MAT258S25 Proof 2

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$$\text{let } g : X \to Y$$
$$\text{let } f : Y \to Z$$

$$X \neq \emptyset \quad Y \neq \emptyset \quad Z \neq \emptyset$$

1. If $f \circ g$ is injective, then g is injective.

   Proof.
   $$P_{f \circ g} = \forall x_0, x_1 \in Z : \ (f \circ g)(x) = (f \circ g)(y) \iff x = y$$

   $$\text{let } x, y \in X$$
   $$g(x) = g(y)$$
   $$(f \circ g)(x) = (f \circ g)(y)$$
   $$P_{f \circ g} \implies x = y$$

   $$\therefore \nexists x, y \in X : \ g(x) = g(y) \land x \neq y$$

   QED

2. If $f \circ g$ is surjective, then f is surjective (P).

   Proof.
   $$P_{f \circ g} = \forall x \in X \exists z \in Z : \ (f \circ g)(x) = z$$
   $$P = \forall y \in Y \exists z \in Z : \ f(y) = x$$

   $$P_{f \circ g} \land \neg P \implies \exists z \in Z \forall y \in Y : \ f(y) \neq z$$
   $$\text{let } x \in X : \nexists z : f(g(x)) = z$$
   $$\nexists z \in Z : \ (f \circ g)(x) = z$$
   $$\sim \neg P_{f \circ g}$$

   $$\neg P \land P_{f \circ g} \implies \neg P_{f \circ g} \therefore P$$

   QED