Basics of Category Theory
Project 2 in Nature of Information, Q4, 2018

Introduction

Category theory can be said to be born out of observation of commonalities between distinct mathematical structures. This theory allows to formalize what we mean by commonalities. In some sense, Category Theory does not help us to solve specific problems, however it well allows to generalize many phenomena. In this respect, it might seem that this theory is not a practitioner's tool, rather it is a tool of generalist. However, the idea of generalization is paramount for human reasoning. We hope that Category theory as one of the tools to study the process of generalization could help us to process more complex concepts using computational methods.

Ideas and Methods

The basis of category theory is a notion of homomorphism between objects. The premise is that one could understand quite a lot about an object just by observing how it interacts with other objects. The objects is where homomorphism start or end.

- Object

A morphism is a mapping between two objects, say f:A->B

- Morphism

There is a elementary morphism which can be interpreted as no change

- 1_A:A->A

Homomorphism could be composed together

- f:A->B and g:B->C could be composed into g\circ f:A->C

The order of composition of several morphisms does not matter. In other words composition is associative.

- (h\circ g)\circ f = h\circ (g\circ f)

Functors

As with most mathematical entities, it is possible to study maps between categories, which somehow preserve rules of these categories.

Given categories C and D, a functor F:C->D maps objects in morphisms of C into objects and morphisms of D respectively.

- For any object x in C, F(x) is in D
- For any morphism f:x->y in C, F(f):F(x)->F(y) is a morphism in D.

The functor mapping should preserve composition of morphisms and identity morphisms:

- F(g\circ f) = F(g)\circ F(f)
- F(1_x) = 1_{F(x)}

The idea of functor allows us to talk of categories which themselves consists of categories. Morphisms of the underlying category is then functors between categories given as objects. This operation serves as an example of going up in abstraction.

Discussions

It is worth investigating if category theory could help us to solve a problem of generalization faced in machine learning.

Further Reading

- Category Theory Course, John Baez