Problem 22.2-7

This problem can be solved so long as at least one wrestler has been designated to be either a babyface or a heel. In the following algorithm, rather than start out with a designation, we will color two groups either red or blue respectively. It is easy to see that if we were given the type of one wrestler we could perform this with that designation rather than the colors. This algorithm will run in $O(n + r)$ time because it runs once through the list of edges in $\Theta(r)$ time creating the adjacency list necessary for the BFS to work, and then executes BFS in $O(n + r)$ time.

```
IS-SEPARABLE(E, n, r, s)
let l[0 .. r] be a new adjacency list
for each edge q ∈ E
    add q to the adjacency list
// The list l will be used as a graph G
for each vertex u ∈ G.V
    u.color = WHITE
s.color = BLUE
Q = ∅
ENQUEUE(Q, s)
while Q ≠ ∅
    u = DEQUEUE(Q)
    for each v ∈ G.Adj[u]
        if v.color == u.color
            return false
        if v.color == white
            if u.color ==BLUE
                v.color = RED
            else
                v.color = BLUE
            ENQUEUE(Q, v)
return true
```

Problem 22.3-5

a) By the definition of a tree edge, v must be discovered by following (u, v), and by the definition of DFS, v will be closed first if it is found after u. A similar argument holds for forward edges, where by definition a forward edge connects a vertex u to a descendant v lower in the tree produced by DFS.

b) There are two possible situations for back edges. First is that a vertex later in the tree has been connected to one of its ancestors, and second is that a node is connected back to itself. In the first case, v will be found before u because it is closer to the root of the tree generated by DFS. In the second case, the discovery time of u and v will be equal, which is accounted for in this statement.

c) An intuitive definition of a cross edge is one that connects trees in the forest produced by DFS. Because of that, u will be discovered and finished before v is ever discovered, otherwise they would be in the same tree, which is a contradiction.

Problem 22.3-8

u v

w

In the above graph, if DFS were called first on vertex u then it will be discovered before vertex v is discovered, but v is not a descendant of u.