NI: Week 5 - Conceptual Spaces as Pointed Convex Spaces

Abstract

The aim of this work is to propose a framework for conceptual spaces in the form of pointed convex spaces. As in general pointed spaces from category theory, maps between convex spaces respect representative points. Moreover, we suggest using representative points as a coarse approximation of its underlying space. Increasing collection of neighborhoods of the representative point could serve as finer approximations of the underlying space.

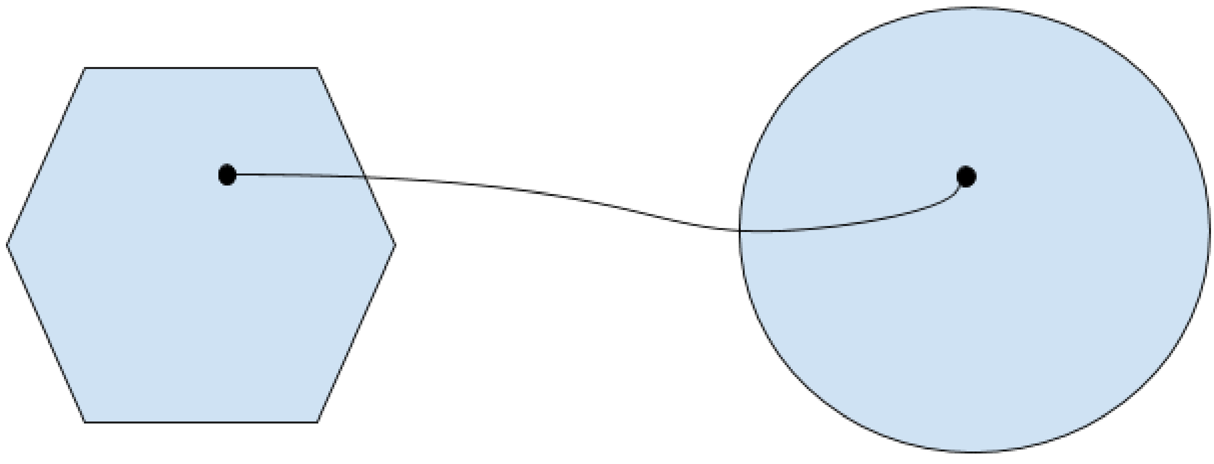
Introduction

Usually conceptual spaces are introduced as a convex spaces with some representative element. The convexity assumption comes from the idea that if two points A and B lie in some conceptual space, then all the points lying in between A and B should also live in the underlying conceptual space. The representative element assumptions mostly comes from human perception of the world. Most humans have their underlying example for such concepts as man, stone, etc. These unique examples could serve as representatives for conceptual space for man, stone, etc. The important question in the theory of Conceptual Spaces is initial birth and dynamics of conceptual spaces and their representative elements. This questions being rather profound, we ask the following simple question. How could conceptual spaces be modeled in a fixed time framework? We propose to look at conceptual spaces as a category of pointed convex spaces. The maps between these spaces should preserve underlying points. Moreover, representative points could act as coarse approximations of underlying spaces.

Ideas and Results

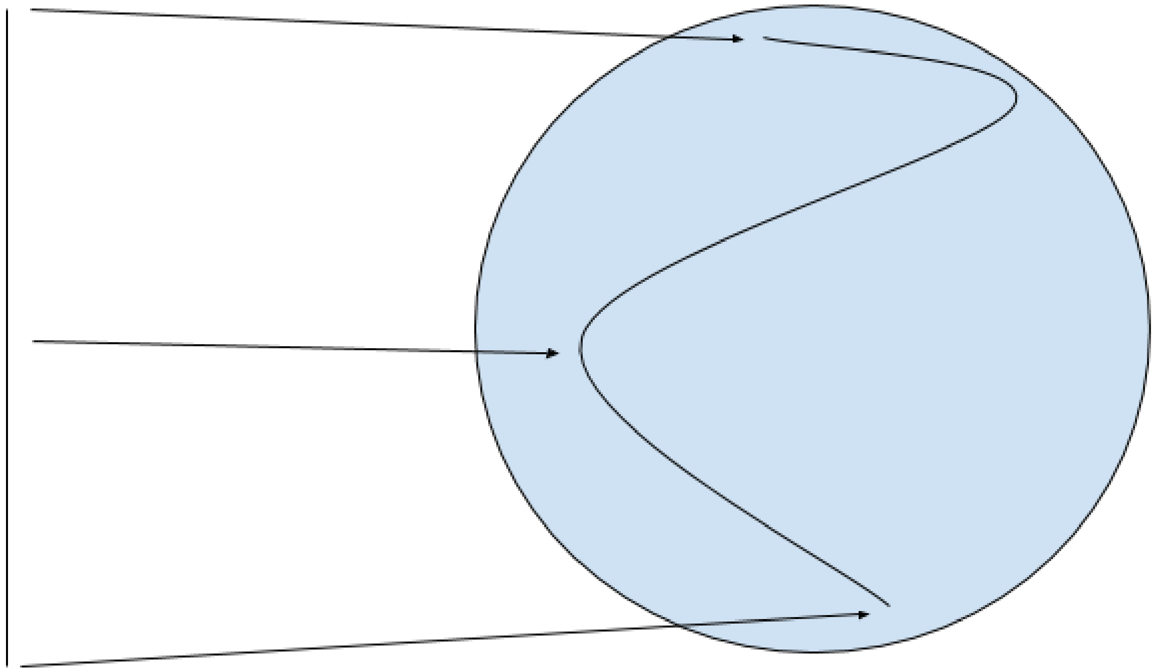
Conceptual Spaces as a Category of Pointed Convex Sets

A theory of pointed spaces plays an important role in algebraic topology. Many important concepts such as homotopy groups are defined using pointed spaces, though in most cases the dependence on points could be eliminated. Then one could leverage on theory of pointed topological spaces to study conceptual spaces. Namely, one could consider conceptual spaces as a category of convex spaces. Here the underlying points can be interpreted as a general notion of representative element. However, underlying points could also be interpreted as a notion of some concept, say that of dog. Then the corresponding category corresponds to the category of different representations of dogs.



Maps in the Given Category

Maps in the given category can interpreted as transformations between different conceptual spaces. Computing convenient maps between conceptual spaces would allow us to go back and forth between conceptual spaces. Furthermore, maps are powerful tools which could model many complex phenomena such as movement. To model a movement we could consider a map between a unit interval [0,1] and some geometrically arisen space.



Points as an Approximation of Underlying Set

Convex space as it is might be quite a complex object to describe and work with. However, the essential information about that convex space could be extractable from its representative point. Moreover, the larger neighborhoods of that representative points might provide finer approximations to the underlying convex space.

Discussions

In the given notes, we wanted to provide a glimpse to idea of modeling conceptual spaces after pointed convex spaces as in category theory. There still much to be done. For instance, how are representative points are chosen? Are there somewhat optimal maps between conceptual spaces? How could more complex processes be presented in the given framework. All of these are quite important questions to answer if we wish to understand conceptual spaces deeper using the given framework.