CS 4330 HW1 Solution

January 31, 2019

0.7

a)

Let R be the relation on $\mathbb{R}$ where xRy iff $|x-y|<5$

Reflexive: $|x-x|<5 \forall x \in \mathbb{R}$

Symmetric: if $|x-y|<5$, $|y-x|<5$ for all $x, y \in \mathbb{R}$

Not transitive: let $x=7, y=5, z=1$, then $|x-y|<5$, $|y-z|<5$ but $|x-z| \not< 5$

b)

Let R be the relation $\leq$ over real number.

Reflexive: $x \leq x \forall x \in \mathbb{R}$

Transitive: if $x \leq y$ and $y \leq z$ then $x \leq z$ for all $x, y, z \in \mathbb{R}$

Not symmetric: $x \leq y$ but it is not necessarily true that $y \leq x \forall x, y \in \mathbb{R}$. For instance, $3 \leq 5$ but $5 \not\leq 3$.

c)

Let xRy, where $x, y \in \mathbb{R}$ iff $xy > 0$.

Not reflexive: Let $x=0$, then $xx \not> 0$ so R is not reflexive

Symmetric: if $xy>0$, $yx>0$ for all $x, y \in \mathbb{R}$

Transitive: if $xy>0, yz>0$ then $xz>0 \forall x, y, z \in \mathbb{R}$

0.11

a)

Base case: If $n=1, S(n)=1=\frac{1}{2}1(1+1)=1$. Therefore, the base case holds.

Suppose $S(n)=1+2+3+....+n=\frac{1}{2}n(n+1)$. We want to prove this holds for $n+1$.

We have

$$\begin{aligned}
S(n+1) &= 1+2+...+n+(n+1) \\
&= \frac{1}{2}n(n+1)+(n+1) && \text{using our induction hypothesis} \\
&= \frac{1}{2}n(n+1)+\frac{1}{2}2(n+1) \\
&= \frac{1}{2}(n+1)(n+2)
\end{aligned}$$

Therefore, by induction, we've proved that $S(n) = \frac{1}{2}n(n+1)$.

b)

Base case: If $n = 1, C(n) = 1^3 = 1 = \frac{1}{4}1^2(1+1)^2 = 1$. Therefore, the base case holds.

Suppose $C(n) = 1^3 + 2^3 + ... + n^3 = \frac{1}{4}n^2(n+1)^2$. We want to prove this holds for $n+1$.

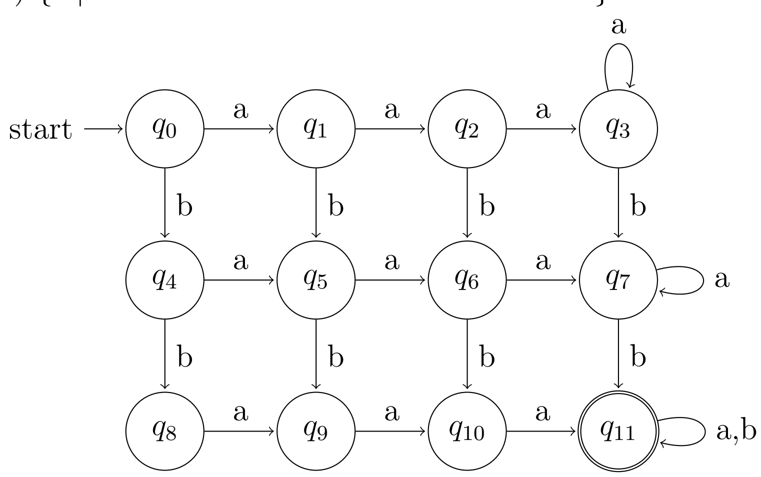
We have

$$\begin{aligned}
C(n+1) &= 1^3+2^3+....+n^3+(n+1)^3 \\
&= \frac{1}{4}n^2(n+1)^2+(n+1)^3 && \text{using our induction hypothesis} \\
&= \frac{1}{4}n^2(n+1)^2+\frac{1}{4}4(n+1)^3 \\
&= \frac{1}{4}(n+1)^2(n^2+4n+4) \\
&= \frac{1}{4}(n+1)^2(n+2)^2
\end{aligned}$$

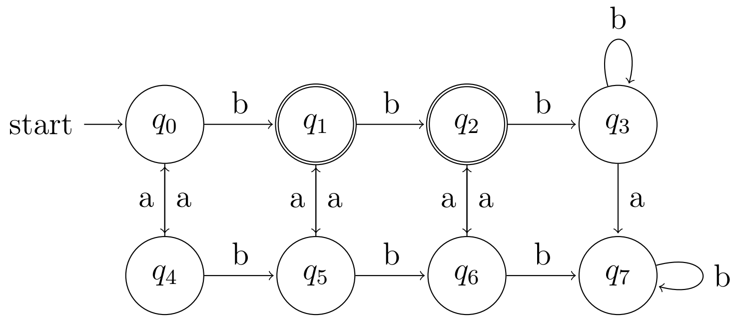
Therefore, by induction, we've proved that $C(n) = \frac{1}{4}n^2(n+1)^2$.

1.4

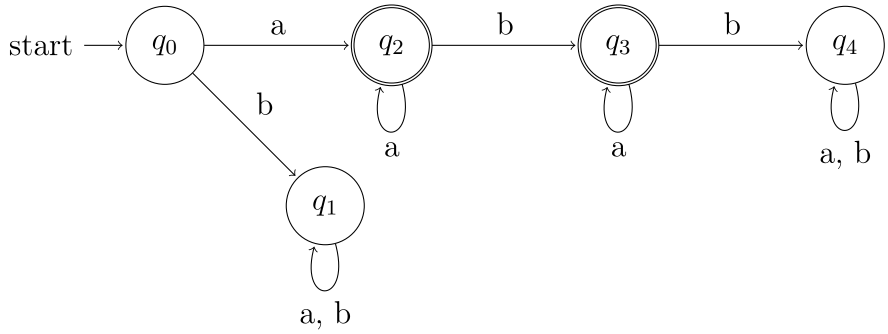
a) $\{w|w$ has at least 3 a's and at least 2 b's$\}$



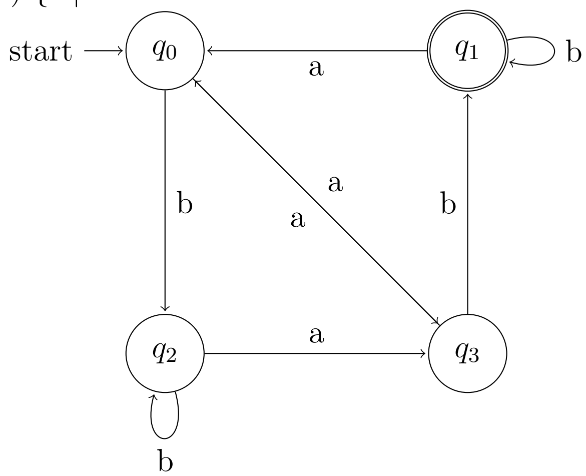
c) $\{w|w$ has an even number of a's and one or two b's$\}$



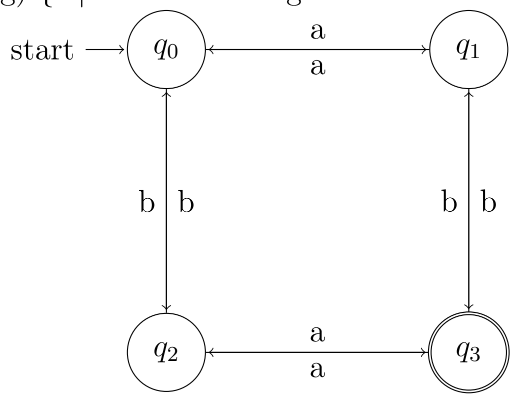
e) $\{w|w$ starts with an a and has at most one b$\}$



f) $\{w|w$ has an odd number of a's and ends with a b$\}$



g) $\{w|w$ has even length and an odd number of a's$\}$

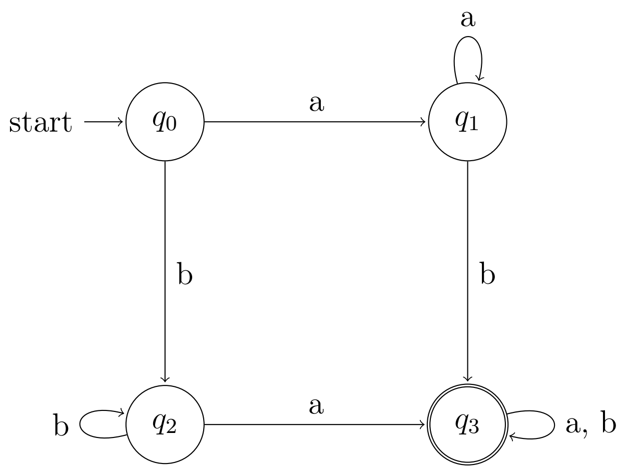


1.5

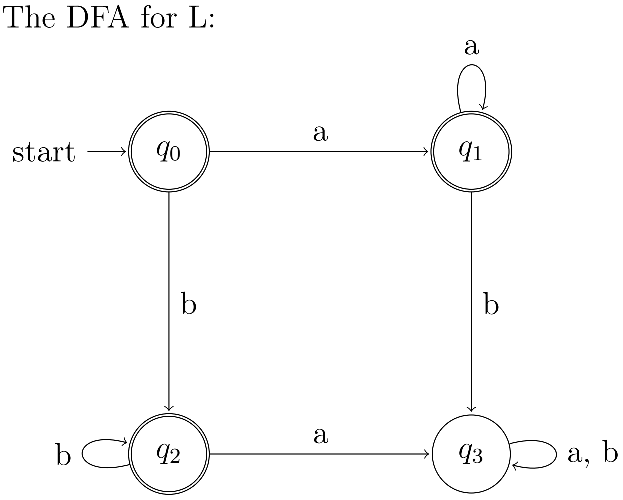
c) $L = \{w|w$ contains neither the substrings ab nor ba$\}$.

$\overline{L} = \{w|w$ contains either the substrings ab or ba$\}$.

DFA for $\overline{L}$:



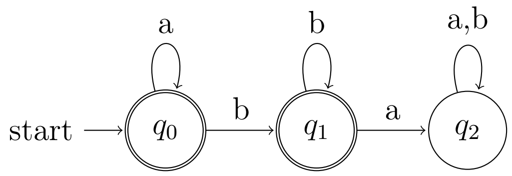
The DFA for L:



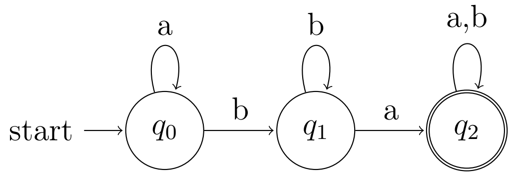
d) $L = \{w|w$ is any string not in $a^*b^*\}$.

$\overline{L} = \{w|w$ is any string in $a^*b^*\}$.

The DFA for $\overline{L}$ is



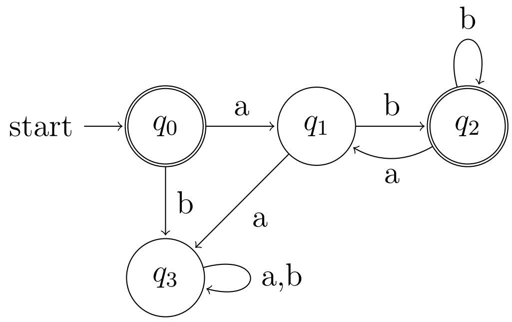
The DFA for L is



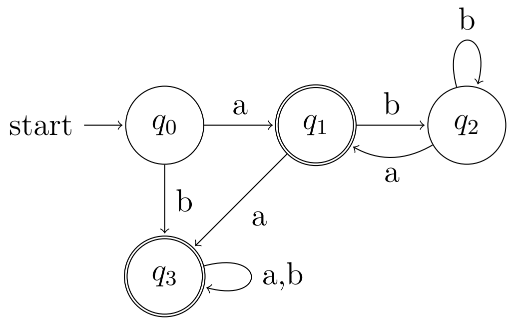
e) $L = \{w|w$ is any string not in $(ab^+)^*\}$.

$\overline{L} = \{w|w$ is any string in $(ab^+)^*\}$.

The DFA for $\overline{L}$ is



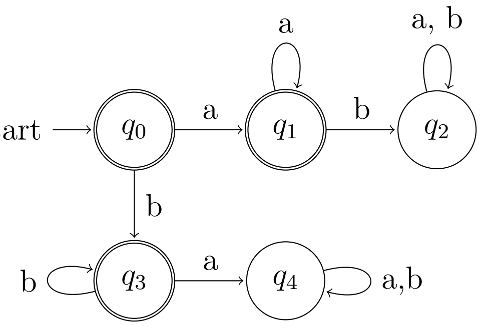
The DFA for L is



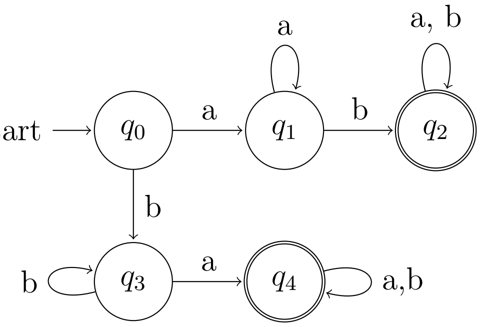
f) $L = \{w | w \text{ is any string not in } a^* \cup b^*\}$.

$\overline{L} = \{w | w \text{ is any string in } a^* \cup b^*\}$.

The DFA for $\overline{L}$ is



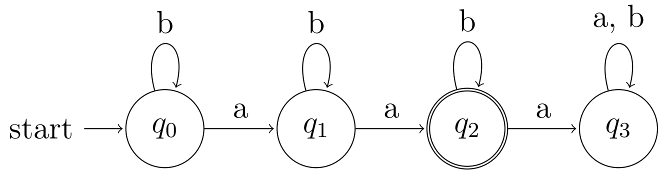
The DFA for L is



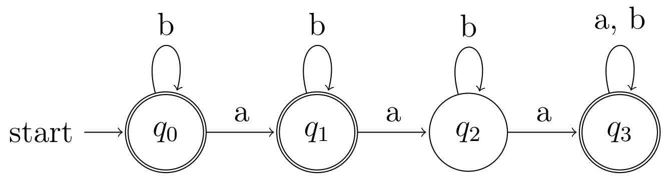
g) $L = \{w | w \text{ is any string that doesn't contain exactly two a'}\}$.

$\overline{L} = \{w | w \text{ is any string that contains exactly two a'}\}$.

DFA for $\overline{L}$:



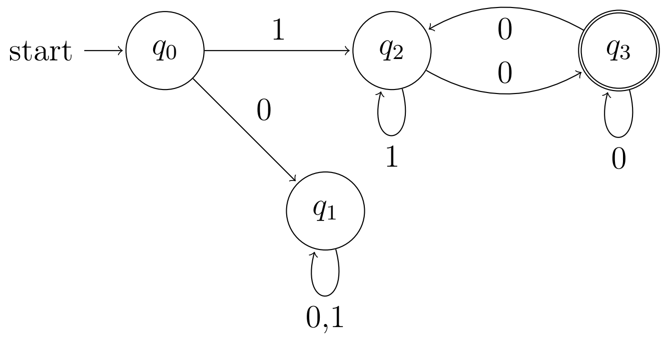
Then, DFA for L is



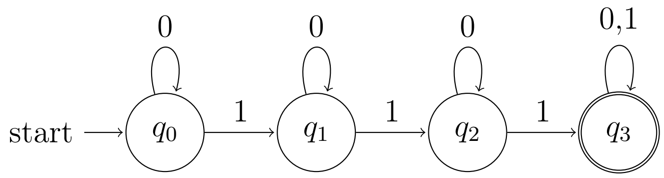
1.6

a)

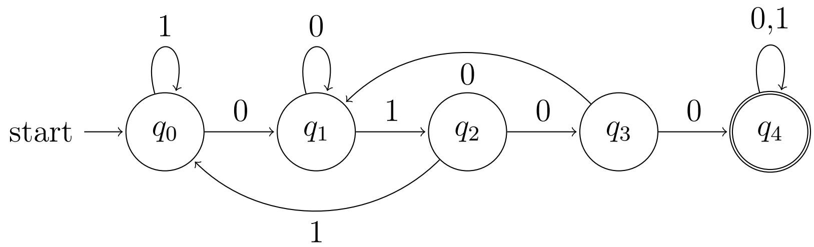
$\{w|w$ begins with a 1 and ends with a $0\}$.



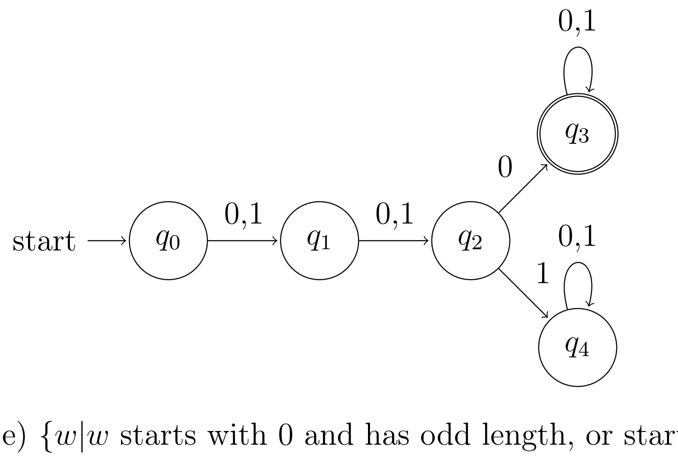
b) $\{w|w$ contains at least three 1s$\}$.



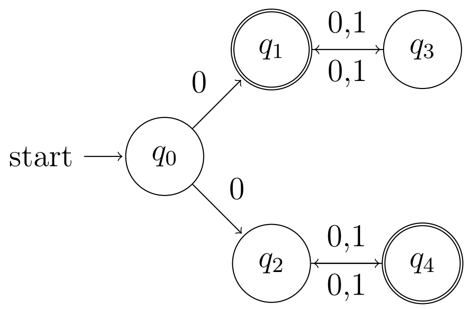
c) $\{w|w$ contains the substring $0101\}$.



d) $\{w|w$ w has length at least 3 and its third symbol is a $0\}$.



e) $\{w|w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$.


start
q_0
0
0
q_1
q_2
0,1
0,1
q_3
0,1
0,1
q_4