



SCHOOL OF ENGINEERING AND TECHNOLOGY

END-SEMESTER EXAMINATION (THEORY)

(Academic Session: 2019 – 20, Semester Term: Aug 2019– Dec 2019)

Name of the Program: M.Tech

Semester: IX

Stream: CSE

PAPER TITLE: Advanced Graph Theory

PAPER CODE: ECS61115

Maximum Marks: 40

Time duration: 3 hours

Total No of questions: 8

Total No of Pages:2

(Any other information required for the student may be mentioned here)

Answer all the Groups

Group A

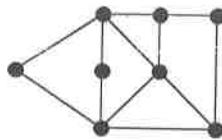
(Answer all the questions)

$5 \times 1 = 5$

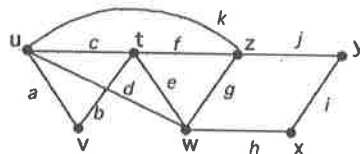
1.

a) Draw the complete bipartite graph $K_{1,5}$

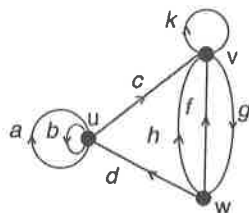
b) Calculate the girth of the following graph



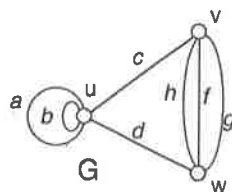
c) Figure out eulerian tour of the following graph



d) Calculate in-degree and out-degree of vertex u, v and w of the following graph



e) Show the subgraph obtained by deleting vertex w:

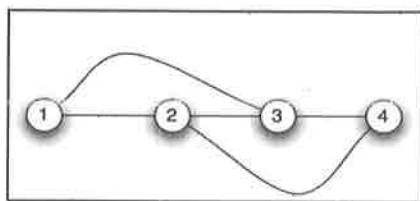


Group B

(Answer three questions)

$$5 \times 3 = 15$$

2. Explain the centrality and the geodesic centrality of any graph $G=(V,E)$ where V the set of vertices and E the set of edges. Calculate normalized centrality and normalized geodesic centrality of the following graph:



3. A graph is bipartite if and only if all its cycles are even. Prove this statement.
4. Every tree with at least one edge has at least two leaves. Prove this statement with a diagram
5. If G is a tree, then prove that $|E|=|V|-1$, where E is the edge set and V is vertex set.

Group C

(Answer two questions)

$$2 \times 10 = 20$$

6. a) Define connectivity, point-connectivity and line-connectivity.
6. b) For any graph G , prove the inequality $\kappa(G) \leq \lambda(G) \leq \delta(G)$, where κ , λ and δ are connectivity, line connectivity and degree of the graph respectively.
7. a) Let G be a connected graph with three or more vertices. Prove that G is 2-connected if and only if for each pair of vertices in G , there are two internally disjoint paths between them.
7. b) Assume for some $k \geq 2$ where k is connectivity; using a diagram prove this assumption holds for every pair of vertices whose distance apart is less than k .
8. Let T be a graph with n vertices. Prove that the following statements are equivalent.
 - 1) T is a tree.
 - 2) T contains no cycles and has $n - 1$ edges.
 - 3) T is connected and has $n - 1$ edges.
 - 4) T is connected, and every edge is a cut-edge.
 - 5) Any two vertices of T are connected by exactly one path.
 - 6) T contains no cycles, and for any new edge e , the graph $T + e$ has exactly one cycle.

