Pitch and Timbre of Periodic Signals (sine, square, sawtooth)

Fourier Synthesis of a Square Wave

Background

From Fourier theory we know that any periodic signal can be generated by adding up sine waves of harmonics of the fundamental frequency with appropriate amplitude and phase:

$y(t) = \sum_k A_k sin(2\pi k f_0 t + \Phi_k)$

For a square wave the amplitudes are 1/k for the odd harmonics and 0 for the even harmonics; all phase terms are zero, thus:

$y(t) = \sum_{k=1,3,5..} \frac{1}{k} sin(2\pi k f_0 t)$

Pitch and Timbre of harmonic (and pseudo-har[redacted]onic) signals

Pitch of Pure Tones

Pitch is often defined as the characteristic of a sound that makes it sound high or low, or that determines its position on a musical scale. Pitch is related to the repetition rate of the waveform of a sound. For a pure tone, this corresponds to the frequency; for a 'complex tone it usually (but not always) corresponds to the fundamental frequency. Frequency is the most important contributor to the sensation of pitch, but not the only one by any means. Other contributors to pitch include intensity, spectrum, duration, amplitude envelope, and the presence of other sounds. Various attempts have been made to establish a psychophysical pitch scale. If, after listening to a 4000-Hz tone followed by a tone of very low frequency, one is asked to tune an oscillator to a pitch halfway between, a likely choice would be around 1000 Hz. On a scale of pitch, then, 1000 Hz is judged halfway between 0 and 4000 Hz. The unit for subjective pitch is the mel the scale which is arranged so that doubling the number of mels doubles the subjective pitch. A scale from 0 to 2400 mels covers the audible range of 20 to 16,000 Hz.

A numerical scale of pitch (in mels) is not nearly so useful as a numerical scale of loudness (in sones), however. Pitch is more often related to a musical scale, where the octave is the "natural" pitch interval that is subdivided into the desired number of steps. Two major theories of pitch perception have been developed; they are usually referred to as the place (or frequency) theory and the periodicity (or time) theory. According to the place theory, the cochlea converts a vibration in time to a vibration in space (along the basilar membrane), and this in turn excites a spatial pattern of neural activity. The place theory explains some aspects of auditory perception but fails to explain others. According to the periodicity theory of pitch, the ear performs a temporal analysis of the sound wave. Presumably, the time distribution of impulses carried along the 35 auditory nerve has encoded into it the temporal structure of the sound wave.

Note on mel scale(s)

There is not really a universally accepted mel scale. Different variants are in use, both with respect to scaling with respect to absolute values:

- when you see values 0-2400 mel: it will imply that 'mel' and 'Hz' will be very similar up to 1000Hz
- when you see values 0-24: it relates to critically spaced mel channels and 1 mel corresponds to about 100Hz below 1kHz

Pitch of Complex Tones

One of the most remarkable properties of the auditory system is its ability to extract pitch from complex tones. When the complex tone consists of a number of harmonically related partials, the pitch corresponds to the "missing fundamental." This pitch is often referred to as pitch of the missing fundamental, virtual pitch, or musical pitch. When the partials are not exactly harmonics of a missing fundamental, we arrive at a "virtual pitch" by some strategy that may weigh several possibilities, and when the choice is difficult the pitch may be ambiguous. Familiar examples of such virtual pitch are the bass notes we hear from loudspeakers of very small size that radiate negligible power at low frequencies, and the subjective strike note of carillon bells, tuned church bells and orchestral chimes.

Timbre

The American National Standards Institute defines timbre as " ... that attribute of auditory sensation in terms of which a listener can judge that two sounds similarly presented and having the same loudness and pitch are dissimilar". According to this definition, timbre is the subjective correlate of all those sound properties that do not directly influence pitch or loudness. These properties include the sound's spectral power distribution, its temporal envelope (as would be shown on an oscilloscope display), rate and depth of amplitude or frequency modulation, and the degree of inharmonicity of its partials. The timbre of a sound therefore depends on many physical variables. It has been shown that from a subjective point of view the sensation of timbre has about three rather orthogonal dimensions. These can be represented by the verbal ranges dullsharp, compact-scattered and colorful-colorless. These subjective dimensions are loosely related to the physical quantities of high-frequency energy in the attack, synchronicity in high-harmonic transients, and spectral power distribution. The concept of timbre plays a very important role in the orchestration of traditional music and in the composition of computer music. There is, however, no [redacted] This poses considerable problems in communicating or teaching the skills of orchestration and computer score writing to student-composers. In the following demonstrations one can hear how spectral make-up, temporal envelope and degree of spectral inharmonicity all have a very specific influence on the perceived timbre of sounds from musical instruments.

(For a demo with musical instruments, see **Demo 28 in AuditoryDemonstrations**)

The Analytical Perspective

Periodic signals can be expressed as a harmonic complex of sine waves.

$y = A \sum_k \alpha_k sin(2\pi k f_0 t + \Phi)$

The same formula can be applied to pseudo harmonic signals, that have either time-varying amplitude (e.g. Gaussian pulse) or time-varying frequency (e.g. chirps).

Periodicity and Rhythm

Background

For a large group of sounds and speech in particular periodicity induces a *tonal* percept, ralso called *pitch*. The pitch percept, however, fades away at the lower end of our hearing spectrum, i.e. around 20Hz.
So we may wonder how do we percieved a periodicity of 10Hz or even 1Hz ?
We definitely do not perceive these slow repititions as a low frequency pitch.
But on the other hand the regular recurrences of patterns in time at such low frequencies give us a sensation of rhythm, like the clapping of hands, the playing of musical notes on an instrument, etc.

With the same demo as before, we now focus on the very low (fundamental) frequency range.
Focus on the 1Hz - 10Hz range, but you may play the same demo with a base frequency above 25Hz as well to hear the contrast.