



# ADAMAS UNIVERSITY

## END-SEMESTER EXAMINATION : JANUARY 2021

(Academic Session: 2020 – 21)

<b>Name of the Program:</b>	B.Tech in Computer Science and Engineering	<b>Semester:</b>	VII
<b>Paper Title :</b>	Number Theory	<b>Paper Code:</b>	SMA44101
<b>Maximum Marks :</b>	40	<b>Time duration:</b>	3 hrs.
<b>Total No of questions:</b>	8	<b>Total No of Pages:</b>	1
<i>(Any other information for the student may be mentioned here)</i>			

### Instructions:

Attempt any three questions from **Section A** (each carrying 4 marks); any **Two Questions** from **Section B** (each carrying 10 marks).

**Section C** is Compulsory (carrying 8 marks).

<b>Section A ( Attempt any Three)      3 × 4 = 12</b>		
<b>1</b>	Using the digraphic cipher that sends the plaintext block $P_1, P_2$ to the ciphertext block $C_1, C_2$ , with $C_1 \equiv 8P_1 + 9P_2 \pmod{26}$ $C_2 \equiv 3P_1 + 11P_2 \pmod{26}$ encrypt the message DO NOT SHOOT THE MESSENGER.	<b>4</b>
<b>2</b>	Solve the following congruence $3x^2 + 9x + 7 \equiv 0 \pmod{13}$ .	<b>4</b>
<b>3</b>	Decipher the message: RTOLKTOIK, which was encrypted by the transformation, $C \equiv 3P + 24 \pmod{26}.$	<b>4</b>
<b>4</b>	Factor 247 using the Pollard Rho method with polynomial $f(x) = x^2 + 1$ and initial guess $x_0 = 1$ .	<b>4</b>
<b>Section B ( Attempt any Two)      2 × 10 = 20</b>		
<b>5</b>	i) Use Gauss's lemma to compute the Legendre symbol $\left(\frac{5}{19}\right)$ . ii) For any positive integer k, there are infinitely many primes of the form $4k + 1$ .	<b>5+5</b>
<b>6</b>	i) Using the Generalized Quadratic Reciprocity Law, determine whether the congruence $x^2 \equiv 231 \pmod{1105}$ is solvable. ii) Show that the only prime p for which $3p+1$ is a perfect square is $p=5$ .	<b>5+5</b>
<b>7</b>	Using the Diffie-Hellman key agreement protocol, find the common key that can be used by two parties with keys $k_1 = 27$ and $k_2 = 31$ when the modulus is $p=103$ and the base $r=5$ .	<b>10</b>
<b>SECTION C is Compulsory      1 × 8 = 8</b>		
<b>8</b>	If the cipher text message: 05041874034705152088235607360468 is produced by RSA cipher with $e=5$ and $n=2881$ , find the plaintext message.	<b>8</b>