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Exercise - 1(A)

1. Evaluate the following limit.

(a) $\lim_{x \to 1} (2x+3)$

$= 2 \times 1 + 3$

5

(b) $\lim_{x \to -7} (2x+5)$

$= 2 \times (-7) + 5$

$= -14 + 5$

$= -9$

(c) $\lim_{x \to 5} \frac{4}{x-7}$

$= \frac{4}{5-7}$

$= \frac{4}{-2}$

$= -2$

(d) $\lim_{h \to 0} \frac{5}{\sqrt{2h+1} + 1}$

$= \frac{5}{\sqrt{2 \times 0 + 1} + 1}$

$\frac{5}{2}$

2. Evaluate.

ⓐ $\lim\limits_{x \to 0} \frac{7x^2+4x}{x} \left[\frac{0}{0} \text{ form}\right]$

$= \lim\limits_{x \to 0} \frac{\cancel{x}(7x+4)}{\cancel{x}} \left[\frac{0}{0} \text{ form}\right]$

$= 7 \times 0 + 4$

$= 4$

ⓑ $\lim\limits_{x \to 5} \frac{x^2-25}{x-5} \left[\frac{0}{0} \text{ form}\right]$

$= \lim\limits_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}}$

$= 10$

ⓒ $\lim\limits_{x \to 1} \frac{x^2-4x+3}{x-1} \left[\frac{0}{0} \text{ form}\right]$

$\cancel{\lim\limits_{x \to 1} \frac{x^2(x-4)+3}{x-1}}$

$\lim\limits_{x \to 1} \frac{x^2-(3+1)x+3}{x-1} \left[\frac{0}{0} \text{ form}\right]$

$\lim\limits_{x \to 1} \frac{x^2-3x-x+3}{x-1}$

$\lim\limits_{x \to 1} \frac{x(x-3(x-3)}{(x-1)}$

$$= \lim_{x \to 0} \frac{(x-3)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= 1-3$$

$$-2 //$$

(d) $\lim_{x \to 2} \frac{x^2-7x+10}{x-2}$ $\left[\frac{0}{0} \text{ form}\right]$

$$= \lim_{x \to 2} \frac{x^2-(5+2)x+10}{x-2}$$

$$= \lim_{x \to 2} \frac{x^2-5x-2x+10}{x-2}$$

$$= \lim_{x \to 2} \frac{x(x-5)-2(x-5)}{(x-2)}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x-5)}{\cancel{(x-2)}}$$

$$= 2-5$$

$$-3 //$$

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ⓔ $\lim_{x \to 1} \frac{x^2+x-2}{x^2-1}$ $\left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x \to 1} \frac{x^2+(2-1)x-2}{x^2-1}$

$= \lim_{x \to 1} \frac{x^2+2x-x-2}{x^2-1}$

$= \lim_{x \to 1} \frac{x(x+2)-(x+2)}{(x+1)(x-1)}$

$= \lim_{x \to 1} \frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

$3/2$

F) $\lim_{x \to 1} \frac{x^2-4x+3}{x^2-3x+2}$ $\left[\frac{0}{0} \text{ form}\right]$

$= \lim_{x \to 1} \frac{x^2-(3+1)x+3}{x^2-(2+1)x+2}$

$= \lim_{x \to 1} \frac{x^2-3x-x+3}{x^2-2x-x+2}$

$= \lim_{x \to 1} \frac{x(x-3)-(x-3)}{x(x-2)-(x-2)}$

$= \lim_{x \to 1} \frac{(x-3)\cancel{(x-1)}}{(x-2)\cancel{(x-1)}}$

$\frac{1-3}{1-2}$

$\frac{\cancel{+}2}{\cancel{+}1} =$

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(g) $\lim_{x \to -3} \left[\frac{x+3}{x^2+4x+3} \right]$ $\left[\frac{0}{0} \text{ form} \right]$

$\lim_{x \to -3} \left[\frac{x+3}{x^2 + (3+1)x + 3} \right]$ $\left[\frac{0}{0} \text{ form} \right]$

$\lim_{x \to -3} \left[\frac{x+3}{x^2 + 3x + x + 3} \right]$

$\lim_{x \to -3} \frac{x+3}{x(x+3)+1(x+3)}$

$\lim_{x \to -3} \frac{\cancel{x+3}}{\cancel{(x+3)}(x+1)}$

$\frac{1}{-3+1}$

$\frac{1}{-2}$

(h) $\lim_{x \to 3} \left[\frac{1}{x-3} - \frac{9}{x^3 - 3x^2} \right]$ $\left[\frac{0}{0} \text{ form} \right]$

$\lim_{x \to 3} \left[\frac{1}{x-3} - \frac{9}{x^2(x-3)} \right]$

$\lim_{x \to 3} \left[\frac{x^2 - 3^2}{x^2(x-3)} \right]$

$\lim_{x \to 2} \left[\frac{(x+3)\cancel{(x-3)}}{x^2 \cancel{(x-3)}} \right]$

$\frac{5}{\ }$

3. Evaluate

(a) $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$ $\boxed{\frac{0}{0} \text{ Form}}$

$= \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$

$= \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \times \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$

$= \lim_{x \to 9} \frac{(\sqrt{x})^2 - 3^2}{(x-9) \times (\sqrt{x} + 3)}$

$= \lim_{x \to 9} \frac{\cancel{(x-9)}}{\cancel{(x-9)} \times (\sqrt{x} + 3)}$

$= \frac{1}{\sqrt{9} + 3}$

$= \frac{1}{6}$ //

(b) $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$

$\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x} \times \frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2}$

$\lim_{x \to 0} \frac{(\sqrt{x+4})^2 - 2^2}{x(\sqrt{x+4} + 2)}$

$\lim_{x \to 0} \frac{x + 4 - 4}{x(\sqrt{x+4} + 2)}$

$\lim$ $\cancel{x}$ $= \frac{1}{\ }$

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Ⓒ $\lim_{x \to 0} \frac{7x}{\sqrt{3x+4}-2}$

$= \lim_{x \to 0} \frac{7x}{\sqrt{3x+4}-2} \times \frac{\sqrt{3x+4}+2}{\sqrt{3x+4}+2}$

$= \lim_{x \to 0} \frac{7x(\sqrt{3x+4}+2)}{(\sqrt{3x+4})^2-2^2}$

$= \lim_{x \to 0} \frac{7\cancel{x}(\sqrt{3x+4}+2)}{3\cancel{x}+\cancel{4}-\cancel{4}}$

$7(\sqrt{4}+2)/3$

$7(4)/3$

$28/3$

ⓓ $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2}$

$= \lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2} \times \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$

$= \lim_{x \to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(\sqrt{x+3})^2-2^2}$

$\frac{(x-1)(\sqrt{x+3}+2)}{x+3-4}$

$= \lim_{x \to 1} \frac{\cancel{(x-1)}(\sqrt{x+3}+2)}{\cancel{(x-1)}}$

$= \sqrt{1+3}+2$

$= 2+2$

$= 4$

(e) $$\lim_{x \to 4} \frac{x^2-16}{\sqrt{3x+4}-4}$$

$$= \lim_{x \to 4} \frac{x^2-16}{\sqrt{3x+4}-4} \times \frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}$$

$$= \lim_{x \to 4} \frac{(x^2-16) \times \sqrt{3x+4}+4}{(\sqrt{3x+4})^2-4^2}$$

$$= \lim_{x \to 4} \frac{(x^2-16) \times \sqrt{3x+4}+4}{3x+4-16}$$

$$= \lim_{x \to 4} \frac{(x^2-16) \cdot \sqrt{3x+4}+4}{3(x-4)}$$

$$= \lim_{x \to 4} \frac{(x+4) \cdot \sqrt{3x+4}+4}{3}$$

$$= \frac{8(\sqrt{12+4}+4)}{3}$$

$$= \frac{8 \cdot (4+4)}{3}$$

$$= \frac{8 \cdot 8}{3}$$

$$\frac{64}{3}$$

f $\lim_{x \to -1} \dfrac{\sqrt{x^2+8} - 3}{x+1}$

$= \lim_{x \to -1} \dfrac{\sqrt{x^2+8} - 3}{x+1} \times \dfrac{\sqrt{x^2+8} + 3}{\sqrt{x^2+8} + 3}$

$= \lim_{x \to -1} \dfrac{(\sqrt{x^2+8})^2 - 3^2}{(x+1)(\sqrt{x^2+8} + 3)}$

$= \lim_{x \to -1} \dfrac{x^2+8-9}{(x+1)(\sqrt{x^2+8} + 3)}$

$= \lim_{x \to -1} \dfrac{x^2-1}{(x+1)(\sqrt{x^2+8} + 3)}$

$= \lim_{x \to -1} \dfrac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(\sqrt{x^2+8} + 3)}$

$= \dfrac{-1-1}{\sqrt{(-1)^2+8} + 3}$

$= \dfrac{-2}{3+3}$

$= \dfrac{-2}{6}$

(g) $\lim_{x \to a} \dfrac{\sqrt{3a-x} - \sqrt{x+a}}{4(x-a)} \times \dfrac{\sqrt{3a-x} + \sqrt{x+a}}{\sqrt{3a-x} + \sqrt{x+a}}$

$\lim_{x \to a} \dfrac{(\sqrt{3a-x})^2 - (\sqrt{x+a})^2}{4(x-a)(\sqrt{3a-x} + \sqrt{x+a})}$

$\lim_{x \to a} \dfrac{3a-x-x-a}{4(x-a)\sqrt{3a-x} + (\sqrt{x+a})}$

$\dfrac{2a-2x}{4(x-a)(\sqrt{3a-x}) + \sqrt{x+a})}$

$$\frac{-\cancel{2}}{{}_2\cancel{4}(\sqrt{3a-x}+\sqrt{x+a})}$$

$$\frac{-1}{2(\sqrt{3a-x}+\sqrt{x+a})}$$

$$\frac{-1}{2(\sqrt{2a}+\sqrt{2a})}$$

$$\frac{-1}{2(\sqrt{2a}(1+1))}$$

$$\frac{-1}{\sqrt{2a}\cdot 2}$$

$$\frac{-1}{4\sqrt{2a}}$$

(b) $$\lim_{x\to a}\frac{\sqrt{2x}-\sqrt{3x-a}}{\sqrt{x}-\sqrt{a}}\times\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}\times\frac{\sqrt{2x}+\sqrt{3x-a}}{\sqrt{2x}+\sqrt{3x-a}}$$

$$\lim_{x\to a}\frac{(\sqrt{2x})^2-(\sqrt{3x-a})^2\ (\sqrt{x}+\sqrt{a})}{(\sqrt{x})^2-(\sqrt{a})^2\times\sqrt{2x}+\sqrt{3x-a}}$$

$$\lim_{x\to a}\frac{(2x-3x+a)(\sqrt{x}+\sqrt{a})}{(x-a)\times(\sqrt{2x}+\sqrt{3x-a})}$$

$$\lim_{x\to a}\frac{(a-x)(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{2x}+\sqrt{3a-a})}$$

$$\lim_{x\to a}\frac{-1(\sqrt{x}+\sqrt{a})}{\sqrt{2x}+\sqrt{2a}}$$

$$\lim_{x\to a}\frac{-1(\sqrt{a}+\sqrt{a})}{\sqrt{2a}+\sqrt{2a}}$$

$$= \frac{-\cancel{2}\sqrt{a}}{\cancel{2}\sqrt{2a}}$$

$$= -\sqrt{\frac{a}{2a}}$$

$$= -\frac{1}{\sqrt{2}} \;//$$

① $$\lim_{x\to 2} \frac{x-\sqrt{8-x^2}}{\sqrt{x^2+12}-4} \times \frac{x+\sqrt{8-x^2}}{x+\sqrt{8-x^2}} \times \frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}$$

$$= \lim_{x\to 2} \frac{x^2-(\sqrt{8-x^2})^2 \times (\sqrt{x^2+12}+4)}{((\sqrt{x^2+12})^2-4^2)(x+\sqrt{8-x^2})}$$

$$= \lim_{x\to 2} \frac{(x^2-8+x^2)\times(\sqrt{x^2+12}+4)}{x^2+12-16\;(x+\sqrt{8-x^2})}$$

$$\cancel{\lim_{x\to 2} \frac{2^2-8+2^2(\sqrt{2^2+12}+4)}{2^2+12-16(2+\sqrt{8-2^2})}}$$

$$\cancel{(6-8)(\sqrt{16}+4)}$$

$$= \lim_{x\to 2} \frac{(2x^2-8)\sqrt{x^2+12}+4}{(x^2-4)(x+\sqrt{8-x^2})}$$

$$= \lim_{x\to 2} \frac{2(x^2-4)}{(x^2-4)} \frac{\sqrt{x^2+12}+4}{(x+\sqrt{8-x^2})}$$

$$= \frac{2(\sqrt{4+12}+4)}{2+\sqrt{8-2^2}}$$

$$= \frac{2\cdot(8)}{2+2}$$

$$= \frac{\cancel{16}\,4}{\cancel{4}}$$

$$4 \;//$$

④ Evaluate

(a) $\lim_{x \to a} \dfrac{x^5 - a^5}{x - a}$

$5a^{5-1}$

$5a^4$ //

(b) $\lim_{x \to 2} \dfrac{x^8 - 256}{x - 2}$

$\lim_{x \to 2} \dfrac{x^8 - 2^8}{x - 2}$

8

(c) $\lim_{x \to a} \dfrac{x^{2/3} - a^{2/3}}{\cancel{4x^3 - 1}\ x - a}$

$\frac{2}{3} a^{2/3 - 1}$

$\frac{2}{3} a^{\frac{2-3}{3}}$

$\frac{2}{3} a^{-1/3}$

$\dfrac{2}{3a^{1/3}}$

$= x^{1/2}$

$= \sqrt{x} \quad \frac{1}{\sqrt{x}}$

$\frac{1}{i^{1/2}}$

$i^{-1/2} = i^{2-1}$

(d) $\lim_{x \to a} \frac{x^{5/2} - a^{5/2}}{x^{3/2} - a^{3/2}}$

$$\lim_{x \to a} \frac{\frac{x^{5/2} - a^{5/2}}{x-a}}{\frac{x^{3/2} - a^{3/2}}{x-a}}$$

$$\frac{\frac{5}{2} a^{5/2 - 1}}{\frac{3}{2} a^{3/2 - 1}}$$

$$\frac{\frac{5}{\cancel{2}} a^{3/2}}{\frac{3}{\cancel{2}} a^{1/2}}$$

$$\frac{5}{3} a^{3/2} \cdot a^{-1/2}$$

$$\frac{5}{3} a^{3/2 - 1/2}$$

$$\frac{5}{3} a$$

$9 + \infty = ?$

$(x+2)(x+3)$

$\infty + 2 = \infty$

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5.

(a) $\lim_{x \to \infty} \frac{x^2 + 7x + 3}{9x^2 + 7x + 2}$ $\left(\frac{\infty}{\infty} \text{ form} \right)$

Dividing neunomitor and denominator by higher degree which is x^2.

$$= \lim_{x \to \infty} \frac{\frac{x^2 + 7x + 3}{x^2}}{\frac{9x^2 + 7x + 2}{x^2}}$$

$$= \lim_{x \to \infty} \frac{\frac{x^2}{x^2} + \frac{7x}{x^2} + \frac{3}{x^2}}{\frac{9x^2}{x^2} + \frac{7x}{x^2} + \frac{2}{x^2}}$$

$$= \lim_{x \to \infty} \frac{1 + \frac{7}{x} + \frac{3}{x^2}}{9 + \frac{7}{x} + \frac{2}{x^2}}$$

$$= \frac{1}{9}$$

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(b) $\lim_{x \to \infty} \frac{4x^2 + x + 1}{3x^2 + 2x + 1} \quad \left(\frac{\infty}{\infty}\right)$

$= \lim_{x \to \infty} \frac{\frac{4x^2}{x^2} + \frac{x}{x^2} + \frac{1}{x^2}}{\frac{3x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2}}$

$= \lim_{x \to \infty} \frac{4/x + 1/x + 1/x}{3 + 2/x + 1/x}$

$= 4/3$

(c) $\lim_{x \to \infty} \frac{x^2 + x - 2}{4x^3 - 1} \quad \left(\frac{\infty}{\infty}\right)$

$= \lim_{x \to \infty} \frac{\frac{x^2}{x^3} + \frac{x}{x^3} - \frac{2}{x^3}}{\frac{4x^3}{x^3} - \frac{1}{x^3}}$

$= \lim_{x \to \infty} \frac{\frac{1}{x} + \frac{1}{x^2} - \frac{2}{x^3}}{4 - \frac{1}{x^3}}$

$= \frac{0}{4}$

$= 0$ //

(d) $\lim_{x \to \infty} (\sqrt{x+1} - \sqrt{x})$ ($\infty - \infty$ form)

$$= \lim_{x \to \infty} \sqrt{x+1} - \sqrt{x} \times \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{(\sqrt{x+1})^2 - (\sqrt{x})^2}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{\cancel{x} + 1 - \cancel{x}}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x^{1/2}}}{\sqrt{\frac{x+1}{x}} + \frac{\sqrt{x}}{\sqrt{x}}}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{x^{1/2}}}{\sqrt{1+\frac{1}{x}} + 1}$$

$$= \frac{\frac{1}{\infty}}{\sqrt{1+\frac{1}{\infty}} + 1}$$

$$= \frac{0}{1+1}$$

$$= \frac{0}{2}$$

$$= 0$$

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ⓒ $\lim_{x \to \infty} \sqrt{x}\,(\sqrt{x+1} - \sqrt{x})$ ($\infty - \infty$ form)

$$= \lim_{x \to \infty} \sqrt{x}(\sqrt{x+1} - \sqrt{x}) \times \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}\,((\sqrt{x+1})^2 - (\sqrt{x})^2}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}\,(\cancel{x} + 1 - \cancel{x})}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}}{\sqrt{x+1} + \sqrt{x}}$$

$$= \lim_{x \to \infty} \frac{\frac{\sqrt{\cancel{x}}}{\sqrt{\cancel{x}}}}{\sqrt{\frac{x+1}{x}} + \frac{\sqrt{\cancel{x}}}{\sqrt{\cancel{x}}}}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1}$$

$$= \frac{1}{1+1}$$

$$\frac{1}{2} \,//$$

(F) $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x+1}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$\lim_{x\to\infty} = \frac{\frac{\sqrt{x^2+1}}{x}}{\frac{x+1}{x}}$

or, $\lim_{x\to\infty} \frac{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}}{1+\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{\sqrt{\frac{1}{\infty}+\frac{1}{x^2}}}{1+\frac{1}{x}}$

$= \frac{\sqrt{\frac{1}{\infty}+\frac{1}{\infty^2}}}{1+\frac{1}{\infty}}$

$= \frac{0}{1}$

$= 0$

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6. If $f(x) = \frac{ax+b}{x-5}$, $\lim_{x\to 0} f(x) = -1$ and $\lim_{x\to\infty} f(x) = 3$ find the value of $f(3)$

Soln

$$\lim_{x\to 0} f(x) = -1$$

$$\lim_{x\to 0} \frac{ax+b}{x-5} = -1$$

$$\frac{a.0+b}{0-5} = -1$$

$$b = 5$$

Again,

$$\lim_{x\to\infty} f(x) = 3$$

$$\lim_{x\to\infty} \frac{ax+b}{x-5} = 3$$

$$\lim_{x\to\infty} \frac{\frac{ax+b}{x}}{\frac{x-5}{x}} = 3$$

$$\lim_{x\to\infty} \frac{a+\frac{b}{x}}{x-\frac{5}{x}}$$

$$\frac{a+0}{1-0} = 3$$

$$a = 3$$

Putting $a=3$ & $b=5$ In $f(x)$

$$f(x) = \frac{3x+5}{x-5} = f(3) = \frac{3\times3+5}{3-5} = -7$$

7. If $\lim_{x\to 1} \frac{x^4-1}{x-1} = \lim_{x\to k} \frac{x^3-k^3}{x^2-k^2}$ find the value of k.

Sol^n

$$\lim_{x\to 1} \frac{x^4-1}{x-1} = \lim_{x\to k} \frac{x^3-k^3}{x^2-k^2}$$

$$\cancel{\lim_{x\to 1}} \; 4\cdot 1^{4-1} = \lim_{x\to k} \frac{\frac{x^3-k^3}{x-k}}{\frac{x^2-k^2}{x-k}}$$

$$4 = \frac{3\cdot k^{\cancel{2}}}{2\cdot \cancel{k}}$$

$$\frac{8}{3} = k$$

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④ $\lim_{x \to 0} \frac{\tan mx}{\tan nx}$ $\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x \to 0} \frac{\frac{\tan mx}{mx} \times mx}{\frac{\tan nx}{nx} \times nx}$$

$$= \frac{\left(\lim_{mx \to 0} \frac{\tan mx}{mx}\right)}{\left(\lim_{nx \to 0} \frac{\tan nx}{nx}\right)} \times \frac{m\cancel{x}}{n\cancel{x}}$$

$$= \frac{1}{1} \times \frac{m}{n}$$

$$= m/n$$

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6. $\lim_{x \to b} \frac{\tan(x-b)}{x^2-b^2}$ $\left(\frac{0}{0} \text{ form}\right)$

$\lim_{x \to b} \frac{\tan(x-b)}{(x-b)} \times \frac{1}{x+b}$

$= \lim_{x-b \to 0} \frac{\tan(x-b)}{(x-b)} \times \frac{1}{x+b}$

$= 1 \times \frac{1}{x+b}$

$= \frac{1}{2b}$

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8; $\lim_{x \to 0} \frac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x}$ $\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x \to 0} \frac{\frac{\sin 2x}{2x} \times 2x + \frac{\sin 6x}{6x} \times 6x}{\frac{\sin 5x}{5x} \times 5x - \frac{\sin 3x}{3x} \times 3x}$$

$$= \lim_{x \to 0} \frac{2x + 6x}{5x - 3x}$$

$$= \lim_{x \to 0} \frac{8x}{2x}$$

$$= 4$$

9; $\lim_{x \to 0} \frac{\tan 3x - 2x}{3x - \sin 2x}$ $\left(\frac{0}{0} \text{ form}\right)$

$$\lim_{x \to 0} \frac{\frac{\tan 3x}{3x} \times x - 2x}{3x - \frac{\sin 2x}{2x} \times 2x}$$

$$= \frac{\lim_{x \to 0} \left(\frac{\tan 3x}{3x} \times 3x\right) - 2x}{\lim_{x \to 0} \left(3x - \frac{\sin 2x}{2x} \times 2x\right)}$$

$$= \lim_{x \to 0} \frac{3x - 2x}{3x - 2x}$$

$$= \lim_{x \to 0} \frac{x}{x}$$

$$1$$

$= \frac{2}{4} = 1/2$

ii) $\lim_{x \to 0} \frac{1-\cos 3x}{1-\cos 5x}$ $\left[\frac{0}{0} \text{ form}\right]$

Soln

$= \lim_{x \to 0} \frac{1-\cos 3x}{1-\cos 5x}$

$= \lim_{x \to 0} \frac{2\sin^2 \frac{3x}{2}}{2\sin^2 \frac{5x}{2}}$

$= \lim_{x \to 0} \frac{\frac{2\sin^2 \frac{3x}{2}}{\left(\frac{3x}{2}\right)^2} \times \left(\frac{3x}{2}\right)^2}{\frac{2\sin^2 \frac{5x}{2}}{(5x/2)^2} \times \left(\frac{5x}{2}\right)^2}$

$= 2 \frac{1}{1}$

14. $\lim_{x \to 0} \frac{\cos mx - \cos nx}{x^2}$ $\boxed{\frac{0}{0} \text{ form}}$

$= \lim_{x \to 0} \frac{2 \cdot \sin\left(\frac{mx+nx}{2}\right) \cdot \sin\left(\frac{nx-mx}{2}\right)}{x^2}$

$= \lim_{x \to 0} \frac{2 \sin x\left(\frac{m+n}{2}\right) \cdot \sin x\left(\frac{m-x}{2}\right)}{x^2}$

$= \lim_{x \to 0} 2 . \frac{\sin\left(\frac{m+n}{2}\right)}{\left(\frac{m+n}{2}\right)} \times \left(\frac{m+n}{2}\right) \cdot \frac{\sin\frac{(m-x)}{2}}{\left(\frac{m-x}{2}\right)} \times \left(\frac{m-x}{2}\right)$

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$$= 2 \cdot \frac{1}{} \cdot \left(\frac{m+n}{2}\right)\left(\frac{m-n}{2}\right)$$

$$= \frac{m^2 - n^2}{2}$$

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16. $\lim_{x \to y} \dfrac{\sin x - \sin y}{x - y}$ $\boxed{\dfrac{0}{0} \text{ form}}$

$$\lim_{x \to y} \frac{2\cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)}{x-y}$$

$$2 \cdot \lim_{x \to y} \cos\left(\frac{x+y}{2}\right) \cdot \frac{\sin\left(\frac{x-y}{2}\right)}{\left(\frac{x-y}{2}\right) \cdot 2}$$

$$2 \cdot \lim_{x \to y} \left\{ \cos\left(\frac{x+y}{2}\right) \cdot \frac{\sin\left(\frac{x-y}{2}\right)}{\left(\frac{x-y}{2}\right)} \right\} \times \frac{1}{2}$$

$$\cancel{2} \cdot \cos\frac{\cancel{2}y}{\cancel{2}} \cdot 1 \cdot \frac{1}{\cancel{2}}$$

$\cos y$ //

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19. $\lim_{x \to a} \dfrac{x\cos a - a\cos x}{(x-a)}$ $\left(\dfrac{0}{0}\text{ form}\right)$

Soln

$= \lim_{x \to a} \dfrac{x\cos a - a\cos x}{(x-a)}$

$= \lim_{x \to a} \dfrac{x\cos a - a\cos a + a\cos a - a\cos x}{(x-a)}$

$= \lim_{x \to a} \dfrac{(x-a)\cos a}{(x-a)} - a\dfrac{(\cos x - \cos a)}{x-a}$

$= \cos a - \lim_{x \to a} \dfrac{-2\cdot\sin\left(\frac{x+a}{2}\right)\cdot\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2} \times 2}$

$= \cos a + a\, \cancel{2}\cdot\sin\dfrac{\cancel{2}a}{\cancel{2}}\cdot\dfrac{1}{\cancel{2}}$

$= \cos a + a\sin a$

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20. $$\lim_{y\to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{y\to 0} \frac{x\sec(x+y) + y\sec(x+y) - x\sec x}{y}$$

$$= \lim_{y\to 0} \frac{y\sec(x+y)}{y} + \frac{x(\sec(x+y) - \sec x)}{y}$$

$$= \lim_{y\to 0} \sec(x+y) + \frac{x}{y}\left\{\frac{1}{\cos(x+y)} - \frac{1}{\cos x}\right\}$$

$$= \lim_{y\to 0} \sec(x+y) + \frac{x}{y}\left\{\frac{\cos x - \cos(x+y)}{\cos(x+y)\cos x}\right\}$$

$$= \lim_{y\to 0} \sec(x+y) + \frac{x}{y}\left\{\frac{2\cdot\sin\frac{2x+y}{2}\cdot\sin\left(\frac{y+x-x}{2}\right)}{\cos(x+y)\cos x}\right\}$$

$$= \lim_{y\to 0} \sec(x+y) + \frac{x}{y}\left\{\frac{2\cdot\sin\left(x+\frac{y}{2}\right)\cdot\sin\left(\frac{y}{2}\right)}{\cos(x+y)\cos x\left(\frac{y}{2}\right)}\right\}$$

$$= \lim_{y\to 0} \sec(x+y) + \frac{x}{\cancel{y}}\left\{\frac{\cancel{2}\cdot\sin\left(x+\frac{y}{2}\right)}{\cos(x+y)\cdot\cos x}\left(\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)\frac{y}{2}\right\}$$

$$= \sec x + \frac{\sin x}{\cos^2 x}$$

$$= \sec x + \tan x\cdot\sec x$$

$$\sec x\,(1+\tan x)$$

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22. $\lim_{x \to c} \dfrac{\sin x - \sin c}{\sqrt{x} - \sqrt{c}}$

$= \lim_{x \to c} \dfrac{\sin x - \sin c}{\sqrt{x} - \sqrt{c}} \times \dfrac{\sqrt{x} + \sqrt{c}}{\sqrt{x} + \sqrt{c}}$

$= \lim_{x \to c} \dfrac{2 \cdot \cos\left(\frac{x+c}{2}\right) \cdot \sin\left(\frac{x-c}{2}\right) \times \sqrt{x} + \sqrt{c}}{(\sqrt{x})^2 - (\sqrt{c})^2}$

$= \lim_{x \to c} \dfrac{2 \cdot \cos\left(\frac{x+c}{2}\right) \cdot \sin\left(\frac{x-c}{2}\right) \cdot (\sqrt{x} + \sqrt{c})}{x - c}$

$= \lim_{x \to c} \dfrac{2 \cdot \cos\left(\frac{x+c}{2}\right) \cdot \sin\left(\frac{x-c}{2}\right) \times \sqrt{a} + \sqrt{c}}{\left(\frac{x-c}{2}\right) \cdot 2}$

$= \cancel{2} \cos \frac{\cancel{2}c}{\cancel{2}} \cdot 1 \times (\sqrt{c} + \sqrt{c}) \cdot \frac{1}{\cancel{2}}$

$\cos c \; 2\sqrt{c}$

$\underline{\underline{2\sqrt{c} \cos c}}$

$$= \lim_{x \to 0}$$

$$= e^2 \cdot 1$$

$$= e^2$$

23) $\lim_{x \to 0} \frac{e^{5x} - 1}{x \cdot 3^x} \quad \left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x \to 0} \left(\frac{e^{5x} - 1}{5x} \times 5 \times \frac{1}{3^x} \right)$$

$$= 1 \times 5 \frac{1}{3^0}$$

$$= 5$$

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26. $\lim_{x \to 0} \frac{\log_e(1+x)}{e^x - 1}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0} \frac{\log_e(1+x)}{e^x - 1}$

$= \lim_{x \to 0} \frac{\frac{\log_e(1+x)}{x}}{\frac{e^x - 1}{x}}$

$= \frac{1}{1} = 1$

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28. $\lim_{x \to 0} \frac{a^x - b^x}{x}$ $\left(\frac{0}{0} \text{ form}\right)$

$\lim_{x \to 0} \frac{a^x - b^x + 1 - 1}{x}$

$= \lim_{x \to 0} \frac{a^x - 1}{x} - \frac{(b^x - 1)}{x}$

$= \log_e a - \log_e b$

$= \log_e \frac{a}{b}$ //

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Exercise 1 (D)

1. Test the continuity or discontinuity of the following functions at the points specified

(a) $f(x) = x^3 + 3x + 2$ at $x = 1$

(a) Left hand limit $x = 1$ is,

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^3 + 3x + 2) = 1^3 + 3 \times 1 + 2 = 6$

Right hand limit $x = 1$ is,

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^3 + 3x + 2) = 1^3 + 3 \times 1 + 2 = 6$

Hence, $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$ are finite & equal

So, $\lim_{x \to 1} f(x)$ exist and $\lim_{x \to 1} f(x) = 6$

Also,

$f(1) = 1^3 + 3 \times 1 + 2 = 6$

$\therefore \lim_{x \to 1} f(x) = f(1)$ Hence $f(x)$ is continuous $x = 1$

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(b) $f(x) = 7 - x^2$ at $x = 0$

Soln

Left hand limit at $x = 0$ is,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} = (7 - x^2) = 7 - 0 = 7$$

Right hand limit at $x = 0$ is,

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (7 - x^2) = 7 - 0 = 7$$

Hence, $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$ are finite & equal.

Thus, $f(0) = 7 - 0^2 = 7$

$$\lim_{x \to 0} f(x) = f(0)$$

Thus, $f(x)$ is continuous at $x = 0$.

(c) Left hand limit at $x = 0$ is,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{1}{3x} = -\infty$$

i.e $\lim_{x \to 0^-} f(x)$ does not exist

d. Here $f(x) = \frac{1}{1-x}$

functional value $= f(1) = \frac{1}{1-1} = \frac{1}{0} = \infty$

i.e $f(1)$ does not exist

So, $f(x)$ is not continuous at $x = 1$

(e) Here, $f(x) = \frac{1}{x-3}, x \neq 3$

Left hand limit $= \lim_{x \to a^-} f(x) = \frac{1}{a-3}$

Right hand limit $= \lim_{x \to a^+} \frac{1}{x-3} = \frac{1}{a-3}$

Functional value $f(a) = \frac{1}{a-3}$

$L.H.L = R.H.L = f(a)$ at $a \neq 3$

f.

Soln

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2-4}{x-2} = 2+2 = 4$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x^2-4}{x-2} = 2+2=4$

$L.H.L = R.H.L$ i.e $\lim_{x \to 2} f(x)$ exist

Functional value $= f(2) = \frac{2^2-4}{2-2} = \frac{0}{0}$

which is an Indeterminate form

Hence $f(x)$ an Indeterminate form

Hence $f(x)$ is discountinuous at $x=2$

g $F(1) = \frac{|1-1|}{1-1} = \frac{0}{0}$ which is Indeterminate form

so, $f(x)$ is discontinuous at $x = 1$

2. Are the following function continuous at the points mentioned.

(a) $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{when } x \neq 2 \\ 4 & \text{when } x = 2 \end{cases}$ at $x = 2$

Sol^n

left hand limit at $x=2$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2-4}{x-2} = 2+2 = 4$

Right hand limit

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x^2-4}{x-2} = 2+2 = 4$

hence L.H.L = R.H.L f(x) are finite are equal at $x=2$

b. $f(x) = \begin{cases} \frac{x^2-3x}{x-3} & x \neq 3 \\ 3 & x = 3 \end{cases}$ at $x = 3$

$= \lim_{x \to 3^-} f(x) = \lim_{x \to 3} \frac{x^2-3x}{x-3} = \lim_{x \to 3^-} x = 3$

$= \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{x^2-3x}{x-3} = \lim_{3^+} x = 3$

hence L.H.L = R.H.L are finite and equal at $x=3$

(c) $f(x) = \begin{cases} \dfrac{x^2-x-6}{x-3} & \text{if } x \neq 3 \\ 3 & \text{if } x = 3 \end{cases}$ at $x=3$

Sol^n

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-x-6}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+2)}{(x-3)}$$

$$= \lim_{x\to 3} (x+2) = 5$$

Again,

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{(x-3)(x+2)}{(x-3)} = 5$$

and $f(3) = 3$

Hence L.H.R = R.H.L $\neq f(3)$

So, $f(x)$ is not continuous at $x=3$.

3. Discuss the continuity of the point specified

(a) $f(x) = \begin{cases} 2-x^2 & \text{for } x \leq 1 \\ x & \text{for } x > 1 \end{cases}$ at $x=1$

$$\text{L.H.L} = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2-x^2) = 2-1^2 =$$

$$\text{R.H.L} = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2-x^2) = 2-1^2 = 1$$

R.H.L = R.H.L $= f(1)$.

So $f(x)$ is continuous at $x=1$.

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b) $f(x) = \begin{cases} 3x^2+5 & \text{for } x \geq 2 \\ 2x+11 & \text{for } x < 2 \end{cases}$ at $x=2$

$R.H.L = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3x^2+5) = 3\cdot 4+5 = 17$

$L.H.L = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x+11) = 2\times 2+11 = 15$

$L.H.L \neq R.H.L$ at $x=2$
so, $f(x)$ discontinuous at $x=2$.

(c) when $x=2$, $f(2)=3$
again $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2x-1 = 4-1 = 3$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x+1 = 2+1 = 3$

$R.H.L = L.H.L = f(2)$ is equal then $f(x)$ is continuous at $x=3$.

d. $f(x) = \begin{cases} 3+2x & \text{for } -3/2 \leq x < 0 \\ 3-2x & \text{for } 0 \leq x < 3/2 \\ -3-2x & \text{for } x \geq 3/2 \end{cases}$ (1) at $x=0$ at $x=3/2$

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At $x = 0$,

$$L.H.L = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (3+2x) = 3+2\times0 = 3$$

$$R.H.L = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (3-2x) = 3$$

$L.H.L = R.H.L$ is continous.

At $x = \frac{3}{2}$

$$L.H.L = \lim_{x \to 3/2^-} f(x) = \lim_{x \to 3/2^-} (3-2x) = 3 - 2\times 3/2 = 0$$

$$R.H.L = \lim_{x \to 3/2^+} f(x) = \lim_{x \to 3/2^+} (-3-2x) = -3 - 2\times\frac{3}{2} = -6$$

$L.H.L \neq R.H.L$

So $f(x)$ is discontinuous at $x = 3/2$.

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4.

a. $f(x) = \frac{x+5}{x-4}$

the function f(x) will not be defined and hence discontinuous at the point where the dominator is 0

$x-4=0$

$x=4$

b. $f(x) = \frac{x^2}{x^2-3x+2}$

the function f(x) will not be defined and hence discontinuous at the points where the denominator is 0

i.e $x^2-3x+2=0$

$(x-1)(x-2)=0$

$x=1,2$

5.

a. $f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ K & \text{if } x=3 \end{cases}$ at $x=3$

sol^n

$\lim_{x\to3} f(x) = \lim_{x\to3} \frac{x^2-9}{x-3} = \lim_{x\to3} x+3 = 6$

f(x) is continuous at $x=3$

$\lim_{x\to3} f(x) = f(3)$

$K=6$

(b) $f(x) = \begin{cases} 2x-1 & \text{if } x<2 \\ k & \text{if } x=0 \\ x+1 & \text{if } x>2 \end{cases}$ at $x=2$

sol^n

$$L.H.L = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2x-1 = 3$$

$$R.H.L = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x+1) = 3$$

$$L.H.L = R.H.L = \cancel{f(2)}$$
$$k = 3$$

(c) $f(x) = \begin{cases} 2ax+3 & \text{if } x<1 \\ 1-ax^2 & \text{if } x\geq 1 \end{cases}$ at $x=1$

$$L.H.L = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2ax+3 = 2a+3$$

$$R.H.L = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 1-ax^2 = 1-a$$

$$L.H.L = R.H.L$$
$$2a+3 = 1-a$$
$$3a = -2$$
$$a = -2/3$$

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6.
a. $f(x) = \begin{cases} 2x+3 & \text{for } x<1 \\ 4 & \text{for } x=1 \\ 6x-1 & \text{for } x>1 \end{cases}$

Is the function f(x) continuous at $x=1$? if not state how can you make it continuous at $x=1$.

soln

$L.H.L = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x+3) = 2\times1+3 = 5$

$R.H.L = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (6x-1) = 6\times1-1 = 5$

functional value $f(1) = 4$

$L.H.L = R.H.L \neq f(1)$

So f(x) is discontinuous at $x=1$. the function f(x) can be made continuous at $x=1$ by redefining as follows.

$f(x) = \begin{cases} 2x+3 & \text{for } x<1 \\ 5 & \text{for } x=1 \\ 6x-1 & \text{for } x>1 \end{cases}$

b. $f(x) = \begin{cases} 2-x^2 & \text{for } x<2 \\ 3 & \text{for } x=2 \\ x-4 & \text{for } x>2 \end{cases}$

Verify that the limit of the function exist at $x=2$ is the function continous at $x=2$? State how can make it continuous

(a) $L.H.L = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2-x^2 = 2-4 = -2$

$R.H.L = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \cancel{f(x)}\, 2-x^2 = 2-4 = -2$

functional value $= f(2) = 3$

hence $L.H.L = R.H.L \neq f(2)$

Here $f(x)$ is discontinuous at $x=3$

the function can be made continuous by redefining as follows.

$f(x) = \begin{cases} 2-x^2 & \text{for } x<2 \\ -2 & \text{for } x=2 \\ x-4 & x>2 \end{cases}$

=