

# Geometric Morphometric Analysis of Projectile Points from the Southwest United States

Robert J. Bischoff<sup>1</sup>

<sup>1</sup>School of Human Evolution and Social Change, Arizona State University, Tempe, AZ

## Abstract

Traditional analyses of projectile points often use visual identification, the presence or absence of discrete characteristics, or linear measurements and angles to classify points into distinct types. Geometric morphometrics provides additional tools for analyzing, visualizing, and comparing projectile point morphology utilizing the whole or parts of the form in either two or three dimensions. This study is an analysis of the effectiveness of geometric morphometric methods for identifying technological similarity in 2D projectile point outlines for previously classified late prehistoric projectile points found in the U.S. Southwest and unclassified projectile points from Tonto Basin, Arizona. Various methods from geometric morphometrics were compared to determine which method best reproduced the original classification scheme. Elliptical Fourier analysis was compared with various configurations of semilandmark and landmark analyses using generalized Procrustes analysis. These methods were applied to the complete projectile point form, and the landmark analysis was also applied to half of the lower quadrant of the projectile point—essentially one corner of the projectile point. The landmark analysis applied to the corner of the projectile point provided the best results. This method was then applied to the Tonto Basin points. Hierarchical clustering was used on the Tonto Basin projectile point morphometric data to explore the variation in shapes between sites. To demonstrate that geometric morphometric methods can be used without relying on typologies, a network analysis of the morphometric distances was also conducted. This network graph produced distinct clusters of technological similarity in projectile point outlines, while also showing the continuous variation between points. These results demonstrate the effectiveness of geometric morphometrics for the 2D analysis of late prehistoric arrow points in the U.S. Southwest.

**Keywords:** American Southwest, Hohokam, Arizona, projectile points, lithics, computational archaeology, geometric morphometrics

23 **Introduction**

24 Geometric morphometrics (GM) is a quantitative approach to studying shape in two or three dimensions  
25 that has recently been adopted in archaeology (see MacLeod, 2017; Okumura and Araujo, 2019; Shott and  
26 Trail, 2010, for overviews). It has numerous advantages over traditional lithic analyses, particularly because  
27 it can overcome the reliance on linear dimensions (Shott and Trail, 2010, pp. 196–197). Lithic artifacts can  
28 be assigned to typologies or directly compared without the use of a typology, as will be demonstrated in this  
29 paper. There are several approaches within GM that provide similar results through different methods. One  
30 of the more traditional approaches is to place landmarks at homologous locations around the object. Land-  
31 marks can be augmented with semilandmarks, which are points placed relative to another using a consistent  
32 rule—usually equidistant spacing between two points (Okumura and Araujo, 2019, pp. 2–4). Another common  
33 approach is to use elliptical Fourier analysis to compare the outlines of objects. Each method has strengths  
34 and weaknesses. A major purpose of this study is to evaluate the effectiveness of these methods for analyz-  
35 ing projectile points in the U.S. Southwest during the late prehistoric period (specifically during the Hohokam  
36 Classic Period—AD 1100–1500).

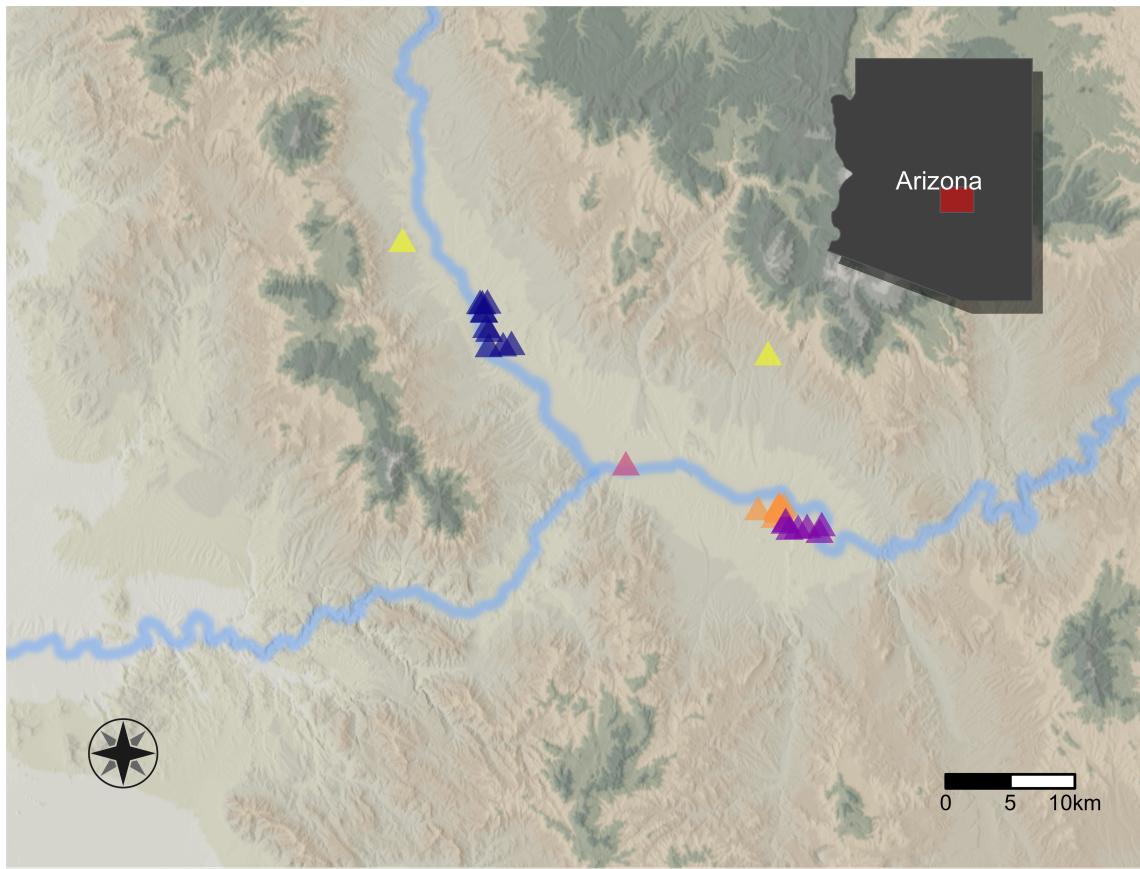
37 Once the method of analyzing the projectile points has been determined, the next step is to determine  
38 how to compare projectile points using the results of the analysis. One approach would be to use an existing  
39 regional typology and to assign projectile points to the closest match (e.g., Kocer and Ferguson, 2017). Another  
40 approach, would be to use cluster analysis to assign projectile points to newly created types [e.g., Petrik2018-  
41 pd; Matzig2021-id]. The final approach would be to ignore typologies and compare the morphometric distance  
42 for each projectile point directly. This is the second primary purpose of this study—to evaluate the effectiveness  
43 of these approaches for use in analyzing projectile points from the Southwest United States.

44 Regional analyses are fundamental parts of archaeology, but there are many challenges to overcome. One  
45 of these challenges is harmonizing the different categorization schemes (i.e., ontologies) used throughout  
46 the region. Another of these challenges, is determining whether the current categories are useful. The U.S.  
47 Southwest has a long history of regional ceramic typologies (e.g., Colton, 1956; W Gladwin and HS Gladwin,  
48 1930; Hargrave, 1932; Kidder, 1915; Martin and Willis, 1940), but there are still disagreements, challenges,  
49 and competing definitions (Duff, 1996). Regional analyses in the Southwest, based in large part on pottery,  
50 have produced many useful insights (e.g., Bernardini, 2005; Clark et al., 2019); Hegmon et al. (2016); Mills  
51 et al. (2013); Peeples (2018)]. However, one type of material culture that has received little attention—in the  
52 Southwest at least—is lithics (i.e., chipped stone). Projectile points are commonly discussed during the archaic  
53 period of the Southwest, and they are common topics in many other areas of the North American continent  
54 and world where they are found, but they are rarely discussed after the appearance of pottery.

55 Despite the over-emphasis on pottery in the Southwest, there are some excellent resources on projectile  
56 point typologies (e.g., Hoffman, 1997; Justice, 2002; Loendorf and Rice, 2004); Sliva (2006)]. However, ad  
57 hoc approaches are common, and these cannot easily be extrapolated beyond specific projects. Even using  
58 existing resources can make comparisons difficult. How does Tagg's (1994, p.111) Type 23 compare to Sliva's  
59 (Sliva, 2006, p. 35) Cohonina Side-notched? There is an answer, but often it is easier to come up with a new  
60 typology schema than to try to harmonize existing work.

61 Another challenge that is not unique to projectile points is that interpretations may differ between analysts.  
62 Exactly when does a base begin curving enough to be called basal notched? Even the difference between a  
63 side-notched and a corner-notched point can, at times, be ambiguous. Not to mention the frustrating situation  
64 where a point appears to have one corner-notch and one side-notch. How should one place this point into  
65 an existing typology? These are questions that can be handled in different ways that differ from analyst to  
66 analyst. Idiosyncrasies and biases are impossible to be rid of entirely, but using approaches such as those  
67 described in this paper can reduce them and increase the reproducibility of the process.

68 By necessity, this paper covers a number of topics. The geographic area is the U.S. Southwest, but the meth-  
69 ods and analysis are applicable to any area. The primary purpose was to explore geometric morphometric  
70 methods using previously typed specimens from the Southwest and untyped specimens from the Tonto Basin.



**Figure 1.** Location of Tonto Basin in the state of Arizona, United States, along with archaeological sites discussed in this paper grouped by site cluster.

71 Another purpose was to analyze the results with and without using typologies. The results demonstrate that,  
 72 in this particular case, a combined landmark and semilandmark approach is most effective and that useful  
 73 analyses can be conducted with and without the use of typologies.

## 74 **Background**

75 In order to test the effectiveness of geometric morphometric methods, I needed a dataset of well-typed pro-  
 76 jective points that could be used as a validation set. I chose to use the typology published by Noel Justice (2002)  
 77 for the simple reason that it is easily accessible and contains numerous illustrations. These illustrations were  
 78 used as type specimens to compare projectile points from Tonto Basin in central Arizona (Figure 1). These  
 79 points were excavated in a series of large cultural resource management projects necessitated by work on  
 80 the Roosevelt Dam. The largest project—the Roosevelt Platform Mound Study—included 129 sites. Most of the  
 81 sites date between AD 1275 and 1325 with occupation continuing until around AD 1450 (Rice, 1998). In the  
 82 original analysis, projectile points were classified according to small and large points and then subdivided  
 83 based on morphological characteristics (Rice, 1994, p.727). The typology used is an excellent demonstration  
 84 of the difficulty in conducting projectile point studies in this area, as the typology is idiosyncratic to this specific  
 85 project, and cannot be easily compared with other datasets. This is not a criticism of the analyst's choice to  
 86 create a new typology, as no existing typology met the needs of the researchers.

87 This is an exploratory analysis designed to minimize the amount of time spent collecting data and to be as  
 88 reproducible as possible. There are a number of research steps that are often not addressed in publications.

89 This missing documentation can make reproducing results challenging. I will describe the rational for relevant  
90 decisions, but the script used to generate the analysis will be included in an RMarkdown document in the  
91 supplemental material (see statement at end of manuscript).

92 One of the key elements of this study is reproducibility, which necessitates automation. Projectile point  
93 analysis often includes assigning a point to a type based on linear metrics—sometimes angular measurements  
94 as well, and the presence or absence of various features (e.g., concave base, serrated blades, corner-notches).  
95 But often, the analyst is left to visually compare the point to various type specimens to identify the closest  
96 match. This method is harder to reproduce and subject to greater human error. Yet, algorithms are only part  
97 of the answer, and human judgment and context are still critical to any analysis. The key is to minimize the  
98 possibilities for error and maximize the opportunities for reproducibility, which I have tried to do here. Thus,  
99 one of the key questions of this research is to determine what input should be left to the analyst and what  
100 can be left to automated or standardized procedures.

## 101 **Data Collection**

102 This study has two sources of data: illustrations of projectile points published by Justice (2002) and images  
103 of projectile points from collections held at Arizona State University. The datasets include 74 illustrations  
104 from Justice's publication and 90 projectile points from Tonto Basin. The 74 illustrations do not include all of  
105 Justice's illustrations or types, as many could not be included because there were so few complete, illustrated  
106 examples.

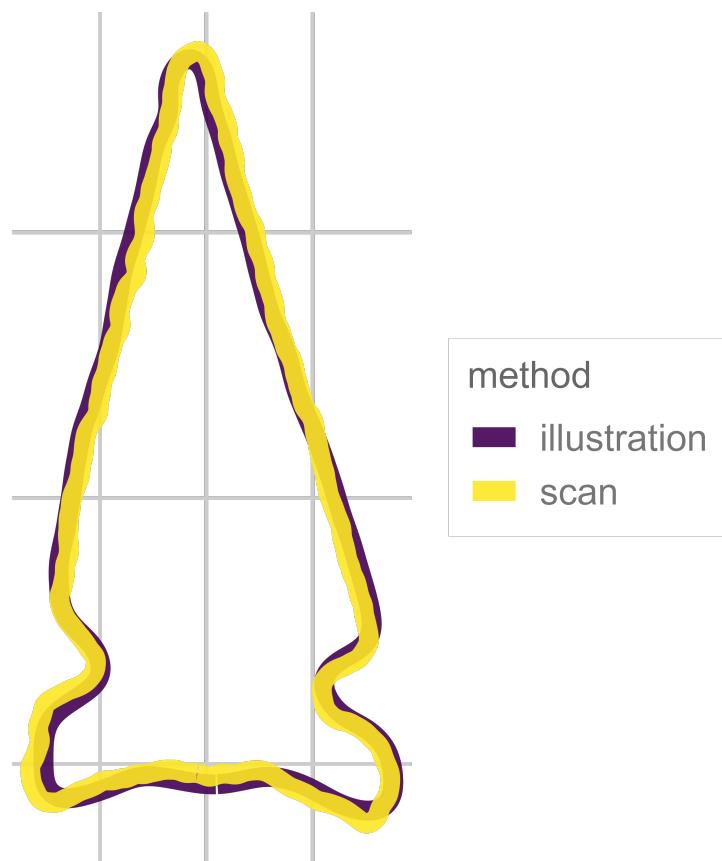
107 It is worthwhile to question how an illustration compares to an image of a physical projectile point obtained  
108 from a flatbed scanner. Fortunately, illustrations have been published for some of the projectile points in this  
109 study. Figure 2 is a comparison of outlines created from an illustration and scan of the same projectile point.  
110 There are subtle differences between the two mediums—the base is slightly more rounded in places than the  
111 scan. These differences are detectable in a morphometrics analysis, however, the differences are minor as  
112 seen in figure 3. These minor differences would not affect the results of the analyses presented here or most  
113 GM analyses. The quality of projectile point illustration can vary, but from this brief comparison there should  
114 be no hesitation using illustrations for 2D morphometric analysis.

115 Justice's projectile point illustrations were scanned, and the illustrations were converted into individual,  
116 solid black outlines and saved as jpeg files using common image-editing software. The open source statistical  
117 software R was used for all analyses (R Core Team, 2022). The Momocs package (Bonhomme et al., 2014)  
118 has an import function to convert jpeg files into outlines. This is a major advantage over manual outlining  
119 processes used in popular GM software, such as tpsDig (James Rohlf, 2015). These outlines form the basis of the geometric  
120 morphometric analyses conducted here with the exception of the landmark analyses.  
121 Landmarking was performed using the tpsDig software. The Tonto Basin projectile point images were created  
122 using a flatbed scanner at 1200 DPI <sup>1</sup>. The images were converted to outlines using the same process as the  
123 projectile point illustrations.

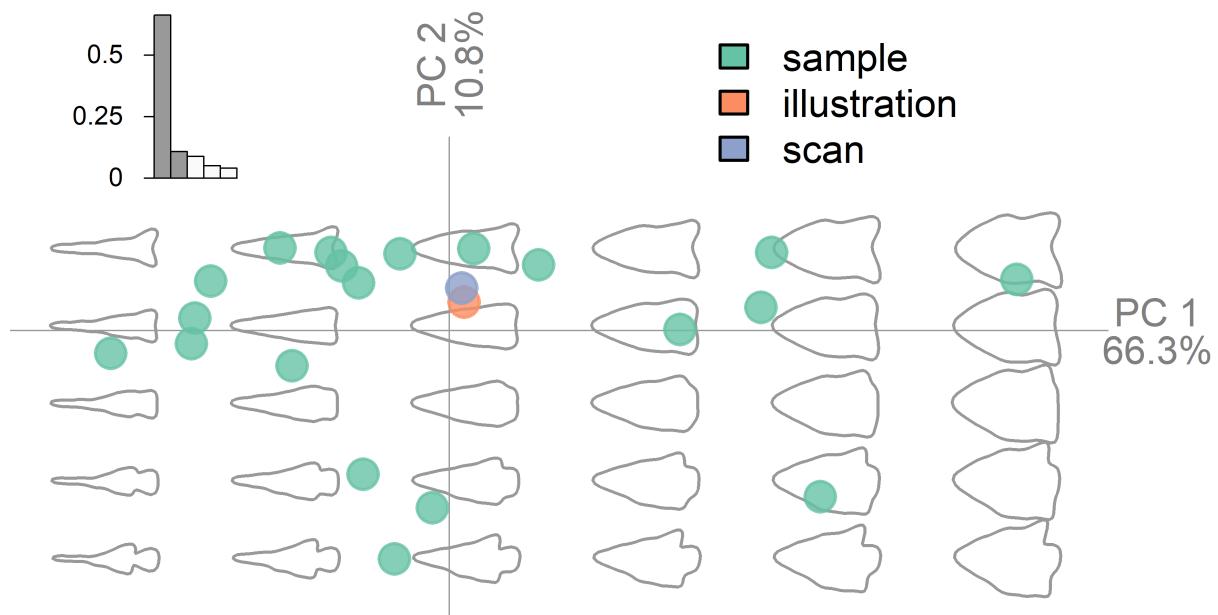
## 124 **Projectile Points of the Southwest**

125 There are a few regional typologies for projectile points in the Southwest: primary examples are Hoffman  
126 (1997), Justice (2002), Loendorf and Rice (2004), and Sliva (2006). Altogether, these four typologies include  
127 129 projectile point types, although many overlap. In some cases, the authors identify correlates of the types  
128 from other typologies. This allows for some harmonization of the different typologies. Many types date to  
129 the Archaic period, and thus predate the primary period I am interested in (AD 1100-1500). Not all of the  
130 projectile points were ascribed dates by the authors. Justice lists 23 projectile points that overlap with the  
131 AD 1100-1500 period (the maximal dates for the Hohokam Classic period). Projectile points have restricted

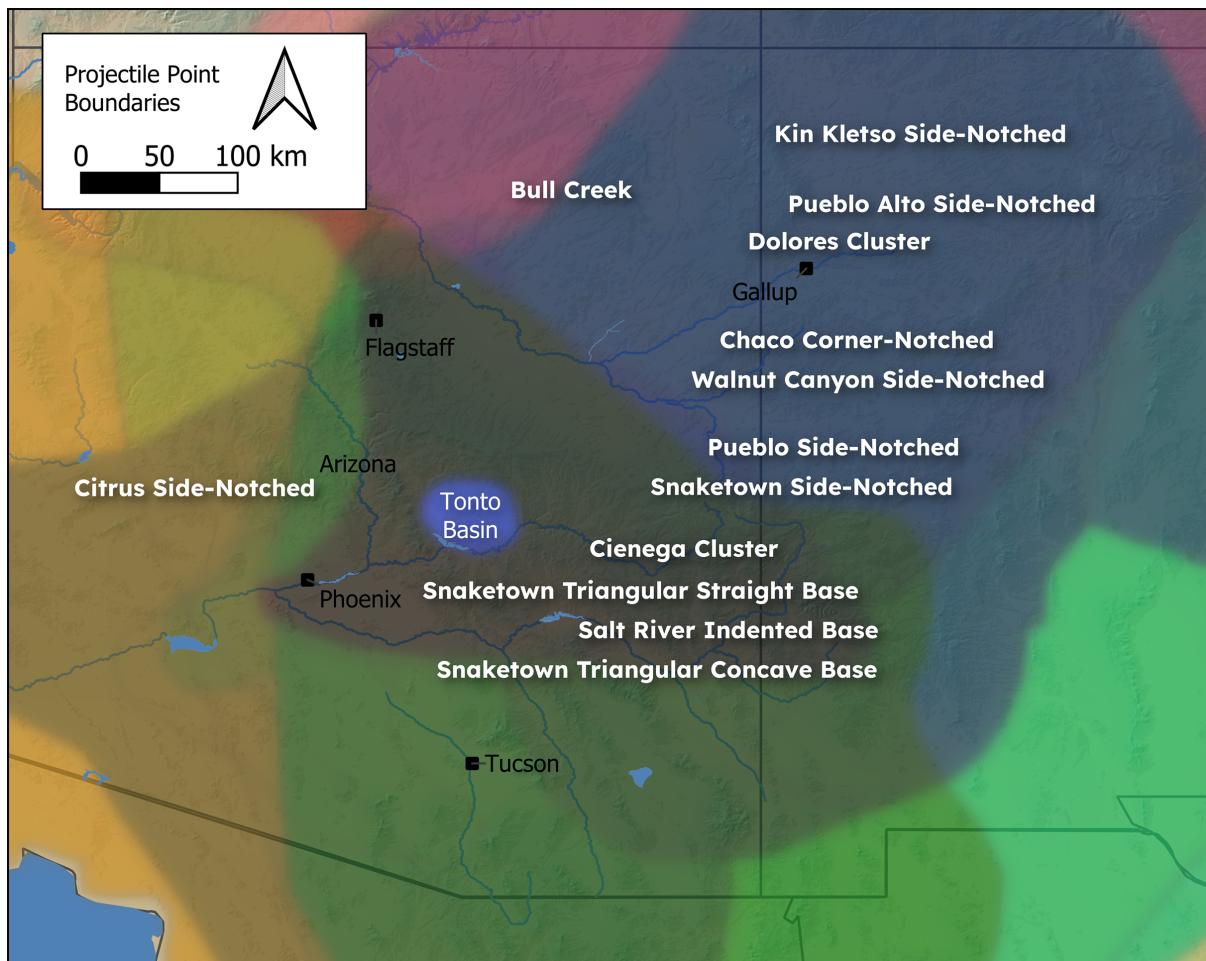
<sup>1</sup> Many of the images were obtained by the author but some were generously contributed by Joshua Watts—see the results of his study here: (Watts, 2013).



**Figure 2.** Comparison of projectile point outlines for an illustration and a scan of the same projectile point  
(Oliver and Simon 1997: Figure 9.3; Specimen 33598)



**Figure 3.** Principal component plot comparing the morphometric differences between a sample of 20 random projectile points and the illustration and scan of the same projectile point. The morphospace is also projected. The morphospace computes the projectile point outlines shown in the figure, which represents how shapes vary along each axis.



**Figure 4.** Location of selected projectile point boundaries defined by Justice (2002) as digitized by Buchanan and colleagues (2019). Darker colors represent greater overlap in the number of projectile point boundaries.

132 geographical boundaries, although these boundaries correspond to much greater areas than ceramic types  
 133 typically do (Buchanan, Hamilton, et al., 2019). Figure 4 shows that several projectile point boundaries defined  
 134 by Justice overlap with the Tonto Basin.

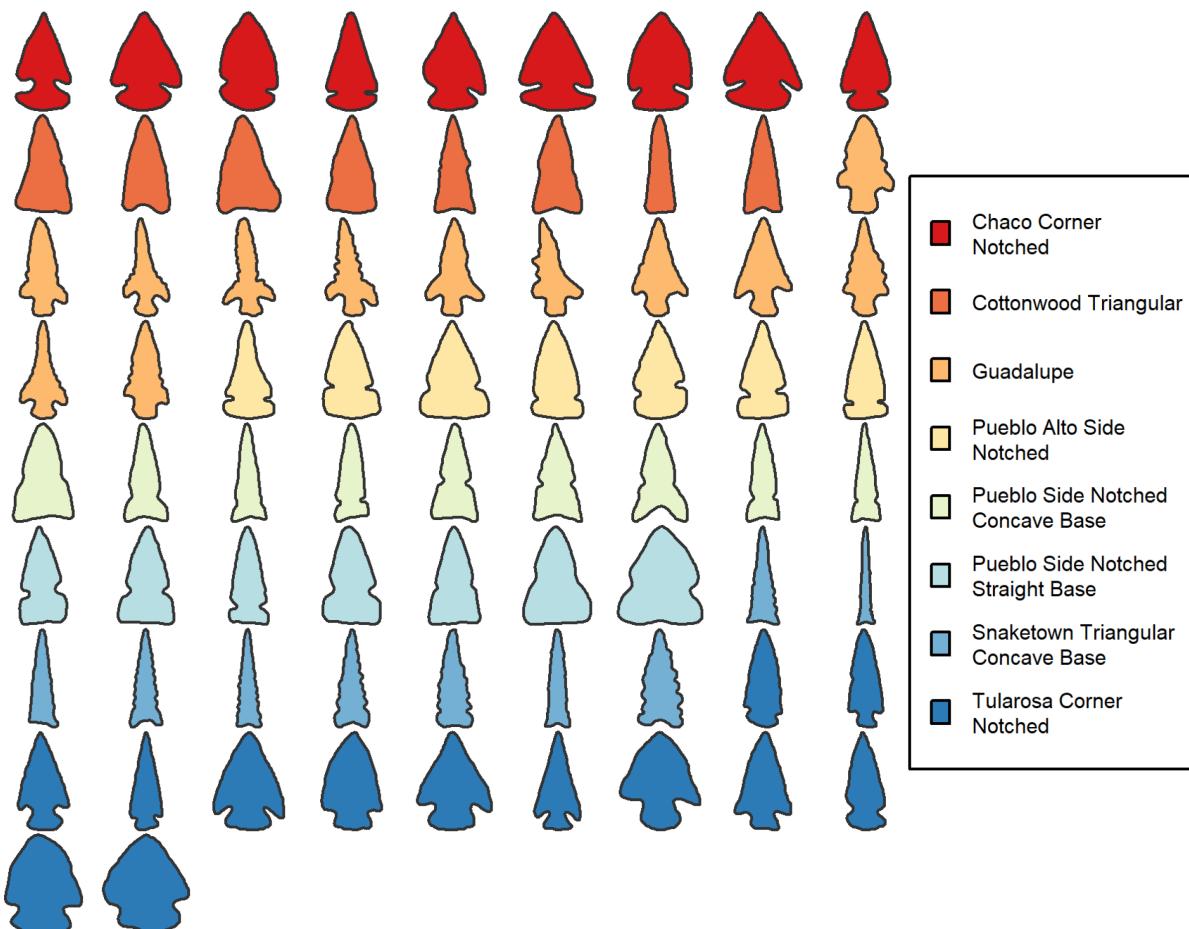
135 For this study, I digitized 74 projectile point images from Justice's publication representing 8 projectile point  
 136 types (table 1). Justice placed each projectile point type into a cluster of related points. Figure 5 shows the  
 137 projectile point outlines by type. The included projectile point types include some Archaic points and types not  
 138 expected to overlap with the Tonto Basin projectile points. Small numbers of archaic points are often found  
 139 at later sites. These are likely the result of collecting activities and not indicative of the continued use of these  
 140 points (see Justice (2002) for numerous examples). These curated archaic types form useful comparisons, as  
 141 they should not match non-archaic projectile points. One limitation of this study is that projectile points must  
 142 be complete, or nearly complete (minor damage to the tip or another part of the point that was judged to not  
 143 significantly impact the shape of the point was ignored). Thus, not all of the illustrations included in Justice's  
 144 book could be included in the outline analyses.

#### 145 **Tonto Basin Projectile Points**

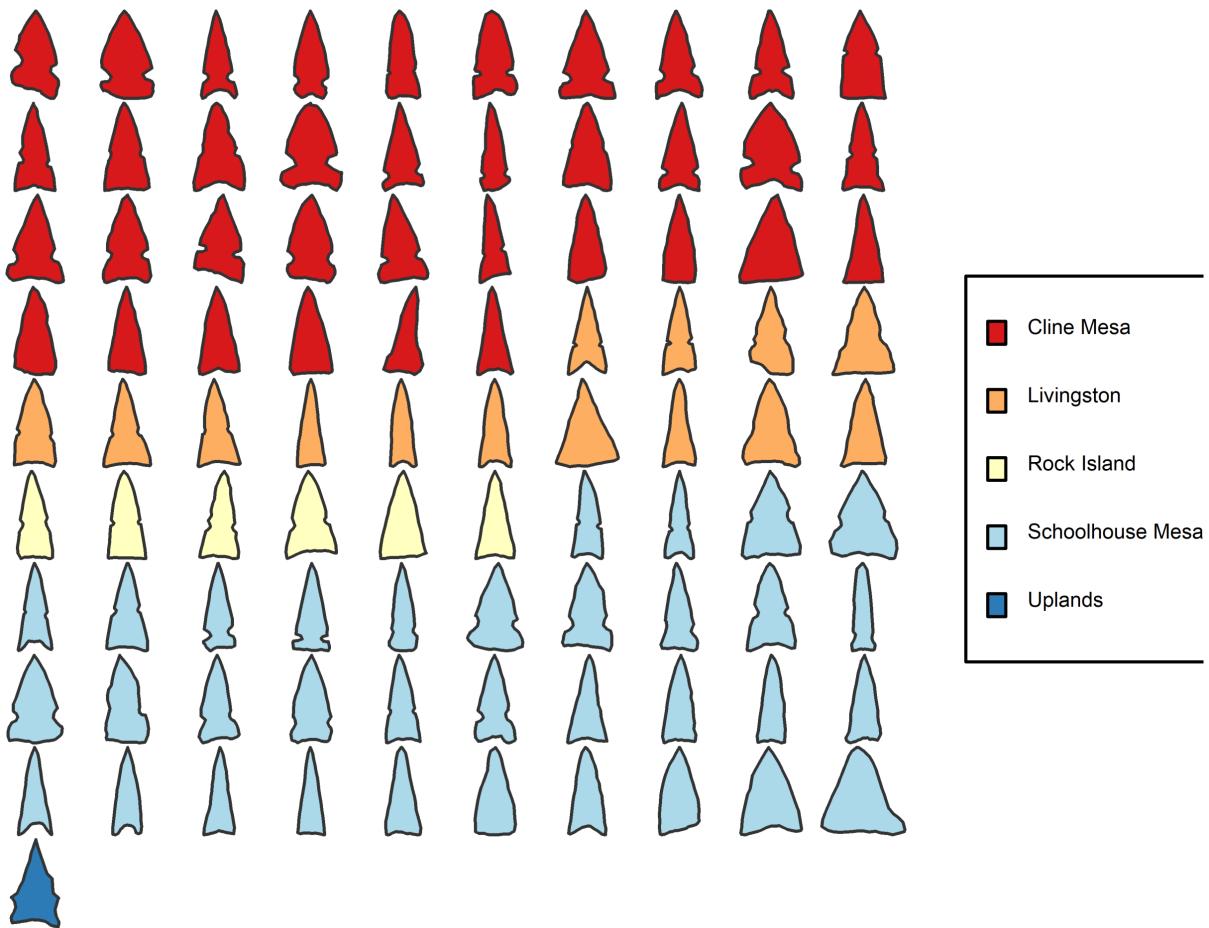
146 The sample of Tonto Basin points used in this study come from the Roosevelt Platform Mound Study. They  
 147 come from 18 different sites that were grouped into five clusters in the original reports (see Rice, 1998, for an  
 148 overview). Figure 1 shows the sites used in this study grouped by site cluster. The majority of these sites were  
 149 occupied during the Roosevelt phase (AD 1275-1325) and early portion of the Gila phase (AD 1325-1450). The

**Table 1.** Cluster Names, Types, and Number of Samples

Cluster	Type	total
Chaco	Chaco Corner Notched	9
Chaco	Pueblo Alto Side Notched	7
Cienega	Tularosa Corner Notched	13
Livermore	Guadalupe	12
Pueblo Side Notched	Pueblo Side Notched Concave Base	9
Pueblo Side Notched	Pueblo Side Notched Straight Base	7
Snaketown	Snaketown Triangular Concave Base	9
Western Triangular	Cottonwood Triangular	8



**Figure 5.** Outlines of projectile point illustrations taken from Justice (2002). Note that the projectile points are not scaled.



**Figure 6.** Outlines of projectile points from sites in the Tonto Basin. Note that the projectile points are not scaled.

150 sites consist primarily of compounds, room blocks, and platform mounds.

151 The projectile points exhibit a variety of forms (see figure 6). Rice (1994) classified Tonto Basin points into  
 152 small and large complexes (likely equivalent to dart and arrow points), and further classified small points into  
 153 the longer Salado series and the shorter Tonto series. These series were further subdivided using a custom  
 154 classification scheme based on blade, tang, and base shape, as well as notch style. This is a logical way to  
 155 classify the points, but it does not easily lend itself to regional comparisons, as other points were not classified  
 156 in the same way. Nearly all of the points in the original sample consisted of side-notched or triangular points.  
 157 Because the results of the analysis discussed below indicated that analyzing the projectile points by shape  
 158 was the most logical choice, only the triangular and side-notched points were used in this study.

## 159 Methods

### 160 Geometric Morphometrics

161 I used two GM methods in this analysis: elliptical Fourier analysis (EFA) and full generalized Procrustes align-  
 162 ment (GPA). Something to keep in mind is that GM methods analyze the form of the object separated from size,  
 163 position, and orientation. Real-world measurements such as length and width are not explicitly included in  
 164 these methods, although relative dimensions, such as length to width ratio are captured in the overall form of  
 165 the object. Measurements such as length and weight can be included in various analyses but are not included  
 166 here. The purpose of this study is to determine whether GM methods alone are sufficient to discriminate

167 between types of projectile points and how they can best be used in the context of the U.S. Southwest.  
168 EFA was developed by Kuhl and Giardina (1982) as a quantitative means for describing a closed outline.  
169 There are a handful of papers that use EFA for lithic studies in archaeology [e.g., Cardillo (2010); Fox (2015);  
170 Gingerich et al. (2014); Hoggard et al. (2019); Iovita (2011); Iovita and McPherron (2011); Matzig et al. (2021)].  
171 The mathematics behind the method are complex to describe, which is one reason the method has not been  
172 adopted as quickly as it should be (see Caple et al., 2017). Caple and colleagues (Caple et al., 2017) provide  
173 an excellent description of EFA for non-mathematicians, and the reader is referred to their treatise for more  
174 details. For my purposes, it is enough to know that EFA analysis requires a closed outline and a number of  
175 harmonics. The harmonics can be thought of as ellipses in a time series used to describe the shape of the  
176 object. EFA creates a series of coefficients—four for each harmonic (A,B,C,D)—which can be used in multivariate  
177 statistics. Most commonly, principal components analysis (PCA) is used to transform the EFA values. The  
178 PCA results can then be used in distance-based methods such as clustering or even network analysis. Three  
179 harmonics can be used to create an oval shape, and 12 harmonics is sufficient for a complex projectile point  
180 outline. The number of harmonics necessary to capture the outline to a certain accuracy can be computed by  
181 first calculating the harmonic power using the formula:

$$HarmonicPower_n = \frac{A_n^2 + B_n^2 + C_n^2 + D_n^2}{2}$$

182 where n is the number of harmonics and A, B, C, and D are the coefficients generated from the EFA. The  
183 harmonic power is first calculated for a maximum number of harmonics and then the desired proportion  
184 (e.g., 99%) of the harmonic power can be used as a baseline to determine which number of harmonics has at  
185 least that much harmonic power.

186 Generalized Procrustes Analysis, or GPA, is primarily a way to align, scale, and rotate landmarks (Gower,  
187 1975). Instead of outlines like EFA, GPA requires landmarks located on homologous locations for each object  
188 (Rohlf and Slice, 1990). As an alternative to landmarks, semilandmarks can be placed at equidistant locations  
189 around the object. Landmarks and semilandmark approaches can be combined. There is substantial discussion  
190 on the validity of certain types of landmarks and the use of semilandmarks as landmarks (e.g., De Groot,  
191 2011; MacLeod, 2017; Okumura and Araujo, 2019; Shott and Trail, 2010). One disadvantage of traditional  
192 landmark analysis, compared to EFA, is that the analyst must be more involved in the selection of the number  
193 and placement of landmarks. Once the landmarks, or semilandmarks are placed on the objects, they are iter-  
194 atively modified to achieve the best possible alignment between shapes without changing the relative positions  
195 between landmarks. This modification is done using the GPA procedure. As with EFA, the next step is usually  
196 to perform a PCA analysis. Landmark analysis using GPA is more common, so far, in archaeological analysis of  
197 stone tools than EFA (e.g., Archer et al., 2018; Bischoff and Allison, 2020; Buchanan, Eren, et al., 2015; Charlin  
198 and González-José, 2018; Fisher, 2018; Gingerich et al., 2014; Herzlinger et al., 2017; Lycett et al., 2010; Riede  
199 et al., 2019; Selden et al., 2020; Shott and Trail, 2010; Smith et al., 2015; Thulman, 2012).

200 The project was initially designed to compare the EFA results with a semilandmark analysis using the full  
201 outline of the projectile point. The projectile point outline consists of a series of coordinates describing the  
202 outline. Semilandmarks can be obtained by sampling the outlines to create an equal number of coordinates,  
203 which are then treated as semilandmarks. Both approaches yielded similar results, but neither achieved sat-  
204 isfactory accuracy. The research design was then modified to include a more traditional landmark analysis to  
205 determine whether it would improve upon the initial design.

206 There are some disadvantages to using landmarks, which is why the EFA/semlandmark approach was  
207 initially favored. The principal disadvantages to using landmarks are reproducibility and accuracy. Landmarks  
208 are more subjective in many ways than the semilandmarks or EFA (see Shott and Trail, 2010, p. 205). The  
209 analyst must decide how many points to place, what topological points should be used as landmarks, and  
210 how many landmarks should be used. The placement of landmarks can vary between analysts and can be  
211 affected by the instruments or software used to collect or create the landmarks. Another major concern is  
212 the loss of detail from not considering the entire outline. Serrated projectile points and points with more than

213 one notch (this occurs more often than one might expect in Southwestern projectile points) are difficult to  
214 capture without including many landmarks, which are only applicable in a minority of situations. Secondary  
215 to these points, but still a concern, is that placing landmarks can be a more time-consuming process, as it is  
216 not as easily subject to automation as semilandmarks or EFA (although see Palaniswamy et al., 2010, for one  
217 of several examples of automation).

218 Despite the disadvantages, landmarks are widely used for good reasons. I see two main advantages to  
219 landmark analysis in the context of projectile point analysis. The first is that the analyst can use their prior  
220 experience to determine what topological locations on the projectile point are most useful for discriminating  
221 between types. Decades of research on projectile points has refined many typologies into useful tools, despite  
222 their limitations. This knowledge can be applied to choosing appropriate landmarks. The second advantage is  
223 that outline analysis requires complete projectile points, whereas landmark analysis can use damaged points.  
224 If chunks of the projectile are missing then the outline is not usable. Possibly, the missing portion could be es-  
225 timated and filled in, but that process is more error prone than estimating missing landmarks. Landmarks can  
226 be placed on reconstructed projectile point illustrations or missing landmarks can be estimated mathemati-  
227 cally (Gunz et al., 2009). Most projectile points suffer from some type of damage and some of the projectile  
228 points I classified as “complete” suffer from minor damage to the tip of the point or elsewhere. The use of  
229 damaged points can greatly increase the available sample size for studies, which is often a major limitation in  
230 projectile point studies.

231 Landmark configurations can vary significantly, depending on what the analysis is designed to measure  
232 and on the point type. Most projectile point landmark analyses incorporate both landmarks and semiland-  
233 marks. The difference being that landmarks are placed on homologous points (notches, corners, etc.) and  
234 semilandmarks are placed equidistantly along a curve or line. Most of the area of a projectile point is usually  
235 in the blade—the portion above the notches. The base of the point, the portion below the notches, is also the  
236 hafting element. For projectile point typologies, the base of the point usually contains the most important el-  
237 ements for determining the type—notching style and basal shape being the two major elements. Thus, if most  
238 of the landmarks or semilandmarks are on the blade margins then the base of the point is not getting as much  
239 coverage. It is more than just tradition that the base gets the most attention. Hafting a point is an important  
240 technological choice, more so than how long the point is. Furthermore, projectile points can be resharpened.  
241 Resharpening the blade margins can modify the shape of the blade and change its appearance. While it is  
242 possible to modify the base of the point and even convert a side-notched point into a corner-notched point  
243 and vice-versa, it is unlikely that this happened regularly with the small arrowpoints used in this study (Loen-  
244 dorf, Rogers, et al., 2019). Because it is necessary to incorporate different landmarking procedures for each  
245 projectile point shape, I separated the projectile points into three classes: side-notched, corner-notched, and  
246 triangular. For this study, I combined stemmed points into the corner-notched category.

247 Because of the vagaries of placing landmarks, I used two configurations in this study. In the first configura-  
248 tion, the full outline was used. In the second configuration, I used what I term the “corner” of the projectile  
249 point. Figure 7 shows the first landmark configurations. For simplicity, I will refer to both landmarks and semi-  
250 landmarks in the following discussion as landmarks. The landmark configuration was designed to place fewer  
251 landmarks along the blade margins and more landmarks along the notches and the base. Separate curves  
252 were placed between the tip of the point and the notches and the notches and the base (or the tip and the  
253 base for triangular projectile points), and landmarks were placed at equidistant locations along the curves.  
254 The second configuration is much sparser (figure 8). The landmarks were placed only on the right side of  
255 the point. For the side-notched and corner-notched projectile points, landmarking started from the top of the  
256 notch, moved to the middle of the notch and then the bottom portion of the notch. For corner-notched points,  
257 this last landmark marked the right corner of the point, but for side-notched points an additional landmark  
258 was needed to mark the base of the point. The final landmark was placed at the center of the basal margin.  
259 Triangular points differed by placing the first landmark in the center of the blade margin. The first approach  
260 contains between 30 and 42 landmarks that cover the entire point outline, whereas the second ranges from

**Table 2.** Linear Discriminant Analysis Results for Projectile Point Shapes

Shape	EFA	semiLdk	Mean
Corner-notched	0.88	0.91	0.90
Side-notched	0.65	0.74	0.70
Triangular	0.76	0.59	0.68
Mean	0.76	0.75	0.76

261 3 to 5 landmarks that cover only a portion of the projectile point. These extremes were chosen to provide  
262 significant contrast between approaches.

## 263 Results

### 264 Justice Projectile Points

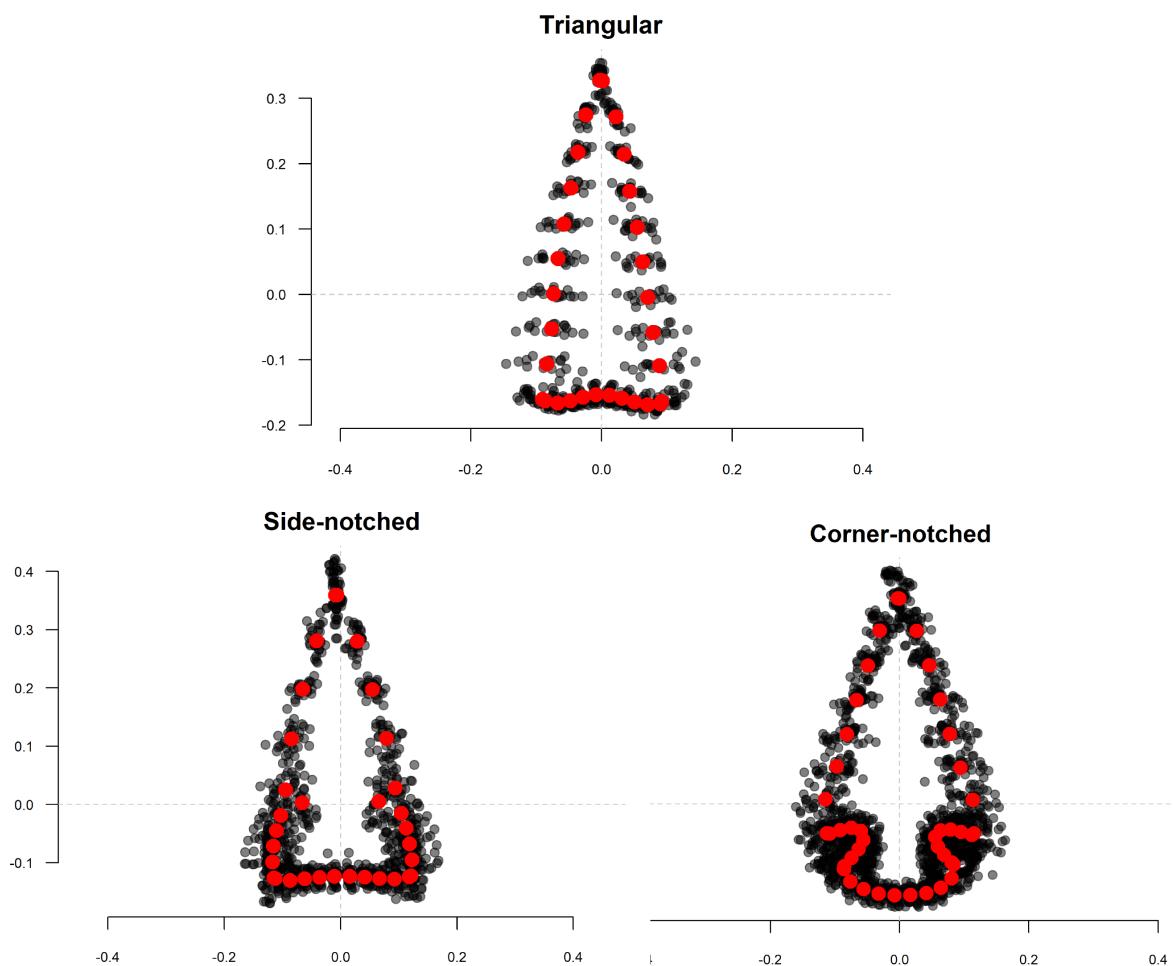
265 The first step in the analysis was to determine how well projectile points typed by Justice could be correctly  
266 assigned using GM methods. Linear discriminant analysis (LDA) was used to type the projectile points using  
267 the GM results. A general target of 0.85 was arbitrarily chosen as a minimum target for acceptable results-  
268 meaning that 85% of the projectile points were classified correctly. As mentioned previously, Justice placed  
269 each projectile point type into a cluster. Presumably, projectile point types in the same cluster should be more  
270 closely related than they are to projectile point types in other clusters. This gives another level of comparison  
271 that was used in addition to the types.

272 The original intent was to compare EFA versus semilandmarks placed at equidistant locations around the  
273 outline. However, these results were unsatisfactory, and a more traditional landmark analysis was also com-  
274 pleted. Because the number and placement of landmarks has a significant impact on the outcome of the  
275 study, two different landmark configurations were used. Tables 2, 3, and 4 show the LDA results by type,  
276 cluster, and by shape—the column and row means are also included. These tables will be referred to in the  
277 sections that follow.

278 Part of the reason the results were unsatisfactory for the EFA and semilandmarks was that the LDA analysis  
279 had trouble discriminating between notched and unnotched projectile points and between side-notched and  
280 corner-notched points. These are some of the most basic distinctions that are made when analyzing projectile  
281 points. While it would be convenient if the analysis did not require an additional step, it is not difficult to  
282 separate the projectile points into these basic shapes prior to the GM analysis. Table 2 shows the accuracy of  
283 EFA and semilandmark analysis for identifying each type of shape. The landmark analysis was not compared  
284 as each shape used a different landmark configuration. The primary challenge was identifying side-notched  
285 from triangular points.

### 286 Elliptical Fourier Analysis

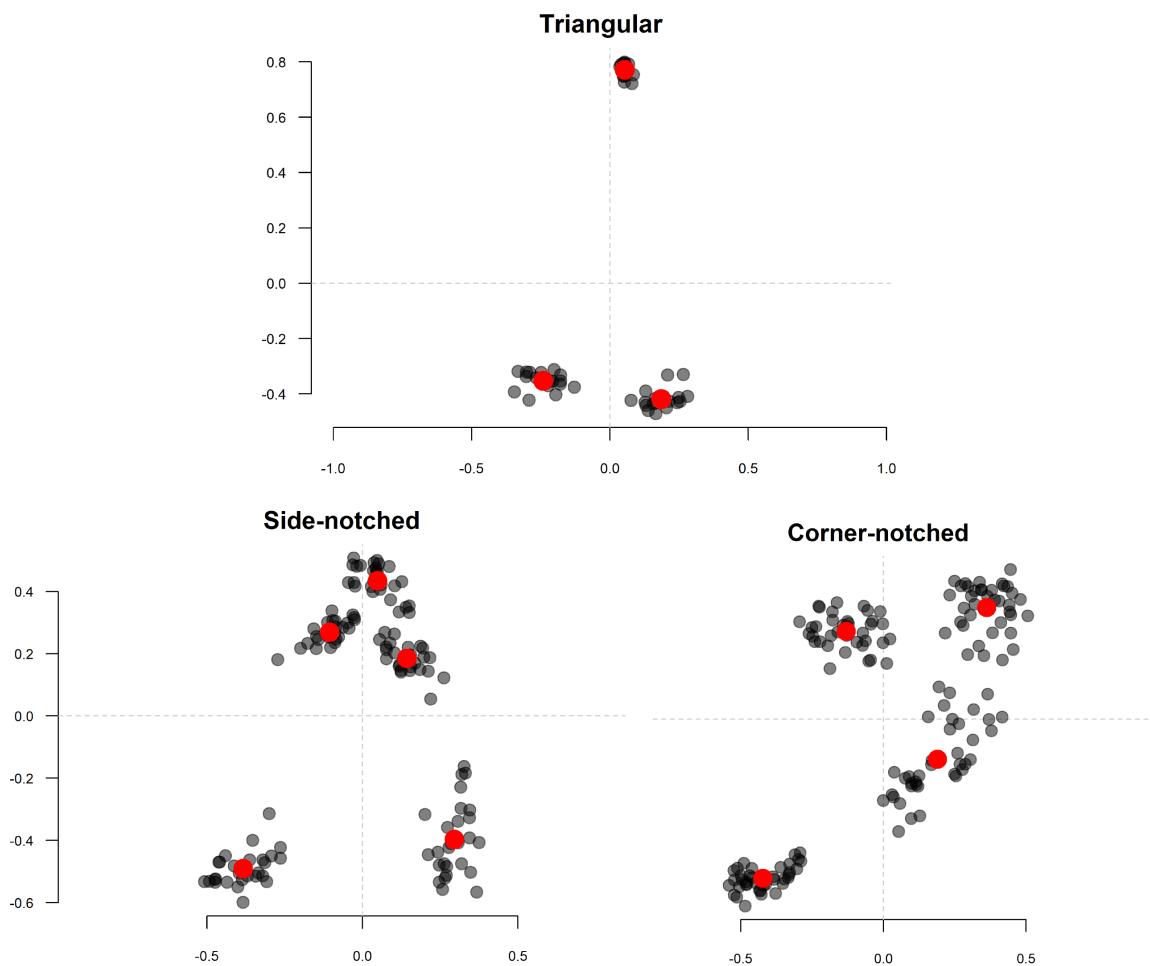
287 The EFA analysis was conducted using the Momocs packages (Bonhomme et al., 2014) in R (R Core Team, 2022).  
288 The first step was to calculate the number of harmonics to use. In this case 12 harmonics described 99% of  
289 the variation in the projectile point outlines. Next, the EFA function was used on each projectile point, and  
290 then a PCA was used to reduce the dimensionality of the data. Figure 9 shows the results of the PCA analysis.  
291 A useful feature of PCA plots using these data is that the morphospace can be plotted with the PCA results.  
292 The morphospace shows how the shapes vary along each axis of the PCA. In this case, PC1 (the first principal  
293 component) varies between short, wide projectile points and long, narrow points. Some of shapes on the top  
294 and bottom left have inverted into impossible shapes, but note that no projectile points fall into these areas.  
295 PC2 varies primarily from stemmed points to side-notched projectile points.



**Figure 7.** Comparison of the full outline landmarks for corner-notched, side-notched and triangular shaped projectile points from Justice's (2002) projectile point illustrations. Red dots indicate the mean location for each of the landmarks.

**Table 3.** Linear Discriminant Analysis Results for Projectile Point Types

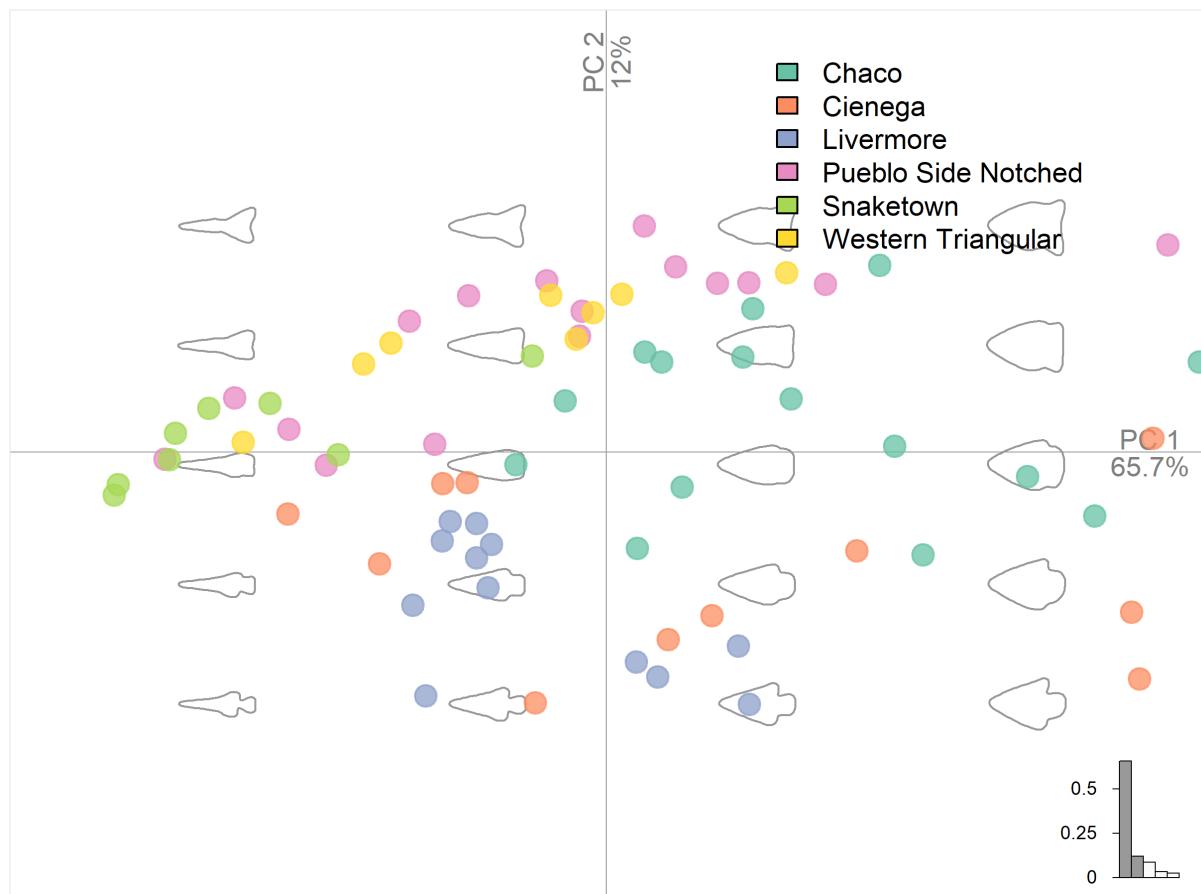
Type	EFA	semiLdk	Ldk	Ldk-corner	Mean
Chaco Corner Notched	0.56	0.89	0.85	0.85	0.79
Cottonwood Triangular	0.38	0.38	1.00	0.75	0.63
Guadalupe	0.75	0.83	0.93	0.93	0.86
Pueblo Alto Side Notched	0.86	0.71	1.00	1.00	0.89
Pueblo Side Notched Concave Base	0.44	0.78	0.73	0.73	0.67
Pueblo Side Notched Straight Base	0.57	0.57	0.43	0.71	0.57
Snaketown Triangular Concave Base	0.78	0.89	0.78	0.89	0.84
Tularosa Corner Notched	0.77	0.69	0.60	0.80	0.72
Mean	0.64	0.72	0.79	0.83	0.74



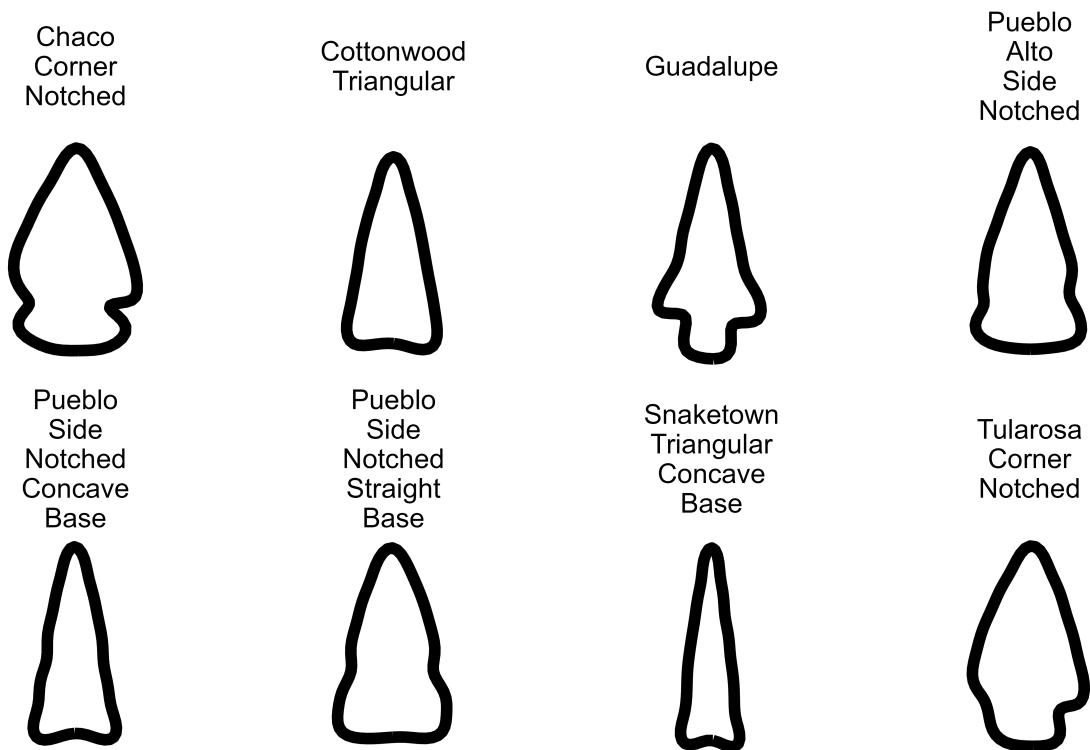
**Figure 8.** Comparison of the projectile point corner's landmarks for corner-notched, side-notched and triangular shaped points from Justice's (2002) projectile point illustrations. Red dots indicate the mean location for each of the landmarks.

**Table 4.** Linear Discriminant Analysis Results for Projectile Point Clusters

Cluster	EFA	semiLdk	Ldk	Ldk-corner	Mean
Chaco	0.81	0.88	0.92	0.92	0.88
Cienega	0.69	0.69	0.60	0.80	0.70
Livermore	0.83	0.75	0.93	0.93	0.86
Pueblo Side Notched	0.75	0.56	0.89	1.00	0.80
Snaketown	0.78	0.89	0.78	0.89	0.84
Western Triangular	0.38	0.50	1.00	0.75	0.66
Mean	0.71	0.71	0.85	0.88	0.79



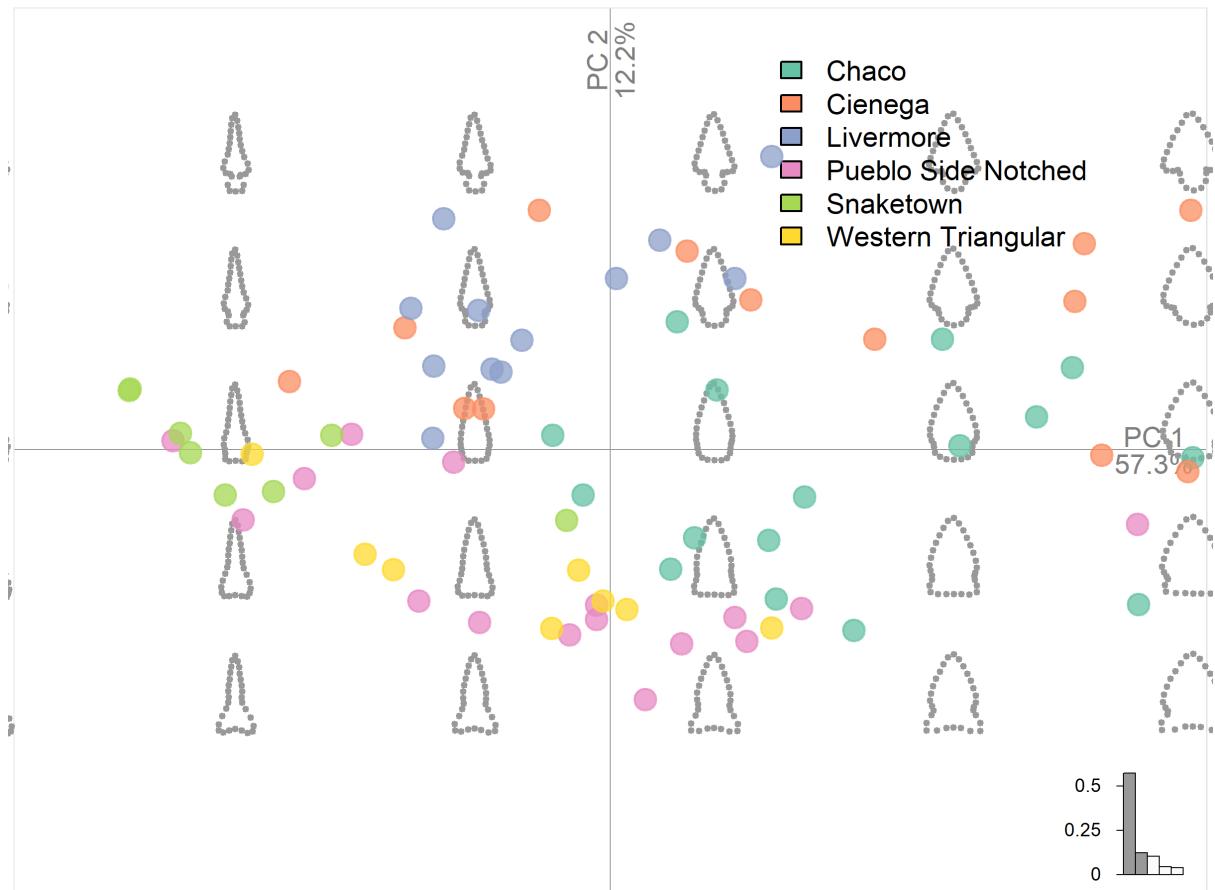
**Figure 9.** Principal components plot showing projectile points from Justice (2002) and the morphospace based on an elliptical Fourier analysis. The projectile points are labeled by the cluster assigned by Justice.



**Figure 10.** Mean shapes by projectile point type using elliptical Fourier analysis.

296 The first objective for the analysis of the Justice projectile points is to determine how well the different  
 297 point types can be discriminated. Meaning, how well can GM methods classify these projectile points into  
 298 their original categories. The LDA results were far from the target goal of 0.85 for most projectile point types,  
 299 and only one type (Pueblo Alto Side Notched-0.86) met the target (see EFA results in Table 3). The LDA results  
 300 were better when the projectile point types were grouped into clusters, as shown in Table 4; however, none  
 301 of the clusters met the target of 0.85. Even more disconcerting were the results shown in Table 2, as only the  
 302 corner-notched projectile points were discriminated with an accuracy greater than the target.

303 The differences in classification accuracy between corner-notched, side-notched, and triangular projectile  
 304 points can perhaps best be explained by examining the mean shapes of each point, as generated through  
 305 EFA. Figure 10 shows the mean shapes for the selected projectile point types. These are the mathematically  
 306 average shapes when all of the projectile points in the type are combined. This has a tendency to average out  
 307 the notches for the side-notched points, as the placement of these notches vary in height. Pueblo Alto Side  
 308 Notched points appear to be an exception to the side-notched problem, as they have the highest classification  
 309 accuracy. Corner notched and stemmed points must, by definition, always have their notches or stems in the  
 310 same location, even though the shape of the notches and stems still varies. This explains why it is easier  
 311 to discriminate them from other point types. As for the side-notched and triangular points, sometimes the  
 312 notches are subtle and the notches are only a small part of the whole form, which is clearly not a strong  
 313 enough element to separate triangular and side-notched points consistently.



**Figure 11.** Principal Components Plots showing projectile points from Justice (2002) and the morphospace based on a semilandmark generalized procrustes alignment. The projectile points are labeled by the cluster assigned by Justice.

### 314 Semilandmarks

315 Analyzing projectile points using semilandmarks is comparable to the EFA analysis. The major analytical choice  
 316 is how many landmarks to use. Each projectile point is represented by a varied number of coordinates that  
 317 represent its outline. The number of points must be standardized so that each projectile point has an identical  
 318 number of coordinates, which is done using the Momocs package. The choice of how many points to use does  
 319 affect the GM analysis. To solve this problem, I tried different numbers of semilandmarks varying from 10 to  
 320 100. More than 100 points appears to no longer have a substantial effect on the results. Each projectile  
 321 point was sampled multiple times and then all of the points were classified using LDA according to the same  
 322 procedures used for the EFA analysis. The results of this test ranged from 65% accuracy (10 points) to 73%  
 323 accuracy (30 points—the number used for the final analysis) with points higher than 30 consistently measuring  
 324 in at 64% accuracy.

325 The PCA components had similar dimensions of variation as the EFA analysis (see figure 11). The major im-  
 326 provement in accuracy (the ‘semiLdk’ column of Tables 2-4) is perhaps due to the better alignment generated  
 327 by the GPA procedure, but that is only speculation.

328 Regardless of the reason, a jump from 64% classification to 72% between the EFA and semilandmark anal-  
 329 ysis is a substantial improvement. It does not reach the target of greater than 85% classification accuracy, but  
 330 it is a step in the right direction. The mean shapes are nearly identical to the EFA analysis, which indicates that  
 331 this method suffers from the same problems with side-notched and triangular projectile points, but it does a  
 332 somewhat better job differentiating corner-notched and side-notched points. Curiously, it does a worse job

333 differentiating triangular points. The side-notches are the likely culprit.

334 **Landmarks**

335 Neither the semilandmarks nor EFA adequately distinguished between point types or between point shapes.  
336 Identifying notches was particularly troublesome. A solution to this problem was to use landmarks and ex-  
337 plicitly identify the notches or lack of notches. No comparison was made between projectile point shapes  
338 (i.e., triangular versus side-notched) using landmark analysis, as initial experiments determined that it was  
339 best to use different landmark procedures for the different shapes. Perhaps machine learning may solve this  
340 problem (see Castillo Flores et al., 2019; MacLeod, 2018; Nash and Prewitt, 2016). Triangular projectile points  
341 require a different approach than side-notched points, and even side-notched and corner-notched/stemmed  
342 points require different procedures.

343 The LDA results for the first landmark configuration (Ldk in Tables 3-4) are much better than EFA and better  
344 than the semilandmark analysis, but still not as accurate as desired. The biggest underperformer by far was  
345 Pueblo Side Notched Straight Base at 0.43. All of the previous analyses struggled to capture basal shape  
346 distinctions, but this analysis struggled more so, as the base is a critical component of this type. What is  
347 particularly notable is that Cottonwood Triangular projectile points were classified perfectly whereas they  
348 were previously the worst performing type in the EFA and semilandmark analyses. As Table 4 shows, the  
349 cluster assignments performed well. If Cienega points were not so problematic, then the results would be  
350 excellent.

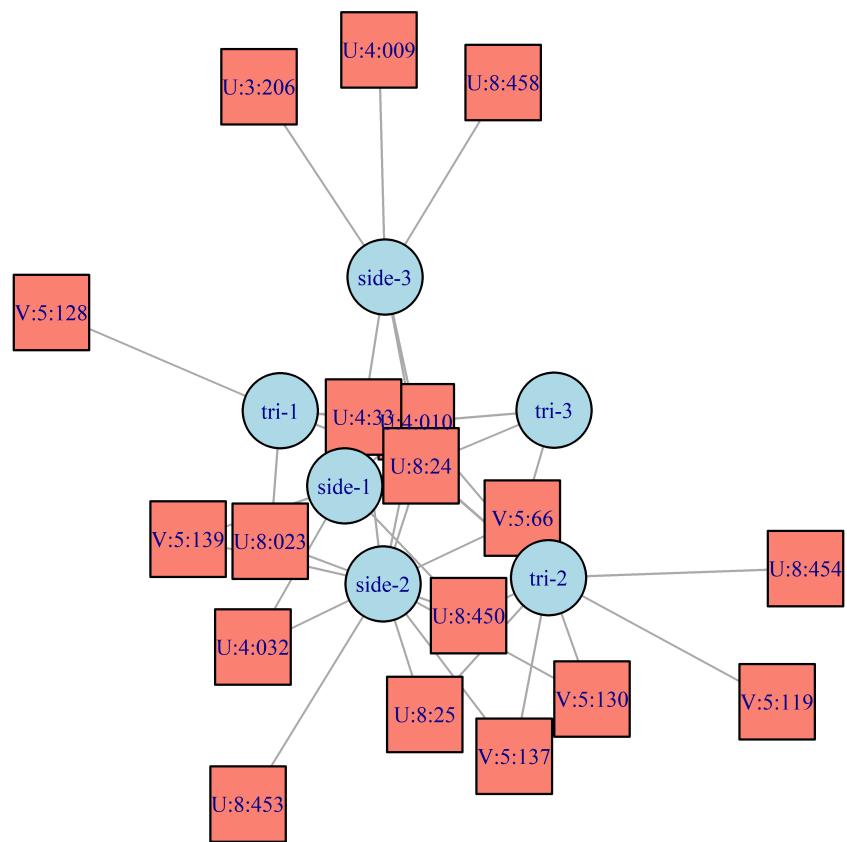
351 The final analysis used the second landmark configuration—the projectile point corners. These results  
352 proved superior to the first landmark configuration and are almost 20% higher in accuracy than the EFA results  
353 on average. The lowest type for accuracy was again Pueblo Side Notched Straight Base, but it improved from  
354 the first landmark configuration to 0.71 from 0.43. The accuracy results were more consistent and accurate.  
355 With some additional experimentation on landmark placement, this configuration could likely achieve better  
356 results and meet the targeted 0.85 accuracy.

357 Not only did landmark analysis provide superior accuracy, but it will also make it easier to use larger sample  
358 sizes. Presumably, notching style is an important attribute that should be captured in the analysis. If EFA or  
359 the semilandmark analysis as conducted here fails to sufficiently emphasize the notches, then these methods  
360 are insufficient for classifying projectile points. While landmark analysis is more time-consuming, the use of  
361 the second configuration does reduce the burden of landmarking.

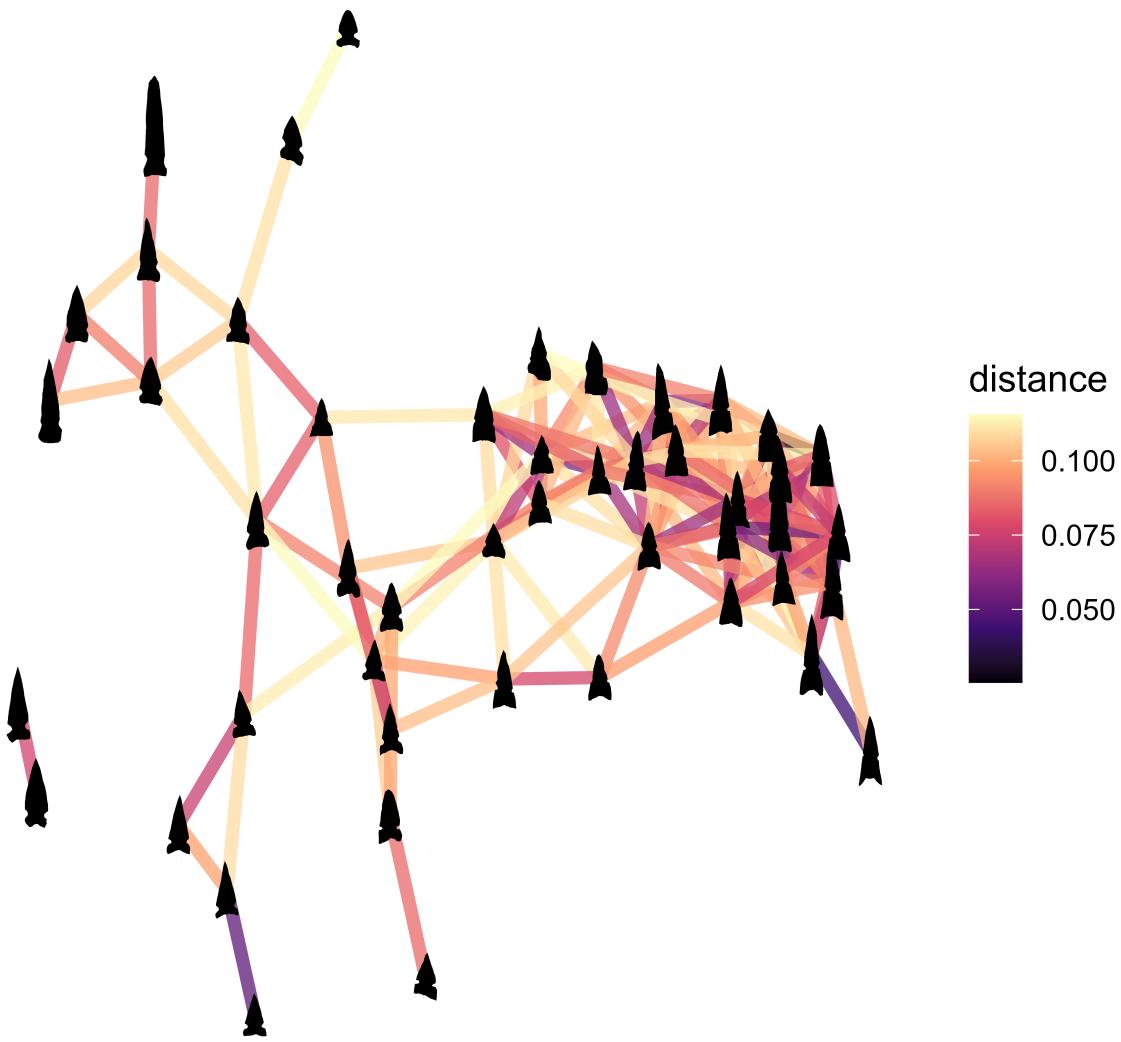
362 **Tonto Basin Projectile Points**

363 The initial intent was to classify the Tonto Basin projectile points using the analysis of the Justice projectile  
364 points; however, the limited sample size limits the validity of the exercise. The analysis was not futile though,  
365 as the second landmark configuration using the corners of the projectile points proved the most effective. I  
366 therefore used the same landmark configuration to analyze the projectile points from Tonto Basin. The results  
367 of the GPA and PCA analysis were used in a hierarchical cluster analysis using Ward's method (see Murtagh and  
368 Legendre, 2014). Figure 12 is a network graph showing the results. This graph shows the assigned projectile  
369 point types from the cluster analysis with connections from every Tonto Basin site where that projectile point  
370 was found to the assigned type. Several sites in Tonto Basin only had one or two types of projectile points  
371 (low sample sizes were again problematic), but some of the larger, well-excavated sites shared all or most of  
372 the projectile point types. It is beyond the purpose of this study to explore the patterns in this data, but the  
373 methods clearly provide useful data for exploratory analysis.

374 The final question I wished to address in this study was whether it was necessary to use a typology in a GM  
375 analysis of projectile points. There are many ways to answer this question, but in short, the answer is no. That  
376 does not mean typologies are not useful, but they can mask important variation. The following is one way to  
377 approach analyzing projectile points without using a typology.

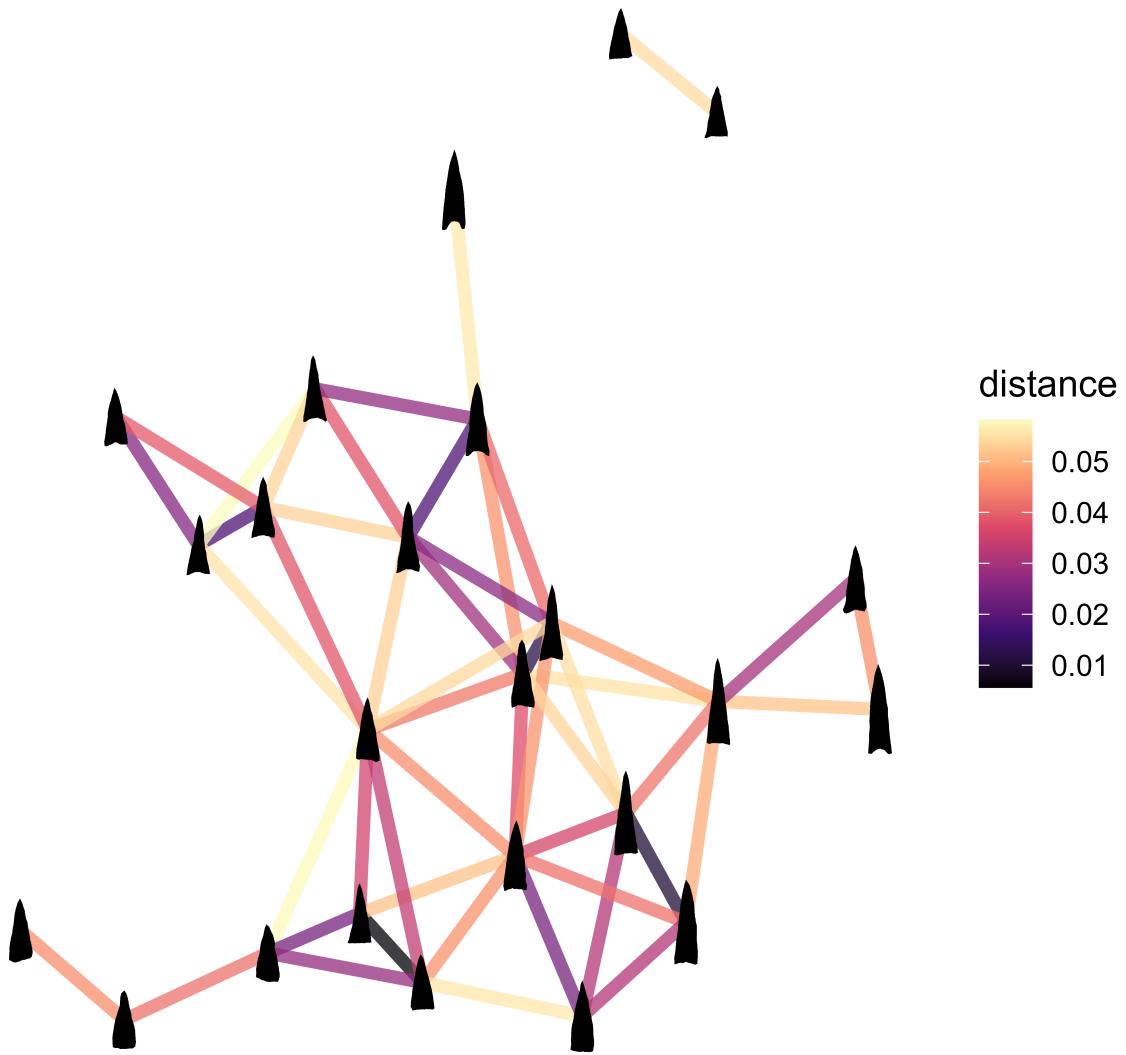


**Figure 12.** Bipartite network graph displaying assigned projectile point clusters for side-notched and triangular projectile points in Tonto Basin and Tonto Basin sites. The circles represent cluster designations by shape (e.g., side = side-notched and tri = triangular). The squares represent sites. The links between squares and circles show which point clusters are found at which sites.



**Figure 13.** Network graph displaying side-notched projectile points from Tonto Basin as nodes with ties showing the morphometric distance (euclidean distance in Principal Component space) between projectile points. Darker colors represent stronger ties. Note that only the strongest 10% of ties are shown.

Because the results of the GM analyses can be projected into multidimensional space, the distance between these values is meaningful and can be directly compared. One way to flatten multidimensional space into two dimensions is to calculate the Euclidean distance between each point and display the results as a network graph, as in figures 13 and 14. This way each point can be compared directly without grouping the projectile points into types. The results are messier than neatly fitting each point into a single type, but subtle variation in morphology is easier to visualize this way. The results should only be interpreted as a visual aid. The closer the projectile points are to each other, the more similar they are in shape, keeping in mind that only the corners of the projectile points (from the notches down for the side-notched points) were used in this analysis. Many of the points clustered closely together, indicating a common shape across the sites. The side-notched points have a particularly large cluster of typical Hohokam side-notched points. Yet there are also a large number of projectile points that do not closely match the other points, which indicates that there is also a lot of variation. This variation may represent idiosyncrasies, exchange, migration, novice knappers, or a different intention for the point. Regardless of the purpose, the GM analysis better captures the “otherness” of a projectile point than classifying a point as other/unknown or worse, forcing it into a category it does not belong in.



**Figure 14.** Network graph displaying triangular projectile points from Tonto Basin as nodes with ties showing the morphometric distance (euclidean distance in Principal Component space) between projectile points. Darker colors represent stronger ties. Note that only the strongest 10% of ties are shown.

392 The clustering analysis provided several different projectile point types and provided an overview of what  
393 sites shared similar projectile points. This type of analysis can be combined with architectural or other data  
394 to look for correlations or patterns that provide insights into the behavior of the people who made and used  
395 these projectiles. Yet the analyst must take care to ensure the clustering groups are appropriately sized and  
396 the results make sense. One way to view the data more closely is to look at a distance network graph to view  
397 the variation in morphometric shape. This way typological distinctions will not mask the variation.

## 398 Conclusion

399 The purpose of this paper was to evaluate geometric morphometric methods for analyzing projectile points in  
400 the Southwestern U.S. These analyses provided significant variation in their results, yet they also demonstrated  
401 positive results. While EFA underperformed all of the other analyses, a different dataset may favor this analysis.  
402 Some of the types performed better for EFA than other types, which suggests that EFA may be the optimal  
403 choice for some datasets. Indeed a recent case study found that EFA performed comparable to or better than  
404 landmark analysis in several case studies (Matzig et al., 2021). A clear result from this exploratory analysis is  
405 that GM analysis is not a one-size-fits-all approach. Better results were obtained from a full outline approach  
406 using semilandmarks, which raises interesting theoretical questions I am unable to address here but would  
407 be worthwhile to pursue further. More traditional landmark approaches performed better, likely because  
408 the outline approaches failed to identify side-notches consistently. In this case, a landmark/semlandmark  
409 method using the corner of the projectile point—from the base to the middle of the basal margin or from the  
410 middle of the blade if the point is triangular—proved to be the most useful method. The main advantage of this  
411 method was that it provided the most accurate reproduction of Justice's original classification of the projectile  
412 points. Another advantage is that broken points are easier to use with this method. If one half of the point  
413 is missing, either the top or the lateral margin, it does not affect the analysis. This increases the number of  
414 points available for analysis tremendously compared to only using whole points. The final advantage, though  
415 minor, is that the landmarking analysis is simple and only requires three to five landmarks.

416 I mentioned how difficult it is to conduct regional analyses with projectile points in the U.S. Southwest. The  
417 main difficulty is harmonizing existing typologies and then fitting new projectile points into this typology. While  
418 the sample size available for this study was too small to attempt classifying the Tonto Basin points according  
419 to Justice's typology, it would be possible to do so with these methods given enough data. However, this pa-  
420 per also demonstrated that it is possible to type projectile points using common clustering methods which  
421 may better capture the variation in projectile point morphology than previously used types. Furthermore,  
422 it is possible to analyze projectile points without resorting to types. The distances between projectile point  
423 morphologies can be computed and compared directly. These distances could even be aggregated and sum-  
424 marized regionally. The main challenge for the regional analysis is obtaining the projectile point outlines or  
425 landmarks. Once these are obtained, thousands of points can be analyzed and assigned to clusters relatively  
426 quickly.

427 Compared to a traditional analysis of linear metrics and weights, a GM analysis can capture much more in-  
428 formation and provide more informative ways to analyze and visualize the data. The visualization capabilities  
429 of GM is one of its greatest strengths, as it allows the analyst to see the data they are working with, visually  
430 validate their results, and share their findings in visually compelling ways. Additionally, this analysis is more  
431 reproducible and adaptable than traditional lithic analyses. While the analyst still has a lot of control over a  
432 GM analysis, the results should be less biased than analyses based on visual type comparisons.

## 433 Acknowledgements

434 Special thanks to Zac Selden, Michael Shott, Bernard Means, and Loren Davis for teaching a geometric mor-  
435 phometric workshop that was my first major introduction to these methods. Joshua Watts, Matthew Peebles,

436 Melissa Powell, Christopher Caseldine, and several volunteers assisted with this research.

## 437 **Data, scripts, and supplementary information availability**

438 All relevant data and scripts are available at the following DOI [10.17605/OSF.IO/ZGE9Q](https://doi.org/10.17605/OSF.IO/ZGE9Q) and on GitHub at  
439 [github.com/bischrob/TontoBasinPoints](https://github.com/bischrob/TontoBasinPoints). The R code used in the analysis is included in the manuscript.Rmd  
440 file used to create this manuscript, although some lines have been commented out to improve efficiency.

## 441 **Conflicts of interest disclosure**

442 The author declares that they comply with the PCI rule of having no financial conflicts of interest in relation to  
443 the content of the article.

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