Homework 02 - Linear Algebra

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1. Let $A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 1 & -1 \\ 2 & 4 & 0 & 3 \\ -3 & 1 & -1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & -2 \\ 4 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 3 & -2 & 0 & 5 \\ 1 & 0 & -3 & 4 \end{bmatrix}$

a. Does the matrix $D = ABC$ exist? If so, then, $d34 = ?$

Ans- Yes, matrix D exists

$A_{4\times4} \otimes B_{4\times2} = M_{[4\times2]}$

$M_{4\times2} \otimes C_{2\times4} = D_{4\times4}$

So, Matrix D exists and, it is 4x4 Matrix.

$A \otimes B = \begin{bmatrix} 1\times1 + 0\times3 + (-1)\times0 + 2\times4 & 1\times2 + 0\times-1 + (-1)\times(-2) + 2\times1 \\ 0\times1 + 3\times3 + 1\times0 + (-1)4 & 0\times2 + 3\times(-1) + 1\times(-2) + (-1)\times1 \\ 2\times1 + 4\times3 + 0\times0 + 3\times4 & 2\times2 + 4\times-1 + 0\times-2 + 3\times1 \\ -3\times1 + 1\times3 + (-1)\times0 + 2\times4 & -3\times2 + 1\times-1 + (-1)\times(-2) + 2\times1 \end{bmatrix}$

$$\begin{bmatrix} 9 & 6 \\ 5 & -6 \\ 26 & -3 \\ 8 & -3 \end{bmatrix} \otimes \begin{bmatrix} 3 & -2 & 10 & 5 \\ 1 & 0 & -3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 9\times3+6\times1 & 9\times-2+6\times0 & 9\times0+6\times-3 & 9\times5+6\times4 \\ 5\times3+(-6)\times1 & 5\times-2+(-6)\times0 & 5\times0+(-6)\times-3 & 5\times5+(-6)\times4 \\ 26\times3+(-3)\times1 & 26\times-2+(-3)\times0 & 26\times0+(-3)\times(-3) & 26\times5+(-3)\times4 \\ 8\times3+(-3)\times1 & 8\times(-2)+(-3)\times0 & 8\times0+(-3)\times(-3) & 8\times5+(-3)\times4 \end{bmatrix}$$

$$D = \begin{bmatrix} 33 & -12 & -18 & 69 \\ 9 & -10 & 18 & 1 \\ 81 & -49 & +9 & 142 \\ 21 & -16 & 9 & 28 \end{bmatrix}$$

$$D_{34}$$

$$D_{34} = 142$$

b. Does the matrix E = BAC exist? If So, then, e22 = ?

Ans - E does not exist. Because number of column in B is 2, whereas number of rows in A is 4. So, they can't be multiplied.

c. Does the matrix F = BCA exist? If So, then, f43 = ?

Ans - Yes, F exists and will be a 4X4 matrix.

$$BC = \begin{bmatrix} 1\times3+2\times1 & 1\times-2+2\times0 & 1\times0+2\times-3 & 1\times5+2\times4 \\ 3\times3+(-1)\times1 & 3\times-2+(-1)\times0 & 3\times0+(-1)\times-3 & 3\times5+(-1)\times4 \\ 0\times3+(-2)\times1 & 0\times-2+(-2)\times0 & 0\times0+(-2)\times-3 & 0\times5+(-2)\times4 \\ 4\times3+(1)\times1 & 4\times-2+1\times0 & 4\times0+1\times-3 & 4\times5+1\times4 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -2 & -6 & 13 \\ 8 & -6 & 3 & 11 \\ -2 & 0 & 6 & -8 \\ 13 & -7 & -3 & 24 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 1 & -1 \\ 2 & 4 & 0 & 3 \\ -3 & 1 & -1 & 2 \end{bmatrix}$$

$$F_{43} = 13x-1 + (-7)x1 + (-3)x0 + 24x-1$$

$$= -45$$

d. Does the matrix $G_1 = ACB$ exist? If So, then, $g_{31} = ?$

Ans- Matrix G_1 does not exist, because no. of columns in A is 4 and no. of rows in C is 2. So, they can't be multiplied.

No. of columns in A $\neq$ no of rows in B

e. Does the matrix $H = CAB$ exist? If So, then, $h_{21} = ?$

Ans- CAB exists. And it will be 2x2 Matrix.

$$CA = \begin{bmatrix} 3 & -2 & 0 & 5 \\ 1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 1 & -1 \\ 2 & 4 & 0 & 3 \\ -3 & 1 & -1 & 2 \end{bmatrix}$$

$$CAB = \begin{bmatrix} -2 & -1 & -10 & 18 \\ -17 & -8 & -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & -2 \\ 4 & 1 \end{bmatrix}$$

$$h_{21} = -17x1 + (-8)x3 + (-5x0) + (1x4)$$

$$= -37$$

f. Does the matrix J = CBA exist? If so, then, h13 = ?

Matrix CBA does not exist, because C⊗B will result in 2×2 matrix, but A is 4×4 matrix, So, they can't be multiplied.

2. Given a=[-4 0 1 3], and b=[1.2 5 6], Calculate the following:

a. Magnitude for a

= Sqrt ((-4)² + (0)² + (1)² + (3)²)

= Sqrt (16 + 0 + 1 + 9)

= Sqrt (26) = 5.09

b. Magnitude for b

= Sqrt [(1)² + (2)² + (5)² + (6)²]

= Sqrt (1 + 4 + 25 + 36]

= Sqrt (66) = 8.12

c. Inner product ab =

(-4x1 + 0x2 + 5x1 + 6x3) = 19

d. Projection of a onto b $= \frac{a.b}{|b|^2} \times b$

$= \frac{19}{66} \times 8.12 = 2.33$

3. calculate Inner & Outer Product for

$$A = \begin{bmatrix} 1 & 2 & 7 & 0 \\ 3 & 0 & -4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & -2 & 0 & 3 \\ 1 & 4 & -2 & 0 \\ 2 & 5 & 8 & -3 \\ 3 & 6 & 9 & 2 \end{bmatrix}$$

Ans- Inner product $= A.B = \sum A_k b_k$

Inner product will be a 2×4 matrix

$1\times5+2\times-2+7\times0+0\times3$ $\quad 3\times5+0\times-2+(-4)\times0+1\times3$

$1\times1+2\times4+7\times-2+0\times0$ $\quad 3\times1+0\times4+(-4)\times(-2)+1\times0$

$1\times2+2\times5+7\times8+0\times-3$ $\quad 3\times2+0\times5+(-4)\times8+1\times-3$

$1\times3+2\times6+7\times9+0\times2$ $\quad 3\times3+0\times6+(-4)\times9+1\times2$

$$\begin{bmatrix} 1 & +5 & 6 & 8 & -8 \\ 18 & 11 & +29 & -25 \end{bmatrix}$$

Outer product = (8 × 16) matrix

$$\begin{bmatrix} 5 & -2 & 0 & 3 & 1 & 4 & -2 & 0 & 2 & 5 & 8 & -3 & 3 & 6 & 9 & 2 \\ 10 & -4 & 0 & 6 & 2 & 8 & -4 & 0 & 4 & 10 & 16 & -6 & 6 & 12 & 18 & 4 \\ 35 & -14 & 0 & 21 & 7 & 28 & -14 & 0 & 14 & 35 & 56 & -21 & 21 & 42 & 63 & 14 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 15 & -6 & 0 & 9 & 3 & 12 & -6 & 0 & 6 & 15 & 24 & -9 & 9 & 18 & 27 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -20 & 8 & 0 & -12 & -4 & -16 & 8 & 0 & -8 & -20 & -32 & 12 & -12 & -24 & -36 & -8 \\ 5 & -2 & 0 & 3 & 1 & 4 & -2 & 0 & 2 & 5 & 8 & -3 & 3 & 6 & 9 & 2 \end{bmatrix}$$

4. Shaw examples of following matrices & calculate their Determinants:

a) Zeroes b) Ones c) Identity

d. Upper Triangular e. Lower Triangular

a. Zeroes - $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3}$

$|A| = 0$

b. ones - $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$

$|B| = 0$

c. Identity - $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{(3\times3)}$

$|C| = 1(1-0) - 0(0-0) + 0(0-0) = \underline{1}$

d. Upper Triangular matrix example:-

$$D = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}_{(3\times3)}$$

$$|D| = 1(1-0) - 1(0-0) + 2(0-0)$$

$$= 1 - 0 + 2 = 3$$

e. Lower Triangular Matrix Example:-

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{(3\times3)}$$

$$|E| = 1(3-0) - 0(2-0) + 0(2-3)$$

$$= 3 - 0 + 0 = 3$$

5) Show a 3×3 matrix and its Transpose matrix.

Ans- Example 3×3 matrix :-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$