MAT258S25 Proof 2

Kishan S Patel

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$$\begin{aligned} &\text{let } g : X \to Y \\ &\text{let } f : Y \to Z \\ &X \neq \emptyset \\ &Y \neq \emptyset \\ &Z \neq \emptyset \end{aligned}$$

1. If $f \circ g$ is injective, then g is injective (P).

   Proof.

$$P = \forall x_1, x_2 \in B, g(x_1) = g(x_2) \iff x_1 = x_2$$

$$\begin{aligned} g &: y \mapsto z \\ f &: x \mapsto y \\ \therefore f \circ g &: a \mapsto c \end{aligned}$$

$$\begin{aligned} \neg P &\implies \exists y_0, y_1 \in B : \ g(y_0) = g(y_1) \wedge y_0 = y_1 \\ &\implies (f \circ g)(x_0) = f(y_0) = f(y_1) \end{aligned}$$

QED