Homework 03 - Probability

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Q No. 2.1.1

Ans - S = { (E, E, E, NE), (E, E, NE, E),
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(NE, NE, E, E), (NE, E, E, NE),
(NE, E, NE, E), (E, NE, E, NE),
(E, NE, NE, NE), (NE, NE, NE, E),
(NE, E, NE, NE), (NE, NE, E, NE),
(NE, NE, NE, NE), (E, E, E, E) }

Q No. 2.3.8

a. $P(A) = \frac{70+16}{100} = .86$

b. $P(B) = \frac{70+9}{100} = .79$

c. $P(A') = 1 - \frac{(70+16)}{100} = .14$

d. $P(A \cap B) = \frac{70}{100} = .70$

e. $P(A \cup B) = \frac{70+16+9}{100} = \frac{95}{100} = .95$

f. $P(A' \cup B) = \frac{70+5+9}{100} = \frac{84}{100} = .84$

Q.3.3.8

Mean of CO_2 in atmosphere

$= .06 \times 350 + 0.1 \times 450 + .47 \times 550 + .37 \times 650$

$= 21 + 45 + 258 + 240.5 = 564.5$ ppm

Variance $= (564.5-350)^2 \times .06 + (564.5-450)^2 \times .1 + (564.5-550)^2 \times .47 + (564.5-650)^2 \times .37$

$\sigma^2 = 2760.615 + 1311.025 + 98.817 + 2704.792$

$= 6875.23$

$\sigma = \sqrt{6875.23} = 82.91$

Q.No. 3.4.1

a. mean or μ of discrete uniform distribution

$$\mu = \frac{a+b}{2} = \frac{675+700}{2} = 687.5\,nm$$

variance of a uniform discrete distribution:-

$$\sigma^2 = \frac{(n^2-1)}{12} = \frac{(26^2-1)}{12} = \frac{675}{12}$$
$$= 56.25$$

b. $$\mu = \frac{75+100}{2} = 87.5$$

$$\sigma^2 = \frac{(n^2-1)}{12} = \frac{(26^2-1)}{12} = \frac{675}{12}$$
$$= 56.25$$

Mean changes, as it is the midpoint of interval, So, it shifts with the range. Variance is Same, because the width of interval or n is Same.

4.1.2 a. $P(X<2)$

$$f(x) = 2/x^3, \quad x>1$$

To calculate CDF, we do integration:–

$$F(x) = \int_1^x \frac{2}{x^3} = 2\left[\frac{x^{-3+1}}{-3+1}\right]_1^x$$

$$= \left[-\frac{1}{x^2}\right]_1^x$$

$$= \left[-\frac{1}{x^2} - \left(\frac{-1}{(1)^2}\right)\right] = -\frac{1}{x^2} + 1$$

$$= 1 - \frac{1}{x^2}, \quad x>1$$

$$P(X<2) = F(2) = 1-\frac{1}{2^2} = 1-\frac{1}{4} = \frac{3}{4}$$

$$= 0.75$$

b. $P(X>5) = 1 - P(X<5)$

$$= 1 - F(5)$$

$$= 1 - \left(1 - \frac{1}{25}\right) = 1 - \frac{24}{25} = \frac{1}{25}$$

$$= .04$$

c. $P(4 < X < 8)$

$$F(8) = 1 - \frac{1}{64} = \frac{63}{64}$$

$$F(4) = 1 - \frac{1}{16} = \frac{15}{16}$$

$$= \frac{63}{64} - \frac{15}{16} = \frac{63-60}{64} = \frac{3}{64} = .04$$

d. $P(X < 4 \text{ or } X > 8)$

$$P(X < 4) = F(4) = 1 - \frac{1}{16} = \frac{15}{}$$

$$P(X \geq 8) = 1 - P(X < 8) = 1 - \left(1 - \frac{1}{64}\right)$$

$$= 1 - \frac{63}{64} = \frac{1}{64}$$

So, $P(X < 4 \text{ or } X \geq 8) = \frac{1}{64} + \frac{15}{16}$

$$= \frac{1+60}{64} = \frac{61}{64}$$

$$= .95$$

e. x such that $P(X < x) = 0.95$

$$F(x) = .95$$

$$1 - \frac{1}{x^2} = \frac{95}{100}$$

$$100(x^2 - 1) = 95x^2$$

$$5x^2 = 100$$

$$x^2 = 20$$

$$x = \sqrt{20} = 4.45$$

Q. No 4.5.1

a. $P(Z < 1.32) = .906582$

b. $P(Z < 3.0) = .998650$

c. $P(Z > 1.45) = 1 - .926471$

$$= .073529$$

d. $P(Z > -2.15) = 1 - P(Z < -2.15)$

$$= 1 - .015778$$

$$= 0.984222$$

e. $P(-2.34 < Z < 1.76)$

$P(Z < 1.76) - P(Z < -2.34)$

$= 0.9608 - 0.0096 = 0.9512$

Q. No. 5.2.8

a. $\mu_y = 1.6 \times 73 = 116.8$

conditional PDF

$$f(y \mid x = 73) = \frac{1}{\sqrt{2\pi \times 100}} \exp\left[-\frac{(y - 116.8)^2}{2(10)^2}\right]$$

$$= \frac{1}{10\sqrt{2\pi}} \exp\left[-\frac{(y - 116.8)^2}{200}\right]$$

b. $P(Y < 115 \mid X = 73)$

$$Z = \frac{y - \mu}{\sigma} = \frac{115 - 116.8}{10} = -.18$$

$P(Z < -.18) = .428576$

c) $E(Y \mid X = 73) = 1.6 \times 73 = 116.8$

Q. No. 5.4.1

$$E(X) = 1 \times \frac{1}{8} + 1 \times \frac{1}{4} + 2 \times \frac{1}{2} + 4 \times \frac{1}{}$$

$$= \frac{1}{8} + \frac{1}{4} + 1 + \frac{1}{2} = \frac{15}{8} = 1.875$$

$$E(Y) = \frac{3}{8} + 1 + \frac{5}{2} + \frac{6}{8} = \frac{3+8+20+6}{8}$$

$$= \frac{37}{8} = 4.625$$

$$E(XY) = 3/8 + 1 + 5 + \frac{3}{2}$$

$$= \frac{3+8+40+24}{8} = \frac{75}{8}$$

$$\text{Cov}(X,Y) = E(XY) - \mu_x \mu_y$$

$$= \frac{75}{8} - \frac{15}{8} \times \frac{37}{8}$$

$$= \frac{45}{64} = .703125 = .70$$

Correlation:

$$\rho_{XY} = \frac{\text{Cov}(X,Y)}{\sqrt{V(X)V(Y)}}$$

$$V(X) = \sum x^2 f(x) - \mu^2$$

$$= 1 \times \frac{1}{8} + 1 \times \frac{1}{4} + 4 \times \frac{1}{2} + 16 \times \frac{1}{8} - (1.875)^2$$

$$= \frac{35}{8} - (1.875)^2$$

$$= 4.375 - 3.515 = .86$$

$$\sigma_x = \sqrt{.86} = .92$$

$$V(Y) = 9 \times \frac{1}{8} + 16 \times \frac{1}{4} + 25 \times \frac{1}{2} + 36 \times \frac{1}{8} - (4.625)^2$$

$$= \frac{177}{8} - (4.625)^2$$

$$= 22.125 - 21.390 = .735$$

$$\sigma_y = \sqrt{.735} = .857 = .86$$

Subsituting the values in eq^n :-

Correlation $= \dfrac{.70}{.92 \times .86} = .8860$

$\therefore \rho_{xy} = .886$