PURM 2020: Generating Time Series Data

8/12/20 Project Overview (revised 8/19/20)

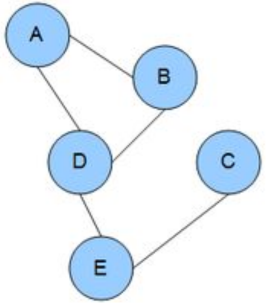
Alan Ismaiel and Jason Shu

Motivation

There exist increasingly many tools to generate large amounts of time series data, which is significantly affordable than collecting the same amount of data.

- However, many of the tools we surveyed use deterministic ways to specify time series
- Other tools used an ARMA approach, which is more random but doesn't intuitively represent relationships between random variables

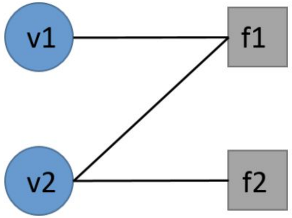
Existing Approaches



Bayesian Graphs/Markov Networks/Mixed Graphs

- Captures similar information as our time series system, but is limited by independence/conditional independence assumptions

Factor Graphs/Machine Learning Models



- Uses existing datasets to generate an approximation of the time series behavior: imprecise and requires data that may not exist

Autoregressive Models/Moving Average Models

- Calculates time series systems as a function in terms of all previous values in the time series, as well as approximations for white noise and error, concepts not considered in our implementation

Existing Tools

- Faker
  - Generates data based on user-defined/pre-existing data types, but has no notion of time or dependence
- DataGenerator
  - Capable of modeling complex probability distributions, but runs on an outdated XML system and has no built-in notion of time series
- Pandas (Time Series Functionality)
  - Good way to organize/analyze time series data, no inherent ability to specify dependencies between variables
- T Simulus
  - Time series data points are hard-coded into JSON objects, or ARMA model is used
  - Simple time series can be composited to make more complex ones
  - Would work better as a platform to plug into *after* determining distribution
- Pyro
  - Claims to be able to represent any computable distribution, built-in generation abilities
  - Complicated input system without inherent representations of independence/dependence

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Our Problem Formulation

To make as expressive a tool as possible, our goal is to take user-inputted probability/independence statements and fully determine distributions using them through a distinct parameterization method.

- To best represent stochastic processes, we aim to encode dependency between time steps (e.g. past results influence future results)
- By utilizing independence specifications, the complexity of defining probability systems can be **exponentially reduced**

Problem Statement

Our considerations of time series can be distinguished into three separate cases:

- **Static Time Series Case**
- **Time Invariant Time Series Case**
- **Time Variant Time Series Case**

Outside of the static case, we define a notion of **relevant time steps**. Put simply, variables defined at a time t are exclusively dependent on a set number of the same variables defined at previous time steps. These previous time step variables defined in a set B_t are the relevant time steps

Problem Statement: Static Case

Static Case

- The notion of time is **not** considered
- **Input:**
  - A set of categorical variables and each variable's values
  - A set of specifications which describe the distribution over the variables
- **Goal:** To find the joint distribution over all variables, or to find whether the specifications are underdetermined or contradictory

Case Examples

- The probability system of drawing a card from a deck 100 times

Problem Statement: Time Invariant Case

Time Invariant Case

- At any time t, the corresponding probability system S_t is under the stationary assumption
- **Stationary Assumption**: Probability distributions don't change when shifted in time

Case Examples

- The probability system of deciding what to wear based only on what was worn the previous day

Problem Statement: Time Variant Case

Time Variant Case

- For all variables generated in a unit of time t, their probabilities are a function of the relevant time steps probability distributions
- Considers the distribution of past time steps to calculate the current one

Case Examples

- The probability system of robot components that have a higher likelihood of wearing down the more time they are functional
- The probability system representing sunny and rainy days that change probabilities based on how many prior days

Solution Approach

Our approach to constructing a solution to this problem takes on the following steps:

1. **Parameterizing** the user input and specifications in an understandable way
2. **Solving** the probability system in terms of the parameterization
3. **Generating** data in the event that the system is fully defined

Detailed Solution

Solutions for the three cases differ, but they generally follow these steps:

1. **Import** user-provided text file containing information about data generation (including variables, variable values, specifications, etc.) into Mathematica
2. **Parse** text file and translate information into standard Mathematica language
3. **Convert** probability symbols (e.g. P[A | B], P[C && D]) to combinations of elementary probabilities, which are denoted by “O parameters”
4. **Solve** the equations for the O parameters. Note whether the system of equations is well-determined, contradictory, or under-determined. If well-determined, map the O values to a distribution
5. **Generate** data from said distribution
6. In a **non-static** case, **repeat step 5** for as many steps as the user would like to generate.

Time Invariant Example

Consider a single boolean variable T, defined with 2 relevant time steps

$P(T_t \cap T_{t-1} \cap T_{t-2})$	o000
$P(T_t \cap T_{t-1} \cap \neg T_{t-2})$	o001
$P(T_t \cap \neg T_{t-1} \cap T_{t-2})$	o010
$P(T_t \cap \neg T_{t-1} \cap \neg T_{t-2})$	o011
$P(\neg T_t \cap T_{t-1} \cap T_{t-2})$	o100
$P(\neg T_t \cap T_{t-1} \cap \neg T_{t-2})$	o101
$P(\neg T_t \cap \neg T_{t-1} \cap T_{t-2})$	o110
$P(\neg T_t \cap \neg T_{t-1} \cap \neg T_{t-2})$	o111

Time Invariant Example

We can assume the following information:

- 1 = o000 + … + o111

Stationary Assumption information:

- P(T_t) = P(T_t-1) = P(T_t-2)
- P(T_t AND T_t-1) = P(T_t-1 AND T_t-2)
- P(T_t AND not T_t-1) = P(T_t-1 AND not T_t-2)
- P(not T_t AND T_t-1) = P(not T_t-1 AND T_t-2)

It can be proven that by defining P(T_t AND T_t-1) = P(T_t-1 AND T_t-2), P(T_t AND not T_t-1) = P(T_t-1 AND not T_t-2), and P(not T_t AND T_t-1) = P(not T_t-1 AND T_t-2), then all the necessary equalities between time steps required for the stationary assumption will hold true. Thus, 4 more equations needed.

Time Invariant Example

Add the following information:

- P(T_t | T_t-1 and T_t-2) = .2
- P(T_t | not T_t-1 and not T_t-2) = .8
- P(T_t | T_t-1 and not T_t-2) = .5
- P(T_t | not T_t-1 and not T_t-2) = .5

Mathematica Output: {{o000 -> {0.0384615}, o001 -> {0.153846}, o010 -> {0.153846}, o011 -> {0.153846}, o100 -> {0.153846}, o101 -> {0.153846}, o110 -> {0.153846}, o111 -> {0.0384615}}}

Notes:

- The amount of relations that must hold for the stationary assumption increases exponentially as the number of relevant time steps increases
- We CANNOT conclude that only a single time step is relevant: the conditional expressions clearly show that both their results could impact the outcome
  - As such, the distribution of these variables currently break rules that would be required to be considered a Markov Chain

Generating Time Invariant Data

From last slide:

{{o000 -> {0.0384615}, o001 -> {0.153846}, o010 -> {0.153846}, o011 -> {0.153846}, o100 -> {0.153846}, o101 -> {0.153846}, o110 -> {0.153846}, o111 -> {0.0384615}}}

Calculate conditional probabilities for current step given past two:

1. P(T_t | T_t-1 && T_t-2) = o000 / (o000 + o100) = .2
2. P(T_t | T_t-1 && not T_t-2) = o001 / (o001 + o101) = .5
3. P(T_t | not T_t-1 && T_t-2) = o010 / (o010 + o110) = .5
4. P(T_t | not T_t-1 && not T_t-2) = o011 / (o011 + o111) = .8

Suppose T_1 was False and T_2 was True. Generate T_3.

From Equation 3 above, P(T_3 | not T_2 && T_1) = .5; generate T_3 using a random number generator.

Suppose T_3 was True. Now generate T_4 using Equation 1, as T_2 and T_3 were True. Repeat generation.

Solution: Time Variant Case

While in the time-invariant case, the stationary assumption guaranteed that distributions remain identical from one time step to another, there is no such assumption for the time-variant case.

- The time-variant case allows probabilities from **past time steps** to influence probabilities in following steps
- Since distributions can change from one step to another, probability quantities from past time steps are represented **symbolically** by the user, and the equations are also solved symbolically
- The literal values of those past probabilities are **substituted** into the symbolic solution before generating data for each step

Time Variant Example

Consider a single boolean variable T, defined with 2 relevant time steps (same as before)

$P(T_t \cap T_{t-1} \cap T_{t-2})$	o000
$P(T_t \cap T_{t-1} \cap \neg T_{t-2})$	o001
$P(T_t \cap \neg T_{t-1} \cap T_{t-2})$	o010
$P(T_t \cap \neg T_{t-1} \cap \neg T_{t-2})$	o011
$P(\neg T_t \cap T_{t-1} \cap T_{t-2})$	o100
$P(\neg T_t \cap T_{t-1} \cap \neg T_{t-2})$	o101
$P(\neg T_t \cap \neg T_{t-1} \cap T_{t-2})$	o110
$P(\neg T_t \cap \neg T_{t-1} \cap \neg T_{t-2})$	o111

Time Variant Example

Additionally, consider the existence of 4 q parameters

$P(T_{t-1} \cap T_{t-2})$	q00
$P(T_{t-1} \cap \neg T_{t-2})$	q01
$P(\neg T_{t-1} \cap T_{t-2})$	q10
$P(\neg T_{t-1} \cap \neg T_{t-2})$	q11

These are defined similarly to the o parameters, albeit without any variables at time step t. We can assume that when generating data for any time t, we will already have the values of these 4 q parameters.
NOTE: Due to the given equation, we can put one q value in terms of the other 3. Therefore, only 3 of these values are necessary to define.

Time Variant Example

Consider 4 different specifications (Mathematica input format in blue):

- P(T_t) = .5*P(T_t-1) + .5*P(T_t-2)
  - o000 + o001 + o010 + o011 == .5 * (**q00** + **q01**) + .5 * (**q00** + **q10**)
- P(T_t and T_t-1) = .9 * P(T_t-1 and T_t-2)
  - o000 + o001 == .9 * **q00**
- P(T_t and T_t-2) = .3
  - o000 + o010 == .3
- P(T_t | T_t-1 AND T_t-2) = .2
  - o000 / (o000 + o001) == .2

This is in addition to the 4 specifications initially given (defining 3 of the **q parameters** in terms of o parameters, and the common given equation)

Time Variant Example

Output:

```
Out[26]= {{o000 → {1. - 0.1 (3. - 2. q00) - 0.8 q00 - 0.1 (-7. q00 + 10. q01) - 0.1 (13. - 20. q00 - 5. q01 - 1. (-7. q00 + 10. q01) - 15. q10) -
    1. (0.1 (-3. + 2. q00) + q10) - 1. (1. - 1. q00 + 0.1 (-3. + 2. q00) - 1. q01 - 0.1 (13. - 20. q00 - 5. q01 - 1. (-7. q00 + 10. q01) - 15. q10) - 1. (0.1 (-3. + 2. q00) + q10)) -
    1. (1. - 0.1 (3. - 2. q00) - 1. q00 - 0.1 (-7. q00 + 10. q01) - 0.1 (13. - 20. q00 - 5. q01 - 1. (-7. q00 + 10. q01) - 15. q10) - 1. (0.1 (-3. + 2. q00) + q10) -
      1. (1. - 1. q00 + 0.1 (-3. + 2. q00) - 1. q01 - 0.1 (13. - 20. q00 - 5. q01 - 1. (-7. q00 + 10. q01) - 15. q10) - 1. (0.1 (-3. + 2. q00) + q10)))} if condition,
  o001 → {1. - 0.1 (3. - 2. q00) - 1. q00 - 0.1 (-7. q00 + 10. q01) - 0.1 (13. - 20. q00 - 5. q01 - 1. (-7. q00 + 10. q01) - 15. q10) - 1. (0.1 (-3. + 2. q00) + q10) -
    1. (1. - 1. q00 + 0.1 (-3. + 2. q00) - 1. q01 - 0.1 (13. - 20. q00 - 5. q01 - 1. (-7. q00 + 10. q01) - 15. q10) - 1. (0.1 (-3. + 2. q00) + q10))} if condition,
  o010 → {0.1 (3. - 2. q00)} if condition, o011 → {1. - 1. q00 + 0.1 (-3. + 2. q00) - 1. q01 - 0.1 (13. - 20. q00 - 5. q01 - 1. (-7. q00 + 10. q01) - 15. q10) - 1. (0.1 (-3. + 2. q00) + q10)} if condition,
  o100 → {0.8 q00} if condition, o101 → {0.1 (-7. q00 + 10. q01)} if condition, o110 → {0.1 (-3. + 2. q00) + q10} if condition,
  o111 → {0.1 (13. - 20. q00 - 5. q01 - 1. (-7. q00 + 10. q01) - 15. q10)} if condition}}
```

Q Conditions (system is conflicting if Qs fall outside these constraints):

```
(q00 > 0 && 0 < q01 < 0.190909 && q00 - 1.42857 q01 < 0 && 0.6 - 0.6 q00 - 1. q01 - q10 < 0 && -0.866667 + 0.866667 q00 + 1. q01 + q10 < 0) ||
  (q00 > 0 && 0.190909 < q01 < 0.290244 && 0.6 - 0.6 q00 - 1. q01 - q10 < 0 &&
    -0.866667 + 0.866667 q00 + 1. q01 + q10 < 0 && -0.75 + q00 + 2.5 q01 < 0) || (q00 > 0 && 0.290244 < q01 < 0.3 &&
    0.6 - 0.6 q00 - 1. q01 - q10 < 0 && -0.866667 + 0.866667 q00 + 1. q01 + q10 < 0 && -0.75 + q00 + 2.5 q01 < 0) ||
  (q00 > 0 && 0.3 < q01 < 0.566667 && -0.866667 + 0.866667 q00 + 1. q01 + q10 < 0 && 0.3 - 0.2 q00 - q10 < 0 && -0.85 + q00 + 1.5 q01 < 0) ||
  (0.190909 < q01 < 0.290244 && q00 - 1.42857 q01 < 0 && -0.866667 + 0.866667 q00 + 1. q01 + q10 < 0 &&
    0.3 - 0.2 q00 - q10 < 0 && 0.75 - q00 - 2.5 q01 < 0) || (0.290244 < q01 < 0.3 && -0.866667 + 0.866667 q00 + 1. q01 + q10 < 0 &&
    0.3 - 0.2 q00 - q10 < 0 && -0.85 + q00 + 1.5 q01 < 0 && 0.75 - q00 - 2.5 q01 < 0)
```

Generating Time Variant Data

STEP 1 (BASE CASE):

- q00 == .3
- q01 == .25
- q10 == .25

{{o000 → {0.06}, o001 → {0.21}, o010 → {0.24},
o011 → {0.04}, o100 → {0.24}, o101 → {0.04}, o110 → {0.01}, o111 → {0.16}}}

From here, deduce the next q values as follows:

- q00 == o000 + o001 == .27
- q01 == o010 + o011 == .28
- q10 == o100 + o101 == .28

{{o000 → {0.054}, o001 → {0.189}, o010 → {0.246}, o011 → {0.061},
o100 → {0.216}, o101 → {0.091}, o110 → {0.034}, o111 → {0.109}}}

STEP 2:

- Replace the previous q values with the ones deduced from step 1

From here, deduce the next q values as follows:

- q00 == o000 + o001 == .243
- q01 == o010 + o011 == .307
- q10 == o100 + o101 == .307

{{o000 → {0.0486}, o001 → {0.1701}, o010 → {0.2514}, o011 → {0.0799},
o100 → {0.1944}, o101 → {0.1369}, o110 → {0.0556}, o111 → {0.0631}}}

STEP 3:

- Replace the previous q values with the ones deduced from step 2

And so on…

Program Demonstration

To program these cases to generate, **Mathematica** is our program of choice

- Possesses exceptional capabilities in solving and notating complex systems of equations
- Extensive and accessible documentation for its various features
- Built in discrete probability distributions and random generation capabilities

In the two weeks since we have begun programming with Mathematica, we have created a foundation to build our time series systems off of, and a working implementation of the static case

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Limitations

There are some limitations to consider in our solution approach

Complex

- Even when utilizing independence statements, these systems grow at an exponential rate, and fully providing functioning input specifications for more complicated systems becomes increasingly difficult.

Costly

- Though we have yet to reach a limitation of Mathematica's high-end solver, we can't confidently claim that one won't exist.

Theoretical

- The more complicated the probability systems get, the more likely users may be inclined to approximate values they don't know, leading to error in the generation

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