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HW04 - Statistics

9.2.6 a. $n = 15$, $\bar{x} = 2.78$, $\sigma = 0.9$

$H_0: \mu = 3$, $H_1: \mu \neq 3$

Since population std is known, we will use z-test score.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{2.78 - 3}{0.9/\sqrt{15}} = -0.9565$$

Critical value for $\alpha = 0.05$

~~we x~~ So, $z_{\alpha/2} = 1.96$

if $|z| > cv$, reject H_0

Since, $0.95 < 1.96$,

we fail to reject H_0.

Since, it is two tailed test

So, P value $= 2 \times P(z < -.95)$

$= 2 \times .171$

$= .342$

Since, P value $> \alpha$ $(.34 > 0.05)$,

we fail to reject H_0.

6. Power = $1 - \beta$

$\mu = 3.25$

$\alpha = 0.05$

σ is known, and its two tailed test

$\therefore Z_{\alpha/2} = 1.96$

Rejection region in terms of $\bar{x}$

$= \mu_0 + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < \bar{x} < \mu_0 - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

Putting the values, we get

$\bar{x}_L = 3 - 1.96 \times 0.23 = 2.54$

$\bar{x}_U = 3 + .45 = 3.45$

So, the region of acceptance is when $\bar{x}$ is between 2.54 to 3.45

$2.54 \leq \bar{x} \leq 3.45$

Now,

$\beta = P(\text{accept } H_0 \mid \mu = 3.25)$

$\therefore Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

Substituting the values, we get:-

$$Z_{\bar{x}_L} = \frac{2.54 - 3.25}{0.23} \doteq -3.08$$

$$Z_{\bar{x}_U} = \frac{3.45 - 3.25}{0.23} \doteq .869 \approx .87$$

$$\beta = P(-3.08 \leq Z \leq .87)$$

$$\beta = 0.807850 - 0.001035$$

$$= 0.808 - 0.001 = .806$$

$$\text{Power} = 1 - \beta \doteq 0.194 = 19\%.$$

(c) $\alpha = 0.05$, $Z_{\alpha/2} = 1.96$

Power $= 90\% = .90$.

$\beta = 1 - \text{Power} = .10$

$Z_{1-\beta} = 1.28$

$\mu = 3.75$, $\mu_0 = 3$

Formula for sample size with σ known:

$$n = \left(\frac{Z_{\alpha/2} + Z_{1-\beta}}{\frac{|\mu - \mu_0|}{\sigma}} \right)^2$$

Putting the values, we get:-

$$n = \left(\frac{1.96 + 1.28}{.75/0.9}\right)^2 = \left(\frac{3.24}{0.83}\right)^2 = (3.90)^2$$
$$= 15.21$$

rounding 15.21, we get 16.

$\therefore n = 16$

10.1.2 ⓐ $\sigma_1 = 0.020$ $\sigma_2 = 0.025$

$\bar{x}_1 = \frac{160.15}{10} = 16.015$ $\bar{x}_2 = \frac{160.05}{10} = 16.005$

$n = 10$ $\alpha = 0.05$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$SE = \sqrt{\frac{(0.020)^2}{10} + \frac{(0.025)^2}{10}}$$

$$= \sqrt{.00004 + .0000625}$$

$$= \sqrt{.0001025} \approx 0.010124$$

$$Z = \frac{16.015 - 16.005}{0.010124} = \frac{0.010}{0.010124}$$

$$= 0.9877$$

Reject H_0: if $|Z| > CV$

$.987 < 1.96$

$\therefore$ we fail to reject H_0

$$P\text{-value} = 2 \times P(Z > .987)$$

$$= 2 \times 0.163543$$

$$= 0.327086 \simeq 0.33$$

$\therefore$ P-value $> \alpha$ $(0.33 > 0.05)$

we fail to reject H_0

(b) CI = 95%, two tailed test

$Z_{\alpha/2} = 1.96$

Region of acceptance for the diff in mean, with CL of 95%.

$$= (\bar{x}_1 - \bar{x}_2) \pm Z_{\alpha/2} \cdot SE$$

Substituting the values, we get:

$$= 0.010 \pm 1.96 \times 0.010124$$

$$= 0.010 \pm 0.01985$$

$$-0.00985 < (\bar{x}_1 - \bar{x}_2) < 0.02985$$

$$\approx -0.010 < (\bar{x}_1 - \bar{x}_2) < 0.030$$

This means that we are 95% confident that the difference in mean volume filled by these two machines lies between -0.010 oz to 0.030 oz.

The number is insignificant. So, In practical terms, the engineers suspicion is right.

(c) Power $= 1 - \beta$, $\mu_1 - \mu_2 = 0.04$

$\alpha = 0.05$, it's two tailed test

$\therefore Z_{\alpha/2} = 1.96$

The rejection region in terms of difference in mean $(\bar{x}_1 - \bar{x}_2)$

$$= (\mu_1 - \mu_2) + Z_{\alpha/2} \cdot SE < (\bar{x}_1 - \bar{x}_2) < (\mu_1 - \mu_2) - Z_{\alpha/2} \cdot SE$$

From, $H_0 : \mu_1 = \mu_2$, $\mu_1 - \mu_2 = 0$

Substituting the values, we get:

$0 \pm 1.96\,(0.010124)$

$= \pm\ 0.0199$

So, the region of acceptance is

$= -0.0199 < (\bar{x}_1 - \bar{x}_2) < 0.0199$

$\therefore\ Z_{(\bar{x}_1 - \bar{x}_2)_L} = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{SE}$

Substituting the value, we get

Lower tail $Z = \dfrac{-0.0199 - 0.04}{0.010124}$

$= -5.914$

upper tail $z = \dfrac{0.0199 - 0.04}{0.010124}$

$= \dfrac{-0.0201}{0.010124} = -1.99$

$\beta = P(\text{accept } H_0 \mid \mu_1 - \mu_2 = 0.04)$

$= P(-5.914 < z < -1.99)$

$= .0233 - 0 = .0233$

$Power = 1 - \beta = 1 - .023 = .977$

$= 97\%$

(d) $\mu_1 - \mu_2 = 0.04$

$\alpha = 0.05$

$\sigma_1 = 0.020 \quad \sigma_2 = 0.025$

$\beta = 0.05, \quad 1 - \beta = 0.95$

$Z_{\alpha/2} = 1.96, \quad Z_{1-\beta} = 1.645$

$$n = \frac{(\sigma_1^2 + \sigma_2^2)(Z_{\alpha/2} + Z_{1-\beta})^2}{(\mu_1 - \mu_2)^2}$$

Putting the values:-

$$\frac{(0.001025)(1.96 + 1.645)^2}{(.04)^2}$$

$$= \frac{(0.001025)(3.605)^2}{0.0016}$$

$$= \frac{0.0133209}{0.0016} = 8.3255$$

$$\approx 9$$

11.2.1 (a) $n = 250$

$\Sigma x = 6322.28$

$\Sigma y = 4757.90$

$\Sigma x^2 = 162674.18$

$\Sigma y^2 = 107679.27$

$\Sigma xy = 125471.10$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 162674.18 - 159884.89$$
$$= 2789.29$$

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$$

$$= 125471.10 - 120323.104$$
$$= 5147.996$$

$$b_1 = \frac{S_{xy}}{S_{xx}} = \frac{5147.996}{2789.29} = 1.8456$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{6322.28}{250} = 25.28872$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{4757.90}{250} = 19.0316$$

$$b_0 = \bar{y} - b_1 \bar{x} = 19.0316 - 1.8456 \times 25.28872$$

$$= -27.7549$$

$\therefore \hat{y} = -27.7549 + 1.8456x$

(b) $x = 30$

Substituting the value of x, we get

$\hat{y} = -27.7549 + 1.85(30)$

$= 27.75$

(c) $Res = y_{obs} - \hat{y}$

$\hat{y} = -27.7549 + 1.8456(25)$

$= 18.5$

$Res = 25 - 18.5 = 6.5$

(d) The prediction is ~~so~~ an under-estimate, because the predicted fat level (18.5) is less than observed fat level (25).