

Evolutionary Invasion Analysis

Specialist --> Generalist

First, we will create our equations and solve for the fixed points of a three-species system, with the specialist as the resident pathogen strategy. We will get rid of the specialist equations and aspects of those equations in other ODEs.

```
In[1]:= xcoopRHSnogen = mu1 * X * ((a1 * Y) / (a1 * Y) + k1) * (R - X) - δ * X;  
ycoopRHSnogen = mu2 * Y * ((a2 * X) / (a2 * X) + k2) * (R - Y) - z3 * s * Y - δ * Y;  
xcompRHSnogen = mu1 * X * (R - X - b1 * Y) - δ * X;  
ycompRHSnogen = mu2 * Y * (R - Y - b2 * X) - z3 * s * Y - δ * Y;  
sRHS = v3 * z3 * s * Y - δ * s;  
fps =  
  Simplify[Solve[{xcoopRHSnogen == 0, ycoopRHSnogen == 0, sRHS == 0}, {X, Y, s}]];  
fpscomp =  
  Simplify[Solve[{xcompRHSnogen == 0, ycompRHSnogen == 0, sRHS == 0}, {X, Y, s}]];
```

Fixed point #6 contains the 3-species equilibrium point.

```
In[8]:= fp3speciescoop = FullSimplify[(fps[[6]] // Normal)];  
fp3speciescomp = FullSimplify[(fpscomp[[6]] // Normal)];
```

Next, we will generate our Jacobian matrices and evaluate them on the 3-species fixed point. Let's also get the determinants and traces of these matrices, for good measure.

```
In[10]:= Jcoop = {{D[xcoopRHSnogen, X], D[xcoopRHSnogen, Y], D[xcoopRHSnogen, s]},  
  {D[ycoopRHSnogen, X], D[ycoopRHSnogen, Y], D[ycoopRHSnogen, s]},  
  {D[sRHS, X], D[sRHS, Y], D[sRHS, s]}} // FullSimplify;  
Jrescoop = Jcoop /. fp3speciescoop;  
Jcomp = {{D[xcompRHSnogen, X], D[xcompRHSnogen, Y], D[xcompRHSnogen, s]},  
  {D[ycompRHSnogen, X], D[ycompRHSnogen, Y], D[ycompRHSnogen, s]},  
  {D[sRHS, X], D[sRHS, Y], D[sRHS, s]}} // FullSimplify;  
Jrescomp = Jcomp /. fp3speciescomp;
```

Let's add a mutant pathogen with a generalist strategy.

```

In[14]:= xcoopRHS = mu1 * X * ((a1 * Y) / (a1 * Y) + k1) * (R - X) - z1 * g * X - δ * X;
ycoopRHS = mu2 * Y * ((a2 * X) / (a2 * X) + k2) * (R - Y) - z3 * s * Y - z2 * g * Y - δ * Y;
xcompRHS = mu1 * X * (R - X - b1 * Y) - z1 * g * X - δ * X;
ycompRHS = mu2 * Y * (R - Y - b2 * X) - z3 * s * Y - z2 * g * Y - δ * Y;
gRHS = v2 * z2 * g * Y + v1 * z1 * g * X - δ * g;
sRHS = v3 * z3 * s * Y - δ * s;
fps4 = Simplify[
  Solve[{xcoopRHS == 0, ycoopRHS == 0, sRHS == 0, gRHS == 0}, {X, Y, s, g}]];
fpscomp4 =
  Simplify[Solve[{xcompRHS == 0, ycompRHS == 0, sRHS == 0, gRHS == 0}, {X, Y, s, g}]];

In[22]:= fp4speciescoop = FullSimplify[(fps4[[3]] // Normal)];
fp4speciescomp = FullSimplify[(fpscomp4[[9]] // Normal)];
Jcoop4 = {{D[xcoopRHS, X], D[xcoopRHS, Y], D[xcoopRHS, s], D[xcoopRHS, g]},
  {D[ycoopRHS, X], D[ycoopRHS, Y], D[ycoopRHS, s], D[ycoopRHS, g]},
  {D[sRHS, X], D[sRHS, Y], D[sRHS, s], D[sRHS, g]},
  {D[gRHS, X], D[gRHS, Y], D[gRHS, s], D[gRHS, g]} } // FullSimplify;
Jaugcoop = Jcoop4 /. fp4speciescoop;
Jcomp4 = {{D[xcompRHS, X], D[xcompRHS, Y], D[xcompRHS, s], D[xcompRHS, g]},
  {D[ycompRHS, X], D[ycompRHS, Y], D[ycompRHS, s], D[ycompRHS, g]},
  {D[sRHS, X], D[sRHS, Y], D[sRHS, s], D[sRHS, g]},
  {D[gRHS, X], D[gRHS, Y], D[gRHS, s], D[gRHS, g]} } // FullSimplify;
Jaugcomp = Jcomp4 /. fp4speciescomp;

```

```

In[28]:= MatrixForm[Jrescoop]

```

```

Out[28]//MatrixForm=

```

$$\begin{pmatrix} -\delta + (1+k_1) \mu_1 \left(R - 2 \left(R - \frac{\delta}{\mu_1 + k_1 \mu_1} \right) \right) & 0 & 0 \\ 0 & -\delta + (1+k_2) \mu_2 \left(R - \frac{2\delta}{v_3 z_3} \right) - \frac{(1+k_2) \mu_2 (R v_3 z_3 - \delta) - v_3 z_3 \delta}{v_3 z_3} & -\frac{\delta}{v_3} \\ 0 & \frac{(1+k_2) \mu_2 (R v_3 z_3 - \delta) - v_3 z_3 \delta}{z_3} & 0 \end{pmatrix}$$

```

In[29]:= MatrixForm[Jaugcoop]

```

```

Out[29]//MatrixForm=

```

$$\begin{pmatrix} -\delta + (1+k_1) \mu_1 \left(R - 2 \left(R - \frac{\delta}{\mu_1 + k_1 \mu_1} \right) \right) & 0 & 0 \\ 0 & -\delta + (1+k_2) \mu_2 \left(R - \frac{2\delta}{v_3 z_3} \right) - \frac{(1+k_2) \mu_2 (R v_3 z_3 - \delta) - v_3 z_3 \delta}{v_3 z_3} & -\frac{\delta}{v_3} \\ 0 & \frac{(1+k_2) \mu_2 (R v_3 z_3 - \delta) - v_3 z_3 \delta}{z_3} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```

In[30]:= MatrixForm[Jrescomp]

```

```

Out[30]//MatrixForm=

```

$$\begin{pmatrix} -\delta + \mu_1 \left(R - \frac{b_1 \delta}{v_3 z_3} - 2 \left(R - \frac{\delta}{\mu_1} - \frac{b_1 \delta}{v_3 z_3} \right) \right) & -b_1 \mu_1 \left(R - \frac{\delta}{\mu_1} - \frac{b_1 \delta}{v_3 z_3} \right) \\ -\frac{b_2 \mu_2 \delta}{v_3 z_3} & -\delta + \mu_2 \left(R - \frac{2\delta}{v_3 z_3} - b_2 \left(R - \frac{\delta}{\mu_1} - \frac{b_1 \delta}{v_3 z_3} \right) \right) - \frac{b_2 \mu_2 v_3 z_3 \delta - \mu_1 (v_3 z_3 \delta + \mu_2 ((-1+b_2) R v_3 z_3 + \delta))}{\mu_1 z_3} \\ 0 & 0 \end{pmatrix}$$

```
In[31]:= MatrixForm[Jaugcomp]
```

```
Out[31]//MatrixForm=
```

$$\begin{pmatrix} -\delta + \mu_1 \left(R - \frac{b_1 \delta}{v_3 z_3} - 2 \left(R - \frac{\delta}{\mu_1} - \frac{b_1 \delta}{v_3 z_3} \right) \right) & -b_1 \mu_1 \left(R - \frac{\delta}{\mu_1} - \frac{b_1 \delta}{v_3 z_3} \right) & 0 & 0 \\ -\frac{b_2 \mu_2 \delta}{v_3 z_3} & -\delta + \mu_2 \left(R - \frac{2 \delta}{v_3 z_3} - b_2 \left(R - \frac{\delta}{\mu_1} - \frac{b_1 \delta}{v_3 z_3} \right) \right) & -\frac{b_2 \mu_2 v_3 z_3 \delta - \mu_1 (v_3 z_3 \delta + \mu_2 ((-1+b_2) R v_3 z_3 + \delta))}{\mu_1 z_3} & 0 \end{pmatrix}$$

We want an augmented matrix of the block-triangular form $\begin{pmatrix} \mathbf{J}_{res} & \mathbf{u} \\ 0 & \mathbf{J}_{mut} \end{pmatrix}$. Importantly, our matrices should match, such that the upper 3x3 block of the augmented Jacobian is identical to the resident Jacobian. We will then define the 0 matrix as a matrix of form (0,0,0) in the bottom row of the augmented matrix, the **u** matrix comprising the top three values of the final column of the matrix, and the **Jmut** matrix is equal to the bottom far right value. These values, in this case, are shown below.

```
In[32]:= Jmutcomp = FullSimplify[-\delta + \frac{v_2 z_2 \delta}{v_3 z_3} + v_1 z_1 \left( R - \frac{\delta}{\mu_1} - \frac{b_1 \delta}{v_3 z_3} \right)]
```

```
Jmutcoop = FullSimplify[-\delta + \frac{v_2 z_2 \delta}{v_3 z_3} + v_1 z_1 \left( R - \frac{\delta}{\mu_1 + k_1 \mu_1} \right)]
```

```
Out[32]=
```

$$R v_1 z_1 - \frac{(b_1 \mu_1 v_1 z_1 - \mu_1 v_2 z_2 + v_3 (\mu_1 + v_1 z_1) z_3) \delta}{\mu_1 v_3 z_3}$$

```
Out[33]=
```

$$R v_1 z_1 + \left(-1 - \frac{v_1 z_1}{\mu_1 + k_1 \mu_1} + \frac{v_2 z_2}{v_3 z_3} \right) \delta$$

The eigenvalues of the augmented matrix are those of the **Jres** and **Jmut** matrices. We have assumed already that the eigenvalues of **Jres** are negative, so we will ignore them. Instead, we are only interested in **Jmut**. Where **Jmut** is positive, then the mutant will be able to invade.

To simplify these Jacobians, let's define $v_1 z_1$ as β_{gen}^E , the infectivity of the generalist on E. coli, $v_2 z_2$ as β_{gen}^S , the infectivity of the generalist on S. enterica, and $v_3 z_3$ as β_{sp}^S , the infectivity of the specialist on S. enterica.

```
In[34]:= simplifiedcoop = betaEgen R + \left( -1 + \frac{betaSgen}{betaSsp} - \frac{betaEgen}{\mu_1 + k_1 \mu_1} \right) \delta // FullSimplify
```

```
Out[34]=
```

$$\text{betaEgen } R + \left(-1 + \frac{\text{betaSgen}}{\text{betaSsp}} - \frac{\text{betaEgen}}{\mu_1 + k_1 \mu_1} \right) \delta$$

```
In[35]:= Reduce[{simplifiedcoop > 0, betaSgen > 0,
  betaSsp > 0, betaEgen > 0, mu1 > 0, R > 0, k1 > 0, delta > 0}, Reals]
```

```
Out[35]=
```

```
betaSsp > 0 &&
  ( ( 0 < betaSgen < betaSsp && k1 > 0 && delta > 0 && R > 0 && betaEgen >
    
$$\frac{-\text{betaSgen } \delta + \text{betaSsp } \delta}{\text{betaSsp } R} \text{ \&\&}$$

    mu1 > (betaEgen betaSsp delta) / (betaEgen betaSsp R + betaEgen betaSsp k1 R +
      betaSgen delta - betaSsp delta + betaSgen k1 delta - betaSsp k1 delta) ) ||
  (betaSgen >= betaSsp && k1 > 0 && delta > 0 && R > 0 && betaEgen > 0 &&
    mu1 > (betaEgen betaSsp delta) / (betaEgen betaSsp R + betaEgen betaSsp k1 R +
      betaSgen delta - betaSsp delta + betaSgen k1 delta - betaSsp k1 delta) ) )
```

These results indicate where the generalist will be able to invade; fundamentally, anywhere that it can infect E, then the generalist should be able to invade. It can invade under certain circumstances when it is better on S than the specialist without being able to infect E, but in that case it is not actually a generalist.

Let's think next about the competitive case. We will again define $v1 \cdot z1$ as β_{gen}^E , the infectivity of the generalist on E. coli, $v2 \cdot z2$ as β_{gen}^S , the infectivity of the generalist on S. enterica, and $v3 \cdot z3$ as β_{sp}^S , the infectivity of the specialist on S. enterica.

```
In[42]:= simplifiedcomp = R betaEgen -
  
$$\frac{(b1 \text{ mu1 betaEgen} - \text{mu1 betaSgen} + (\text{mu1} + \text{betaEgen}) \text{ betaSsp}) \delta}{\text{mu1 betaSsp}} // \text{FullSimplify}$$

```

```
Out[42]=
```

```
betaEgen R - 
$$\frac{(b1 \text{ betaEgen mu1} - \text{betaSgen mu1} + \text{betaSsp} (\text{betaEgen} + \text{mu1})) \delta}{\text{betaSsp mu1}}$$

```



```

In[57]:= Jcoop = {{D[xcoopRHSnosp, X], D[xcoopRHSnosp, Y], D[xcoopRHSnosp, g]},
  {D[ycoopRHSnosp, X], D[ycoopRHSnosp, Y], D[ycoopRHSnosp, g]},
  {D[gRHS, X], D[gRHS, Y], D[gRHS, g]}} // FullSimplify;
Jrescoop = Jcoop /. fp3speciescoop;
Jcomp = {{D[xcompRHSnosp, X], D[xcompRHSnosp, Y], D[xcompRHSnosp, g]},
  {D[ycompRHSnosp, X], D[ycompRHSnosp, Y], D[ycompRHSnosp, g]},
  {D[gRHS, X], D[gRHS, Y], D[gRHS, g]}} // FullSimplify;
Jrescomp = Jcomp /. fp3speciescomp;

In[61]:= xcoopRHS = mu1 * X * ((a1 * Y) / (a1 * Y) + k1) * (R - X) - z1 * g * X - delta * X;
ycoopRHS = mu2 * Y * ((a2 * X) / (a2 * X) + k2) * (R - Y) - z3 * s * Y - z2 * g * Y - delta * Y;
xcompRHS = mu1 * X * (R - X - b1 * Y) - z1 * g * X - delta * X;
ycompRHS = mu2 * Y * (R - Y - b2 * X) - z3 * s * Y - z2 * g * Y - delta * Y;
gRHS = v2 * z2 * g * Y + v1 * z1 * g * X - delta * g;
sRHS = v3 * z3 * s * Y - delta * s;
fps4 = Simplify[
  Solve[{xcoopRHS == 0, ycoopRHS == 0, sRHS == 0, gRHS == 0}, {X, Y, s, g}]];
fpscomp4 =
  Simplify[Solve[{xcompRHS == 0, ycompRHS == 0, sRHS == 0, gRHS == 0}, {X, Y, s, g}]];

In[69]:= fp4speciescoop = FullSimplify[(fps4[[6]] // Normal)];
fp4speciescomp = FullSimplify[(fpscomp4[[5]] // Normal)];
Jcoop4 = {{D[xcoopRHS, X], D[xcoopRHS, Y], D[xcoopRHS, g], D[xcoopRHS, s]},
  {D[ycoopRHS, X], D[ycoopRHS, Y], D[ycoopRHS, g], D[ycoopRHS, s]},
  {D[gRHS, X], D[gRHS, Y], D[gRHS, g], D[gRHS, s]},
  {D[sRHS, X], D[sRHS, Y], D[sRHS, g], D[sRHS, s]}} // FullSimplify;
Jaugcoop = Jcoop4 /. fp4speciescoop;
Jcomp4 = {{D[xcompRHS, X], D[xcompRHS, Y], D[xcompRHS, g], D[xcompRHS, s]},
  {D[ycompRHS, X], D[ycompRHS, Y], D[ycompRHS, g], D[ycompRHS, s]},
  {D[gRHS, X], D[gRHS, Y], D[gRHS, g], D[gRHS, s]},
  {D[sRHS, X], D[sRHS, Y], D[sRHS, g], D[sRHS, s]}} // FullSimplify;
Jaugcomp = Jcomp4 /. fp4speciescomp;

In[75]:= MatrixForm[Jrescoop]
Out[75]//MatrixForm=

$$\begin{pmatrix} -\delta - \frac{z1 \left( -((1+k2) \mu2 v1 z1 \delta) + (1+k1) \mu1 ((1+k2) \mu2 (R v1 z1 + R v2 z2 - \delta) - v2 z2 \delta) \right)}{(1+k2) \mu2 v1 z1^2 + (1+k1) \mu1 v2 z2^2} + (1+k1) \mu1 \left( R - \frac{2 \left( -((1+k2) \mu2 z1}{(1+k2) \mu2 v1 z1^2 + (1+k1) \mu1 v2 z2^2} \right)}{(1+k2) \mu2 v1 z1^2 + (1+k1) \mu1 v2 z2^2} \right) \\ 0 \\ \frac{v1 z1 \left( -((1+k2) \mu2 v1 z1 \delta) + (1+k1) \mu1 ((1+k2) \mu2 (R v1 z1 + R v2 z2 - \delta) - v2 z2 \delta) \right)}{(1+k2) \mu2 v1 z1^2 + (1+k1) \mu1 v2 z2^2} \end{pmatrix}$$


```

In[76]:= **MatrixForm[Jaugcoop]**

Out[76]//MatrixForm=

$$\begin{pmatrix} -\delta - \frac{z1 \left(-(1+k2) \mu2 v1 z1 \delta + (1+k1) \mu1 \left((1+k2) \mu2 (R v1 z1 + R v2 z2 - \delta) - v2 z2 \delta \right) \right)}{(1+k2) \mu2 v1 z1^2 + (1+k1) \mu1 v2 z2^2} + (1+k1) \mu1 \left(R - \frac{2 \left(-(1+k2) \mu2 z1}{(1+k2) \mu2 v1 z1^2 + (1+k1) \mu1 v2 z2^2} \right)}{1} \right) & 0 \\ 0 & 0 \\ \frac{v1 z1 \left(-(1+k2) \mu2 v1 z1 \delta + (1+k1) \mu1 \left((1+k2) \mu2 (R v1 z1 + R v2 z2 - \delta) - v2 z2 \delta \right) \right)}{(1+k2) \mu2 v1 z1^2 + (1+k1) \mu1 v2 z2^2} & 0 \end{pmatrix}$$

In[77]:= **MatrixForm[Jrescomp]**

Out[77]//MatrixForm=

$$\begin{pmatrix} -\delta - \frac{z1 \left(\mu1 \mu2 R \left((-1+b1) v1 z1 + (-1+b2) v2 z2 \right) + \mu1 \left(\mu2 - b1 b2 \mu2 - b1 v1 z1 + v2 z2 \right) \delta + \mu2 \left(v1 z1 - b2 v2 z2 \right) \delta \right)}{\mu1 z2 \left(b1 v1 z1 - v2 z2 \right) + \mu2 z1 \left(-v1 z1 + b2 v2 z2 \right)} + \mu1 \left(R - \frac{2 \left(v1 z1 \left(\mu1 \mu2 R \left((-1+b1) v1 z1 + (-1+b2) v2 z2 \right) + \mu1 \left(\mu2 - b1 b2 \mu2 - b1 v1 z1 + v2 z2 \right) \delta + \mu2 \left(v1 z1 - b2 v2 z2 \right) \delta \right) \right)}{\mu1 z2 \left(b1 v1 z1 - v2 z2 \right) + \mu2 z1 \left(-v1 z1 + b2 v2 z2 \right)} \right) & -\frac{b2 \mu2 \left(v1 z1 \left(-z1 + z2 \right) \delta + \mu1 z2 \left(-R v1 z1 + \delta \right) + \mu2 z1 \left(R v1 z1 - b2 v2 z2 \right) \right)}{\mu1 z2 \left(b1 v1 z1 - v2 z2 \right) + \mu2 z1 \left(-v1 z1 + b2 v2 z2 \right)} \\ 0 & 0 \end{pmatrix}$$

In[78]:= **MatrixForm[Jaugcomp]**

Out[78]//MatrixForm=

$$\begin{pmatrix} -\delta - \frac{z1 \left(\mu1 \mu2 R \left((-1+b1) v1 z1 + (-1+b2) v2 z2 \right) + \mu1 \left(\mu2 - b1 b2 \mu2 - b1 v1 z1 + v2 z2 \right) \delta + \mu2 \left(v1 z1 - b2 v2 z2 \right) \delta \right)}{\mu1 z2 \left(b1 v1 z1 - v2 z2 \right) + \mu2 z1 \left(-v1 z1 + b2 v2 z2 \right)} + \mu1 \left(R - \frac{2 \left(v1 z1 \left(\mu1 \mu2 R \left((-1+b1) v1 z1 + (-1+b2) v2 z2 \right) + \mu1 \left(\mu2 - b1 b2 \mu2 - b1 v1 z1 + v2 z2 \right) \delta + \mu2 \left(v1 z1 - b2 v2 z2 \right) \delta \right) \right)}{\mu1 z2 \left(b1 v1 z1 - v2 z2 \right) + \mu2 z1 \left(-v1 z1 + b2 v2 z2 \right)} \right) & -\frac{b2 \mu2 \left(v1 z1 \left(-z1 + z2 \right) \delta + \mu1 z2 \left(-R v1 z1 + \delta \right) + \mu2 z1 \left(R v1 z1 - b2 v2 z2 \right) \right)}{\mu1 z2 \left(b1 v1 z1 - v2 z2 \right) + \mu2 z1 \left(-v1 z1 + b2 v2 z2 \right)} \\ 0 & 0 \end{pmatrix}$$

Again, we want an augmented matrix of the block-triangular form $\begin{pmatrix} J_{res} & u \\ 0 & J_{mut} \end{pmatrix}$. Importantly, our matrices should match, such that the upper 3x3 block of the augmented Jacobian is identical to the resident Jacobian. We will then define the 0 matrix as a matrix of form (0,0,0) in the bottom row of the augmented matrix, the u matrix comprising the top three values of the final column of the matrix, and the Jmut matrix is equal to the bottom far right value. These values, in this case, are shown below.

In[*]:= **Jmutcoop = FullSimplify[**

$$-\delta + \frac{v3 z3 \left((1+k2) \mu2 R v1 z1^2 - (1+k1) \mu1 z2 \left(R v1 z1 - \delta \right) + v1 z1 \left(-z1 + z2 \right) \delta \right)}{(1+k2) \mu2 v1 z1^2 + (1+k1) \mu1 v2 z2^2} \Big]$$

Jmutcomp =

$$\text{FullSimplify} \left[-\delta + \frac{v3 z3 \left(v1 z1 \left(-z1 + z2 \right) \delta + \mu1 z2 \left(-R v1 z1 + \delta \right) + \mu2 z1 \left(R v1 z1 - b2 v2 z2 \right) \right)}{\mu1 z2 \left(-b1 v1 z1 + v2 z2 \right) + \mu2 z1 \left(v1 z1 - b2 v2 z2 \right)} \right]$$

Out[*]=

$$-\delta + \frac{v3 z3 \left((1+k2) \mu2 R v1 z1^2 - (1+k1) \mu1 z2 \left(R v1 z1 - \delta \right) + v1 z1 \left(-z1 + z2 \right) \delta \right)}{(1+k2) \mu2 v1 z1^2 + (1+k1) \mu1 v2 z2^2}$$

Out[*]=

$$-\delta + \frac{v3 z3 \left(v1 z1 \left(-z1 + z2 \right) \delta + \mu1 z2 \left(-R v1 z1 + \delta \right) + \mu2 z1 \left(R v1 z1 - b2 v2 z2 \right) \right)}{\mu1 z2 \left(-b1 v1 z1 + v2 z2 \right) + \mu2 z1 \left(v1 z1 - b2 v2 z2 \right)}$$

Let's again define $v1*z1$ as β_{gen}^E , the infectivity of the generalist on E. coli, $v2*z2$ as β_{gen}^S , the infectivity of the generalist on S. enterica, and $v3*z3$ as β_{sp}^S , the infectivity of the specialist on S. enterica.

```
In[*]:= simplifiedcoop =
  -δ + (betaSsp ((1 + k2) mu2 R betaEgen² - (1 + k1) mu1 (betaSgen / v2) (R betaEgen - δ) +
    betaEgen (- (betaEgen / v1) + (betaSgen / v2) ) δ)) /
    ((1 + k2) mu2 betaEgen² + (1 + k1) mu1 betaSgen²) // FullSimplify

Out[*]=
  -δ +
  betaSsp (  $\frac{\text{betaSgen} (1+k1) \text{mu1} \delta}{v2} + \frac{\text{betaEgen} \text{betaSgen} (-(1+k1) \text{mu1} R) + \delta}{v2} + \text{betaEgen}^2 \left( (1+k2) \text{mu2} R - \frac{\delta}{v1} \right)$  )
  -----
  betaSgen² (1 + k1) mu1 + betaEgen² (1 + k2) mu2

In[*]:= Reduce[{simplifiedcoop > 0, betaSsp > 0, betaEgen > 0, betaSgen > 0,
  mu1 > 0, mu2 > 0, v1 > 0, v2 > 0, k1 > 0, k2 > 0, δ > 0}, Reals]

Out[*]=
  mu1 > 0 && betaEgen > 0 && k1 > 0 && k2 > 0 && betaSgen > 0 && mu2 > 0 &&
  (  $\left( 0 < v2 < \frac{\text{betaSgen} \text{mu1} + \text{betaSgen} k1 \text{mu1}}{\text{betaEgen} \text{mu2} + \text{betaEgen} k2 \text{mu2}} \right.$  && betaSsp > 0 && v1 > 0 &&
    δ > 0 && R <  $\frac{-\text{betaEgen} \text{betaSgen} \text{betaSsp} v1 \delta - \text{betaSgen} \text{betaSsp} \text{mu1} v1 \delta - \text{betaSgen} \text{betaSsp} k1 \text{mu1} v1 \delta + \text{betaEgen}^2 \text{betaSsp} v2 \delta + \text{betaSgen}^2 \text{mu1} v1 v2 \delta + \text{betaSgen}^2 k1 \text{mu1} v1 v2 \delta + \text{betaEgen}^2 \text{mu2} v1 v2 \delta + \text{betaEgen}^2 k2 \text{mu2} v1 v2 \delta}{(-\text{betaEgen} \text{betaSgen} \text{betaSsp} \text{mu1} v1 - \text{betaEgen} \text{betaSgen} \text{betaSsp} k1 \text{mu1} v1 + \text{betaEgen}^2 \text{betaSsp} \text{mu2} v1 v2 + \text{betaEgen}^2 \text{betaSsp} k2 \text{mu2} v1 v2)}$  ) ||
    ( v2 ==  $\frac{\text{betaSgen} \text{mu1} + \text{betaSgen} k1 \text{mu1}}{\text{betaEgen} \text{mu2} + \text{betaEgen} k2 \text{mu2}}$  && betaSsp >
       $\frac{\text{betaSgen}^2 \text{mu1} v2 + \text{betaSgen}^2 k1 \text{mu1} v2 + \text{betaEgen}^2 \text{mu2} v2 + \text{betaEgen}^2 k2 \text{mu2} v2}{\text{betaEgen} \text{betaSgen} + \text{betaSgen} \text{mu1} + \text{betaSgen} k1 \text{mu1}}$  &&
      v1 > -  $\left( \frac{\text{betaEgen}^2 \text{betaSsp} v2}{(-\text{betaEgen} \text{betaSgen} \text{betaSsp} - \text{betaSgen} \text{betaSsp} \text{mu1} - \text{betaSgen} \text{betaSsp} k1 \text{mu1} + \text{betaSgen}^2 \text{mu1} v2 + \text{betaSgen}^2 k1 \text{mu1} v2 + \text{betaEgen}^2 \text{mu2} v2 + \text{betaEgen}^2 k2 \text{mu2} v2)} \right)$  && δ > 0 ) ||
      ( v2 >  $\frac{\text{betaSgen} \text{mu1} + \text{betaSgen} k1 \text{mu1}}{\text{betaEgen} \text{mu2} + \text{betaEgen} k2 \text{mu2}}$  && betaSsp > 0 && v1 > 0 && δ > 0 &&
        R >  $\frac{-\text{betaEgen} \text{betaSgen} \text{betaSsp} v1 \delta - \text{betaSgen} \text{betaSsp} \text{mu1} v1 \delta - \text{betaSgen} \text{betaSsp} k1 \text{mu1} v1 \delta + \text{betaEgen}^2 \text{betaSsp} v2 \delta + \text{betaSgen}^2 \text{mu1} v1 v2 \delta + \text{betaSgen}^2 k1 \text{mu1} v1 v2 \delta + \text{betaEgen}^2 \text{mu2} v1 v2 \delta + \text{betaEgen}^2 k2 \text{mu2} v1 v2 \delta}{(-\text{betaEgen} \text{betaSgen} \text{betaSsp} \text{mu1} v1 - \text{betaEgen} \text{betaSgen} \text{betaSsp} k1 \text{mu1} v1 + \text{betaEgen}^2 \text{betaSsp} \text{mu2} v1 v2 + \text{betaEgen}^2 \text{betaSsp} k2 \text{mu2} v1 v2)}$  ) ) ) ) )
```


These results indicate that anywhere both bacteria can grow and the generalist can infect both strains, then a specialist can invade depending on v_2 , v_1 , the infectivity of the specialist on S , the dilution, and the amount of resources.

In[*]:= **simplifiedcomp** =

$$-\delta + (\text{betaSsp} (\text{betaEgen} (- (\text{betaEgen} / v_1) + (\text{betaSgen} / v_2)) \delta + \mu_1 (\text{betaSgen} / v_2) \\ (-R \text{betaEgen} + \delta) + \mu_2 (\text{betaEgen} / v_1) (R \text{betaEgen} - b_2 \delta)) / \\ (\mu_1 (\text{betaSgen} / v_2) (-b_1 \text{betaEgen} + \text{betaSgen}) + \mu_2 (\text{betaEgen} / v_1) \\ (\text{betaEgen} - b_2 \text{betaSgen})) // \text{FullSimplify}$$

Out[*]=

$$-\delta + (\text{betaSsp} (\text{betaEgen} \text{betaSgen} v_1 (\mu_1 R - \delta) - \\ \text{betaSgen} \mu_1 v_1 \delta + b_2 \text{betaEgen} \mu_2 v_2 \delta + \text{betaEgen}^2 v_2 (-\mu_2 R + \delta))) / \\ ((b_1 \text{betaEgen} - \text{betaSgen}) \text{betaSgen} \mu_1 v_1 + \text{betaEgen} (-\text{betaEgen} + b_2 \text{betaSgen}) \mu_2 v_2)$$

In[*]:= **Reduce**[{**simplifiedcomp** > 0, **b1** > 0, **b2** > 0, **mu1** > 0, **mu2** > 0, **betaSsp** > 0, **betaEgen** > 0, **betaSgen** > 0, **v1** > 0, **v2** > 0, **δ** > 0, **R** > 0}, **Reals**]

Out[*]=

$$\text{betaEgen} > 0 \ \&\& \ v_1 > 0 \ \&\& \ \text{betaSgen} > 0 \ \&\& \left(\left(0 < v_2 < \frac{\text{betaSgen} v_1}{\text{betaEgen}} \ \&\& \right. \right. \\ \left. \left(0 < \mu_1 < \frac{\text{betaSgen} v_1 - \text{betaEgen} v_2}{v_1} \ \&\& \left(0 < \mu_2 < \frac{\text{betaSgen} \mu_1 v_1}{\text{betaEgen} v_2} \ \&\& \right. \right. \right. \\ \left. \left(0 < b_2 < \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_2 v_2} \ \&\& \left(0 < b_1 < \right. \right. \right. \\ \left. \left. \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \right. \right. \\ \left. \&\& R > 0 \ \&\& \text{betaSsp} > \right. \\ \left. (b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2) / (-\text{betaEgen} \text{betaSgen} v_1 - \right. \\ \left. \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2) \ \&\& \right. \\ \delta > (-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \text{betaEgen}^2 \text{betaSsp} \\ \mu_2 R v_2) / (-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - b_1 \text{betaEgen} \\ \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \text{betaSgen} \text{betaSsp} \mu_1 \\ v_1 + \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \\ \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2) \left. \right) \ \&\& \left(b_1 > \right. \\ \left. \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \right. \\ \left. \&\& R > 0 \ \&\& \text{betaSsp} > 0 \ \&\& \right. \\ 0 < \delta < (-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \text{betaEgen}^2 \\ \text{betaSsp} \mu_2 R v_2) / (-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - \\ b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 -$$

$$\begin{aligned}
& \left(\left(\left(\left(\left(\frac{\text{betaSgen betaSsp mu1 v1} + \text{betaEgen}^2 \text{ betaSsp v2} + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \text{betaEgen}^2 \text{ mu2 v2} - \text{b2 betaEgen betaSgen mu2 v2} + \text{b2 betaEgen} \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \text{betaSsp mu2 v2} \right) \right) \right) \right) \right) \right) \vee \left(\frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \leq \right. \\
& \left. \text{b2} \leq \frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} \right) \& \\
& \text{b1} > 0 \& \& \text{R} > 0 \& \& \text{betaSsp} > 0 \& \& 0 < \delta < \\
& \left(-\text{betaEgen betaSgen betaSsp mu1 R v1} + \text{betaEgen}^2 \text{ betaSsp mu2 R v2} \right) / \\
& \left(-\text{betaEgen betaSgen betaSsp v1} - \text{b1 betaEgen betaSgen mu1 v1} + \right. \\
& \left. \text{betaSgen}^2 \text{ mu1 v1} - \text{betaSgen betaSsp mu1 v1} + \right. \\
& \left. \text{betaEgen}^2 \text{ betaSsp v2} + \text{betaEgen}^2 \text{ mu2 v2} - \right. \\
& \left. \left. \left. \left. \left. \text{b2 betaEgen betaSgen mu2 v2} + \text{b2 betaEgen betaSsp mu2 v2} \right) \right) \right) \right) \vee \\
& \left(\text{b2} > \frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} \right) \& \\
& \text{b1} > 0 \& \& \text{R} > 0 \& \& \left(0 < \text{betaSsp} < \left(\text{b1 betaEgen betaSgen mu1 v1} - \right. \right. \\
& \left. \left. \text{betaSgen}^2 \text{ mu1 v1} - \text{betaEgen}^2 \text{ mu2 v2} + \right. \right. \\
& \left. \left. \text{b2 betaEgen betaSgen mu2 v2} \right) / \left(-\text{betaEgen betaSgen v1} - \right. \right. \\
& \left. \left. \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + \text{b2 betaEgen mu2 v2} \right) \right) \& \\
& 0 < \delta < \left(-\text{betaEgen betaSgen betaSsp mu1 R v1} + \text{betaEgen}^2 \right. \\
& \left. \text{betaSsp mu2 R v2} \right) / \left(-\text{betaEgen betaSgen betaSsp v1} - \right. \\
& \left. \text{b1 betaEgen betaSgen mu1 v1} + \text{betaSgen}^2 \text{ mu1 v1} - \text{betaSgen} \right. \\
& \left. \text{betaSsp mu1 v1} + \text{betaEgen}^2 \text{ betaSsp v2} + \text{betaEgen}^2 \text{ mu2 v2} - \right. \\
& \left. \left. \left. \left. \left. \text{b2 betaEgen betaSgen mu2 v2} + \text{b2 betaEgen betaSsp mu2 v2} \right) \right) \right) \right) \vee \\
& \left(\text{betaSsp} \geq \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \right. \\
& \left. \left. \text{betaEgen}^2 \text{ mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \right. \\
& \left(-\text{betaEgen betaSgen v1} - \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + \right. \\
& \left. \left. \left. \left. \left. \text{b2 betaEgen mu2 v2} \right) \right) \right) \right) \& \delta > 0 \right) \right) \vee \left(\text{mu2} = \frac{\text{betaSgen mu1 v1}}{\text{betaEgen v2}} \right) \& \\
& \left(0 < \text{b2} < \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \right) \& 0 < \text{b1} < \\
& \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2} - \text{b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \& \\
& \text{R} > 0 \& \& \text{betaSsp} > \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \\
& \left. \text{betaEgen}^2 \text{ mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \\
& \left(-\text{betaEgen betaSgen v1} - \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + \right. \\
& \left. \left. \left. \left. \left. \text{b2 betaEgen mu2 v2} \right) \right) \right) \right) \& \delta > 0 \right) \vee
\end{aligned}$$

$$\begin{aligned}
& \left(b2 > \frac{\text{betaEgen betaSgen } v1 + \text{betaSgen } \mu1 \, v1 - \text{betaEgen}^2 \, v2}{\text{betaEgen } \mu2 \, v2} \& \right. \\
& \quad b1 > 0 \& R > 0 \& \text{betaSsp} > \left(b1 \text{ betaEgen betaSgen } \mu1 \, v1 - \text{betaSgen}^2 \right. \\
& \quad \quad \mu1 \, v1 - \text{betaEgen}^2 \, \mu2 \, v2 + b2 \text{ betaEgen betaSgen } \mu2 \, v2 \Big) / \\
& \quad \quad \left(-\text{betaEgen betaSgen } v1 - \text{betaSgen } \mu1 \, v1 + \text{betaEgen}^2 \, v2 + \right. \\
& \quad \quad \quad \left. b2 \text{ betaEgen } \mu2 \, v2 \right) \& \delta > 0 \Big) \Big) \Big) \Big) || \\
& \left(\frac{\text{betaSgen } \mu1 \, v1}{\text{betaEgen } v2} < \mu2 < \frac{\text{betaSgen}^2 \, v1 - \text{betaEgen betaSgen } v2}{\text{betaEgen } v2} \& \right. \\
& \quad \left(\left(0 < b2 < \frac{\text{betaSgen}^2 \, \mu1 \, v1 + \text{betaEgen}^2 \, \mu2 \, v2}{\text{betaEgen betaSgen } \mu2 \, v2} \& 0 < b1 < \right. \right. \\
& \quad \quad \frac{\text{betaSgen}^2 \, \mu1 \, v1 + \text{betaEgen}^2 \, \mu2 \, v2 - b2 \text{ betaEgen betaSgen } \mu2 \, v2}{\text{betaEgen betaSgen } \mu1 \, v1} \& \\
& \quad \quad R > 0 \& \left(0 < \text{betaSsp} < \left(b1 \text{ betaEgen betaSgen } \mu1 \, v1 - \right. \right. \\
& \quad \quad \quad \text{betaSgen}^2 \, \mu1 \, v1 - \text{betaEgen}^2 \, \mu2 \, v2 + \\
& \quad \quad \quad b2 \text{ betaEgen betaSgen } \mu2 \, v2 \Big) / \left(-\text{betaEgen betaSgen } v1 - \right. \\
& \quad \quad \quad \text{betaSgen } \mu1 \, v1 + \text{betaEgen}^2 \, v2 + b2 \text{ betaEgen } \mu2 \, v2 \Big) \& \\
& \quad \quad 0 < \delta < \left(-\text{betaEgen betaSgen betaSsp } \mu1 \, R \, v1 + \text{betaEgen}^2 \right. \\
& \quad \quad \quad \text{betaSsp } \mu2 \, R \, v2 \Big) / \left(-\text{betaEgen betaSgen betaSsp } v1 - \right. \\
& \quad \quad \quad b1 \text{ betaEgen betaSgen } \mu1 \, v1 + \text{betaSgen}^2 \, \mu1 \, v1 - \text{betaSgen} \\
& \quad \quad \quad \text{betaSsp } \mu1 \, v1 + \text{betaEgen}^2 \, \text{betaSsp } v2 + \text{betaEgen}^2 \, \mu2 \, v2 - \\
& \quad \quad \quad b2 \text{ betaEgen betaSgen } \mu2 \, v2 + b2 \text{ betaEgen betaSsp } \mu2 \, v2 \Big) \Big) \Big) || \\
& \quad \left(\text{betaSsp} \geq \left(b1 \text{ betaEgen betaSgen } \mu1 \, v1 - \text{betaSgen}^2 \, \mu1 \, v1 - \right. \right. \\
& \quad \quad \text{betaEgen}^2 \, \mu2 \, v2 + b2 \text{ betaEgen betaSgen } \mu2 \, v2 \Big) / \\
& \quad \quad \left(-\text{betaEgen betaSgen } v1 - \text{betaSgen } \mu1 \, v1 + \right. \\
& \quad \quad \quad \left. \text{betaEgen}^2 \, v2 + b2 \text{ betaEgen } \mu2 \, v2 \right) \& \delta > 0 \Big) \Big) \Big) || \\
& \left(b2 > \frac{\text{betaEgen betaSgen } v1 + \text{betaSgen } \mu1 \, v1 - \text{betaEgen}^2 \, v2}{\text{betaEgen } \mu2 \, v2} \& \right. \\
& \quad b1 > 0 \& R > 0 \& \text{betaSsp} > \\
& \quad \left(b1 \text{ betaEgen betaSgen } \mu1 \, v1 - \text{betaSgen}^2 \, \mu1 \, v1 - \text{betaEgen}^2 \, \mu2 \, v2 + \right. \\
& \quad \quad b2 \text{ betaEgen betaSgen } \mu2 \, v2 \Big) / \left(-\text{betaEgen betaSgen } v1 - \right. \\
& \quad \quad \text{betaSgen } \mu1 \, v1 + \text{betaEgen}^2 \, v2 + b2 \text{ betaEgen } \mu2 \, v2 \Big) \& \delta > \\
& \quad \left(-\text{betaEgen betaSgen betaSsp } \mu1 \, R \, v1 + \text{betaEgen}^2 \, \text{betaSsp } \mu2 \, R \, v2 \right) / \\
& \quad \left(-\text{betaEgen betaSgen betaSsp } v1 - b1 \text{ betaEgen betaSgen } \mu1 \, v1 + \right. \\
& \quad \quad \text{betaSgen}^2 \, \mu1 \, v1 - \text{betaSgen betaSsp } \mu1 \, v1 + \\
& \quad \quad \text{betaEgen}^2 \, \text{betaSsp } v2 + \text{betaEgen}^2 \, \mu2 \, v2 -
\end{aligned}$$

$$\begin{aligned}
& \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \& \\
& R > 0 \& \left(\left(0 < \text{betaSsp} < \left(b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \right. \right. \right. \\
& \quad \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + \\
& \quad \left. \left. b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 \right) / \left(-\text{betaEgen} \text{betaSgen} v_1 - \right. \right. \\
& \quad \left. \left. \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2 \right) \& \right. \\
& \quad \left. 0 < \delta < \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \text{betaEgen}^2 \right. \right. \\
& \quad \left. \left. \text{betaSsp} \mu_2 R v_2 \right) / \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - \right. \right. \\
& \quad \left. \left. b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \text{betaSgen} \right. \right. \\
& \quad \left. \left. \text{betaSsp} \mu_1 v_1 + \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - \right. \right. \\
& \quad \left. \left. b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2 \right) \right) \mid \mid \\
& \left(\text{betaSsp} \geq \left(b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \right. \right. \\
& \quad \left. \left. \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 \right) / \right. \\
& \quad \left. \left(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \right. \right. \\
& \quad \left. \left. \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2 \right) \& \delta > 0 \right) \mid \mid \\
& \left(b_2 = \frac{\text{betaEgen} \text{betaSgen} v_1 + \text{betaSgen} \mu_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen} \mu_2 v_2} \& 0 < b_1 < \right. \\
& \quad \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \& \\
& \quad R > 0 \& \text{betaSsp} > 0 \& 0 < \delta < \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \right. \\
& \quad \left. \text{betaEgen}^2 \text{betaSsp} \mu_2 R v_2 \right) / \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - \right. \\
& \quad \left. b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \right. \\
& \quad \left. \text{betaSgen} \text{betaSsp} \mu_1 v_1 + \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - \right. \\
& \quad \left. \left. b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2 \right) \right) \mid \mid \\
& \left(\frac{\text{betaEgen} \text{betaSgen} v_1 + \text{betaSgen} \mu_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen} \mu_2 v_2} < b_2 < \right. \\
& \quad \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_2 v_2} \& \left(\left(0 < b_1 < \right. \right. \\
& \quad \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \\
& \quad \& R > 0 \& \text{betaSsp} > 0 \& \\
& \quad \left. 0 < \delta < \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \text{betaEgen}^2 \text{betaSsp} \right. \right. \\
& \quad \left. \left. \mu_2 R v_2 \right) / \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - b_1 \text{betaEgen} \right. \right. \\
& \quad \left. \left. \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \text{betaSgen} \text{betaSsp} \mu_1 \right. \right. \\
& \quad \left. \left. v_1 + \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \right. \right. \\
& \quad \left. \left. \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2 \right) \right) \mid \mid \left(b_1 > \right.
\end{aligned}$$

[illegible]

$$\begin{aligned}
& \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen betaSgen mu}_2 v_2}{\text{betaEgen betaSgen mu}_1 v_1} \\
& \& R > 0 \& \text{betaSsp} > 0 \& \\
& 0 < \delta < \left(-\text{betaEgen betaSgen betaSsp mu}_1 R v_1 + \text{betaEgen}^2 \right. \\
& \quad \text{betaSsp mu}_2 R v_2) / (-\text{betaEgen betaSgen betaSsp } v_1 - \\
& \quad b_1 \text{betaEgen betaSgen mu}_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \\
& \quad \text{betaSgen betaSsp mu}_1 v_1 + \text{betaEgen}^2 \text{betaSsp } v_2 + \\
& \quad \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen betaSgen mu}_2 v_2 + b_2 \text{betaEgen} \\
& \quad \left. \text{betaSsp mu}_2 v_2) \right) || \left(\frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2}{\text{betaEgen betaSgen mu}_2 v_2} \leq \right. \\
& \quad b_2 \leq \frac{\text{betaEgen betaSgen } v_1 + \text{betaSgen mu}_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen mu}_2 v_2} \& \\
& b_1 > 0 \& R > 0 \& \text{betaSsp} > 0 \& 0 < \delta < \\
& \left(-\text{betaEgen betaSgen betaSsp mu}_1 R v_1 + \text{betaEgen}^2 \text{betaSsp mu}_2 R v_2) / \right. \\
& \left(-\text{betaEgen betaSgen betaSsp } v_1 - b_1 \text{betaEgen betaSgen mu}_1 v_1 + \right. \\
& \quad \text{betaSgen}^2 \mu_1 v_1 - \text{betaSgen betaSsp mu}_1 v_1 + \\
& \quad \text{betaEgen}^2 \text{betaSsp } v_2 + \text{betaEgen}^2 \mu_2 v_2 - \\
& \quad \left. b_2 \text{betaEgen betaSgen mu}_2 v_2 + b_2 \text{betaEgen betaSsp mu}_2 v_2) \right) || \\
& \left(b_2 > \frac{\text{betaEgen betaSgen } v_1 + \text{betaSgen mu}_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen mu}_2 v_2} \& \right. \\
& b_1 > 0 \& R > 0 \& \left((0 < \text{betaSsp} < (b_1 \text{betaEgen betaSgen mu}_1 v_1 - \right. \\
& \quad \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + \\
& \quad b_2 \text{betaEgen betaSgen mu}_2 v_2) / (-\text{betaEgen betaSgen } v_1 - \\
& \quad \text{betaSgen mu}_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen mu}_2 v_2) \& \\
& \quad 0 < \delta < (-\text{betaEgen betaSgen betaSsp mu}_1 R v_1 + \text{betaEgen}^2 \\
& \quad \text{betaSsp mu}_2 R v_2) / (-\text{betaEgen betaSgen betaSsp } v_1 - \\
& \quad b_1 \text{betaEgen betaSgen mu}_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \text{betaSgen} \\
& \quad \text{betaSsp mu}_1 v_1 + \text{betaEgen}^2 \text{betaSsp } v_2 + \text{betaEgen}^2 \mu_2 v_2 - \\
& \quad b_2 \text{betaEgen betaSgen mu}_2 v_2 + b_2 \text{betaEgen betaSsp mu}_2 v_2) \& || \\
& \quad (\text{betaSsp} \geq (b_1 \text{betaEgen betaSgen mu}_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \\
& \quad \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen betaSgen mu}_2 v_2) / \\
& \quad (-\text{betaEgen betaSgen } v_1 - \text{betaSgen mu}_1 v_1 + \\
& \quad \text{betaEgen}^2 v_2 + b_2 \text{betaEgen mu}_2 v_2) \& \delta > 0)) \& \& \& \& \& || \\
& \mu_2 = \frac{\text{betaSgen}^2 v_1 - \text{betaEgen betaSgen } v_2}{\text{betaEgen } v_2} \& \left(\left(0 < b_2 < \right. \right. \\
& \quad \left. \frac{\text{betaEgen betaSgen } v_1 + \text{betaSgen mu}_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen mu}_2 v_2} \& 0 < b_1 < \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \& \\
& R > 0 \& \text{betaSsp} > \left(\frac{b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2)} \& \delta > 0 \right) || \\
& \left(b_2 > \frac{\text{betaEgen} \text{betaSgen} v_1 + \text{betaSgen} \mu_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen} \mu_2 v_2} \& \right. \\
& \left. b_1 > 0 \& R > 0 \& \text{betaSsp} > \left(\frac{b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2)} \& \delta > 0 \right) \right) || \\
& \left(\mu_2 > \frac{\text{betaSgen}^2 v_1 - \text{betaEgen} \text{betaSgen} v_2}{\text{betaEgen} v_2} \& \left(\left(0 < b_2 < \frac{\text{betaEgen} \text{betaSgen} v_1 + \text{betaSgen} \mu_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen} \mu_2 v_2} \& 0 < b_1 < \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \& \right. \right. \right. \\
& \left. \left. R > 0 \& \left(\left(0 < \text{betaSsp} < \left(\frac{b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2)} \& \right. \right. \right. \right. \\
& \left. \left. \left. 0 < \delta < \left(\frac{-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \text{betaEgen}^2 \text{betaSsp} \mu_2 R v_2}{(-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \text{betaSgen} \text{betaSsp} \mu_1 v_1 + \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2)} \right) \right) \right) || \right. \\
& \left. \left(\text{betaSsp} \geq \left(\frac{b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2)} \& \delta > 0 \right) \right) || \right. \\
& \left. \left(b_2 = \frac{\text{betaEgen} \text{betaSgen} v_1 + \text{betaSgen} \mu_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen} \mu_2 v_2} \& 0 < b_1 < \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \& \right. \right.
\end{aligned}$$

$$\begin{aligned}
& R > 0 \ \&\& \beta_{\text{Ssp}} > 0 \ \&\& 0 < \delta < \left(\frac{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 R v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} \mu_2 R v_2}{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} v_1 - b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} v_2 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Ssp}} \mu_2 v_2} \right) \mid \mid \\
& \left(\frac{\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 + \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Egen}}^2 v_2}{\beta_{\text{Egen}} \mu_2 v_2} < b_2 < \right. \\
& \quad \frac{\beta_{\text{Sgen}}^2 \mu_1 v_1 + \beta_{\text{Egen}}^2 \mu_2 v_2}{\beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2} \ \&\& \left(\left(0 < b_1 < \right. \right. \\
& \quad \frac{\beta_{\text{Sgen}}^2 \mu_1 v_1 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{\beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1} \\
& \quad \&\& R > 0 \ \&\& \beta_{\text{Ssp}} > 0 \ \&\& \\
& \quad 0 < \delta < \left(\frac{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 R v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} \mu_2 R v_2}{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} v_1 - b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} v_2 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Ssp}} \mu_2 v_2} \right) \mid \mid \left(b_1 > \right. \\
& \quad \frac{\beta_{\text{Sgen}}^2 \mu_1 v_1 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{\beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1} \\
& \quad \&\& R > 0 \ \&\& \beta_{\text{Ssp}} > \\
& \quad \left(\frac{b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Egen}}^2 \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{-\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 - \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Egen}}^2 v_2 + b_2 \beta_{\text{Egen}} \mu_2 v_2} \right) \ \&\& \\
& \quad \delta > \left(\frac{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 R v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} \mu_2 R v_2}{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} v_1 - b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} v_2 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Ssp}} \mu_2 v_2} \right) \mid \mid \\
& \quad \left(b_2 \geq \frac{\beta_{\text{Sgen}}^2 \mu_1 v_1 + \beta_{\text{Egen}}^2 \mu_2 v_2}{\beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2} \ \&\& b_1 > 0 \ \&\& R > 0 \ \&\& \right. \\
& \quad \beta_{\text{Ssp}} > \left(\frac{b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Egen}}^2 \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{-\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 - \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Egen}}^2 v_2 + b_2 \beta_{\text{Egen}} \mu_2 v_2} \right) \ \&\& \delta > \\
& \quad \left(\frac{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 R v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} \mu_2 R v_2}{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} v_1 - b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} v_2 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Ssp}} \mu_2 v_2} \right) \mid \mid
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2 +}{\right. \\
& \quad \left. b_2 \beta_{Egen} \beta_{Ssp} \mu_2 v_2) \right) \Bigg| \Bigg| \left(\mu_1 > \right. \\
& \quad \left. \frac{\mu_1 v_1 - \beta_{Egen} v_2}{v_1} \& \left(\left(0 < \mu_2 < \frac{\beta_{Sgen}^2 v_1 - \beta_{Egen} \beta_{Sgen} v_2}{\beta_{Egen} v_2} \& \right. \right. \right. \\
& \quad \left. \left. b_2 < \frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_2 v_2} \& \left(\left(0 < b_1 < \right. \right. \right. \right. \\
& \quad \left. \left. \frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_1 v_1} \right. \right. \\
& \quad \left. \& R > 0 \& \beta_{Ssp} > \right. \\
& \quad \left(b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 - \beta_{Sgen}^2 \mu_1 v_1 - \beta_{Egen}^2 \mu_2 \right. \\
& \quad \quad \left. v_2 + b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2) / (-\beta_{Egen} \beta_{Sgen} v_1 - \right. \\
& \quad \quad \left. \beta_{Sgen} \mu_1 v_1 + \beta_{Egen}^2 v_2 + b_2 \beta_{Egen} \mu_2 v_2) \& \right. \\
& \quad \delta > (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} \mu_1 R v_1 + \beta_{Egen}^2 \beta_{Ssp} \\
& \quad \quad \mu_2 R v_2) / (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} v_1 - b_1 \beta_{Egen} \\
& \quad \quad \beta_{Sgen} \mu_1 v_1 + \beta_{Sgen}^2 \mu_1 v_1 - \beta_{Sgen} \beta_{Ssp} \mu_1 \\
& \quad \quad v_1 + \beta_{Egen}^2 \beta_{Ssp} v_2 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \\
& \quad \quad \beta_{Sgen} \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Ssp} \mu_2 v_2) \Bigg| \Bigg| \left(b_1 > \right. \\
& \quad \left. \frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_1 v_1} \right. \\
& \quad \left. \& R > 0 \& \beta_{Ssp} > 0 \& \right. \\
& \quad 0 < \delta < (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} \mu_1 R v_1 + \beta_{Egen}^2 \\
& \quad \quad \beta_{Ssp} \mu_2 R v_2) / (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} v_1 - \\
& \quad \quad b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 + \beta_{Sgen}^2 \mu_1 v_1 - \\
& \quad \quad \beta_{Sgen} \beta_{Ssp} \mu_1 v_1 + \beta_{Egen}^2 \beta_{Ssp} v_2 + \\
& \quad \quad \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2 + b_2 \beta_{Egen} \\
& \quad \quad \beta_{Ssp} \mu_2 v_2) \Bigg) \Bigg) \Bigg| \Bigg| \left(\frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_2 v_2} \leq \right. \\
& \quad \left. b_2 \leq \frac{\beta_{Egen} \beta_{Sgen} v_1 + \beta_{Sgen} \mu_1 v_1 - \beta_{Egen}^2 v_2}{\beta_{Egen} \mu_2 v_2} \& \right. \\
& \quad \left. b_1 > 0 \& R > 0 \& \beta_{Ssp} > 0 \& 0 < \delta < \right. \\
& \quad \left(-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} \mu_1 R v_1 + \beta_{Egen}^2 \beta_{Ssp} \mu_2 R v_2) / \right. \\
& \quad \left(-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} v_1 - b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 + \right. \\
& \quad \quad \beta_{Sgen}^2 \mu_1 v_1 - \beta_{Sgen} \beta_{Ssp} \mu_1 v_1 + \\
& \quad \quad \beta_{Egen}^2 \beta_{Ssp} v_2 + \beta_{Egen}^2 \mu_2 v_2 - \\
& \quad \quad \left. b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Ssp} \mu_2 v_2) \Bigg) \Bigg| \Bigg|
\end{aligned}$$

$$\left(b_2 > \frac{\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 + \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Egen}}^2 v_2}{\beta_{\text{Egen}} \mu_2 v_2} \&\& \right.$$
$$b_1 > 0 \&\& R > 0 \&\& \left((0 < \beta_{\text{Ssp}} < (\beta_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Egen}}^2 \mu_2 v_2 + \right.$$
$$\beta_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2) / (-\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 - \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Egen}}^2 v_2 + \beta_2 \beta_{\text{Egen}} \mu_2 v_2) \&\&$$
$$0 < \delta < (-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 R v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} \mu_2 R v_2) / (-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} v_1 -$$
$$\beta_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} v_2 + \beta_{\text{Egen}}^2 \mu_2 v_2 -$$
$$\beta_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2 + \beta_2 \beta_{\text{Egen}} \beta_{\text{Ssp}} \mu_2 v_2)) ||$$
$$(\beta_{\text{Ssp}} \geq (\beta_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Egen}}^2 \mu_2 v_2 + \beta_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2) /$$
$$(-\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 - \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Egen}}^2 v_2 + \beta_2 \beta_{\text{Egen}} \mu_2 v_2) \&\& \delta > 0)) \Bigg) ||$$
$$\left(\mu_2 = \frac{\beta_{\text{Sgen}}^2 v_1 - \beta_{\text{Egen}} \beta_{\text{Sgen}} v_2}{\beta_{\text{Egen}} v_2} \&\& \left((0 < b_2 < \right.$$
$$\frac{\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 + \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Egen}}^2 v_2}{\beta_{\text{Egen}} \mu_2 v_2} \&\& \left((0 < b_1 < \right.$$
$$\frac{\beta_{\text{Sgen}}^2 \mu_1 v_1 + \beta_{\text{Egen}}^2 \mu_2 v_2 - \beta_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{\beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1}$$
$$\&\& R > 0 \&\& \beta_{\text{Ssp}} >$$
$$(\beta_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Egen}}^2 \mu_2 v_2 + \beta_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2) / (-\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 - \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Egen}}^2 v_2 + \beta_2 \beta_{\text{Egen}} \mu_2 v_2) \&\&$$
$$\delta > (-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 R v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} \mu_2 R v_2) / (-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} v_1 - \beta_1 \beta_{\text{Egen}}$$
$$\beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} v_2 + \beta_{\text{Egen}}^2 \mu_2 v_2 - \beta_2 \beta_{\text{Egen}}$$
$$\beta_{\text{Sgen}} \mu_2 v_2 + \beta_2 \beta_{\text{Egen}} \beta_{\text{Ssp}} \mu_2 v_2)) || \left(b_1 >$$
$$\frac{\beta_{\text{Sgen}}^2 \mu_1 v_1 + \beta_{\text{Egen}}^2 \mu_2 v_2 - \beta_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{\beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1}$$
$$\&\& R > 0 \&\& \beta_{\text{Ssp}} > 0 \&\&$$
$$0 < \delta < (-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 R v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} \mu_2 R v_2) / (-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} v_1 - \beta_1 \beta_{\text{Egen}}$$
$$\beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} v_2 + \beta_{\text{Egen}}^2 \mu_2 v_2 - \beta_2 \beta_{\text{Egen}}$$

[illegible]

$$\begin{aligned}
& \text{betaSgen mu1 v1 + betaEgen}^2 \text{ v2 + b2 betaEgen mu2 v2) \&\& } \\
& \delta > \left(\frac{-\text{betaEgen betaSgen betaSsp mu1 R v1 + betaEgen}^2 \text{ betaSsp mu2 R v2}}{-\text{betaEgen betaSgen betaSsp v1 - b1 betaEgen betaSgen mu1 v1 + betaSgen}^2 \text{ mu1 v1 - betaSgen betaSsp mu1 v1 + betaEgen}^2 \text{ betaSsp v2 + betaEgen}^2 \text{ mu2 v2 - b2 betaEgen betaSgen mu2 v2 + b2 betaEgen betaSsp mu2 v2}} \right) || \left(\text{b1} > \right. \\
& \left. \frac{\text{betaSgen}^2 \text{ mu1 v1 + betaEgen}^2 \text{ mu2 v2 - b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \right. \\
& \left. \&\& \text{R} > 0 \&\& \text{betaSsp} > 0 \&\& \right. \\
& 0 < \delta < \left(\frac{-\text{betaEgen betaSgen betaSsp mu1 R v1 + betaEgen}^2 \text{ betaSsp mu2 R v2}}{-\text{betaEgen betaSgen betaSsp v1 - b1 betaEgen betaSgen mu1 v1 + betaSgen}^2 \text{ mu1 v1 - betaSgen betaSsp mu1 v1 + betaEgen}^2 \text{ betaSsp v2 + betaEgen}^2 \text{ mu2 v2 - b2 betaEgen betaSgen mu2 v2 + b2 betaEgen betaSsp mu2 v2}} \right) || \\
& \left(\text{b2} = \frac{\text{betaEgen betaSgen v1 + betaSgen mu1 v1 - betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} \&\& \text{b1} > \right. \\
& \left. \frac{\text{betaSgen}^2 \text{ mu1 v1 + betaEgen}^2 \text{ mu2 v2 - b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \&\& \right. \\
& \left. \text{R} > 0 \&\& \text{betaSsp} > 0 \&\& 0 < \delta < \left(\frac{-\text{betaEgen betaSgen betaSsp mu1 R v1 + betaEgen}^2 \text{ betaSsp mu2 R v2}}{-\text{betaEgen betaSgen betaSsp v1 - b1 betaEgen betaSgen mu1 v1 + betaSgen}^2 \text{ mu1 v1 - betaSgen betaSsp mu1 v1 + betaEgen}^2 \text{ betaSsp v2 + betaEgen}^2 \text{ mu2 v2 - b2 betaEgen betaSgen mu2 v2 + b2 betaEgen betaSsp mu2 v2}} \right) || \right. \\
& \left(\frac{\text{betaEgen betaSgen v1 + betaSgen mu1 v1 - betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} < \right. \\
& \left. \text{b2} < \frac{\text{betaSgen}^2 \text{ mu1 v1 + betaEgen}^2 \text{ mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \&\& \text{b1} > \right. \\
& \left. \frac{\text{betaSgen}^2 \text{ mu1 v1 + betaEgen}^2 \text{ mu2 v2 - b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \&\& \right. \\
& \left. \text{R} > 0 \&\& \left(0 < \text{betaSsp} < \left(\text{b1 betaEgen betaSgen mu1 v1 - betaSgen}^2 \text{ mu1 v1 - betaEgen}^2 \text{ mu2 v2 + b2 betaEgen betaSgen mu2 v2} \right) / \left(-\text{betaEgen betaSgen v1 - betaSgen mu1 v1 + betaEgen}^2 \text{ v2 + b2 betaEgen mu2 v2} \right) \&\& \right. \right. \\
& \left. 0 < \delta < \left(\frac{-\text{betaEgen betaSgen betaSsp mu1 R v1 + betaEgen}^2 \text{ betaSsp mu2 R v2}}{-\text{betaEgen betaSgen betaSsp v1 - b1 betaEgen betaSgen mu1 v1 + betaSgen}^2 \text{ mu1 v1 - betaSgen betaSsp mu1 v1 + betaEgen}^2 \text{ betaSsp v2 + betaEgen}^2 \text{ mu2 v2 - b2 betaEgen betaSgen mu2 v2 + b2 betaEgen betaSsp mu2 v2}} \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\text{betaSsp mu1 v1} + \text{betaEgen}^2 \text{ betaSsp v2} + \text{betaEgen}^2 \text{ mu2 v2} - \right. \\
& \quad \left. \text{b2 betaEgen betaSgen mu2 v2} + \text{b2 betaEgen betaSsp mu2 v2} \right) || \\
& \left(\text{betaSsp} \geq \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \right. \\
& \quad \left. \left. \text{betaEgen}^2 \text{ mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \right. \\
& \quad \left(-\text{betaEgen betaSgen v1} - \text{betaSgen mu1 v1} + \right. \\
& \quad \left. \left. \text{betaEgen}^2 \text{ v2} + \text{b2 betaEgen mu2 v2} \right) \& \delta > 0 \right)) || \\
& \left(\text{b2} \geq \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \& \text{b1} > 0 \& \text{R} > 0 \& \right. \\
& \quad \left(\left(0 < \text{betaSsp} < \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \right. \right. \\
& \quad \left. \left. \text{betaEgen}^2 \text{ mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \right. \\
& \quad \left(-\text{betaEgen betaSgen v1} - \text{betaSgen mu1 v1} + \right. \\
& \quad \left. \left. \text{betaEgen}^2 \text{ v2} + \text{b2 betaEgen mu2 v2} \right) \& \right. \\
& \quad \left. 0 < \delta < \left(-\text{betaEgen betaSgen betaSsp mu1 R v1} + \text{betaEgen}^2 \right. \right. \\
& \quad \left. \left. \text{betaSsp mu2 R v2} \right) / \left(-\text{betaEgen betaSgen betaSsp v1} - \right. \right. \\
& \quad \left. \left. \text{b1 betaEgen betaSgen mu1 v1} + \text{betaSgen}^2 \text{ mu1 v1} - \text{betaSgen} \right. \right. \\
& \quad \left. \left. \text{betaSsp mu1 v1} + \text{betaEgen}^2 \text{ betaSsp v2} + \text{betaEgen}^2 \text{ mu2 v2} - \right. \right. \\
& \quad \left. \left. \text{b2 betaEgen betaSgen mu2 v2} + \text{b2 betaEgen betaSsp mu2 v2} \right) \right) || \\
& \left(\text{betaSsp} \geq \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \right. \\
& \quad \left. \left. \text{betaEgen}^2 \text{ mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \right. \\
& \quad \left(-\text{betaEgen betaSgen v1} - \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + \right. \\
& \quad \left. \left. \text{b2 betaEgen mu2 v2} \right) \& \delta > 0 \right)))) || \left(\text{mu2} == \frac{\text{betaSgen mu1 v1}}{\text{betaEgen v2}} \& \right. \\
& \quad \left(\left(0 < \text{b2} < \frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} \& 0 < \text{b1} < \right. \right. \\
& \quad \left. \left. \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2} - \text{b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \& \right. \right. \\
& \quad \left. \left. \text{R} > 0 \& \text{betaSsp} > \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \right. \right. \\
& \quad \left. \left. \text{betaEgen}^2 \text{ mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \right. \\
& \quad \left(-\text{betaEgen betaSgen v1} - \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + \right. \\
& \quad \left. \left. \text{b2 betaEgen mu2 v2} \right) \& \delta > 0 \right)) || \\
& \left(\frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} < \right. \\
& \quad \left. \text{b2} < \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \& \text{b1} > \right. \\
& \quad \left. \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2} - \text{b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \& \right)
\end{aligned}$$

$$\begin{aligned}
& R > 0 \ \&\& \beta_{\text{Ssp}} > \left(\frac{b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Egen}}^2 \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{(-\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 - \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Egen}}^2 v_2 + b_2 \beta_{\text{Egen}} \mu_2 v_2)} \right) \ \&\& \delta > 0 \Big) \Big| \Big| \\
& \left(b_2 \geq \frac{\beta_{\text{Sgen}}^2 \mu_1 v_1 + \beta_{\text{Egen}}^2 \mu_2 v_2}{\beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2} \ \&\& b_1 > 0 \ \&\& R > 0 \ \&\& \right. \\
& \quad \beta_{\text{Ssp}} > \left(\frac{b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Egen}}^2 \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{(-\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 - \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Egen}}^2 v_2 + b_2 \beta_{\text{Egen}} \mu_2 v_2)} \right) \ \&\& \delta > 0 \Big) \Big) \Big| \Big| \left(\mu_2 > \frac{\beta_{\text{Sgen}} \mu_1 v_1}{\beta_{\text{Egen}} v_2} \ \&\& \right. \\
& \quad \left. \left(0 < b_2 < \frac{\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 + \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Egen}}^2 v_2}{\beta_{\text{Egen}} \mu_2 v_2} \ \&\& 0 < b_1 < \frac{\beta_{\text{Sgen}}^2 \mu_1 v_1 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{\beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1} \ \&\& \right. \right. \\
& \quad R > 0 \ \&\& \left(0 < \beta_{\text{Ssp}} < \left(\frac{b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Egen}}^2 \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{(-\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 - \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Egen}}^2 v_2 + b_2 \beta_{\text{Egen}} \mu_2 v_2)} \right) \ \&\& \right. \\
& \quad 0 < \delta < \left(\frac{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 R v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} \mu_2 R v_2}{(-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} v_1 - b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} v_2 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Ssp}} \mu_2 v_2)} \right) \Big) \Big| \Big| \\
& \quad \left(\beta_{\text{Ssp}} \geq \left(\frac{b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Egen}}^2 \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{(-\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 - \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Egen}}^2 v_2 + b_2 \beta_{\text{Egen}} \mu_2 v_2)} \right) \ \&\& \delta > 0 \right) \Big) \Big| \Big| \\
& \quad \left(b_2 = \frac{\beta_{\text{Egen}} \beta_{\text{Sgen}} v_1 + \beta_{\text{Sgen}} \mu_1 v_1 - \beta_{\text{Egen}}^2 v_2}{\beta_{\text{Egen}} \mu_2 v_2} \ \&\& 0 < b_1 < \frac{\beta_{\text{Sgen}}^2 \mu_1 v_1 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2}{\beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1} \ \&\& \right. \\
& \quad R > 0 \ \&\& \beta_{\text{Ssp}} > 0 \ \&\& 0 < \delta < \left(\frac{-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 R v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} \mu_2 R v_2}{(-\beta_{\text{Egen}} \beta_{\text{Sgen}} \beta_{\text{Ssp}} v_1 - b_1 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_1 v_1 + \beta_{\text{Sgen}}^2 \mu_1 v_1 - \beta_{\text{Sgen}} \beta_{\text{Ssp}} \mu_1 v_1 + \beta_{\text{Egen}}^2 \beta_{\text{Ssp}} v_2 + \beta_{\text{Egen}}^2 \mu_2 v_2 - b_2 \beta_{\text{Egen}} \beta_{\text{Sgen}} \mu_2 v_2 + b_2 \beta_{\text{Egen}} \beta_{\text{Ssp}} \mu_2 v_2)} \right) \Big) \Big| \Big|
\end{aligned}$$

$$v_2 = \frac{\beta_{Sgen} v_1}{\beta_{Egen}} \quad \&\& \mu_1 > 0 \quad \&\& \left(\left(0 < \mu_2 < \frac{\beta_{Sgen} \mu_1 v_1}{\beta_{Egen} v_2} \quad \&\& \right. \right.$$
$$\left. \left(\left(0 < b_2 < \frac{\beta_{Egen} \beta_{Sgen} v_1 + \beta_{Sgen} \mu_1 v_1 - \beta_{Egen}^2 v_2}{\beta_{Egen} \mu_2 v_2} \quad \&\& \left(\left(0 < b_1 < \frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_1 v_1} \quad \&\& \right. \right. \right. \right.$$
$$R > 0 \quad \&\& \beta_{Ssp} > (b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 - \beta_{Sgen}^2 \mu_1 v_1 -$$
$$\beta_{Egen}^2 \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2) /$$
$$(-\beta_{Egen} \beta_{Sgen} v_1 - \beta_{Sgen} \mu_1 v_1 + \beta_{Egen}^2 v_2 +$$
$$b_2 \beta_{Egen} \mu_2 v_2) \quad \&\& \delta > (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} \mu_1 R v_1 +$$
$$\beta_{Egen}^2 \beta_{Ssp} \mu_2 R v_2) / (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} v_1 -$$
$$b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 + \beta_{Sgen}^2 \mu_1 v_1 -$$
$$\beta_{Sgen} \beta_{Ssp} \mu_1 v_1 + \beta_{Egen}^2 \beta_{Ssp} v_2 + \beta_{Egen}^2 \mu_2 v_2 -$$
$$b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Ssp} \mu_2 v_2) \Big) ||$$
$$\left(b_1 > \frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_1 v_1} \quad \&\& \right.$$
$$R > 0 \quad \&\& \beta_{Ssp} > 0 \quad \&\& 0 < \delta < (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} \mu_1 R v_1 +$$
$$\beta_{Egen}^2 \beta_{Ssp} \mu_2 R v_2) / (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} v_1 -$$
$$b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 + \beta_{Sgen}^2 \mu_1 v_1 -$$
$$\beta_{Sgen} \beta_{Ssp} \mu_1 v_1 + \beta_{Egen}^2 \beta_{Ssp} v_2 + \beta_{Egen}^2 \mu_2 v_2 -$$
$$b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Ssp} \mu_2 v_2) \Big) \Big) ||$$
$$\left(b_2 = \frac{\beta_{Egen} \beta_{Sgen} v_1 + \beta_{Sgen} \mu_1 v_1 - \beta_{Egen}^2 v_2}{\beta_{Egen} \mu_2 v_2} \quad \&\& \right.$$
$$b_1 > \frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_1 v_1} \quad \&\&$$
$$R > 0 \quad \&\& \beta_{Ssp} > 0 \quad \&\&$$
$$0 < \delta < (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} \mu_1 R v_1 + \beta_{Egen}^2 \beta_{Ssp} \mu_2 R v_2) /$$
$$(-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} v_1 - b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 +$$
$$\beta_{Sgen}^2 \mu_1 v_1 - \beta_{Sgen} \beta_{Ssp} \mu_1 v_1 +$$
$$\beta_{Egen}^2 \beta_{Ssp} v_2 + \beta_{Egen}^2 \mu_2 v_2 -$$
$$b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Ssp} \mu_2 v_2) \Big) ||$$
$$\left(\frac{\beta_{Egen} \beta_{Sgen} v_1 + \beta_{Sgen} \mu_1 v_1 - \beta_{Egen}^2 v_2}{\beta_{Egen} \mu_2 v_2} < b_2 < \right.$$
$$\frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_2 v_2} \quad \&\&$$

$$\begin{aligned}
& b1 > \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2 - b2 \text{betaEgen} \text{betaSgen} \mu2 v2}{\text{betaEgen} \text{betaSgen} \mu1 v1} \& \& \\
& R > 0 \& \& \left((0 < \text{betaSsp} < (b1 \text{betaEgen} \text{betaSgen} \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \right. \\
& \quad \text{betaEgen}^2 \mu2 v2 + b2 \text{betaEgen} \text{betaSgen} \mu2 v2) / \\
& \quad (-\text{betaEgen} \text{betaSgen} v1 - \text{betaSgen} \mu1 v1 + \text{betaEgen}^2 v2 + \\
& \quad b2 \text{betaEgen} \mu2 v2) \& 0 < \delta < (-\text{betaEgen} \text{betaSgen} \text{betaSsp} \\
& \quad \mu1 R v1 + \text{betaEgen}^2 \text{betaSsp} \mu2 R v2) / (-\text{betaEgen} \text{betaSgen} \\
& \quad \text{betaSsp} v1 - b1 \text{betaEgen} \text{betaSgen} \mu1 v1 + \text{betaSgen}^2 \mu1 v1 - \\
& \quad \text{betaSgen} \text{betaSsp} \mu1 v1 + \text{betaEgen}^2 \text{betaSsp} v2 + \text{betaEgen}^2 \mu2 v2 - \\
& \quad b2 \text{betaEgen} \text{betaSgen} \mu2 v2 + b2 \text{betaEgen} \text{betaSsp} \mu2 v2) \& \& \\
& \quad (\text{betaSsp} \geq (b1 \text{betaEgen} \text{betaSgen} \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \text{betaEgen}^2 \\
& \quad \mu2 v2 + b2 \text{betaEgen} \text{betaSgen} \mu2 v2) / (-\text{betaEgen} \text{betaSgen} v1 - \\
& \quad \text{betaSgen} \mu1 v1 + \text{betaEgen}^2 v2 + b2 \text{betaEgen} \mu2 v2) \& \delta > 0) \& \& \left. \right) \& \& \\
& \left(b2 \geq \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2}{\text{betaEgen} \text{betaSgen} \mu2 v2} \& \& b1 > 0 \& \& R > 0 \& \& \right. \\
& \quad \left((0 < \text{betaSsp} < (b1 \text{betaEgen} \text{betaSgen} \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \text{betaEgen}^2 \right. \\
& \quad \mu2 v2 + b2 \text{betaEgen} \text{betaSgen} \mu2 v2) / (-\text{betaEgen} \text{betaSgen} v1 - \\
& \quad \text{betaSgen} \mu1 v1 + \text{betaEgen}^2 v2 + b2 \text{betaEgen} \mu2 v2) \& \& \\
& \quad 0 < \delta < (-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu1 R v1 + \text{betaEgen}^2 \\
& \quad \text{betaSsp} \mu2 R v2) / (-\text{betaEgen} \text{betaSgen} \text{betaSsp} v1 - \\
& \quad b1 \text{betaEgen} \text{betaSgen} \mu1 v1 + \text{betaSgen}^2 \mu1 v1 - \\
& \quad \text{betaSgen} \text{betaSsp} \mu1 v1 + \text{betaEgen}^2 \text{betaSsp} v2 + \text{betaEgen}^2 \mu2 v2 - \\
& \quad b2 \text{betaEgen} \text{betaSgen} \mu2 v2 + b2 \text{betaEgen} \text{betaSsp} \mu2 v2) \& \& \left. \right) \& \& \\
& \quad (\text{betaSsp} \geq (b1 \text{betaEgen} \text{betaSgen} \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \\
& \quad \text{betaEgen}^2 \mu2 v2 + b2 \text{betaEgen} \text{betaSgen} \mu2 v2) / \\
& \quad (-\text{betaEgen} \text{betaSgen} v1 - \text{betaSgen} \mu1 v1 + \text{betaEgen}^2 v2 + \\
& \quad b2 \text{betaEgen} \mu2 v2) \& \delta > 0) \& \& \left. \right) \& \& \left(\mu2 = \frac{\text{betaSgen} \mu1 v1}{\text{betaEgen} v2} \& \& \right. \\
& \quad \left. \left(\left(0 < b2 < \frac{\text{betaEgen} \text{betaSgen} v1 + \text{betaSgen} \mu1 v1 - \text{betaEgen}^2 v2}{\text{betaEgen} \mu2 v2} \& \& \right. \right. \right. \\
& \quad \left. \left. 0 < b1 < \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2 - b2 \text{betaEgen} \text{betaSgen} \mu2 v2}{\text{betaEgen} \text{betaSgen} \mu1 v1} \& \& \right. \right. \\
& \quad \left. \left. R > 0 \& \& \text{betaSsp} > (b1 \text{betaEgen} \text{betaSgen} \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \right. \right. \\
& \quad \left. \left. \text{betaEgen}^2 \mu2 v2 + b2 \text{betaEgen} \text{betaSgen} \mu2 v2) / (-\text{betaEgen} \text{betaSgen} \right. \right. \\
& \quad \left. \left. v1 - \text{betaSgen} \mu1 v1 + \text{betaEgen}^2 v2 + b2 \text{betaEgen} \mu2 v2) \& \delta > 0 \right) \& \& \left. \right) \& \&
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} < \text{b2} < \right. \\
& \quad \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \& \\
& \quad \text{b1} > \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2} - \text{b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \& \\
& \quad \text{R} > 0 \& \text{betaSsp} > \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \\
& \quad \quad \left. \text{betaEgen}^2 \text{ mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \left(-\text{betaEgen betaSgen} \right. \\
& \quad \quad \left. \text{v1} - \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + \text{b2 betaEgen mu2 v2} \right) \& \delta > 0 \left. \right) \mid \mid \\
& \left(\text{b2} \geq \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \& \text{b1} > 0 \& \text{R} > 0 \& \right. \\
& \quad \text{betaSsp} > \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \\
& \quad \quad \left. \text{betaEgen}^2 \text{ mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \\
& \quad \left(-\text{betaEgen betaSgen v1} - \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + \right. \\
& \quad \quad \left. \text{b2 betaEgen mu2 v2} \right) \& \delta > 0 \left. \right) \mid \mid \left(\text{mu2} > \frac{\text{betaSgen mu1 v1}}{\text{betaEgen v2}} \& \right. \\
& \left. \left(\left(0 < \text{b2} < \frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} \& \right. \right. \right. \\
& \quad 0 < \text{b1} < \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2} - \text{b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \& \\
& \quad \text{R} > 0 \& \left(\left(0 < \text{betaSsp} < \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \right. \right. \\
& \quad \quad \left. \text{betaEgen}^2 \text{ mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \\
& \quad \quad \left(-\text{betaEgen betaSgen v1} - \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + \right. \\
& \quad \quad \left. \text{b2 betaEgen mu2 v2} \right) \& 0 < \delta < \left(-\text{betaEgen betaSgen betaSsp} \right. \\
& \quad \quad \left. \text{mu1 R v1} + \text{betaEgen}^2 \text{ betaSsp mu2 R v2} \right) / \left(-\text{betaEgen betaSgen} \right. \\
& \quad \quad \left. \text{betaSsp v1} - \text{b1 betaEgen betaSgen mu1 v1} + \text{betaSgen}^2 \text{ mu1 v1} - \right. \\
& \quad \quad \left. \text{betaSgen betaSsp mu1 v1} + \text{betaEgen}^2 \text{ betaSsp v2} + \text{betaEgen}^2 \text{ mu2 v2} - \right. \\
& \quad \quad \left. \text{b2 betaEgen betaSgen mu2 v2} + \text{b2 betaEgen betaSsp mu2 v2} \right) \mid \mid \\
& \quad \left(\text{betaSsp} \geq \left(\text{b1 betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \text{betaEgen}^2 \right. \right. \\
& \quad \quad \left. \text{mu2 v2} + \text{b2 betaEgen betaSgen mu2 v2} \right) / \left(-\text{betaEgen betaSgen v1} - \right. \\
& \quad \quad \left. \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + \text{b2 betaEgen mu2 v2} \right) \& \delta > 0 \left. \right) \mid \mid \\
& \left(\text{b2} = \frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} \& \right. \\
& \quad 0 < \text{b1} < \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2} - \text{b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \& \\
& \quad \text{R} > 0 \& \text{betaSsp} > 0 \& 0 < \delta < \left(-\text{betaEgen betaSgen betaSsp mu1 R v1} + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\text{betaEgen}^2 \text{betaSsp} \mu_2 R v_2}{(-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - \right. \\
& \quad \left. b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \right. \\
& \quad \left. \text{betaSgen} \text{betaSsp} \mu_1 v_1 + \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - \right. \\
& \quad \left. b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2) \right) || \\
& \left(\frac{\text{betaEgen} \text{betaSgen} v_1 + \text{betaSgen} \mu_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen} \mu_2 v_2} < b_2 < \right. \\
& \quad \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_2 v_2} \& \left(\left(0 < b_1 < \right. \right. \\
& \quad \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \& \\
& \quad R > 0 \& \text{betaSsp} > 0 \& 0 < \delta < \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \right. \\
& \quad \left. \text{betaEgen}^2 \text{betaSsp} \mu_2 R v_2 \right) / \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - \right. \\
& \quad \left. b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \right. \\
& \quad \left. \text{betaSgen} \text{betaSsp} \mu_1 v_1 + \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - \right. \\
& \quad \left. b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2) \right) || \\
& \left(b_1 > \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \& \right. \\
& \quad R > 0 \& \text{betaSsp} > \left(b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \right. \\
& \quad \left. \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2) / \right. \\
& \quad \left(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + \right. \\
& \quad \left. b_2 \text{betaEgen} \mu_2 v_2) \& \delta > \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \right. \\
& \quad \left. \text{betaEgen}^2 \text{betaSsp} \mu_2 R v_2 \right) / \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - \right. \\
& \quad \left. b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \right. \\
& \quad \left. \text{betaSgen} \text{betaSsp} \mu_1 v_1 + \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - \right. \\
& \quad \left. b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2) \right) || \\
& \left(b_2 \geq \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_2 v_2} \& b_1 > 0 \& R > 0 \& \right. \\
& \quad \text{betaSsp} > \left(b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \right. \\
& \quad \left. \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2) / \left(-\text{betaEgen} \text{betaSgen} v_1 - \right. \right. \\
& \quad \left. \left. \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2) \& \right. \right. \\
& \quad \delta > \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \text{betaEgen}^2 \text{betaSsp} \mu_2 R v_2 \right) / \\
& \quad \left(-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 + \right. \\
& \quad \left. \text{betaSgen}^2 \mu_1 v_1 - \text{betaSgen} \text{betaSsp} \mu_1 v_1 + \right. \\
& \quad \left. \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - \right. \\
& \quad \left. b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2) \right) ||
\end{aligned}$$

$$\begin{aligned}
& \left(v2 > \frac{\text{betaSgen } v1}{\text{betaEgen}} \& \left(\left(0 < \mu1 \leq \frac{-\text{betaEgen } \text{betaSgen } v1 + \text{betaEgen}^2 v2}{\text{betaSgen } v1} \& \right. \right. \right. \\
& \quad \left. \left(0 < \mu2 < \frac{\text{betaSgen } \mu1 v1}{\text{betaEgen } v2} \& \right. \right. \\
& \quad \left. \left(0 < b2 < \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2}{\text{betaEgen } \text{betaSgen } \mu2 v2} \& b1 > \right. \right. \\
& \quad \left. \left. \left. \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2 - b2 \text{betaEgen } \text{betaSgen } \mu2 v2}{\text{betaEgen } \text{betaSgen } \mu1 v1} \& \right. \right. \right. \\
& R > 0 \& \left(\left(0 < \text{betaSsp} < \left(b1 \text{betaEgen } \text{betaSgen } \mu1 v1 - \right. \right. \right. \\
& \quad \text{betaSgen}^2 \mu1 v1 - \text{betaEgen}^2 \mu2 v2 + \\
& \quad \left. b2 \text{betaEgen } \text{betaSgen } \mu2 v2 \right) / \left(-\text{betaEgen } \text{betaSgen } v1 - \right. \\
& \quad \left. \text{betaSgen } \mu1 v1 + \text{betaEgen}^2 v2 + b2 \text{betaEgen } \mu2 v2 \right) \& \\
& \quad 0 < \delta < \left(-\text{betaEgen } \text{betaSgen } \text{betaSsp } \mu1 R v1 + \text{betaEgen}^2 \right. \\
& \quad \left. \text{betaSsp } \mu2 R v2 \right) / \left(-\text{betaEgen } \text{betaSgen } \text{betaSsp } v1 - \right. \\
& \quad \left. b1 \text{betaEgen } \text{betaSgen } \mu1 v1 + \text{betaSgen}^2 \mu1 v1 - \text{betaSgen} \right. \\
& \quad \left. \text{betaSsp } \mu1 v1 + \text{betaEgen}^2 \text{betaSsp } v2 + \text{betaEgen}^2 \mu2 v2 - \right. \\
& \quad \left. b2 \text{betaEgen } \text{betaSgen } \mu2 v2 + b2 \text{betaEgen } \text{betaSsp } \mu2 v2 \right) \left. \right) \mid \mid \\
& \left(\text{betaSsp} \geq \left(b1 \text{betaEgen } \text{betaSgen } \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \right. \right. \\
& \quad \left. \text{betaEgen}^2 \mu2 v2 + b2 \text{betaEgen } \text{betaSgen } \mu2 v2 \right) / \\
& \quad \left(-\text{betaEgen } \text{betaSgen } v1 - \text{betaSgen } \mu1 v1 + \right. \\
& \quad \left. \text{betaEgen}^2 v2 + b2 \text{betaEgen } \mu2 v2 \right) \& \delta > 0 \left. \right) \left. \right) \mid \mid \\
& \left(b2 \geq \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2}{\text{betaEgen } \text{betaSgen } \mu2 v2} \& b1 > 0 \& R > 0 \& \right. \\
& \quad \left(\left(0 < \text{betaSsp} < \left(b1 \text{betaEgen } \text{betaSgen } \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \right. \right. \right. \\
& \quad \left. \text{betaEgen}^2 \mu2 v2 + b2 \text{betaEgen } \text{betaSgen } \mu2 v2 \right) / \\
& \quad \left(-\text{betaEgen } \text{betaSgen } v1 - \text{betaSgen } \mu1 v1 + \right. \\
& \quad \left. \text{betaEgen}^2 v2 + b2 \text{betaEgen } \mu2 v2 \right) \& \\
& \quad 0 < \delta < \left(-\text{betaEgen } \text{betaSgen } \text{betaSsp } \mu1 R v1 + \text{betaEgen}^2 \right. \\
& \quad \left. \text{betaSsp } \mu2 R v2 \right) / \left(-\text{betaEgen } \text{betaSgen } \text{betaSsp } v1 - \right. \\
& \quad \left. b1 \text{betaEgen } \text{betaSgen } \mu1 v1 + \text{betaSgen}^2 \mu1 v1 - \text{betaSgen} \right. \\
& \quad \left. \text{betaSsp } \mu1 v1 + \text{betaEgen}^2 \text{betaSsp } v2 + \text{betaEgen}^2 \mu2 v2 - \right. \\
& \quad \left. b2 \text{betaEgen } \text{betaSgen } \mu2 v2 + b2 \text{betaEgen } \text{betaSsp } \mu2 v2 \right) \left. \right) \mid \mid \\
& \left(\text{betaSsp} \geq \left(b1 \text{betaEgen } \text{betaSgen } \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \right. \right. \\
& \quad \left. \text{betaEgen}^2 \mu2 v2 + b2 \text{betaEgen } \text{betaSgen } \mu2 v2 \right) / \\
& \quad \left(-\text{betaEgen } \text{betaSgen } v1 - \text{betaSgen } \mu1 v1 + \text{betaEgen}^2 v2 + \right. \\
& \quad \left. b2 \text{betaEgen } \mu2 v2 \right) \& \delta > 0 \left. \right) \left. \right) \left. \right) \mid \mid \left(\mu2 == \frac{\text{betaSgen } \mu1 v1}{\text{betaEgen } v2} \& \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\left(0 < b2 < \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2}{\text{betaEgen} \text{betaSgen} \mu2 v2} \ \&\& b1 > \right. \right. \\
& \quad \left. \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2 - b2 \text{betaEgen} \text{betaSgen} \mu2 v2}{\text{betaEgen} \text{betaSgen} \mu1 v1} \ \&\& \right. \\
& \quad R > 0 \ \&\& \text{betaSsp} > (b1 \text{betaEgen} \text{betaSgen} \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \\
& \quad \text{betaEgen}^2 \mu2 v2 + b2 \text{betaEgen} \text{betaSgen} \mu2 v2) / \\
& \quad (-\text{betaEgen} \text{betaSgen} v1 - \text{betaSgen} \mu1 v1 + \text{betaEgen}^2 v2 + \\
& \quad \left. b2 \text{betaEgen} \mu2 v2) \ \&\& \delta > 0 \right) || \\
& \left(b2 \geq \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2}{\text{betaEgen} \text{betaSgen} \mu2 v2} \ \&\& b1 > 0 \ \&\& R > 0 \ \&\& \right. \\
& \quad \text{betaSsp} > (b1 \text{betaEgen} \text{betaSgen} \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \\
& \quad \text{betaEgen}^2 \mu2 v2 + b2 \text{betaEgen} \text{betaSgen} \mu2 v2) / \\
& \quad (-\text{betaEgen} \text{betaSgen} v1 - \text{betaSgen} \mu1 v1 + \text{betaEgen}^2 v2 + \\
& \quad \left. b2 \text{betaEgen} \mu2 v2) \ \&\& \delta > 0 \right) || \left(\mu2 > \frac{\text{betaSgen} \mu1 v1}{\text{betaEgen} v2} \ \&\& \right. \\
& \left(0 < b2 < \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2}{\text{betaEgen} \text{betaSgen} \mu2 v2} \ \&\& \left(0 < b1 < \right. \right. \\
& \quad \left. \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2 - b2 \text{betaEgen} \text{betaSgen} \mu2 v2}{\text{betaEgen} \text{betaSgen} \mu1 v1} \right. \\
& \quad \&\& R > 0 \ \&\& \text{betaSsp} > 0 \ \&\& \\
& \quad 0 < \delta < (-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu1 R v1 + \text{betaEgen}^2 \text{betaSsp} \\
& \quad \mu2 R v2) / (-\text{betaEgen} \text{betaSgen} \text{betaSsp} v1 - b1 \text{betaEgen} \\
& \quad \text{betaSgen} \mu1 v1 + \text{betaSgen}^2 \mu1 v1 - \text{betaSgen} \text{betaSsp} \mu1 \\
& \quad v1 + \text{betaEgen}^2 \text{betaSsp} v2 + \text{betaEgen}^2 \mu2 v2 - b2 \text{betaEgen} \\
& \quad \left. \text{betaSgen} \mu2 v2 + b2 \text{betaEgen} \text{betaSsp} \mu2 v2) \right) || \left(b1 > \right. \\
& \quad \left. \frac{\text{betaSgen}^2 \mu1 v1 + \text{betaEgen}^2 \mu2 v2 - b2 \text{betaEgen} \text{betaSgen} \mu2 v2}{\text{betaEgen} \text{betaSgen} \mu1 v1} \right. \\
& \quad \&\& R > 0 \ \&\& \text{betaSsp} > \\
& \quad (b1 \text{betaEgen} \text{betaSgen} \mu1 v1 - \text{betaSgen}^2 \mu1 v1 - \text{betaEgen}^2 \mu2 \\
& \quad v2 + b2 \text{betaEgen} \text{betaSgen} \mu2 v2) / (-\text{betaEgen} \text{betaSgen} v1 - \\
& \quad \text{betaSgen} \mu1 v1 + \text{betaEgen}^2 v2 + b2 \text{betaEgen} \mu2 v2) \ \&\& \\
& \quad \delta > (-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu1 R v1 + \text{betaEgen}^2 \text{betaSsp} \\
& \quad \mu2 R v2) / (-\text{betaEgen} \text{betaSgen} \text{betaSsp} v1 - b1 \text{betaEgen} \\
& \quad \text{betaSgen} \mu1 v1 + \text{betaSgen}^2 \mu1 v1 - \text{betaSgen} \text{betaSsp} \mu1 \\
& \quad v1 + \text{betaEgen}^2 \text{betaSsp} v2 + \text{betaEgen}^2 \mu2 v2 - b2 \text{betaEgen}
\end{aligned}$$

$$\left(\frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_2 v_2} \&\& b_1 > 0 \&\& R > 0 \&\& \right) \left(\frac{\beta_{Ssp} > (b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 - \beta_{Sgen}^2 \mu_1 v_1 - \beta_{Egen}^2 \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2)}{(-\beta_{Egen} \beta_{Sgen} v_1 - \beta_{Sgen} \mu_1 v_1 + \beta_{Egen}^2 v_2 + b_2 \beta_{Egen} \mu_2 v_2)} \&\& \delta > (-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} \mu_1 R v_1 + \beta_{Egen}^2 \beta_{Ssp} \mu_2 R v_2)}{(-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} v_1 - b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 + \beta_{Sgen}^2 \mu_1 v_1 - \beta_{Sgen} \beta_{Ssp} \mu_1 v_1 + \beta_{Egen}^2 \beta_{Ssp} v_2 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Ssp} \mu_2 v_2)} \right) \Big| \Big|$$
$$\left(\mu_1 > \frac{-\beta_{Egen} \beta_{Sgen} v_1 + \beta_{Egen}^2 v_2}{\beta_{Sgen} v_1} \&\& \left(0 < \mu_2 < \frac{\beta_{Sgen} \mu_1 v_1}{\beta_{Egen} v_2} \&\& \left(0 < b_2 < \frac{\beta_{Egen} \beta_{Sgen} v_1 + \beta_{Sgen} \mu_1 v_1 - \beta_{Egen}^2 v_2}{\beta_{Egen} \mu_2 v_2} \&\& \left(0 < b_1 < \frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_1 v_1} \&\& R > 0 \&\& \beta_{Ssp} > \frac{(b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 - \beta_{Sgen}^2 \mu_1 v_1 - \beta_{Egen}^2 \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2)}{(-\beta_{Egen} \beta_{Sgen} v_1 - \beta_{Sgen} \mu_1 v_1 + \beta_{Egen}^2 v_2 + b_2 \beta_{Egen} \mu_2 v_2)} \&\& \delta > \frac{(-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} \mu_1 R v_1 + \beta_{Egen}^2 \beta_{Ssp} \mu_2 R v_2)}{(-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} v_1 - b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 + \beta_{Sgen}^2 \mu_1 v_1 - \beta_{Sgen} \beta_{Ssp} \mu_1 v_1 + \beta_{Egen}^2 \beta_{Ssp} v_2 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Ssp} \mu_2 v_2)} \right) \Big| \Big| \left(b_1 > \frac{\beta_{Sgen}^2 \mu_1 v_1 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2}{\beta_{Egen} \beta_{Sgen} \mu_1 v_1} \&\& R > 0 \&\& \beta_{Ssp} > 0 \&\& 0 < \delta < \frac{(-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} \mu_1 R v_1 + \beta_{Egen}^2 \beta_{Ssp} \mu_2 R v_2)}{(-\beta_{Egen} \beta_{Sgen} \beta_{Ssp} v_1 - b_1 \beta_{Egen} \beta_{Sgen} \mu_1 v_1 + \beta_{Sgen}^2 \mu_1 v_1 - \beta_{Sgen} \beta_{Ssp} \mu_1 v_1 + \beta_{Egen}^2 \beta_{Ssp} v_2 + \beta_{Egen}^2 \mu_2 v_2 - b_2 \beta_{Egen} \beta_{Sgen} \mu_2 v_2 + b_2 \beta_{Egen} \beta_{Ssp} \mu_2 v_2)} \right) \Big| \Big|$$

$$\left(\begin{aligned} & \text{betaSgen mu2 v2 + b2 betaEgen betaSsp mu2 v2}) \Big) \Big) \Big) \Big) || \\ & \left(\text{b2} = \frac{\text{betaEgen betaSgen v1 + betaSgen mu1 v1 - betaEgen}^2 \text{v2}}{\text{betaEgen mu2 v2}} \&\& \text{b1} > \right. \\ & \quad \frac{\text{betaSgen}^2 \text{mu1 v1 + betaEgen}^2 \text{mu2 v2 - b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \&\& \\ & \quad \text{R} > 0 \&\& \text{betaSsp} > 0 \&\& 0 < \delta < (-\text{betaEgen betaSgen betaSsp mu1 R v1 +} \\ & \quad \text{betaEgen}^2 \text{betaSsp mu2 R v2}) / (-\text{betaEgen betaSgen betaSsp v1 -} \\ & \quad \text{b1 betaEgen betaSgen mu1 v1 + betaSgen}^2 \text{mu1 v1 -} \\ & \quad \text{betaSgen betaSsp mu1 v1 + betaEgen}^2 \text{betaSsp v2 + betaEgen}^2 \text{mu2 v2 -} \\ & \quad \text{b2 betaEgen betaSgen mu2 v2 + b2 betaEgen betaSsp mu2 v2}) \Big) \Big) || \\ & \left(\frac{\text{betaEgen betaSgen v1 + betaSgen mu1 v1 - betaEgen}^2 \text{v2}}{\text{betaEgen mu2 v2}} < \right. \\ & \quad \text{b2} < \frac{\text{betaSgen}^2 \text{mu1 v1 + betaEgen}^2 \text{mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \&\& \text{b1} > \\ & \quad \frac{\text{betaSgen}^2 \text{mu1 v1 + betaEgen}^2 \text{mu2 v2 - b2 betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \&\& \\ & \quad \text{R} > 0 \&\& ((0 < \text{betaSsp} < (\text{b1 betaEgen betaSgen mu1 v1 -} \\ & \quad \text{betaSgen}^2 \text{mu1 v1 - betaEgen}^2 \text{mu2 v2 +} \\ & \quad \text{b2 betaEgen betaSgen mu2 v2}) / (-\text{betaEgen betaSgen v1 -} \\ & \quad \text{betaSgen mu1 v1 + betaEgen}^2 \text{v2 + b2 betaEgen mu2 v2})) \&\& \\ & \quad 0 < \delta < (-\text{betaEgen betaSgen betaSsp mu1 R v1 + betaEgen}^2 \\ & \quad \text{betaSsp mu2 R v2}) / (-\text{betaEgen betaSgen betaSsp v1 -} \\ & \quad \text{b1 betaEgen betaSgen mu1 v1 + betaSgen}^2 \text{mu1 v1 - betaSgen} \\ & \quad \text{betaSsp mu1 v1 + betaEgen}^2 \text{betaSsp v2 + betaEgen}^2 \text{mu2 v2 -} \\ & \quad \text{b2 betaEgen betaSgen mu2 v2 + b2 betaEgen betaSsp mu2 v2})) || \\ & \quad (\text{betaSsp} \geq (\text{b1 betaEgen betaSgen mu1 v1 - betaSgen}^2 \text{mu1 v1 -} \\ & \quad \text{betaEgen}^2 \text{mu2 v2 + b2 betaEgen betaSgen mu2 v2}) / \\ & \quad (-\text{betaEgen betaSgen v1 - betaSgen mu1 v1 +} \\ & \quad \text{betaEgen}^2 \text{v2 + b2 betaEgen mu2 v2})) \&\& \delta > 0)) \Big) \Big) || \\ & \left(\text{b2} \geq \frac{\text{betaSgen}^2 \text{mu1 v1 + betaEgen}^2 \text{mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \&\& \text{b1} > 0 \&\& \text{R} > 0 \&\& \right. \\ & \quad ((0 < \text{betaSsp} < (\text{b1 betaEgen betaSgen mu1 v1 - betaSgen}^2 \text{mu1 v1 -} \\ & \quad \text{betaEgen}^2 \text{mu2 v2 + b2 betaEgen betaSgen mu2 v2}) / \\ & \quad (-\text{betaEgen betaSgen v1 - betaSgen mu1 v1 +} \\ & \quad \text{betaEgen}^2 \text{v2 + b2 betaEgen mu2 v2})) \&\& \end{aligned} \right)$$

$$\begin{aligned}
& 0 < \delta < \left(\frac{-\text{betaEgen} \text{betaSgen} \text{betaSsp} \mu_1 R v_1 + \text{betaEgen}^2 \text{betaSsp} \mu_2 R v_2}{-\text{betaEgen} \text{betaSgen} \text{betaSsp} v_1 - b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 + \text{betaSgen}^2 \mu_1 v_1 - \text{betaSgen} \text{betaSsp} \mu_1 v_1 + \text{betaEgen}^2 \text{betaSsp} v_2 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSsp} \mu_2 v_2} \right) \mid \mid \\
& \left(\text{betaSsp} \geq \frac{(b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2)}{(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2)} \&\& \delta > 0 \right) \mid \mid \left(\mu_2 = \frac{\text{betaSgen} \mu_1 v_1}{\text{betaEgen} v_2} \&\& \right. \\
& \left. \left(0 < b_2 < \frac{\text{betaEgen} \text{betaSgen} v_1 + \text{betaSgen} \mu_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen} \mu_2 v_2} \&\& 0 < b_1 < \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \&\& \right. \right. \\
& R > 0 \&\& \text{betaSsp} > \frac{(b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2)}{(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2)} \&\& \delta > 0 \mid \mid \\
& \left. \frac{\text{betaEgen} \text{betaSgen} v_1 + \text{betaSgen} \mu_1 v_1 - \text{betaEgen}^2 v_2}{\text{betaEgen} \mu_2 v_2} < b_2 < \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_2 v_2} \&\& b_1 > \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2 - b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_1 v_1} \&\& \right. \\
& R > 0 \&\& \text{betaSsp} > \frac{(b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2)}{(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2)} \&\& \delta > 0 \mid \mid \\
& \left. b_2 \geq \frac{\text{betaSgen}^2 \mu_1 v_1 + \text{betaEgen}^2 \mu_2 v_2}{\text{betaEgen} \text{betaSgen} \mu_2 v_2} \&\& b_1 > 0 \&\& R > 0 \&\& \text{betaSsp} > \frac{(b_1 \text{betaEgen} \text{betaSgen} \mu_1 v_1 - \text{betaSgen}^2 \mu_1 v_1 - \text{betaEgen}^2 \mu_2 v_2 + b_2 \text{betaEgen} \text{betaSgen} \mu_2 v_2)}{(-\text{betaEgen} \text{betaSgen} v_1 - \text{betaSgen} \mu_1 v_1 + \text{betaEgen}^2 v_2 + b_2 \text{betaEgen} \mu_2 v_2)} \&\& \delta > 0 \right) \mid \mid \left(\mu_2 > \frac{\text{betaSgen} \mu_1 v_1}{\text{betaEgen} v_2} \&\& \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(0 < b2 < \frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} \ \&\& \ 0 < b1 < \right. \right. \\
& \quad \left. \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2} - b2 \text{ betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \ \&\& \right. \\
& \quad R > 0 \ \&\& \left(\left(0 < \text{betaSsp} < \left(b1 \text{ betaEgen betaSgen mu1 v1} - \right. \right. \right. \\
& \quad \quad \text{betaSgen}^2 \text{ mu1 v1} - \text{betaEgen}^2 \text{ mu2 v2} + \\
& \quad \quad b2 \text{ betaEgen betaSgen mu2 v2} \bigg) / \left(-\text{betaEgen betaSgen v1} - \right. \\
& \quad \quad \left. \left. \text{betaSgen mu1 v1} + \text{betaEgen}^2 \text{ v2} + b2 \text{ betaEgen mu2 v2} \right) \ \&\& \right. \\
& \quad \quad 0 < \delta < \left(-\text{betaEgen betaSgen betaSsp mu1 R v1} + \text{betaEgen}^2 \right. \\
& \quad \quad \left. \text{betaSsp mu2 R v2} \right) / \left(-\text{betaEgen betaSgen betaSsp v1} - \right. \\
& \quad \quad \left. b1 \text{ betaEgen betaSgen mu1 v1} + \text{betaSgen}^2 \text{ mu1 v1} - \text{betaSgen} \right. \\
& \quad \quad \left. \text{betaSsp mu1 v1} + \text{betaEgen}^2 \text{ betaSsp v2} + \text{betaEgen}^2 \text{ mu2 v2} - \right. \\
& \quad \quad \left. b2 \text{ betaEgen betaSgen mu2 v2} + b2 \text{ betaEgen betaSsp mu2 v2} \right) \bigg) \ || \\
& \quad \left(\text{betaSsp} \geq \left(b1 \text{ betaEgen betaSgen mu1 v1} - \text{betaSgen}^2 \text{ mu1 v1} - \right. \right. \\
& \quad \quad \text{betaEgen}^2 \text{ mu2 v2} + b2 \text{ betaEgen betaSgen mu2 v2} \bigg) / \\
& \quad \quad \left(-\text{betaEgen betaSgen v1} - \text{betaSgen mu1 v1} + \right. \\
& \quad \quad \left. \left. \text{betaEgen}^2 \text{ v2} + b2 \text{ betaEgen mu2 v2} \right) \ \&\& \ \delta > 0 \right) \bigg) \ || \\
& \left(b2 = \frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} \ \&\& \ 0 < b1 < \right. \\
& \quad \left. \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2} - b2 \text{ betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \ \&\& \right. \\
& \quad R > 0 \ \&\& \text{betaSsp} > 0 \ \&\& \ 0 < \delta < \left(-\text{betaEgen betaSgen betaSsp mu1 R v1} + \right. \\
& \quad \quad \text{betaEgen}^2 \text{ betaSsp mu2 R v2} \bigg) / \left(-\text{betaEgen betaSgen betaSsp v1} - \right. \\
& \quad \quad b1 \text{ betaEgen betaSgen mu1 v1} + \text{betaSgen}^2 \text{ mu1 v1} - \\
& \quad \quad \text{betaSgen betaSsp mu1 v1} + \text{betaEgen}^2 \text{ betaSsp v2} + \text{betaEgen}^2 \text{ mu2 v2} - \\
& \quad \quad \left. b2 \text{ betaEgen betaSgen mu2 v2} + b2 \text{ betaEgen betaSsp mu2 v2} \right) \bigg) \ || \\
& \left(\frac{\text{betaEgen betaSgen v1} + \text{betaSgen mu1 v1} - \text{betaEgen}^2 \text{ v2}}{\text{betaEgen mu2 v2}} < b2 < \right. \\
& \quad \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2}}{\text{betaEgen betaSgen mu2 v2}} \ \&\& \left(\left(0 < b1 < \right. \right. \\
& \quad \left. \frac{\text{betaSgen}^2 \text{ mu1 v1} + \text{betaEgen}^2 \text{ mu2 v2} - b2 \text{ betaEgen betaSgen mu2 v2}}{\text{betaEgen betaSgen mu1 v1}} \right. \\
& \quad \ \&\& \ R > 0 \ \&\& \text{betaSsp} > 0 \ \&\& \\
& \quad \left. 0 < \delta < \left(-\text{betaEgen betaSgen betaSsp mu1 R v1} + \text{betaEgen}^2 \text{ betaSsp} \right. \right. \\
& \quad \quad \left. \text{mu2 R v2} \right) / \left(-\text{betaEgen betaSgen betaSsp v1} - b1 \text{ betaEgen} \right. \\
& \quad \quad \left. \text{betaSgen mu1 v1} + \text{betaSgen}^2 \text{ mu1 v1} - \text{betaSgen betaSsp mu1} \right.
\end{aligned}$$

