from syntax to semantics in computational reasoning?

One example is a notion of reinforcement learning, where a policy P is able to generate a series of rewards. And depending on the values of these rewards we are able to infer effectiveness of a given policy.

Idea of Compositionality

The idea of compositionality can be thought as a relationship between simple elements which make up a more complex object.

Q: Why do we might need to use compositionality in building more advanced computational reasoning systems?

It is quite clear that any attempt to build a computational reasoning will involve dealing with an immense number of types of inputs, concepts and world models. Compositionality could help us to make sense of this complexity.

Discussion

In this note we have discussed three tools used in Category theory and tried to think of possible applications of these tools and ideas in the context of theory of computational reasoning. We hope that further development of ideas and techniques in this direction will help to attract more people in working in this direction.

Further Reading:

- Category Theory Course, John Baez
- A Subtle Introduction to Category Theory, W. J. Zeng
- W H A T I S A P P L I E D C A T E G O R Y T H E O R Y ?