NI: Week 4 - Universal and Useful Representations

Abstract
This work ponders about a concept of representation from machine learning and mathematics. We try to distinguish between two kinds of representations, *universal* and *useful*. Universal representations have ability to give rise to other representations. Useful representations are formed to facilitate a learning agent to accomplish goals and earn internal rewards.

Motivation
For a long time, I have been wishing to study information, reasoning and intellect from mathematical/geometric perspective. However, all my wishes used to be cut by a single question:

"What if my work will not be published?"

This thought have been hunting me for a long time. Every time I wished to start some research on this direction, I clearly saw that my ideas and results were far from publishable. However, thinking about publishing process for some time, I realized that the very purpose of publishing is spreading new knowledge. An no new knowledge could ever be created without people trying new things. This made me realize a vanity of my doubts, and made me write my weekly papers. None of these weekly papers could be of publishing quality, however, in writing each of them I will be working towards my dream of uncovering truth behind of information, computation and intellect using help of mathematics/geometry. Let us begin our journey.

Introduction
A concept of representation plays an important role in fields of Machine Learning and Mathematics. The general idea of representation could be described as a mapping from some complicated or misunderstood space to simpler or understood space. Let us give some examples of representations employed in mathematics and computer science:

Group representations
A group representation is a mapping:

$$G \rightarrow Aut(V)$$

Here the group G might be quite complicated, but my mapping it into the group of linear automorphisms of a vector space V, it is possible to understand G better thanks to a simpler structure of vector spaces and linear maps.

Representations in Deep Learning
The whole concept of a deep neural network can be formulated as a tool for representation extraction. After going through a deep neural network a datapoint, say image, becomes a simple vector. So in some sense neural networks working with images can be viewed as the following maps:

(Images) -> (Euclidean Space)

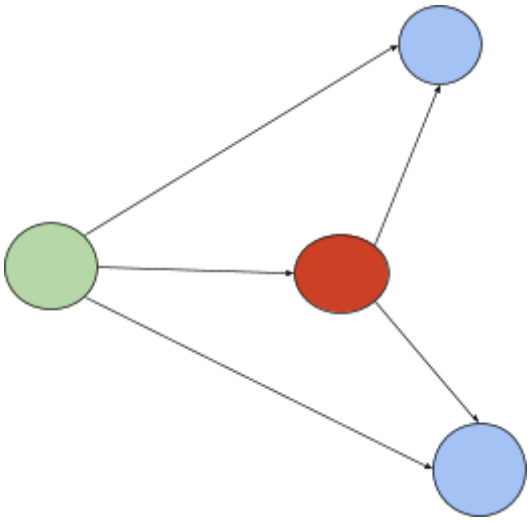
As in previous examples, images are pretty complicated objects viewed from computational intelligence standpoint. However, computers could easily manipulate with elements of euclidean space.

One of the main challenges for current computational intelligence is to come up with ways to represent more complex objects or processes and be able to manipulate these representations. For example a single image might be quite a simple object, however such processes such as driving a car, or making rational decisions in everyday situations are relatively more complicated.

Ideas and Results

To understand a process of representation better, we would like to focus on two types of representations: universal and useful.

Universal representations are representations which give rise to other representations.



In this diagram, an object we wish to represent is painted green. The blue circles are its two representations. The red circle is a universal representation. Having an access to universal representations it is possible to access all other representations. This kind of universality should require that the universal representation should be quite rich. However, for objects which are encountered fairly rarely, it might be not be worthwhile to develop a universal representation. In this case we work with useful representations.

Useful Representations are sufficiently expressive representations to accomplish some task at hand. For instance, a job of deep neural network for image classification is to minimize the misclassification error. In order to achieve this task, a neural network can be thought as to develop its internal representations for images. The more important or more frequent an object becomes, the more expressive useful representations tend to be. In more general setting given some agent A who receives some internal reqard R, the representation of an object X is constructed so that R is maximized.

Discussion

We shared some thoughts on universal and useful representations on objects. Though it was easy to define these concepts, the real question is about constructions of these representations. How are universal representations are constructed? How are useful representations are constructed? What kind of tools do we need to construct these representations? All of these questions remain open for future research.