|  |  |  |  |
| --- | --- | --- | --- |
| Image result for adamas university logo | **ADAMAS UNIVERSITY**  **END SEMESTER EXAMINATION**  (Academic Session: 2020 – 21) | | |
| **Name of the Program:** | **M.Tech** | **Semester:** | I |
| **Paper Title:** | **Advanced Graph Theory** | **Paper Code:** | **ECS61115** |
| **Maximum Marks:** | **50** | **Time Duration:** | **3 Hrs** |
| **Total No. of Questions:** |  | **Total No of Pages:** |  |
| *(Any other information for the student may be mentioned here)* | 1. At top sheet, clearly mention Name, Univ. Roll No., Enrolment No., Paper Name & Code, Date of Exam. 2. All parts of a Question should be answered consecutively. Each Answer should start from a fresh page. 3. Assumptions made if any, should be stated clearly at the beginning of your answer. | | |

|  |  |  |  |
| --- | --- | --- | --- |
| **Group A**  **Answer All the Questions (5 x 1 = 5)** | | | |
| 1 | Draw the complete bipartite graph K1,5 | **Remember** | **CO1** |
| 2 | Calculate the girth of the following graph | **Knowledge** | **CO2** |
| 3 | Figure out Eulerian tour of the following graph | **Knowledge** | **CO3** |
| 4 | Calculate in-degree and out-degree of vertex u, v and w of the following graph | **Understand** | **CO4** |
| 5 | Find the subgraph obtained by deleting vertex w: | **Knowledge** | **CO5** |
| **Group B**  **Answer All the Questions (5 x 2 = 10)** | | | |
| 6 a) | Explain the centrality and the geodesic centrality of any graph G=(V,E) where V the set of vertices and E the set of edges. Calculate normalized centrality and normalized geodesic centrality of the following graph: | **Knowledge** | **CO1** |
| **(OR)** | | | |
| 6 b) | If G is 2-connected, then prove that any two vertices of G lie on a common cycle. | **Apply** | **CO1** |
| 7 a) | Agraph is bipartite if and only if all its cycles are even. Prove this statement. | **Apply** | **CO2** |
| **(OR)** | | | |
| 7 b) | If G is a block with v*≥*3, then prove any two edges of G lie on a common cycle with a diagram. | **Apply** | **CO2** |
| 8 a) | Every tree with at least one edge has at least two leaves. Prove this statement with a diagram | **Apply** | **CO3** |
| **(OR)** | | | |
| 8 b) | Let G = (V, E) be a non-empty, non-trivial graph. Prove that G has at least one pair of vertices with equal degree. | **Apply** | **CO3** |
| 9 a) | If G is a tree, then prove that |E|=|V|-1, where E is the edge set and V is vertex set. | **Apply** | **CO4** |
| **(OR)** | | | |
| 9 b) | Let *𝛋* be the line connectivity and *Kp* be the line connectivity and p be any vertex, then 𝛋 (*Kp*)= p- 1. Explain it. | **Knowledge** | **CO4** |
| 10 a) | Let graph G be k-edge-connected if G is connected and every edge-cut has at least k edges. Explain it with an diagram | **Knowledge** | **CO5** |
| **(OR)** | | | |
| 10 b) | Let G be a graph. Show that the edge-connectivity 𝛋e(G) is less than or equal to the minimum degree 𝛿min (G) | **Apply** | **CO5** |
| **Group C**  **Answer All the Questions (7 x 5 = 35)** | | | |
| 11 a) | Define connectivity, point-connectivity and line-connectivity with diagrams. | **Remember** | **CO1** |
| **(OR)** | | | |
| 11 b) | For any graph G, prove the inequality *𝛋(G) ≤𝛌*(G) ≤𝛅(G), where 𝛋, 𝛌 and 𝛅 are connectivity, line connectivity and degree of the graph respectively. | **Apply** | **CO1** |
| 12 a) | Let G be a con­nected graph with three or more vertices. Prove that G is 2-connected if and only if for each pair of vertices in G, there are two internally disjoint paths between them | **Apply** | **CO2** |
| **(OR)** | | | |
| 12 b) | Assume for some k ≥2 where k is connectivity; using a diagram prove this assumption holds for every pair of vertices whose distance apart is less than k. | **Understand** | **CO2** |
| 13 a) | Compare these two graphs G1 and G2 are isomorphic. | **Understand** | **CO3** |
| **(OR)** | | | |
| 13 b) | If V1 and V2 are two sub graphs in G and *V1* and *V2* have *m* and *n* points, then G= *Kmn* . Compare among star biograph *K1,7* and complete bipartitegraphs *K3,6 and K6,3* | **Understand** | **CO3** |
| 14 a) | 1. Let G be a graph. Prove that the edge-connectivity 𝛋e(G) is less than or equal to the minimum degree 𝛿min (G). 2. Every Hamiltonian graph is 2-connected. Every non-Hamiltonian 2-connected graph has a theta subgraph. | **Understand** | **CO4** |
| **(OR)** | | | |
| 14 b) | Let e be any edge of a k-connected graph G for k ≥3. Prove that the edge-deletion subgraph G-e is (k-1)-connected. | **Apply** | **CO4** |
| 15 a) | Let u and v be distinct, non-adjacent vertices in a connected graph G. Prove that the maximum number of internally disjoint u-v paths in G equals the minimum number of vertices needed to separate u and v | **Apply** | **CO4** |
| **(OR)** | | | |
| 15 b) | Let G be any graph. If G contains a u-v path of length 2, prove that G contains k internally disjoint u-v paths. |  | **CO4** |
| 16 a) | Let G be a connected graph. Prove that  (1) *G* is Eulerian (2) Every point of G has even degree (3) The set of lines of G can be partitioned into cycles | **Apply** | **CO5** |
| **(OR)** | | | |
| 16 b) | If a graph G is 2-edge­ connected then prove with necessary and sufficient condition that G is a cycle or a Whitney-Robbins synthesis from a cycle | **Apply** | **CO5** |
| 17 a) | Let u and v be any two non-adjacent vertices of a connected graph G. Let Puv be a collection of internally disjoint u-v paths in G, and let Suv be a u-v separating set of vertices in G. Then show that **⏐**Puv**⏐** ≤ **⏐**Suv**⏐** | **Understand** | **CO5** |
| **(OR)** | | | |
| 17 b) | Let u and v be distinct, non-adjacent vertices in a connected graph G. Prove that the maximum number of internally disjoint u-v paths in G equals the minimum number of vertices needed to separate u and v | **Apply** | **CO5** |