



# ADAMAS UNIVERSITY

## END SEMESTER EXAMINATION

(Academic Session: 2020 – 21)

<b>Name of the Program:</b>	M.TECH (Environmental Engineering)	<b>Semester:</b>	II
<b>Paper Title:</b>	Numerical Methods	<b>Paper Code:</b>	ENV21014
<b>Maximum Marks:</b>	50	<b>Time Duration:</b>	3 Hrs
<b>Total No. of Questions:</b>	17	<b>Total No of Pages:</b>	3
(Any other information for the student may be mentioned here)	<ol style="list-style-type: none"> <li>At top sheet, clearly mention Name, Univ. Roll No., Enrolment No., Paper Name &amp; Code, Date of Exam.</li> <li>All parts of a Question should be answered consecutively. Each Answer should start from a fresh page.</li> <li>Assumptions made if any, should be stated clearly at the beginning of your answer.</li> </ol>		

<b>Group A</b> <b>Answer All the Questions (5 x 1 = 5)</b>											
1	What is the intermediate value property?	<b>Re</b>	<b>CO1</b>								
2	What is interpolation and extrapolation? Discuss with an example.	<b>U</b>	<b>CO4</b>								
3	What is order and degree of $y''^2 + 3xy = 1 + x^2$ ?	<b>Re</b>	<b>CO3</b>								
4	Classify the PDE $f_{xx} + f_{yy} + f_x + f_y = 0$ .	<b>U</b>	<b>CO3</b>								
5	Which part of the $(x, y)$ plane is the following equation to be elliptic? $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + (x^2 + 4y^2) \frac{\partial^2 u}{\partial y^2} = 2 \sin(xy)$	<b>Re</b>	<b>CO3</b>								
<b>Group B</b> <b>Answer All the Questions (5 x 2 = 10)</b>											
6 a)	i) What is the rate of convergence of the Newton-Raphson method for any real function $f(x)$ ? ii) If $\pi$ is taken 3.14 in the place of 3.14156, then find the relative error?	<b>Re</b>	<b>CO1</b>								
<b>(OR)</b>											
6 b)	What will be the rounding off values of the numbers of 43.38256, 0.0326457 and 0.2537623 to four significant digits?	<b>Re</b>	<b>CO1</b>								
7 a)	Find the interval in which a real root of the equation $x^3 - 2x - 5 = 0$ lies.	<b>Re</b>	<b>CO1</b>								
<b>(OR)</b>											
7 b)	Tell the pros and cons of the Newton Raphson method.	<b>Re</b>	<b>CO1</b>								
8 a)	If $f(x)$ is given by <table border="1" style="margin: 10px auto; width: 80%;"> <tr> <td><math>x</math></td><td>0</td><td>0.5</td><td>1</td></tr> <tr> <td><math>f(x)</math></td><td>1</td><td>0.8</td><td>0.5</td></tr> </table> Then using Trapezoidal rule, find $\int_0^1 f(x)dx$ .	$x$	0	0.5	1	$f(x)$	1	0.8	0.5	<b>Re</b>	<b>CO3</b>
$x$	0	0.5	1								
$f(x)$	1	0.8	0.5								

(OR)

8 b)	Summarize the Newton divided difference table for the following data: <table><tr><td><math>x</math></td><td>-1</td><td>0</td><td>1</td><td>3</td></tr><tr><td><math>y</math></td><td>2</td><td>1</td><td>0</td><td>-1</td></tr></table>	$x$	-1	0	1	3	$y$	2	1	0	-1	U	CO3
$x$	-1	0	1	3									
$y$	2	1	0	-1									
9 a)	Find the cubic polynomial which takes the following values: <table><tr><td><math>x</math></td><td>0</td><td>1</td><td>2</td></tr><tr><td><math>f(x)</math></td><td>1</td><td>2</td><td>1</td></tr></table> Hence or otherwise find $f'(3)$ .	$x$	0	1	2	$f(x)$	1	2	1	Re	CO3		
$x$	0	1	2										
$f(x)$	1	2	1										

(OR)

9 b)	Tell the Gaussian 2-point quadrature formula.	Re	CO3
10 a)	Given $3\frac{dy}{dx} + \sqrt{y} = e^{0.1x}$ , $y(0.3) = 5$ , and the step size is of $h = 0.3$ . Estimate of $\frac{dy}{dx}(0.9)$ by applying Euler's method.	Ap	CO4

(OR)

10 b)	Applying Taylor's series method to approximate $y$ when $x = 0.2$ for $\frac{dy}{dx} = x^2y - 1$ , $y(0) = 1$ .	Ap	CO4
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**Group C**

**Answer All the Questions (7 x 5 = 35)**

11 a)	The bacteria concentration in a reservoir varies as $C = 4e^{-2t} + e^{-0.1t}$ . Using Newton-Raphson method, find the time required for the bacteria concentration to be 0.5.	Re	CO1
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(OR)

11 b)	Find a real root of the following equation using Secant method $x\log_{10}x = 1.9$ , correct to 3 decimal places.	Re	CO1																
12 a)	Illustrate the mean radiation dose at an altitude of 3000 ft by fitting an exponential curve to the given data: <table><tr><td>Altitude (x)</td><td>50</td><td>450</td><td>780</td><td>1200</td><td>4400</td><td>4800</td><td>5300</td></tr><tr><td>Dose of radiation (y)</td><td>28</td><td>30</td><td>32</td><td>36</td><td>51</td><td>58</td><td>69</td></tr></table>	Altitude (x)	50	450	780	1200	4400	4800	5300	Dose of radiation (y)	28	30	32	36	51	58	69	U	CO2
Altitude (x)	50	450	780	1200	4400	4800	5300												
Dose of radiation (y)	28	30	32	36	51	58	69												

(OR)

12 b)	<p>One entry in the following table is incorrect and <math>y</math> is the cubic polynomial in <math>x</math>. Apply the difference table to locate and correct the error:</p> <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>y</td><td>25</td><td>21</td><td>18</td><td>18</td><td>27</td><td>45</td><td>76</td><td>123</td></tr></table>	x	0	1	2	3	4	5	6	7	y	25	21	18	18	27	45	76	123	Ap	CO2				
x	0	1	2	3	4	5	6	7																	
y	25	21	18	18	27	45	76	123																	
13 a)	<p>A Chemical company, wishing to study the effect of extraction time(<math>t</math>) on the efficiency of an extraction operation (<math>e</math>) obtained from the data shown in the following table:</p> <table><tr><td><math>t</math></td><td>27</td><td>45</td><td>41</td><td>19</td><td>3</td><td>39</td><td>19</td><td>49</td><td>15</td><td>31</td></tr><tr><td><math>e</math></td><td>57</td><td>64</td><td>80</td><td>46</td><td>62</td><td>72</td><td>52</td><td>77</td><td>57</td><td>68</td></tr></table> <p>Build a straight line to the given data by the method of least</p>	$t$	27	45	41	19	3	39	19	49	15	31	$e$	57	64	80	46	62	72	52	77	57	68	Ap	CO2
$t$	27	45	41	19	3	39	19	49	15	31															
$e$	57	64	80	46	62	72	52	77	57	68															

	squares.																
(OR)																	
13 b)	Apply the method of difference to calculate $\Delta^2(\frac{1}{x^2+5x+6})$ .	Ap	CO2														
14 a)	Use the Lagrange's formula to find the form of $f(x)$ , given <table><tr><td><math>x</math></td><td>1</td><td>3</td><td>4</td><td>6</td></tr><tr><td><math>f(x)</math></td><td>-3</td><td>9</td><td>30</td><td>132</td></tr></table> And hence find $f(4.5)$ .	$x$	1	3	4	6	$f(x)$	-3	9	30	132	U	CO3				
$x$	1	3	4	6													
$f(x)$	-3	9	30	132													
(OR)																	
14 b)	The following data gives corresponding value of pressure and specific volume of a super-heated steam: <table><tr><td>Volume (v)</td><td>1.0</td><td>1.2</td><td>1.4</td><td>1.6</td><td>1.8</td><td>2.0</td></tr><tr><td>Pressure (p)</td><td>0</td><td>0.128</td><td>0.544</td><td>1.296</td><td>2.432</td><td>4.00</td></tr></table> Find the rate of change of pressure with respect to volume when $v = 2$ .	Volume (v)	1.0	1.2	1.4	1.6	1.8	2.0	Pressure (p)	0	0.128	0.544	1.296	2.432	4.00	Re	CO3
Volume (v)	1.0	1.2	1.4	1.6	1.8	2.0											
Pressure (p)	0	0.128	0.544	1.296	2.432	4.00											
15 a)	Explain the Simpson's 1/3 rd rule to compute the integral $I = \int_0^{0.6} e^{-x^2} dx$ , by taking six subintervals.	U	CO3														
(OR)																	
15 b)	Find $\int_0^1 \frac{dx}{1+x}$ correct to three decimal places using the Romberg's method. Hence find the value of $\log_e 2$ .	Re	CO3														
16 a)	Illustrate three point Gaussian quadrature formula to compute $\int_1^2 \frac{dx}{1+x^3}$	U	CO3														
(OR)																	
16 b)	From the table below, for what values of $x, y$ is minimum? Also, find this value of $y$ . <table><tr><td><math>x</math></td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td><math>y</math></td><td>0.205</td><td>0.240</td><td>0.259</td><td>0.262</td><td>0.250</td><td>0.224</td></tr></table>	$x$	3	4	5	6	7	8	$y$	0.205	0.240	0.259	0.262	0.250	0.224	Re	CO3
$x$	3	4	5	6	7	8											
$y$	0.205	0.240	0.259	0.262	0.250	0.224											
17 a)	Solve the following differential equation by using Euler's method for $x = 1, h = 0.2$ , $\frac{dy}{dx} = xy, y(0) = 1$ .	Ap	CO4														
(OR)																	
17 b)	Apply R-K method of order four to calculate $y(0.2)$ , given that $\frac{dy}{dx} = x + y^2, y(0) = 1$ , taking $h = 0.1$	Ap	CO4														

\*\*\*\* End \*\*\*\*