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The Income Structure:

We are working with the following model:

$$y_{it} = \alpha_i + y^P_{i,t} + y^T_{it}$$

And Permanent Income Dynamics: $y^P_{it} = \rho y^P_{it-1} + \xi_{it}$.

ξ_{it} : Shock/ Innovation to permanent Income.

For simplicity (notational) : $y^T_{it} = \gamma_{it}$

The OLS Regression: $y_{it} = \beta_0 + \beta_1 (age)_{it} + \varepsilon_{it}$ (By Tonetti, & Blundell too).

First, the calculation of (ρ), [The Autoregressive Parameter].

The permanent component of income follows an AR(1) process:

$$y^P_{it} = \rho\, y^P_{it-1} + \xi_{it}.$$

As per Tonetti, finding the variance of y^P_{it} at time $t=0$.

Then,

$$y^P_{it} = \rho y^P_{it-1} + \xi_{it}.$$

Taking Variance on Both sides:

$$Var(y^P_{it}) = Var(\rho y^P_{it-1}) + Var(\xi_{it})$$

(Orthogonality holds true).

Then,

$$Var(y^P_{it}) = \rho^2 Var(y^P_{it-1}) + \sigma^2_{\xi}$$

And for the stationary variance of y_{it}^p,

following the stationarity condition, $|\rho| < 1$,

Then, $\lim_{t \to \infty} Var(y_{it}^p) =$ (some constant).

Then let,

$$Var(y_{it}^p) = Var(y_{it-1}^p) = Var(y_{i,0}^p) = Var\, y_p .$$

Then,

$$Var\, y_p = \rho^2 Var\, y_p + \sigma_{\xi}^2$$

$$\Rightarrow \boxed{Var\, y_p = \frac{\hat{\sigma}_{\xi_p}^2}{1 - \hat{\rho}^2}}$$ holds true at $t=0$.

And for the calculation of the Autoregressive Parameter i.e. 'ρ',

Again, having the AR(1) process,

$$y_{it}^p = \beta_0 + \beta_1 y_{it-1}^p + \varepsilon_{i,t} .$$

$$\Downarrow$$

$$y_{it}^p = \rho y_{it-1}^p + \xi_{it} .$$

$$\Downarrow$$

$$\hat{\rho} = \frac{Cov(y_{it}^p, y_{it-1}^p)}{Var(y_{it-1}^p)} .$$ (Analogous to calculation of $\hat{\beta}_1$).

And from the data, the sample counterpart can be considered as:

$$\hat{\rho} = \frac{\sum_t (y_{it}^p - \bar{y})(y_{it-1}^p - \bar{y})}{\sum (y_{it-1}^p - \bar{y})^2} \quad ; \bar{y} = \text{Mean of } y_{it}^p .$$

And again, $|\rho| < 1$ would ensure stationarity.

For the calculation of $\sigma_\alpha^2 = Var(\alpha_i)$ [The individual-specific component].
The model which we have:

$$y_{it} = \alpha_i + y_{it}^P + y_{it}^T$$

In the data, α_i is not directly visible.
We have considered the variance of fitted values, to be indicative of the ~~...~~ variance of individual-specific component.
It has been taken from the regression of $\log y_{it} \sim age_{it}$.

~~Analogy: In a linear model~~

For the transitory component's variance,

$$y_{it} = \alpha_i + y_{it}^P + y_{it}^T$$

$$\Downarrow$$

$$\varepsilon_{it}$$

$$\downarrow$$

Calculating the variance.

* can be calculated using the residuals (code available in the repository).