ROUTLEDGE ADVANCES IN EXPERIMENTAL AND COMPUTABLE ECONOMICS

Agent-Based Computational Economics

How the idea originated and where it is going

Shu-Heng Chen

ROUTLEDGE

Agent-Based Computational Economics

This book aims to answer two questions that are fundamental to the study of agent-based economic models: what is agent-based computational economics and why do we need agent-based economic modeling? This book provides a review of the development of agent-based computational economics (ACE) from the perspective of how artificial economic agents are designed under the influences of complex sciences, experimental economics, artificial intelligence, evolutionary biology, psychology, anthropology, and neuroscience.

The book begins with a historical review of ACE by tracing its origins. From a modeling viewpoint, ACE brings truly decentralized procedures into market analysis, from a single market to the whole economy. The book also reviews how experimental economics and artificial intelligence have shaped the development of ACE. For the former, it discusses how ACE models can be used to analyse the economic consequences of cognitive capacity, personality, and cultural inheritance. For the latter, the book covers the various tools used to construct artificial adaptive agents, including reinforcement learning, fuzzy decision rules, neural networks, and evolutionary computation.

This book will be of interest to graduate students researching computational economics, experimental economics, behavioural economics, and research methodology.

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Agent-Based Computational Economics

How the idea originated and where it is going

Shu-Heng Chen

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Preface

The idea of this book is to review the development of agent-based modeling in economics from a perspective that the author considers most generic. It is, therefore, not a survey of the application domains of agent-based modeling in economics, which itself can be a subject of interest, but now also becomes difficult given its quick expansion. The perspective taken in this book centers on *the idea of using agents as a bottom-up design for the study of emergent complexity*. The book takes John von Neumann's contribution to cellular automata as a starting point to see how this idea grows and evolves; in particular, how the use and hence the design of agents changes after constant interactions with other disciplines: computer science, artificial intelligence, experimental economics, behavioral economics, evolutionary economics, and econometrics. These constant interactions enrich the design of agents with the coexistence of several different principles, from the original simple design to more complex and intelligent design. They will be presented in this book with various illustrations from agent-based macroeconomic models to agent-based microeconomic models, from artificial financial markets to evolution of technology. This perspective, while it may be narrow, is focused enough to distinguish this book from other similar work in the literature.

The plan of the book began in May, 2008, when the author was generously invited by Prof. Kumaraswamy Velupillai to the University of Trento to give a two-day workshop on agent-based modeling in economics and finance. The skeleton of the book emerged as a preparation for the workshop. During the workshop, the author further benefited from discussions with Stefano Zambelli, Charlotte Bruun, Francesco Luna, and Stephen Kinsella, which helped grow many fine details. In fact, they are all experts on agent-based modeling in economics, although the skeleton of the book is not extensive enough to accommodate all of their contributions in this area.

From October to November 2009, the author was honorably invited as a visiting professor to Trento to give a course on Heterogeneous and Multi-Agent Modeling in Economics for the second-year PhD students at the Interdepartmental Centre for Research Training in Economics and Management (CIFREM). The lecture was given in a very interactive and stimulating environment. Prof. Kumaraswamy Velupillai attended all of my lectures, and encouraged me to prepare my lecture into a book format, generously inviting me to submit a book proposal for his editing series on Routledge Advances in Experimental and Computable Economics. This invitation gave the author the impetus to start a book project.

Around this time and in the following years, the author was luckily invited to give tutorials in summer schools or plenary speeches in international conferences. These invitations provided the author further momentum to carry out the book project, to lecture on some preliminary versions of the book and, most importantly, to receive feedback from audiences. These events were:

- The First Chinese Forum on Intelligent Finance, Chinese Academy of Sciences, Beijing, China, February 26–28, 2009.
- The Summer School of the 15th International Conference Computing in Economics and Finance, University of Technology, Sydney, Australia, July 14, 2009.
- The Central European University Summer School on Complex Systems and Social Simulations, Budapest, Hungary, July 23, 2009.
- APCTP (Asia Pacific Center for Theoretical Physics) School on Econophysics, Pohang, Korea, August 24–27, 2009.
- Facing Crisis: International Seminar on How to Develop Methods of Economic Research, Beijing, China, September 10–11, 2009.
- International Conference on How and Why Economists and Philosophers Do Experiments: Dialogue between Experimental Economics and Experimental Philosophy, Kyoto Sangyo University, Kyoto, Japan, March 27–28, 2010.
- Sino-foreign-interchange Workshop on Intelligence Science and Intelligent Data Engineering, Harbin, China, June 3–5, 2010.
- The 16th International Conference on Computing in Economics and Finance, City University London, UK, July 13–17, 2010.
- The Second Edition of the International Workshop on Managing Financial Instability in Capitalist Economies (MAFIN 2010), Reykjavik University, Reykjavik, Iceland, September 23–25, 2010.
- Conference on Quantitative Behavioral Finance, University of Nice Sophia Antipolis, Nice, France, December 8–10, 2010.
- First Workshop on Quantitative Finance and Economics, International Christian University, Tokyo, February 21–23, 2011.
- International Conference on Nonlinear Economic Dynamics and Financial Market, South China Normal University and Guangzhou University, Guangzhou, China, March 31–April 2, 2011.
- Third International Conference on Econophysics and Summer School on Teaching and Enterprise, Department of Physics and School of Science, Loughborough University, UK, September 24–29, 2011.
- Lecture Series on Agent-Based Computational Economics: A Historical and Interdisciplinary Review, School of Management, Harbin Institute of Technology, Harbin, China, November 7–9, 2011.
- Third International Workshop on Managing Financial Instability in Capitalist Economies, Genoa, Italy, September 19–21, 2012.

– The Fifth Edition of Epistemological Perspectives on Simulation, Trinity University, San Antonio, Texas, October 10–12, 2012.
– Workshop on Computational Finance and Economics, Mexican Central Bank, Mexico City, Mexico, October 17, 2012.
– Eleventh International Conference of Socionetwork Strategies: Understanding Complex Society from Agent-Based Simulation, Research Institute for Socionetwork Strategies, Kansai University, Osaka, Japan, February 27, 2014.
– Fifth World Congress on Social Simulation, São Paulo, Brazil, November 4–7, 2014.
– First Cross-Straits Symposium on Economic Frontier and Policy Simulation in China, China Southern Normal University, Guangzhou, China, November 15–16, 2014.

The author is greateful to Heping Pan, Carl Chiarella, Xue-Zhong (Tony) He, George Kampis, László Gulyás, Guocheng Wang, Sobei Oda, Lei Xu, Wei Zhao, Giulia Iori, Marco Raberto, Seunghwan Kim, Woo-Sung Jung, Gabjin Oh, Jorgen Vitting Andersen, Taisei Kaizoji, Mauro Politi, Duo Wang, Fahuai Yi, Feodor Kusmartsev, Zhong-Yu Wang, Silvano Cincotti, Yu Zhang, Dante Suarez, Alexandrova Kabadjova Biliana, Edward Tseng, Kazuhito Ogawa, Jaime Sichman, Zhiqiang Dong, and Lianqing Peng for their generous invitations and kind arrangement.

The book in its manuscript form has been used as lecture materials for a one-semester course given at the Master of Finance Program in Tianjin University, in years 2012 and 2014. This class probably has the most devoted students in China. The teaching experience in this class is challenging but breathtaking. The book has substantial context on the natural allied relationship between agent-based computational economics and experimental economics. The leadership of Wei Zhang has helped Tianjin University build the strongest academic environment for this new research paradigm. On this occasion, the author is particularly grateful to Wei Zhang and his colleagues at College and Management and Economics, including Xiong Xiong, Yongjie Zhang, Da Ren, Xu Feng, Dehua Shen, and many others, for providing the author with a very stimulating and inspiring research-oriented teaching environment.

While writing the book, the author witnessed and was accompanied by the fast-growing agent-based communities in both economics and social sciences. The author constantly benefited from participation at some major events organized by the Society for Computational Economics, the Society for Economic Science with Heterogeneous Agents, the NYC Computational Economics and Complexity Workshop, the Pan-Asian Association for Agent-based Approach in Social Systems Sciences, the International Foundation for Autonomous Agents and Multiagent Systems, the Computational Social Science Society of the Americas, the IEEE Computational Intelligence Society, and the Asia-Pacific Econophysics Conference. The author would like to give thanks to a number of active members who have not just helped the author to learn this subject, but also contributed themselves to the shining history of the communities. In addition to those who have already been mentioned above, they are David Kendrick, Leigh Tesfatsion, Thomas Lux, Jasmina Arifovic, Robert Marks, Hans Amman, Blake LeBaron, Barkley Rosser, Alan Kirman, Herbert Dawid, Cars Hommes, Nick Vriend, John Duffy, Robert Axtell, Frank Westerhoff, Giovanni Dosi, Maruo Gallegati, Domenico Delli Gatti, Massimo Ricottilli, Pietro Terna, Jason Barr, Troy Tassier, Leanne Ussher, Chris Ruebeck, Alan Isaac, Myong-Hun Chang, Andreas Pape, Nigel Gilbert, Claudio Cioffi-Revilla, William Griffin, David Sallach, Bruce Edmonds, Scott Moss, William Rand, William Lawless, Flaminio Squazzoni, Van Dyke Parunak, Akira Namatame, Yuji Aruka, Hiroshi Deguchi, Takao Terano, Shingo Takahashi, Aki-Hiro Sato, and Siew Ann Cheong.

The book was mainly written at the AI-ECON Research Center, National Chengchi University. The center is one of most active research units in promoting agent-based computational economics. The author is very blessed by being surrounded by many supportive colleagues, students, post-docs, and visitors. Many of them became co-authors in a number of the author's other projects. Here, acknowledgements are specifically given to Shu G. Wang, Wei-Lin Mao, Chu-Chia Lin, Chung-Ming Kuan, Been-Lon Chen, Mei-Lie Chu, Yih-Chyi Chuang, Sun-Chong Wang, Sai-Ping Li, Chen-Yuan Tung, Nai-Shing Yen, Reuy-Ming Liao, Lee-Xieng Yang, Ray-May Hsung, Chia-Hsuan Yeh, Wo-Chiang Lee, Wei-Yuan Lin, Chueh-Yung Tsao, Tzu-Wen Kuo, Ya-Chi Huang, Bin-Tzong Chie, Chung-Ching Tai, Chia-Ling Chang, Hung-Wen Lin, Ye-Rong Du, Chia-Yang Lin, Ying-Fang Kao, Nicolas Navet, Tina Yu, Tong Zhang, Michael Kampouridis, Tong-Kui Yu, Jie-Jun Tseng, Tzai-Der Wang, Kuo-Chuan Shih, Umberto Gostoli, and Ragupathy Venkatachalam. I am in great debt to them and to my family, without whom it would have been impossible to complete the book. Finally, I am grateful to Lisa Thomson and Routledge's professional editing team for being so supportive and helping me accomplish the last mile of the book.

Shu-Heng Chen
March 15, 2015

Part I

Ideas and structures of the book

1 Economics in an interdisciplinary context

Humans are heterogeneous in many ways. Nothing can be more evident than this simple fact. Yet, in mainstream economics, the device of the homogeneous agent or, more formally, the *representative agent*, has been employed for quite a long, yet uneasy, period of time. Psychologists, on the other hand, have acknowledged the heterogeneity of agents right from the beginning. Various developments in psychometric testing simply show us that humans are empirically different. They are not just bounded rational; they are heterogeneous in cognitive capacity as well as personality. Moreover, anthropologists and sociologists show us that, when put in a social context, they are under different sets of beliefs or norms. From the viewpoint of genetic biology, some human heterogeneities are inherited from parents or ancestors. Nevertheless, mainstream economics has long been silent on all of these *human factors*, assuming that they are not *economically sensible*. The empirical evidence accumulated in recent years, however, shows the significance of cognitive capacity, personality, emotion, cultural inheritance, and social norms, from micro to macro. Nevertheless, the modeling techniques which can incorporate agents who are heterogeneous in these dimensions and demonstrate the emergent aggregate behavior through their interactions are less well established in economics.

The purpose of this book is to place the study of economics in an interdisciplinary framework so that the underlying mathematical or computational modeling can be grounded in various kinds of empirical evidence ranging from genetic biology to neural sciences, sociology, psychology, and, of course, experimental economics. In fact, this interdisciplinary modeling has already existed by different names among people with different backgrounds. For people with a conventional economics, psychology, or mathematics background, its familiar name is *behavioral economics*; for people with a mixed background of economics and computer sciences or computer engineering, its familiar name is *agent-based computational economics*; for the recent immigrants from physics to their “colony” in economics, it is called *econophysics*. Each of its names represents an origin of its development. Behavioral and econophysic modeling is more analytically demanding, whereas agent-based computational economic modeling is computationally intensive.

Regardless of different names, models with these tags and origins share a great common feature, i.e., they can each replace the conventional representative agent

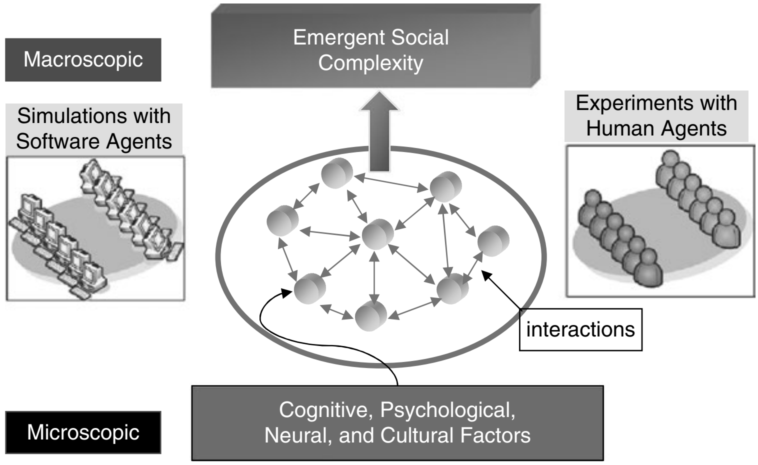

Figure 1.1 Microfoundations and macroeconomics.

Source: Adapted from Chen and Wang (2011), Figure 2.

model and provide an alternative *microfoundation*. Figure 1.1 shows this common feature. We will come back to this figure and elaborate on its essence in Section 1.1. Here, we only provide a brief list to exemplify the microfoundational work already done in each of the three research areas.

Behavioral macroeconomics

There is a series of works on behavioral macroeconomics by George Akerlof, the 2001 Nobel Laureate in Economics. The most notable features of this are his Nobel Prize lecture (Akerlof, 2002), his American Economic Association Presidential address (Akerlof, 2007), and his advice on the current financial tsunami (Akerlof and Shiller, 2009). This series can be augmented by a number of macroeconomic laboratory experiments (Duffy, 2009).

Agent-based computational economics

Agent-based computational economics, almost since its beginning, has been devoted to the study of macroeconomic issues. Leigh Tesfatsion, on her Iowa State University web page, http://www.econ.iastate.edu/tesfatsi/amulmark.htm, has a collection of these studies. Among them, Chen (2003), Delli Gatti *et al.* (2008), LeBaron and Tesfatsion (2008), and Delli Gatti *et al.* (2011) provide various illustrations with different motives. Due to the financial crisis which occurred in 2008–2009, attention has been paid to agent-based computational economic modeling as an alternative approach to maintaining better tabs on the increasingly complex and intertwined economy (Buchanan, 2009; Farmer and Foley, 2009). In addition, a series of conferences were organized in the year 2010 to reflect on the crises in economic theory with regard to the economic crisis of 2007–2009. In 2009, George Soros pledged to give 50 million dollars over ten years to set up the Institute of New Economic Thinking as a reaction to his feeling that "false theory" has resulted in tremendous damage to the world economy. Agent-based economic modeling is considered to be a candidate for an alternative.

Econophysics

In physics, during the late nineteenth century a fundamentally new approach referred to as *statistical mechanics* was advanced by James Maxwell (1831–1879), Ludwig Boltzmann (1844–1906), Josiah Gibbs (1839–1903), and others. This approach, which significantly contributed to the study of molecular dynamics, was also formally introduced to the study of economics and even the social sciences in the 1990s.[1] This new field is broadly known as *econophysics* or *sociophysics*.[2] An econophysics approach to macroeconomics can be exemplified by a series of work done by Masano Aoki (Aoki, 1996, 2002a; Aoki and Yoshikawa, 2006).

1.1 The interdisciplinary framework

Figure 1.1 has all the ideas to be included in this book, albeit expressed in a highly simplified way. Let us start with the middle part of the figure, which intends to picture *a system of interacting agents*.[3] For a physicist, this picture may be read as a *particle system*, with two important departures:

Heterogeneous agents

First, agents (particles) are not homogeneous; instead, they are heterogeneous. Abandoning the device of the representative agent is exactly the concept conveyed at the beginning of this book. In Part VI, we will provide corroborative evidence and discussions as to why heterogeneous agents should not be viewed as an exception but as a rule in the future of economic modeling.

Interactions

Second, the relations among the agents (particles) are not just random bumping but *social* in the sense that these agents mutually *influence* each other, so that their behaviors change along with these interaction processes. These agents are, in general, not independent. This feature allows us to accommodate concerns from anthropology, religion, culture, sociobiology, and evolutionary psychology.

Social networks

Although the interactions among agents can be erratic, they may not be entirely random. Implicitly or explicitly, the interactions take place through social networks. The topologies of social networks can be another crucial factor for the interactions, and the topologies, in general, are endogenously determined.

Homo sapiens

Let us move to the bottom of Figure 1.1, which describes these individual agents. In conventional economics, the description of the agents is simple: *Homo economicus* or economic man. They are identically infinitely smart, hyperrational, self-interested, unemotional, and utility-maximizing agents. While these creature have been surviving in mainstream economics for decades, economists are now becoming more interested in knowing *Homo sapiens*—emotional beings (Thaler, 2000).[4] This broader interest has brought about significant growth in interdisciplinary engagement between economists and other social scientists or even scientists. Psychology and computer science both come into play from this side.

Psychological fundamentals

On the one hand, we certainly hope to give a more realistic description of the human agents by at least not missing their essential dimensions; on the other hand, we want to make this description programmable. The former motivates an increasing number of economists to learn from psychologists and coherently ties economics and psychology in an unprecedented way. This interdisciplinary collaboration between the two has also promoted a new subfamily in economics, namely *behavioral economics*. Economists are now more alert to the social consequences of widely documented agents' behavioral *biases*. More recently, psychology has helped economists to reshape a proper definition or representation of an individual economic agent. A series of recent studies indicates that *cognitive capacity* (the intelligence quotient) and *personality* are two important missing elements in conventional characterizations of economic agents. In fact, these two human factors should be thought of as the *fundamentals* of the economy; they are certainly more concrete than *preference*, a very controversial idea, both historically and currently.

Artificial agents

We program the artificial agents to reflect various kinds of psychological fundamentals, behavioral rules, or behavioral biases. Artificial agents is not a term commonly used in behavioral economics, although all the models inevitably start with some artificial agents. The whole of Part III is devoted to this construct, but most materials introduced there were produced in the earlier stages of agent-based computational economics when it was still distinct from behavioral economics. In agent-based computational economics the focus of artificial agents is on learning, whereas in behavioral economics the focus is on preference and utility. In the future, the gap between the two will be narrowed as *behavioral agent-based computational economic models* are gradually developed. Chapter 19 presents one case in point.[5]

1.2 Organization of the book

Normally, the table of contents of a book suggests that the reader can read the book in a sequential order. While the table of contents must be unique, that kind of suggestion is not. Therefore, in this section, we elaborate on the organization of the book and suggest some alternative tables of contents which could be used by different readers with different purposes or different pursuits.

1.2.1 Two fundamental questions

The book tries to answer two questions which we consider to be quite fundamental to the study of agent-based economic models, namely, *what* and *why*? What is agent-based computational economics? Why do we need agent-based economic modeling of the economy? These two questions are generally shared by other social scientists who are also interested in agent-based modeling. Therefore, they are better addressed in a broader background, i.e., *agent-based computational social sciences*. To answer the first question, it would be nice if we could start with some very simple agent-based social or economic models which, however, all have the essences of agent-based models. Chapter 4 serves this purpose. It is mainly composed of the three simplest agent-based social models, namely *Schelling's Segregation Model*, *Conway's Game of Life*, and *Wolfram's Edge of Chaos*. This chapter can help beginners to quickly grasp what an agent-based social model is.

The most direct way to address the second question is to ask whether we can have a collection of successful agent-based models in the social sciences. By success, we mean that these models are capable of explaining or predicting some social phenomena which are hard to capture using the conventional models of the respective disciplines or are able to provide new insights. While we cannot be absolutely sure what these models are, Chapter 2 does make such an attempt. In addition to that, Epstein (2008) provides a long list of answers to the issues involved, and in Chapter 2 we shall review some of them.

1.2.2 Novelty discovery: toward autonomous agents

The book will start with a concrete example of agent-based (economic) modeling, namely *cellular automata* (Chapter 4). The reason we choose cellular automata as our kick-off example is partially because we consider *a model of agents* to be the first part of agent-based modeling. However, to clearly indicate our departure from *Homo economicus* to *Homo sapiens*, we would like to provide a simple historical background on the development of economic agents in economics; specifically, from *algorithmic (behavioral) agents* to *autonomous agents* (Chapter 5). This will quickly lead us to see that part of the economic agents is defined by the associated algorithms. In fact, Chapters 5 to 7 provide many more illustrations on the algorithmic aspects of economic agents, as they are called algorithmic agents.

Autonomous agents are first exemplified in Chapter 6 via an artificial intelligence tool called genetic programming (GP), while the foundation work for autonomous agents is not given until later in Part IV. This line of exposition is then further extended to Chapter 8. Chapter 8 can be read together with Section 14.5 and Part VIII, and are all concerned with a central theme of the book, which I shall refer to as *the legacy of Marshall*. Together they demonstrate one unique feature of agent-based modeling, i.e., its capability of modeling *intrinsically constant changes*. One essential ingredient of triggering constant change is equipping agents with a *novelty-discovering* or *chance-discovering* capability so that they may constantly exploit the surrounding environment, which causes the surrounding environment to act or react, and hence change constantly.

1.2.3 Microstructure dynamics

If economics is about constant change, and that happens because autonomous agents keep on searching for chance and novelties, then change in each individual and change in the microstructures must accompany the holistic picture of constant change. A number of chapters in this book attempt to have *microstructure dynamics* as their focus. Part V illustrates the rich microstructure dynamics in agent-based financial markets. Chapter 14 is mainly devoted to the study of microstructure dynamics in light of the *statistical mechanical approach* (Section 14.4). With this approach, the set of behaviors or strategies is finite or bounded. A finite set allows us to study the microstructure dynamics on solid ground, but it inevitably implies the absence of novelties and their discovery, which is the other focus of the book. Section 15.4, therefore, extends the analysis of the microstructure dynamics into an infinite set so that rich microstructure dynamics are embedded within the novelty-discovering processes.

These chapters are connected by two hypotheses, namely the *market fraction hypothesis* in Chapter 14 and the *dinosaurs hypothesis* in Section 15.4. The two hypotheses are further connected by using genetic programming to formulate and test them (Section 15.4).

1.2.4 Agent engineering

A large part of the book is concerned with the design of software agents used in agent-based modeling. In general, this task is known as *agent engineering*. On the one hand, the book reviews a number of tools which have been used to design agents with different degrees of sophistication; on the other hand, the book also addresses how to use these tools properly. The latter subject involves the empirical grounds of agent engineering. The behavior of human agents observed in experimental economics provides one empirical ground. Using this empirical ground to build software agents naturally ties software-agent simulations and human-agent experiments together.

The tools used to build software agents are mainly introduced in Part IV, which covers reinforcement learning (Chapter 10), artificial neural networks (Chapter 12), and evolutionary computation (Chapter 13). In this repertoire, do agents follow reinforcement learning to learn? Or do they learn as predicted by artificial neural networks? When is evolutionary computation a more sensible description of learning? A number of chapters contribute to the study of these issues. Chapter 7 is concerned with the idea of *calibrating artificial agents* using data from human-subject experiments. Similar to Chapter 7, Chapter 16 introduces work using real data (field data) to estimate the parametric behavioral rules, and is not necessarily restricted to learning.

1.2.5 Experimental economics

A large part of the book is also written to reflect the intertwined connection between experimental economics (EE) and agent-based computational economics (ACE). Several different developments of algorithmic agents are all inspired or related to experimental economics. The double auction (DA) market (Chapter 8) is probably the most illuminating illustration of the connection between agent-based computational economics and experimental economics. Having said that, we notice that the DA market is the context in which various versions of agents, crossing both realms of EE and ACE, have been proposed. The motivation behind inventing *zero-intelligence agents* consists of replicating the market behavior observed in the double auction market experiments (Section 8.3). The *programmed agents* or *human-written agents* are part of the tournament-like offline experiments (Section 9.2). The idea of *calibrated agents* is first introduced to replicate human choice behavior in the multi-armed bandit experiment (Chapter 7). Finally, autonomous agents are also inspired by both online and offline human experiments in double auction markets (Sections 9.3 and 9.4). Needless to say, the idea of algorithmic agents is enriched by interaction with observations from experimental economics.

1.2.6 Econophysics

It is fair to say that agent-based modeling was first used by physicists, though known by different names, including cellular automata, the kinetic model, percolation model, Ising model, etc. The recent massive economic and financial applications of these models by physicists have contributed to a significant part of the field known as *econophysics* (Chen and Li, 2012). In Chapter 4, we present the *cellular automata tradition* of ACE. The tradition initiated by von Neumann (1903–1957; von Neumann, 1966) is then passed on to Thomas Schelling (Schelling, 1978), John Conway, Stephen Wolfram (Wolfram, 1994), Peter Albin (1934–2005; Albin, 1975, 1998), Duncan Foley, Joshua Epstein, and Robert Axtell (Epstein and Axtell, 1996), and further down to the arising of the spatial agent-based models extensively applied in geography, city planning, and ecology. This series of literature enables us to see the connection between the particle system in physics and agent-based modeling in economics. They together serve as a gateway leading to the current development of complex science and the later more general development of complex networks (Chapter 22).

Econophysics, in spirit, also concurs with the *randomization approach* or the *maximum entropy approach* in agent-based modeling (Section 8.5.1). The capability of this approach to replicate complex financial dynamics systems shows that some aggregate phenomena generated from human-agent systems with the complex motives and behavioral rules of humans can be rather well approximated by a system with rather simple agents characterized by simple motives and simple rules. In a sense, it indicates that adding more complex strategies to the agent-based models may have little by way of macroscopic effect since these complex strategies may interact in such a way that they mutually annihilate each others' forces. It is this possibility that prompts us to think about a general physical system which is equipped with the most rudimentary forces but can overarch several seemingly unrelated social phenomena, for example from pedestrian counterflow, the Schelling segregation model (Vinkovic and Kirman, 2006), the El Farol Bar problem (minority games), and then to financial markets.[6]

Notes

1 What may interest both economists and physicists is that the early study of molecules in physics was motivated by observing interactions among humans (Ball, 2006).

2 Galam (2004) gives a personal account of the origin of sociophysics, but a more interesting and even earlier review of the interdisciplinary relation between classical physics and classical economics was documented by Cottrell *et al.* (2009).

3 The size of the system, which can be another important consideration in this book, does not have to be as small or finite as the one drawn here.

4 Thaler (2000) particularly characterizes the shift in the interest by distinguishing the *normative* description of human behavior from the *positive* description of human behavior.

5 Nonetheless, there is another major difference which we would like to point out here, i.e., heterogeneity. Despite the findings of so many *anomalies*, behavioral economics does not necessarily resist the device of a representative agent. In fact, the device of the representative agent is still extensively used in various behavioral economic models, in particular, behavioral macroeconomics, since that may make it easier for us to present the aggregate consequence of a certain class of behavioral biases by not *averaging them out*. For example, Stracca (2004) states that "what matters for aggregate market prices is the behavior of the representative agent, so we do not have to care, in principle, about behavioral biases that cancel out in the aggregate" (p. 378). However, neoclassical economics used to consider exactly the opposite, namely, these biases will cancel each other out when being summed up. Therefore, it seems important to *show*, rather than to *assume*, that these biases will not go away in the aggregates. For that reason, we believe that *heterogeneous behavioral economic models* should be more persuasive than the homogeneous ones, or, naturally, be the next step or the extension of the latter (Thaler, 2000).

6 It is possible to simulate the financial time series using *social force models for pedestrian dynamics* (Parisi, 2010). The social force model is one kind of agent-based model which is not much different from the particle system in physics. The agents (particles) in this system have simple objectives and follow simple rules.

2 Agent-based modeling in the social sciences

Over the last decade, there has been much evidence of agent-based modeling and simulation being extensively used among different social science disciplines. This tendency has enabled agent-based social scientists to find a common language among them to facilitate the resultant interdisciplinary communication and collaboration, which in turn has defined a number of common interests shared by the social scientists. This gathering has also caused the emergence of a new discipline across the social sciences, which is known as *computational social sciences* (CSS).

2.1 What is it?

Computational social science presents a comprehensive view of the social sciences, the study of social phenomena. However, it does not use or follow any single-disciplinary viewpoint or framework to examine these social phenomena. While the social phenomena exemplified in computational social science include voting, identity, segregation, social exclusion, discrimination, financial crises, urban dynamics, social networks, leadership, congestion, disease transmission, gossip and mass media, culture and social norms, interpersonal relations, and prosocial behavior, we do not study them in the way that they are treated in the parent disciplines to which they conventionally belong, be they economics, sociology, the political sciences, management, or psychology. Instead, we study each of these phenomena as a social process and place these social processes (emergent processes) together into a *coherent framework*, in which they can be communicative with each other as if there were only one social science.

To do so, we search for the generic properties or common ground of these social processes. This coherent framework is agent-based modeling and simulation.[1] The social science studied using agent-based modeling and simulation is known as *computational social science*. Different names also exist, such as agent-based social sciences (Trajkovski and Collins, 2009), bottom-up social sciences (Epstein and Axtell, 1996), algorithmic (behavioral) social sciences (Saunders-Newton, 2006; Velupillai, 2009), generative social sciences (Epstein, 2007), and complex adaptive social systems (Miller and Page, 2007).

Several different attempts have been made to provide a review of this rapidly accumulating literature.[2] Among the many existing reviews or work on

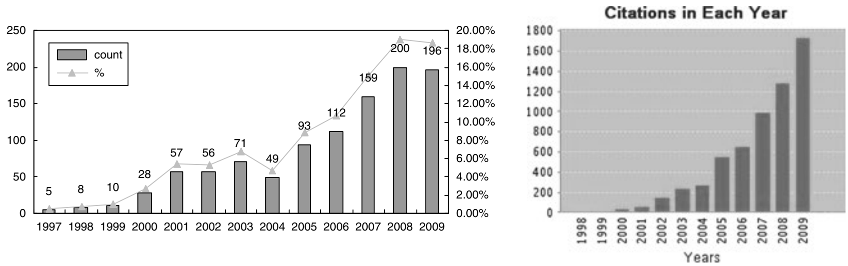

Figure 2.1 Number of published papers in ABSS (left panel) and citations in each year (right panel).

Source: Adapted from Chen, Yang, and Yu (2011).

computational social science, the collection of work resulting from the efforts of Nigel Gilbert (Gilbert, 2010) can be considered to be one of the most comprehensive. In this four-volume collection, Gilbert inclues 66 articles on computational social science. The overall collection places an article on cellular automata, one of the origins of agent-based modeling (see Chapter 4 of this book), as its leading chapter. Pioneering work built upon cellular automata (checkerboards) by James Sakoda and Thomas Schelling (the checkerboard model) is also included (see again Chapter 4 of this book). Many other articles are classif ed according to the arena in which the distinguishing features and signif cant contributions of agent-based modeling can be found, such as the formation (emergence) of markets, opinions, groups (segregation), networks, norms, organizations, leaders, and pro-social behavior. In addition, there are sections containing collections devoted to foundational and methodological issues, plus one section devoted to the modeling of cognitive and psychological agents. This four-volume collection gives a concrete demonstration of what computational social science is and summarizes various research directions that have been developing and unfolding since the 1970s.

By retrieving data from the Social Sciences Citation Index database, Chen, Yang, and Yu (2011) found a total of 1051 papers on agent-based social simulation (ABSS) that had been published during 1997–2009. Figure 2.1, in the left panel, illustrates the number of published papers in ABSS, and in the right panel shows the annual citations of the published papers in ABSS. The results appear to suggest that the number of papers in ABSS has increased distinctly since 2001, and that respective citations have also increased with each passing year.

2.1.1 Three constituents of CSS

The critical feature that makes agent-based modeling so relevant for the social sciences is that social behavior and social dynamics involve many details, which are nontrivial but are frequently oversimplif ed by alternative paradigms, such as equation-based models or variable-based models. The following three constituents of an agent-based model can be lucidly illustrated by one of the classics, namely Schelling's segregation models, which will be detailed in Section 4.1.

Software agents

First are the details about individuals. This partially explains why CSS is referred to as *algorithmic social sciences*, because each agent (actor) is represented by an algorithm or a computational program. This algorithm (program) corresponds to the decision rules, behavioral models, or even preferences that characterize the agents. In a sense, it is a simple model of a man, in light of Herbert Simon (Augier and March, 2004). In borrowing the term from computer science, one may also refer to CSS as *software agents* or *autonomous agents*.

Embeddedness

Second are the details of the environment within which the agents are embedded. In Schelling's segregation model (Section 4.1), the embeddedness is a *two-dimensional cellular automaton* (a city) which defines the geography of the space in which agents live. The geography (topology) of the city further defines a *social network* for each agent. In addition to the geographies or social networks, other embeddedness includes institutions, cultures, histories, etc.

Aggregation (emergence)

Finally, with these details, individuals interact through the embeddedness and the resulting patterns and macrobehaviors, also known as the emergent properties, are normally hard to predict. This also explains why CSS is referred to as "bottom-up social sciences." In Schelling's segregation model, the segregation phenomenon as an aggregation phenomenon is a sum of the interactions of fairly tolerant people. Obviously, this is not a linear scaling up. "From the bottom up" normally refers to the surprising phenomena that would not be predicted from the model itself, which focuses on the actions of individual agents rather than overarching downward-focused principles.[3]

2.1.2 ACE scissors

The design of an agent-based model, therefore, is composed of two parts, the *behavioral part* (software agents) and the *institutional part* (embeddedness). These two parts then jointly determine the emergent outcomes. These two parts together provide "scissors" for understanding social phenomena and conducting policy designs.

To fulfill the aforementioned purpose, one can use an agent-based model to identify the cause of an emergent phenomenon. For example, is financial volatility mainly attributed to the agents' behaviors, such as herding tendency, or attributed to institutional arrangements, such as the trading matching rule, or both (the combined effect)? This analytical framework would, therefore, be very much different from conventional policy analysis, which largely leaves out the behavioral considerations or simply assumes that agents are all rational. Instead, the ACE scissors naturally open the sensitivity issue of a policy design: is the design robust to different behavioral assumptions? As the Chinese proverb goes: an orange becomes a trifoliate orange after crossing the Huai River. Whether a policy design is proper may crucially depend on the agents' behaviors in which it intends to intervene. In the spirit of the proverb, the same design when applied to the area south of the Huai River can result in oranges being grown, but when applied to the area north of the Huai River, it can only result in trifoliate oranges (whose fruit is bitter and not edible raw) being grown.

2.1.3 The third way

While deduction and induction are the two familiar types of reasoning, one has to realize that agent-based computational modeling and simulation constitute neither a method of deduction (theory) nor a method of induction (statistical inference). The distinction from the usual deduction and induction has been well acknowledged by economists and social scientists (Axelrod, 1997a; Axelrod and Tesfatsion, 2006; Gallegati and Richiardi, 2009). Axelrod (1997a) proposed that agent-based social simulation can be considered as the third approach, i.e., in addition to deduction and induction, to science.

> Simulation in general, and ABM [agent-based modeling] in particular, is a third way of doing science in addition to deduction and induction. Scientists use deduction to derive theorems from assumptions, and induction to find patterns in empirical data. Simulation, like deduction, starts with a set of explicit assumptions. But unlike deduction, simulation does not prove theorems with generality. Instead, simulation generates data suitable for analysis by induction. Nevertheless, unlike typical induction, the simulated data come from a rigorously specified set of assumptions regarding an actual or proposed system of interest rather than direct measurements of the real world. Consequently, simulation differs from standard deduction and induction in both its implementation and its goals. Simulation permits increased understanding of systems through controlled computational experiments.
>
> (Axelrod and Tesfatsion, 2006, p. 1650)

While Herbert Simon, to my knowledge, did not write directly on this issue, he did notice the limitation of normal induction.

> Students are always told that they can't run a successful experiment if they don't have a hypothesis ... I believe that is a very bad criterion for the design of experiments ... If you look down the list of outstanding discoveries in the physical sciences or the biological sciences—look at Nobel awards in those fields—you will note that a considerable number of the prizes are given to people who had the good fortune to *experience a surprise*.
>
> (Simon *et al.*, 1992, p. 22; emphasis added)

At this point, agent-based simulations are related to Simon's comment since some emergent phenomena coming out of agent-based simulation bring us novelties and surprises, which inspire us to make hypotheses of these observations. In this sense, some economists, such as Gallegati and Richiardi (2009), also relate agent-based social simulation to what Charles Peirce (1839–1914) called *abduction*. Peirce advocated that there is a type of logical reasoning beyond deduction and induction. He called this unique type of reasoning abduction, and suggested that it was the logic of discovery (Peirce, 1997). While, for many philosophers of science, abduction is treated as a part of induction, Peirce forcefully distinguished between the two by indicating that induction is about the test of an established hypothesis using observations, and that abduction is about the formation of the hypothesis.[4]

In addition to simulation and abduction, others have suggested the term *computational paradigm* to distinguish agent-based modeling from the conventional scientific paradigms (Hoekstra, Kroc, and Sloot, 2010). Wolfram (2002) even calls it "a new kind of science," to which we shall come back later in Section 4.3.1. Very much sharing the same view of simulation and computation, Gintis (2012) makes the following remark, related to his series of agent-based general equilibrium models (for more details, see Section 3.2).

> Those unused to working with complex dynamics systems may object that a computational proof is no proof at all. In fact, a computational proof may not be a mathematical proof, but it is a scientific proof: it is evidential rather than tautological proof, and depends on induction rather than deduction. The natural sciences, in which complex systems abound, routinely use mathematical models that admit no closed-form analytical solutions, ascertain their properties through approximation and simulation, and justify these models by virtue of how they conform to empirical reality.
>
> (Gintis, 2012, p. 60)

2.2 Why?

Why do we need agent-based modeling in economics or, generally, in the social sciences? Briefly, there are three reasons for this. Let us spell them out first, and then elaborate on each of the three.

1 Agent-based modeling and simulation refer to a repertoire of tools to make complex systems easier to study.
2 Agent-based modeling and simulation constitute a set of new instruments; their invention, like many other instruments, enables us to observe objects which are otherwise difficult to see, and hence expand the *interval* by which a science is defined.
3 Agent-based modeling and simulation make the experimental social sciences possible.

2.2.1 Universal literacy

Regarding the first point, in his keynote speech given at the 2010 Computational Social Science Society of America annual meeting, Uri Wilensky, the founder of Netlogo, pointed out that agent-based modeling can help reduce the barrier or threshold for studying complex (adaptive) systems. Using the predator–prey model and forest fires as two illustrations, Wilensky showed how the complex phenomena conventionally studied by high mathematics, such as differential equations, can be much more easily approached by agent-based modeling (Wilensky and Reisman, 2006; Goldstone and Wilensky, 2008; Sengupta and Wilensky, 2011). By using agent-based modeling, not only can we make complex adaptive systems have high accessibility for general people, i.e., a lower threshold, but they can also allow us to explore or address more questions than we would be able to with the conventional approaches, such as differential equations, i.e., a high ceiling. Hence, agent-based modeling helps enhance *universal literacy* by introducing a *low threshold* and *high ceiling*.

In fact, as we shall see in Section 4.3.1, in light of the similar argument of computational irreducibility or "a new kind of science" (Wolfram, 2002), agent-based modeling is probably the "right mathematics" to do science (Borrill and Tesfatsion, 2011). Wilensky's argument has been further substantiated by a group of people who are engaged in K-12 complex systems education.

2.2.2 Higher resolution and yet better integration

As to the second point, each science, to be well defined, needs to decide its boundary, which is an *interval*, starting from the lowest level (the microscopic level), then passing through the middle levels (mesoscopic levels), and ending at the highest level (macroscopic level). A rough example in physics is the interval from "small" physics to "big" physics. This interval, however, is not constant and, to some extent, its expansion can be regarded as scientific progress. In economics, for example, the progress can be characterized as a move further up to the level of the world economy (international economics) or a move further down into the level of neurons (neuroeconomics). However, the interval cannot be broadened in a fragmentary manner such that the bottom and the top do not talk to each other. Mobility into different levels and their successful integration into a coherent body is, nonetheless, constrained by technology: the computer, the database, the fMRI, and various fine machines enabling us to measure them. Advancing to larger intervals and the coherent integration of different levels within this interval is progress in science.

As we shall see, agent-based economic modeling normally involves a lot of *functional details* which enable us to broaden the interval between economics and the social sciences in a coherent manner. Students of economics and the social sciences may experience very few of these functional details. One example is what Leigh Tesfatsion called "the procurement processes" (Tesfatsion, 2006), which are what make a more realistic economy behave, be it efficient or not. Reading through a series of ACE studies, students may easily be motivated to cook their own models since it will not be too hard to find some imperfections in previous models. The students do not have to worry about the solutions of their models because ACE models, by definition, are algorithmic and computational, and they are what Wilensky called "low threshold, high ceiling" models. This enhancement of universal literacy may easily cause the various economic models to flourish.

The usual defense for agent-based modeling is its superiority to its alternatives, mainly top-down system dynamics or equation-based systems. This superiority can mean a better understanding (explanations) of social phenomena, better forecasting of the future, and other things.[5]

2.2.3 *Scalable extensions and replications of human-subject experiments*

Finally, the third answer for using agent-based modeling is that it is an extension of *experimental economics* or *experimental social sciences*.[6] It had been held for a long while that economics was not an experimental discipline. The following reservation given by Samuelson and Nordhaus (1985) is well known:

> Economists cannot perform controlled experiments like chemists or biologists because they can't easily control other important factors. Just like astronomists or meteorologists, they usually have to solely use their observation.
>
> (Samuelson and Nordhaus, 1985, p. 8)

The background underlying the change from the early hesitation to embrace this discipline to its later recognition, in particular by the Bank of Sweden Nobel Memorial Prize and the Swedish Academy in the year 2002, has been documented in Fontaine and Leonard (2005), which gives an extensive review of the idea of experiments in economics, which is not just limited to laboratory experiments, but also to policy experiments as well as thought experiments.

They particularly mentioned computation as a form of experimentation. In this vein, the rise of experimental economics provides us with a good case to expect the coming of agent-based computational economics. As we shall see later, if one can agree on the promises delivered by experimental economics, then it would be easier to accept ACE so long as one realizes that the latter serves to fully deliver the promises of the former.

Having said that, we acknowledge the limitations of experiments with human subjects. The most obvious limitation is *money*. Subjects need to be paid in typical economic experiments. Therefore, the direct cost is the remuneration paid to the subjects in an experiment. Even though the direct cost only increases linearly with the number of subjects, stringent budget constraints can allow most labs to run experiments only with a limited number of subjects (Winter, 2009), a limited number of scenarios, and a limited number of repetitions. Unless we are sure that the experiments we run are *size-independent*, have *little noise*, and are *robust* to a wide range of perturbations, we may not be able to run enough experiments before obtaining sensible results.

In addition to the budget constraints, human agents are not as controllable as initially thought. Running an experiment for many consecutive hours can easily tire human subjects, which adds an additional constraint to experiments. Hence, it is difficult to run any experiment beyond three hours. The third constraint is the physical space of the experimental lab. Currently, it is hard to see any experimental lab which can host more than 100 subjects. This automatically puts an upper constraint on the size of an experiment. Although online web-based experiments may not require a physical lab, not all kinds of experiments can be easily conducted without the physical presence of the subjects.

Hence, replacing human agents with software agents seems to be an attractive alternative when the aforementioned constraints are stringent. Using software agents, one can easily enlarge the size beyond experiments with human subjects, for example by expanding a double auction experiment with a total of 20 agents to a total of 2000 agents. In addition to this direct expansion, using software agents allows us to design some "experiments" that are hard to conduct with human subjects due to any of the constraints mentioned above or other ethical reasons.[7]

Of course, a fundamental challenge for this attraction is whether human agents are replaceable. Is there a free lunch? The ten-dollar hourly rate which we pay for each human subject may enable us to learn something real about *Homo sapiens*, but can we gain the same quality of knowledge from the software counterparts? To answer this question, we have to know to what extent and under what specific circumstances the software agent is computationally or behaviorally equivalent to *Homo sapiens*. Without properly addressing this question, straightforward extensions of human-subject experiments using software agents can be premature. In fact, a great deal of effort has been made to address this question, and part of its development will be reviewed in this book (see Parts III, IV, and V).

2.3 Agent-based modeling in different disciplines

A cursory review of the use of agent-based modeling in various disciplines of the social sciences is given in this section.[8]

2.3.1 Anthropology, archeology, ethnology, and history

An early collection made by Timothy Kohler and George Gumerman (Kohler and Gumerman, 2000) includes agent-based modeling studies from archeologists, anthropologists, and ethnologists. They demonstrate how recent developments in modeling societies have endowed researchers with the freedom to move from the more traditional analytical techniques to an agent-centered, evolutionary, and generative understanding of how social phenomena emerged and work through time.

In these disciplines, agent-based models are used to simulate the past, some of which we already know from history and some of which happened prehistorically so that we may never know about them. One of the most ambitious projects is to use agent-based modeling to study "big history." Epstein and Axtell (1996) is probably the best illustration in this direction. The idea of the project is whether one can replicate a list of key features observed on a large scale in human history, such as population, immigration, famine, war, trade, disease, and social networks. What distinguishes this work is that software agents are programmed with multiple functions, whereas in most other agent-based models software agents are programmed in a rather low dimension.

While forecasting the future is always challenging, if we have time we will eventually know what the true answer is as times goes on, and hence the accuracy of our forecast. This "easiness," therefore, has no comparison with "forecasting" the past where the true answer may never be known, and we cannot reverse time. Unfortunately, in archeology, this is the best we can do, i.e., very often, we have to "forecast" what happened in prehistorical societies. Given this challenge, the contribution of agent-based modeling is to help us have a system such that all the fragmentary information can be more effectively pieced together so that the missing parts can be recovered to some extent, in particular when the missing parts were generated by humans in complex adaptive systems and their recovery is beyond what straightforward linear interpolation can do.

In this case, agent-based models are used to unravel archaeological mysteries by bringing "the missing complexity." The most well-cited work in this direction is what is known as Artificial Anasazi (Axtell *et al.*, 2002; Diamond, 2002; Gumerman *et al.*, 2003; Kohler, Gumerman, and Reynolds, 2005; Kohler *et al.*, 2008; Janssen, 2009). This project attempts to explain the history of the ancient Puebloan peoples (the Anasazi) that inhabited the Four Corners area in the American Southwest between 1800 BC and 1300 AD and who disappeared from the region in the space of a few years with no evidence of enemy invasions or dramatic environmental catastrophes.

2.3.2 Demography

Demography is the study of (human) population, both in terms of its size and structure. This size and structure change over time, and their projection can be a basis upon which many public policies are built, for example, retirement benefits. Unlike many other disciplines reviewed in this section, simulation had already been applied in this discipline long before agent-based modeling was introduced. In particular, in demography, both macrosimulation and microsimulation are carried out.[9] In fact, soon after Guy Orcutt's pioneering proposal of using microsimulation to study socioeconomic systems (Orcutt, 1957; Orcutt *et al.*, 1961), the microsimulation approach was already being used in demographic studies in the 1960s. A good survey of microsimulation in demography can be found in van Imhoff and Post (1998) and Morand *et al.* (2010).

Microsimulation acknowledges the great heterogeneities of individuals, including their age, sex, family status, education, etc., and the roles of these attributes in forming their decisions as to mate searching, marriage (and divorce), sex, pregnancy, residence, and health care. These individual decisions will then together determine mortality, nuptiality, fertility, and migration, variables that are related to population change. Therefore, it builds the micro-founded behavioral rules using empirical data, and through simulation generates aggregate behaviors, including population projection. This layout makes microsimulation very similar to agent-based modeling, and possibly makes agent-based modeling easier to accept for demographers than in other disciplines where equation-based modeling plays a dominant role.

Agent-based modeling extends microsimulation in several directions, such as adding downward causations, including social interactions with unobserved heterogeneities (social norms or networks), carrying out thought experiments, developing explanation mechanisms, etc. These differences between agent-based modeling and microsimulation have been well discussed in recent literature on agent-based demographic models (Billari and Prskawetz, 2003; Billari *et al.*, 2006). These extensions have resulted in some advances in demographic research, such as marriage (Billari *et al.*, 2007) and population projection (Griffith, Swanson, and Knight, 2012). In addition, agent-based modeling is not necessarily a substitute for microsimulation, for it can be a complement to it and the hybridization of the two can enhance demographic study and population projection (Wu and Birkin, 2012).

2.3.3 Entomology and ethology

If we consider social science broad enough to cover the social behaviors of insects or animals, it would be worth mentioning that agent-based simulation has also been applied to modeling their social behavior. Craig Reynolds's *boids* project is one of most illuminating examples from the early days (Reynolds, 1987). The boids project explores how the simple behaviors of individual birds combine to produce *flocking*. In this model, the birds obey only three rules:

1 *Avoidance*: If a bird is about to crash into another bird, it turns around.
2 *Attraction*: If a bird is far away from other birds, it heads towards the nearest bird.
3 *Alignment*: Otherwise, a bird will fly in the same direction as the bird next to it.

These three rules, later on, were also used in simulating the emergence of fish schools (Parrish and Viscido, 2005), and are, in fact, consistent with what biologists had found in their experimental studies of schools of fish (Partridge, 1981).

In the late 1990s, agent-based modeling was applied to study the social structure of non-human primates. One of the most cited agent-based models in this area is known as "DomWorld" (dominance world), and was proposed by Charlotte Hemelrijk (Hemelrijk, 1999, 2000; Bryson, Ando, and Lehmann, 2012). DomWorld provides an explanation of systematic differences in social organization observed in closely related primate species.

One interesting aspect of these agent-based models of non-human primates is the study of *group decision-making*; in particular, how group decisions are achieved when individual members may have different priorities for their own interest (Pratt *et al.*, 2005; Sellers, Hill, and Logan, 2007). Of course, group decision-making in human systems is also ubiquitous, but agent-based modeling of this class of social behavior is not commonly seen, particularly in economics.

2.3.4 Ecology

Agent-based modeling, alternatively known as individual-based learning, probably has a longer history in ecology than in economics. Its history can be traced back to the 1970s or even 1960s, while it was through the visionary paper by Huston, DeAngelis, and Post (1988) that the use of agent-based modeling in ecology became a self-conscious discipline; the equivalent acknowledgment of agent-based modeling in economics was not available at that time. Grimm and Railsback (2005) document well the historical development of agent-based ecology or individual ecology, and give many illuminating examples of successful replications of ecological patterns, such as the well-known lynx–hare cycle, using individual modeling.

What is coincidental is that when illustrating the difference between the equation-based approach and the agent-based approach, Wilensky chose the Lotka–Volterra equation as the working example. This classical Lotka–Volterra predator–prey model, standing at the heart of ecology, does not allow for the characteristic trait of individuals in the model (of the population growth rate); neither does it allow for spatial considerations and the resultant local interactions. When agent-based ecologists pursued "a genuinely new and different way of doing ecology," these are what they consider important. For them, agent-based modeling has to do with understanding, not simplifying, the complexity of nature (Grimm and Railsback, 2005).

2.3.5 Epidemiology

Modern theoretical epidemiology begins with the research on the spread of malaria by Ronald Ross (1857–1932), the 1902 Nobel Laureate in Medicine. Building upon the work of Ronald Ross (Ross, 1915; Ross and Hudson, 1917), Anderson McKendrick (1876–1943) and William Kermack (1898–1970) published their seminal work on theoretical epidemiology, known as the *Kermack–McKendrick model* or the *SIR model*. The pivotal role that this model has in epidemiology is probably equivalent to the role of the Lokta–Volterra equation in ecology. They both used differential equations to give a fundamental description of the essential processes observed in epidemiology or ecology. Very similar to the agent-based ecological models in relation to the Lotka–Volterra equation, the agent-based epidemiological model works as a complement to the early well-established Kermack–McKendrick (compartmental) models (Bian, 2004; Eubank *et al.*, 2004; Ferguson *et al.*, 2005; Auchincloss and Roux, 2008; Roche, Guegan, and Bousquet, 2008; El-Sayed *et al.*, 2012), and now both these equation-based and agent-based models are recognized as two primary types of disease-spread models.

2.3.6 Geography

Geography, by its nature, deals with highly distributed spatial systems. This nature inevitably makes agent-based modeling a relevant and even powerful tool for geography. In particular, as we shall see later (Chapter 4), one of the pioneering applications of agent-based modeling to social science begins in city dynamics (Schelling, 1971). The cellular automata model used in Schelling (1971) has become a foundation for studying geographical or spatial dynamics (Batty, 2007). The spatial agent-based models are further enriched and developed with the empirical data available from geographical information systems (GIS; Gimblett, 2002; Brown *et al.*, 2005; Baynes and Heckbert, 2010; Heckbert *et al.*, 2010). The integration of GIS and ABM has become a research paradigm to simulate many social, ecological, and environmental processes in a spatial context. Disaster management is one such extension, and criminology is another case in point (Groff, 2008; Liu and Eck, 2008).

The application of agent-based modeling to disaster management systems is mainly due to the attempt to smooth information flow so as to enhance a timely relief operation. To do so, it is desirable to have the whole disaster management system designed in an autonomous decentralized manner, and agent-based modeling is well suited for achieving this goal (Sadik *et al.*, 2010).

A very comprehensive and updated review of the significance of agent-based models to geographical systems can be found in the collection produced by Batty *et al.* (2012). This collection addresses the very fundamental issue of the role of agent-based models in the rising awareness of the increasing complexity of geographical systems, which is capable of distinguishing the strong sciences from the weak sciences, and the models which can *predict* from the models which can *inform*. Issues of geographical systems, including energy, security, epidemics, crime, poverty, migration, aging, urbanization, housing and financial markets, transportation, crowd movement, floods, climate change, and disaster management are discussed using various agent-based models. Given the edge-crossing nature of many of these issues, sometimes we need to integrate or couple various agent-based models so that the dialogues with different pieces of information can be enriched. This trend of further research efforts corresponds well with what we mean by "higher resolution and yet better integration" (Section 2.2.2).

2.3.7 Political sciences and international relations

Computer simulation of political science is not new. Ithiel de Sola Pool (1917–1984), the founder of the political science department at MIT, was considered to be a pioneer in this area. He gave the first computer simulation of decision-making in international crises, i.e., the outbreak of World War I (Pool, 1965). He also gave the first major computer simulation of the American electorate based on public opinion data (Abelson, Pool, and Popkin, 1965). His contributions are well acknowledged, e.g., see Deutsch, Platt, and Senghaas (1971).

After Pool's pioneering work, we have Thomas Schelling in the late 1960s and Robert Axelrod in the mid 1980s. They continued the social simulation approach to dealing with issues related to conflicts, competition, and cooperation (Schelling, 1969, 1971; Axelrod, 1984). Their studies, built upon cellular automata and programmed agents (actors), also laid the foundation for the burgeoning of the agent-based political science in the 1990s, as nicely surveyed in Johnson (1999), Cederman (2001), Kollman and Page (2006), and Susumu *et al.* (2007).

Some noticeable advances include the agent-based modeling of state formation and stability (Cederman, 1997), the rise and fall of nationalism (Cederman, 1997), preference aggregation (the Tiebout model; Kollman, Miller and Page, 1997), the size of wars (Cederman, 2003), ethnic and cultural violence (Lim, Metzler, and Bar-Yam, 2007), and, probably the most focused one, voting and multiparty competition (Fowler and Smirnov, 2005; Laver and Sergenti, 2011).

In international relations, Susumu Yamakage and his colleagues at the University of Tokyo have applied the agent-based modeling technique to the Cuban Missile Crisis in 1962 and conflicts in Northeast African countries (Yamakage *et al.*, 2007). In his study on the Missile Crisis, agent-based modeling is used to simulate the group decision-making process based on a tape recording of a National Security Council meeting called by John Kennedy. This kind of agent-based modeling involves agents' *dialogs*, or how the consensus and decision was reached through the influence of consecutive dialogs.

2.3.8 Management science

Agent-based modeling has also become quite important in management science. The application of agent-based modeling to management and organizations and its significance are well elucidated in a recent collection addressing the relation between complexity and management (Allen, Maguire, and McKelvey, 2011).

2.3.9 Sociology

While sociology has a much less analytical and modeling tradition as compared to economics, the first two articles on using agent-based models in the social sciences were published in the inaugural issue of the *Journal of Mathematical Sociology* (Sakoda, 1971; Schelling, 1971). Of the two authors, Thomas Schelling is largely recognized as an economist, but James Sakoda is undoubtedly a sociologist. The attempt to make sociology analytical and mathematical and hence a part of hard science had long existed before the advent of agent-based modeling. The main pursuit made by James Coleman and the establishment of the field of mathematical sociology in the late 1960s, as well as the launch of the the *Journal of Mathematical Sociology*, have all helped sociology move toward a suitable formalism. The early development of agent-based modeling in sociology has been well surveyed in Macy and Willer (2002), and the most recent developments have been surveyed by Squazzoni (2012).

Eighteen studies published from the year 1996 to the year 2001 were surveyed by Macy and Willer (2002). They classified these studies into two kinds of emergence, namely *emergent structure* and *emergent social order*. The former refers to the formation of social differentiation or homogenization (integration) through social influence and selection pressure, in the form of segregation, cultural clusters, stratification, diffusion, coordination, and the sudden collapse of norms, beliefs and institutions, whereas the latter refers to trust, cooperation, pro-social behavior, and collective action. They further used a kind of decision tree to assign each study attribute. This decision tree takes the following three elements explicitly into account: networks (spatial or social), learning (individual and social), and parameter manipulation (behavioral or environmental). This decision tree is in effect generic when one tries to do taxonomic work on the agent-based research in other disciplines.

Squazzoni (2012) applies the same taxonomy to extend the review by including studies published in the last decade. He, however, started with the emergent order first (his Chapter 2) by focusing on the emergence of pro-social behavior with various cooperation-enhancement mechanisms, followed by the emergence of social structure with a focus on social influence (his Chapter 3). In addition to this main body of literature, a genealogical study is also conducted by tracing the origin of agent-based ideas in sociology. There, he mentions the influence of some early works by James Coleman, Raymond Boudon, Herbert Simon, Fredrick Hayek, Thomas Schelling, and Mark Granovetter. This book is also one of the very few to make a connection between laboratory experiments and agent-based modeling (the subject discussed in Section 2.2.3).

2.4 The ten that make it new

In this chapter, we give a cursory look at the use of agent-based modeling in various major disciplines of the social sciences. By no means are we trying to give an exhaustive coverage here. In fact, some, such as education, law, linguistics, psychology, and social work, have not even been mentioned. However, we hope that the limited survey offered here is sufficient for us to see how social scientists are motivated by the use of agent-based modeling.

While each of these disciplines has its own conventional and well-established methodologies, such as equation-based, variable-based, statistically based, experimentally based, or microsimulation-based methodologies, the appearance of agent-based models can work in a complementary manner. As we have seen earlier in this chapter, and shall see more in the following chapters, *details matter*, not necessarily due to the prediction concern but more because of the understanding concern, since *they are the major driver for the use of agent-based modeling in the social sciences*. These microdetails can be manifested in many different forms in different disciplines. Many times they are beyond data availability and mathematical tractability, and hence agent-based models become the last resort.

Readers with enough patience to go through the cited articles in the various aforementioned disciplines may find that the advent of computational social science or the use of agent-based modeling in social science points to the following ten changes in social science research:

1. from equation-based to agent-based;
2. from analytical derivation to computer simulation;
3. from factors to actors;
4. from macrosimulation to microsimulation, and further to agent-based simulation;
5. from the modeling of population to the modeling of individuals;
6. from spatially free setting to spatially explicit modeling (situated modeling);
7. from statistical identification and estimation of social patterns to searching for the underlying generative mechanism;
8. from forecasting to understanding and explanation;
9. from policy applications to thought or theoretic-oriented experiments;
10. from small-scale laboratory experiments to large-scale laboratory experiments.

On each of these ten, one can find some assertions being made by the articles cited in this chapter. No elaboration shall be given here. We would, however, like to cite Van Dyke Parunak, Savit, and Riolo (1998) for an in-depth illustration of the first point, and Gilbert and Troitzsch (1999) for a comprehensive review of the three-stage evolution of simulation in the social sciences, the fourth point above.

Finally, we are fully aware that agent-based modeling faces different degrees of resistance in different disciplines, partially depending on the strength of their incumbents. Some fundamental questions or consideration of social simulation, or simply simulation, are essential for a comprehensive understanding of the role of simulation in science, its relation to theory, and its contribution to knowledge discovery. On this aspect, one can find a large number of studies with very contrasting viewpoints, for example, Lehtinen and Kuorikoski (2007) and Velupillai and Zambelli (2010). Lehtinen and Kuorikoski (2007) distinguish simulation from computation and argue that, based on the publications of the top journals, economists have not been ready to accept agent-based simulation. They are aware of the acceptance of agent-based simulation in physics, and the reason that it has not been accepted in economics is because economics is rather peculiar in having a love for the "perfect model." It is only if we view simulations as attempts to provide direct representations of real systems, and not abstract models, that the epistemology of simulation can make sense. However, at the very end, they still keep a slice of the positive expectations for the future of agent-based modeling in economics, which is largely consistent with what has been said in Section 2.2.3.

> The recent acceptance of behavioral and experimental economics within the mainstream reflects economists' increasing willingness to break away from these methodological constraints and to make use of results from

> experimental sources. Perhaps this will also mean that computerized quasi-experiments may one day find acceptance within economic orthodoxy.
>
> (Lehtinen and Kuorikoski, 2007, p. 326)

We have now presented the big picture of agent-based modeling in the social sciences. In the following, we shall provide a focused review of the use of agent-based modeling in economics. We start this job by tracing its origins. Part II of the book will trace the footprints along four trails. Some of these four are also shared by other disciplines, but there is at least one which is unique to economics, i.e., *market origin*. We shall start with this one (Chapter 3) and then continue with the rest.

Notes

1 The claim that agent-based modeling can help put the social sciences together is frequently made by many social scientists, such as Kohler (2000).

2 See, for example, Bandini, Manzoni, and Vizzari (2009), Meyer, Lorscheid, and Troitzscho (2009), Heath, Hill, and Ciarallo (2009), and Nikolai and Madeyand (2009). Bandini, Manzoni, and Vizzari (2009) and Nikolai and Madeyand (2009) provide a comprehensive survey of agent-based social simulation (ABSS) platforms. Their goal is to help researchers better choose a toolkit that suits their purposes. Nikolai and Madeyand (2009) have also created a corresponding page entitled *ABSS Software Comparison* in Wikipedia based on their research. Meyer, Lorscheid, and Troitzscho (2009) performed a co-citation analysis to visualize the intellectual structure of social simulation and its development.

3 While the agent-based models exemplified by cellular automata are made up of just two levels, the cell level and the checkerboard level, general agent-based models are not restricted to only two levels, and are referred to as "systems of systems of systems" by Jeffrey Johnson. In general modeling, interaction between the levels is still rarely seen in agent-based social science (see Chapter 24).

4 A concise introduction to Peirce's theory of abduction can be found in Fann (1970).

5 In addition to comprehension and forecasting, Epstein (2008) provides a list of an additional 16 answers to the question "Why model?" This list enables us to be able to evaluate the usefulness of agent-based models as opposed to equation-based models, by not attaching too much weight to their forecasting performance. Among his list of the 16 reasons for modeling, *guide data collection* is the one which deserves more attention, since we normally assume that theory comes after the data. However, here, Epstein (2008) reminds us that it is the opposite that is the case, as without models it is not always clear what data to collect. For economists, theory preceding data is best illustrated by Simon Kuznets's work on gross national product (GNP), which was clearly motivated and guided by Keynes's *General Theory* (Keynes, 1936).

6 Of course, experimental economics is the most obvious; many agent-based models proposed in the 1990s sought to *understand* the human behavior observed in experiments (see Chapter 6). Nonetheless, the research method using laboratory experiments is applied not only to economics but also generally to other disciplines in the social sciences (Webster and Sell, 2007; Morton and Williams, 2010; Druckman *et al.*, 2011).

7 However, if one performs a literature survey and lists all experimental studies on the one hand and all simulation studies on the other hand, it may not be hard to see that these two areas overlap only in a limited domain. In other words, agent-based modeling and experimental economics are more complementary to each other rather than just being a scaling-up or a scaling-down of each other.

8 It will be more interesting to review the advances of computational social sciences by exemplifying some of their successes or superiorities, and one of the most convincing ways to do so is to provide a list of research questions which agent-based modeling seems to tackle more successfully than the existing approaches. The degree of persuasion can be multiplied if one can show that the success is not just established in a few disciplines but in many other disciplines. However, this requires a more extensive and deep literature review, which is beyond the scope of this chapter. We nonetheless believe that the short review provided in this section is still useful for any readers who would like to take part in this adventure on their own.

9 The differences between macrosimulation and microsimulation as used in demography have been detailed in van Imhoff and Post (1998).

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Part II

Origins of ACE

There are several origins (traditions) of agent-based modeling in economics and social sciences. In this part of the book, we shall give a comprehensive and interdisciplinary review of ACE by tracing its four origins. The four origins of ACE considered in this part are, in chronological order, the *market origin* (with a long history), the *cellular automata origin* in the 1970s, the *economic tournament origin* (the *game theory origin*) in the 1980s, and the *experimental economics origin* in the 1990s.

These four origins are, of course, not independent of each other. They can be imagined to be four gates to the same castle, with the market origin being the main gate. While tourists may enter the castle through different gates, their experiences of the castle will be similar if they all explore the castle long enough. The four origins selected above are then very much like the answer to the question, "Where did you start your tour in ACE?" Hence, the answer may be Peter Albin and Ducan Foley's model of the non-tâtonnement process (Albin, 1992; Albin and Foley, 1992), Thomas Schelling's segregation model (Schelling, 1971), Robert Axelrod's simulation of the iterated prisoner's dilemma tournament (Axelrod, 1987), Jasmina Arifovic's simulation of cobweb experiments (Arifovic, 1994), or a long list like this. Certainly, these four origins are by no means exhaustive, and different tour guides may have different arrangements. However, as long as we visit the main gate (the markets origin) and explore the whole castle, our choices of the other three origins will have little effect.

Among the four origins, the most important and familiar one for economists is the *market origin*, a derivative of the historically long pursuit for a real construction (procurement processes) and hence a real understanding of markets. This part will begin with this origin (Chapter 3). Chapter 4 will be the cellular automata origin, which makes agent-based modeling a truly interdisciplinary subject, overarching biology, life sciences, computer science, physics, mathematics and logic, and social sciences. The third origin, the tournament origin or the game theory origin, is very much related to evolutionary game theory and, more generally, evolutionary economics. This game theory origin is also extensively shared by other agent-based social sciences, such as ecology (Grimm and Railsback, 2005), political sciences (Laver and Sergenti, 2011), and sociology (Squazzoni, 2012). Chapter 5 will show how agent-based simulation can advance our understanding of the evolutionary nature of games or markets by tracing this game theory origin. Finally, there is an increasingly large group of economists who are interested in using computer agents (software agents) to simulate human-subject behaviors observed in the laboratory. Chapter 6 will trace this experimental economics origin.

3 The markets origin

When Leon Walras (1834–1910) proposed his competitive general equilibrium model in his magnum opus *Elements of Pure Economics* in 1874 and characterized the economy as a set of equations and unknowns, a seed for a long pursuit was planted. The fundamental quest is whether the price discovery process, also known as the tâtonnement process, can actually replace the true market process, and whether the Walrasian auctioneer, or the equivalent advanced supercomputer, can actually do the job of resources allocation as the natural markets do.

Herbert Scarf, one of the founders of the general equilibrium model, once stated:

> In my opinion, the major attraction of markets over centralized calculation, for Gorbachev and his economic reformers, is not so much the mathematical difficulty of a single equilibrium calculation; it is rather that these computations must be performed over and over again in real time, in the face of constantly changing economic circumstances. The economic is in continual flux, with new possibilities constantly emerging, and mathematical solutions to the equilibrium equations will at best represent the solutions to yesterday's problem. If we are to be responsive to the novel conditions of daily life—and to engage the energies and skills of millions of self-interested economic actors—it may be necessary to use the market as an algorithm for solving the equilibrium equations rather than solving these equations themselves on the computer.
>
> (Scarf, 1990, p. 379)

In the history of economic analysis, this quest has been pursued by economists in different forms, including:

1 the debate on the possibility of socialist calculation (Boettke, 2000);
2 the silence of "markets" in economic theory (McMillan, 2002; Mirowski, 2007);
3 the aggregation problem over adaptive interacting heterogeneous agents (Kirman, 1992; Stoker, 1993; Blundell and Stoker, 2005; Gallegati *et al.*, 2006b); and
4 the mathematics suitable for social sciences (Velupillai, 2010; Borrill and Tesfatsion, 2011).

Possibility of socialist calculation

The first form of the quest, the socialist calculation debate, stands in an important position in the history of economic thought. It involves the great economists such as Ludwig von Mises (1881–1973), Friedrich Hayek (1899–1992), Abba Ptachya Lerner (1903–1982), and Oscar Lange (1904–1965). Under the Walrasian formulation, an economy can be seen as a set of equations. Thus, there should be no need for prices. Using information about available resources and people's preferences, it should be possible to calculate the optimal solution for resource allocation. Oscar Lange (Lange, 1936, 1937) even proposed the tâtonnement procedure as the actual computing algorithm for the operation of a centrally planned economy. Nevertheless, Friedrich Hayek responded that the system of equations required too much information that would not be easily available and the ensuing calculations would be too difficult. This is partly because individuals possess useful knowledge but do not realize its importance or do not have the incentive to transmit it (Hayek, 1945). He contended that the only rational solution is to utilize all the dispersed knowledge in the market place through the use of price signals.

Hayek's advocacy of the market for the fusion and use of knowledge (Hayek, 1945) provides an important intellectual inspiration for agent-based modeling through prediction markets (Vriend, 2002; see also Section 9.6.1). Probably for the same reason, sociologist Flaminion Squazzoni also includes Hayek as one of the predecessors of agent-based computational sociology (Squazzoni, 2012).

Silence of "markets"

As to the second form of the quest, one may be surprised to find that economics has little coverage of markets. The strong analytic form of economics inevitably clothes markets with a uniform, hiding their great and wild varieties. In his attempt to demystify the market, John McMillan (1951–2005) provides the following observation:

> Textbook economic theory does not dispel the markets-are-magical notion, for it says little about how markets go about doing their job. Although economics is in large part the study of markets, the textbooks depict them abstractly. The supply-and-demand diagram, expounded in countless Economics 101 lectures, is a bloodless account of exchange. It leaves unexplained much of what needs to be explained. It tells us what prices can do, but is silent on how they are set. Supply and demand bypasses questions of how buyers and sellers get together, what other dealings they have, how buyers evaluate what they are buying, and how agreements are enforced.
>
> (McMillan, 2002, p. 8)

Regardless of the forms in which the issue is presented, they all point to some undesirable consequences when the originally decentralized processes is assumed away or oversimplified in economics.

The strong motivation of having decentralized processes as the backbone of economics has then encouraged research in using agent-based models in economics. Examples abound: Albin and Foley (1992), Marks (1992), Vriend (1995), Bell (1998), Kirman and Vriend (2001), Wilhite (2001), McFadzean, Stewart, and Tesfatsion (2001), Tesfatsion (2002), Riechmann (2002), Gode, Spear and Sunder (2004), Axtell (2005), and Gintis (2006, 2007, 2010). To some extent, they all intended to replace the Walrasian auctioneer by a decentralized process or the procurement process (Tesfatsion, 2006).

> To move from Walrasian to agent-based modeling, the Walrasian Auctioneer has to be replaced by agent-driven procurement processes . . . this replacement is *by no means a small perturbation* of the model.
>
> (Tesfatsion, 2006, p. 847; emphasis added)

However, before we start looking at the agent-based modeling of the non-tâtonnement process, we should point out that agent-based modeling can also work well with the tâtonnement process. It is not the presence or absence of the Walrasian auctioneer per se that distinguishes the use of equation-based models from the use of agent-based models. It is just that the equation-based model is very hard to apply to the non-tâtonnement process, because price in this case may no longer be a single point over time but a distribution over space and time. It is hard for the equation-based model to work out how the price distribution will evolve over time. Having said that, we certainly can apply agent-based model to the tâtonnement process, and this chapter begins with a section on agent-based modeling of the tâtonnement process (Section 3.1) before moving to agent-based non-tâtonnement processes (Section 3.2). Then, by integrating a number of markets, a full-fledged version of agent-based non-tâtonnement processes can be developed into an agent-based macroeconomic model (Section 3.4).

In Section 3.3, we will take a detour from the Walrasian non-tâtonnement process to examine *auctions*. The reason for taking this detour is three-fold. First, as opposed to bilateral bargaining, the auction is an alternative exchange institution; hence, it provides an addition to what we will review in Section 3.2. Second, the review of some agent-based auction models enables us not only to see how one can use agent-based modeling to penetrate into the detailed operation of markets to observe market behaviors, but also to expose ourselves to thinking about the design or evolution of institutions, which, to some extent, echoes well with the viewpoint of "economists as engineers" (Roth, 2002) or "reinventing the bazaar" (McMillan, 2002). Third, this experience will be useful when we move back to large-scale agent-based non-tâtonnement processes (macroeconomics), because we will find that various exchange institutions have been proposed to build the constituent markets.

The focus of this chapter is on the market origin of ACE. It begs the question as to why and how ACE started in this very old discipline of economics, which has been dominated by the equation-based approach almost since its beginning. When we move from one section to another, we shall see that the drive for the paradigm shift toward agent-based modeling from the original equation-based modeling in economics is essentially the same as many other sister disciplines, as we briefly reviewed in Section 2.3. In a word, it is all about *heterogeneity*. The desire to effectively take care of heterogeneity in demographics, ecology, epidemics, entomology, and geography is essentially the same as that in economics, although each has different objects to deal with. Section 3.5 is devoted to the market origin directly from the perspective of heterogeneity.

3.1 Agent-based modeling of the tâtonnement process

3.1.1 Equation-based models

The tâtonnement process proposed by Leon Walras (1834–1910) is a top-down price adjustment heuristic working on the system of excess demand functions. This process does not allow trading until all the plans of economic agents are mutually consistent and all markets are clear. When these plans are not mutually consistent, economic agents are required to revise and resubmit their plans on the basis of the new prices announced by the Walrasian auctioneer.

Let us assume a pure exchange economy with M commodities. By convention and Walras's law, we use the last commodities as the numeraire and the $M-1$ excess demand functions $D_1, D_2, \ldots, D_{M-1}$ as the functions of the prices of the first $M-1$ commodities, which are represented in Equation (3.1):

$$\begin{aligned} D_1(P_1(0), P_2(0), \ldots, P_{M-1}(0)) &\gtrless 0 \\ D_2(P_1(0), P_2(0), \ldots, P_{M-1}(0)) &\gtrless 0 \\ \vdots \qquad & \quad \vdots \\ D_{M-1}(P_1(0), P_2(0), \ldots, P_{M-1}(0)) &\gtrless 0 \end{aligned} \tag{3.1}$$

Let us also assume that the initial price set by the auctioneer is $P_1(0), P_2(0), \ldots, P_{M-1}(0)$, abbreviated by $\{P_i(0)\}_{i=1}^{M-1}$. As shown by the inequality in (3.1), this initial set of prices does not clear any of the markets, leaving some with excess demands and some with excess supplies. The proposed tâtonnement process is a kind of *diagonal method*, which is to first find the price that can clear its own market, given that the prices of other markets remain unchanged. This is equivalent to finding a set of prices $(P_1(1), P_2(1), \ldots, P_{M-1}(1))$, abbreviated by $\{P_i(1)\}_{i=1}^{M-1}$, that can satisfy Equation (3.2):

$$\begin{aligned} D_1(P_1(1), P_2(0), \ldots, P_{M-1}(0)) &= 0 \\ D_2(P_1(0), P_2(1), \ldots, P_{M-1}(0)) &= 0 \\ \vdots \qquad & \quad \vdots \\ D_{M-1}(P_1(0), P_2(0), \ldots, P_{M-1}(1)) &= 0 \end{aligned} \tag{3.2}$$

The reason that Equation (3.2) *may* be achievable is because of the following heuristic (for convenience, call it Heuristic 1): excess demand tends to adjust downward if the own price increases and tends to adjust upward if the own price decreases. Of course, it is likely that when all these diagonal changes are set together, all markets are back to the state of disequilibrium, as shown in Equation (3.3):

$$\begin{aligned}
D_1(P_1(1), P_2(1), \ldots, P_{M-1}(1)) &\gtrless 0 \\
D_2(P_1(1), P_2(1), \ldots, P_{M-1}(1)) &\gtrless 0 \\
\vdots \qquad & \vdots \\
D_{M-1}(P_1(1), P_2(1), \ldots, P_{M-1}(1)) &\gtrless 0
\end{aligned} \tag{3.3}$$

However, even so, as Walras argued, $\{P_i(1)\}_{i=1}^{M-1}$ may stand closer to the equilibrium prices than $\{P_i(0)\}_{i=1}^{M-1}$, due to another heuristic (Heuristic 2) that diagonal disturbance is stronger than off-diagonal disturbance. Hence, by applying the diagonal approach again, we come up with $\{P_i(2)\}_{i=1}^{M-1}$ which satisfies Equation (3.4):

$$\begin{aligned}
D_1(P_1(2), P_2(1), \ldots, P_{M-1}(1)) &= 0 \\
D_2(P_1(1), P_2(2), \ldots, P_{M-1}(1)) &= 0 \\
\vdots \qquad & \vdots \\
D_{M-1}(P_1(1), P_2(1), \ldots, P_{M-1}(2)) &= 0
\end{aligned} \tag{3.4}$$

Even though these prices together still fail to clear the markets (Equation 3.5), they may get closer to the equilibrium prices.

$$\begin{aligned}
D_1(P_1(2), P_2(2), \ldots, P_{M-1}(2)) &\gtrless 0 \\
D_2(P_1(2), P_2(2), \ldots, P_{M-1}(2)) &\gtrless 0 \\
\vdots \qquad & \vdots \\
D_{M-1}(P_1(2), P_2(2), \ldots, P_{M-1}(2)) &\gtrless 0
\end{aligned} \tag{3.5}$$

It is then hoped that eventually the price sequence will converge to the equilibrium price (Equation 3.6), which is why this process is called the trial-and-error or groping process.[1]

$$\lim_{j \to \infty} P_i(j) = P_i^*, \quad i = 1, 2, \ldots, M-1. \tag{3.6}$$

Even though both heuristics above may seem plausible, since they both operate on the macro (the market) level, one may still be interested in knowing how plausibly these two heuristics can be micro-founded. This is the first step in moving from equation-based thinking to agent-based thinking, and an inquiry in this direction leads us to the famous Sonnenschein–Mantel–Debreu Theorem, which is normally called "bad news" in general equilibrium theory (Mas-Colell, Whinston, and Green, 1995). Basically, it says that without further assumptions (i.e., the assumptions already required for the *existence* of competitive equilibrium) on the nature of the preferences, endowments, or technologies (if production is also taken into account) of the individuals, we cannot say anything more about the behavior of these excess demand functions with regard to their uniqueness, comparative statics, and stability. In other words, there is no guarantee that one can grow these heuristics from the bottom up.

The tâtonnement process implicitly depends on the individuals' responses to the newly proposed prices; therefore, it can be handled by an agent-based version as well. Of course, if linear scale-up of individuals is the only thing one can do, then it can be done extraneously. Nonetheless, by using agent-based modeling, we no longer need to work with the excess demand functions, which means that when these functions are difficult to derive, agent-based models become another resort without stringent assumptions, such as perfect rationality, gross substitute, etc. We now turn to one such example, adapted from Riechmann (2002).

3.1.2 Agent-based models

By following Riechmann (2002), we consider an economy composed of N consumers, M commodities, M firms (one firm for each commodity). The consumers under this manna-from-heaven kind of budget constraint Y_i ($i = 1, 2, \ldots, N$) can choose a bundle of M commodities which serve their best interest in the usual sense (Equation 3.7):

$$\max_{q_{i,1}, q_{i,2}, \ldots, q_{i,M}} U_i(q_{i,1}, q_{i,2}, \ldots, q_{i,M})$$

$$\text{s.t.} \sum_{j=1}^{M} P^e_{i,j} q_{i,j} \leq Y_i. \tag{3.7}$$

The firms are given exogenously a linear supply function:

$$P_j = B_j + \alpha_j Q_j, \quad j = 1, 2, \ldots, M. \tag{3.8}$$

In this economy, the Walrasian auctioneer will leave the consumers to drive the groping process on their own. At the beginning, the auctioneer does not announce any possible suggested price. The consumers have to figure out the possible prices of all M commodities on their own $P^e_{i,j}$ ($j = 1, 2, \ldots, M$), and use these expectations to solve the utilization maximization problem (Equation 3.7) and propose their planned optimal bundle, $q^*_{i,1}, q^*_{i,2}, \ldots, q^*_{i,M}$ ($i = 1, 2, \ldots, N$). After receiving the proposed bundles of all consumers, the Walrasian auctioneer will set the price by equating the aggregate demand to aggregate supply for each M.

$$P_j^* = B_j + \alpha_j \sum_{i=1}^{N} q_{i,j}^*, \alpha_j \geq 0, \quad j = 1, 2, \ldots, M. \tag{3.9}$$

Nonetheless, these market-clearing prices may deviate from consumers' expectations ($P_j^* \neq P_{i,j}^e$, for some j); as a result, some with misperception may run out their manna,

$$\sum_{j=1}^{M} P_{i,j}^* q_{i,j}^* > Y_i,$$

and some may have surplus.

Since the plans are not mutually consistent, transactions cannot be implemented, and all consumers have to revise their proposal on the basis of their updated expectations of the prices. This inevitably involves modeling of expectation behavior. Agent-based modeling has great flexibility to accommodate various possible behavioral models or learning algorithms, as we shall see in the subsequent chapters of this book. Here, it is sufficient to just leave a general form as Equation (3.10):

$$P_{i,j}^e(t) = f(P_j^*(t-1), \quad P_{i,j}^e(t-1)\ldots), \quad j = 1, 2, \ldots, M. \tag{3.10}$$

The index t here refers to the tth iteration of the groping process. We have no intention to color this equation explicitly with further deliberate thoughts, such as the use of other consumers' (neighbors') past expectations or the forming of the expectations system-wise. Experienced readers know that there is a lot to add here. With this general form, the consumers come up with their new (revised) expectations and newly proposed bundles. Given these bundles, the Walrasian auctioneer then announces another new set of market-clearing prices. Again, this new set may be invalid with consumers' misperception and consumption; hence the iteration will cycle until we finally reach a set of prices with which everyone's expectations are fulfilled and the plan and the realization are mutually consistent.

The tâtonnement process described above is *bottom-up*. The distinguishing feature of this bottom-up design is that the Walrasian auctioneer does not have to work out the possible equilibrium price on his own; instead he "invites" all market participants to engage in price predictions and, hopefully, to pool together the information widely distributed over all corners of the economy, assuming that there are some social networks to enhance the flow of the information. In every single iteration, he only needs to find the roots of the M equations and leave the crowd to compute the rest. In a popular term, he is taking full advantage of the *wisdom of the crowd*. This also reminds us of the legacy of Hayek (Hayek, 1945) and its engineering application, namely, prediction markets (see more in Section 9.6.1). Of course, it does not mean that this will necessarily enhance the stability or speed up the convergence.[2] However, this bottom-up design does allow us to explicitly examine the critical role of information flow and processing even in the setup of competitive general equilibrium environment.

Of course, moving in this direction, one can see that when consumers almost take over the essential computing job, the Walrasian auctioneer may become "unemployed." So, in the next section, we shall see how agent-based models are applied to the context without the Walrasian auctioneer.

3.2 Agent-based modeling of the non-tâtonnement process

3.2.1 Why a non-tâtonnement process?

Over the last two decades, economists have used agent-based computer simulation to study the market process from a decentralized perspective, which is known as the *non-tâtonnement process*, an alternative to the tâtonnement process. Albin and Foley (1992) is a pioneering work in this direction. The fundamental limitation or unattractiveness of the tâtonnement process has long been noticed in mathematical economics, and the two major non-tâtonnement processes, the Edgeworth process and the Hahn process, have been well studied. A survey of this literature can be found in Fisher (1983), who suggested a simple alternative name for the non-tâtonnement process: *trading process*.

Mathematical economists are interested in the non-tâtonnement process because not allowing trade to happen in disequilibrium is too far away from reality. As for agent-based economics, there are different and deeper reasons for the pursuit. First of all, agent-based models may not necessarily agree with the notion of equilibrium usually formulated under the neoclassical equation-based formalism. In fact, defining equilibrium itself in agent-based models is a challenging exercise.[3] Therefore, while reality is still an important concern, whether those trades happen in "disequilibrium" certainly depends on the concept of equilibrium we have in mind.

Second, agent-based economics certainly has its own justifications for its interest in the non-tâtonnement process: a theoretical one and a practical one. The theoretical one is the primary one and can be traced back to the debate on the possibility of socialist calculation, as we have mentioned in the beginning of the chapter. It was argued that the prevalence of various bilateral, trilateral, quadrilateral, or multilateral tradings as trading institutions should be evaluated from an evolutionary perspective as they also evolve over time. This point has recently been given a refreshing interpretation by Axtell (2005). On the basis of an argument from computational complexity theory, Axtell (2005) pointed out that the non-tâtonnement process dominates the tâtonnement one in terms of computational complexity. In economic terms, tâtonnement process as an institution is more costly than its alternative; hence natural selection will avoid it and choose the alternative. By the same token, given the fixed resources, the non-tâtonnement process can accommodate a large-scale market, allowing more traders to trade more commodities than the tâtonnement process. Hence, it is this computational power of the non-tâtonnement process that interests agent-based economists. In this vein, agent-based modeling of the non-tâtonnement process can be regarded as a continuation of the legacy of Hayek (Gintis, 2013).

The practical justification is based on *spatially explicit thinking*. As seen earlier in Chapter 2, spatially explicit thinking is the major driver for the use of agent-based models in ecology, epidemiology, and geography. There is no exception for economics in terms of the non-tâtonnement process (we shall see more in the next section and in Chapters 4 and 22). The only difference between economics and the aforementioned disciplines is that we are interested not only in physical space but also in social space (social networks).

3.2.2 Literature overview

The development of agent-based modeling of the non-tâtonnement process has a short history, but there seems to be a self-organized research agenda, thanks particularly to a series of recent inspiring work by Herbert Gintis (Gintis, 2006, 2007, 2010, 2012, 2013). The research agenda starts with the simplest economy of pure exchange, two goods, and Cobb–Douglas utility functions. Since this pure exchange model has no production side and has no formal way to replenish the goods being consumed, only durable goods are considered or, alternatively speaking, agents are not allowed to consume before the entire trading sequence is over. Even with this primitiveness, these pioneering models do explicitly include one essential element which was rarely seen in most economic models at that time, i.e., *networks*; hence these simple models are complexified in that direction. In the subsequent stage, these models are developed into a large economy with many commodities and a much larger number of agents. For example, in Axtell (2005), the economy has one million agents with 20,000 commodities.

Then came Herbert Gintis who pushed forward the research on agent-based modeling of the non-tâtonnement process in several directions. Most elementarily, Gintis augmented the earlier pure exchange models with production, and there are two versions of this augmentation, from *production without firms* to *production with firms*. The former starts with a focus on the Scarf economy, which uses the Leontief utility function to fail the gross substitute assumption and hence the stability result of the Walrasian tâtonnement process. This extreme case is then replaced with the more general hybrid CES (constant elasticity of substitution) utility function, which includes both the Leontief function, when the elasticity goes to infinity, and the Cobb–Douglas function, when the elasticity goes to zero, for special cases.

With other added features, particularly *learning*,[4] this agent-based non-tâtonnement model with production and generalized utility function is able to demonstrate several important aspects:

1. a process from the distributed private prices to the quasi-public prices and its contribution to the efficiency and stability of the economy;
2. the emergence of money goods and the contribution of money to the efficiency of the economy;

3 its contribution to the slow progress in the literature of Walrasian dynamics, particularly by bringing in the legacy of Hayek (Hayek, 1945).

Then the model of production without firms is extended to the case of having firms as the units of production; as a result, the commodity market, labor market, and capital market all become constituents of the model. This extension is naturally connected to the very latest developments of agent-based macroeconomic models (Section 3.4).

The rest of this section will proceed in line with this development by discussing bilateral exchange with networks (Section 3.2.3) first and then production without firms (Section 3.2.4). Since production with firms is part of agent-based macroeconomic models, it will be generally reviewed in Section 3.4.

3.2.3 Bilateral exchange with networks

Before we proceed further to the literature, it would be useful to keep one fundamental question in mind: *What are the additional elements needed to move from the tâtonnement process to the non-tâtonnement process?* To answer this question, we start with a general setup by considering a population of N agents, indexed by $i = 1, 2, \ldots, N$, and a total of M commodities in the market, indexed by $m = 1, 2, \ldots, M$. Each of the agents is characterized by their endowments $\mathbf{W}_i$ and utility functions $\mathbf{U}_i$, where $\mathbf{U}_i = U_i(x_{i,1}, x_{i,2}, \ldots, x_{i,M})$, and $\mathbf{W}_i = \{w_{i,1}, w_{i,2}, \ldots, w_{i,M}\}$. With the initial endowment $\mathbf{W}_i$, their initial utility is:

$$\mathbf{U}_i(\mathbf{W}_i(0)) = U_i(w_{i,1}(0), w_{i,2}(0), \ldots, w_{i,M}(0)).$$

Agent i can change his endowment portfolio through bilateral trade. A *bilateral trading process* is described as follows:

Bilateral exchange protocol

1 Some *assignment rules*, either deterministic or stochastic, determine an agent, say, agent i, to be a trader in action.
2 Some *searching and matching rules*, deterministic or stochastic, spatially explicit or implicit, socially explicit or implicit, determine an agent j as the trading partner of agent i.
3 Some *bargaining rules*, naive or deliberate, myopic or forward looking, truth-telling or strategic, fair or greedy, static or adaptive, will guide agent i to determine a mutually acceptable term of trade (price) with agent j, who may follow the same or different bargaining rules.
4 Agents i and j implement the trade if the deal is made; otherwise, no trade happens. For the former case, agent i's current endowment is updated to

$$\mathbf{W}_i(t) = \mathbf{W}_i(t-1) + \Delta_i(t) = \mathbf{W}_i(t-1) + (\Delta_{i,1}(t), \Delta_{i,2}(t), \ldots, \Delta_{i,M}(t)).$$

Here, we have indexed the endowment by the event time, and t refers to the endowment after the tth iteration of the trade cycle. Similar adjustment holds for agent j, except that his event time (number of trade iterations) may be different, which can be further distinguished by t_i and t_j:

$$\Delta_i(t_i) + \Delta_j(t_j) = 0.$$

5 Back to (1), unless some termination criteria are met.

From the description above, we can identify at least three elements which are generally absent in the tâtonnement process. These elements are *search* (protocol 2), *networks* (protocol 2), and *bargaining and matching* (protocol 4). These new elements do not necessarily call for agent-based models. What makes the agent-based models unique is that they can help build the critical details of the spatial and social relationships and build search rules and bargaining heuristics which are in spirit of bounded rationality. Although the spatial and social relationships are not part of the non-tâtonnement processes reviewed by Fisher (1983), we know that search rules must depend on these details explicitly or implicitly.

Hence, one of the contributions of the agent-based models of the non-tâtonnement process is to examine the old issues in a new light, such as the price convergence and its speed and particularly the spatial and social relationships of traders. The new light also guides us to some issues which have not been studied by the conventional mathematical models, such as search costs, advertising expenditures, and wealth distribution. Below we will briefly review the pioneering work done by Albin and Foley (1992), Bell (1998), and Wilhite (2001).

General description

These models can be characterized by 5-tuples:

$$\begin{aligned}&(N, M, \{\mathbf{U}_i\}_{i=1}^{N}, \{\mathbf{W}_i(0)\}_{i=1}^{N}, \mathbf{G}(\mathbf{V}, \mathbf{E})),\\ &\text{where } \mathbf{V} = \{1, 2, , \ldots, N\}, \quad \mathbf{E} = \{b_{i,j} : i, j \in \mathbf{V}\}.\end{aligned} \tag{3.11}$$

$\mathbf{G}(\mathbf{V}, \mathbf{E})$ denotes the embedded network.[5] In Equation (3.11), $b_{ij} = 1$ if there exists an edge (connection, relation) between agents i and j; otherwise it is zero. They start with an economy composed of two durable goods[6] ($M = 2$) and N agents with identical preferences (utility functions) $\mathbf{U}_i = \mathbf{U}_j$, $i \neq j$, but with different initial endowments, $\mathbf{W}_i(0) \neq \mathbf{W}_j(0)$, $i \neq j$, which leads to a heterogeneity in marginal rate of substitution (MRS) or a distribution of MRS. This heterogeneity in MRS or the distribution of MRS provides the basic incentive for trades. The next question is how these agents with heterogeneous MRS can find and match each other with a mutually acceptable price. This is where the network, $\mathbf{G}(\mathbf{V}, \mathbf{E})$, plays an explicit role. We will consider the roles of different networks.

Networks

Concerning the choice of $\mathbf{G}(\mathbf{V}, \mathbf{E})$, the ring network (see Section 22.3.2) seems to be the most favorable beginning, probably next to the random network. Therefore, it would be interesting to know the attractive features of the ring network. If it is because of its striking manifestation of locality, then the checkerboard model (see Chapter 4) should be equally attractive. However, the checkerboard model was considered by none of the following three: Albin and Foley (1992) only focused on the ring network, but Bell (1998) went further to include the star network. Then Wilhite (2001) further added the small-world network.[7]

It is interesting to notice the design of the network topologies introduced by Wilhite (2001). The choice of network topology is not purely based on the literature in network science, but on a unique economic consideration regarding the extent of a market. The four network topologies (fully connected network, local disconnedted network, local connected network, and small-world network) correspond to a completely open global market, a closed local market, and two local but connected markets with different degrees of connections. This setting actually allows us to have an economic geographic underpinning to the model.

Search and matching

In addition to these different choices of networks, $\mathbf{G}(\mathbf{V}, \mathbf{E})$ also plays different functional roles in the trading process. In Wilhite (2001), the network serves as a precondition for chance discovery. Agents are assumed to know the MRS of all agents belonging to the same group, and can only trade with the agents within the same group. Bell (1998) assumed that an agent can only trade with the adjacent agents (neighbors or linked agents) defined by the network. Albin and Foley (1992) assumed that agents need to "promote" (advertise) their products to their neighbors before they can possibly trade with them.

Bargaining

Agents in all these models are not strategic, maybe for the consideration of model simplicity. They are not only myopic but also not greedy (having no bargaining strategy) at all. They are myopic in the sense that all pairs of agents can only accept welfare-enhancement deals. Hence, after the tth (t_i and t_j) iterations of trades, it must be true that

$$\mathbf{U}_i(\mathbf{W}_i(t_i)) \geq \mathbf{U}_i(\mathbf{W}_i(t_i - 1)), \text{ and } \mathbf{U}_j(\mathbf{W}_j(t_j)) \geq \mathbf{U}_j(\mathbf{W}_j(t_j - 1)). \tag{3.12}$$

The price is simply set to be either the market-clearing price of a two-person economy (Bell, 1998; Wilhite, 2001) or somewhere between the reservation prices of the two sides (Albin and Foley, 1992), which are truth-revealing. This is equivalent to assuming that there is a well-established culture or norm to decide the fair price. Under this circumstance, there is no need for the agents to consider any arbitrage behavior. Forecasting the forthcoming prices, buying low and selling high so as to maximize their utilities in the long run, are not considered in these models. Furthermore, in Wilhite (2001) there is an issue of information transparency. If all agents are globally connected, then the MRSs of all agents become publicly known, and each agent only needs to figure out what would be the most favorable deal in the market and try to make a deal with the potential partner.[8]

Market performance

The most interesting result of these agent-based studies is the effect of network topologies on the speed of convergence to the Walrasian equilibrium. The superiority of the star network relative to the ring network was found by Bell (1998), and the superiority of the small-world network relative to the fully connected network was found by Wilhite (2001). Wilhite (2001) actually balanced the convergence speed and the search intensity and found that, while the small-world network may converge to the Walrasian equilibrium price more slowly, it is less costly if we also take search intensity into account. In fact, Bell (1998) and Wilhite (2001) can be regarded as one of the earliest studies showing the significance of network topologies in economic performance. In particular, Wilhite (2001) showed the economic efficiency of the small-world network.

In addition to the convergence speed, Wilhite (2001) also identified the important nodes (middle men), such as the centrality of the agents, and found some positive connections between centrality and wealth. The appearance of middle men not only helps wealth creation (in terms of utility) in the society, but also contributes to the inequality of wealth (again in terms of utility of final holdings).

3.2.4 Production without firms

In this section, we begin to introduce the Gintis models. We start with the models that have production but no firms. Even with this focus, we still see different versions of the models. What we plan to do is to start with the common structure of the Gintis models. We then add remarks to highlight some varieties emanating from this common structure. As before, we begin with the very basic question: what makes the Gintis model essentially different from the models reviewed in Section 3.2.3? Primarily speaking, the Gintis models have three added elements, namely *production*, *private price*, and *learning*.[9]

Production

There are two assumptions which are frequently used in the literature of production economy without firms, and Gintis followed them in most of his work. The first assumption, which can be called the *dependence assumption*, is that *production goods* are distinguished from *consumption goods* in the following sense: the agent does not consume the goods which he produces, and does not produce those goods for which he has demand. Hence, production is mainly for exchange purposes. The second assumption, which can be called the *specification assumption*, is that each agent has his specific talent that can only produce one good, even though he can consume many goods.[10] Hence, with the two assumptions above, for an economy of N agents and M commodities, we can denote the commodity which is produced by agent i as m_i, $m_i \in \{1, 2, \ldots, M\}$, and the quantity of m_i that agent i produces as y_{m_i}.[11]

Private price

Private price is another important part which distinguishes Gintis's work from other earlier work on the non-tâtonnement process. As we have seen in Section 3.2.3, earlier work essentially has no bargaining behavior involved, and all agents are happy with institutionally or culturally formed pricing rules given the transparency of their endowments and preferences. Gintis's model does not rely on this transparency; in particular, one could argue that, with the absence of the Walrasian auctioneer (the central agency), any widely acceptable pricing rule must emerge from the bottom, and the top-down determination of this rule becomes less convincing. What Gintis did is to assume that agents are able to have their own subjective beliefs or expectations of the prices, say,

$$\mathbf{P}_i^e(t) = (P_{i,1}^e(t), P_{i,2}^e(t), \ldots, P_{i,M}^e(t)). \tag{3.13}$$

To show the significance of (3.13) he even gave it a name, *private price*, as opposed to *public price*, which is officially announced by the Walrasian auctioneer. Here, we index the price expectations by t to indicate that agent i may constantly revise his expectations over time through learning. However, since the discussion is about a snapshot of the time flows, we shall avoid the index to make the presentation less loaded.

Equation (3.13) can be considered to be agent i's personal opinion about the economy. Another agent j may come up with his own different opinion, say,

$$\mathbf{P}_j^e(t) = (P_{j,1}^e(t), P_{j,2}^e(t), \ldots, P_{j,M}^e(t)). \tag{3.14}$$

This personal opinion has two effects. First, agent i will use it as the true price vector to figure out the optimal bundle and maximize his utility, as solving the utility-maximization problem,

$$\begin{aligned} &\max_{x_{i,1}, x_{i,2}, \ldots, x_{i,M}} U_i(x_{i,1}, x_{i,2}, \ldots, x_{i,M}), \\ &\qquad \text{s.t.} \sum_{m=1}^{M} P_{i,m}^e x_{i,m} \le P_{i,m_i}^e y_{m_i}, \end{aligned} \tag{3.15}$$

to derive the optimal bundle[12]

$$\mathbf{X}_i^* = (x_{i,1}^*, x_{i,2}^*, \ldots, x_{i,m_i-1}^*, 0, x_{i,m_i+1}^*, x_{i,M}^*). \tag{3.16}$$

Second, his later trading decision will also be based upon his private prices, as we shall see in the trading protocol below. Needlessly to say, this personal opinion can initially be heterogeneous among agents; in other words, agent i may need to negotiate with a potential trading partner, agent j, with his different private prices. Therefore, this causes us to have a bargaining procedure which is more complex than the one in Section 3.2.3. Now, let us look at Gintis's proposed protocol.

Gintis's bilateral exchange protocol

1 **Beginning:** Randomly choose j, such that $x^*_{i,m_j} > 0$, and $x^*_{j,m_i} > 0$, and pair agent i with agent j (double coincidence).[13]

2 Agent i proposes an equal value of exchange,

$$x^*_{i,m_j} = \frac{P^e_{i,m_i} x_{j,m_i}}{P^e_{i,m_j}},$$

i.e., to satisfy agent j's need up to x_{j,m_i} in exchange for his own need x^*_{i,m_j}.

3 Agent j will consider the offer if

$$P^e_{j,m_i} x_{j,m_i} \geq P^e_{j,m_j} x^*_{i,m_j}; \tag{3.17}$$

otherwise, he will decline the offer. Since the offer is in a form of *take-it-or-leave-it*, this will be the **End of Trade**.

4 Agent j will accept the proposal if the condition (3.17) is satisfied, and

$$x_{j,m_i} \leq x^*_{j,m_i} \tag{3.18}$$

is also satisfied.

5 However, if only the saturation condition (3.18) is not satisfied, the proposal will still be accepted, but the trading volume will be downward adjusted; i.e., agent i will only receive a maximum of

$$x_{i,m_j} = \frac{P^e_{i,m_i} x^*_{j,m_i}}{P^e_{i,m_j}} \leq x^*_{i,m_j}. \tag{3.19}$$

6 **End of Trade:** Back to **Beginning**.

While Gintis's bilateral exchange protocol is a special case of the general protocol outlined in Section 3.2.3, a remark needs to be made here. It is the principle of *double coincidence* governing the searching and matching at the outset. Agent i, chosen to be active, needs to find the partner j who not only produces what he wants but also needs what he produces. Assuming that agent i demands m^i commodities ($1 \leq m^i \leq M - 1$), based on the protocol, agent i may need to go through this loop for a minimum of m^i times if other agents do not come to him. One example of this searching and matching process, as suggested in Gintis (2006), is given in Figure 3.1.

The purpose of giving this example is not to show an ideal one, but to show that a great variety can be easily extended from the illustrated one, although more or

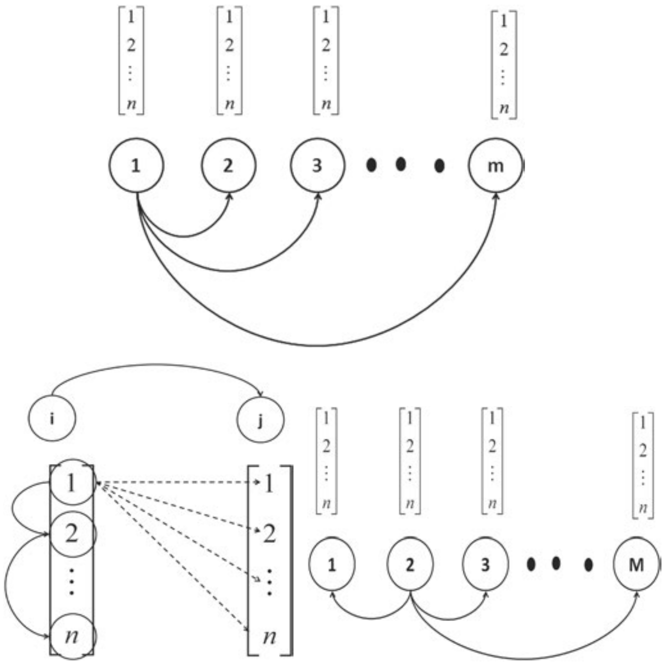

Figure 3.1 A well-ordered trading schedule.

Notes: In the case of a well-ordered trading schedule, the M commodities are ordered from $1, 2, 3, \ldots$ to M, and we assume that for each commodity, there are exactly n producers ($nM = N$), and they are also ordered from $1, 2, 3, \ldots$ to n. The assignment rule starts with the first agent in the first market being the active agent. Following the ordered sequence, this agent will then search for one partner for each market from market 2 to market M, except for the markets in which he has no demand (the top panel). When he finishes, the second agent of the first market will become active, then the third agent in the first market, and so on (the lower-left panel). After the completion of all agents in the first market, then it starts with the first agent in the second market (the lower-right panel) and proceeds in the same manner, one agent after another and one market after another. The assignment continues until all agents in all markets are done.

less in the same cumbersome way. However, this exercise is worth doing because it compels us to think of the interaction of agents through the embedded configurations, which is the essence of agent-based models. Take this protocol as an example. While Gintis did not explicitly give the underlying spatial configurations of agents, based on the given mobility, it is not hard to see that only something closer to a fully connected network can support the required mobility.

Learning

The private prices hold by each agent, $\{\mathbf{P}_i^e(t)\}_{i=1}^N$, will then be reviewed and updated through a given learning mechanism. The learning mechanism proposed by Gintis is composed of both individual learning and social learning. The individual learning allows agents to learn from their own experience, and the social learning allows agents to learn from others' experience. The individual learning suggested by Gintis is a kind of greedy algorithm or gradient descent algorithm. Obviously, an unreasonably high or low private price will make trades too difficult or too easy to happen. By the gradient descent algorithm, agents will adjust upward or downward their private prices based on their trading experience.

> [W]hen a good is underpriced compared to its equilibrium price, a shopper who is willing to pay more and a seller who is willing to charge more will both do better, on average, than other agents whose private prices are nearer the reigning quasi-public price structure.
>
> (Gintis, 2006, p. 9)

As to social learning, Gintis suggested an individualized version of replicator dynamics. Replicator dynamics, originating from biology (Taylor and Jonker, 1978), is frequently used in game theory (Samuelson, 1997), and its major idea is reproduction or imitation. With this operator, the private prices of the well-performing agents will be imitated by the ill-performing agents. In addition to imitation, Gintis allowed a small perturbation (mutation) to the replicates. This combined use of imitation and mutation makes the proposed learning algorithm a kind of evolutionary algorithm as frequently used in agent-based computational economics. There is more of this combined use to come in subsequent chapters (see, for example, Chapters 6 and 13).

Summary

Now, we have almost finished the description of the Gintis model of production without firms. As a summary, the flow of the model is shown in Figure 3.2. All

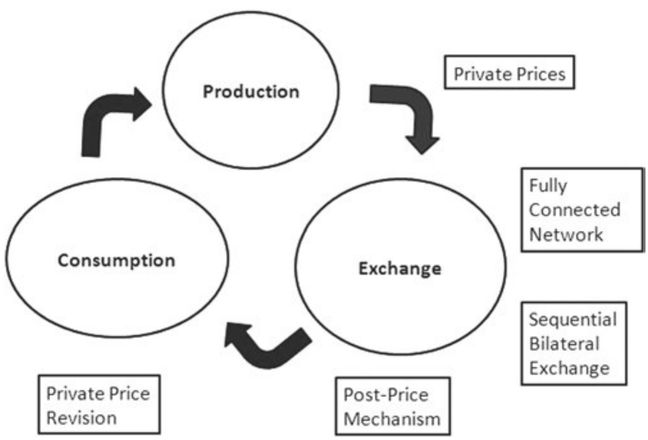

Figure 3.2 Flow of the economy.

economic activities are separated into three stages: production, exchange, and consumption. The production stage is simple: each agent is just endowed with a unit of $y_{m_i}(t)$ $(i = 1, 2, \ldots, N)$, directly from their labor. There is no economic decision about the division between work and leisure. We simply assume that all endowed talents are used in production. Then the total supply of each commodity is determined as follows. For convenience, we divide the agents into M segments by the commodities which they produce and there are n_j producers for the jth commodity; $\sum_{j=1}^{M} n_j = N$. We then reindex them by the commodity sequence, $1, 2, \ldots, n_1$ (the first commodity), $n_1 + 1, \ldots, n_1 + n_2$ (the second commodity), and so on and so forth. Then

$$Q_j^s(t) = \sum_{i=(n_1+\cdots+n_{j-1})+1}^{(n_1+\cdots+n_{j-1})+n_j} y_i(t), \quad j = 1, 2, \ldots, M, \tag{3.20}$$

where n_j is the number of producers for commodity j, and $n_0 = 0$. The aggregate demand for each commodity is

$$Q_j^d(t) = \sum_{i=1}^{N} x_{i,j}^*(\mathbf{P}_i^e(t)), \quad j = 1, 2, \ldots, M. \tag{3.21}$$

However, here we shall not equate Q_j^s with Q_j^d to derive the competitive equilibrium price. In fact, the private prices $\mathbf{P}_i^e(t)$ $(i = 1, 2, \ldots, N)$ have already been determined at the end of the last cycle, and now they will drive the following sequences of bilateral exchanges as described in Gintis's bilateral exchange protocol (Figure 3.1). When the bilateral exchange is done, the economy enters into the consumption stage and the utility of each agent is determined as

$$\mathbf{U}_i(\mathbf{X}_i(t)), \quad i = 1, 2, \ldots, N. \tag{3.22}$$

Notice that $\mathbf{X}_i(t)$, the realized one, may not be the same as $\mathbf{X}_i^*(t)$, the planned one, due to misperceived private prices and a sequence of "bad luck," such as running out of search time (number of trials) before making a deal or the match being made by rationing (Equation 3.19). On the basis of the difference between $\mathbf{X}_i(t)$ and $\mathbf{X}_i^*(t)$, agent i will revise their private prices through individual learning (3.23) and social learning, including imitation (3.24) and mutation (3.26):

$$\mathbf{P}_i^e\left(t + \frac{1}{2}\right) = \mathbf{P}_i^e(t) + \underbrace{\Delta\mathbf{P}_i(\mathbf{X}_i(t), \mathbf{X}_i^*(t))}_{\text{gradient descent}}, \quad i = 1, 2, \ldots, N, \tag{3.23}$$

$$\mathbf{P}_i^e\left(t + \frac{3}{4}\right) = \mathbf{P}_{\iota(i,\text{random}(i))}^e\left(t + \frac{1}{2}\right), \quad i = 1, 2, \ldots, N. \tag{3.24}$$

The ι is an imitation operator, which is a sup indicator of the two: i and a generated random number, *conditioned on* i, say j.[14] The ι function then compares the performance or the fitness of the two, and outputs the one with higher performance,[15] i.e.,

$$\mathbf{P}_i^e\left(t+\frac{3}{4}\right)=\begin{cases}\mathbf{P}_i^e(t+\frac{1}{2}), & \text{if } \iota(i,j)=i,\\ \mathbf{P}_j^e(t+\frac{1}{2}), & \text{if } \iota(i,j)=j.\end{cases} \tag{3.25}$$

Finally, the imitated private prices are under some small chance of mutation:

$$\mathbf{P}_i^e(t+1)=\mathbf{P}_i^e\left(t+\frac{3}{4}\right)+\underbrace{\Delta\mathbf{P}_i}_{\text{mutation}},\quad i=1,2,\ldots,N. \tag{3.26}$$

Then the time index moves to $t+1$ and the next cycle starts.

This model has been applied to three theoretical settings, each with a familiar type of utility function, in each of which the Walrasian competitive equilibrium is known. The three utility functions are the Leontief utility function, the Cobb–Douglas utility function, and the hybrid CES utility function. The major intellectual thread connecting these three is the property of gross substitute, which plays a crucial role in competitive equilibrium analysis (Mas-Colell, Whinston, and Green, 1995). Since the major results are largely shared by these three applications, in the following we shall first focus on the case of the Leontief utility function, followed by a general discussion of the holistic picture of these three.

3.2.5 Scarf economy

The Leontief utility function in equilibrium analysis was motivated by Scarf (1960), and this study has been intensively followed by Gintis (Gintis, 2006, 2007, 2012). Scarf (1960) used his proposed model to show that there can be a total failure (global instability) of the tâtonnement process to converge to the multiple market competitive equilibrium. This work is particularly striking in light of the earlier result obtained by Arrow, Block, and Hurwicz (1959), which showed that the competitive equilibrium is globally stable when all goods are gross substitutes. Much before the above-mentioned finding, the traditional view maintained by John Hicks (Hicks, 1946) is that the phenomenon of instability is either due to income effect or due to complementarity, and it is precisely the latter upon which Scarf's example is built. In fact, Scarf considered a special utility function which makes full use of perfect complementarity—the *Leontief utility function*.

Due to its profoundness, the Scarf economy has been constantly pursued in both analytical economics (Hirota, 1981, 1985, 2003) and experimental economics (Anderson *et al.*, 2004). Of course, experimental economists may not be interested in the tâtonnement process; instead, the frequently tested institution is the double auction, a non-tâtonnement process. Nevertheless, the inquiries pursued by experimental economists are still very much theoretically motivated. They are interested in knowing the relation between the tâtonnement process and the non-tâtonnement process; in particular, whether the former can help to predict the latter. "To what extent are tâtonnement models reliable when applied to non-tâtonnement institutions?" (Anderson *et al.*, 2004, p. 210). The main experimental result is that the tâtonnement process can have a fairly good prediction or application to the non-tâtonnement process:

> Existing experimental results suggest that in spite of seeming difficulties such models are surprisingly reliable.
>
> (Anderson *et al.*, 2004, p. 210)

> When the model predicts convergence the data converge; when the model predicts orbits, the data orbit in the direction predicted by the model.
>
> (Anderson *et al.*, 2004, p. 209)

Scarf economy with many goods

An economy is composed of N goods, denoted by $j = 1, 2, \ldots, N$. Corresponding to these N goods, there are N types of agents.[16] Agents of type i are initially endowed with w_i units of good i $(i = 1, 2, \ldots, N)$, and zero units for the other $N - 1$ goods. Let $\mathbf{W}_i$ be the endowment vector of i, then

$$\mathbf{W}_i = (\underbrace{0, 0, \ldots, 0}_{i-1\ \ 0s}, w_i, \underbrace{0, \ldots, 0}_{N-i\ \ 0s}), \quad i = 1, 2, \ldots, N. \tag{3.27}$$

Regardless of the agent types, they are commonly shared with the same utility function, namely, the Leontief utility function:

$$U(x_1, x_2, \ldots, x_N) = \min \left\{ \frac{x_1}{w_1}, \frac{x_2}{w_2}, \ldots, \frac{x_N}{w_N} \right\}. \tag{3.28}$$

Given a price vector $(P_1, P_2, \ldots, P_N)$, an individual utility maximization problem for the type i agents can be defined as below:

$$\max U_i(x_1, x_2, \ldots, x_N) = \max_{\{x_i\}_{i=1}^N} \left(\min \left\{ \frac{x_i}{w_i} \right\}_{i=1}^N \right), \tag{3.29}$$

subject to

$$\sum_{j=1}^N P_j x_j = \sum_{j=1}^N P_j w_j = P_i w_i. \tag{3.30}$$

Solving Equations (3.29) and (3.30) will lead to the optimal bundle of consumption as follows:

$$x_{i,j}^* = \lambda_i^* w_j, \quad j = 1, 2, \ldots, N, \tag{3.31}$$

where

$$\lambda_i^* = \frac{P_i w_i}{\sum_{j=1}^{N} P_j w_j}. \tag{3.32}$$

Without losing generality, let us consider an economy where there is only one agent per type. With this simplification, the aggregate demand for each good j can be derived as follows:

$$Q_j^d = \sum_{i=1}^{N} x_{i,j}^* = \sum_{i=1}^{N} \lambda_i^* w_j. \tag{3.33}$$

Then the market clearing condition for the whole economy of the multiple markets is as follows:

$$Q_j^d = Q_j^s = w_j, \quad j = 1, 2, \ldots, N. \tag{3.34}$$

Solving the system of equations (3.34) will lead to the simultaneous determination of n equilibrium prices:

$$P_j^* = \frac{1}{w_j}, \quad j = 1, 2, \ldots, N. \tag{3.35}$$

If we consider the last good (good N) as the numeraire ($P_N = 1$), then with the according normalization,

$$P_j^* = \frac{w_N}{w_j}, \quad j = 1, 2, \ldots, N. \tag{3.36}$$

Scarf economy with three goods

With this general setting, Scarf (1960) considered a special version of this model with only three goods and hence only three types of agents, $i, j = 1, 2, 3$. The Leontief utility function (3.28) is further restricted to only two goods. One is what the agent is initially endowed, but the other has to be acquired from the market through trading. Specifically, they are as follows:

$$\begin{aligned}
U_1(x_1, x_2) &= \min\left\{\frac{x_1}{w_1}, \frac{x_2}{w_2}\right\}, \\
U_2(x_2, x_3) &= \min\left\{\frac{x_2}{w_2}, \frac{x_3}{w_3}\right\}, \\
U_3(x_3, x_1) &= \min\left\{\frac{x_3}{w_3}, \frac{x_1}{w_1}\right\}.
\end{aligned} \tag{3.37}$$

Given a vector of the prices (P_1, P_2, P_3), the demand for each commodity of each type of agent is given as follows:

$$x_{1,j}^* = \lambda_1^* w_j \quad (j = 1, 2), \tag{3.38}$$
$$x_{2,j}^* = \lambda_2^* w_j \quad (j = 2, 3), \tag{3.39}$$
$$x_{3,j}^* = \lambda_3^* w_j \quad (j = 1, 3), \tag{3.40}$$

where

$$\lambda_1^* = \frac{P_1 w_1}{\sum_{j=1,2} P_j w_j}, \; \lambda_2^* = \frac{P_2 w_2}{\sum_{j=2,3} P_j w_j}, \; \lambda_3^* = \frac{P_3 w_3}{\sum_{j=1,3} P_j w_j}. \tag{3.41}$$

By solving the market clearing condition (3.42),

$$\begin{bmatrix} w_1 & 0 & w_1 \\ w_2 & w_2 & 0 \\ w_3 & 0 & w_3 \end{bmatrix} \begin{bmatrix} \lambda_1^* \\ \lambda_2^* \\ \lambda_3^* \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}, \tag{3.42}$$

one can have the following market-clearing prices for the three markets:

$$\begin{bmatrix} P_1^* \\ P_2^* \\ P_3^* \end{bmatrix} = \begin{bmatrix} \frac{1}{w_1} \\ \frac{1}{w_2} \\ \frac{1}{w_3} \end{bmatrix}. \tag{3.43}$$

By using good 3 as the numeraire $(P_3 = 1)$, the above equilibrium price can be normalized as

$$\begin{bmatrix} P_1^* \\ P_2^* \\ P_3^* \end{bmatrix} = \begin{bmatrix} \frac{w_3}{w_1} \\ \frac{w_3}{w_2} \\ 1 \end{bmatrix}. \tag{3.44}$$

Example

If $(w_1, w_2, w_3) = (10, 20, 400)$, then $(P_1^*, P_2^*, P_3^*) = (40, 20, 1)$.

Example: Tâtonnement process fails

Gintis (2007) gave an example of the Walrasian tâtonnement process as follows:

$$\begin{aligned} P_1^*(t+1) &= P_1^*(t) + 0.01(\lambda_1^* + \lambda_3^* - 1)w_1, \\ P_2^*(t+1) &= P_2^*(t) + 0.01(\lambda_1^* + \lambda_2^* - 1)w_2. \end{aligned} \tag{3.45}$$

Then, starting from a perturbation of (P_1^*, P_2^*, P_3^*) by $(\Delta_1, \Delta_2, \Delta_3) = (3, -2, 0)$, Gintis (2007) showed that the above price adjustment process (3.45) drives the subsequent prices to move on an ellipse around the equilibrium prices in a counterclockwise manner (see his Figure 1).

Example: Tâtonnement process fails badly

Gintis (2007) further showed that the case can fail even worse if *market noise* is added to the price. An example is that the received prices upon which the agent makes decisions are premultiplied by a random factor uniformly distributed over [0.8, 1.2]. Denote the noisy version of the price by

$$\tilde{P}_j = \mu_j P_j, \quad \mu_j \sim [0.8, 1.2], \quad j = 1, 2; \quad \tilde{P}_3 = P_3^* = 1. \tag{3.46}$$

This noisy price will then replace the noise-free price to enter into the demand function, Equations (3.38) and (3.41), and the disturbed multipliers become:

$$\tilde{x}_{1,j}^* = \tilde{\lambda}_1^* w_j \quad (j = 1, 2), \tag{3.47}$$

$$\tilde{x}_{2,j}^* = \tilde{\lambda}_2^* w_j \quad (j = 2, 3), \tag{3.48}$$

$$\tilde{x}_{3,j}^* = \tilde{\lambda}_3^* w_j \quad (j = 1, 3), \tag{3.49}$$

where

$$\tilde{\lambda}_1^* = \frac{\tilde{P}_1 w_1}{\sum_{j=1,2} \tilde{P}_j w_j}, \quad \tilde{\lambda}_2^* = \frac{\tilde{P}_2 w_2}{\sum_{j=2,3} \tilde{P}_j w_j}, \quad \tilde{\lambda}_3^* = \frac{\tilde{P}_3 w_3}{\sum_{j=1,3} \tilde{P}_j w_j}. \tag{3.50}$$

If we continue to apply the tâtonnement process as declared in Equation (3.45), then the price dynamics become Equation (3.51):

$$\begin{aligned} P_1^*(t+1) &= P_1^*(t) + 0.01(\tilde{\lambda}_1^* + \tilde{\lambda}_3^* - 1)w_1, \\ P_2^*(t+1) &= P_2^*(t) + 0.01(\tilde{\lambda}_1^* + \tilde{\lambda}_2^* - 1)w_2. \end{aligned} \tag{3.51}$$

Gintis (2007) simulated one of these noisy Scarf economies and showed that a small amount of noise can completely destabilize the economy with persistently increasing deviations away from the original Walrasian economy (see his Figure 2).

3.2.6 Decentralized dynamics

Gintis's agent-based model of the non-tâtonnement process was first applied to the Scarf economy where some basic properties of the decentralized dynamics were established (Gintis, 2006, 2007). These properties have been further corroborated by applications of this model to other theoretical settings, including with the Cobb–Douglas utility function (Gintis, 2010) and with the hybrid CES utility function (Gintis, 2012). The decentralized dynamics of the Walrasian economy were mainly concerned with the convergence properties of the market economy, including its prices, quantities (excess demand or excess supply), and various efficiency measures. The emergence of quasi-public prices and the emergence of money were also studied as part of the convergence processes. Second, comparative dynamic analysis was conducted to see the effects of private prices, money, and learning. Finally, the stability of the economy was tested with exogenously given nominal shocks.

Random agents

Before we proceed further, let us just make a quick note of the heterogeneity of the economy; that is, Gintis applied the maximum entropy principle in his design of agents. This principle was applied not only to their initial private prices, but also to their utility functions, whether Cobb–Douglas or CES. For example, in Gintis (2010) he considered the following Cobb–Douglas utility function:

$$U_i(x_{i,1}, x_{i,2}, \ldots, x_{i,M}) = \prod_{j=1}^{M} (x_{i,j})^{\alpha_{i,j}}, \tag{3.52}$$

where $\sum_{j=1}^{M} \alpha_{i,j} = 1$.

Excluding the commodity produced by agent i himself, each agent can have at most $M - 1$ commodities in his utility function:

$$\begin{cases} \alpha_{i,j} = 0, & \text{if } j = m_i, \\ \alpha_{i,j} \geq 0, & \text{if } j \neq m_i. \end{cases} \tag{3.53}$$

When the entropy maximization principle is applied, the number of commodities entering in agent i's utility function, say, k, is randomly generated from the uniform distribution, i.e., $k \in \{1, 2, 3, \ldots, M - 1\}$. For each agent of the k type, these positive k coefficients of α are first sampled from uniform [0, 1] and then renormalized, so that their sum is one. In this way, the random agents generated do not require us to have any specific knowledge about them and they have the largest diversity. This entropy maximization technique has been used very often in the design of agents, and we will discuss it again in Section 8.5.1.

Convergence

Let $\mathbf{P}_j^e(t) = \{P_{i,j}^e(t)\}_{i=1}^N$ be the private price vector of market j at time t,

$$\sigma_j(t) = \sqrt{\frac{\mathrm{Var}(\mathbf{P}_j^e(t))}{N - 1}}, \quad j = 1, 2, \ldots, M \tag{3.54}$$

be its standard deviation, and

$$\sigma(t) = \{\sigma_j(t)\}_{j=1}^{M}. \tag{3.55}$$

Furthermore, let $\bar{P}_j(t)$ be the mean transaction prices in market j at time t, which is the average of the transaction price over all successful bilateral exchanges. Notice that, based on our trading protocol, these prices must be a large sample from $\mathbf{P}^e_j(t)$. To study market convergence, two measures are proposed: the normalized standard deviation,

$$\frac{\sigma(t)}{\bar{\mathbf{P}}(t)} = \left\{ \frac{\sigma_j(t)}{\bar{P}_j(t)} \right\}_{j=1}^{M},$$

and the normalized deviation of market price from the Walrasian equilibrium price,

$$\mathbf{dp}(t) = \frac{|\bar{\mathbf{P}}(t) - \mathbf{P}^*|}{\mathbf{P}^*} = \left\{ \frac{|\bar{P}_j(t) - P^*_j|}{P^*_j} \right\}_{j=1}^{M}.$$

The convergence has two phases. In the first phase, the standard deviation of private prices $\sigma(t)$ or the normalized one $\frac{\sigma(t)}{\bar{\mathbf{P}}(t)}$ declines dramatically to a low level, indicating the emergence of a *quasi-public price*, i.e., a price approximately shared by most agents. In the second phase, this quasi-public price converges further to the Walrasian competitive equilibrium price, although it may take a much longer time to do so. One particular interesting result is that, even in the Scarf economy where the Walrasian tâtonnement process converges to a limit cycle, rather than to the competitive equilibrium prices, the quasi-public prices generated by the agent-based model can still converge to a small neighborhood of the Walrasian tâtonnement process with a small fluctuation (Gintis, 2012).[17]

In addition to the price behavior, one can also study the Gintis model from the quantity aspect primarily based on the difference between the planned consumption and realized consumption. The competitive equilibrium implies that

$$\mathbf{X}^*_i(t) = \mathbf{X}_i(t), \quad \forall\, t \text{ and } \forall\, i = 1, 2, \ldots, N. \tag{3.56}$$

The absolute value of the difference of the two vectors componentwise gives either the excess demand or the excess supply of agent i for all commodities. Based on the dependence assumption mentioned in Section 3.2.4, the absolute difference shows the excess supply only for the commodity which he produces, i.e., m_i; for the rest, it shows the excess demand:

$$|\mathbf{X}^*_i(t) - \mathbf{X}_i(t)|_j = \begin{cases} x^*_{i,j}(t) - x_{i,j}(t) & \text{if } j \neq m_i \text{ (excess demand)} \\ x_{i,j}(t) - 0 & \text{if } j = m_i \text{ (excess supply)} \end{cases} \tag{3.57}$$

By summing these absolute differences over all i, one can have the excess aggregate demand, $Z_j^D(t)$, and the excess aggregate supply, $Z_j^S(t)$:

$$Z_j^D(t) = \sum_{\{i:m_i \neq j\}} |\mathbf{X}_i^*(t) - \mathbf{X}_i(t)|_j, \quad j = 1, 2, \ldots, M, \tag{3.58}$$

$$Z_j^S(t) = \sum_{\{i:m_i = j\}} |\mathbf{X}_i^*(t) - \mathbf{X}_i(t)|_j, \quad j = 1, 2, \ldots, M. \tag{3.59}$$

Depending on the trading protocol and the distribution of the private prices, it is likely that one can have both excess aggregate demand and excess aggregate supply for the same market.[18] This simply indicates the failure of matching either in terms of private prices or in terms of trading pairs. The degree of mismatch may decline with increasing search intensity or the emergence of the quasi-public price. However, so long as the mismatch occurs, the excessiveness can still coexist in both sides of the market; it is just a matter of magnitude.[19]

Efficiency measure

In addition to prices [$\frac{\sigma(t)}{\mathbf{P}(t)}$ and $\mathbf{dp}(t)$] and quantities [$\mathbf{Z^D}(t)$ and $\mathbf{Z^S}(t)$], the convergence of the economy can also be studied from the efficiency viewpoint. Let us denote the utility of agent i under the Walrasian competitive equilibrium by $\mathbf{U}^*{}_i$; then one possible measure of efficiency is based on how much of this full utility is realized,

$$\Xi_1(t) = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{U}_i(t)}{\mathbf{U}^*{}_i}.$$

Alternatively, one can use the planned consumption bundle as the basis to see how much of that utility is realized,

$$\Xi_2(t) = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{U}_i(t)}{\mathbf{U}^*{}_i(t)}.$$

If the emergences of the quasi-public prices are cohesive and that cohesiveness can be accompanied by decreasing frequencies of matching failures, then $\Xi_2(t)$ is expected to be close to one. Furthermore, if the quasi-public prices can converge to the Walrasian competitive equilibrium prices, then we can expect the convergence of $\Xi_1(t)$ to one as well. As we shall see later, there are two factors that can contribute to this process; one is a behavioral factor, and the other is an institutional factor. The former refers to learning, and the latter refers to money. Learning can help to shape the cohesive quasi-public prices, and money can enhance the trading protocol and reduce the failures of searching and matching.

Private price

The stability and the robustness of the Walrasian market economy is partially attributed to the use of private prices, the decentralized decisions. The reliance on the private prices ameliorates the possible effects of shocks to the economy; if all agents depend on one public price, their actions are easily become synchronized, and the effect of shocks can, therefore, be amplified.

> Public prices, by contrast, lead all agents to adjust in the same direction in a given period, thus creating a situation in which instability and chaos are likely outcomes.
>
> (Gintis, 2006, p. 14)

> Quasi-public prices have much of the virtue of public prices in that they support a relatively high level of economic efficiency, while at the same time acting as shock absorbers in the face of random exogenous perturbations to the economy.
>
> (Gintis, 2010, p. 18)

To show the significance of the private price, Gintis brought the public prices back to the model (Gintis, 2007). In this case, the public prices are derived using the Walrasian auctioneer's price adjustment equation, something similar to Equation (3.45), by feeding in the excess demand (3.58) and the excess supply (3.59) of the markets.[20] When the public prices are available, agents can replace their own prices with public prices, and these agents are called *public-price agents*. Gintis found that when the proportion of these public-price agents is high up to a degree, the convergence properties of prices can be altered, and prices can become rather unstable (see Gintis, 2007, Figure 4).

Money

Money is another important subject in general equilibrium analysis. Gintis demonstrated three features of money in his agent-based model. First, it shows how one or more than one commodities, known as money, can emerge from the bilateral exchange economy. Second, it shows what makes money. Third, it shows how much efficiency an economy can gain from the monetary institution. The first two features are not completely new; using agent-based models to simulate the emergence of money has already been done by Marimon, McGrattan, and Sargent (1990), Basci (1999), Staudinger (1999), and Duffy (2001) in the context of the well-known Kyotaki–Wright model (Kiyotaki and Wright, 1989). Nevertheless, Gintis's works were mainly concerned with the possibility that the commodity with a high storage cost can become the medium of exchange due to speculative reason, also called the speculative equilibrium.

Gintis was not curious about the possibility of speculative equilibrium, and his simulation only found the fundamental equilibrium; i.e., only the commodity with the lowest storage cost can become money, which he called fiat money. What really interested him was how the emergence of money can reduce the searching and matching failures and be welfare enhancing. His proposed full-fledged Walrasian economy enables us to have a qualitative evaluation of the possible gain from a monetary economy; in particular, one can do this kind of experiment with market maturation characterized by the number of goods traded in the market.[21]

Trading protocol with money

However, to have an evolutionary process toward a monetary economy, the previous trading protocol is not enough. Agents need to be less myopic or more intelligent, and we need a protocol which allows for more extended behavior. To do so, one first needs an extended budget constraint, with an inventory of goods.

The original budget constraint (3.15) does not explicitly include the idea of inventory; it only considers the flow of goods (production), not the stock of goods (inventory), but that can be easily extended by allowing for the accumulation of the unsold in the previous periods. For example, Gintis (2010) works with the following concept of stock. Let us denote the inventory of the commodity produced by agent i by $q_{m_i}(t)$. Let us further distinguish the inventory before the trade and after the trade by $q^b_{m_i}(t)$ and $q^a_{m_i}(t)$. Hence,

$$q^b_{m_i}(t) = y_{m_i}(t) + \beta_{m_i} q^a_{m_i}(t-1), \quad \text{where } 0 \leq \beta_{m_i} \leq 1. \tag{3.60}$$

In this way, the budget constraint is now in a form of inventory, and the discount rate β_{m_i} can be interpreted as a storage cost, as it indicates the perishing nature of the good. Then the original utility maximization problem is modified with the following extension:

$$\begin{aligned} \max_{x_{i,1},x_{i,2},\ldots,x_{i,M}} \quad & U_i(x_{i,1}, x_{i,2}, \ldots, x_{i,M}), \\ \text{s.t.} \sum_{m=1}^{M} & P^e_{i,m} x_{i,m} \leq P^e_{i,m_i} q^b_{m_i}. \end{aligned} \tag{3.61}$$

The above generalization is very useful for dealing with the money good, since each agent can consider trading some goods, j, which he has no demand for, as long as its discount rate is low enough ($\beta_j \gg 0$). To continue, let us divide the commodity set for each agent i into three subsets, namely the production set, the consumption set, and others. The last refers to the goods that agent i neither produces nor has demand for. These three sets are denoted by $\mathbf{m^y}_i$, $\mathbf{m^c}_i$, and $\mathbf{m^h}_i$ (others). Based on the specialization assumption, $\mathbf{m^y}_i = \{m_i\}$.

The idea of money originates from the possibility that agent i would like to trade with agent j for goods outside of their production and consumption; i.e., the goods are not necessarily in the form of a pair (m_i, m_j). For example, if agent j is not interested in the commodity m_i offered by agent i, agent i may still consider any other commodities in $\mathbf{m^c}_j$ which agent i can offer, i.e., anything belonging to the set

$$\mathbf{m}_{i,j} = \mathbf{m^h}_i \cap \mathbf{m^c}_j.$$

If this possibility exist, agent i may like to trade with the agents who can supply any goods belonging to $\mathbf{m^h}_i$, and he can certainly trade with the above agent j using any of his inventory in $\mathbf{m}_{i,j}$ if his inventory of m_i has run out. Hence, what should really interest agent i is

$$\mathbf{H}_i = \bigcup_{\{j:m_j \in \mathbf{m^c}_i\}} \mathbf{m}_{i,j} = \bigcup_{\{j:m_j \in \mathbf{m^c}_i\}} (\mathbf{m^h}_i \cap \mathbf{m^c}_j). \tag{3.62}$$

With this possible width of trading, the budget constraint in the optimization problem in (3.61) needs to be further adjusted to

$$\max_{x_{i,1},x_{i,2},\ldots,x_{i,M}} U_i(x_{i,1}, x_{i,2}, \ldots, x_{i,M}),$$

$$\text{s.t.} \sum_{m=1}^{M} P^e_{i,m} x_{i,m} \leq P^e_{i,m_i} q^b_{m_i} + \sum_{h_i \in \mathbf{H}_i} P^e_{i,h_i} q^b_{h_i}, \tag{3.63}$$

The new extended budget constraint (3.63) simply indicates that agent i, in addition to what he produced but failed to sell, may also stockpile "speculative" commodities for which he has no intrinsic demand but which can facilitate his acquisition of consumption goods in future trades.

The reasoning of this kind of speculative trading can be iterated further, and, in the end, the two commodity sets $\mathbf{m^c}_i$ and $\mathbf{m^h}_i$ may overlap, which means that the consumption good may also become speculative. To manage the complexity, the priority of the pairs of traded commodities needs to be set first, and it may also entail more modeling details of speculative activities of agents, very much in the research direction of Marimon, McGrattan, and Sargent (1990) and Duffy (2001).

Learning

As we have mentioned earlier, one distinguishing feature of the Gintis model is that the agents are adaptive. In this part, Gintis combined individual learning with social learning. As we shall see elsewhere in the book, these two learning schemes and their hybridization are considered crucial in the emergent dynamics. Gintis found that neither individual learning nor social learning alone can bring the economy close enough toward the competitive equilibrium; the prices, quantities (excess demand or supply), and efficiency behave poorly as compared to those in the baseline model when both types of learning are included. The situation can be worse in the case where imitation is absent. Qualitatively speaking, when only one type of learning is present, the only thing that remains unchanged is the emergence of quasi-public prices, although their cohesiveness becomes much weaker.

Summary

Gintis's series of work inspires a new framework with which to study Walrasian dynamics. From what has been presented, the various work on modeling and simulating the Walrasian dynamics can be viewed as a relay for the general equilibrium analysis thriving between the 1950s and the 1970s and clothes the general equilibrium analysis with a strong flavor of realism.

3.3 Auctions

Highly distributed bilateral trade serves as a good start for understanding the nature of primitive and less organized markets. However, the market, as an institution, evolves with time: organized markets emerge from the original primitive markets or from nowhere (McMillan, 2002; Roth, 2002; Mirowski, 2007; Backhouse, 2010). As a result, the determination of prices varies among different exchange institutions; in addition to bilateral *bargaining* and *posted price*, there is *auction* (see Figure 3.3).

Auction, bargaining, and posted price are considered to be the three major exchange institutions. Their coexistence is an obvious fact; however, their existences are the result of evolution to facilitate exchanges. Will one exchange institution replace the others in the future, and why? These questions have been addressed theoretically. For example, from the viewpoint of evolutionary stability, Lu and McAfee (1996) and Kultti (1999) addressed the evolutionary dominance of the three exchange institutions and found that bargaining is inferior to auction and posted price, while the latter two are equivalent.

Since its inception, agent-based modeling has been applied to study not only decentralized bilateral trading but also auctions, as well as addressing the fundamental questions therein. The agent-based modeling of auctions is a unique

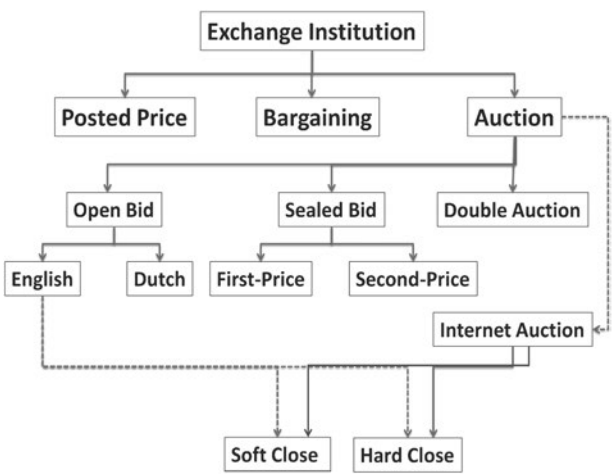

Figure 3.3 Varieties of exchange institution.

research area, because auction, as a part of human civilization, has existed for thousands of years, not to mention its recent invigoration and innovation thanks to Internet auctions. Later theoretical developments, such as the revenue equivalence theorem, revenue isomorphism, and Nash equilibrium bidding, and the observed real bidding behaviors, such as winner's curse and bid shading, enlarge the depth and width of the auction institution. The growing bodies of practice, theory, field work, and experimental studies provide a rich set of data and ideas (theory and hypothesis) for agent-based models to explore. This scale of abundant materials distinguishes agent-based modeling of auctions from many others, such as agent-based modeling of the non-tâtonnement process (Section 3.2).

In fact, the parallel developments of auctions in practice, theory, and experimentation enable us to more easily see the two facets of agent-based models, i.e., the institutional aspect and the behavioral aspect. From the institutional aspect, the fundamental questions are: Why are some goods sold with posted prices, while others are auctioned?[22] In what form should the goods be auctioned, open or sealed (Milgrom and Weber, 1982; McAfee and McMillan, 1987; Kagel and Levin, 2002), first-price or second-price (Rothkopf, Teisber, and Kahn, 1990)? From the behavioral aspect, the issues are: Can bidders learn Nash equilibrium strategies (Harrison, 1989)? How do bidders learn under different auction formats? What are the underlying behavioral rules which drive agents to overbid (winner's curse) or underbid (bidder shading)? How is risk preference contributing to the bidding behavior? Why do some subjects remain "inexperienced" and hence constantly suffer from winner's curse? Do subjects with lower cognitive capability tend to be "inexperienced"? How does the size of bidders contribute to the observed bidding pattern under different bidding formats? It is also interesting to note that different learning tools can lead to different behaviors.

3.3.1 Basic structure

The basic structure of auction is as follows. There are N agents attending an auction. Each of the N agents (bidders) has a private value, say P_i $(i = 1, 2, \ldots, N)$, for the object being auctioned, and P_i is independently and uniformly sampled from an interval with a minimum of $P_{\min}$ and a maximum of $P_{\max}$. Alternatively, let P^* be the middle point of this interval, and let $\epsilon (= P^* - P_{\min} = P_{\max} - P^*)$ be the deviation; then the interval can be rewritten as $[P^* - \epsilon, P^* + \epsilon]$. For this private-value auction, if agent i with a bid of b_i wins the bid, then his payoff, π_i, should be the difference between P_i and what he pays, i.e., the cost of acquiring the object, c_i. c_i, in turn, depends on the auction format. For a *first-price sealed-bid auction*, $c_i = b_i$, hence the payoff $\pi_i = P_i - c_i = P_i - b_i$. Kagel, Harstad, and Levin (1987) considered auctions under two kinds of information conditions: Under the first condition, all agents know that all P_i are uniformly drawn from $[P^* - \epsilon, P^* + \epsilon]$; under the second condition, the agents know only the range of the interval, i.e., 2ϵ or simply ϵ, but not P^*. An auction under the first condition is known as an *independent private-value* (IPV) auction; the other, an *affiliated private-value* (APV) auction.

An alternative to the private-value auction is the *common-value* (CV) auction. In a common-value auction, the value of the subject being auctioned is identical for all agents, such as the resale value of an object in an open market. However, agents do not know this common value; instead, they have to estimate the true value on their own. To see how similar the common-value auction and the private-value auction are, we can use the same notations as above, but with different meanings. Now let P^* be the common value of the object, and let P_i be agent i's estimate of P^*. As above, P_i is uniformly sampled from $[P^* - \epsilon, P^* + \epsilon]$. If agent i wins the bid by bidding b_i, under the first-price auction, his payoff $\pi_i = P^* - b_i$.

After auctions were formalized using the framework of game theory, the bidding behavior was studied in a game-theoretic manner. One of the most important developments is to characterize the theoretical bidding behavior as the *risk neutral Nash equilibrium* (RNNE) bidding function. The RNNE bidding functions corresponding to the above three auctions (independent, affiliated, common) are as follows:[23]

$$b_i(P_i) = \begin{cases} \frac{N-1}{N} P_i + \frac{1}{N}(P^* - \epsilon) & \text{if IPV,} \\ P_i - \frac{N}{2}\epsilon & \text{if APV,} \\ P_i - \epsilon & \text{if CV.} \end{cases} \tag{3.64}$$

However, this theoretical bidding function can only serve as a benchmark, since they are not well supported by human-subject experiments. Among many documented results, the most famous anomaly may be the *winner's curse*, indicating that winners tend to overbid substantially to incur losses. Attempts, therefore, have to be made to understand various bidders' "errors" and to develop more realistic human bidding behavior. Hence, the inquiry on whether we can use agent-based simulation to shed light on the human-subject experimental results and then evaluate various auction designs becomes a drive for the agent-based modeling of auctions. In the following, we shall review some of the works in this direction.

3.3.2 Learning Nash bidding strategy

The agent-based research on auctions started from Andreoni and Miller (1995), just a few years after the agent-based models were applied to the decentralized non-tâtonnement process (Albin and Foley, 1992). In this initial stage, Andreoni and Miller (1995) addressed the issue of whether RNNE bidding strategies can possibly be acquired by artificial adaptive agents. They studied the RNNE bidding strategy for both first-price and second-price auctions applied in the three classes of auction, namely independent private-value auction, affiliated private-value auction, and common-value auction.

They considered a population of 40 agents. These 40 agents were divided into either 4-bidder groups or 8-bidder groups. Auction games were then repeatedly run within these groups, as done in the human experimental auctions. In each iteration, P^*, as specified in Section 3.3.1, was randomly drawn from a given interval. This is to make sure that the agents can develop not just a *single bid* to fit a specific P^*, but a *bidding function* over different P^*s. The agents were then examined to see whether they are able to learn the RNNE bidding function as given in Equation (3.64).

In light of Equation (3.64), Andreoni and Miller (1995) assumed that each agent follows a two-parameter bidding function as follows:

$$b_i(P_i) = \begin{cases} \beta_{i,1} P_i + \beta_{i,2}\epsilon & \text{if APV or CV,} \\ \beta_{i,1} P_i + \beta_{i,2}(P^* - \epsilon) & \text{if IPV.} \end{cases} \tag{3.65}$$

Whether agents are able to learn the RNNE bidding function is equivalent to seeing whether β_1 and β_2 are equal to the corresponding coefficients in Equation (3.64).

The learning behaviors of the agents are modeled to reflect a kind of social learning by employing the single-population genetic algorithm which uses the three genetic operators: reproduction, crossover, and mutation.[24] The population size of the genetic algorithm (GA) is 40. Each individual (chromosome) is a binary string which encodes the two parameters of the parametric bidding function, $\beta_{i,1}$ and $\beta_{i,2}$. In a one-to-one mapping, each individual bidder corresponds to one of these 40 bidding functions. The fitness (performance) of each bidder (bidding function) is based on the profits as a result of a number of auctions (not a single one).

One advantage of the above representation is that all three types of auction share the same search space, i.e., a two-dimensional space (two parameters) with the same range. This commonality can, therefore, exclude the possibility of the search space being the cause of any differences in the capability of learning the three Nash bidding functions. "This means that any difference in the ability of the algorithm to find Nash equilibria will not be attributable to the dimensions of the search space, but rather to the informational differences in the auctions" (Andreoni and Miller, 1995, p. 47).

With this setting, they were able to use agent-based simulation to shed some light on the theoretical and experimental auctions in two regards, namely *bidding errors* and the *equivalence theorem*. As to the bidding errors, they found that learning the RNNE bidding function is difficult, even though it is easier to learn in the affiliated private-value auction in comparison with the other two auctions. The difficulty of learning the bidding function under the independent private-value auction can be attributed to, if not the search space, the correlation between the two received signals, i.e., P_i and $P^* - \epsilon$, "the highly correlated information in independent-values auctions, rather than the flat payoff space, is complicating adaptive learning" (Andreoni and Miller, 1995, p. 55). They also analyzed why the Nash bidding function is difficult to learn under the common-value auction and attributed it to the event chance.

The most significant implication of the bidding errors may be the breakdown of the revenue equivalence theorem. They found that, under the independent private-value auction, the revenue from the first-price auction is higher than the revenue from the second-price auction, which is consistent with the less frequent adoption of the second-price auction.

3.3.3 Bidder heterogeneity

Andreoni and Miller's agent-based auction model is built upon *social learning*. Under social learning, there tends to be a more homogenous population of bidders;[25] therefore, it would be difficult to fit the results with a population of heterogeneous bidders, which may be encountered in experimental auctions. In this regard, Noe, Rebello, and Wang (2012) proposed an agent-based model built upon *individual learning* to capture bidder heterogeneity. This individual learning is represented by augmented multi-population genetic algorithms (Arifovic, 1994; Vriend, 2001).[26]

In their multi-population GA, each agent maintains a fixed-size strategy pool with a size of 80 chromosomes. Each of the chromosomes represents a number in the [0, 1] interval with an increment of 1/500 from 0 to 1. These 80 strategies (80 values of bids) have an equal chance of being activated to represent the agent to bid against other agents in each auction (see Figure 3.4, which shows the kind of common-value auction detailed below.) After a number of times (e.g., 100 times, i.e., 100 auctions), all the pools will be updated based on the performance of their chromosomes over these 100 trials. The evaluation of each chromosome (strategy, bid), regardless of being sampled or not, is done in a counterfactual manner by assuming the same history of the last 100 auctions, i.e., by assuming that every other opponent would make the same sequence of bids again.

They then placed these artificial agents in the context of a common-value auction and focused on the common-value auction only (no independent private-value auction or affiliated-value auction). They considered two different designs. In the

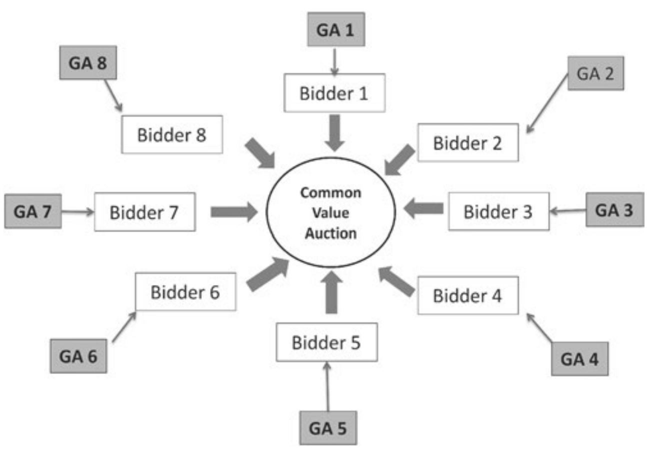

Figure 3.4 Multi-population genetic algorithms in the common-value auction: an illustration with eight bidders.

first design, the value of the item is fixed; i.e. P^* is a constant. To be specific, $P^* = 0.5$ in their simulation, but bidders only know the range of this value, which is between 0 and 1. In the second design, P^* is uncertain and is uniformly drawn from the same [0, 1] interval. The agents may know the range of this randomly drawn value, but not the distribution.[27] The second design is purported to evaluate the impact of uncertainty on bidding.

Using the results from their augmented multi-population GA, they distinguished the agents into two fundamental types: those who are well trained and those who are not, or, alternatively, those who follow the pure Nash strategy and those who adopt mixed strategies.[28] They then provided an explanation about what may cause the emergence of these two groups of agents, and how the distribution of these two groups is affected by the auction format. With this analysis, GA is not just a tool for simulating learning and evolution, but more importantly, a tool for helping us gain understanding of the relations between institutions and learning (or how institutions may affect learning).

The well-trained or trained agents are characterized by a converged Nash homogeneous population; i.e., all their 80 chromosomes eventually become identical and converge to the Nash equilibrium (or very close to Nash), b^*:

$$b^* = \begin{cases} P^* - \Delta & \text{if the first-price auction,} \\ P^*, & \text{if the second-price auction,} \end{cases} \tag{3.66}$$

where Δ is a tick of the price, which in their setting is 1/500. In this case, no matter which chromosome is sampled from the population, it is always the Nash (b^*); hence, the agent almost surely bids the pure Nash equilibrium. The not-well-trained agents are the complement to the trained agents; in particular, their 80 bidding strategies are uniformly distributed over a specific interval, say [0, 0.5]; hence, in each auction, they may bid randomly in the interval, like zero-intelligence agents (Gode and Sunder, 1993).[29]

The first-price auction and the second-price auction result in very different distributions of the two types of agents, trained and untrained (zero-intelligence). The first-price auction comes up with a great majority of trained agents (95 percent and higher), whereas the second-price auction comes up with a great majority of untrained agents (also around 95 percent). The explanation for the emergence of the different distributions of these two groups under the different auction formats is known as the "flat maximum" (Harrison, 1989), which refers to a phenomenon in which almost all of the chromosomes of a population have the same payoff and the same fitness. This phenomenon applies well to auctions, because losers, no matter how much they bid, will receive zero payoffs. When the fitness landscape becomes flat, learning with feedback becomes impossible and agents act as if they were trapped in a swamp.

Nonetheless, the indicated flatness is not exogenously given but endogenously emerges. In the case of the first-price auction, the emergence of a majority of trained agents, who bid Nash, exacerbates the flat maximum of the fitness landscape for other agents, who happened to learn slowly and now find it even more difficult to learn. In the first-price auction, overbidding only incurs losses and thus will be wiped out in the long run. However, in the second-price auction, overbidding will not be wiped out, as long as the second highest price is less than P^*. Therefore, the emergence of exceptionally few agents who bid aggressively (overbid) poses a flat maximum for other agents who did not learn to bid aggressively, have even more difficulty learning in any direction, and hence underbid randomly.

Uncertainty

When the value of an item being auctioned becomes uncertain, the degree of bidder heterogeneity is increased in both the first-price and the second-price auctions. The mesoscopic structures of the two types of agents are dramatically altered; now a convergent population is less likely to emerge even in the first-price auction. Although agents can still acquire a pure strategy, they, apart from a very few, tend to underbid in a substantial way. While uncertainty tends to lower the average of bids, it also increases their diversity. Hence the auction ends up with an increase in the revenue when the number of bidders is reasonablly large.[30] The second-price auction can produce more revenue than the first-price auction; however, since the revenue from the second-price auction is more volatile than that from the first-price auction, sellers' preference for the first-price auction may remain unchanged.

Social learning versus individual learning

When a sealed auction is run under the social learning scheme, we almost always end up with a highly homogeneous population, i.e., all 80 bidders bid the same value, while not necessarily Nash. In other words, bidder heterogeneity disappears completely. Maybe the most dramatic change happens in the second-price auction where agents to coordinate themselves to learn the Nash equilibrium.

3.3.4 English auctions

While most agent-based auction markets deal with sealed-bid auctions, one of the most popularly used auctions in the real world is the English auction. For example, eBay, Amazon, and Yahoo all employ English auction schemes for their Internet auctions. English auctions receive relatively less attention in the agent-based literature. Duffy and Unver (2008) is one of the few, and this article provides not only an agent-based model of English auctions but also an explanation for last-minute bidding (*sniping*). Hence, their proposed agent-based model is not just a model of English auctions, but also a model adapting two frequently used formats in Internet auctions, namely, the *hard-close format*, adopted by eBay, and the *soft-close format* (automatic extension), adopted by Amazon.[31]

Bidding in time

In terms of modeling, one difference between sealed bids and open bids is how they proceed: the former is *discrete* in time, while the latter is *continuous* in time. The discrete-time models do not have to address timing (when to bid) as part of the bidding strategies, but the continuous-time models in general do have to do so. Therefore, in addition to how much to bid, the continuous-time modeling has to deal with when to bid, which makes the bidding strategies even more sophisticated.

To deal with time, there are several possibilities. One is to augment the strategy with a waiting-time distribution function (such as an exponential distribution) and the waiting-time parameter can be fixed or state dependent. Alternatively, event time can be used to discretize time into many tiny intervals (Farmer, Patelli, and Zovko, 2005), bids can be submitted in any of these tiny intervals, and the decision about when to submit is part of the bidding strategies.

The latter approach, i.e., the time discretization, is the modeling strategy applied by Duffy and Unver (2008). Their approach to dealing with time can be conceived as extending a single-run sealed-bid auction to a multiple-round version. In the original version, after all bids are submitted, the bidder with the highest bid is the winner, and the auction is over. In the extended version, the information received, such as the current bid,[32] will be announced; then bidders, based on the information received and the trading strategies employed, can resubmit, or simply pass, in the next run, until time is up or the auction comes to the final round.

Bidding strategy representation

However, this discrete-time approximation does not free us from the issue of the timing decision: Should one bid early or late? This issue is particularly motivated by the famous *sniping* phenomenon observed in Internet auctions. Snipers, like the background players in the Santa Fe Institute double auction tournament game (Section 9.2), do not reveal their information at the initial time, but wait until the last moment to jump in. Hence, waiting and not releasing private information so as to make a successful bid at the critical moment is an essential part of the strategy and thus cannot be left outside the strategy modeling.

Duffy and Unver (2008) tackle this issue by combining when to bid and how much to bid into one single strategy; hence, the given bidding strategy tells bidders when and how much to bid. So, taking the eight-interval discretization ($T = 8$) as an example, an illustration of the bidding schedule can be depicted as (3.67):

$$\left[\begin{array}{c|c|c|c|c|c|c|c|c} t & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ b_{i,t} & \alpha_{i,1} & \alpha_{i,2} & \alpha_{i,3} & \alpha_{i,4} & \alpha_{i,5} & \alpha_{i,6} & \alpha_{i,7} & \alpha_{i,8} \end{array}\right], \tag{3.67}$$

where $b_{i,t} = \alpha_{i,t} P_i$ and $\alpha_{i,t} \in [0, 1]$. Here, α is a discount of bidder i's private value of the item, P_i, and α is between zero and one. Zero means that bidder i does not submit a bid in that moment, while one means that the full value has been bid.

Representation (3.67) assumes that bidder i follows an offline strategy; what he bids does not depend on the information released so far, such as the current bid. This representation is easier to code but less flexible and requires a higher cognitive loading for the bidder, since the bidder cannot have a spontaneous reaction to the market. Alternatively, one can set bids generally as a function of time and the information released so far:

$$b_{i,t} = b_i(T, Z_t \mid t), \tag{3.68}$$

where Z_t is the information observed up to the time t (the tth interval, out of a total of T intervals). This representation is more flexible because it allows bidders to react to the current bid, but may be more demanding to code.

Finite state automata

Duffy and Unver took the offline approach rather than the online approach.[33] In fact, what they did is a compromise between the online flexibility and the offline simplicity. Therefore, even though timely information about the current round may be ignored due to the offline constraint (3.67), information about the previous round (histories) may still be taken into account. However, the only information they took into account was whether the rival bidders in the previous auction made a late submission (submitting in the final periods, including the extension periods). This information is given as an external input in their augmented bidding model built upon a finite state machine.[34]

Sniping and revenue equivalence

As in earlier studies, the learning behavior of the agents was driven by the genetic algorithm. In this case, a single-population genetic algorithm (social learning) is applied to evolve a population of 30 bidding strategies. With this setup, they studied a rather small-scale auction from two bidders up to five bidders. The basic result is that, while the hard-close auction and the soft-close auction can differ in late bidding frequency (sniping frequency) and revenue, these differences become insignificant when the number of bidders increases to a scale of four or five. Hence, the difference between these two institutional designs can be trivial when a large number of bidders is involved. However, when the number of bidders is as small as two or three, the sniping phenomenon is more frequently seen in the hard-close auction than in the soft-close auction. The above late-bidding pattern may be affected by the distribution of independent values. Generally, so long as the distribution is not too centered, it is very likely to observe the difference in late bidding between hard close and soft close.

3.3.5 Reserve prices

In addition to letting buyers determine the final price, sellers can participate in auctions in a more active way. A typical way to do so is to determine a *reserve* *price*, below which the seller is not required to sell. The role of the reserve price can be understood as follows:

> Should reserve prices be used, setting a minimum price below which the item will not sell? eBay leaves this up to the seller. Theory says a reserve price is in the seller's interest when the bidding competition is weak. *If set at the right level*, it can drive the price up higher than the competition would.
>
> (McMillan, 2002, p. 78; emphasis added)

The key issue, of course, is how to set the reserve price right. Under the given game-theoretic framework of auctions, obtaining the optimal reserve price can be analytically intractable. Yet, the computational approach using agent-based simulation provides a viable alternative, and Boyer, Brorsen, and Zhang (2014) was the first agent-based model to address this issue.

Their analysis was, in fact, much broader than just determining the right level of the reserve price, if used; they tried to understand, from the revenue-maximization viewpoint, when an auction with a reserve price is preferred and when an alternative, such as posted-price selling, is preferred. In a way, their analysis extends the theoretic analysis of the institutional competition between the auction market and the posted-price market (Wang, 1993, 1998) to the common-value auction (versus posted-price selling) by incorporating the reserve price.

Like Duffy and Unver (2008), Boyer, Brorsen, and Zhang (2014) examined an auction market with a rather small number of bidders, say only two or three. This small-scale design can be justified mainly because the differences among many auction designs may become insignificant when there is high competition with a large number of bidders. Therefore, the issue can be nontrivial only when one has a small number of bidders.

As in the general setting of the common-value auction (Section 3.3.1), let P^* be the common value of the item being auctioned and P_i be agent i's estimate of P^*. P_i is uniformly sampled from the interval $[P^* - \epsilon, P^* + \epsilon]$. Alternatively, we can write

$$P_i = P^* + \mu_i, \tag{3.69}$$

where $\mu_i \sim [-\epsilon, \epsilon]$. Generally, u_i is assumed to be independent of u_j for $i \neq j$; denote this relation by $u_i \perp u_j$. Given P_i, the bidding function used by agent i is parameterized by strategy variable x_i, where $x_i \in [-\underline{\theta}_1, \overline{\theta}_1]$, and the parameters $-\underline{\theta}_1$ and $\overline{\theta}_1$ define the search space of each agent i. It is further assumed that the space is symmetric, by setting $\underline{\theta}_1 = \overline{\theta}_1 = \theta_1$. Altogether, this bidding function can be written as follows:

$$\begin{aligned} b_i(P_i) &= P_i - x_i \\ &= (P^* + \mu_i) - x_i, \end{aligned} \tag{3.70}$$

where $\mu_i \sim U[-\epsilon, \epsilon]$, $\mu_i \perp \mu_j (i \neq j)$, $x_i \in [-\theta_1, \theta_1]$.

However, unlike the sellers in the common-value auctions previously discussed, the seller in Boyer, Brorsen, and Zhang (2014) also plays a role by setting either his reserve price or posted price. Hence, a description of the seller behavior needs to be included. Like the common value P^*, the item to be sold is assumed to have a *salvage value*, denoted by C^*; hence, if the transaction fails, then the seller shall have at least this salvage value as his revenue. However, as the buyer is not certain of P^*, the seller is also not certain of C^*. This uncertainty can be characterized as

$$C = C^* + \nu, \tag{3.71}$$

where $\nu \sim U[-\eta, \eta]$. Furthermore, ν is assumed to be independent of μ_i, $\forall i$. The reserve price function or the posted price function is then characterized by a strategy variable y, where $y \in [\underline{\theta}_2, \overline{\theta}_2]$. By assuming symmetry, $\underline{\theta}_2 = \overline{\theta}_2 = \theta_2$. In this given search space, the seller needs to figure out the right reserve price or the posted price. Putting them together, the price function becomes

$$\begin{aligned} s(C^*) &= C - y \\ &= (C^* + \nu) - y, \end{aligned} \tag{3.72}$$

where $\nu \sim U[-\eta, \eta]$, $\nu \perp \mu_i (\forall i)$, $y \in [-\theta_2, \theta_2]$.

Notice that although Equation (3.72) can be both the reserve price and the posted price, the generated revenues are different. When it is used as the reserve price, the seller will have s as his a minimum revenue, since $\exists i$ s.t. $b_i > s$. On the other hand, when it is used as the posted price, the seller will just have s as his revenue if one buyer takes his offer. This difference does not imply that the reserve price will generate higher revenue for the seller, since the price determined under these two different trading institutions may be different, which also causes the probability of achieving a successful transaction and hence the expected revenue to be different.

Buyers with Equation (3.70) and the seller with Equation (3.72) are then placed in the game-like situation. Each of them attempts to react to the others' decisions on either b_j (x_j) or s (y), while searching for their maximum expected revenue. This process can be understood as an evolving tuple: $(b_1(t), b_2(t), s(t))$, or, equivalently, $(x_1(t), x_2(t), y(t))$. This coevolutionary game or interaction of agents can be modeled with many possible algorithms, such as genetic algorithms, as frequently seen in earlier agent-based auction models. Nevertheless, Boyer, Brorsen, and Zhang (2014) tried something different, known as *particle swarm optimization* (PSO).

Like a genetic algorithm, PSO is a population-based learning (search) algorithm. In the context of Boyer, Brorsen, and Zhang (2014), each individual buyer maintains a set of strategies, $\{x_{i,k}(t)\}_{k=1}^{K}$ $(i = 1, \ldots, N)$, as well as the seller, $\{y_k(t)\}_{k=1}^{K}$. These strategies are paired by k (one can imagine that there are K parallel auction markets running simultaneously). The agent (buyer or seller) then reviews and revises these strategies based on the performance of each individual strategy, allowing them to learn from each other through numerical crossover and mutation. If the learning is restricted in the same pool maintained by the agent, then each agent may learn only from his own experience and not from others. This individual-learning described above is structurally not different from Figure 3.4, except that both the buyers and the seller are engaged. In sum, Boyer and Brorsen applied a multi-population PSO to the learning behavior of the buyers and the seller, and we shall briefly summarize what they found.

The most basic finding is that there is no absolute dominance of one exchange institution over the other. As far as revenue is concerned, both the auction market with a reserve price and the posted-price market can be preferable, depending on the market uncertainty presented to the buyers and the seller. Basically, if there is no uncertainty ($\eta = 0$), i.e., if the seller knows the exact salvage value (C^*) of the auctioned item, then posted-price selling is more favorable than using a reserve price in an auction market. However, when the degree of uncertainty increases up to a degree that $\eta \approx \epsilon$, the difference between the auction market and the post-price market becomes small or insignificant. When the degree of uncertainty further increases ($\eta \gg \epsilon$), the use of reserve price can dominate the use of posted price.

Qualitatively, the addition of one buyer to the market, which makes the competition keener, leads to the same result. Hence, this finding in spirit is very similar to the earlier findings by Wang (1993), in the case of the private-value auction, and by Wang (1998), in the case of the affiliated auction. However, Wang (1993, 1998) did not consider the reserve price. The essential feature of Boyer, Brorsen, and Zhang (2014) is the justification of the use of the reserve price in the common-value auction. Using the agent-based model, Boyer, Brorsen, and Zhang (2014) confirmed the familiar experience that setting the reserve price helps reduce the chance of bid shading and can enhance the welfare of the seller when the seller has perfect information about the salvage value. However, the gain from setting the reserve price diminishes with the seller's uncertainty about the salvage value.

3.4 Macroeconomics

Earlier, in Section 3.2, we saw that agent-based modeling can be used to reformulate the Walrasian general equilibrium model and fully bring in the decentralization processes composed of many heterogeneous agents. In Section 3.3, we also saw that the trading protocols and procurement procedures can be further distinguished based on the trading institutions. Now, if we further bring firms, banks, and central authorities into this new formulation and allow them to interact with each other in different markets governed by different trading institutions, we will move to a stage where a new formulation of macroeconomics—agent-based macroeconomics—begins. In this section, we will continue the discussion of market origin and see its manifestation in macroeconomic models.

3.4.1 Time for change?

During a visit to the London School of Economics in November, 2009, Queen Elizabeth II, whose personal fortune was estimated to have fallen £25 million in the credit crunch, asked why no one had predicted the credit crunch. In fact, Bezemer (2010) showed that most mainstream equilibrium models, such as the dynamic stochastic general equilibrium (DSGE) models, failed to predict the coming of this crunch, while some nonconventional models did. On July 20, 2010, a number of leading economists were invited to give a congressional testimony to the Committee on Science and Technology, US House of Representatives, to discuss what could be wrong with the then-dominant macroeconomic models, in particular, the DSGE model.

These economists were, Robert Solow (Solow, 2010), Sidney Winter (Winter, 2010), Scott Page (Page, 2010), David Colander (Colander, 2010), and V. V. Chari (Chari, 2010). Apart from the last one, who defended the DSGE model, all gave serious criticisms of the DSGE model. At that point, what was wrong with the current macroeconomic model was no longer simply an academic issue, it had become a public affair drawing extensive attention from the public and media. The September, 2009 issue of *Nature* had a special edition for the financial crisis, calling for a change in macroeconomic modeling (Buchanan, 2009). The July, 2010 issue of *The Economist* had a column addressing the failure of the conventional macroeconomic model and asking whether agent-based modeling could do it differently.[35]

Jean-Claude Trichet, the previous President of the European Central Bank (ECB), in his opening address delivered at the 2010 ECB Central Banking Conference, even said:

> First, we have to think about how to characterise the homo economicus at the heart of any model. The atomistic, optimising agents underlying existing models do not capture behaviour during a crisis period. We need to deal better with heterogeneity across agents and the interaction among those heterogeneous agents. We need to entertain alternative motivations for economic choices. Behavioural economics draws on psychology to explain decisions made in crisis circumstances. Agent-based modelling dispenses with the optimisation assumption and allows for more complex interactions between agents. Such approaches are worthy of our attention.
>
> (Trichet, 2010)

3.4.2 Change is already happening

Agent-based modeling, as an alternative to the current mainstream macroeconomic modeling, has already been ongoing for a number of years. However, this change did not come out of the blue; instead, it has been a long process of the cumulative use of agent-based modeling first in *market modeling* and gradually extending to macroeconomics. The natural extension of the agent-based modeling of the non-tâtonnement process developed agent-based modeling of macroeconomics as an alternative to the mainstream macroeconomic models, such as the DGSE model. The earlier studies from Albin and Foley (1992) all the way down to Tesfatsion (2006) can be read as preparatory steps to knowing how to operate

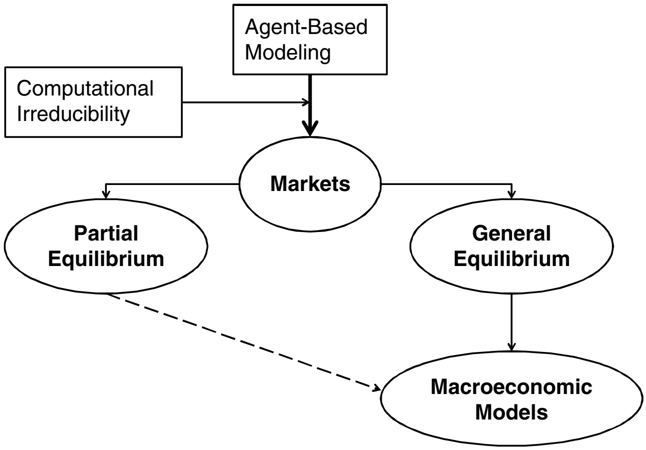

Figure 3.5 A route to agent-based modeling in economics: from markets to general equilibrium and macroeconomy.

Notes: The flows above indicate that the early application of agent-based modeling to markets branches into two directions. The one which works on the whole system of markets, known as general equilibrium analysis, has agent-based modeling of macroeconomics as the immediate descendant, whereas the one which focuses only on one specific market, also known as partial equilibrium analysis, has an indirect effect on it. The junction shows computational irreducibility as a theoretical underpinning of the agent-based modeling of markets, an idea to be further discussed in Section 4.3.1.

various elements required in the decentralized markets, upon which full-scale agent-based macroeconomic models also depend (see Figure 3.5). As one reads the literature, one can expect the familiar procedures or algorithms for searching, networking, bargaining, and matching behavior to reappear in the agent-based macroeconomic models.

3.4.3 A minimal model

The development of agent-based macroeconomic model also benefits from the developments of other agent-based economic models for overlapping generations, double auction markets, fish markets, oligopolistic competition, electricity markets (Bunn and Oliveria, 2001; Weidlich, 2008), financial markets, prediction markets, labor markets, spatial dynamics, networks, housing markets, and behavioral game theory (Some of these models will be reviewed in subsequent chapters.) Of course, an agent-based macroeconomic model is not a jumble of these individual dimensional models. In fact, it is not clear on how to integrate these "components" into a macroeconomic giant, nor its feasibility, nor its desirability. Perhaps the first step is to come up with a minimal model as a benchmark and then to include additional features when needed. Hence, the first research question placed in this line of research is: *What are the minimal elements?* or *What is the minimal model?*

Among many possible additions to the agent-based macroeconomic models, some quickly stand out to be the necessary elements for a minimal model. They are *decentralized markets* and *balance-sheet consistency*. Balance-sheet consistency, also termed *stock-flow consistency*, requires the *closedness* of a model in the sense that any change in an agent's balance sheet should be counterbalanced by changes of the opposite sign in other agents' balance sheets. These two elements are well expected, since they are simply inherited from the agent-based non-tâtonnement process. The former is related to institution and is to replace the Walrasian auctioneer; the latter is related to behavior and is to replace the assumption of *Homo economicus* or strong rationality. The balance sheet specifically does not require agents to have any form of rationality but satisfatory budget constraints; hence, it is the budget-constraint condition in the multi-agent setting.[36]

Of course, to make this model interesting or relevant in the macro sense, the model is expected to generate some aggregates related to the core issues of macroeconomics. Therefore, we also need *a minimal set of questions* requiring an interesting agent-based macroeconomic model to answer. The minimal set of questions should include those related to business cycles (depression and financial crisis), growth, and income distribution. To answer these questions, the minimal elements should include *a minimal set of markets* and *a minimal set of agents*. The markets should include goods markets, labor markets, and financial markets; the agents should include households, firms, banks, central bank, and government. This minimal model is shown in Figure 3.6.

Table 3.1 summarizes the presence of various agents and markets in 13 selected agent-based macroeconomic models. By convention, it is desirable to further distinguish firms into consumption-goods firms (C) and production-goods firms (K). Four out of the 13 models mentioned actually do so. Apart from this distinction,

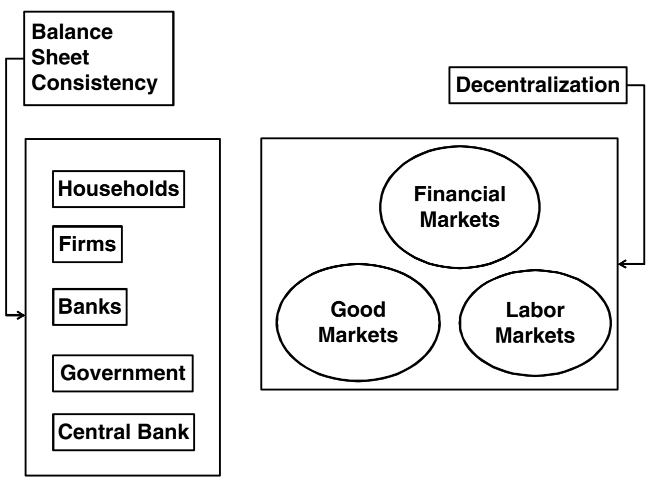

Figure 3.6 The minimal macroeconomic model.

Table 3.1 The agents in various agent-based macroeconomic models

Models	*Hs*	*Fs*		*U*	*Bs*	*CA*	
		C	*K*			*CB*	*G*
Wright (2005)	*Y*	N	N	N	N	N	N
Gintis (2007)	*Y*	**Y**	N	N	N	Y	Y
Russo *et al.* (2007)	*Y*	*Y*	N	N	N	N	Y
Bruun (2008)	*Y*	*Y*	*Y*	N	Y	N	N
Raberto, Teglio, and Cincotti (2008)	*Y*	Y	N	Y	N	Y	N
Chan and Steiglitz (2008)	*Y*	*Y*	*Y*	Y	Y	Y	N
Mandel *et al.* (2010)	*Y*	**Y**	N	N	N	Y	Y
Lengnick (2013)	*Y*	*Y*	N	N	N	N	N
Kinsella, Greiff, and Nell (2011)	*Y*	*Y*	N	N	*Y*	N	Y
Ashraf, Gershman, and Howitt (2011)	*Y*	N	N	N	*Y*	Y	Y
Delli Gatti *et al.* (2011)	*Y*	*Y*	N	N	*Y*	N	N
Cincotti, Raberto, and Teglio (2012)	*Y*	*Y*	Y	N	*Y*	Y	Y
Dosi *et al.* (2013)	Y	*Y*	*Y*	N	Y	Y	Y

Note: *Italic text* refers to the case where heterogeneous agents are modeled, rather than a single agent, a representative agent, or a single aggregate. **Bold** refers to the case where there are multiple sectors of firms, i.e., more than one industry among the firms.

Table 3.2 The markets in various agent-based macroeconomic models

Models	*Goods*		*Labor*	*Credit*	*Stock*
	C	*K*			
Wright (2005)	Y	N	*Y*	N	N
Gintis (2007)	*Y*	N	*Y*	N	N
Russo *et al.* (2007)	*Y*	N	*Y*	N	N
Bruun (2008)	*Y*	Y	*Y*	N	Y
Raberto, Teglio, and Cincotti (2008)	Y	N	Y	Y	Y
Chan and Steiglitz (2008)	Y	Y	Y	Y	N
Mandel *et al.* (2010)	*Y*	N	*Y*	N	N
Lengnick (2013)	*Y*	N	*Y*	N	N
Kinsella, Greiff, and Nell (2011)	Y	N	*Y*	N	N
Ashraf, Gershman, and Howitt (2011)	*Y*	N	*Y*	N	N
Delli Gatti *et al.* (2011)	*Y*	N	*Y*	*Y*	N
Cincotti, Raberto, and Teglio (2012)	*Y*	N	*Y*	*Y*	Y
Dosi *et al.* (2013)	Y	*Y*	Y	*Y*	N

Note: *Italic text* refers to the case where the market is operated through a decentralized searching and matching process. Some markets uniquely presented in a few models are not taken into account in this table, such as the deposit market (Chan and Steiglitz, 2008) and the firesale market (Ashraf, Gershman, and Howitt, 2011).

labor union (U) is added to two out of the 13 models. Hence, together, seven types of agents are identified in Table 3.1.

As a companion, Table 3.2 is a summary of the kinds of markets existing in the 13 models. A little surprise is that none of the 13 models includes all of the five markets, while they commonly have consumption goods markets and labor markets.

3.4.4 Decentralization and networks

By decentralization, what we mean is that both sides of the markets are emerging from the bottom, and there is no exogenously given aggregate demand or supply directly imposed on the market.[37] We certainly do not mean that all markets have to be decentralized; in fact, decentralization is a matter of degree, which varies among markets. For example, the labor market and the goods market, in principle, are less centralized and more localized than the financial market. Hence, while the financial market can be applied with the clearing-house mechanism, the labor market and the goods market are generally operated under the individually searching, bargaining, and matching process (Figure 3.7).

The difference in the degree of decentralization may call for different network topologies or different network formation processes (Figure 3.7). Of course, for a minimal model, one can always start with the *random network* with random meeting and matching process, but many agent-based macroeconomic models have already bypassed this minimal criterion. In some cases, a specific network topology taking into account the geographical factors has already been applied, such as the checkerboard model used by Bruun (2008). Some other studies have even advanced to the level that the network topology is not exogenously given, but endogenously determined. For example, in Mandel *et al.* (2010) and Lengnick (2013), the searching and matching process is based on a network topology which itself is constantly reviewed and revised. In this process, connecting, disconnecting, and reconnecting are the results of search behavior, because agents may only keep a finite number of links, e.g., one job and a few shops, in their personal business networks. Therefore, search means that all possible potential links have to compete with the incumbent links.

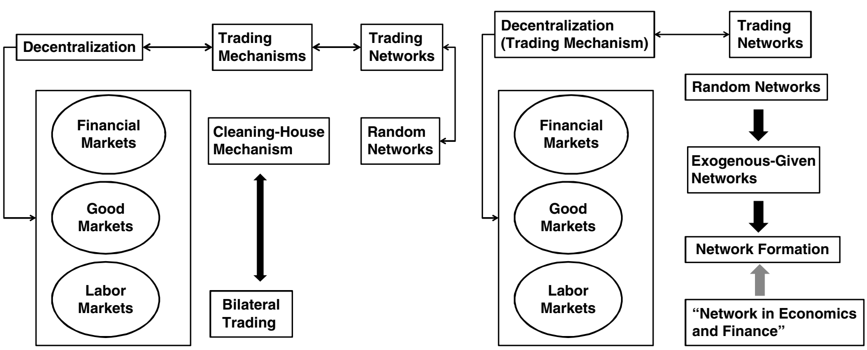

Figure 3.7 The institutional aspect of the minimal macroeconomics model.

Notes: The left panel shows the minimal macroeconomic model from the institutional aspect, including both the trading mechanism and the network topology. The right panel shows how the minimal macroeconomic model can be extended to include more deliberate network topologies.

Despite this more deliberate design, the networks appearing in the agent-based macroeconomic models are mainly used to drive the basic searching and matching processes. The use of networks in the labor market and the goods market is still distant from the recent research focusing on job network and trading network; specifically, the role of the social network in labor market research has not been fully incorporated in the current agent-based macroeconomic models. Likewise, while some agent-based macroeconomic models have already been able to generate financial contagion through endogenously evolving networks, their connections to the research focusing on interbank networks have not been established.[38] The use of networks in agent-based macroeconomics is still at a primitive stage.

3.4.5 Bounded rationality

For a minimal model, do we still need utility function, production function, and social welfare function (Figure 3.8)? In neoclassical macroeconomics, these are core elements. Without them, all optimizing behaviors lose grounds. What happens to the agent-based macroeconomics models when agents are generally assumed to be boundedly rational? As we shall see, in agent-based macroeconomic models, utility function plays a quite negligible role, while production function is still indispensable, even though there are some extreme cases where none of these functions are assumed (Kinsella, Greiff, and Nell, 2011).

In fact, many agent-based macroeconomic models do not have the utility function at all (see Table 3.3). Decisions can be made by heuristics without this function. On the other hand, the production function is the starting point for firms, because the production plan and the demand for inputs depend on its availability. Profits are ill-defined without it. Nonetheless, the production functions employed

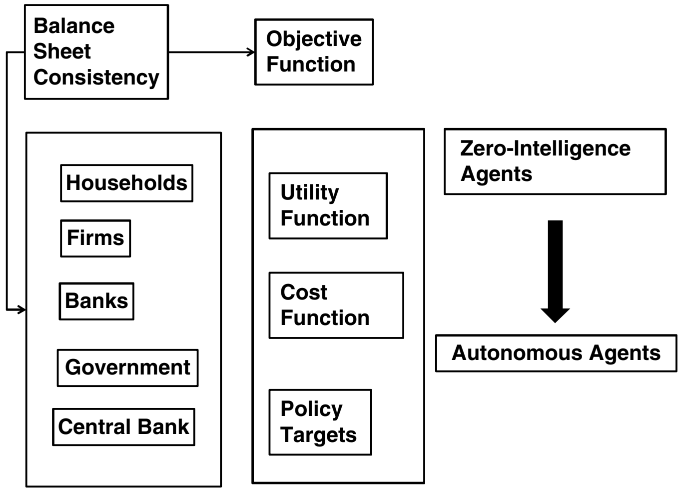

Figure 3.8 The behavioral aspect of the minimal macroeconomic model.

Table 3.3 Bounded rationality and designs of artificial agents

Models	*Households*		*Firms*			
	C	*S*	*P*	*Q*	*W*	*L*
Wright (2005)	0	0	—	—	0	0
Gintis (2007)	4	1	2	4	2	4
Russo *et al.* (2007)	1	1	2	2	2	2
Bruun (2008)	2	2	1	2	1	2
Raberto, Teglio, and Cincotti (2008)	2	2	4	4	1	4
Chan and Steiglitz (2008)	2	2	2	2	1	2
Mandel *et al.* (2010)	0	0	2	2	2	2
Lengnick (2013)	2	2	2	2	0	2
Kinsella, Greiff, and Nell (2011)	2	2	—	2	2	2
Ashraf, Gershman, and Howitt (2011)	2	2	2	2	2	2
Delli Gatti *et al.* (2011)	2	2	2	2	2	2
Cincotti, Raberto, and Teglio (2012)	2	2	3	3	2	4
Dosi *et al.* (2013)	—	—	2	2	—	—

Note: Under the household column, "0" denotes randomly behaving agents (zero-intelligence agents), "1" denotes full consumption and forced saving, "2" denotes the use of heuristics with no objective function explicitly involved, "3" denotes the use of heuristics explicitly guided by an objective function, and "4" denotes optimizing behavior. Under the firm column, we consider four aspects of firms' decisions: pricing (P), production (Q), wage (W), and employment (L). "0" denotes the cases where agents are behaving randomly; "1" denotes the cases where agents are decision-takers and the decision is made exogenously; "2" denotes the cases where agents follow some kind of heuristic to make decisions; "3" denotes the cases where agents' heuristics are specifically based on management sciences; "4" denotes the cases where agents are optimizing. When agents are pooled together as an aggregate and not modeled individually, the above classifications are not applicable; the sign "—" is used instead.

by most agent-based macroeconomic models are simple: they are either a simple linear function (particularly when only one input is taken into account) or the Leontief production function (when more than one input are taken into account). These simple forms of production function not only make the demand for inputs easier to figure out but also make the production technology easier to operate.

Table 3.3 provides a summary of the kinds of bounded-rational agents used in the literature. While this summary is based only on the 13 models indicated, the features should be enough to apply generally to other models. Basically, we can have different classes of agents, from the simple to the deliberate, depending on how their decisions are determined.

Households

Take the household as an example. In the simplest case (the class denoted by "0"), we have randomly behaving or zero-intelligence agents. For these agents, the consumption share and hence the saving rate are randomly determined.[39] As we move to the upper level (the class denoted by "2"), the consumption and saving decisions are made through different heuristics which are amenable to different considerations, such as consumption smoothness, precautionary saving, and social conformity. Notice that in this level, while consumers implicitly have an objective to pursue, for example, consumption smoothness, the objective function (utility function) is not explicitly made here. The class denoted by "4" represents the optimizing agents, whose consumption is derived from utility maximization under a given utility function.[40] In between, we have class "1" referring to the agents who intend to spend all their income (i.e., no saving), and class "3" referring to the agents who use heuristics to search for a higher value under a given objective function.[41]

Firms

The firms in most agent-based macroeconomic models are profit maximizing in the sense that they are, at least, adaptive in finding various profit-enhancing operational characteristics. Firms' operational characteristics include the decisions made on pricing, quantities, inventory, wages, employment, investment, financing, and research and development (R&D). Not all agent-based macroeconomic models have all of these characteristics, but most of them have the operational characteristics regarding price adjustment, wage adjustment, production, and employment behavior.

Pricing in most models is determined by a *mark-up rule*. The markup rate changes over time with economic conditions, such as market share, sales projections, general market prices, and profit. Wage in most models is also under a proportional incremental adjustment based on the economic conditions, such as vacancy rate. The vacancy-rate-based wage adjustment is the most popular in these models. In some cases where the posted-price mechanism is replaced by bilateral (employer–employee) bargaining, the bargaining power of both sides depends on the current state of the labor market and the idiosyncratic characters of employers or employees (Russo *et al.*, 2007; Kinsella, Greiff, and Nell, 2011), such as the duration of being unemployed. In some other cases where the wage is determined by the labor union (Raberto, Teglio, and Cincotti, 2008; Chan and Steiglitz, 2008) or is rigid (Bruun, 2008), the firms' decisions on wages are not part of their operational characteristics. Institutional arrangements, such as the minimum wage, are also considered by some models as part of their adaptive rule of wage adjustment (Mandel *et al.*, 2010; Delli Gatti *et al.*, 2011).

Production is frequently driven by sales projection and hence a targeted production. Given the targeted production level, the demand for various production factors are figured out, while in most models, labor turns out to be the only variable input considered. As to the relationship between production and employment, there are three possibilities. First, both decisions are individually made based on the agents' own heuristics (Gintis, 2007); second, the production decision is made first and then the employment decision is made accordingly through the production function (Russo *et al.*, 2007; Bruun, 2008; Chan and Steiglitz, 2008; Mandel *et al.*, 2010; Ashraf, Gershman, and Howitt, 2011; Delli Gatti *et al.*, 2011; Cincotti, Raberto, and Teglio, 2012); third, the other way round, the production is based on the a priori decision made on employment (Lengnick, 2013). Some studies do deal with other possible inputs, particularly capital, and the tradeoff between labor and capital (Gintis, 2007; Chan and Steiglitz, 2008; Mandel *et al.*, 2010; Kinsella, Greiff, and Nell, 2011; Ashraf, Gershman, and Howitt, 2011; Cincotti, Raberto, and Teglio, 2012; Dosi *et al.*, 2013).

Even though most firms follow some kind of heuristics to make their decisions and are classified as class "2" in Table 3.3, formal learning models are also applied in some of these models, such as replicator dynamics (Gintis, 2007), reinforcement learning (Chan and Steiglitz, 2008), and genetic algorithms (Mandel *et al.*, 2010). Hence, boundedly rational agents in these models range from the very primitive zero-intelligence agents to autonomous agents (Figure 3.8), which will be detailed in Part IV. Some of these heuristics are based on ideas borrowed from management science and hence their formulation can be far more deliberate than the usual and more intuitive class "2" (Cincotti, Raberto, and Teglio, 2012); class "3" is used to make a distinction for these models.

Finally, it is quite common that an agent-based macroeconomic model does not have all markets modeled in an agent-based fashion. For example, in Dosi *et al.* (2013), there is a labor market, but the market is completely formed in an aggregate manner so that one cannot see the wage-offer or contract-offer decision made by the firms, and neither do we see the formation of job-searching behavior or the reservation wage of the households. Wage in this model is a public price (index) as if it were determined by a central authority with a given formula.

3.4.6 Granularity

The minimal model above only provides a skeleton; it does not specify the granulation under the skeleton. This is probably the most difficult part of the model design. The granulation issues include the number of goods in the goods market (single-sector economy or multi-sector economy), the number of hierarchies in the supply chain (level two or more), the number of firms in each market (oligopolistic competition or monopoly), and the number of households. In addition to number or size, granularity also includes diversity, i.e., heterogeneity, such as heterogeneity in labor (productivity), households (preferences), and firms (scale and product differentiation).

Table 3.4 provides a summary of the scales of the 13 agent-based macroeconomic models, measured by the number of agents involved. This rough description simply gives readers a flavor of how "macro" the current agent-based macroeconomic models are. From what has been revealed in this table, the largest model can have about 5000 households, 250 firms, and 100 banks, in addition to the central authorities.[42] These numbers also set the maximum possible heterogeneity. Furthermore, up to this point, the hierarchies remain very basic—from households and firms to markets to the whole economy. It would be interesting to know whether one can possibly scale up these very humble models to some realistic degree, as in the inquiry made by LeBaron and Tesfatsion (2008):

> One of the greatest potential contributions that ACE could make to macroeconomic theory is permitting the constructive exploration of scale effects

Table 3.4 The numbers of various agents in the agent-based macroeconomic models

Models	*H*	*F*		*U*	*B*	*CA*	
		C	*K*			*CB*	*G*
Wright (2005)	1000	0	0	0	0	0	0
Gintis (2007)	5000	100–140	0	0	0	1	1
Russo *et al.* (2007)	500	100	0	0	0	0	1
Bruun (2008)	1800	160	40	0	1	0	0
Raberto, Teglio, and Cincotti (2008)	1000	1	0	1	0	1	0
Chan and Steiglitz (2008)	1100	10	5	1	5	1	0
Mandel *et al.* (2010)	—	—	0	0	0	1	1
Lengnick (2013)	1000	100	0	0	0	0	0
Kinsella, Greiff, and Nell (2011)	500	150	0	0	75	0	1
Ashraf, Gershman, and Howitt (2011)	2400	50	0	0	5	1	1
Delli Gatti *et al.* (2011)	500	100	0	0	10	0	0
Cincotti, Raberto, and Teglio (2012)	2000	20	1	0	3	1	1
Dosi *et al.* (2013)	—	200	50	0	1	1	1

Note: From the left to the right are the number of household agents (under the column "H"), consumption-goods firms ("F-C"), capital-goods firms ("F-K"), unions ("U"), banks ("B"), central bank ("CA-CB") and government ("CA-G"). Dosi *et al.* (2013) did not treat households individually—the households behave as an aggregate entity. The number of agents in Mandel *et al.* (2010) is not available from the paper.

> without the external imposition of artificial coordination devices. What does it matter if an economy has 10,000 versus 300 million participants? What macroeconomic purposes are served by small-scale models, and which require a scale closer to empirical reality? Do macroeconomies exhibit important regularities that simply cannot be generated using small scale models?
>
> (LeBaron and Tesfatsion, 2008, p. 248)

While they are quite optimistic on this possibility, as they said, "[T]he question is not whether this can be done, but whether it should be done, and for what purposes" (LeBaron and Tesfatsion, 2008, p. 248). Real large-scale macroeconomic models are yet to be seen and explored.

3.4.7 Emergent macroeconomics

The minimal model can generate emergent properties, which manifests the strengths and the advantages of agent-based macroeconomic models over equation-based macroeconomic models. Agent-based macroeconomic models can generate the dynamic behavior of almost all macroeconomic variables conventionally studied by the equation-based models. These variables are normally related to economic fluctuation and growth, such as GDP, consumption, investment, wage, unemployment rate, interest rate, and inflation rate. Therefore, in this regard, the agent-based macroeconomic models have gradually made themselves into alternatives to the conventional equation-based macroeconomic models.[43] However, agent-based macroeconomic models can do more than that. The additional strengths and advantages of agent-based macroeconomic models come from their displacement of the representative agents and the Walrasian auctioneer. These two displacements lead to a number of features that distinguish agent-based macroeconomic models.

First, households and firms in agent-based macroeconomic models are no longer simplified into a single representative agent; instead, they are each individually modeled. Their complex interaction processes may further contribute to their evolving heterogeneities. The latter enables us to observe the kinds of heterogeneities that interest economists, such as income distribution, wealth distribution, firm size distribution, firms' growth rate distribution, price distribution, profit-rate distribution, wage distribution, and the indexes used to characterize these distributions, including the Herfindahl–Hirschman index and the Gini index (Wright, 2005; Russo *et al.*, 2007; Bruun, 2008; Lengnick, 2013; Kinsella, Greiff, and Nell, 2011; Delli Gatti *et al.*, 2011). The three aspects of macroeconomics, growth, fluctuation, and distribution, can now be studied in one unified framework. Over the last half of the twentieth century, great efforts have been made to model the phenomenon of growth with cycles; however, incorporating the distribution behavior of households and firms into the growth-with-cycles model is definitely a challenge for the equation-based macroeconomic models. Hence, the model which can accommodate this further integration is definitely a milestone for macroeconomics.

Second, the inclusion of the labor market in a decentralized manner enables us to further see the details of labor markets, such as the dynamics (time series) of vacancy rate, labor contract duration, and unemployment duration (Russo *et al.*, 2007; Ashraf, Gershman, and Howitt, 2011). Needless to say, these labor market details are generally hard to obtain in the equation-based models. Therefore, in addition to unemployment rate, one also learns the labor market structure underpinning the unemployment rate.

Third, since firms are modeled individually, agent-based macroeconomic models can simultaneously demonstrate the industry dynamics along with the macroeconomic dynamics. The macrodynamics are, therefore, generated with the underpinning competitive dynamics of firms. The emergent growth-with-cycles is now accompanied by various characteristics of firms, such as their size, growth rate, bankruptcy rate, duration, and markups (Wright, 2005; Russo *et al.*, 2007; Bruun, 2008; Chan and Steiglitz, 2008; Mandel *et al.*, 2010; Ashraf, Gershman, and Howitt, 2011; Delli Gatti *et al.*, 2011; Dosi *et al.*, 2013). This kind of two-in-one feature is again hard to observe in equation-based macroeconomic models.

Fourth, in agent-based models, the relationships among all the economic variables are emergent and not assumed or imposed. These include the famous Philips curve, Beveridge curve, and Okun's law. Unlike the conventional equation-based models which take these relationships as given, agent-based models allow these relations to be studied as endogenously emergent properties (Russo *et al.*, 2007; Lengnick, 2013; Delli Gatti *et al.*, 2011).

Last, in addition to its substitute and complementary relationships with the conventional equation-based models, agent-based macroeconomic models have far-reaching implications for economic models. This is so because they can serve as a platform to integrate various markets currently studied through agent-based modeling yet in an isolated manner, such as agent-based financial markets, housing markets, electricity markets, educational markets, innovation networks, etc. Needless to say, financial markets, real estates, energy, human capital, and innovation all play important roles in the operation of a macroeconomy, but studying them in an isolated manner may limit our understanding of their possible interactions with other markets. Currently, a number of agent-based macroeconomic models already have a built-in stock market (Bruun, 2008; Raberto, Teglio, and Cincotti, 2008), although these built-in ones are much simpler than the general agent-based financial markets (Chapters 14 and 15).

3.5 Interacting heterogeneous agents

From the previous section, we can see that various heterogeneities, whether given or emergent, are one of the essences of agent-based computational economics. We shall conclude this chapter with a short remark on this essence.

3.5.1 Heterogeneous agents

For many economists, their path to the study of agent-based modeling is partially due to their dissatisfaction with the mainstream economic methodology built upon *representative agents*. Hartley (1997) provides a lengthy discussion on this "troubling" concept. There are many reasons for going against the device of representative agents, from both empirical and theoretical aspects.

The main empirical grounds are the ample empirical evidence showing the existence of great heterogeneity and diversity at the micro level, from households, firms, traders, to other decision-makers. Nonetheless, a solid understanding of this diversity, such as the wealth distribution of households, the size distribution of firms, and the optimistic and pessimistic forecasting distributed among financial practitioners, is still lacking. Therefore, there is a need to search for a more suitable methodology to study the distributive behavior of an economy.

Furthermore, given the great diversity at the micro level, the relationship between the macro (aggregates) and the micro (individuals) becomes much more complex than that held in the representative-agent economy. The exact relationship between the micro and macro has actually presented economic theorists with a new challenge. All of these together motivated the formation of the heterogeneous-agent approaches or the agent-based paradigm for economics in the 1990s.[44]

3.5.2 Distributed artifical intelligence

Computer scientists have been devoted to paradigm shifts for a long time, and agent-based modeling or multi-agent systems play a pivotal role in these transitions; hence, in addition to the heterogeneous-agents origin, there is an origin from the artifical intelligence (AI) perspective. Here we shall provide a brief remark on *distributed artificial intelligence* and review the origins of ACE from an AI perspective.

Agent-based modeling is also known as *multi-agent systems* (MAS) in the realm AI. Computer scientists may have a perspective different from social scientists on tracing the intellectual origins of MAS. Wooldridge (2009) gave a review of the emergence of the multi-agent system from an AI viewpoint. He took Alan Newell and Herbert Simon's *rule-based expert system* (production system) as the origin (Newell and Simon, 1972), because it motivates the later development of the *blackboard system*, which enables individual agents to interact with each other through the blackboard and hence a "primitive" society of agents is formed. The blackboard system in fact employs the same idea on which the current complex *social-intelligence designs* are based.[45] That includes a public domain of shared information and the involvement, via the blackboard, of heterogeneous agents each of whom has its expertise or expert domains. This architecture allows problem decomposition and parallelism, which certainly facilitates problem-solving.

The blackboard system is primitive because it is not a real decentralized system and has not really taken advantage of the idea of networks. Because of this, the advantages of parallelism are not fully exploited. The later development of MAS in fact is to network the heterogeneous agents and enable them to "talk" (reacting to a received message by sending a message). Up to this point, one can see succinctly that the nature of computation is *sociality*, which is what has already been encapsulated in *cellular automata* (Chapter 4). Computation in a social manner has been known as *distributed artificial intelligence* since the early 1980s.

Notes

1 Tâtonnement means groping in French.

2 In fact, as has been shown in a number of simulations in Riechmann (2002), the market can fail badly, as well as depending on the slope of supply curve, i.e., α_j in Equation (3.8), and the specific learning algorithms employed by the agents. One attempt of Riechmann (2002) is to shed light on the *learnability issue* in light of the ways to learn, characterized by different learning algorithms. One departure from the standard tâtonnement process is that, in Riechmann (2002), agents are allowed to trade even though the plans are not mutually consistent. If they run into deficit, they will be punished in some form which entices them to learn from observed errors. Agents then learn to improve their utility by using *genetic algorithms* (see more in Sections 6.1.3 and 13.4).

3 One example is the ACE Trading World proposed by Tesfatsion (2006). After inviting readers to define equilibrium from a resultant dynamically complete economic model, she then added:

> In my experience, economics students are generally intrigued but flummoxed when presented with this type of exercise because it is radically different from the usual economic problems their professors has asked them to consider. In particular, they find it difficult to specify procurement processes driven solely by agent interactions and to define a correspondingly appropriate concept of equilibrium.
>
> (Tesfatsion, 2006, p. 853)

4 Learning was not considered to be an element in the early works, such as Albin and Foley (1992) and Wilhite (2001). As we shall see in Section 3.2.3, agents in the early works are truth-telling and accept a *fair* price, given this transparency. Since strategic behavior and speculations are not involved, these studies have limited room for learning.

5 See more in Chapter 22. Network is a subject which plays a quite crucial role in agent-based models. In this book, it will be addressed in separate chapters later on. Those readers who are not familiar with networks may find some terms or notations appearing too abruptly, but please either bear with us at this moment or just jump directly to the appropriate place in the text.

6 There is a good reason for the restriction to durable goods, as explained in the literature, for example:

> It is important to remember that, while this stage of development permits trading out of equilibrium, most of the work to be discussed does not permit consumption or production to take place until equilibrium has been reached. One must think of participants as swapping titles to commodity stocks while prices (and, of course, possessions) adjust. Only after the music stops do people go home and enjoy what they have. Such a model is obviously most suited to pure exchange with no firms.
>
> (Fisher, 1983, p. 27)

Obviously, durable goods can sustain long enough until the music stops.

7 See Section 22.3.3 for details.

8 However, to define the "best possible deal," one needs to specify the possible underlying bargaining behavior. The involvement of strategic bargaining behavior can make the best possible deal or the best trading partner difficult to identify, or other related procurement procedures difficult to proceed. Hence, assuming simple naive behavior can greatly simplify the model.

9 And the price to add these three elements is the omission of *networks*.

10 We believe that these two assumptions are largely innocuous; they just help make our presentation easier, and later on when we have models of production with firms, both of these assumptions can be removed. In fact, there is a third assumption: all goods are not durable but perishable, although they may perish slowly, depending on the perishing rate or the discount rate; hence it is possible to have inventories of them. For the purpose of studying the emergence of money, Gintis actually considered goods with zero discount rate.

11 While the commodity is no longer manna from heaven and has to be produced using labor input, the disutility of labor has not been explicitly taken into account. Hence, y_{m_i} in this production model without firms is just exogenously given and essentially naturally endowed. Its endogeneity will be discussed when we come to the production models with firms.

12 Both Equations (3.15) and (3.16) are formed and derived from the two assumptions above, the dependence assumption and the specification assumption.

13 x^*_{i,m_j} and x^*_{j,m_i} are from Equations (3.15) and (3.16) above.

14 Due to agents' possible heterogeneities, adding restrictions about their similarities enables us to select those agents who are comparable to i.

15 The fitness measure used by Gintis is utility, but he also noticed that agents with different utility functions may not be that comparable:

> There is nothing intrinsically desirable about using utility as the fitness criterion. Because utility functions are heterogeneous and individuals who prefer goods with low prices do better than agents who prefer high-priced goods independent of the trading prowess, there is significant noise in the imitation dynamic.
>
> (Gintis, 2010, pp. 6–7)

16 To be consistent with the notations used above, we shall see that in this section $N = M$.
17 Gintis (2007) argued that both the tâtonnement process and the non-tâtonnement process as studied by Anderson *et al.* (2004) fail to work on the Scarf economy because of the presence of the public price. If the decentralized process is realized through the private price, then the Scarf economy can be saved. Therefore, it remains interesting to study Gintis's proposal in the experimental context, in addition to his agent-based simulation.
18 The central assumption of the Hahn process is that the market is organized well enough that after trade we can have unsatisfied consumers or unsatisfied producers, but not both (Fisher, 1983). This assumption may be violated under our trading protocol.
19 While the importance of search intensity has been acknowledged (see particularly Wilhite, 2001), most studies of agent-based non-tâtonnement models treat search intensity as an exogenous setting—Albin and Foley (1992) being the only exception. For example, each agent is exogenously given finite times of trading trials in one market. If he does not make any progress in this finite number of trials, his realized consumption of that commodity may be low to zero; likewise, his production may all become his inventory.
20 As mentioned earlier, since the assumption of the Hahn process may be violated, one needs to work out an excess demand by simultaneously taking into account $Z_j^D(t)$ and $Z_j^S(t)$.
21 Gintis (2012) actually provided such a simulation and found that "[w]hile with six goods and one style the relative efficiency of money is only 150 per cent, for nine goods and twenty styles (180 goods), the relative efficiency is 1,200 per cent" (p. 57). Of course, in this setting, the number of goods is exogenously given; a more challenging attempt is then to endogenize the number of goods traded.
22 There have already been a number of studies from theoretical viewpoints to find the conditions under which posted price is preferred to auction (Wang, 1993; Campbell and Levin, 2006).
23 Here, we adopt the approximate and simpler version used in the simulation of Andreoni and Miller (1995). For the exact and longer version, interested readers are referred to Kagel and Levin (1986) and Kagel, Harstad, and Levin (1987).
24 For more details about genetic algorithms, see Sections 6.1.3 and 13.4.
25 Social learning will not necessarily lead to a homogeneous population, but see their footnote 16.
26 In fact, to make a comparison, they also considered the same model but with social learning (single-population GA) and examined the different effects of these two different kinds of learning.
27 All of these assumptions are not explicitly made, but are consistent with the setting of the given search space.
28 The first taxonomy may suit the case where P^* is a constant, but for the case where P^* varies, the second taxonomy may be more appropriate.
29 Again, this analogy is only suitable for the case where P^* is a constant.
30 Notice that this result is not entirely consistent with experimental studies (Chen, Katuscak, and Ozdenoren, 1997).
31 The *hard-close format* refers to the case where the duration of the auction ends exactly on the originally specified termination time without exception, whereas the *soft-close format* refers to the case where the duration of the auction will be automatically extended if there are bids submitted in the last few minutes before the termination time.
32 The current bid may not be the highest bid because, in internet auctions, bidder i can submit a *proxy bid*, $P_{i,\max}$, so that an increment Δ can automatically be added to the current bid for bidder i ($b_{i,t+1} = P_t + \Delta$) as long as this sum is still not greater than the proxy. That is, $P_t + \Delta \leq P_{i,\max}$. With this proxy bidding, $P_{i,\max}$ may be the highest (but unobservable) bid.

33 In fact, they used an even more simplified version by considering only three possible values of α, $\alpha \in \{1, 2/3, 1/3\}$, and allowed each of these αs to appear only once in the time intervals. Hence, they only needed to code the time, when these alphas are applied, with a three-digit code, such as (0,2,3) or (8,4,5).

34 The finite state machine will be detailed in Section 12.9. Basically, what we need are state-dependent or history-dependent decision rules to inform agents what to do, given what has happened. In fact, as we shall see in Chapter 13, there are many possible ways to implement this idea, such as instance-based rules (Section 12.8) and decision trees (Section 12.10).

35 See "Agents of change: Conventional economic models failed to foresee the financial crisis. Could agent-based modelling do better?" in *The Economist*, 22 July 2010.

36 This minimal rationality is first introduced in Becker (1962). See more discussions in Section 8.3.

37 However, not all agent-based macroeconomic models can meet this criterion. One example is Raberto, Teglio, and Cincotti (2008).

38 See Part VII for a more systematic review of the networks in the general context of economics and finance.

39 For randomly behaving agents, the support (range) for the possible behavior, instead of being arbitrarily given, can be further tailored to the empirical data (Mandel *et al.*, 2010).

40 Gintis (2007) is the only one which has optimizing consumers; nevertheless, the optimizing consumers only decide the demand for each commodity under the assumption of zero saving. In other words, in his model, there is no further optimizing behavior to decide the share of consumption and saving. At this point, the saving decision in most agent-based macroeconomic models is very much heuristic based.

41 For class "1" agents, saving can still happen in a form of forced saving when planned consumption fails.

42 Of course, the number of households or firms can be very superficial if one ignores their behavioral design (Section 3.4.5). For example, Cincotti, Raberto, and Teglio (2012) considered 20 firms, but all the firms were cost-minimizing firms under a given planned production, and with a rather delicate variant of the Cobb–Douglas production function, cost minimization leads to a moderate amount of the firms' demand for production and investment.

43 One issue that concerns us is whether the generated macroeconomic time series data can replicate well the macroeconomic time series observed in the real economy or the so-called stylized features. This issue, known as validation or calibration, becomes an econometric part of agent-based modeling and will be addressed in Chapter 16. However, at this stage, many studies use agent-based macroeconomic modeling mainly as a tool for thought experiments or policy scenario simulations; studies devoted to stylized-fact replications are quite limited (Wright, 2005; Delli Gatti *et al.*, 2011).

44 This is best exemplified by the great heterogeneities demonstrated in those studies mentioned in Section 3.4.7. For additional macroeconomic textbooks written within this new background, the interested reader is referred to Aoki and Yoshikawa (2006) and Delli Gatti *et al.* (2008).

45 Social intelligence or collective intelligence, sometimes also known as *swarm intelligence* (Bonabeau and Meyer, 2001), has become an interdisciplinary research subject across various social and technical disciplines. Inquiries into emergent social intelligence are basically concerned with the design of an interaction platform so as to expect and observe the emergent social intelligence. One example is the design of *e-participation* or, more specifically, *prediction markets* (Wolfers and Zitzewitz, 2004). The social intelligence of prediction markets, when applied to predicting election outcomes, has already shown superior performance compared to the results of polls (Berg, Nelson, and Rietz, 2003). For more discussions, see Section 9.6.

4 Cellular automata

The intellectual inquiry behind cellular automata can be very biological. It is about how cells, as building blocks, can develop into coherent organisms. While a mathematical formalism of the cellular growth processes was already given by D'Arcy Thompson (1860–1948; Thompson, 1917), the speculation that organic development might be susceptible to computation comes much later.[1] Alan Turing (Turing, 1952) and John von Neumann are the two pioneers, and it is von Neumann who actually demonstrated the first cellular automata, a 29-state self-reproducing cellular automaton, in the 1950s.[2] Von Neumann drew some of his inspiration from his colleague in the Manhattan project, Stanislaw Marcin Ulam (1909–1984). At that moment, Ulam was studying the *growth of crystals* using a simple lattice network approach. He suggested to von Neumann as early as 1950 that simple cellular automata could be found in sets of local rules that generated mathematical patterns in two- and three-dimensional space where global order could be reproduced from local action.[3]

There is always a strong desire to *visualize* the interacting dynamics of individuals and to watch the change in the society. However, direct presentation of the interacting dynamics can be difficult unless it can be displayed in a low-dimensional space. The *cellular automaton*, despite its unique intellectual motivation, is probably the earliest model by which the idea of agent-based economics is manifested. Specifically, Schelling's well-known *spatial proximity model* can be regarded as the first agent-based computational economic model (Schelling, 1969, 1971; see Section 4.1).[4] Another pioneering application of cellular automata to social sciences was made by James Sakoda (1916–2005; Sakoda, 1971), who, in effect, already had the checkerboard design in his unpublished dissertation (Sakoda, 1949). Nevertheless, neither Thomas Schelling nor James Sakoda referred to cellular automata.[5] The formal concept of cellular automata was not known to them in the 1970s. The first person who explicitly classified *checkerboard models* under the cellular automata framework was the economist Peter Albin (Albin, 1975). Later on, the checkerboard model was treated as a special kind of cellular automata (Hegselmann and Flache, 1998; Batty, 2007; Castellano, Fortunato, and Loreto, 2009).

A cellular automaton is a collection of agents situated on a grid of specified shape, usually a one-dimensional row, or two-dimensional rectangle (checkerboard). Each agent is characterized by a set of *network-based decision rules*, which indicate how an agent's decision or choice is determined by the respective network. While the network can be global and local, agents' behavior, by and large, tends to only be affected by the local network and little by the global network. Specifically, this local network is characterized by a set of *neighbors* whose behavior in the past may affect the agent's behavior in the future.

Cellular automata play an important role in the rise of complex sciences, because they constitute one of the earliest popularly used models for demonstrating how complex patterns may emerge from the local interactions of agents who follow very simple rules. An early example was given by the famous *Game of Life*, invented by Cambridge mathematician John Conway in 1970 (Section 4.2). Later examples were contributed by Stephen Wolfram, who in 1983 published the first of a series of papers systematically investigating a very basic but essentially unknown class of cellular automata, which he termed *elementary cellular automata* (Section 4.3). Using a concept from dynamical systems, he first classified cellular automata into four categories. The unexpected complexity of the behavior of these simple rules, demonstrated by Class III and Class IV, led Wolfram to suspect that *complexity in nature may be due to similar simple mechanisms*. Epstein and Axtell (1996) is probably the first study introducing this sense of complex science in the context of the social sciences. In Epstein and Axtell (1996), the fundamental collective behaviors, such as group formation, cultural transmission, combat, and trade, are seen to emerge from the interaction of individual agents following a few simple rules.

Clearly, the cellular automaton is an agent-based model, but it is distinguished from other agent-based models by its visible *spatial structure*, which makes this model very suitable for models of spatial dynamics, such as city dynamics. It is also a special case of graph-based models (Chapter 22), whose edges connecting the nodes (cells) are regularly and locally formed. Hence, the Euclidean distance between two cells is preserved when the edge-based distance is applied, which makes it suitable for models of locally discrete choices, i.e., the discrete choices made by agents who are locally interacting with their neighbors. Schelling's spatial proximity model shares both of these features (Section 4.1).[6]

4.1 Segregation

Thomas Schelling, the 2005 Nobel Prize Laureate in Economics, used both one- and two-dimensional cellular automaton models to show how an undesirable macro behavior can emerge from interaction among individuals' rational choices. In his spatial proximity model, there are two types of agents: say, red and green. Each agent is placed in a cell of the cellular automaton (see Figure 4.1). They have neighbors defined by the Moore neighborhood, i.e., the occupants of the adjacent squares (see Figure 4.1, the black circle). Both types of agent can accept agents of different types to be their neighbors, but there is a *limit*. Supposing that the agent dislikes being the minority in the community, then the limit is 50 percent; the agent would be indifferent to the combination of

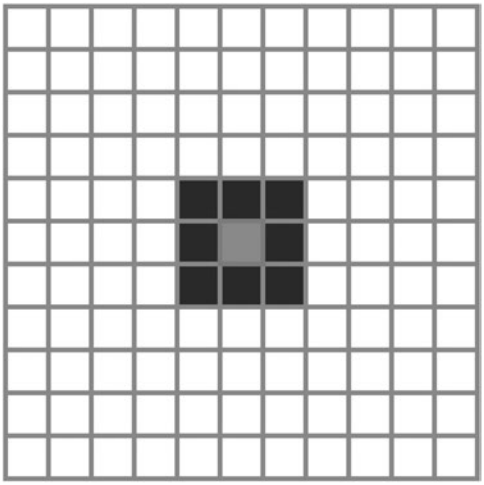

Figure 4.1 Agent in a checkerboard and his Moore neighborhood.

his neighbors as long as the neighbors of different types account for no more than 50 percent of his neighbors. If the agent becomes a minority, he will move to the nearest satisfactory place, called the *myopic best response* by Schelling. The simulation shows that an initially integrated society would converge to a segregated one, even though *no single individual has a strict preference for segregation*.

In Figures 4.2 and 4.3, we demonstrate a simulation of the segregation model using social simulation software known as *NetLogo*. Figure 4.2 presents the initial configuration where green and red agents are randomly distributed on the checkerboard, and Figure 4.3 depicts the convergent configuration after several iterations. This example shows that an initially integrated society would converge to a segregated one, even though no single individual has a strict preference for segregation; each agent simply does not want to be a minority, but has no problem living with unlike agents.

What was interestingly shown in Schelling's model is the idea broadly referred to as to the *emergent property*. Regardless of the rich content of the term itself, by being restricted to Schelling's case, we observe that a collective behavior of segregation can emerge from a group of agents who prefer integration to segregation. In fact, Pancs and Vriend (2007) revealed an even bigger surprise. What Pancs and Vriend (2007) did was to change the *conditional preference*, i.e., the preference for the proportion of unlike neighbors conditioned on the agent remaining as a majority. Originally, this preference, set by Schelling, was indifferent. Pancs and Vriend changed it to a preference for a higher proportion of the unlike people, and the higher the better. Nonetheless, even with this more tolerant behavior for unlikeness segregation still emerges.

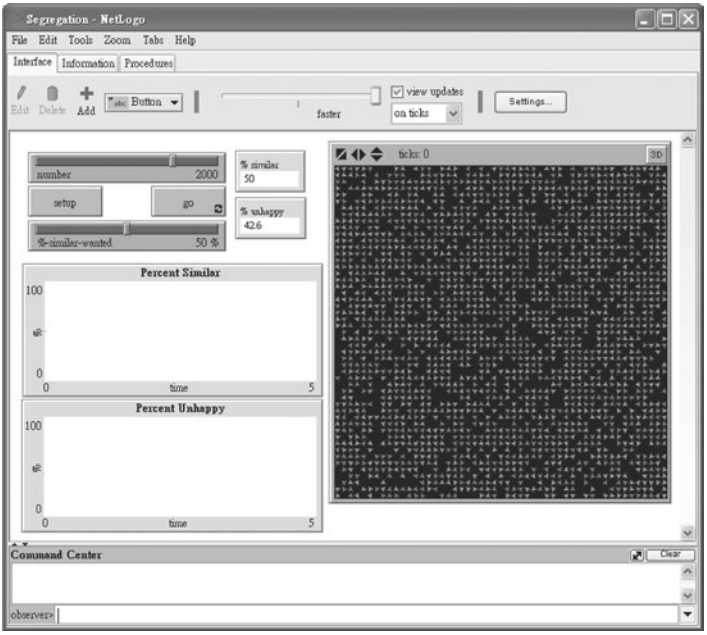

Figure 4.2 Schelling's segregation model: a NetLogo demonstration (initial setup).

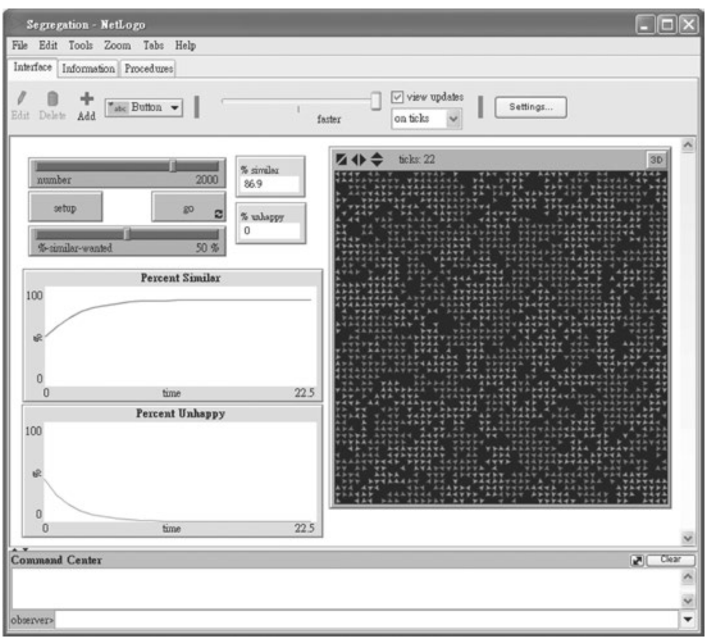

Figure 4.3 Schelling's segregation model after several iterations.

The Sakoda–Schelling checkerboard model tried to relate individual decisions to the emergent aggregate pattern, particularly when the latter is usually not what was intended by the former. In other words, the aggregate pattern may surprise each individual, while at the same time presenting a challenge to the limits of the linear scaling or linear thinking. Hence, the Sakoda–Schelling model features a number of things that one might normally find hard to obtain from a top-down or linear-scaling-up manner, as conventional macroeconomics may tend to do. Here, a truly decentralized process has shown how central planning without the details of interaction patterns may overlook the complexity of coordination and aggregation. Later on, the physical space (the three-dimensional geography) was further relaxed to consider a more general social space, as Robert Axelrod (Axelrod, 1997b) has done (see Section 20.1).

4.2 Game of Life

The Game of Life, also called Life, was invented by the Cambridge mathematician John Conway. In the late 1960s, motivated by Stanislaw Ulam's work, Conway began to use two-dimensional cellular automata to work on self-producing automata or artificial life. Life is a kind of cellular automaton, which provides a set of genetic laws for all cells in a two-dimensional checkerboard. These laws concern the birth, survival, and death of cells. The purpose is to simulate the rise, fall, and steady performance of organisms. Conway did not publish on his invented game; instead, it is Martin Gardner whose publication made this game well known to the public (Gardner, 1970).

More precisely, the Game of Life is a two-state, two-dimensional cellular automaton. The rules for each agent (cell) are very simple:

- Any live cell with fewer than two neighbors dies of loneliness.
- Any live cell with more than three neighbors dies of crowding.
- Any dead cell with exactly three neighbors comes to life.
- Any live cell with two or three neighbors lives, unchanged, to the next generation.

Like the segregation model, the Game of Life can also be simulated in many agent-based simulation platforms. Figure 4.4 demonstrates one of the runs using the software Netlogo (Wilensky, 1998).[7] As we can see from this figure, the simple rules in the Game of Life can generate many different "surviving patterns." Many of them have a name, such as blinker, block, and beehives.[8] Some patterns are still, but some are able to "walk" (such as the "glider" discussed below), just like we have animals and plants in the universe. If one starts with a very large universe and randomly initiates some living cells, then it is very interesting to watch the complex dynamics generated from it; in this case, Figure 4.4 is just a snapshot.[9]

The set of rules above is abbreviated as "2,3/3" in the literature, as a parametric rule. The part before the slash, "2,3," gives the condition for survival, whereas the part after the slash, "3," gives the condition for resurrection. With the two sets of

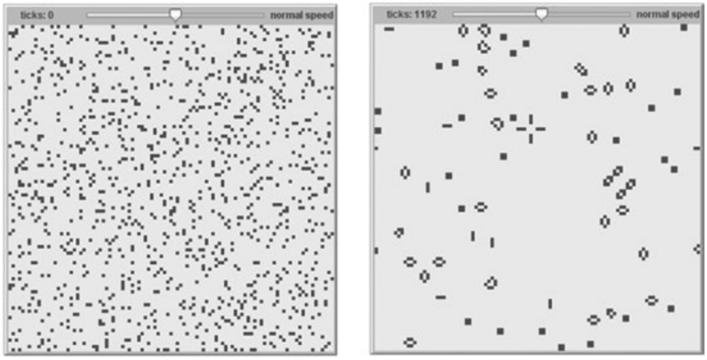

Figure 4.4 Game of Life: Netlogo simulation.

Notes: The Game of Life is initialized by placing a set of agents randomly in a two-dimensional checkerboard (the left panel). Then each agent follows the given rules to live, die, or rejuvenate itself. The right panel then shows the emergent pattern of scattering small clusters after more than 1000 iterations.

parameters, a families of rules can be used for experiments. John Conway noticed that not all the rules are interesting, in the sense of *unpredictability*. The set of rules 2,3/3 above was carefully chosen after trying many other possibilities, some of which caused the cells to die too fast and others of which caused too many cells to be born. Life 2,3/3 balances these tendencies, making it hard to tell whether a pattern will die out completely, form a stable population, or grow forever (Gardner, 1970).

Limit of growth

One of the hardest and most intriguing issues arising from Life 2,3/3 is whether we can have growth without limits. To elaborate on this issue, we start with a finite number of agents randomly distributed in a finite subset of the two-dimensional checkerboard, as a small village in the universe, such as Figure 4.4 (the left panel). We may expect that some kind of initial configuration will trigger an expansion process by making the initial small village spread to a larger domain. However, the question is: can it spread without a boundary? The original conjecture made by Conway himself is negative, i.e., it is impossible to have growth without limits. However, he was also open to objections and, for that, he was prepared to offer $50 for an initial setting which can have growth without limits. His invitation was then answered by William Gosper, who discovered a configuration known as the glider gun,[10] and earned the pecuniary award from Conway.[11] In addition to gliders, there are other patterns able to cross the screen, such as spaceships and flotillas.

What we learn from the Game of Life and other cellular automata in this chapter, and agent-based models in general, is that a system composed of simple homogeneous agents who follow simple deterministic rules can still lead to emergent patterns or dynamics which are almost impossible to predict, except when using computer simulations. When the checkerboard is large enough, we may have various still or moving lives, which may interact randomly; for example, one glider may bump into another glider, or a glider may cross a block, a blinker, etc. New patterns may emerge after these meetings, this process may never end, and one may hardly predict what will happen next.

While all cells are given the same "genetic materials," highly heterogeneous patterns may emerge from these homogeneous settings. Furthermore, the heterogeneous patterns that emerge from the homogeneous settings can have various forms of interactions, which, depending on our interpretations, can be understood as specialization or integration, competition or cooperation, the emergence of team production or hierarchical organization. While it is one of the earliest popularized agent-based models, Life 2,3/3 already implicitly has many ideas which distinguish agent-based modeling from variable-based or equation-based modeling. It truly provides the possibility of making biologists, computer scientists, and social scientists meet together.

4.3 Elementary cellular automata

The cellular automata which were introduced in the previous sections are both in two-dimensional lattices. This seems to be natural for the spatial modeling of geographical dynamics or city dynamics. However, there is also a one-dimensional version, called *elementary cellular automata*, which was studied by Stephen Wolfram in the 1980s. His pioneering study and many follow-ups in this line enable us to see the rich behavior of cellular automata and have formally established the theoretical connections among dynamical systems, universal computability, and cellular automata (CA). In this section, we shall briefly review the development of CA in the 1980s along these lines.

Before we start, it is useful to formally present the following *neighbors-based decision rule*:

$$X_{i,t+1} = f(\mathbf{X}(N_{i,t})), \tag{4.1}$$

where $N_{i,t}$ is the set of *neighbors* of agent i, which can include agent i himself, and $\mathbf{X}(N_{i,t})$ refers to the *state* of these neighbors in period t. There are two major elements in (4.1), namely, a set of neighbors and a set of states. The neighbors of agent i are the agents who are *close* to agent i, but it does not necessarily mean that they are close in *physical distance*. More generally, they may be close in *social distance*.[12] However, in cellular automata, neighbors are normally defined by their physical distance, for example, the Moore neighborhood (Figure 4.5, left panel) and the von Neumann neighborhood (Figure 4.5, right panel). One can see that both neighborhoods are characterized by the set of neighbors who are immediately adjacent to agent i. The state variables refer to the decision made by agent i. Basically, what Equation (4.1) says is that agent i's decision in period $t+1$ depends on the decisions made by his neighbors in period t, or more generally, up to period t. That is why it is refered to as the *neighbors-based decision rule*.

In a very simple setting, let us consider a one-dimensional lattice, as shown in Figure 4.6. Each agent, in addition to himself, is assumed to have only two

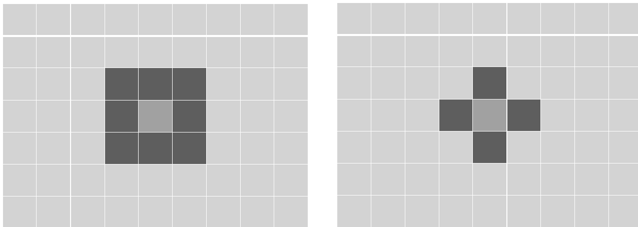

Figure 4.5 Neighborhoods in cellular automata.

Note: The two most frequently used neighborhoods in cellular automata: on the left is the Moore neighborhood, and on the right is the von Neumann neighborhood.

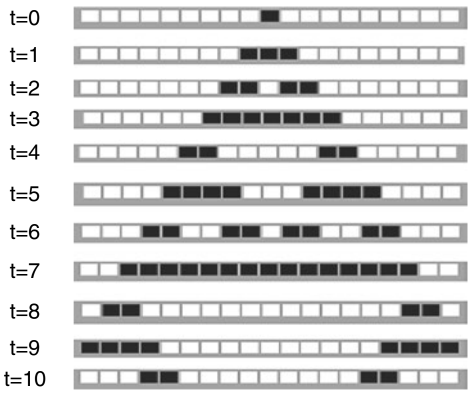

Figure 4.6 Wolfram's elementary cellular automata: Rule 126.

Note: A cellular automaton with finite cells is normally operated by connecting the rightmost cell with the leftmost cell as an atoll; hence these two cells are neighbors to each other.

neighbors, the one to the left and the one to the right. Let us also assume that there is a binary choice available to each agent, either "1" or "0" (say, be an optimist or be a pessimist). In this binary setting, a decision rule for each agent is simply a mapping from a binary string with length 3 to a binary variable:

$$X_{i,t+1} = f : \{1,0\} \times \{1,0\} \times \{1,0\} \rightarrow \{1,0\}. \tag{4.2}$$

The triplets refer to the decisions made by his left neighbor, agent i, and his right neighbor in period t, respectively. Since there are only a total of 2^3 possible combinations of the decisions made by the agents, a rule can be represented by an eight-bit binary string, as exemplified by Equation (4.3):

$$f_{126} : \underbrace{111}_{0} \; \underbrace{110}_{1} \; \underbrace{101}_{1} \; \underbrace{100}_{1} \; \underbrace{011}_{1} \; \underbrace{010}_{1} \; \underbrace{001}_{1} \; \underbrace{000}_{0} \tag{4.3}$$

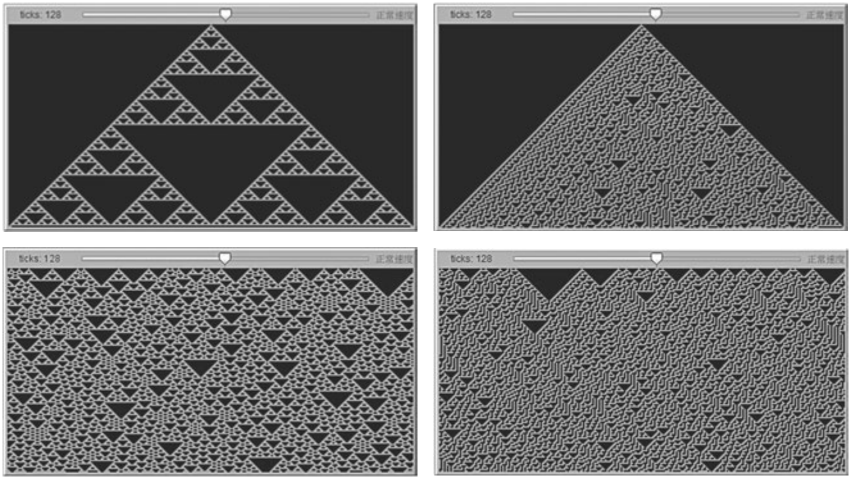

Figure 4.7 Dynamics of elementary cellular automata: Rule 126 and Rule 30.

Notes: The dynamics of Rule 126 (left) and Rule 30 (right). There are two different initial configurations. The upper panels correspond to the case where all cells but the middle one are black, whereas the lower panels correspond to the case where the color of each cell is randomly determined. With the random initialization (the lower two panels), there is a recurring series of downward facing triangles, but their size and and location seem to change over time and space in such a way that exact repetition is not found.

In Equation (4.3), the eight strings from the left to the right spell out the eight possible combinations and the decision in response to each combination. For example, if all three neighbors (including i himself) chose “1,” i.e., “111,” in the previous period t, agent i will reverse the decision in this period $t + 1$ to “0.” However, if all three chose “0” (“000”), then agent i will still choose “0.” If we further assume that all agents follow the same rule, for example Equation (4.3), then the transitions from the current states of all agents to the next states are determined as demonstrated in Figure 4.6.

In Figure 4.6, state one has color black, and state zero has color white. In the initial period ($t = 0$), there is only one agent having state one, while all others chose “0.” Then, by the neighbors-based decision rule, Equation (4.3), we can trace the state dynamics from period 0 to periods 1, 2, 3, . . . , row by row, as shown in Figure 4.6; a larger and better picture is available by using NetLogo (Figure 4.7), which demonstrates the *time–space pattern* of the rule in Equation (4.3). Obviously, something interesting there immediately catches our eyes.

In the 1980s, Stephen Wolfram systematically studied a class of rules like Equation (4.3). Given the structure described above, it is not hard to see that there are a total of 128 (2^{2^3}) neighbors-based decision rules. This can be done by simply altering one or more output bits in Equation (4.3). For example, by changing the fourth bit of rule (4.3) from one to zero, one obtains the following rule:

$$f_{110}: \underbrace{111}_{0}\underbrace{110}_{1}\ \underbrace{101}_{1}\ \underbrace{100}_{0}\ \underbrace{011}_{1}\ \underbrace{010}_{1}\ \underbrace{001}_{1}\ \underbrace{000}_{0} \tag{4.4}$$

and by replacing the first two 1s in (4.3) by zeros, one has the following new rule instead:

$$f_{30}: \underbrace{111}_{0}\ \underbrace{110}_{0}\ \underbrace{101}_{0}\ \underbrace{100}_{1}\ \underbrace{011}_{1}\ \underbrace{010}_{1}\ \underbrace{001}_{1}\ \underbrace{000}_{0} \tag{4.5}$$

In addition, by replacing the first and the fourth 1s in (4.3) by zero, we have another rule as follows:

$$f_{54}: \underbrace{111}_{0}\ \underbrace{110}_{0}\ \underbrace{101}_{1}\ \underbrace{100}_{1}\ \underbrace{011}_{0}\ \underbrace{010}_{1}\ \underbrace{001}_{1}\ \underbrace{000}_{0} \tag{4.6}$$

All these rules were numbered by Wolfram, from 0 to 255, and, based on their time–space dynamics and the theory of dynamic systems, are classified into four classes, namely fixed points (limit points), period cycles (limit cycles), strange attractors (chaos), and "edge of chaos," described as follows:

- Class I: Nearly all initial patterns quickly converge to a uniform homogeneous final state, with all agents either being in a state of 0 or 1.[13]
- Class II: Nearly all initial patterns quickly converge to a stable structure (vertical stripes) or a continually recurring repetition of short cycles.
- Class III: Nearly all initial patterns evolve in a pseudo-random, chaotic, or apparently irregular way; typical clusters evolve at intervals (see Figure 4.7).
- Class IV: Nearly all initial patterns evolve into structures that interact in a complex and interesting way. While the generated structures can be unstable and nonperiodic (like Class III), the automata belonging to this class will also generate laterally shifted patterns, that is, oblique lines or stripes. This class possibly contains universal automata.[14]

Rule (4.3) is numbered 126, and rule (4.5) is numbered 30. They are both categorized as Class III (strange attractors). Rule (4.4) numbered 110 and rule (4.4) numbered 54 are both categorized as Class IV (edge of chaos).[15] It is not our purpose to give a full-length treatment of the relationship between dynamical systems and one-dimensional cellular automata, nor its further connection with the theory of computation and formal languages. For that detour, interested readers can find some references in Section 4.6. The key message to deliver here is two-fold. First, complex unpredictable patterns can emerge from very simple homogeneous interacting behavior. Right from Chapter 2 of Wolfram (2002), Wolfram gave a quite lengthy discussion of the behavior generated by Rule 30 (pp. 27–30) and Rule 110 (pp. 32–8), showing how extremely simple rules can generate highly complex,

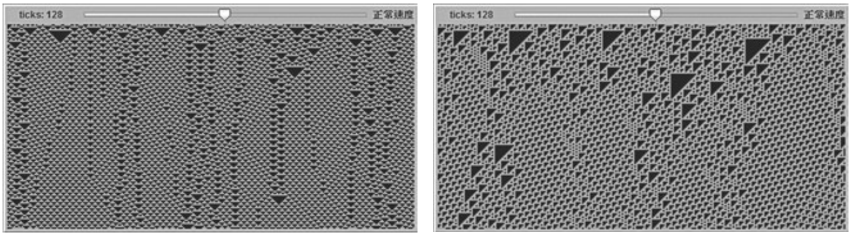

Figure 4.8 Dynamics of elementary cellular automata: Rule 54 and Rule 110.

Note: The dynamics of Rule 54 (left) and Rule 110 (right). The color of each cell is initially randomly determined.

random, unpredictable patterns. Second, a small change in an individual rule, say a one-bit change from f_{126} to f_{110}, may fundamentally change the nature of the system dynamics (from Class III to Class IV).[16]

Now, to provide a little entertainment, let us close this section with three brief "proverbs" as the essence of agent-based modeling.

– Small is big: sometimes a seemingly small and inconsequential local event in a system can be amplified to cause global and substantial change.
– Details matter.[17]
– Unpredictable is predictable (or always anticipate the unanticipated).

4.3.1 A new kind of economics?

In his very provocative book *A New Kind of Science* (Wolfram, 2002), Stephen Wolfram described many of phenomena observed in his cellular automata simulation, such as the simulation of Rule 30 shown in Figure 4.7, as what he termed *computationally irreducible*. According to Wolfram (2002), if the behavior of a system is not obviously simple, then it will generally be computationally irreducible, which means that the only way to predict its evolution is to run the system itself. There is no simple set of equations that can look into its future.

By distinguishing the phenomena known as computationally reducible from those known as computationally irreducible, Wolfram (Wolfram, 2002) argues that the conventional sciences are mainly efforts devoted to computationally reducible phenomena. A new kind of science can then be considered to be a paradigm shift toward the study of computationally irreducible phenomena. Wolfram's proposal is not limited to physics or biology. If one applies this irreducibility characterization to economics or social sciences, one can equally perceive a new kind of economics or a new kind of social science; for example, see Borrill and Tesfatsion (2011).

> Not only in practice, but now also in theory, we have come to realize that the only option we have to understand the global properties of many *social*

> *systems* of interest is to build and run computer models of these systems and observe what happens.
>
> (Borrill and Tesfatsion, 2011, p. 230; emphasis added)

The argument based on computational irreducibility, therefore, has been taken by Borrill and Tesfatsion (2011) to further support the agent-based modeling methodology as the right mathematics for the social sciences, including economics.[18]

An immediate specific example of phenomena which are not well studied by the "old" science of economics because of their computational irreducibility is *markets*. Earlier, in Chapter 3, we mentioned the silence of the study of markets in economics, as eloquently described by Mirowski (2007). In Mirowski (2007), markets in the history of economic theory have either been taken for granted, or have been simplified into just exchange behavior, or have been lumped together as abstract entities. What has been largely ignored are many critical details referred to as procurement procedures, as nicely described in Tesfatsion (2006) as well as in Chapter 3 in this book.

4.4 Opinion dynamics and market sentiment

Up to the present time, the agents (cells) which we observed in the previous sections have been homogeneous in the sense that they have identical behavioral rules. Of course, one may easily extend homogeneous cellular automata to heterogeneous CA, where agents actually follow different rules. This extension can be particularly useful when CA is applied to the social sciences, where agents may usually have different preferences, beliefs (belief formations), and decision rules. For example, agents in the Schelling model can have different tolerance levels or tipping points, as his later tipping model suggested (Schelling, 1972). Also, in Schelling's model, the agent's state (attribute), be it green or red, will remain heterogeneous.

Of course, not all attributes of agents are exogenously given; instead, some can endogenously evolve to be heterogeneous as well. For example, the agents' preferences or opinions may not be fixed and can be affected by their neighbors. In this section, we will apply CA to examine opinion dynamics or market sentiment, within which agents may or may not have the same opinion (belief, sentiment) and may or may not have the same belief formation processes. In a simple binary setting, agents in this CA will switch their beliefs or opinions, say, from being positive to negative or the other way around. The switching decision is determined by their network-based decision rules and local information; in a sense, it is very similar to the rules driving Wolfram's elementary cellular automata.

Let us consider a two-dimensional lattice having 10×10 cells, with each of them denoted by (i, j)—the ith row and the jth column—see Figure 4.9. Each cell is occupied by one agent, and his opinion (belief, expectation, attitude) of the future prospects for the economy at time t is denoted by $x_{ij}(t)$, which is a binary digit: "1" (being optimistic) or "-1" (being pessimistic). In addition, let $\mathbf{X}(t)$ be

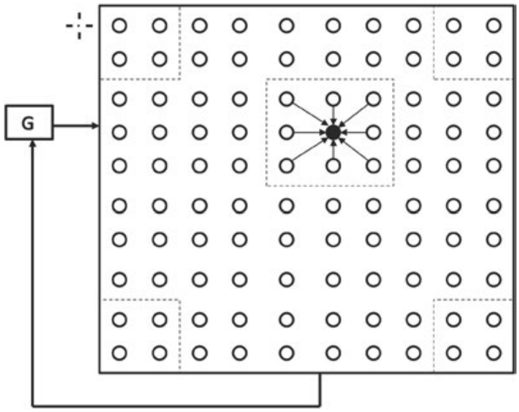

Figure 4.9 Market sentiment: local and global.

a matrix that is a collection of all agents' opinions, i.e., $\mathbf{X}(t) = [x_{ij}(t)]$. Let us also assume that each agent's expectation in the next period is under the influence of his neighbors' current expectations. Here, neighbors are defined by the Moore neighborhood (see Figure 4.9).

Equations (4.7) and (4.8) give the expectation formation rule, f_{ij}, for each individual. Basically, f_{ij} is a weighted average of the observed local market sentiment and the received global market sentiment $G(t)$. The local market sentiment is simply the average opinion held by an agent's neighbors, and the agent can gain personal access to it because of the locality. The global market sentiment is provided by a central agency based on a census with sample S_t and a sample size of N_{S_t} (Equations 4.10 and 4.11). While the global market sentiment can be derived using a much larger sample average, as in Equation (4.11), than the one from local neighbors, the agent cannot have a direct verification of its quality. Hence, the weight $\beta_{ij}(t)$ can be considered to be the confidence that agent ij has for this "officially" announced global market sentiment.

$$x_{ij}(t+1) = \begin{cases} 1 & \text{if } f_{ij}(\mathbf{X}(t),t) > 0, \\ x_{ij}(t) & \text{if } f_{ij}(\mathbf{X}(t),t) = 0, \\ -1 & \text{if } f_{ij}(\mathbf{X}(t),t) < 0. \end{cases} \tag{4.7}$$

$$\begin{aligned} f_{ij}(\mathbf{X}(t),t) &= (1-\beta_{ij}(t))\frac{\sum_{N_{ij}} x_{ij}(t)}{\#(N_{ij})} + \beta_{ij}(t)G(t) \\ &= (1-\beta_{ij}(t))(2p_{ij}(t)-1) + \beta_{ij}(t)G(t), \end{aligned} \tag{4.8}$$

where

$$p_{ij}(t) = \frac{\#\{x_{ij}(t) = 1 \mid x_{ij} \in N_{ij}\}}{\#(N_{ij})}. \tag{4.9}$$

$$G(t) = \begin{cases} 1 & \text{if } g(\mathbf{X}(t)) > 0, \\ 0 & \text{if } g(\mathbf{X}(t)) = 0, \\ -1 & \text{if } g(\mathbf{X}(t)) < 0, \end{cases} \tag{4.10}$$

where

$$g(\mathbf{X}(t)) = \frac{\sum_{x_{ij}(t) \in S_t} x_{ij}(t)}{N_{S_t}}. \tag{4.11}$$

Equations (4.7) to (4.11) then enable a simple cellular automaton to simulate the sentiment dynamics. An essential question concerns how the central agency should announce the global sentiment. Should it directly distribute what it has from the census to the market, as it could do by following Equation (4.10), or should it manipulate it in some other way if the known situation is not good or very bad? In other words, should *bad news* be hidden or distorted in such a way that its possible negative impacts can be avoided? To motivate this issue, let us recall the familiar kind of argument by quoting Fischer (1991), which shows the great significance of *fear* in economics:

> There is nothing exceptional in the result that changes in expectations affect the equilibrium of the economy; the interesting feature is that those changed expectations (animal spirits) may be correct and thus self-justifying. This is a rigorous justification of the notion that optimism itself may be sufficient to create a boom, or that *all we have to fear is fear itself.*
>
> (Fischer, 1991, p. 32; emphasis added)

To not allow the self-fulfilled negative expectations to manifest themselves, there is the possibility that the central agency would like to manipulate the information on hand and change the way it is announced. Let us consider a very extreme alternative behavioral rule and replace rule (4.10) with rule (4.12):

$$G^1(t) = 1, \forall t. \tag{4.12}$$

The difference between $G(t)$ (rule 4.10) and $G^1(t)$ is that the former announcement is based on the census and is unbiased in that sense, whereas the latter is not: it is always biased toward a positive announcement, regardless of the census result. Therefore, here, a strong degree of manipulation has been imposed. The question is: is it desirable to do this?

Chen (1997a) first assumes that $\beta_{ij}(t)$ is constant and homogeneous among all agents, i.e., $\beta_{ij}(t) = \beta$:

$$\begin{aligned} f_{ij}(\mathbf{X}(t), t) &= (1-\beta)\frac{\sum_{N_{ij}} x_{ij}(t)}{\#(N_{ij})} + \beta G(t) \\ &= (1-\beta)(2p_{ij}(t) - 1) + \beta G(t). \end{aligned} \tag{4.13}$$

He then simulated the sentiment model with respect to different values of β using G and G^1. Denote the fraction of optimistic agents by

$$p(t) = \frac{\#\{x_{ij}(t) = 1\}}{N},$$

where N is the total population size (number of cells). It is found that when the initial diversity of the agents' expectations is large, as measured by the entropy of $p(0)$, and β is high, the use of G^1 can have a dramatic effect in driving $p(t)$ toward a "good" equilibrium, i.e., an equilibrium with a majority of agents with optimistic expectations. This implies that the tendency for the central agency to manipulate the census will increase with the agents' confidence in the announcement and the initial diversity of agents' opinions.

However, the assumption of a constant β is strong because it simply says that, no matter how the central agency manipulates the census, the agents' confidence in it will never change. Alternatively put, this constancy assumption violates the Lucas critique (Lucas, 1976), which basically asserts that agents' behavioral response to government policy should not be invariant to the policy per se. Therefore, a variant β is preferred considering that agents are able to learn the credibility of the public announcement and react upon it.

4.4.1 Cellular automata with Bayesian learning agents

Now, this is the first time that we address the learning of agents in CA models. Agents in the segregation model, the Game of Life, and elementary cellular automata, do not learn in the sense of modifying their behavioral rules. They only collect local information on which their decisions are based, but the decision rules never change. Now, rule change or rule switching is an important part of social behavior. If these behaviors are not random, then some kind of learning model is required to articulate how learning happens. Learning becomes a core issue in the development of economic theory mainly because the idea of boundedly rational agents is very much not static but dynamic. Bounded rationality is a concept in a limited time-space, and hence it is naturally dynamic within that time-space.

Modeling the learning behavior of agents is itself evolving. In economics, this business has for quite a long time been dominated by Bayesian learning or its variants. Many learning models, such as recursive least squares, the Kalman filter, and the moving average, can all be interpreted in the spirit of Bayesian learning. The influence of psychology and computer science on the learning literature in economics comes much later. Reinforcement learning and artificial intelligence were not formally introduced to economic models of learning until the 1990s (see Section 7.3 and Chapter 10). We shall come to these two subjects later in this book, but, before that, we shall first start with mainstream Bayesian learning.

The idea of Bayesian learning is to allow agents to have their own prior beliefs (prior distributions), which are then constantly updated with the information received. Given the unique mapping between the moment generation function and the probability density, distribution updating can also be represented in terms of moment updating, which means that agents have to constantly update their subjective first moment, second moment, and up to infinite high-order moments. Of course, no one can seriously do this in modeling, and hence some kinds of approximation have to be made. In the simplest settings, only the first moment needs to be taken care of. In some more deliberate settings, the second moment can also be incorporated. The Kalman filter is an example.

Mean (first moment) updating within the Kalman filter can normally be represented as a weighted average of the prior mean and posterior adjustment. Equation (4.14) provides an illustration of the belief (trust) dynamics. In this case, $\beta_{ij}(t-1)$ is the a priori trust, whereas $\beta_{ij}(t)$ is the posterior trust. The posterior adjustment depends on two things: in plain English, surprise and reaction to surprise. The former is commonly known as error, whereas the latter is known as the Kalman gain.

Error occurs when the agent finds the central agency to be more reliable or less reliable from his most recent experience. The original discrepancy between his own recent experience of the local market sentiment, $p_{ij}(t)$, and his received information from the central agency, $G(t)$, is given in Equation (4.17). This original discrepancy is then mapped to the adjustment to the agent's trust in the central agency's announcement, $\Delta\beta_{ij}(t)$, in the form of Equation (4.16). This mapping is monotonically decreasing, suggesting that the larger the discrepancy, the lower the additional increment in trust; actually, it is symmetric around the origin and can even be negative when the discrepancy is too large. In that case, what the agent experiences from his surroundings is exactly the opposite of what the central agency releases. $\Delta\beta_{ij}$ is then the error related to the perceived trustworthiness of the central agency.

$$\beta_{ij}(t) = \omega_{ij}(t)\beta_{ij}(t-1) + \kappa_{ij}(t)\Delta\beta_{ij}(t), \tag{4.14}$$

where

$$\omega_{ij}(t) + \kappa_{ij}(t) = 1 \tag{4.15}$$

and

$$\Delta\beta_{ij}(t) = \begin{cases} 0.4 & \text{if } 0 < \delta_{ij}(t-1) \le 0.2, \\ 0.2 & \text{if } 0.2 < \delta_{ij}(t-1) \le 0.4, \\ 0.0 & \text{if } 0.4 < \delta_{ij}(t-1) \le 0.6, \\ -0.2 & \text{if } 0.6 < \delta_{ij}(t-1) \le 0.8 \\ -0.4 & \text{if } 0.8 < \delta_{ij}(t-1), \end{cases} \tag{4.16}$$

$$\delta_{ij}(t) = \left| p_{ij}(t) - \frac{1+G(t)}{2} \right|. \tag{4.17}$$

The Kalman gain exists because of the uncertainty of the surprise (error) or, alternatively, the quality of information. In plain English, it depends on how seriously we are going to take our "surprise." In the stochastic optimal control of linear dynamical systems, the Kalman gain tends to converge to a constant and to zero when the environment is stationary, which means that after a number of iterations the agent has already learned or discovered the underlying law and since then any error occurring has been completely attributed to noise, which the agent should completely ignore (a zero Kalman gain). However, in the potentially highly nonlinear CA environment, we adopt a more psychological approach. Basically, we condition the Kalman gain ($\kappa_{ij}(t)$) on the current state of the agent's trust in the central agency ($\beta_{ij}(t)$). The state-dependent Kalman gain is given in Equation (4.18):

$$\kappa_{ij}(t) = \begin{cases} 0.1 & \text{if } 0.48 < \beta_{ij}(t-1) \leq 0.6, \\ 0.3 & \text{if } 0.36 < \beta_{ij}(t-1) \leq 0.48, \\ 0.5 & \text{if } 0.24 < \beta_{ij}(t-1) \leq 0.36, \\ 0.3 & \text{if } 0.12 < \beta_{ij}(t-1) \leq 0.24, \\ 0.1 & \text{if } 0 < \beta_{ij}(t-1) \leq 0.12. \end{cases} \tag{4.18}$$

The essence of Equation (4.18) is simple. It basically says that when the agent is very sure of the quality of the received information, either positively (very high trust) or negatively (very low trust), he tends to have a low Kalman gain, which basically means that he ignores his inconsistent experience, if any. On the other hand, the further away he is from the sure state, the larger the gain. This setting corresponds to the case of the zero Kalman gain where the agent is quite sure of having discovered the underlying law. So, under this quite intuitive consideration, the Kalman gain function also has a symmetric form.[19]

In the cellular automata model with Bayesian learning agents, the degree of trust in the central agency, $\beta_{ij}(t)$, can differ among different agents due to their different surroundings and hence their different local dynamics. At the end of the simulation, we may summarize this heterogeneity with an indicator showing the *credibility* of government by taking the average of all the people's weights assigned to the central agency's announcement, i.e.,

$$\tilde{\beta}(t^*) = \frac{\sum_{ij} \beta_{ij}(t^*)}{N}, \tag{4.19}$$

where $\beta_{ij}(t^*)$ refers to the degree of trust at the specific moment, t^*, when the whole CA system has converged to a fixed configuration. This gives the CA dynamics a political-economy performance measure which may interest the central agency. An accompanying measure which may also interest the central agency is the sentiment index per se. Let p^* be the percentage of optimists, i.e., the agents whose x^*_{ij} is positive when the system has converged to the stable configuration.

Table 4.1 Performance comparison between rule G^1 and rule G

$p(0) = 0.1$	0.00	(-0.38^*)
$p(0) = 0.2$	0.00	(-0.38^*)
$p(0) = 0.3$	0.00	(-0.35^*)
$p(0) = 0.4$	0.28^*	(-0.16^*)
$p(0) = 0.5$	0.50^*	(0.01^*)

Note: This simulation result is based on the prior distribution $\beta_{ij}(0) \sim \text{Uniform}[0, 0.5]$. Those values outside the parentheses are the differences in the probability of achieving the target between using the G^1 rule and using the G rule; those values inside the parentheses are the differences in credibility $\bar{\beta}$ between using the G^1 rule and using the G rule. Asrterisks indicate the values which are significantly different from zero (at the statistical significance level of 0.01).

Chen (1997a) then simulated this Bayesian version of cellular automata, characterized by Equations (4.7), (4.8), (4.14), (4.15), (4.16), (4.17), and (4.18), by comparing the two policy rules, i.e. $G(t)$ [Equation (4.10)] and G^1 [Equation (4.12)] in terms of the two measures above, i.e., credibility and the sentiment index. Under the Bayesian setting, an a priori distribution characterized by $\beta_{ij}(0)$ is required for each ij. Here, it is assumed that $\beta_{ij}(0)$ is uniformly sampled from a uniform distribution over the interval [0, 0.5].[20]

The result, which is based on 1000 independent runs of the simulation, is shown in Table 4.1. Here, the G rule is taken as the benchmark, and only the superiority (inferiority) of G^1 is shown in the table. The number outside the parentheses is the difference in the sentiment index, and the number inside the parentheses is the difference in the credibility index. As to the sentiment index, we first estimate the probability of having a sentiment index greater than 70 percent (at least 70 percent of agents are in an optimistic state, $p^* > 0.7$), and figure out the gain in this likelihood while replacing rule G with G^1.[21]

The result shows that G^1 (manipulation) can be a favorable policy rule only if the initial configuration has almost a balanced number of optimists and pessimists ($p(0) = 0.4$, 0.5). Only in this highly mixed state can the manipulation through the central agency successfully drive the "economy" out of a stalemate toward a desirable state. However, if the initial configuration is overwhelmingly dominated by pessimists ($p(0) = 0.1$, 0.2, 0.3), then the central agency will not make anything different (a zero gain) through manipulation except lose its credibility in a substantial way. The latter occurs when the trust in government is constantly adjusted downward through Bayesian updating in the presence of a contradictory experience.

4.4.2 Heuristics for intervention

The above simulation of evolutionary cellular automata is based on the assumption that the central agency can precisely pinpoint the initial market sentiment $p(0)$, i.e., the sentiment right after a shock, say a financial scandal of a big listed company. However, the initial responses to this shock are held by agents widely distributed over the two-dimensional lattice, and the central agency may not be capable of monitoring all agents in the streets and corners. All they have is the sample by which they can roughly gauge the current market sentiment.

With this limited information, the decision to manipulate can be made only upon this estimated market sentiment. A set of new policy rules is then formulated with this estimation, $\hat{p}(t)$, as shown in Equation (4.20):

$$G_h(t) = \begin{cases} 1 & \text{if } \hat{p}(t) = \frac{1}{2}(1 + g(\mathbf{X}(t)) > h/10, \\ 0 & \text{if } \hat{p}(t) = \frac{1}{2}(1 + g(\mathbf{X}(t)) = h/10, \\ -1 & \text{if } \hat{p}(t) = \frac{1}{2}(1 + g(\mathbf{X}(t)) < h/10. \end{cases} \tag{4.20}$$

Equation (4.20) generalizes G and G^1 by imposing a condition on intervention (manipulation), G_h. When the parameter h is set to zero, the rule is G^1 (unconditional intervention); when the parameter h is set to 5, the rule is G (unconditional laissez faire). By varying h, the decision on the intervention is further conditioned on the initial estimate of the market sentiment. Generally speaking, when the market sentiment is low, intervention becomes less likely, whereas when the market sentiment is very close to the stalemate zone, intervention becomes more likely. Hence, the action to intervene is determined and characterized by a threshold parameter h.

Chen (1997a) then simulated this cellular automaton composed of both boundedly rational agents, who follow Bayesian learning rules [Equations (4.14) to (4.18)], and the central authority, who follows simple heuristics [Equation (4.20)]. For each heuristic (h = 1, 2, 3, and 4), 1000 trials are conducted. The market sentiment and the credibility of the central authority, in the steady state, are then averaged and compared with those of the baseline (rule G, zero intervention). Table 4.2 shows the superiority or inferiority results.

As we have experienced in Table 4.1, when the initial market sentiment is low ($p(0) = 0.1, 0.2, 0.3$), there is no need for intervention, because, regardless of the heuristic applied, the chance of meeting the target, i.e., $p^* > 0.7$ (see footnote [21]), improves little; on the contrary, it simply causes damage to the credibility of the central authority. In these cases, obviously honesty (rule G) is the best policy.

Table 4.2 Performance comparison between rule G_h and rule G

	G_1		G_2		G_3		G_4	
$p(0) = 0.1$	0.00	(−0.02)	0.00	(0.00)	0.00	(0.00)	0.00	(0.00)
$p(0) = 0.2$	0.00	(−0.04)	0.00	(−0.01)	0.00	(0.00)	0.00	(0.00)
$p(0) = 0.3$	0.00	(−0.12)	0.00	(−0.03)	0.00	(0.00)	0.00	(0.00)
$p(0) = 0.4$	0.27	(−0.16)	0.31	(−0.15)	0.22	(−0.10)	0.09	(−0.04)
$p(0) = 0.5$	0.50	(0.01)	0.50	(0.01)	0.46	(0.01)	0.33	(0.00)
NET	0.44		0.63		0.59		0.38	

Note: This simulation result is based on the prior distribution $\beta_{ij}(0) \sim \text{Uniform}[0, 0.5]$. Those values outside the parentheses are the differences in the probability of achieving the target between using the G_h rule and using the G rule; those values inside the parentheses are the differences in credibility between using the G_h rule and using the G rule.

However, when the situation moves closer to the stalemate zone ($p(0) = 0.4$), a tradeoff between market sentiment and credibility exists. Furthermore, when it comes exactly to the stalemate zone, Pareto improvement can happen for all heuristics; in this case, laissez faire (rule G) becomes the worst policy. If one assigns equal weights to both performance measures (market sentiment and credibility), and also assumes a uniform distribution over all initial states, then the net effect of each heuristic is given in the final row of Table 4.2. There we see rules G_2 or G_3 are better than rules G_1 and G_4 (G_1 is even uniformly dominated by G_2, and hence it is not even admissible).

By following the heuristic G_2, the central authority will try to stabilize the initially turbulent economy with some manipulations or interventions as long as the market sentiment index is not lower than 20 percent, which means that, when the initial shock to the economy is not too big, the central authority shall take some measures before it is too late. This kind of behavior may be applicable to understanding frequent operations of saving the economy, such as save the markets, save the stock price, save the currency, and save the big company. On the other hand, if the initial shock is too big and has resulted in a significant impact such that the market sentiment is even less than 20 percent, any effort to intervene in the market can be made to no avail. In this case, the central authority shall leave the market to self-adjust to the level consistent with the economic fundamentals (laissez faire).

The essence of this analysis is that local shocks may have global effects, and this should be considered a generic property of agent-based models (cellular automata). Through the agent-based simulation, one can find the boundary within which intervention should matter and beyond which intervention should fail. However, drawing this boundary is difficult in the real world, and it is probably this uncertainty that results in the long-standing controversy, intervention or not, being hard to settle.

Hilbert's "23 problems" in economics?

At the beginning of the last century, the great mathematician David Hilbert (1862–1943) presented a list of 23 mathematical problems that seemed highly inaccessible based on the methods available at the time. These problems have fascinated generations of mathematicians (Browder, 1976). In a similar vein, one may wonder if there is an equivalent list of unsolved problems in economics which are equally inaccessible based on the methods available at this time. If so, what are these problems, and what will be the methods making these problems less inaccessible? Although not with the same degree of rigor that Hilbert used to formulate his problems, we argue that one candidate is *market intervention*: whether the government should ever try to intervene in the stock market, the foreign exchange market, etc. The candidacy of this issue may be well justified by the development of the literature (Sarno and Taylor, 2001; Khan and Bateau, 2011). However, it is cellular automata models specifically and agent-based models in general that enable us to see the computationally irreducible nature of the market intervention problem.[22] This nature of the market intervention problem is, however, largely out of the question under the conventional equation-based modeling of economic policy.

4.5 Other physics-oriented agent-based models

The development of agent-based models is, to some extent, under the substantial influence of physical models. In addition to cellular automata as one origin of ACE, others include kinetic models, percolation models, and Ising models.

4.5.1 Kinetic model

The kinetic theory of gases has been used in the study of wealth and income distribution. In this model, money-exchange trading was treated like the elastic scattering process in physics. This kinetic model of income distribution was first studied by John Angle during the 1980s, and was referred to differently as the *inequality process*. Angle's inequality process is motivated by the surplus theory of social stratification in economic anthropology, rather than by anything in physics (Angle, 1986). Later on in the 2000s, this model was independently studied again by physicists Adrian Dragulescu and Victor Yakovenko, who caused the model to become well known among econophysicists (Dragulescu and Yakovenko, 2000; Chakraborti and Chakrabarti, 2000). In a series of studies, Arnab Chatterjee and Bikas Chakrabarti showed how wealth distribution can change from the Gibbs distribution to the Gamma distribution and further to the Pareto distribution by manipulating different saving behavior (Chatterjee, Chakrabartia, and Manna, 2004; Chatterjee, Chakrabarti, and Stinchcombe, 2005). The kinetic model, therefore, becomes the most parsimonious model which is able to account for the empirical phenomena of wealth distribution. Some economists, however, are very critical of this model partially due to its lack of a realistic description of economic behavior (Gallegati *et al.*, 2006b; Yakovenko and Rosser, 2009).

4.5.2 Percolation models

Percolation theory was invented by Paul Flory (1910–1985), who published the first percolation theory in 1941, to explain polymer gelation (Flory, 1941). Percolation theory has been applied by Rama Cont and Jean-Philipe Bouchaud to study the herding effect in financial markets (Cont and Bouchaud, 2000). Their model, known as the Cont–Bouchaud model, is probably the first agent-based model of a financial market built by explicitly taking into account the network effect. Despite its physical origin, the operation of this model can be interpreted mathematically as a *random graph* with a given probability that determines the existence of a link between any two points of the graph. This probability parameter, also called the percolation parameter, plays a critical role in this model as determining the distribution of the cluster size and the fluctuation of the price. Many further variations of this model and its applications to other fields, such as marketing, have been well surveyed in Samanidou *et al.* (2007).

4.5.3 Ising models

While Ising models, cellular automata, and percolation models originated from different physical observations, an equivalence relationship among the three can be established (Domany and Kinzel, 1984). After brief reviews of the applications of cellular automata and percolation models, we shall do the same here for Ising models. The Ising model originated from the dissertation of Ernst Ising (1900–1998). Ising studied a linear chain of magnetic moments, which are only able to take two positions or states, either up or down, and which are coupled by interactions between nearest neighbors. In physics, the model is strikingly successfully in the search for the transition between ferromagnetic and paramagnetic states. In addition to physics, the Ising model is also widely used in biology and the social sciences. In economics, Hans Follmer (Follmer, 1974) was the first to explicitly use an *Ising model* to model the social interactions of consumers and the resultant random but *interdependent* preferences. He showed that with the presence of even short-range interaction the microeconomic characteristics may no longer determine the macroeconomic phase. Other economic applications of the Ising model include financial markets (Iori, 1999, 2002; Sornett and Zhou, 2006) and tax evasion (Zaklan, Westerhoff, and Stauffer, 2009).

4.6 Further explorations

4.6.1 Historical notes

One question which one may encounter frequently but usually will not be given a definite answer is: who began the enterprise of ACE? One natural candidate is John von Neumann, due to his pioneering study on cellular automata (von Neumann, 1966). However, there is some evidence showing that, while originating his cellular automata, von Neumann might not have involved his understanding of economics in his masterpiece and not have thought about its possible economic applications.[23] Others, such as James Sakoda or Thomas Schelling, as mentioned in Section 4.1, are also good candidates, but their checkerboard models were not motivated by automata theory and hence may be viewed as a "serendipitously one-shot play" rather than a systematic intent to advance into a new paradigm. These considerations lead us to place Peter Albin as another possible candidate. His two essential articles on ACE (Albin, 1982, 1992), reprinted in Albin (1998), plus his first book (Albin, 1975), may qualify him for this position.

Albin (1982), reprinted as Chapter 2 of Albin (1998), may be considered to be the first article in the history of economic analysis to address the computable foundation of economics, while his treatment is less rigorous and less extensive compared to Velupillai (2000, 2010). Later on, in his preface to Albin (1998) and in the introductory chapter authored by Ducan Foley, they proposed a *Chomsky–Wolfram synthesis* as a framework to address the complexity in economics. In this effort, they were trying to find a thread passing through John von Neumann, Alan Turing, Noam Chomsky, Kurt Godel, John Conway, and Stephen Wolfram.

The thread, called the *automata-theoretic foundation* of economics, also nicely connects computer science, linguistics, and dynamic systems.

4.6.2 Further reading

Chapter 2 of Sigmund (1993) gives one of the best non-technical historical reviews of the scientific pursuit of cellular automata and artificial life. While the chapter mainly focuses on John Conway, the author treats John von Neumann as a key figure and elucidates how his research was affected by Kurt Godel, Alan Turing, and Stanislaw Ulam, and also summarizes his posthumous influence on the research on artificial life. John von Neumann's bibliography, written by Norman Macrae (Macrae, 1992), is also helpful reading to see a number of von Neumann's academic milestones; this is particularly helpful because von Neumann is usually only known for his work on game theory (von Neumann and Morgenstern, 1944) by most economists, while his contributions in his last decade of life, such as von Neumann (1958); von Neumann (1966), are largely ignored by economists.

Griffeath and Moore (2003) collect some recent advancements in cellular automata, covering life sciences, mathematics and logic, physics, economics, and art. Adamatzky (2010) is a collection updating the progress made on the Game of Life during the four decades following the publication of Gardner (1970).

Schelling's segregation model motivated a lot of follow-up studies, and has contributed quite significantly to the recent progress in urban and regional studies as well as the real estate pricing model (Card, Mas, and Rothstein, 2008; Zhang, 2011). It has also been used in models of financial markets; for a survey, see Kandhai, Qiu, and Sloot (2013). The Schelling model has become very influential in public policy pursuing integration. Dodge (2012) shows some "case studies" in Illinois and Singapore with regard to external intervention to prevent the segregation process, a process sensitive to the "tipping point" (see Dodge, 2012, Chapter 19).

Albin (1992), reprinted as Chapter 6 of Albin and Foley (1992), is an application of Conway's Game of Life to the multi-person prisoner's dilemma game, and by using the idea from the Game of Life he was able to find that rules can lead to the emergence of global cooperation, instead of universal defection.

4.6.3 Software implementation: NetLogo

Most agent-based models introduced in this chapter can be implemented in NetLogo.

- Segregation: http://ccl.northwestern.edu/netlogo/models/Segregation
- Game of Life: http://ccl.northwestern.edu/netlogo/models/Life
- Elementary cellular automata: http://ccl.northwestern.edu/netlogo/models/CA1DElementary
- Kinetics: http://ccl.northwestern.edu/netlogo/models/EnzymeKinetics
- Percolation: http://ccl.northwestern.edu/netlogo/models/Percolation
- Ising model: http://ccl.northwestern.edu/netlogo/models/Ising.

Simulation exercise: Segregation for more than two agents

Since Thomas Schelling's segregation model only considers two types of agents, what would happen if we were to change the parameters, for example by considering three, four, or five different types of agents? Do you think that the basic self-organized segregation pattern as observed in Schelling will remain unchanged?

Simulation exercise: Segregation when agents are heterogeneous in preference

The simulation of the Schelling model presented in this chapter assumes that all agents have the same tolerance threshold, say 0.5. What may happen if we assume that they have heterogeneous thresholds? This issue is in fact related to the other Schelling model, known as the *neighborhood tipping model* (Schelling, 1972).

Notes

1 Thompson (1917) shows how seemingly very different animals can actually be transformed into one another through the distortion of a coordinate system under the given physical constraints. By this, he argued that natural selection is not the sole explanation for evolution.

2 Von Neumann started a manuscript entitled "Theory of Automata: Construction, Reproduction and Homogeneity" in 1952. Up until his death in 1957, he was working on the notion that computers through their software could embody a set of rules or instructions that would enable them to reproduce their structure. This work was left in manuscript form at his death and then edited by his student and colleague Arthur Burks (von Neumann, 1966).

3 For a historical review of cellular automata, see the references provided in Section 4.6.

4 Schelling described the experience of this work as follows:

> What I did not know when I did the experiments with my twelve-year-old son using copper and zinc pennies was that what I was doing later became known as "agent-based computational models," or "agent-based computational economics."
>
> (Schelling, 2007, p. xi)

5 Edgar Codd (1923–2003), a PhD at the University of Michigan, took the self-reproduction of cellular automata as his thesis subject, and published the first book using *Cellular Automata* as the title (Codd, 1968). In this book, he showed that a system similar to von Neumann's could be constructed with only eight possible states.

6 It should be noticed that most cellular automata used in complex sciences do not have moving agents, so agents are identical to the cells in which they live.

7 See www.bitstorm.org/gameoflife/ for a demonstration of the game.

8 See Sigmund (1993) for a longer list of the names.

9 As Sigmund (1993) put it, the Game of Life is not a two-person game or a one-person game, but actually a no-person game; everybody in this game is just an *onlooker*, who only has to push the button (to initialize) and then watch the show.

10 Bill Gosper's glider gun can be seen at www.youtube.com/watch?v=pvUiA-Q-3hM&feature=related.

11 Quite coincidentally, the year that Conway proposed his wager on the limit of growth was also about the time the same inquiry was pursued by a group of economists and the well-known Club of Rome (Meadows, Randers, and Meadows, 2004). While von Neumann did not use his invented cellular automata to study economic growth, he did

propose a multi-sector model of economic growth in the context of general equilibrium in 1937 (von Neumann, 1945). The idea of *balanced growth* in von Neumann's growth model may be studied together with the growth phenomena observed in the Game of Life, such as the glider gun.

12 This should be clear when we come to social networks (Chapter 22).

13 Depending on the initial pattern, it is possible for the behavior of a single rule to fall into two different classes; hence the qualification *nearly all initial patterns* is added here.

14 For example, Rule 110 in this class has been shown to be able to perform universal computation, but Rule 54, also in this class, is unknown yet.

15 One has to note that all cellular automata with a finite number of cells eventually repeat themselves, even though the cycle can be astronomically large.

16 This motivates the name "edge of chaos," i.e., a slight change of the rule on this edge will either result in a stable pattern, say Class II, or a chaotic pattern, say Class III.

17 Richard Thaler used the term *choice architecture* to demonstrate how some changes in choice designs, layouts, etc. can significantly improve the quality of human decisions. See Thaler and Sunstein (2008).

18 In the 22 chapters devoted to a volume (Zenil, 2013) celebrating a decade since the publication of *A New Kind of Science*, some authors also propose a new kind of their own discipline, such as a new kind of finance (Maymin, 2013), a new kind of philosophy (Tagliabue, 2013), and a new kind of cosmology (Vidal, 2013).

19 Again, in plain English this setting is based on the intuition that to learn that someone you trust is actually lying to you is a very slow process in the beginning. The spirit of Bayesian learning is maintained in this way.

20 The uniform distribution assumption seems to be reasonable for a democratic society where agents' preference for the ruling party may have a quite wide range, from positive to neutral to negative.

21 70 percent is a sensible target if our application domain has something to do with a political election.

22 Velupillia (2007) made this point even more sharply and precisely. He showed an impossibility theorem on policy: basically, an effective theory of policy is impossible for a complex economy.

23 See, for example, Albin (1998), p. xv.

5 Economic tournament origin

In this chapter, through a series of work by Robert Axelrod on the repeated prisoner's dilemma tournament and the later work by John Rust on the Santa Fe double auction tournament, we shall begin to trace the origin of two essential elements of agent-based computational economics: *autonomous agents* and the *open coevolutionary game*. As we shall see, these two elements are intertwined. In fact, the open coevolutionary game cannot be nontrivially formulated without the notion of autonomous agents or novelty-discovering agents.

5.1 Novelty-discovering agents

Novelty-discovering agents are agents which, *with minimal external supervision*, are able to search for, to discover, and to take full advantage of hidden patterns, chances, and opportunities. With this capability, they of course can adapt to a changing environment. Due to their minimal dependence on external supervision, they are also called *autonomous agents* in agent-based computational economics. The essence of economics is the existence of these agents and their behavior. Various versions of new economics can be developed without the conventional device of fully rational agents or *Homo economicus*, but can hardly be convincing if agents are deprived of the capability to discover novelties.

It would probably be fair to say that the tools available for economists to build novelty-discovering agents or autonomous agents with a high degree of autonomy were rather limited before the early 1990s. In their pioneering paper introducing novelty-discovering agents to economics (Holland and Miller, 1991), John Holland and John Miller began with the following:

> Economic analysis has largely avoided questions about the way economic agents make choices when confronted by a perpetually novel and evolving world ... This is so ... because standard tools and formal models are ill-tuned for answering such questions. However, recent advances ... in the subdiscipline of artificial intelligence called machine learning, offer new possibilities.
>
> (Holland and Miller, 1991, p. 365)

The idea of novelty-discovering agents was introduced first to game theory, and then to economics through attempts at *tournament automation* pioneered by Robert Axelrod in his automated *iterated prisoner's dilemma* (IPD) tournament (Axelrod, 1987). It was later continued by Martin Andrews and Richard Prager (Andrews and Prager, 1994), two Cambridge engineers, in their automated double auction tournament, originally held by the Santa Fe Institute (Rust, Miller, and Palmer, 1993, 1994; see Section 9.3).

Tournament automation is designed to replace the man-made submissions to a tournament with machine-made (robot-made) submissions. It is preferable to automate tournaments because normally tournaments are designed to tackle open-ended questions. For example, in the aforementioned tournaments, the objective is to know the characterization of the viable strategies in the associated competing environments. However, there is no closed final answer to these questions; hence, running these tournaments in an open *evolutionary* or *coevolutionary* manner seems to be the only legitimate way to do it. This approach to studying social sciences and making social science computational has been well expounded by Robert Axelrod (Axelrod, 1997a).

To support this way of doing social sciences, Axelrod used genetic algorithms to automate his early organized iterated prisoner's tournament (Axelrod, 1984). Since then the genetic algorithm has been used as a tool to involve novelty-discovering agents in many agent-based computational economic models, including agent-based financial markets—see LeBaron (2006) for a survey. Later on, Andrews and Prager continued Axelrod's idea to automate double auction tournaments using genetic programming (Section 9.3). Genetic programming was then used by Shu-Heng Chen and his colleagues as a tool to build agent-based financial markets with novelty-discovering agents (Chapter 15; Chen and Yeh, 2001, 2002; Chen and Liao, 2005; Chen, Liao, and Chou, 2008).[1]

The reason for giving such a brief review is to point out that tournament automation is the origin of novelty-discovering agents, which are currently extensively used in agent-based computational economics.

5.2 Tournament-based economic analysis

Using the tournament approach to obtain insights into complicated dynamic games was initiated by Robert Axelrod at the University of Michigan (Axelrod, 1984). The tournament approach solicited entries from professionals and amateurs, each trying to develop a strategy for the prisoner's dilemma that would do well in the environment provided by all the submissions. Axelrod organized two rounds of the tournament. The first one was organized in 1979, and 14 entries were received with an extra one, a *zero-intelligence player*, being added.[2] The winner was Anatol Rapoport, who submitted the strategy TIT-FOR-TAT. The strategy is simple: it cooperates on the first iteration, then replicates whatever the opponents did on the previous iteration. In the second tournament, 62 entries were received. The winner was again TIT-FOR-TAT. These two competitions were reported in Axelrod (1984). To celebrate the 20th anniversary, the iterated prisoner's dilemma competition was rerun separately at the Congress of Evolutionary Computation in 2004 (June 19–23, Portland, OR, USA) and Computational Intelligence and Games in 2005 (April 4–6, Essex, UK). There were 223 entries in the 2004 event, and 192 entries in the 2005 event. The results were reported in Kendall, Yao, and Chong (2007).

Almost a decade after Axelrod's iterated prisoner's dilemma tournament, the same idea was applied once again to another familiar economic environment, the double auction market, in particular after intensive study through experiments with human subjects (Smith, 1991a). The first DA tournament was held at the Santa Fe Institute (SFI) in 1990. A share of $10,000 was offered to the writers of algorithms that could perform well in a double auction competition. The tournament invited participants to submit trading strategies (programs) and tested their performance relative to other submitted programs in the Santa Fe Token Exchange, an artificial market which is operated by the double auction mechanism. A total of 25 different programs based on various design principles was proposed, and the best-performing one was the Kaplan program, submitted by Todd Kaplan, then a student at the University of Minnesota.

Through analysis of the winners' programs in these agent-based tournaments, one may gain insights into and acquire knowledge of the viable strategies. We call this tournament approach *programmed-agent-inspired economic analysis*, as also demonstrated in Figure 5.1 (left panel). The examples of extracted knowledge are TIT-FOR-TAT in the prisoner's dilemma tournament and *wait in the background* in the double auction tournament. Nevertheless, one limitation of these agent-based tournaments is that they are basically *closed* in the sense that re-entries and new entries are not permitted. Not only are the human-written programs, once submitted, not able to be revised and resubmitted, but new submissions are also not acceptable. Under such circumstances, one cannot help but wonder whether we could gain different insights had we been given a different set of submissions. This issue has been noted from the beginning of the tournament approach. For example, Robert Axelrod once wondered "whether the amount of cooperation I observed was due to the prior expectations of the people who submitted the rules" (Axelrod, 1997a, p. 6). This thought has been even more systemically raised in

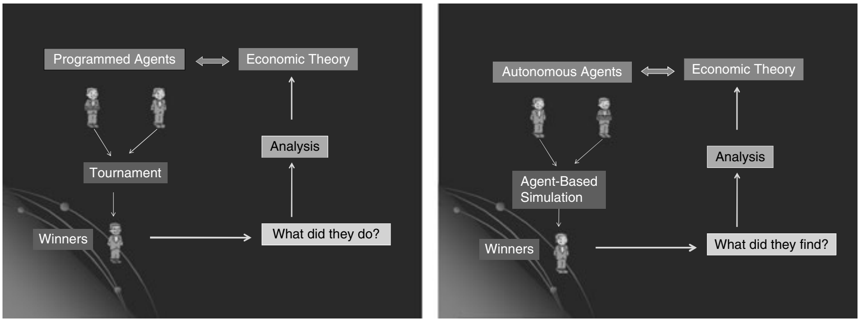

Figure 5.1 Economic analysis inspired by programmed agents and autonomous agents.

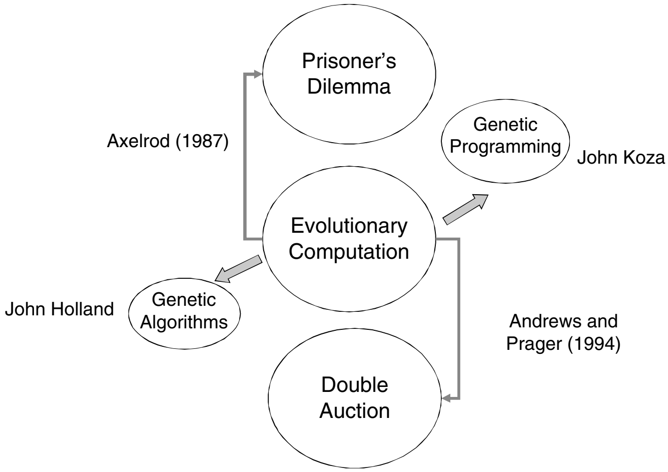

Figure 5.2 Automated open tournaments.

Note: The diagram shows that genetic algorithms and genetic programming, two branches of evolutionary computation, have been applied respectively by Robert Axelrod (Axelrod, 1987) and Andrews and Prager (Andrews and Prager, 1994) to automate the iterated prisoner's dilemma tournament and the double auction tournament.

analysis of the double auction tournament (Rust, Miller, and Palmer, 1993). There, they stated:

> The open question is whether strategies exist that are capable of dominating Kaplan's "wait in the background" strategy over a nontrivial range of environments. If we were to run another DA tournament, it seems likely that entrants would attempt to beat Kaplan by developing more sophisticated delay and "endgame" strategies rather than reverting to truthtelling mode after a fixed amount of time.
>
> (Rust, Miller, and Palmer, 1993, p. 192)

One solution to this conundrum is to institutionalize the tournament so that it can be constantly run and rerun. One example is the *trading agent competition*, held annually since the year 2000 (Wellman, Greenwald, and Stone, 2007). However, this may demand more commitment and devotion. A convenient alternative to introducing an *open* tournament is to *automate* the submissions, as shown in Figure 5.2. This attempt was again initiated by Robert Axelrod (Axelrod, 1987), which led to the idea of autonomous agents.[3]

5.3 Automated open tournaments

Using computers to generate something which is nontrivial is certainly not a novel experience. For mathematical logicians, the familiar automatic theorem proving, as endorsed by Kurt Godel's well-known completeness theorem, is one example.

In this spirit, if a theorem that can be proven by a human can also be proven by a computer, then the idea is to replace humans with computers to generate a set of submissions for the tournaments.[4] By so doing, one can save laborious efforts by humans and hence facilitate automation in an open tournament. This is basically what Robert Axelrod did in his pioneering application of autonomous agents in the social sciences.

With the participation of autonomous agents, Robert Axelrod considered two conceptually distinct styles of open IPD tournaments, which later on became stereotypes for automated open tournaments. The first style was based on matching autonomous agents with a fixed set of human-written programmed agents (Figure 5.3), whereas the second style was based on leaving autonomous agents to play against each other. To distinguish the two, we shall call the former an *evolutionary tournament* and the latter a *coevolutionary tournament*. In an evolutionary tournament, autonomous agents are adapted to a fixed environment, and the most effective strategy can be considered to be the optimal strategy, *optimal with respect to the fixed set of opponents*. The use of a genetic algorithm in this case is more like the application of a numerical optimization algorithm that searches for the optimal solution. In a coevolutionary tournament, each agent adapts to the adaptations of other agents. The process may not even converge, and the effective strategy can change over time.

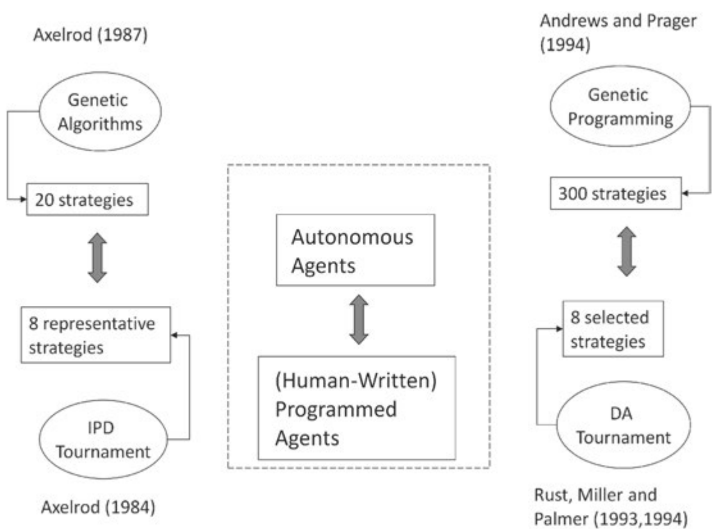

Figure 5.3 Automated evolutionary tournament.

Notes: The evolutionary tournament is one form of open tournament, which seeks to match the autonomous agents with a fixed set of human-written programs, as shown in the middle of the figure. The figure also shows the earliest automated evolutionary IPD tournament and DA tournament, conducted respectively by Robert Axelrod (Axelrod, 1987) and Andrews and Prager (Andrews and Prager, 1994).

Axelrod presents the results of both styles of tournaments. In his evolutionary tournament, it was found, with a little surprise, that TIT-FOR-TAT was no longer the most effective strategy with respect to a fixed set of eight representative human-written programs which were retrieved from the 62 entries in his second IPD tournament (Figure 5.3).[5] What were found to be even more effective were the autonomous agents, who are able to distinguish the exploitable representatives from other less exploitable representatives, and then to exploit the former at the cost of having less cooperation with the latter, but still netting the two with a positive gain. Of course, without a fixed set of opponents and a sufficiently large number of repetitions, this kind of strategy is not readily available. Nevertheless, its existence indicates that the effective strategy derived from the closed tournament may not be complete.

From Axelrod's evolutionary tournaments, we can more easily see the meaning and the role of autonomous agents in economics. Autonomous agents are purposive. To achieve their purposes, they can discover and exploit hidden patterns and opportunities to some extent *without direct external intervention from humans (model builders)*. In other words, these agents are able to learn and adapt *on their own*. In Axelrod's case above, the autonomous agents were constantly looking for chances, opportunities, and patterns; hence, it is probably just a matter of time before a way to outperform TIT-FOR-TAT or any other fixed strategy will be found.

Then, in his simulation of the coevolutionary IPD tournaments, the capability of the agents to adapt to the changing environment is further illustrated. In this case, each autonomous agent plays the IPD with each of the 20 agents including its own twin rather than the eight representative human-written programs. Each agent's discovery and exploitation of the environment may bring in a new environment which in turn creates new opportunities for other opponents for further discovery and exploitation. This cycle can be indefinite and echoes well Alfred Marshall's biological description of the economy as "constantly changing" (Marshall, 1924):

> Economics, like biology, deals with a matter, of which the inner nature and constitution, as well as outer form, are constantly changing.
>
> (Marshall, 1924, p. 772)

When programmed agents are replaced by autonomous agents in an open coevolutionary environment, the economic theory originally inspired by the programmed agents can now be further inspired by the autonomous agents. Figure 5.1 (right panel) demonstrates the idea of *autonomous-agent-inspired economic theory*. While this idea has been carried out several times, few have actually completed the circle and asked what was found by these agents in the end. Is there any finding that is both surprising and sensible?

One difficulty is that it is not easy to translate the syntax rules that have evolved into their semantic expressions and to extract knowledge from the surviving computer-generated programs. The difficulty can be multiplied when the environment is coevolutionary, where the acquired rules (knowledge) constantly depend on what other agents have learned and can be mixed with many kinds of luck or noise. Unless we are fortunate enough to get into some stable states, such as Nash equilibria, it would be hard to make sense of such transient knowledge in the constantly coevolving environment.

5.4 Concluding remarks

One complete implementation of autonomous-agent-inspired economic theory was provided by Chen and Yu (2011). In a double auction environment, they dispatched the autonomous agents to play a set of truth-tellers, who reveal their reservation price during the bargaining process. They then asked what are the best bargaining rules found in this theoretical environment. Using genetic programming to evolve these autonomous agents in a large number of iterations and under different market topologies, they found a general rule called the *optimal procrastination rule*.[6]

To understand whether these rules are applicable to real human behavior, Chen, Shih, and Tai (2012) further set up human-subject experiments to see whether subjects (mainly students) can learn what the autonomous agents found, i.e., the optimal procrastination rules. Not surprisingly, the behavior of human subjects is quite heterogeneous. They differ in the capability to learn this rule, and also differ in the time required to learn the rule. Chen, Shih, and Tai (2012) provided a further test as to what may cause this difference by examining the causal relationship between the human subjects' cognitive capacity and learning performance, and the analysis was further extended to their personality.

Up to this point, we have seen the involvement of human-subject experiments as a further extension of the tournament origin. As we shall see in the next chapter, human-subject experiments in economics or experimental economics have served as another important origin for agent-based modeling. As we have already mentioned in Section 2.2.3, agent-based computational economics can be considered not only as a software replication of human behavior but also as a scaled-up version of human-subject experiments. Besides, human subjects can be involved as an enhancement for the design of agent-based modeling. The latter enhancement technique is also known as *participatory agent-based simulation* (Ramanath and Nigel Gilbert, 2004). Hence, we now turn to the fourth origin of agent-based computational economics, the economic-experiment origin.

Notes

1 As in the earlier chapter on the market origin, we shall see again that genetic algorithms and genetic programming are extensively used in the studies reviewed in this chapter. A more systematic introduction to this technical background will be given in Chapter 13.

2 Axelrod did not use the term zero-intelligence, but the idea is the same. It is a randomly behaving agent who defects or cooperates with equal probability. See Chapter 8 for a comprehensive treatment of the zero-intelligence agent.

3 The work by Andrews and Prager on the double auction tournament (Andrews and Prager, 1994) is in parallel to the work by Axelrod on the iterated prisoner's dilemma

tournament, as shown in Figure 5.2 here and in Figure 5.3 later. They both used state-of-the-art technology to automate the existing tournament. However, since Andrews and Prager (1994) is also a pioneering study on the agent-based double auction market, in fact, the first application of genetic programming to double auction markets, we shall leave it to Chapter 9 along with discussions of other models of agent-based double markets.

4 Of course, this is still a very idealized situation, because by the completeness theorem the resources (time and space) are assumed to be unbounded.

5 At the beginning, Axelrod might have been surprised by this result, as he wrote:

> Because TIT FOR TAT had done best in the computer tournament itself, I did not think that it would be possible to do much better with an evolutionary process. But as noted earlier, in about a quarter of the simulation runs with sexual reproduction, the population did indeed evolve substantially better strategies—strategies that were quite different from TIT FOR TAT.
>
> (Axelrod, 1997a, p. 26)

6 See Section 9.4 for further details.

6 Agent-based modeling of economic experiments

While a large number of agent-based economic and social models are built upon the cellular automaton as it illustrates that complex patterns can be formed from the working of simple rules, a separate development of the ACE literature without reference to CA appears in the middle of the 1990s. This line, which we shall survey in this chapter, is mainly motivated by *economic experiments*, and we shall call it the *fourth origin of agent-based modeling in economics*.

Instead of demonstrating that complex patterns may emerge from the workings of simple rules, the attempt of this development is to replicate, and hence possibly explain, the outcomes of some economic experiments with human subjects. This step is certainly a milestone in the short history of agent-based modeling, because, in the spirit of the Turing test (Turing, 1950), this is what we may expect from artificial economics.[1] This wave of ACE, therefore, functions by providing an algorithmic foundation for experimental economics.

Macroeconomic experiments

In this chapter, three early examples are provided. They are all agent-based models of human-subject experiments. These experiments were performed in a very familiar economic context: one was from the *cobweb model*, and the other two were from the *overlapping generations model*. Within these familiar theoretical models, a number of *macroeconomic experiments* have been conducted since the late 1980s. These experiments are not macro in the sense of large scale, but more in the sense of their *microfoundations*.[2] They have involved the laboratory approach to studying the learning, adaptation, expectation, and coordination behaviors of human subjects, and to seeing the possible implications for industrial and macroeconomic dynamics, including the market price, inflation, and the exchange rate.

Agent-based models of microeconomic experiments also started at almost the same time (the mid 1990s), even though microeconomic experiments have a longer history than macro ones. However, we will leave the introduction to this development to Part III. With this separation, Chapter 6 will mainly focus on the *mirroring function* of agent-based models in relation to economic experiments, i.e., how the agent-based model using *heterogeneous agents* can develop a plausible alternative economic theory. While the necessity of replacing homogeneous agents with heterogeneous agents has long been discussed in economic theory (see also the introduction to Part II), this chapter has a very sharp focus on the following research question: *Can the heterogeneous-agent (multi-agent) system be reduced to the homogeneous-agent (single-agent) system?*

This question is critical because the motto of agent-based modeling, particularly when it is applied to the social sciences, is that *details matter* (see Section 4.3). In fact, it is the attempt to take into account these details that distinguishes agent-based modeling from the conventional equation-based modeling, which is particularly evident in ecology and epidemiology (Section 2.3). When these details represented by more dimensions of individuals are considered, very naturally, agents are heterogeneous. Although a rigorous analysis is still lacking, this chapter tends to indicate that by using heterogeneous agents one can better replicate the macroeconomic dynamics observed in economic experiments.

6.1 Agent-based modeling of cobweb experiments

6.1.1 The cobweb model

The cobweb model is a familiar playground in which to investigate the effects of production decisions on price dynamics. In this model consumers base their decisions on the current market price, but producers decide how much to produce based on past prices. Agricultural commodities serve as a good example of the cobweb model.[3] This model plays an important role in macroeconomics, because it is the place in which the concept of rational expectations originated (Muth, 1961).[4] Moreover, it is also the first neoclassical macroeconomic prototype to which an agent-based computational approach was applied (Arifovic, 1994). This section will first briefly formulate the cobweb model and then review the work on the agent-based modeling of the cobweb model.

Consider a competitive market composed of n firms which produce the same goods by employing the same technology and which face the same cost function described in Equation (6.1):

$$c_{i,t} = xq_{i,t} + \frac{1}{2}ynq_{i,t}^2, \tag{6.1}$$

where $q_{i,t}$ is the quantity supplied by firm i at time t, and x and y are the parameters of the cost function. Since at time $t-1$ the price of the goods at time t, P_t, is not available, the decision about the optimal $q_{i,t}$ must be based on the expectation (forecast) of P_t, i.e., $P_{i,t}^e$. Given $P_{i,t}^e$ and the cost function $c_{i,t}$, the expected profit of firm i at time t can be expressed as follows:

$$\pi_{i,t}^e = P_{i,t}^e q_{i,t} - c_{i,t}. \tag{6.2}$$

Given $P_{i,t}^e$, $q_{i,t}$ is chosen at a level such that $\pi_{i,t}^e$ can be maximized and, according to the first-order condition, is given by

$$q_{i,t} = \frac{1}{yn}(P_{i,t}^e - x). \tag{6.3}$$

Once $q_{i,t}$ is decided, the aggregate supply of the goods at time t is fixed and P_t, which sets demand equal to supply, is determined by the demand function:

$$P_t = A - B \sum_{i=1}^{n} q_{i,t}, \tag{6.4}$$

where A and B are parameters of the demand function.

Given P_t, the actual profit of firm i at time t is:

$$\pi_{i,t} = P_t q_{i,t} - c_{i,t}. \tag{6.5}$$

The neoclassical analysis simplifies the cobweb model by assuming the homogeneity of market participants, i.e., a representative agent. In such a setting, it can be shown that the homogeneous rational expectations equilibrium price (P^*) and quantity (Q^*) are (Chen and Yeh, 1996, p. 449):

$$P_t^* = \frac{Ay + Bx}{B + y}, \quad Q_t^* = \frac{A - x}{B + y}. \tag{6.6}$$

6.1.2 The cobweb theorem and cobweb experiments

The neoclassical analysis based on homogeneous agents provides us with a limited understanding of the price dynamics or price instability in a real market, since firms' expectations of the prices and the resultant production decisions must in general be heterogeneous. A question typically asked in the 1980s was: would actual price converge to the homogeneous rational expectations equilibrium (HREE) price, even though agents are bounded rational and may not follow rational expectations?[5] Earlier studies show that in general the market will not converge to the HREE (Ezekiel, 1938; Bray, 1982; Marcet and Sargent, 1989). These studies indicate that, depending on the so-called *cobweb ratio*, the market dynamics can be separated into the *stable* case and the *unstable* case. Using our notations introduced above, we can write down the stability condition as follows:

$$B/y \begin{cases} < 1, & \text{price dynamics stable,} \\ \geq 1, & \text{price dynamics unstable.} \end{cases} \qquad n = 1, 2, \ldots, N. \tag{6.7}$$

B appearing in Equation (6.4) refers to the demand sensitivity of price to quantity, and y appearing in Equation (6.3) to the supply sensitivity of price to quantity. If the former is smaller than the latter, then the price will converge to the HREE price via the proposed learning dynamics; otherwise, it will not.

Cobweb experiments

However, the theoretic finding based on the representative agent's learning dynamics did not predict the laboratory results well. Stable results for parameterizations that would result in an unstable price (explosive price) are frequently reported in cobweb experiments (Carlson, 1967; Wellford, 1989; Johnson and Plott, 1989), which failed the prediction based on the proposed learning dynamics of the representative agent. For example, one of the most frequently cited cobweb experiments, conducted by Charissa Wellford (Wellford, 1989), consisted of the implementation of 12 experimental sessions, each involving 5 participants with a duration of 30 periods. She simulated the stable and unstable cases. Her results showed that the unstable case did not result in divergent behavior as the above-mentioned learning dynamics predicted. Instead, the market price converged toward the HREE, even though the price path variance in the unstable case was greater than that in the stable case.

The results of the cobweb experiments beg the question: *What makes the market stable?* Can we replicate the cobweb experimental results with bounded rational agents, at least qualitatively? This question was first addressed by Jasmina Arifovic, a Chicago PhD, in the mid 1990s (Arifovic, 1994). Two years later, Chen and Yeh (1996b) also replicated this result, with a different setup. What is shared in these two studies is a market composed of agents (firms) with initially heterogeneous decisions (quantities) or beliefs (prices). These agents then learn and adapt via a social or individual learning process driven by *evolutionary algorithms* (Chapter 13), such as *genetic algorithms* (Section 13.4) or *genetic programming* (Section 13.4).

6.1.3 Agent-based cobweb models: genetic algorithms

Using genetic algorithms to model the adaptive behavior of firms' production, Arifovic (1994) proposed the first agent-based model of the cobweb model. We do not intend to provide a full-fledged version of the genetic algorithm here (see Section 13.4 for the details). We only want to highlight some points with regard to its relevance to building *an agent-based model of industry dynamics*.

The genetic algorithm is a population-based stochastic search algorithm. The population is composed of a number of *chromosomes*. In different applications, these chromosomes represent different things. For example, in Arifovic's case, each chromosome simply represents a *number*, i.e., a quantity-supplied decision made by the firm. Hence, the population can be regarded as a pool of all firms' decisions on the quantity supplied. Denote this population by POP_t as a collection of all firms' supply in time t, i.e.,

$$POP_t = \{q_{1,t}, q_{2,t}, \ldots, q_{n,t}\}. \tag{6.8}$$

Some firms decide to produce a large quantity, whereas some firms decide to supply a small quantity. The heterogeneity of firms is characterized by the range and diversity of the quantity supplied in this population.

The population is time variant and is indexed by t; it evolves as

$$POP_t \rightarrow POP_{t+1} \rightarrow POP_{t+2} \rightarrow \cdots \tag{6.9}$$

The evolving population is driven by *selection*, namely, by following *the survival of the fittest principle*. Under this principle, decisions contributing to making profits prosper, while decisions leading to loss will be supplanted or revised. population then represents the learning process of the entire industry.

$$POP_t \Rightarrow POP_{t+1} \Rightarrow POP_{t+2} \Rightarrow \ldots$$
$$POP_t : (q_{1,t}, q_{2,t}, \ldots, q_{n,t}) \rightarrow \sum_{i=1}^{n} q_{i,t} = Q_t \rightarrow P_t$$
$$\Downarrow$$
$$(\pi_{1,t}, \pi_{2,t}, \ldots, \pi_{n,t})$$
$$\Downarrow$$
$$POP_{t+1} : (q_{1,t+1}, q_{2,t+1}, \ldots, q_{n,t+1}) \rightarrow \sum_{i=1}^{n} q_{i,t+1} = Q_{t+1} \rightarrow P_{t+1}$$
$$\Downarrow$$
$$(\pi_{1,t+1}, \pi_{2,t+1}, \ldots, \pi_{n,t+!})$$
$$\Downarrow$$
$$POP_{t+2} : (q_{1,t+2}, q_{2,t+2}, \ldots, q_{n,t+2})$$

Figure 6.1 A sketch of market dynamics by genetic algorithms.

The details of the industrial dynamics, Equation (6.9), are given by a number of genetic operators. The most basic one is *reproduction* or *imitation*, which can make Equation (6.9) proceed in a manner similar to *replicator dynamics* or *Boltzmann–Gibbs molecular dynamics*. Decisions leading to higher profits will become popular, whereas decisions leading to larger losses will become extinct. An additional two operators, *crossover* and *mutation*, will generate new decisions on the basis of old ones (experience). Leaving behind the technical details, what underlies the operation of these genetic operators are the intensive social interactions that are required for idea exchange and imitation. Hence, it is also referred to as *social learning*. A sketch of the process is given in Figure 6.1.

Arifovic (1994) applied two versions of genetic algorithms to her agent-based model. The *basic GA* involves three genetic operators: reproduction, crossover, and mutation. Arifovic found that in each simulation of the basic GA, individual quantities and prices exhibited fluctuations for the entire duration and did not result in convergence to the rational expectations equilibrium values, which is quite inconsistent with experimental results with human subjects (Wellford, 1989). To enhance the stability and the convergence of the price dynamics, she added the *election operator* to the basic GA. This *augmented GA* then works as expected.[6]

The results of the simulations show that the augmented GA converges to the rational expectations equilibrium values for all sets of cobweb model parameter values, including both stable and unstable cases, and can capture several features of the experimental behavior of human subjects better than other simple learning algorithms.

6.1.4 Agent-based cobweb models: genetic programming

Arifovic did not explicitly deal with firms' formation of expectations; instead, in her model, firms' expectations are implicitly encapsulated into firms' production decisions. To directly work on firms' expectations, one has to start with how expectations are formed or what expectations are based on. A standard answer is to consider a firm's looking at the past market price and trying to figure out the coming one. Technically, we assume that each firm is initially given an arbitrary forecasting function of price as follows:

$$P^e_{i,t} = f_{i,t}(P_{t-1}, P_{t-2}, \ldots), \quad i = 1, 2, \ldots, n. \tag{6.10}$$

As time goes on, each agent will change her own forecasting by learning from herself (individual learning) or learning from others (social learning). The learning dynamics can be simply represented as follows:

$$\begin{bmatrix} P^e_{1,t} \\ P^e_{2,t} \\ \vdots \\ P^e_{n,t} \end{bmatrix} \rightarrow \begin{bmatrix} P^e_{1,t+1} \\ P^e_{2,t+1} \\ \vdots \\ P^e_{n,t+1} \end{bmatrix} \rightarrow \begin{bmatrix} P^e_{1,t+2} \\ P^e_{2,t+2} \\ \vdots \\ P^e_{n,t+2} \end{bmatrix} \rightarrow \cdots \tag{6.11}$$

With their expectations of the price in the forthcoming period, firms can then decide their quantity in an optimum manner by Equation (6.3). The rest of the dynamics are then the same as seen in the previous section. Figure 6.2 demonstrates the equivalent dynamics; notice that the only difference is that we now add a middle step to directly model the firms' price expectations before their decision on quantity.

The question here concerns how to provide the learning dynamics of firms' price expectations. While this question is the same in spirit as the one which Arifovic (1994) presented (Section 6.1.3), technically, it is rather different. The one dealt with by Arifovic is basically *numbers*, which evolve from a set of numbers to another

$$POP_t \Rightarrow POP_{t+1} \Rightarrow POP_{t+2} \Rightarrow \ldots$$

$$POP_t : (P^e_{1,t}, P^e_{2,t}, \ldots, P^e_{n,t})$$

$$\rightarrow (q_{1,t}, q_{2,t}, \ldots, q_{n,t}) \rightarrow \sum_{i=1}^{n} q_{i,t} = Q_t \rightarrow P_t$$

$$\Downarrow$$

$$(\pi_{1,t}, \pi_{2,t}, \ldots, \pi_{n,t})$$

$$\Downarrow$$

$$POP_{t+1} : (P^e_{1,t+1}, P^e_{2,t+1}, \ldots, P^e_{n,t+1})$$

$$\rightarrow (q_{1,t+1}, q_{2,t+1}, \ldots, q_{n,t+1}) \rightarrow \sum_{i=1}^{n} q_{i,t+1} = Q_{t+1} \rightarrow P_{t+1}$$

$$\Downarrow$$

$$(\pi_{1,t+1}, \pi_{2,t+1}, \ldots, \pi_{n,t+!})$$

$$\Downarrow$$

$$POP_{t+2} : (P^e_{1,t+2}, P^e_{2,t+2}, \ldots, P^e_{n,t+2})$$

Figure 6.2 A sketch of market dynamics by genetic programming.

set of numbers. However, the one presented here involves *functions*, which evolve from a set of functions to another set of functions. This question becomes rather challenging when these functions are imposed with no restrictions on their size and shape. In fact, most studies of learning in the economics literature deal only with parametric models in which both the size and the shape are fixed; few actually suggest what to do when learning can generally happen without such tight restrictions.

However, as we shall see more in Chapter 13, this issue is not that easily approachable without the help of *formal language theory* or *automata theory*. In fact, one effective tool to powerfully present how this general learning can be possibly formalized is genetic programming, which can be regarded as an application of *context-free grammar* in an evolutionary manner. The application of genetic programming to the cobweb model started from Chen and Yeh (1996b). In Chen and Yeh (1996b), the learning of the population of firms is driven by genetic programming, as shown in Figure 6.3. From one generation, say, $\mathbf{P^e}_t$, to the next generation, $\mathbf{P^e}_{t+1}$, the same set of genetic operators, namely reproduction, crossover, and mutation, is applied here, although the implementation details are different from the usual binary coding or real coding representation used in genetic algorithms.

Chen and Yeh (1996) compared the learning performance of GP-based learning agents with that of GA-based learning agents. They found that, like GA-based learning agents, GP-based learning agents can also learn the homogeneous rational expectations equilibrium price under both the stable and unstable cobweb case. However, the phenomenon of price euphoria, which did not occur in Arifovic (1994), does show up quite often in the early stages of the GP experiments. This is mainly because agents in their setup were initially endowed with very limited information as compared to Arifovic (1994). Nevertheless, GP-based learning can quickly coordinate agents' beliefs so that the emergence of price euphoria is only temporary. Furthermore, unlike Arifovic (1994), chen and Yeh (1996) did not use the election operator. Without the election operator, the rational expectations equilibrium is exposed to potentially persistent perturbations due to agents' adoption of the new, but untested, rules. However, what shows up in chen and Yeh (1996) is that the market can still bring any price deviation back to equilibrium. Therefore, the self-stabilizing feature of the market, known as the invisible hand, is more powerfully replicated in their GP-based artificial market.

$$\mathbf{P^e_t} \xrightarrow{\text{GP}} \mathbf{P^e_{t+1}} \xrightarrow{\text{GP}} \mathbf{P^e_{t+2}} \xrightarrow{\text{GP}} \cdots$$

$$\begin{bmatrix} P_t \\ \mathbf{P^e_t} \end{bmatrix} \rightarrow \begin{bmatrix} P_{t+1} \\ \mathbf{P^e_{t+1}} \end{bmatrix} \rightarrow \begin{bmatrix} P_{t+2} \\ \mathbf{P^e_{t+2}} \end{bmatrix} \rightarrow \cdots$$

where

$$\mathbf{P^e_t} = \begin{bmatrix} P^e_{1,t} & P^e_{2,t} & \ldots & P^e_{n,t} \end{bmatrix}$$

Figure 6.3 Genetic programming and firms' learning dynamics.

The self-stabilizing feature of the market demonstrated in chen and Yeh (1996) was further tested with two complications. In the first case, Chen and Yeh (1997b) introduced a population of speculators to the market and examined the effect of speculation on market stability. In the second case, the market was perturbed with a structural change characterized by a shift in the demand curve, and Chen and Yeh (2000) then tested whether the market could restore the rational expectations equilibrium. The answer to the first experiment is generally negative, i.e., speculators do not enhance the stability of the market. On the contrary, they destabilize the market. Only in special cases when trading regulations, such as the transaction cost and position limit, were tightly imposed could speculators enhance the market stability. The answer for the second experiment is, however, positive. Chen and Yeh (2000) showed that GP-based adaptive agents could detect the shift in the demand curve and adapt to it. Nonetheless, the transition phase was nonlinear and nonsmooth; one can observe slumps, crashes, and bursts in the transition phase. In addition, the transition speed is uncertain. It could be fast, but could be slow as well.

This series of studies on the cobweb model enriches our understanding of the self-stabilizing feature of the market. The market has its limit, beyond which it can become unstable with crazy fluctuations. However, imposing trading regulations may relax the limit and enhance market stability. One is still curious to know where the self-stabilizing capability comes from in the first place. Economists have known for a long time that it comes from the free competition principle, or the survival-of-the-fittest principle. In GA or GP, this principle is implemented through *selection pressure*. Chen (1997b) studied the role of selection pressure by replacing the usual proportionate selection scheme with one based on an approximate uniform distribution, showing that if selection pressure is removed or alleviated, then the self-stabilizing feature is lost. In a word, selection pressure plays the role of the invisible hand in economics.

It is interesting to know whether the time series data generated by the artificial market can replicate some dynamic properties observed in the real market. Chen and Kuo (1999) and Chen and Yeh (2000) started the analysis of the time series data generated from the artificial market. The time series data employed were generated by simulating the agent-based cobweb model with the presence of speculators. It was found that many stylized features well documented in financial econometrics can in principle be replicated from GP-based artificial markets, including leptokurtosis, non-IIDness, and volatility clustering. Furthermore, Chen and Yeh (2000) performed a CUSUMSQ test, a statistical test for structural change, on the data. The test indicated the presence of structural changes in the data, which suggested that the complex interaction process of these GP-based producers and speculators can even generate endogenous structural changes.

6.2 Agent-based modeling of inflation experiments

While there are several approaches to introducing dynamic general equilibrium structures to economics, the overlapping generations model (hereafter, OLG) may be regarded as the most popular in current macroeconomics. Over the last three decades, the OLG model has been extensively applied to studies of savings, bequests, the demand for assets, prices of assets, inflation, business cycles, economic growth, and the effects of taxes, social security, and budget deficits. In the following, we shall first provide a brief illustration of a simple OLG model of inflation, a *two-period* OLG model.

6.2.1 Two-period overlapping generations model

A simple OLG model can be described as follows. It consists of overlapping generations of two-period-lived agents. At time t, N young agents are born. Each of them lives for two periods $(t, t+1)$. At time t, each of them is endowed with e^1 units of a perishable consumption good, and with e^2 units at time $t+1$ $(e^1 > e^2 > 0)$. Presumably e^1 is assumed to be greater than e^2 in order to increase the likelihood (not ensure) that agents will choose to hold money from period 1 to 2 so as to push value forward. An agent born at time t consumes in both periods. The term c_t^1 is the consumption in the first period (t); c_t^2, that in the second period $(t+1)$. All agents have identical preferences given by

$$U(c_t^1, c_t^2) = \ln(c_t^1) + \ln(c_t^2). \tag{6.12}$$

In addition to the perishable consumption good, there is an asset called *money* circulating in the society. The nominal money supply at time t, denoted by H_t, is exogenously determined by the government and is held distributively by the old generation at time t. For convenience, we shall define h_t as $\frac{H_t}{N}$, i.e., the nominal per capita money supply.

This simple OLG gives rise to the following agent's maximization problem at time t:

$$\max_{(c_{i,t}^1, c_{i,t}^2)} \quad \ln(c_{i,t}^1) + \ln(c_{i,t}^2) \tag{6.13}$$

$$\text{s.t.} \quad c_{i,t}^1 + \frac{m_{i,t}}{P_t} = e^1, \quad c_{i,t}^2 = e^2 + \frac{m_{i,t}}{P_{t+1}}, \tag{6.14}$$

where $m_{i,t}$ represents the nominal money balances that agent i acquires in time period t and spends in time period $t+1$, and P_t denotes the nominal price level in time period t. Since P_{t+1} is not available in period t, what agents can actually do is to maximize their expected utility $E(U(c_t^1, c_t^2))$ by regarding P_{t+1} as a random variable, where $E(.)$ is the expectation operator. Because of the special nature of the utility function and budget constraints, the first-order conditions for this expected utility maximization problem reduce to the certainty equivalence form (6.15):

$$c_{i,t}^1 = \frac{1}{2}(e^1 + e^2 \pi_{i,t+1}^e), \tag{6.15}$$

where $\pi^e_{i,t+1}$ is agent i's expectation of the inflation rate $\pi_{t+1}(\equiv \frac{P_{t+1}}{P_t})$. This solution tells us the optimal decision for savings for agent i given her expectation of the inflation rate, $\pi^e_{i,t+1}$.

Suppose the government deficit G_t is all financed through seigniorage and is constant over time ($G_t = G$). We can then derive the dynamics (time series) of the nominal price $\{P_t\}$ and inflation rate $\{\pi_t\}$ from Equation (6.15). To see this, let us denote the savings of agent i at time t by $s_{i,t}$. Clearly,

$$s_{i,t} = e^1 - c^1_{i,t}. \tag{6.16}$$

From Equation (6.14), we know that

$$m_{i,t} = s_{i,t} P_t, \quad \forall i, t. \tag{6.17}$$

In equilibrium, the nominal aggregate money demand must equal nominal money supply, i.e.,

$$\sum_{i=1}^{N} m_{i,t} = H_t = H_{t-1} + G \cdot P_t, \quad \forall t. \tag{6.18}$$

The second equality of Equation (6.18) says that the money supply in period t is the sum of the money supply in period $t-1$ and the nominal deficit in period t, $G \cdot P_t$. This equality holds, because we assume that the government deficits are all financed by seigniorage.

Summarizing Equations (6.17)–(6.18), we obtain

$$\sum_{i=1}^{N} s_{i,t} P_t = \sum_{i=1}^{N} s_{i,t-1} P_{t-1} + G \cdot P_t. \tag{6.19}$$

The price dynamics are hence governed by the following equation:

$$\pi_t = \frac{P_t}{P_{t-1}} = \frac{\sum_{i=1}^{N} s_{i,t-1}}{\sum_{i=1}^{N} s_{i,t} - G}. \tag{6.20}$$

Now suppose that each agent has perfect foresight, i.e.,

$$\pi^e_{i,t} = \pi_t, \quad \forall i, t. \tag{6.21}$$

By substituting the first-order condition (6.15) into Equation (6.19), the paths of equilibrium inflation rates under perfect foresight dynamics are then

$$\pi_{t+1} = \frac{e^1}{e^2} + 1 - \frac{2g}{e^2} - \left(\frac{e^1}{e^2}\right)\left(\frac{1}{\pi_t}\right), \tag{6.22}$$

where $g = \frac{G}{N}$ is the real per capita deficit.

At the steady state ($\pi_{t+1} = \pi_t$), Equation (6.22) has two real stationary solutions (fixed points), a low-inflation stationary equilibrium, π_L^*, and a high-inflation one, π_H^*, given by

$$\pi_L^* = \frac{1 + \frac{e1}{e2} - \frac{2g}{e2} - \sqrt{(1 + \frac{e1}{e2} - \frac{2g}{e2})^2 - 4\frac{e1}{e2}}}{2}, \tag{6.23}$$

$$\pi_H^* = \frac{1 + \frac{e1}{e2} - \frac{2g}{e2} + \sqrt{(1 + \frac{e1}{e2} - \frac{2g}{e2})^2 - 4\frac{e1}{e2}}}{2}. \tag{6.24}$$

Despite their popularity, OLG models are well known for their *multiplicity of equilibria*; in our case, the coexistence of two inflation equilibria: Equations (6.23) and (6.24). Things can be even more intriguing if these equilibria have different welfare implications. In our case, the one with the higher inflation rate, π_H^*, is the Pareto-inferior equilibrium, whereas the one with the lower inflation rate, π_L^*, is the Pareto-superior equilibrium. It can be further shown that the high inflation steady state is stable under rational expectations, whereas the low inflation steady state is stable under first-order adaptive expectations (Marcet and Sargent, 1989).

6.2.2 OLG experiments of inflation

The two equilibria of the two-period OLG model raise the issue of *equilibrium selection*. This can be particularly interesting because the two equilibria, high inflation and low inflation, do have different welfare implications. To solve this problem, experiments with human subjects have been applied to see which equilibrium is more likely to appear (Marimon and Sunder, 1993, 1994; Bernasconi and Kirchkamp, 2000). The results consistently show that the low-inflation equilibrium is chosen. However, similar to what we have seen in Section 6.1, this result of converging to the low-inflation equilibrium cannot be fully predicted by some familiar learning algorithms, such as *recursive least squares*, under the representative-agent regime. Marcet and Sargent (1989) have shown that the representative agent using recursive least squares fails to converge in some cases.

Can agent-based models better replicate these experimental results? The answer, to some extent, is yes. The general idea is very similar to the one in the agent-based cobweb models (Section 6.1). Homogeneous rational expectation (perfect foresight) is replaced by bounded rational agents with heterogeneous decisions (consumption or saving) (Arifovic, 1995) or beliefs (price expectations; Bullard and Duffy, 1999; Chen and Yeh, 1999). This population of heterogeneous decisions or beliefs is then revised either via individual learning or social learning.

6.2.3 Agent-based OLG models of inflation

To see whether decentralized agents are able to coordinate intelligently to single out a Pareto-superior equilibrium rather than be trapped in a Pareto-inferior equilibrium, Arifovic (1995) proposed the first agent-based modification of an OLG model of inflation. The idea is almost the same as her work on the cobweb model

(Arifovic, 1994). Instead of the quantity-supplied decision ($q_{i,t}$), what concerns agents here are the saving or consumption decision ($s_{i,t}$ or $c^1_{i,t}$). The genetic updating procedure described in the cobweb model (Figure 6.1) is almost ready to apply directly to the evolution of individuals' decisions in the OLG model except that in the OLG learning occurs between nonoverlapping generations of grandparents and grandchildren. This special feature, called the the *nonoverlapping information structure* (Bullard and Duffy, 1998a), is due to the fact that the parents' fitness values are not available when newborn agents are young. This difference is also sketched in Figure 6.4, which shows that the $(t+2)$th population iteration is the successor (offspring) of the tth population rather than the $(t+1)$th one.

In her study, GA-based agents were shown to be able to select the Pareto-superior equilibrium. She further compared the simulation results based on GAs with those from laboratories with human subjects, concluding that GAs were superior to other learning schemes, such as recursive least squares. On the one hand, GAs under the conditions which result in the divergence of the least squares algorithm still converge. On the other hand, GA patterns fluctuate more than least-squares patterns and hence capture better the fluctuating convergent pattern observed from the experimental data.

This line of research was continued further in Dawid (1996), Bullard and Duffy (1998a, b, 1999), and Birchenhall and Lin (2002). On the basis of what to encode, Bullard and Duffy (1999) made the distinction between two implementations of GA learning: namely, learning *how to optimize* (Arifovic, 1995) and learning *how to forecast* (Bullard and Duffy, 1999). It was found that these two implementations lead to the same result: agents can indeed learn the Pareto-superior equilibrium. The only difference is the speed of convergence. The "learning how to forecast" version of genetic algorithm learning converges faster than the "learning how to optimize" implementation studied by Arifovic (1995). Nevertheless, a robustness analysis showed that coordination was more difficult when the number of inflation values considered (search space) by agents was higher, when government deficits

$$
\begin{array}{l}
POP_t : (c^1_{1,t}, c^1_{2,t}, \ldots, c^1_{N,t}) \Rightarrow \sum_{i=1}^{N} S_{i,t} = S_t \Rightarrow P_t \\
\Downarrow \\
POP_{t+1} : (c^1_{1,t+1}, c^1_{2,t+1}, \ldots, c^1_{N,t+1}) \Rightarrow \sum_{i=1}^{n} S_{i,t+1} = S_{t+1} \Rightarrow P_{t+1} \\
\Downarrow \\
POP_t : (c^2_{1,t+1}, c^2_{2,t+1}, \ldots, c^2_{2,t+1}) \\
\Downarrow \\
\left\{ \begin{array}{l}
POP_t : (U_{1,t}, U_{2,t}, \ldots, U_{N,t}) \\
\Downarrow \\
POP_{t+2} : (c^1_{1,t+2}, c^1_{2,t+2}, \ldots, c^1_{N,t+2})
\end{array} \right.
\end{array}
$$

Figure 6.4 A sketch of evolutionary learning dynamics in the OLG model.

increased, and when agents entertained inflation rate forecasts outside the bounds of possible stationary equilibria.

Chen and Yeh (1999) generalized the "learning how to forecast" version of GA learning in Bullard and Duffy (1999) with GP. In Bullard and Duffy (1999), what learning agents learn is just a number for the inflation rate rather than the pattern of motion of the inflation rate, which is a function. Chen and Yeh (1999) considered it too restrictive to learn just a number. From Grandmont (1985), if the equilibrium of an OLG is characterized by limit cycles or strange attractors rather than by fixed points, then what agents need to learn is not just a number, but a functional relationship, such as $\pi_t = f(\pi_{t-1}, \pi_{t-2}, \ldots)$. Chen and Yeh (1999) therefore generalized Bullard and Duffy's (1999) evolution of "beliefs" from a sequence of populations of *numbers* to a sequence of populations of *functions*. Genetic programming serves as a convenient tool to make this extension.

The basic result observed in Chen and Yeh (1999) is largely consistent with Arifovic (1994) and Bullard and Duffy (1999), namely, agents being able to coordinate their actions to achieve the Pareto-superior equilibrium (Figure 6.5). Furthermore, their simulations showed that the convergence is not sensitive to the initial rates of inflation. Hence, the Pareto-superior equilibrium has a large domain of attraction. A test on a structural change (a change in deficit regime) was also conducted. It was found that GP-based agents were capable of converging very fast to the new low-inflationary stationary equilibrium after the new deficit regime was imposed. However, the basic result was not insensitive to the dropping of the survival-of-the-fittest principle. When that golden principle was not enforced, we experienced dramatic fluctuations of inflation and occasionally the appearance of superinflation. The agents were generally worse off.

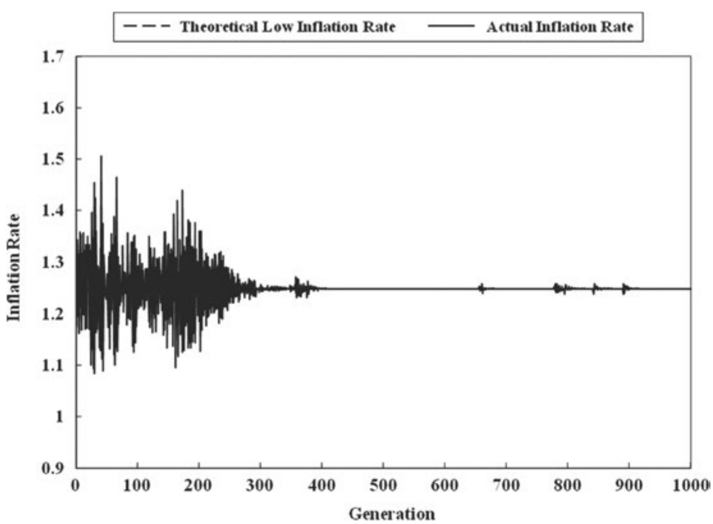

Figure 6.5 Time series of inflation in an agent-based OLG model.

6.2.4 Agent-based OLG models of cycles

Bullard and Duffy (1998b) studied a more complicated version of the two-period OLG model based on Grandmont (1985). They considered the following utility function for the households:

$$U(c_t^1, c_t^2) = \frac{\ln(c_t^1)^{1-\rho_1}}{1-\rho_1} + \frac{\ln(c_t^2)^{1-\rho_2}}{1-\rho_2}. \tag{6.25}$$

Under time-separable preferences and provided that the value of the coefficient of relative risk aversion for the old agent (ρ_2) is high enough and that of the young agents is low enough (ρ_1), Grandmont (1985) showed that stationary perfect-foresight equilibria may also exist in which the equilibrium dynamics are characterized either as *periodic* or *chaotic trajectories* for the inflation rate, and these complicated stationary equilibria are also Pareto optimal. To have these possibilities, they set ρ_2 equal to 2 and then increased the value of this preference parameter up to 16 by increments of 0.1, while fixing ρ_1 at 0.5 in all cases.

The forecast rule considered by Bullard and Duffy (1998b) is to use the price level that was realized $k+1$ periods in the past as the forecast of the next period's price level, namely,

$$P_{i,t}^e = P_{t-k-1}, \quad k \in [0, \bar{k}]. \tag{6.26}$$

In their case, $\bar{k}$ was set to 256, which allows the agents to take actions consistent with a periodic equilibrium of an order as high as 256. Alternatively, agent i's forecast of the gross inflation factor between dates t and $t+1$ is given by

$$\pi_{i,t}^e = \frac{P_{i,t}^e}{P_{t-1}} = \frac{P_{t-k-1}}{P_{t-1}}. \tag{6.27}$$

As usual, the lifetime utility function was chosen as the fitness function to evaluate the performance of a particular forecast rule. Instead of roulette wheel selection, tournament selection was applied to create the next generation.

It was found that the stationary equilibria on which agents coordinate were always relatively simple—either a steady state or a low-order cycle. For low values of ρ_2, in particular those below 4.2, they observed convergence to the monetary steady state in every experiment, which is the same prediction made by the limiting backward perfect-foresight dynamics. As ρ_2 was increased further, the limiting backward perfect-foresight dynamics displayed a bifurcation, with the monetary steady state losing stability and never regaining it for values of $\rho_2 \geq 4.2$. However, in their system with learning, the monetary steady state was always a limit point in at least one of the ten experiments conducted for each different value of ρ_2. In addition, for $\rho_2 \geq 4.2$, their system often converged, in at least one experiment, to a period-two stationary equilibrium, even in cases in which that equilibrium, too, had lost its stability in the backward perfect-foresight dynamics.

It is difficult, however, for an economy comprised of optimizing agents with initial heterogeneous beliefs to coordinate on especially complicated stationary equilibria, such as the period-k cycles where $k \geq 3$. In particular, the period-three cycle that is stable in the backward perfect-foresight dynamics for values $\rho_2 \geq 13$ was never observed in their computational experiments. Interestingly enough, three is the last entry of *Sarkovskii's ordering*, whereas one, two, and four are the first few entries.[7]

They also found that the time it took agents to achieve coordination tended to increase with the relative risk aversion of the old agents over a large portion of the parameter space. Usually, it was the case when the system converged to the period-two cycle. Moreover, when cycles exist, the transient dynamics of their systems could qualitatively display complication dynamics for long periods of time before eventually reverting to relatively simple, low-periodicity equilibria.

6.2.5 Agent-based OLG models of sunspots

A phenomenon related to cyclical equilibria is *sunspot equilibria*. The sunspot variable is the variable which has no intrinsic influence on an economy, i.e., it has nothing to do with an economy's fundamentals. Sunspot equilibria exist if the sunspot variable can impact the economy simply because a proportion of agents believe so and act according to their belief. Azariadis and Guesnerie (1986) showed that the connection between cyclical and sunspot equilibria is very close. They proved that a two-state stationary sunspot equilibrium exists if and only if a period-two equilibrium exists. Dawid (1996) started with an OLG model of inflation comparable to Bullard and Duffy (1998b).

He studied an economy whose households have the following utility function:

$$U(c_t^1, c_t^2) = 0.1[c_t^1]^{0.9} + 10 - \left[\frac{10}{1+c_t^2}\right]^2. \tag{6.28}$$

This utility function has the property that the concavity with respect to c_t^1 is much smaller than the concavity with respect to c_t^2, which is necessary for the existence of a periodic equilibrium (Grandmont, 1985).

He first found that in cases where periodic equilibria exist, households' beliefs were successfully coordinated to the period-two cycle rather than the steady state. He then assumed all households to be sunspot believers and showed that households' beliefs converged to the sunspot equilibrium. In that case, the observed values of the price levels are completely governed by something which has nothing to do with the economy's fundamentals. Finally, he relaxed the assumption by simulating an explicit contest between sunspot believers and sunspot agnostics. The simulation showed that in most cases, the population consisted, after a rather short period, only of households whose actions depended on the value of the sunspot variable.

6.2.6 Robustness issues

Birchenhall and Lin (2002) provided perhaps the most extensive coverage of robustness checks ever seen in agent-based macroeconomic experiments. Their work covers two different levels of GA designs: one is genetic operators, and the other is architecture. For the former, they consider different implementations of the four main GA operators, i.e., selection, crossover, mutation, and election. For the latter, they consider a single-population GA (social learning) versus a multi-population GA (individual learning). They found that Bullard and Duffy's results are sensitive to two main factors: the election operator and architecture. Their experimental results in fact lend support to some early findings, e.g., the significance of the election operator (Arifovic, 1994) and the different consequences of social learning and individual learning (Vriend, 2001). What is particularly interesting is that individual learning reduces the rate of convergence to the same belief. This is certainly an important finding, because most studies on the convergence of GAs to Pareto optimality are based on the social learning version.

6.3 Agent-based modeling of foreign exchange experiments

Another popular class of OLG models to which an agent-based approach is applied is the OLG model of foreign exchange rates, which is a version of the two-country OLG model with fiat money (Kareken and Wallace, 1981).

6.3.1 The Kareken–Wallace model

There are two countries in the model. The residents of both countries are identical in terms of their preferences and lifetime endowments. The basic description of each country is the same as the single-country OLG model. Each household of generation t is endowed with e^1 units of a perishable consumption good at time t, and e^2 of the good at time $t+1$, and consumes c_t^1 of the consumption good when young and c_t^2 when old. Households in both countries have common preferences given by

$$U(c_t^1, c_t^2) = \ln(c_t^1) + \ln(c_t^2). \tag{6.29}$$

The government of each country issues its own unbacked currency, $H_{1,t}$ and $H_{2,t}$. Households can save only through acquiring these two currencies. There are no legal restrictions on holdings of foreign currency. Thus, the residents of both countries can freely hold both currencies in their portfolios. A household in generation t solves the following optimization problem at time t:

$$\max_{(c_{i,t}^1, m_{i,1,t})} \quad \ln(c_{i,t}^1) + \ln(c_{i,t}^2) \tag{6.30}$$

$$\text{s.t.} \quad c_{i,t}^1 + \frac{m_{i,1,t}}{P_{1,t}} + \frac{m_{i,2,t}}{P_{2,t}} = e^1, \quad c_{i,t}^2 = e^2 + \frac{m_{i,1,t}}{P_{1,t+1}} + \frac{m_{i,2,t}}{P_{2,t+1}}, \tag{6.31}$$

where $m_{i,1,t}$ is household i's nominal holdings of currency 1 acquired at time t, $m_{i,2,t}$ is household i's nominal holdings of currency 2 acquired at time t, $P_{1,t}$ is the nominal price of the good in terms of currency 1 at time t, and $P_{2,t}$ is the nominal price of the good in terms of currency 2 at time t. The savings of household i at time t, $s_{i,t}$, are:

$$s_{i,t} = e^1 - c_{i,t}^1 = \frac{m_{i,1,t}}{P_{1,t}} + \frac{m_{i,2,t}}{P_{2,t}}. \tag{6.32}$$

The exchange rate e_t between the two currencies is defined as $e_t = P_{1,t}/P_{2,t}$. When there is no uncertainty, the return on the two currencies must be equal,

$$R_t = R_{1,t} = R_{2,t} = \frac{P_{1,t}}{P_{1,t+1}} = \frac{P_{2,t}}{P_{2,t+1}}, \quad t \geq 1, \tag{6.33}$$

where $R_{1,t}$ and $R_{2,t}$ are the gross real rate of return between t and $t+1$, respectively. Rearranging (6.33), we obtain

$$\frac{P_{1,t+1}}{P_{2,t+1}} = \frac{P_{1,t}}{P_{2,t}}, \quad t \geq 1. \tag{6.34}$$

From Equation (6.34) it follows that the exchange rate is constant over time:

$$e_{t+1} = e_t = e, \quad t \geq 1. \tag{6.35}$$

Savings demand derived from the household's maximization problem is given by

$$s_{i,t} = \frac{m_{i,1,t}}{P_{1,t}} + \frac{m_{i,2,t}}{P_{2,t}} = \frac{1}{2}\left[e^1 - e^2\frac{1}{R_t}\right]. \tag{6.36}$$

Aggregate savings of the world in time period t, S_t, are equal to the sum of their savings in terms of currency 1, $S_{1,t}$, and in terms of currency 2, $S_{2,t}$. With the homogeneity assumption, we can have

$$S_{1,t} = \sum_{i=1}^{2N} \frac{m_{i,1,t}}{P_{1,t}} = \frac{2Nm_{1,t}}{P_{1,t}}, \tag{6.37}$$

and

$$S_{2,t} = \sum_{i=1}^{2N} \frac{m_{i,2,t}}{P_{2,t}} = \frac{2Nm_{2,t}}{P_{2,t}}. \tag{6.38}$$

The equilibrium condition in the loan market requires that

$$S_t = S_{1,t} + S_{2,t} = N\left[e^1 - e^2\frac{P_{1,t+1}}{P_{1,t}}\right] = \frac{H_{1,t} + H_{2,t}e}{P_{1,t}}. \tag{6.39}$$

Equation (6.39) only informs us of the real saving in terms of the real-world money demand. This equation alone cannot determine the households' real demands for each currency. Hence, this equation cannot uniquely determine a set of prices $(P_{1,t}, P_{2,t})$, and leaves the exchange rate indeterminate as well. This is known as the famous *indeterminacy of exchange rate proposition*. The proposition says that if there exists a monetary equilibrium in which both currencies are valued at some exchange rate e, then there exists a monetary equilibrium at any exchange rate $\hat{e} \in (0, \infty)$ associated with a different price sequence $\{\hat{P}_{1,t}, \hat{P}_{2,t}\}$ such that

$$R_t = \frac{P_{1,t}}{P_{1,t+1}} = \frac{P_{2,t}}{P_{2,t+1}} = \frac{\hat{P}_{1,t}}{\hat{P}_{1,t+1}} = \frac{\hat{P}_{2,t}}{\hat{P}_{2,t+1}}, \tag{6.40}$$

and

$$S_t = \frac{H_{1,t} + H_{2,t}e}{P_{1,t}} = \frac{H_{1,t} + H_{2,t}\hat{e}}{\hat{P}_{1,t}}, \tag{6.41}$$

where

$$\hat{P}_{1,t} = \frac{H_{1,t} + \hat{e}H_{2,t}P_{1,t}}{H_{1,t} + eH_{2,t}}, \quad \hat{P}_{2,t} = \frac{\hat{P}_{1,t}}{\hat{e}}. \tag{6.42}$$

Rearranging Equation (6.39), one can derive the law of motion of $P_{1,t}$:

$$P_{1,t+1} = \frac{e^1}{e^2}P_{1,t} - \frac{H_{1,t} + eH_{2,t}}{Ne^2}. \tag{6.43}$$

For any given exchange rate e, this economy with constant supplies of both currencies, H_1 and H_2, has a steady-state equilibrium, namely

$$P_{1,t+1} = P_{1,t} = P_1^* = \frac{H_1 + eH_2}{N(e^1 - e^2)}. \tag{6.44}$$

Like e, the level of P_1^* is also indeterminate. In addition, since households are indifferent between the currencies that have the same rates of return in the homogeneous-expectations equilibrium, the OLG model in which agents are rational does not provide a way of determining the portfolio $\lambda_{i,t}$, which is the fraction of the savings placed in currency 1.

6.3.2 OLG experiments on exchange rate

Arifovic (1996) gives the first experiment based on the Kareken–Wallace model. Two sets of endowments are considered: $(e^1, e^2) = (10, 1)$ and $(10, 4)$. The nominal money supply per capita, h_1 and h_2, are both set to be 10. The experiment was run twice; each run consisted of two sessions corresponding to each set of endowments. Ten subjects participated in the first run, whereas eight subjects participated in the second run. From Section 6.3.1, we know that the perfect foresight stationary equilibrium consumption is $(c_{i,t}^1, c_{i,t}^2) = (5.5, 5.5)$ and $(7, 7)$, respectively. The experimental result shows that $(c_{i,t}^1, c_{i,t}^2)$ converges to this stationary equilibrium consumption. However, the exchange rate fluctuated over a range between 0.5 and 2 in all of the experimental sessions. It did not converge to a constant exchange rate.

6.3.3 Agent-based OLG models of the exchange rate

In order to examine the behavior of the exchange rate and the associated price dynamics, Arifovic (1996) initiated the agent-based modeling of the exchange rate in the context of the OLG model. In the OLG model of the exchange rate, the households have two decisions to make when they are young, namely, saving $(s_{i,t})$ and portfolio $(\lambda_{i,t} \equiv \frac{m_{i,1,t}}{P_{1,t}s_t})$. These two decisions are encoded by the concatenation of two binary strings, the first of which is encoded as $s_{i,t}$, whereas the second is encoded as $\lambda_{i,t}$. The length of a binary string, l, is 30: The first 20 elements of a string encode the first-period consumption of agent i of generation t; the remaining 10 elements encode the portfolio fraction of agent i:

$$\underbrace{010100\ldots110}_{\text{20 bits: } s_{i,t}}\underbrace{101\ldots001}_{\text{10 bits: } \lambda_{i,t}}$$

The single-population augmented genetic algorithm is then applied to evolve these decision rules in a way very similar to what has been done in the overlapping generations model (Figure 6.4), as we summarize in Figure 6.6. Basically, the nonoverlapping learning structure is carried out again here. The collection of the decisions of generation t, POP_t, is reviewed and revised at the end of period $t+1$, which constitutes the population of the decision rules of the generation at time $t+2$, POP_{t+2}.

$$\boxed{\begin{array}{l}
POP_t : \{(c_{1,t}^1, \lambda_{1,t}), (c_{2,t}^1, \lambda_{2,t}) \ldots, (c_{N,t}^1, \lambda_{N,t})\} \\
\Rightarrow \left(\frac{\sum_{i=1}^{n} \lambda_{i,t} s_{i,t}}{H_{1,t}}, \frac{\sum_{i=1}^{N} (1-\lambda_{i,t}) s_{i,t}}{H_{2,t}} \right) \Rightarrow (P_{1,t}, P_{2,t}, R_{1,t-1}, R_{2,t-1}, e_{t-1}) \\
POP_{t+1} : \{(c_{1,t+1}^1, \lambda_{1,t+1}), (c_{2,t+1}^1, \lambda_{2,t+1}) \ldots, (c_{N,t+1}^1, \lambda_{N,t+1})\} \\
\Rightarrow \left(\frac{\sum_{i=1}^{N} \lambda_{i,t+1} s_{i,t+1}}{H_{1,t+1}}, \frac{\sum_{i=1}^{N} (1-\lambda_{i,t+1}) s_{i,t+1}}{H_{2,t+1}} \right) \Rightarrow (P_{1,t+1}, P_{2,t+1}, R_{1,t}, R_{2,t}, e_t) \\
\Downarrow \\
POP_t : (c_{1,t}^2, c_{2,t}^2, \ldots, c_{N,t}^2) \quad c_{i,t}^2 = e^2 + \frac{m_{i,1,t}}{P_{1,t+1}} + \frac{m_{i,2,t}}{P_{2,t+1}} \\
\Downarrow \\
\left\{ \begin{array}{l} POP_t : (U_{1,t}, U_{2,t}, \ldots, U_{N,t}) \\ \Downarrow \\ POP_{t+2} : \{(c_{1,t+2}^1, \lambda_{1,t+2}), (c_{2,t+2}^1, \lambda_{2,t+2}) \ldots, (c_{N,t+2}^1, \lambda_{N,t+2})\} \end{array} \right.
\end{array}}$$

Figure 6.6 A sketch of evolutionary learning dynamics in the OLG model of exchange rates.

While Equation (6.35) predicts the constancy of the exchange rate, genetic algorithm simulations conducted by Arifovic (1996) indicated that there is no sign of a constant exchange rate; instead, the exchange rate fluctuates persistently. Adaptive economic agents in this model can, in effect, endogenously generate *self-fulfilling arbitrage opportunities*, which in turn make exchange rates continuously fluctuate.

The fluctuating exchange rate was further examined using formal statistical tests in both Arifovic (1996) and Arifovic and Gencay (2000). First, in Arifovic (1996), the stationarity test (the Dickey–Fuller test) was applied to examine whether the exchange rate series is nonstationary. The result of the test did not indicate nonstationarity. Second, Arifovic and Gencay (2000) analyzed the statistical properties of the exchange rate returns, i.e., the logarithm of e_t/e_{e-1}. The independence tests (the Ljung–Box–Pierce test and the Brock–Dechert–Scheinkman test) clearly rule out the lack of persistence (dependence) in the return series. Third, they plotted the phase diagrams of the return series and found that there is a well-defined attractor for all series. The shapes of the attractors are robust to the changes in the OLG model parameters as well as to the changes in the GA parameters. Fourth, to verify that this attractor is chaotic, the largest two Lyapunov exponents were calculated. The largest Lyapunov exponent is positive in all series, which supports the claim that the attractors under investigation are chaotic. Finally, volatility clustering was also found to be significant in the return series. This series of econometric examinations confirms that agent-based modeling is able to replicate some stylized facts known in financial markets.

6.3.4 Agent-based OLG models of currency collapse

Arifovic (2002) considered a different application of GAs to modeling the adaptive behavior of households. In this model, agents simply have no endowment when they are old, i.e., $e^2 = 0$. With this assumption, by Equation (6.36), the optimal saving is

$$s_{i,t} = \frac{1}{2}e^1, \tag{6.45}$$

which is fixed and is independent of the rate of return. Therefore, agent i is required to make a decision only on his portfolio, i.e., $\lambda_{i,t}$. In Arifovic (2002), instead of directly modeling the portfolio decision, Arifovic turned to model the exchange-rate forecasting behavior of agents. This makes sense because the exchange rate can be written as a function of the portfolio[8]

$$e_t = \frac{P_{1,t}}{P_{2,t}} = \frac{1-\bar{\lambda}_t}{\bar{\lambda}_t}, \tag{6.46}$$

where

$$\bar{\lambda}_t = \frac{\sum_{i=1}^{2N} \lambda_{i,t}}{2N}. \tag{6.47}$$

Or, alternatively,

$$\bar{\lambda}_t = \frac{1}{1+e_t}. \tag{6.48}$$

However, since e_t is not known, each agent i will form his forecast, $\hat{e}_{i,t}$, and, based on his forecast, his portfolio will be determined as[9]

$$\lambda_{i,t} = \frac{1}{1+\hat{e}_{i,t}}. \tag{6.49}$$

The forecasting models of exchange rates employed by agents are simple moving-average models:

$$\hat{e}_{i,t} = \sum_{j=1}^{T} \frac{e_{t-j}}{T}. \tag{6.50}$$

They differ in the rolling window size T, which is endogenously determined and can be time variant. What is encoded by GAs is the size of the rolling window rather than the usual savings and portfolio decision. Simulations with this new coding scheme resulted in the convergence of the economies to a single-currency equilibrium, i.e., the *collapse* of one of the two currencies. This result was not found in Arifovic (1996). This study, therefore, shows that *different implementations of GA learning may have non-trivial effects on the simulation results*. In one implementation, one can have persistent fluctuation of the exchange rate (Arifovic, 1996); in another case, one can have a single-currency equilibrium (Arifovic, 2002).

Following the design of Franke (1998), Arifovic (2002) combined two different applications of GA learning. In addition to the original population of agents, who are learning how to forecast, she added another population of agents, who are learning how to optimize, as was done in Arifovic (1996).[10] Nevertheless, unlike Franke (1998), these two populations of agents did not compete with each other. Instead, they underwent separate genetic algorithm updating. Simulations with these two separate evolving populations did not converge to a single currency equilibrium, but were characterized instead by persistent fluctuation.

6.3.5 Agent-based OLG models of capital flight

A different scenario of currency collapse is also shown in Arifovic (2001), which is an integration of the OLG model of exchange rate with the OLG model of inflation. In this model, the governments of both countries have constant deficits (G_i, $i = 1, 2$) which were financed via seigniorage:

$$G_i = \frac{H_{i,t} - H_{i,t-1}}{P_{i,t}}, \quad i = 1, 2. \tag{6.51}$$

Combining Equations (6.39) and (6.51) gives the condition for the monetary equilibrium in which both governments finance their deficits via seigniorage:

$$G_1 + G_2 = S_t - S_{t-1}R_{t-1} = S_t - \frac{H_{1,t-1} + eH_{2,t-1}}{P_{1,t}}. \tag{6.52}$$

This integrated model inherits the indeterminacy of the exchange rate from the OLG model of the exchange rate and the indeterminacy of the inflation rate from the OLG model of inflation. Any constant exchange rate $\hat{e}$ [$\hat{e} \in (0, \infty)$] is an equilibrium that supports the same stream of government deficits (G_1, G_2), and the same equilibrium gross rate of return (and thus the same equilibrium savings), as has been shown in Equation (6.41).

$$G_1 + G_2 = S_t - \frac{H_{1,t-1} + eH_{2,t-1}}{P_{1,t}} = S_t - \frac{H_{1,t-1} + \hat{e}H_{2,t-1}}{\hat{P}_{1,t}}, \tag{6.53}$$

where

$$\hat{P}_{1,t} = \frac{H_{1,t-1} + \hat{e}H_{2,t-1}P_{1,t}}{H_{1,t-1} + eH_{2,t-1}}.$$

The existence of these equilibrium exchange rates indicates that the currencies of both countries are valued despite the difference in the two countries' deficits. In fact, in equilibrium the high-deficit country and the low-deficit country experience the same inflation rate, and hence so do their currencies' rates of return. Nonetheless, since the high-deficit country has a higher money supply, if both currencies are valued, then the currency of the high-deficit country will eventually drive the currency of the low-deficit country out of households' portfolios. Given this result, it might be in the interests of a country with lower deficits to impose a degree of capital control.

Arifovic (2001) showed that agent-based dynamics behave quite differently from the above homogeneous rational expectations equilibrium analysis. In her agent-based environment, the evolution of households' decision rules for savings and portfolios results in a flight away from the currency used to finance the larger of the two deficits. In the end, households hold all of their savings in the currency used to finance the lower of the deficits. Thus, the economy converges to the equilibrium in which only the low-deficit currency is valued. The currency of the country that finances the larger of the two deficits becomes valueless, and we have a single-currency equilibrium again:

$$R_{1,t} = \frac{\bar{\lambda}_{t+1}}{\bar{\lambda}_t} = \frac{1 - \bar{\lambda}_{t+1}}{1 - \bar{\lambda}_t} = R_{2,t} \tag{6.54}$$

6.4 Concluding remarks

The lesson gained from the early experiences using agent-based simulation shows that different ways of modeling agents' behaviors may lead to different results.

Take the models of foreign exchange rates as an example (Section 6.3). In one case, the exchange rate can constantly fluctuate with both currencies surviving, and in the other case, a single-currency equilibrium (currency collapse) occurs. Hence, unless, we can have considerable control of agents' behavior, one could say that everything seems to be possible. In other words, a great variation of the results can be attributed to agents' behavior, and a great reduction of this kind of uncertainty can be possible only if we can be assured to exclude a large set of "unrealistic" behavior. However, our wealth of knowledge does not always authorize us to do so. Besides, when the mapping between agents' behavior and the model's behavior becomes complex, it will be challenging to provide an explanation for the mapping itself.

While, unlike equation-based modeling, agent-based modeling cannot offer any analytical proof, it does raise the issue that the two resultant dynamics could be different to such a significant degree that the former is no longer an approximation of the latter. We also see that once individuality and heterogeneity is taken into account, there exists a plethora of models which generally lead to different answers. In this case, one cannot resist the temptation to probe into which one is true. However, a second thought will lead us to realize that this may not be the immediate goal to pursue, considering that proving which one is true is also challenging in some cases. Then, given this uncertainty, what is the use of agent-based modeling?

From our viewpoint, agent-based modeling helps us see the nature of uncertainty or indeterminism. It is exactly because the way in which the tiny seed is planted, of which we cannot be assured, makes the kind of tree we obtain uncertain. On the other hand, when we are lucky enough, we can always have the same result regardless of the models we try; then the social embeddedness is robust enough to endow us with a structuralism interpretation.

Even though a complete satisfactory solution may not be feasible, given the sensitivity of the agent-based simulation results to the design of the artificial agents, the design issue is worth a more thorough treatment. Part III is a review of several existing treatments for the issue.

Notes

1 Turing strongly believed that there was nothing the brain could do that a well-designed computer could not. For a more comprehensive survey of the Turing test, the interested reader is referred to Saygin, Cicekli, and Akman (2000).
2 We are just beginning to see efforts devoted to large agent-based macroeconomic models. At this point, the literature is still developing.
3 The name "cobweb" was first suggested by Nicolas Kaldor (Kaldor, 1934). It is so named because price and output movements resemble a cobweb. Nerlove (2010) gives a simple but nice review of the development of the idea of the cobweb model and a famous diagram to demonstrate it.
4 However, John Muth (1930–2005) might not agree with the use and the interpretation of his invented rational expectations in macroeconomics; his later interests in economics probably leaned more toward Herbert Simon's bounded rationality. See Sent (2002) for a lengthy discussion of this.

5 Essentially, the literature on learning in macroeconomics is devoted to this question. See, for example, Sargent (1993) and Evans and Honkapohja (2001).

6 Some technical details are provided here, while a systematic introduction will be given in Chapter 13. The election operator involves two steps. First, crossover is performed. Second, the potential fitness of the newly generated offspring is compared with the actual fitness values of its parents. Among the two offspring and two parents, the two individuals with the highest fitness are then chosen. The purpose of this operator is to overcome difficulties related to the way mutation influences the convergence process, because the election operator can bring the variance of the population rules to zero as the algorithm converges to the equilibrium values.

7 Sarkovskii's ordering (Sarkovskii, 1964) is an ordering of all natural numbers. The ordering starts from a sequence of all odd numbers except one in an increasing manner, followed by that same sequence premultiplied by 2, and then premultiplied by 2^2, 2^3, . . . , all the way up. This list will exhaust all natural numbers except powers of 2, which will be placed at the end of the ordering in a decreasing manner. Hence, the ordering goes as follows.

$$3 \triangleright 5 \triangleright 7 \triangleright \cdots \triangleright 2 \cdot 3 \triangleright 2 \cdot 5 \triangleright 2 \cdot 7 \triangleright \cdots \triangleright 2^2 \cdot 3 \triangleright 2^2 \cdot 5 \triangleright 2^2 \cdot 7 \triangleright \cdots \triangleright 2^3 \triangleright 2^2 \triangleright 2 \triangleright 1 \quad (6.55)$$

8 To see this, simply note that

$$P_{1,t} = \frac{H_{1,t}}{\sum_{i=1}^{2N} \lambda_{i,t} s_{i,t}} = \frac{H_{1,t}}{2N \frac{e^1}{2} \frac{\sum_{i=1}^{2N} \lambda_{i,t}}{2N}} = \frac{H_{1,t}}{N e^1 \bar{\lambda}_t} \quad (6.56)$$

and, similarly,

$$P_{2,t} = \frac{H_{2,t}}{N e^1 (1 - \bar{\lambda}_t)}. \quad (6.57)$$

Then dividing $P_{1,t}$ by $P_{2,t}$ and assuming that $H_{1,t} = H_{2,t}$, we have Equation (6.46).

9 Of course, this immediately raises the issue of whether Equation (6.48) can be a base for individual decisions (6.49). One convenient justification for this is that agents use this as a heuristic by taking (6.48) as a focal point and assuming every other agent will behave in the same way.

10 Only the portfolio $\lambda_{i,t}$ is encoded, and $s_{i,t}$ is set to the optimal constant $\frac{e^1}{2}$.

Part III

Designing artificial economic agents

Part II gives a historical background of agent-based modeling in economics. The *cellular automata tradition* shows how complex patterns can be formed using simple agents interacting with each other in a social network by following simple rules. Nonetheless, most of the time, agents are *homogeneous* in the sense that they follow the *same* rules, for example, agents in the Schelling's segregation model. The *economic experiment tradition*, on the other hand, is then largely built upon the setting of heterogeneous agents, and *population-based computational intelligence* was employed as a tool to operate these heterogeneous-agent systems.

In a similar vein, Part III can be read as a continuation of the economic experiment tradition, except for the following two distinctions. The minor one is that, instead of agent-based modeling of macroeconomic experiments, this part will focus more on the agent-based modeling of *microeconomic experiments*. We even start at a finer level, i.e., *individual experiments* (Chapter 7). A specific example of the former are the *double auction market experiments*, whereas a specific example of the latter are the *N-armed bandit experiments* (Section 7.2).

The major distinction is that this part will consider a more comprehensive relation between agent-based modeling and experimental economics. Instead of just having software agents mimic human agents (the *mirroring* function), we can make the two have both *competitive* and *collaborative* relations by simultaneously presenting software agents to human agents. This more integrated framework was not apparent in the mid 1990s. Its gradual transparency to us depends on our better grasp of software agents. This part will review several versions of software agents, all of which appeared in the mid 1990s (Chapter 8). We believe that this glossary of software agents makes us be able to see better the relation between human agents and software agents, and hence to design and make use of the integrating system in a productive way (Section 9.5).

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7 Calibrated artificial agents

Presumably, software agents are naturally related to human agents since the former are frequently designed to replicate or mirror the latter at different degrees of precision. Brian Arthur provided the two earliest guidelines for this mirroring function (Arthur, 1991, 1993). Following the idea of calibration in general economic modeling, he also proposed *calibrating* artificial agents, i.e., calibrating the designs of software agents in light of real data. He suggested two criteria on which the calibration is based. The first one is a *statistical criterion*, whereas the second one is an *AI* one, namely the *Turing test*.

In this chapter, we shall focus only on the statistical one, since this is the one which has been discussed at length in his paper (Arthur, 1993) and has now been taken into account by many agent-based economic model-builders. *Statistically calibrated artificial agents* provide important references to the current integration of agent-based computational economics (software agents), experimental economics (human agents), neuroeconomics, and econometrics. Some questions proposed in his early work still remain challenging, even though we are now more capable of addressing them than when they were first proposed 15 years ago.

The AI criterion, the Turing test, was neither well discussed in his own paper nor seriously followed by the later literature. The only two exceptions are Ecemis, Bonabeau, and Ashburn (2005) and Arifovic, McKelvey, and Pevnitskaya (2006). However, it has gradually become clear that calibration exclusively based on some specific statistical procedures may have its limits; therefore, the AI approach as a complement may still be important. We shall discuss it in Section 7.4.

7.1 Challenges proposed

The two papers by Arthur (1991, 1993) are quite fundamental in the literature on *economic agent engineering*. They articulated the most fundamental challenge which still exists even now:

> An important question then for economics is how to construct economic models that are based on an actual human rationality that is bounded or limited. As an ideal, we would want to build our economic models around theoretical agents whose rationality is bounded in exactly the same way

> human rationality is bounded, and whose decision-making behavior matches or replicates real human decision-making behavior.
>
> (Arthur, 1993, p. 2)

The proposal he made for the question above is to build the software agents upon empirical grounds. To implement that proposal, he further suggested a *statistical approach* to designing software agents, i.e., first, to *parameterize* software agents in terms of their decision algorithms, and, second, to *calibrate* them.

> The approach I suggest here is ... to construct, within a given decision context, theoretical economic agents that match or mimic actual human behavior observed in that context. We can do this by representing our agents as using *parameterized decision algorithms* that are *calibrated empirically* to match actual human decisions in that context. Neoclassical models containing such "calibrated artificial agents" can then be constructed. These, we could claim, would display behavior grounded upon actual rather than idealized human behavior.
>
> (Arthur, 1993, p. 2; emphasis added)

The second important question is the choice of the parametric software agents or the choice of the parametric decision algorithms. Although he did suggest using those learning algorithms already widely used in economics, he did not believe in the existence of the universal economic agent which is characterized as a universal decision algorithm that is applicable to all economic problems. Instead, it is *context-dependent*; the behavior can differ considerably from one decision problem to another.[1] In Arthur (1991, 1993), the context considered by him was an *N-armed bandit problem*; within this context, *reinforcement learning* (RL) was chosen as the parametric learning algorithm. Specifically, a two-parameter reinforcement learning model was calibrated with data from human-subject experiments (see Section 7.3).

The data used by Arthur was from an *individual-choice psychology experiment* (a two-armed bandit experiment) conducted by Laval Robillard at Harvard in 1952–1953 and reported in Bush and Mosteller (1955). This work can, therefore, be considered to be the first publication using human subject experiments to design software agents. Below, we shall give a brief review of the reinforcement learning model (Section 7.3); here we shall provide a remark on the empirical meaning of the software agent built by Brian Arthur.

The two parameters in Arthur's model can both be interpreted as a *speed of learning*. This behavior appears frequently in the economic literature. Intuitively speaking, it indicates how fast the agent gives up explorations of other alternatives and becomes locked in to a specific action. If the speed of learning is fast, then his learning will be dominated by his early experience, and it is likely that he may run the risk of locking in to a suboptimal choice.

7.2 N-armed bandit problem

A mathematical formulation of the reinforcement learning model can best be presented with the familiar *N-armed bandit problem*.[2] In the N-armed bandit problem, the agent is provided with a set of N alternatives from which he has to choose one. The consequence of choosing the nth alternative is a payoff r_n which is random, $n = 1, 2, \ldots, N$. The stochastic structure of r_n is unknown to the agent. In a simple setting r_n is *independent and identically distributed* (iid) with a density function $f_n(r)$ and an expectation μ_n. Suppose that the agent repeatedly plays the N-armed bandit problem H times with a sequence of decisions (choices) $d_1, d_2, \ldots, d_H$, where $d_i \in \{1, 2, \ldots, N\}$. Then

$$\pi_H = \sum_{i=1}^{H} r_{d_i}, \tag{7.1}$$

where r_{d_i} is the payoff of the decision made in the ith round. If the agent is further assumed to be risk neutral, then his goal is to maximize the accumulated payoffs over the H rounds, π_H.

The N-armed bandit problem is interesting because to make a sensible decision the agent has to learn the underlying stochastic nature of the payoffs associated with each choice by simply trying these alternatives many times. However, the conundrum of this problem is that he may not be able to afford trying all these alternatives sufficiently long enough before he can have a good approximation (estimation) of the underlying distribution. Because for each time he embarks on an adventure (tries something new or less familiar), he may give up a gain which he feels quite confident of acquiring based on past experience. This conundrum is also known as the tradeoff between *exploration* and *exploitation* in the machine learning literature. On the one hand, the agent should try out as many alternatives as possible and should not become satisfied too soon, and on the other hand, the agent should also be practically satisfied with the best alternative he has experienced.

Given the description above, the decision algorithm involved in this multi-armed bandit problem is the design of a *sampling plan*, which allows one to find the best alternative with as few samples as possible by not making many unnecessary observations at nonpromising argument values. While the N-armed bandit problem has been intensively studied mathematically, its uncertain nature colors this problem with a heavy psychological consideration. Uneasiness, fear, and bewilderment can all arise; in particular, in the real world there is no guarantee that the problem is repeated. When the iid assumption is dropped and the payoff distribution changes over time, the more uncertain and vague environment may leave more room for emotion than for reasoning.

7.3 Reinforcement learning: Arthur's version

Given the possibly very uncertain and vague environment as characterized by the N-armed bandit problem, it is desirable for agents to have a simple heuristic method to adapt quickly, while still performing robustly, to the unknown environment. Reinforcement learning, as we shall see, is simple enough to satisfy this requirement. RL is a heuristic method motivated by *behavioral psychology*. Behavioral psychology, also known as *behaviorism*, is a theory of learning based upon the idea that all behaviors are acquired through *conditioning*. Reinforcement learning has its roots in *classical conditioning*, in particular, Pavlovian models of reward learning in animals. Pavlovian conditioning is so basic to how animals adapt to their environment that it is shown by virtually all animals, from simple multicellular organisms to humans.

The essence of reinforcement learning is very simple: *choices that have led to good outcomes in the past are more likely to be repeated in the future*. The RL model to be presented below, despite its technical variations or extensions, is essential in order to implement this *law of effect*.

In returning to the N-armed bandit problem, there are N alternatives for the agent to choose from. His choice of the nth alternative ($n = 1, 2, \ldots, N$) depends on its *strength*, denoted by $q_n(t)$. Let

$$Q(t) = \sum_{n=1}^{N} q_n(t)$$

be the total strength of the N alternatives; then the probability of choosing n at time t is

$$p_n(t) = \frac{q_n(t)}{Q(t)}. \tag{7.2}$$

The probability is directly proportional to strength. Strength is reinforced according to the success achieved:

$$q_n(t+1) = \begin{cases} q_n(t) + \pi(t) & \text{if } n \text{ is activated at time } t, \\ q_n(t) & \text{if } n \text{ is not activated,} \end{cases} \qquad n = 1, 2, \ldots, N. \tag{7.3}$$

Equation (7.3) only updates the strength of the chosen alternative n at time t, which leads to a payoff $\pi(t)$, $\pi(t) > 0$. If the alternative n is not chosen at time t, then its strength will remain unchanged. From this formula, one will not be surprised to see that those alternatives which have been not chosen for a number of periods may have even less chance of being chosen in periods to come. This property of path dependence or the lock-in effect corresponds to a stylized fact of learning known as the *power law of practice*: learning curves tend to be steep initially, and then flatter.

The RL model above is almost in line with Arthur's model except that $Q(t)$ is renormalized so that $Q(t) = Qt^{\nu}$.[3] With this normalization, the original parameter-free RL model becomes a two-parameter RL model, and the two parameters are Q and ν. Both parameters, in a sense, can be connected to the *speed of learning*, as we mentioned in Section 7.1. Using data from human-subject experiments, he found a calibrated value 0 for the parameter ν ($\hat{\nu} = 0$), which is evidence of *fast learning* and features the possibility of converging to suboptimal behavior. In fact, this is what was observed in Robillard's experiment. Since this observation is important for our further discussion, we shall elaborate on this human subject experiment.

7.4 Early experiments on learning and choice-making

7.4.1 Robillard's experiment

Robillard conducted seven series of experiments with human subjects. Each series is a two-armed bandit problem. There are two options for human subjects, A and B. The probability of obtaining a one unit payoff from option A is f_A and of getting nothing is $1 - f_A$; likewise, f_B and $1 - f_B$ for option B. Robillard then considered several different designs with different compositions of f_A and f_B, denoted by $(f_A : f_B)$. It is clear that as long as $f_A > f_B$, the choice consistent with the expected gain maximization is always A. If all the participants (there are ten in his experiment) are rational in this sense, then eventually we will see that 100 percent of the subjects choose A. Unfortunately, this ideal situation did not present itself in any of Robillard's experiments. Even for the easiest case (0.8 : 0), the proportion of choices of A only came up to 90 percent, and for the most difficult case (0.6 : 0.3), the proportion of choices of A did not even pass above 70 percent. Can reinforcement learning, then, replicate this suboptimal behavioral pattern? This leads to the third question raised by Arthur.

> What would it mean to calibrate a behavioral algorithm? In designing an algorithm to represent human behavior in a particular context, we would be interested not only in reproducing statistically the characteristics of human choice, but also in reproducing the "style" in which humans choose, possibly even the ways in which they might depart from perfect rationality. The ideal would be algorithmic behavior that could pass the Turing test of being indistinguishable from human behavior with its foibles, departures and errors, to an observer who was not informed whether the behavior was algorithm-generated or human-generated (Turing 1956). Calibration ought not to be merely a matter of fitting parameters, but also one of building human-like qualitative behavior into the algorithm specification itself.
>
> (Arthur, 1993, p. 3)

In the passage above, Arthur actually addressed an issue which is still not much settled in the development of agent-based computational economics. While empirical validation is receiving increasing attention among ACE researchers, it is still not entirely clear what are the acceptable criteria and the implementable procedures. In the passage above, Arthur actually proposed two criteria, *a statistical criterion* and *an AI criterion*. For the latter, he refers to the Turing test proposed by Alan Turing at a much earlier stage.

Turing test

Of course, the AI one is much broader than the statistical one, and its implementation may be harder; hence, it has drawn much less attention from ACE economists. Ecemis, Bonabeau, and Ashburn (2005) and Arifovic, McKelvey, and Pevnitskaya (2006) are the only examples known to us. Arifovic, McKelvey, and Pevnitskaya (2006) pointed out that the development of social science theories can be likened to the task of building a computer to mimic human behavior, or equivalently, to building a computer that will pass the Turing test in the range of behavior covered by the theory. Thus, a social science theory can be deemed to be successful when it is no longer possible for a computer judge to tell the difference between behavior generated by humans and that generated by the theory (i.e., by a machine).

Simulation-based test

Arthur himself did not make a serious attempt in relation to the AI criterion; instead, his approach is very statistical. Using a Monte Carlo simulation of the behavior of calibrated agents, one can actually have a distribution of the learning trajectories of the calibrated agents. This distribution serves as a test statistic for the null that the observed learning trajectories from humans are from this distribution. The test results show that six of the seven Robillard trajectories fall well within the distribution of automata trajectories. The only one for which the null is rejected is the experiment (0.8 : 0). It is probably because this payoff scheme "has a close to deterministic outcome and for deterministic payoffs humans appear to speed up learning once they become convinced that actions produce the same payoff each time they are undertaken" (Arthur, 1993, p. 13).

Minimal degree of reasoning

The reinforcement learning model is an intelligent system with a minimal degree of reasoning efforts. To use this model, the agent only needs to react to the payoffs received from his choices in a repeated context. He does not have to know the possibly complex structure between the inputs and outputs. He does not have to reason about the causes of the payoffs and develop any inference system on which his choice can be based. However, given the initial success of the RL model on the replication of human learning trajectories and many similar successes coming later, a pertinent question is: *Did human agents really make minimal reasoning efforts in Robillard's experiment?* If so, why don't they reason harder? If not, what were they actually doing? If they were not reinforced by payoffs just like animals, why did the RL model do so well? These questions are important for designing a human-like reasoning-based intelligent system.

Four possibilities

Because Robillard's experiment was performed in the 1950s, it is difficult to get back to figuring out the answer. There are a few possibilities. The first one is that the environment is so uncertain and vague that human agents put in this situation may not even have a clue about what to learn. Most of the time, they are probably wandering around with no focus in mind.[4] The second possibility is that the RL model happened to fit this dataset well, and its generality to other similar situations has not been tested. The third possibility is that human agents did make some degree of reasoning effort, while they just *acted as if* they were RL learners. The last possibility is that there are other learning algorithms which involve greater degrees of reasoning, but their efficacy has not been tested with RL simultaneously in this dataset.

The first one can be excluded. In taking the seven group learning trajectories together, there is little doubt that they were learning as shown by higher and higher proportions of choosing option A. These learning trajectories cannot possibly be generated by the zero-intelligence agents (Gode and Sunder, 1993; see also Section 8.3). Nevertheless, as we have mentioned, when the difference in payoffs is not evident, such as the design of (0.6 : 0.3), the evidence of learning is much weaker.[5] However, this weak learning can also be captured by the calibrated artificial agents.

The second possibility is formally addressed by Arthur and posed in the following way:

> More convincing than statistical tests of fit would be tests of whether the algorithm can replicate human behavior in quite different choice problems than those for which it was calibrated.
>
> (Arthur, 1993, p. 13)

He then used Herrnstein's experiment as an illustration, to which we now turn.

7.4.2 Herrnstein's experiment

Arthur proceeded to apply the RL model to another dataset from an experiment conducted by the modified N-armed bandit problem of Herrnstein *et al.* (1993). In this experiment, the payoffs to actions A and B, r_A and r_B, are deterministic, but they are not fixed. Instead, they are *frequency dependent*. In other words, r_A and r_B depend on the frequency of actions taken. Specifically,

$$r_A = 3^{1.9-3x}; \quad r_B = 3^{0.8-4.6(1-x)}, \tag{7.4}$$

where x is the frequency of the A-choice in the last 20 trials. The payoff scheme (7.4) was, of course, not known to the human subjects. A plot of the payoff scheme will show that choice A is superior to choice B when $x < 0.75$, and is inferior to B when $x > 0.75$. The optimizing behavior which maximizes the frequency-weighted payoff

$$xr_A + (1-x)r_B$$

would set x^* to be 0.33. However, human subjects were observed to choose the action with the higher payoff at their current frequency, which led to a convergence toward choosing A 75 percent of the time. In fact, the frequency of A-choice in the last 50 of 400 trials for 8 human subjects ranges from 0.45 to 0.75. Such collectively suboptimal behavior is also replicated by the calibrated RL agents, although with a degree of bias. Therefore, we may hypothesize that, when the decision environment is complex and vague to an extent, agents will no longer make any efforts at deliberate reasoning; they simply reinforce what might work based on their experiences. Hence, the complexity of the environment has caused a very minimal degree of reasoning effort, and the N-armed bandit problems, both Robillard's and Herrnstein's, are already sufficiently complex and vague to defy any serious reasoning efforts. Is this hypothesis true? There is some other evidence showing that this may not be the case, and the classic experiment by Feldman (1962) can be used to support the third possibility.

7.4.3 Feldman's experiment

Feldman's experiment (Feldman, 1962) was an experiment on subjects' prediction involving a random series of binary symbols, say, A and B. In this experiment, the subjects were presented with a series, say,

$$AAABBAABBABBBBA\ldots$$

They were then required to ask what the next symbol is, given the series observed so far. Subjects were not given any information on the process which the experimenter was using to generate the next symbol. In addition to their predictions, the subjects were asked to "think aloud" and give the reason for each of their predictions. The intriguing part of this experiment is that while the series are completely random and symbols are generated by a Bernoulli sequence, Feldman found that each subject was quick to spot patterns in the sequence of As and Bs and to form a hypothesis on the process generating the sequence. For example, "You are following with two A's and two B's." Subjects would then stick to this hypothesis as long as it predicted well, and allowed for exceptions fairly liberally. Nevertheless, if the holding hypothesis performed badly over a run of predictions, they would change or drop it in favor of a different one.

This observation indicates that human agents are indeed reasoning, while not in a strict *deductive* way but in an *inductive* way. In other words, their mental processing of the external environment may be more sophisticated than simple reinforcement learning. Since the binary series prediction problem is very similar in structure to the two-armed bandit problem, it is likely that human subjects in Robillard's experiment might also do something beyond simple reinforcement learning. In this case, human agents with some degree of sophisticated reasoning may behave very similarly to human agents with reinforcement learning, i.e., they are *observationally equivalent*.

The last possibility is that while the RL model can replicate the behavior of human subjects, there may be other models that can do even better, but were not used in Arthur (1993) to make a comparison. Alternatively, they may perform equally well, i.e., they are observationally equivalent. To check the third and the fourth possibilities, one needs to carry out econometric tests to assess the fit of various learning algorithms to the experimental data.

7.5 Concluding remark

In any case, in his pioneering research project on the Santa Fe artificial stock market, Arthur actually used Feldman's experiment to motivate a different kind of learning for the artificial agents, known as genetic algorithms. In Part IV, we shall give a full repertoire of the algorithms for learning and intelligent behavior, ranging from reinforcement learning to genetic algorithms, but, before that, we need to know that calibration is not the only concept applied to artificial agents, there are other concepts developed in parallel which are beyond calibration. In the next chapter, we shall examine these concepts.

Notes

1 As we shall see more on this in Part IV; the different algorithms introduced there do correspond to different cognitive tasks.

2 It is called the N-armed bandit problem owing to the fact that it is comparable to finding the best slot machine out of a finite number of such machines.

3 The normalization includes the updated $q_n(t)$. So

$$q_n^{\text{norm}}(t) = \frac{q_n(t)Qt^{\nu}}{Q(t)},$$

where $q_n^{\text{norm}}(t)$ is the normalized $q_n(t)$.

4 Chen and Hsieh (2011) examined how reinforcement learning can be applied to an order-book-driven futures market. A three-parameter reinforcement learning model is fitted (calibrated) to each individual subject of the experiment. While they did find evidence of learning and the null hypothesis that subjects were randomly behaved (zero-intelligence agents) is rejected, the estimated parameters of some individuals do not lead to sensible behavior. It was then suspected that some subjects do not have a focus on what is to be learned. This is so because although their auction experiment does include a binary choice, a limited order and a market order, it is not that appearing as the typical binary choice in the two-armed bandit problem. Hence, some subjects may simply have no clue through the entire duration of the experiment.

5 Using current brain imaging tools, such as the electroencephalogram (or EEG), positron emission topography (PET), or functional magnetic resonance imaging (fMRI), one may actually provide a formal test on this possibility. See, for example, Haier *et al.* (1992) and Lo and Repin (2002). See also Chapter 18.

8 Zero-intelligence agents in the DA markets

8.1 Principles beyond calibration

In Chapter 7, we have seen how early individual experiments are applied as the empirical ground for the design of software agents, in particular, parameterized software agents. However, the calibrated design is only part of the business. There are several other designs which are motivated very differently and hence do not explicitly involve calibration. The three which we would like to present in this chapter are *zero-intelligence agents, programmed agents*, and *autonomous agents*. Quite coincidentally, all these notions of software agents are presented in the literature as *agent-based modeling of double auction market experiments*. Here, we will see a design philosophy much broader than just mirroring. Instead of treating the human-agent system separately from the software-agent system, we may consider the integration of the two as well. In the latter case, human agents and software agents can compete and collaborate with each other. Furthermore, we shall see that a gray area exists ing between human agents and software agents. This gray area makes the calibrated design studied in Chapter 7 less straightforward.

This chapter is arranged as follows. We first give a very brief introduction to double auction markets and related experiments (Section 8.2). A more detailed account of the institutional aspect of double auction markets will be given in Section 9.2.

Simplicity as a benchmark

Starting from Section 8.3, we begin with various notions of software agents which may not necessarily be designed in line with the calibrated agents. Here we see the coexistence of other design principles. In Section 8.3, we experience the *simplicity principle* that leads to the design of simple agents. While a natural, technical notion of simple agents is agents who are *algorithmically* simple, algorithmic complexity, alternatively known as *Kolmogorov complexity* (Li and Vitanyi, 2008), has never been taken seriously in various constructions of simple agents, among which the most popular is the *zero-intelligence agent* of Gode and Sunder (1993).[1] From the design viewpoint, Gode and Sunder (1993) addressed two important questions. First, does there exist a design of agent, called a *benchmark design*, such that we can isolate the institutional significance (structural significance) from the behavioral significance? By isolation we mean that the results from this benchmark design embedded within a specific institution will carry over to other designs of agents, i.e., the results are robust with respect to other designs of agents. Given this existence, the next question concerns what this benchmark design is. Gode and Sunder (1993) is very influential in the sense that it convinces many economists of the existence of this benchmark design and, furthermore, this benchmark design must be a simple or even the simplest design.[2] This has motivated them to propose a randomly behaved agent.[3] Since the zero-intelligence (ZI) agent is first proposed and applied in the agent-based double auction, in the next section we shall quickly review this theoretical environment.

8.2 Double auction markets and experiments

In the double auction market, both sides of markets (buyers and sellers) are able to submit prices, bids from buyers, and asks from sellers, to signify how much they want to buy or sell for a certain number of units of the trading target. The bids and asks will be matched by first ranking bids in descending order and ranking asks in ascending order. If the highest bid is greater than the lowest ask, then a transaction can take place, and the price can be settled somewhere between the bid and ask, say, in the middle. If the matching is continuous in time, it is called a *continuous-time double auction* or online matching; if the matching is discrete in time, it is called a *discrete-time double auction* or batch matching.

The origins of the double auction are not well known, but it is recognized that this form of auction has roots that go back to ancient Egypt and Mesopotamia. This double auction mechanism has been practically applied to many markets. The pit of the Chicago commodities market is an example, and the New York Stock Exchange is another.[4] This market mechanism also gives the earliest idea for economic experiments (Smith, 1991a), and has been very efficient in achieving the equilibrium price. This result in a sense nicely confirms the well-known *invisible hand* or *Hayek hypothesis*.[5]

8.3 Gode–Sunder model

There are two research questions in agent-based double auction markets. The first is: To achieve the degree of market efficiency which we observed from market experiments with human subjects, what is the *minimum degree of intelligence* required for our artificial agents? In other words, if we want to *replace* the human agents in the market experiment with software agents, how smart should we expect these software agents to be? The Gode–Sunder zero-intelligence agent is mainly an answer to these two questions.

Given the fact that the double auction market has been shown to be so efficient, what individual traders actually knew, learned, or did during the trading process has been considered completely *irrelevant*. They are so negligible that Gode and Sunder were even motivated to test a hypothesis that intelligence is completely irrelevant for the market efficiency realized by the double auction market (Gode and Sunder, 1993).

To do so, Gode and Sunder propose what is known as *zero-intelligence agents*. They modeled a zero-intelligence agent as a *randomly behaving agent*. This agent, when making a bid or ask decision, simply randomly picks any price from all those that will not impose a loss on the agent. This constraint is to ensure that *obvious stupidity* can be avoided. Hence, it is, in fact, a zero-intelligence-*constrained* (ZIC) agent, to be distinguished from the zero-intelligence-*unconstrained* agents. Not only are these agents unable to learn, but they also basically behave completely randomly. Even so, Gode and Sunder have shown that this kind of zero-intelligence agent can perform as well as human agents in double auction experiments.

Gode and Sunder (1993) was one of the earliest agent-based double auction markets. If we go back to the early 1990s, the term "agent-based computational economics" had not yet appeared. Nonetheless, the elements of ACE were already shown in the Gode–Sunder simulation model, mainly from the specification of the behavioral rules of software agents to the emergent outcome from their interactions. In Gode and Sunder (1993), these software agents simply behave randomly, while the emergent outcome is a highly efficient market. This result was surprising.

> Adam Smith's invisible hand may be more powerful than some may have thought; it can generate aggregate rationality not only from individual rationality but also from individual irrationality.
>
> (Gode and Sunder, 1993, p. 119)

This setting essentially binds agents' behavior only by their *budget constraints*. In other words, given the budget constraints and institutional arrangements, the rationality of the individual may not matter. Hence, zero-intelligence, as pointed out by Gode and Sunder (1993), can be traced back to Gary Becker (Becker, 1962), who showed that a budget constraint is sufficient to guarantee the proper slope of the supply and demand curves.

In addition to double auction markets, ZI agents have also been applied to other ACE models, including those for general equilibrium and financial markets. For the former, the applications are extended from the original partial equilibrium models to general equilibrium models; for the latter, the simple continuous double auction market is extended to order-book-driven markets and other related market mechanisms. While the concrete implementations of the idea of ZI need to be tailored to different applications, the basic idea is almost the same.[6] Recently, it has been further applied to other ACE models, such as prediction markets (Othman, 2008; Klingert and Meyer, 2010; Othman and Sandholm, 2010), macroeconomics (Ussher, 2009a, b), and trading networks (Tseng *et al.*, 2008, 2009). Randomly behaving agents are also extensively used in other models where the term zero-intelligence agents may be replaced by others, such as noise traders, irrational traders, etc.

8.4 Near zero-intelligence agents

8.4.1 Cliff's augmentation

While it sounds appealing, Gode and Sunder's strong argument on zero intelligence was demonstrated to lack generality by Cliff (1997). Using an analysis of the probability functions underlying the DA markets populated by Gode and Sunder's ZI traders, Cliff (1997) showed that the validity of the zero-intelligence "theorem" is largely a matter of coincidence. Roughly speaking, only in a market whose supply and demand curves are *mirror symmetric* can the ZI traders trade at the theoretical equilibrium price. In more general cases, ZI traders can easily fail. The failure of the ZI traders indicates a need for bargaining mechanisms that are more complex than the simple stochastic generation of bids and asks.

It now becomes clear that it is not in all cases that zero-intelligence agents are found to work. In addition to the setting of an imbalanced (asymmetric) demand and supply schedule (Cliff and Bruten, 1997), they also fail to achieve a competitive equilibrium in other extended settings, such as multiple interlinked markets (Bosch-Domenech and Sunder, 2000). A list of what human subjects can do (in experimental economics) but zero-intelligence agents cannot is given by Duffy (2006). When the ZI agents fail, various enhanced versions to make the ZI agent smarter have been proposed. This forces us to answer how much additional intelligence is required to make it work, which leads to various versions of near zero-intelligence agents (Cliff and Bruten, 1997; Duffy and Unver, 2006; Crockett, 2008).

The general idea for making near zero-intelligence agents is to parameterize the zero-intelligence agent. The parameterization depends on the specific application domain. After the domain is fixed, we impose a stronger restriction on the parameter space; in other words, the search space is now restricted or the search is biased. However, the guidance given to this guided search must be minimal, as a slight perturbation to the zero-intelligence agents. Thus, the borderline is not entirely clearly, and whether the "intelligence" of near zero-intelligence agents is really closer to zero may not be that straightforward, as we shall continue discussing in Section 8.5. Despite this being so, the essential message of these efforts remains the same: markets populated by simple agents may be sufficient to perform something remarkable. This feature, therefore, gives quite strong support for and a demonstration of the KISS (*keep it simple, stupid*) principle, as originally advocated by Robert Axelrod (Axelrod, 1997a) and influential in ACE.[7]

8.4.2 Truth-teller

The near zero-intelligence agent cannot be that near to zero if the zero-intelligence agent itself is not really zero. In fact, it may not be difficult to find another candidate who may seem closer to zero than the ZI agent. One obvious example is the truth-telling agent or *truth-teller*. Truth-telling traders simply use their *reservation prices* (own values or costs) as their bids or asks, no more or no less. In other words, traders will accept any offer so long as they are not loss-making ones. Obviously, this "honest" way of making deals can enhance the frequency of successful trades; nonetheless, the profits gained from these trades can be lower or nil. This strategy is simple in both *algorithmic* and *computational* senses. It requires very limited cognitive resources (working memory). It has often been used as a benchmark for the double auction tournament. The question is whether agents who follow truth-teller can also be considered to be zero intelligent. Or, the zero-intelligence agent has to be a randomly behaving agent. After all, what does zero intelligence mean?

8.5 Simplicity, intelligence, and randomness

8.5.1 Randomly behaving (entropy-maximization) agents

The notion of the zero-intelligence agent is not only well received by economists, but also by physicists, specifically, econophysicists (Bouchaud, Mezard, and Potters, 2002; Farmer, Patelli, and Zovko, 2005), although they are not identically motivated. The reason that physicists favor the device of the zero-intelligence agent is because the strategic behaviors of financial agents are generally poorly known and are difficult to model. Therefore, in the vein of the law of large numbers, they simply assume that these complexifications will cancel each other out so that altogether their aggregate behaviors are observationally equivalent to the randomly behaving agents (zero-intelligence agents). In a sense, doing this is also generally related to the application of the *maximum entropy principle* in agent design, i.e., to be "maximally non-committal with regard to missing information" (Jaynes, 1957, p. 620).

This principle is nicely applicable to Gode and Sunder's zero-intelligence agent. Since normal traders would not propose or accept a deal which would obviously lead to economic loss or not lead to welfare improvement, under no further information on what else they will do, the design of the zero-intelligence agent is *minimally prejudiced* in the sense of Jaynes (1957). Hence, one way to formalize and to generalize the zero-intelligence agent is to explore the applicability of the entropy maximization principle to design the agent. Doing so is also consistent with the KISS principle in spirit since the entropy maximization principle per se is an information-theoretic foundation for model simplicity.

In sum, the idea of the zero-intelligence agent or the randomly behaving agent seems to be a very natural way to start when one has no clue how agents actually behave, wants to have an easy start without too many complications, or wants to have a benchmark as a comparison basis for other more developed models. However, none of the reasons above has anything to do with cognitive salience. Yet the term zero intelligence is a term related to cognitive capability.[8]

8.5.2 Trinity?

We now attempt to put the several ideas concerning agents that have been discussed so far together and take a closer look at their relationships, which may otherwise be largely taken for granted in many applications. We start with the idea of simple agents or agents following simple rules (automata), then agents with

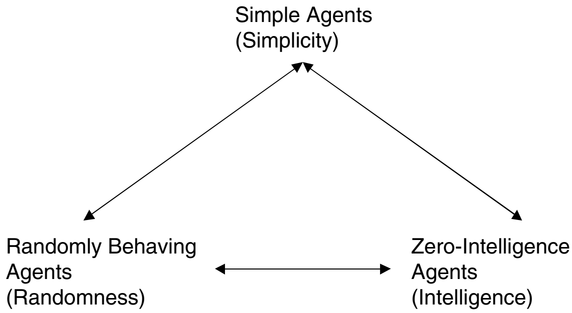

Figure 8.1 Relationships among the three notions of agents.

low-cognitive capability (zero-intelligence agents), and finally end up with randomly behaving agents and entropy-maximizing agents. The basic question is: *Are they identically the same?* We have this question simply because in many applications they are assumed to be so. However, what we encounter here is exactly an example of the agent as an interdisciplinary concept, and terms such as "simple," "naive," and "random" cross several disciplines, from computer science, psychology, and neuroscience to mathematics and physics. Can these different terms or concepts belonging to different disciplines be unified on a higher ground, or are they independent? In the following, we shall reflect upon each of the three bilateral relationships depicted in Figure 8.1.

8.5.3 Simplicity and randomness

For us, a natural technical notion of simplicity required for developing the idea of a simple agent is built upon computer science, i.e., *algorithmic simplicity* or, as more frequently used, *algorithmic complexity*.[9] To put the concept in a rough but easy-to-understand way, this notion requires us to focus on the program length which we use to describe the agent, also known as the *description length*; the shorter the simpler. Hence, simple agents are agents who are *algorithmically* simple. We can then ask whether the randomly behaving agents are simple in terms of the program length of the underlying pseudo-random number generators. There are several different versions of pseudo-random number generators, from the initial attempt by John von Neumann (von Neumann, 1951) to the breakthrough made by Makoto Matsumoto and Takuji Nishimura (Matsumoto and Nishimura, 1998) and to the long-standing challenge of cryptographically secure pseudo-random number generators, each leading to a different quality of the pseudo-random number being generated.[10] The point is that they are not simple, like one-line code, and tend to get longer the better they are at behaving randomly.

With this background, it is not hard to see that randomly behaving agents are not that simple as some might have initially thought. In the context of a double auction, other agents, such as the truth-teller, simply bid or ask with their reservation price (the redemption value for buyers or the minimum cost for sellers). Given the token value table, one can almost write down such truth-teller programs in one line of code, and it is not hard to see that a market exclusively composed of truth-tellers can realize full market efficiency, i.e., the maximum sum of the consumers' and producers' surpluses. Hence, is a randomly behaving agent simple? Are there any artificial agents that are simpler or much more simple than randomly behaving agents? How does one make a choice among simple agents? Without answers to these questions, a serious implementation of the KISS principle can become problematic.

8.5.4 Simplicity and intelligence

What is the relationship between simplicity (or complexity) and cognitive capability? Are simple agents necessarily related to agents with low cognitive capability (naive agents)? There are several measures of intelligence which have been used in empirical economics and experimental economics (some are to be further reviewed in Chapter 17). These studies provide us with some useful observations to help think about the relationship between simple agents and naive agents; in particular, those experiments based on the notion of *iterated dominance*. In the iterated dominance game, the number of steps required to eliminate all dominated strategies is called the *depth of reasoning*, and the interesting feature of these games is that they allow us to develop a *step-by-step* computation in the acquisition of a dominant strategy. How deliberate a strategic behavior is can then be connected to this depth.

Differentiating the complexity of games based on the depth of reasoning or steps of iterated reasoning is not new (Camerer, 2003). Various models of cognitive hierarchy have been proposed to model players' behavior observed in behavioral game experiments (see also Section 19.3), that include the theory of mind (Stahl and Wilson, 1995), Machiavellian intelligence (Ohtsubo and Rapoport, 2006), level-k reasoning (Crawford and Iriberri, 2007b), and cognitive hierarchy (Camerer, Ho, and Chong, 2004). Within this context, Ohtsubo and Rapoport (2006), in a *beauty contest game* (Nagel, 1998), found a positive relationship between the depth of reasoning and intelligence, using a measure known as the "Imposing Memory Task." Further research in this direction may shed more light on our maintained hypothesis that naive agents tend to be algorithmically simple and may have difficulties learning or discovering complex rules that can perform a number of iterations or simulate what other agents think.

The relationship between simple agents and naive agents can become even more complicated when one also incorporates the considerations of learning, adaptation, and evolution. In this dynamic setting, agents with high cognitive capability have the potential to be algorithmically complex, but may be satisfied with using simple rules if those are viable strategies.[11] On the other hand, naive agents can learn some complex rules, but that learning capability has a tighter limit. To be more precise, let us employ the Chomsky hierarchy from computer science as a metaphor.[12] The Chomsky hierarchy is a hierarchy of formal grammars and the formal languages generated by the grammars. The hierarchy arranges four classes of grammars and the four generated languages in monotonically increasing order, so that any class of grammars (languages) appearing in this order is a *proper subset* of the one coming after it. It also implies a hierarchy of the automata which can recognize the grammars. By the same monotonic order, the automata at a higher level can simulate the automata at a lower level, but not vice versa.

If the cognitive capability of agents (mind and brain) can be arranged in a similar hierarchy as the Chomsky hierarchy of grammar and automata, then agents with higher cognitive capability can simulate what agents with lower cognitive capability can think, but not vice versa. In other words, smarter agents can behave like simple naive agents, even though they are not that simple. In this way, our maintained hypothesis is broken, and simple agents are not necessarily naive.[13]

8.5.5 Intelligence and randomness

Finally, would naive agents be randomly behaving agents, and would randomly behaving agents necessarily be naive? By definition, the behavior of the entropy-maximizing (EM) agent should not leave any pattern for observers to detect except those constraints; otherwise, destroying these patterns will further contribute to the increase in entropy. From a strategic viewpoint, doing so may not be trivial. Technically speaking, what one needs is a program which would generate any kind of behavior except repeating itself, i.e., any possible pattern will be self-detected and self-annihilated. Based on our discussion earlier about the pseudo-random number generator, it would be surprising to know that the agent who is even unable to memorize and to learn can have all his behavioral patterns *automatically* self-detected and self-annihilated. So far, we are not aware of any experimental studies using human subjects to test whether a great degree of consciousness is required to generate highly patternless behavior. Naive agents may have difficulties making any decision on a sensible basis, but that does not immediately imply that decisions without logic, reasoning, judgments, or fast and frugal heuristics must be patternless. A man who is drunk might walk in a way that appears "random," but in fact he could just follow a sine-like curve. Hence, naive agents and randomly behaving agents are conceptually not tightly coupled.

8.6 Concluding remarks

Our viewpoint and the discussion regarding the triangular relationship as demonstrated in Figure 8.1 may not be complete. Room for further iterations, of course, exists. Nonetheless, the point is that the equality relationship which is normally assumed in this triangle has not been carefully addressed and soundly established. Based on the discussion above, the three agents are motivated by different disciplines. The simple agent can be sensibly constructed using computation theory (algorithmic complexity), randomly behaving agents can be meaningfully generalized using information theory (entropy), and zero-intelligence agent or agents with low cognitive capability can be much clarified if they are grounded in cognitive psychology (intelligence quotient), cognitive neuroscience, and computational neuroscience. Then a higher ground to integrate or hybridize the three may exist in an interdisciplinary research incorporating algorithmic information theory and computational neuroscience. However, at this moment, they are very much independent and are three competing options with regard to the benchmarking decision.

Notes

1 By the definition of algorithmic randomness, an object is called random if there is no room for its further compression. For completely randomly behaved agents, we have no deterministic rule to describe what they did and hence to predict what they will do; therefore, their behavior must be presented as is and cannot be further compressed. In this sense, they are algorithmically random as well.
2 See, for example, Duffy (2006).
3 See also Sunder (2004) for his own description of the discovery of the zero-intelligence agent.
4 The NYSE, however, does not use a pure order book market mechanism (the double auction mechanism). It also uses the specialist (market-maker) system.
5 For a detailed discussion of the Hayek hypothesis, see Section 9.6.
6 The zero-intelligence agent has been very influential in agent-based computational economics and finance. General reviews can be found in Duffy (2006) and Ladley (2012). See also Barr *et al.* (2008) for quite an extensive discussion on the use of zero-intelligence agents in the future of ACE.
7 Having said that, we must also point out the opposition to this principle. The equally well-known alternative is the KIDS (*keep it descriptive, stupid*) principle, proposed by Bruce Edmonds and Scott Moss (Edmonds and Moss, 2004). It was argued that social simulation models are different from analytical mathematical models, hence the pursuit of simplicity should also change accordingly. The contrast between KISS and KIDS is an ongoing research issue in the methodology of social simulation. The interested reader is referred to the special issue on "The Methodology of Simulation Models" of the *Journal of Artificial Societies and Social Simulation* (Vol 12, No. 4, 2009).
8 For a formal addressing of the cognitive capacity of artificial agents, see Chapter 19.
9 Other frequently used terms for algorithmic complexity include Kolmogorov complexity, program size complexity, and minimum description length. In computation theory, one way to measure the complexity of an object is based on the minimum program length required for describing or replicating the object. This notion was first introduced independently by Andrey Kolmogorov (1903–1987), Greg Chaitin, and Ray Solomonoff (1926–2009) in the 1960s, and is now known as *algorithmic information theory*. For an introduction, the interested reader is referred to Li and Vitanyi (2008).
10 A comprehensive introduction to pseudo-random number generators can be found in Gentle (2010).
11 The *fast and frugal heuristics* advocated by Gerd Gigerenzer (Gigerenzer, 2008) are illustrations of these situations.
12 An introduction to the Chomsky hierarchy and its relationship to computation theory can be found in Linz (2006). See also Section 13.4.1.
13 It seems that we can go further to map the cognitive hierarchy to the Chomsky hierarchy. However, we have reason to hesitate to do so. The cognitive capability as defined and measured by psychologists can become stable with age and hence can be treated as an exogenous variable. However, various models of cognitive hierarchy do not necessarily assume that the cognitive hierarchy is exogenously given; on the contrary, it can be endogenously changed and upgraded when the subjects become more experienced.

9 Autonomous agents in the DA markets

9.1 Programmed agents and autonomous agents

9.1.1 Programmed agents

According to Gode and Sunder (1993), the invisible hand even exists in a market composed of non-purposive agents (individual irrationality). However, our *Homo sapiens* are definitely purposive. When placed in a well-defined experiment like the DA market, *Homo sapiens* are naturally attracted by transaction gains, and it is not likely that blind bidding and asking will be a sensible way for them to react upon the information they acquired.[1]

The weakness of the ZI agents is that they are not purposive, but human agents are. Hence, it helps us little to see the market dynamics from a game-theoretic viewpoint, be it static or evolutionary. One of the research questions of the agent-based double auction market is exactly about how to win or survive. The purposive traders not only will not bid or ask randomly, but may even develop some strategies to trade, be they sophisticated or simple. In fact, an inquiry into an effective characterization of the "optimal" trading strategies used in the double auction market has led to a series of tournaments, known as the *Santa Fe Double Auction tournament* (see also Section 5.2).

This chapter begins with the Santa Fe Double Auction (SFDA) market (Section 9.2). The SFDA tournament provides another early example of agent-based double auction markets. Differing from the Gode–Sunder one, the SFDA market is composed of *programmed agents*, whose strategic behavior rules are hand-written by *Homo sapiens* (the participants in the SFDA tournament). This SFDA design gives the software agents a dual role. On the one hand, they are programmed agents (machine codes); on the other hand, they are incarnations of *Homo sapiens*. The subtle difference between the two is decision-making *online* versus *offline*.

Online versus offline

Human agents in the double auction experiments make online decisions. They receive immediate feedback and are pressed to react. Human-written programs are generated offline, so time pressure is not imminent; however, participants receive no immediate feedback while writing their programs. Therefore, they have to largely rely on their mind power to guide them to write and rewrite the program. It is more like a *deductive* process. Accordingly, double auction experiments and double auction tournaments provide us with two different ways to observe human decision-making processes. The online decision is more inductive, and, possibly, simple but spontaneous, while the offline decision is more deductive, and, possibly, complex but less adaptive.

9.1.2 Autonomous agents

Autonomous agents are to be distinguished from human-written programmed agents. The latter refer to the artificial agents whose behavioral rules or algorithms are written by humans, whereas the former refer to the ones whose behavioral rules are automatically generated by computers. Although the autonomous agent is quite an important development in ACE, the line between autonomous agents and non-autonomous agents is not clear.

In fact, being *autonomous* or not is just a matter of degree. It is very much dependent on the degree of autonomy with which the agents are endowed. Hence, the least-squares learning agents frequently used in macroeconomics can be considered to be autonomous agents because they are able to learn the parameters of the underlying system without further external supervision (Evans and Honkapohja, 2001). However, if the underlying law of motion is not linear but nonlinear, these agents may not be able to jump out of the linear trap on their own. With this clarification, it probably would be fair to say that the tools available for economists to build autonomous agents with a high degree of autonomy were rather limited before the early 1990s, as implied by the quotation we borrowed from Holland and Miller (1991)—see Section 5.1).

In Chapter 5, we also indicated that automated market tournaments depend on autonomous agents. However, both the parameterized agents (Chapter 7) and the zero-intelligence agents (Chapter 8) are not motivated by the idea of great autonomy, and hence are distant from the requirement of being autonomous. This chapter, in addition to the programmed agents in the SFDA markets, will introduce the design of autonomous agents within the context of double auction markets. This should lead us to see work by Martin Andrews and Richard Prager (Andrews and Prager, 1994).

The key ingredient of Andrews and Prager (1994) is the *autonomous agent*, driven by genetic programming. These autonomous agents, by design, are purported to search for better deals to gain from. In the very foundation of classical economics, these agents (autonomous agents) contribute to the discovery and exploitation of hidden patterns and opportunities. The reactions of their opponents further lead to change in the economy, which in turn creates new opportunities for further exploitation. This infinite cycle, while it may not be the whole, is an essential part of Alfred Marshall's biological description of an economy as "constantly changing" (Marshall, 1924). Hence, Andrews and Prager (1994) enables us to move one step toward *a genuine economic model of change*, which is an important goal of ACE. Their work will be reviewed in Section 9.3. However, Andrews and Prager (1994) only focused on the evolutionary part of the agent-based DA market; they did not go further to complete the coevolutionary part. This work was later on completed by Chen and Tai (2003). Their work and the related development (Chen and Tai, 2010; Chen and Yu, 2011) will be sketched in Section 9.4. This chapter will end up with extensions to other market platforms which may involve the use of program agents or autonomous agents: one is the U-MART project (Section 9.5), and the other is the prediction market (Section 9.6).

9.2 Santa Fe double auction markets

The purpose of this section is to give an introduction to the SFDA markets (Rust, Miller, and Palmer, 1993, 1994). The entire operational protocol is summarized in Figure 9.1. The structure of the SFDA market is very similar to the Arizona *continuous time* experimental DA market (Smith, 1962, 1991b).

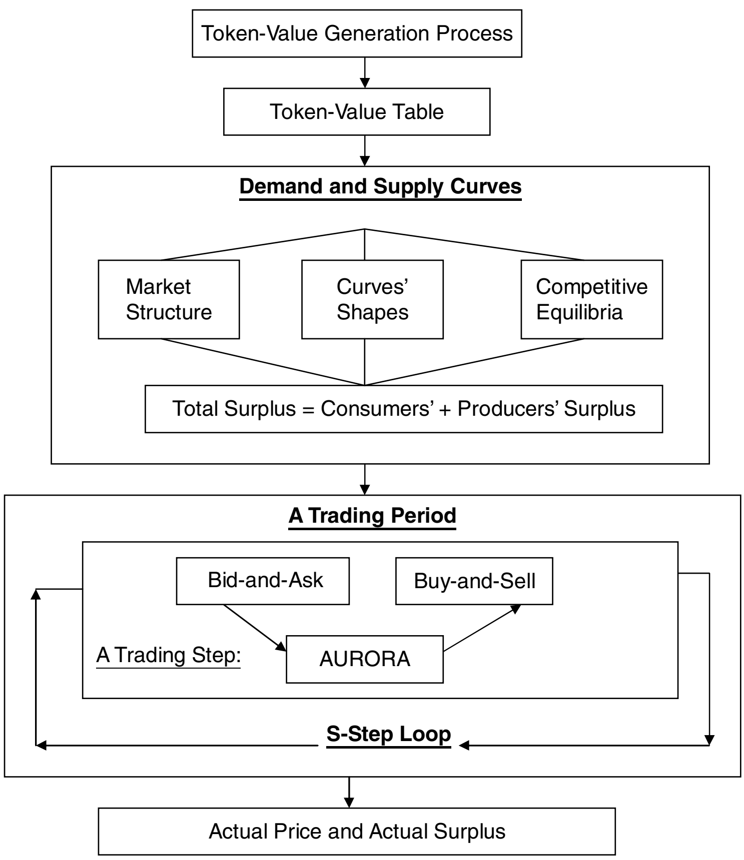

Figure 9.1 Flow chart of the SFI double auction market.

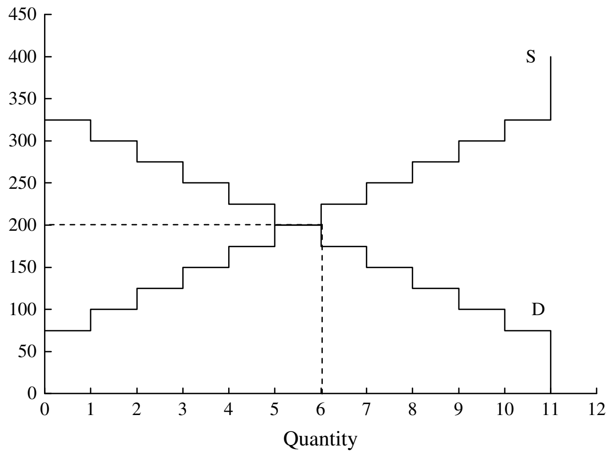

Figure 9.2 Demand and supply curve.

In the experimental market, the subjects are divided into a group of *sellers* and a group of *buyers*. Sellers are given a number of units of an arbitrary commodity, conventionally called *tokens*, and each unit has a *limit price* (below which it cannot be sold), which is *private* (i.e., known only to the seller of that unit). Buyers are given the rights and means to buy a number of units, and for each unit they are given a *private limit price* above which they must not pay. The array of sellers' limit prices determines the market supply curve, and the array of buyers' limit prices determines the market demand curve. An example with 11 units to trade is depicted in Figure 9.2.

In the experiments, traders *quote* (*bid* and *offer*) prices by typing them into their *computer terminals*: the quotes are then distributed to the other traders, and at any time a buyer can accept a seller's offer or a seller can accept a buyer's bid. This continuous trading process is broken into *discrete periods* or *days*: at the start of each day, new allocations of selling or buying rights are distributed to the traders.

9.2.1 Information

At the start of a DA game, the monitor broadcasts *public information* to the traders. Next, the monitor sends each trader a packet of *private information*.

Public information includes:

– the number of buyers;
– the number of sellers;
– the number of rounds, periods, and time steps;
– the number of tokens each agent will have; and
– the joint distribution F from which the traders' token values are drawn.

At the end of each bid-and-ask step (see Section 9.2.3), the trader is informed of:

- each others' bids and asks;
- the current bid (highest outstanding bid);
- the current ask (lowest outstanding ask);
- the holder of the current bid; and
- the holder of the current ask.

At the end of each buy-and-sell step (see Section 9.2.3), the trader is informed of:

- whether there is a successful trade; if so,
- the trading price.

Private information includes:

- traders' realized token values.

9.2.2 Token value generation processes

Token values are represented by T_{jk} where j indexes the trader, and k indexes the token assigned to the trader. Tokens are randomly generated according to

$$T_{jk} = \begin{cases} A + B + C_k + D_{jk} & \text{if } j \text{ is a buyer,} \\ A + C_k + D_{jk} & \text{if } j \text{ is a seller,} \end{cases} \tag{9.1}$$

where

$$A \sim U[0, R_1], \tag{9.2}$$

$$B \sim U[0, R_2], \tag{9.3}$$

$$C_k \sim U[0, R_3], \tag{9.4}$$

and

$$D_{j,k} \sim U[0, R_4]. \tag{9.5}$$

Each of the four digits of the gametype variable corresponds to $\{R_1, R_2, \ldots, R_4\}$ according to the base-3 coding

$$R_i = 3^{k(i)} - 1, \tag{9.6}$$

where $k(i)$ is the ith digit of the gametype. Notice that when

$$R_1 = R_2 = R_3 = 0, \tag{9.7}$$

we have a model of standard independent private values where tokens are independently uniformly distributed on the interval [0, R_4].

For example, two runs of a token-value gametype 6453 with four buyers and four sellers, and with four tokens assigned to each trader, is shown in Table 9.1. Here, "6453" implies

$$A \sim U[0,728],\ B \sim U[0,80],\ C_k \sim U[0,242],\ D_{j,k} \sim U[0,27].$$

The competitiveness of each position token is indicated by signs "+," "−," or "=" appearing inside the brackets. "+" denotes an intra-marginal (competitive) position, whereas "−" denotes an extra-marginal (non-competitive) position. The position exactly in between, which is not different from the competitive equilibrium (CE) price, P^*, is denoted by "=." In this specific example, there is a unique P^*, 691, for the second token-value table, whereas there is only a CE interval for P^*, (806, 814), for the first one.

In addition to P^*, once a table of token values is generated, the shapes of the demand and supply curves are also determined, and hence the *total surplus* (TS), where

$$TS = \sum_{j \in \text{buyer}, k} (T_{j,k} - P^*)^+ + \sum_{j \in \text{seller}, k} (T_{j,k} - P^*)^-, \tag{9.8}$$

"+" above refers to the positive part of the function, and "−" above refers to the negative part of the function. With this token-value generation mechanism, one can automate the generation of various demand and supply schedules. Figure 9.3 shows the 20 markets randomly generated by this mechanism.

Table 9.1 Token table: (gametype 6453)

	Buyers				*Sellers*			
Token	*B1*	*B2*	*B3*	*B4*	*S1*	*S2*	*S3*	*S4*
First run								
T1	845(+)	859(+)	863(+)	860(+)	736(+)	735(+)	740(+)	729(+)
T2	834(+)	828(+)	841(+)	842(+)	756(+)	762(+)	768(+)	742(+)
T3	818(+)	801(−)	823(+)	804(−)	806(+)	818(−)	814(−)	806(+)
T4	786(−)	794(−)	806(−)	788(−)	869(−)	872(−)	870(−)	881(−)
Second run								
T1	754(+)	760(+)	761(+)	751(+)	651(+)	666(+)	646(+)	661(+)
T2	722(+)	708(+)	717(+)	719(+)	661(+)	675(+)	659(+)	665(+)
T3	691(=)	705(+)	690(−)	702(+)	680(+)	683(+)	680(+)	693(−)
T4	681(−)	678(−)	689(−)	691(=)	779(−)	776(−)	788(−)	774(−)

Note: Two runs of gametype 6453. Due to the stochastic nature of the token-value generation process, even with the same gametype, the table of token values can quite likely be different.

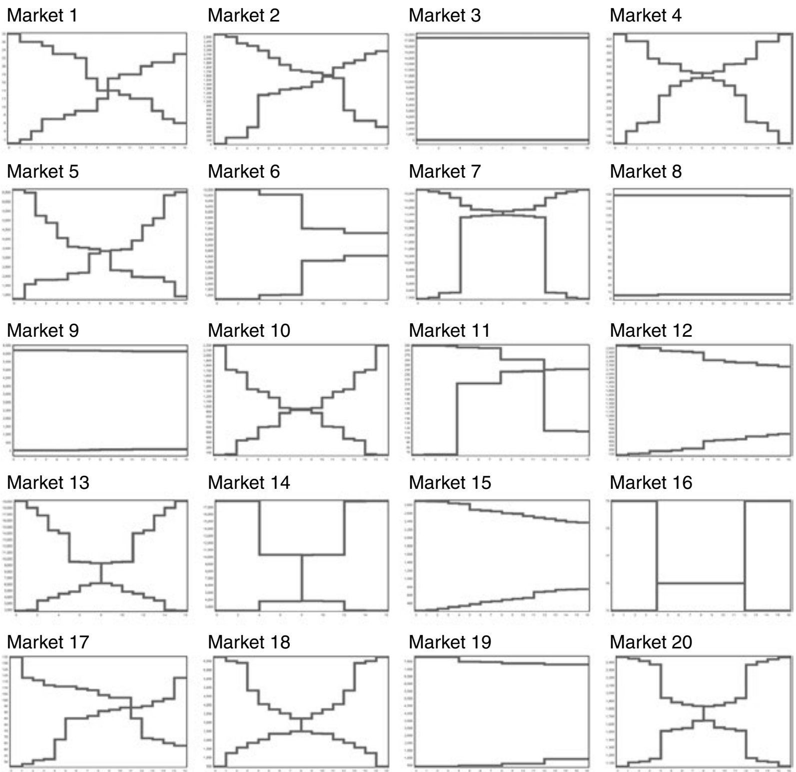

Figure 9.3 Twenty markets randomly generated by the token-value generation mechanism.

9.2.3 Trading procedure

An individual DA game is divided into one or more *rounds*, and each round is further divided into one or more *periods* (*days*). Time is discretized into alternating bid/ask (BA) and buy/sell (BS) steps. A trading period is simply a set of *S* alternating BA and BS *steps*. Transactions are cleared according to the AURORA rule.

1 The BA step:
   The DA market opens with a BA step in which all *software agents* are allowed to simultaneously post bids and asks based upon the *bargaining program* (to be detailed in Section 9.2.4). The bids and asks in one trading period are exemplified in Figure 9.4.
2 The BS step:
   During the BS step, either player can accept the other's bid or ask. If an acceptance occurs, a transaction is executed. If both parties accept each other's

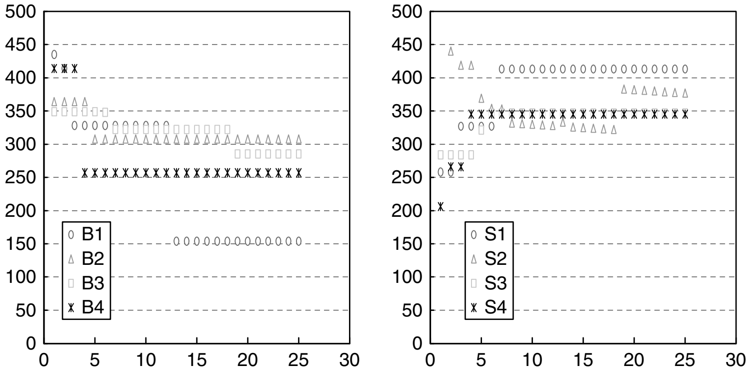

Figure 9.4 Bids and asks in one trading period.

Note: Results from a single run of the electronic DA market with 4 buyers, 4 sellers, and 25 steps in a single trading period based on the first run of gametype 6453 (see Table 9.1).

offers, the auctioneer *randomly* chooses a price *between* the current bid and the current ask to determine the transaction price.

3 The AURORA rule:

The AURORA rules were inspired by the rules used in the AURORA electronic trading system developed by the Chicago Board of Trade. The AURORA rules stipulate that only the holders of current bid (CB) or current ask (CA) are allowed to trade. By the AURORA rule, the actual transaction price (P) is *randomly* determined as follows:

$$P \sim f[CA, CB], \tag{9.9}$$

where f is a probability density function. For example, f can be a *uniform* or a *triangle* distribution. Figure 9.5 is the time series of the actual price, the current bid and the current ask in one trading period (25 steps) of a DA market with four buyers and four sellers.

9.2.4 A glossary of programmed agents

Given the environment as specified in Sections 9.2.1 to 9.2.3, each trader bases its auctions (bids or asks) upon the information available for them, both publicly and privately. For illustration, 11 programmed agents are briefly introduced below.

Truth-teller

Truth-telling traders simply bid/ask with their reservation prices (Section 8.4).

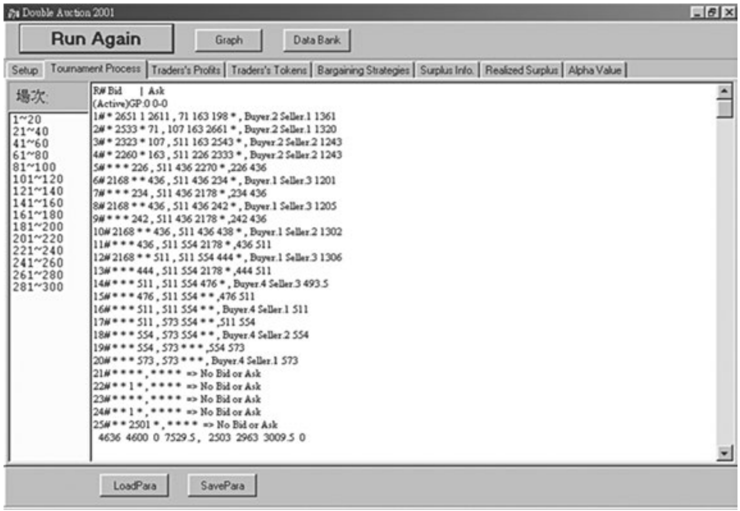

Figure 9.5 Bids, asks, and matches in one trading period.

Notes: There are 25 steps in one trading period (day). The figure shows the bids and asks of each trader during that day. By the AURORA rule, when a match happens, the matched pair is shown with the final transaction price. The symbol "$*$" refers to the case where the trader chooses to pass (skip) in the respective period.

Skeleton

Skeleton was provided by the SFDA to all participants. The participants can include this program or a modified version of it as a part of their own developed programs. The Skeleton trader will judge the market scenarios based on the current bid (the highest bid) and the current ask (the lowest ask) in the previous trading step. This information together with his values (costs) can help him to judge whether the market is *lucrative enough* to enter, and that in turn depends on the *profit margins*. Different trading strategies expect different profit margins. For the Skeleton buyer, if the current bid is smaller than both the current ask and his current token value, then the market scenario is suitable for trading because in this case there is still room for making a profit by proposing a better (more competitive) offer. Otherwise, he will not enter the market.

If he decides to enter the market, a bid or ask will be proposed. To gain more profits, the price is set in a manner that is *just competitive enough* to win the deal. For example, the Skeleton buyer will not bid a price directly up to the current ask; instead, he will propose a weighted average between the current bid and the current ask, probably attaching more weight to the current bid by incorporating the competitive pressure from the seller's side. Incremental increases in bidding are quite common. What makes the Skeleton strategy unique is that these increments are random and biased toward the current bid, instead of toward the current ask or the current token value. In this sense, the Skeleton can be more aggressive and patient than other incremental bidding behavior.[2]

Kaplan

The Kaplan strategy was designed and submitted to the SFDA tournament by Todd Kaplan, who was then a PhD student in economics at the University of Minnesota. It is a so-called "background trader" strategy in the sense that the trader remains silent until the market bid and the market ask are close enough to imply a trading opportunity. When this opportunity emerges, the Kaplan trader will jump out and steal it. In spite of the simplicity of its tactic, the Kaplan strategy turned out to be the winner of the SFDA tournament.

The Kaplan strategy, as a modified version of Skeleton, looks into more information pertaining to the entry decision. In addition to the current bid and ask, it also keeps track of the minimum and maximum transaction prices from the previous period, which can help it to immediately recognize some handsome deals or bad deals. Furthermore, *the time left for trading* (the remaining time) is also taken into account. The enter decision will require the trader to be more *patient* when there is plenty of time to trade. For example, if the current ask is higher than the maximum price in the previous trading period or if the spread (the gap between the current ask and bid) is not narrow enough, then the Kaplan trader will prefer to *wait*. On the other hand, if time is running out, the entry decision will be rather active even though the profit margin is not satisfied.[3] Once the enter decision is made, the Kaplan buyer will bid the current ask or his current token value, depending on which one is smaller. "In practice this implies that Kaplan's program eventually defaults to truthtelling mode when confronting patient opponents who delay making 'serious' bids and and asks" (Rust, Miller, and Palmer, 1993, p. 170).

Ringuette

The Ringuette strategy, another strategy submitted to the SFDA tournament, was designed by Marc Ringuette, a computer scientist at Carnegie Mellon University. Ringuette's entry rule is similar to Kaplan's, and the trader is also a background trader. Again, consider the case of a buyer. The entry decision also depends on the relative position of the current bid, the current ask, and the current token value (the same as the Kaplan strategy). If the three, from the current bid to the current token value, appear in an increasing order, and their distance supports a profit margin, then the Ringuette trader will enter the market. In other words, the Ringuette buyer will wait until the first time when the current bid exceeds the current ask less a profit margin. The Ringuette strategy is a simple rule of thumb, but it was awarded the second place in the SFDA tournament (Rust, Miller, and Palmer, 1994).

Zero-intelligence constrained

The ZIC traders were proposed by Gode and Sunder (1993). ZIC traders send random bids or asks to the market in a range bounded by their reservation prices.

Although ZIC traders can avoid transactions which incur losses, they do not have any goals or tactics during the trading process. Therefore, they are regarded as "zero-intelligence."

Zero-intelligence plus

The zero-intelligence plus (ZIP) strategy is derived from Cliff and Bruten (1997). A ZIP trader forms bids or asks with a chosen profit margin, and will try to raise or lower its profit margin by inspecting its own status, the last shout price, and whether the shout prices are accepted or not. Once the profit margin is chosen, the ZIP trader will gradually adjust its current shout price to the target price.

Markup

The Markup trading strategy is drawn from Zhan and Friedman (2007). Markup traders set up certain markup rates and consequently determine their shout prices. Zhan and Friedman (2007) showed that the market efficiency will be maximized when traders all have 0.1 markup rates.

Gjerstad–Dickhaut

The Gjerstad–Dickhaut (GD) strategy is proposed by Gjerstad and Dickhaut (1998). A GD trader scrutinizes the market history and calculates the possibility of successfully making a transaction with a specific shout price by counting the frequencies of past events. After that, the trader simply chooses a price as her bid/ask if it maximizes her expected profits.

Bayesian game against nature

The Bayesian game against nature (BGAN) strategy was proposed by Friedman (1991). BGAN traders treat the double auction environment as a game against nature. They form beliefs in other traders' bid/ask distributions and then compute the expected profit based on their own reservation prices. Hence their bids/asks simply equal their reservation prices minus/plus the expected profit. Bayesian updating procedures are employed to update BGAN traders' prior beliefs.

Easley–Ledyard

The Easley–Ledyard (EL) strategy was devised by Easley and Ledyard (1993). EL traders balance the profit and the probability of successfully making transactions by placing aggressive bids or asks in the beginning, and then gradually decrease their profit margin when they observe that they might lose chances based on other traders' bidding and asking behavior.

Empirical

The Empirical strategy was inspired by the empirical Bayesian traders proposed in Chan *et al.* (1999). The Empirical trader works in the same way as Friedman's

BGAN but develops its belief by constructing histograms from opponents' past shout prices.

These strategies are, to a certain degree, various types of trading strategies observed in financial market studies. Some of them are simple rules of thumb, such as the Kaplan, ZIP, or EL strategies, while the others are quite sophisticated in their decision processes, such as the GD, BGAN, and Empirical strategies. From the viewpoint of adaptivity, some of them are adaptive in the sense that they adjust in response to the market situations, while the others are non-adaptive by repeating the same behavior regardless of the environment.

Despite their distinct features, none of these strategies is autonomous because their trading tactics are predefined according to some fixed principles. In the following section, we will introduce the autonomous trading agent, whose principle is to constantly exploit the environment.

9.3 Andrews–Prager model

In either IPD tournaments (Chapter 5) or DA tournaments, a practical issue of building autonomous agents is the *representation* of strategies. In his application of a GA, Robert Axelrod represented the IPD strategies as history-dependent actions: each agent responds to the two-sided actions of the three previous runs, say, CC|CD|DD ("C" for cooperation and "D" for defection). Since there are a total of 2^6 (= 64) such histories, a strategy is represented by a 64-bit string, and there are a total of 2^{64} possible strategies. While this number is big, the representation power given by this finite string is still rather limited. One can easily find some human-written programs not belonging to this gigantic set. This shortcoming presents a fundamental challenge to the building of autonomous agents, namely, *how to represent the class of human-written programs as submitted to the tournaments*.

The solution requires expertise beyond statistical learning. In general, it involves areas which are less familiar to economics, such as knowledge representation, formal languages, mathematical logic, etc. One attempt to move forward from the fixed-length string described above, is to consider human-written programs as strings of symbols (alphabets) with variable lengths. This can be quite feasible, for example, if all the programs are written in the program language LISP (Section 13.4.5). If so, they can be placed in the context of *formal languages* in computer science (Section 8.5.4), and can be generated by the appropriate grammars or production rules with a given initial symbol (Section 13.4.1).

The idea of using formal languages to represent strategies and hence to build autonomous agents in economic tournaments was first attempted by Andrews and Prager (1994)—Figures 5.2 and 5.3. Andrews and Prager applied the same evolutionary operators used in genetic algorithms to a *context-free language* so that different strings (programs) are connected with each other by evolutionary paths, of which each step is a perturbation to an alphabet of the string at that time. This technique, as we have seen several times in Sections 6.1.4 and 6.2.3, is known as *genetic programming*, invented by Nichael Cramer (Cramer, 1985) and further developed by John Koza (Koza, 1992a).

Table 9.2 Information available for traders (terminal set)

Index	*Terminal*	*Interpretation*
1	PMax	The highest transaction price on the previous day
2	PMin	The lowest transaction price on the previous day
3	PAvg	The average transaction price on the previous day
4	PMaxBid	The highest bidding price on the previous day
5	PMinBid	The lowest bidding price on the previous day
6	PAvgBid	The average bidding price on the previous day
7	PMaxAsk	The highest asking price on the previous day
8	PMinAsk	The lowest asking price on the previous day
9	PAvgAsk	The average asking price on the previous day
10	Time1	The number of auction rounds left for today
11	Time2	The number of auction rounds that have no transaction
12	HT	The highest token value
13	NT	The second highest token value
14	LT	The lowest token value
15	Pass	Pass the current auction round
16	CASK	The lowest asking price in the previous auction round
17	CBID	The highest bidding price in the previous auction round
18	Constant	Randomly generated constant number

Table 9.3 Logic and mathematical operators (function set)

Function				
+	−	*	%	min
>	exp	abs	log	max
sin	cos	if-then-else	if-bigger-then-else	

A context-free grammar starts with a set of symbols, which can be further separated into a set of terminal symbols and a set of non-terminal symbols. In genetic programming these two sets are also known as the terminal set and function set, respectively. The former normally includes data to be processed, such as variables or constants, whereas the latter have operators to process the data, such as arithmetic and logic operators. As an illustration, Table 9.2 is the terminal set and Table 9.3 is the function set.[4] These two sets are what Andrews and Prager (1994) employed to represent and generate bidding strategies in their double auction tournament.

Given the information (Table 9.2) and the way to operate it (Table 9.3), various bidding strategies can be formed. Two examples are the following:[5]

```
(min PMinBid HT)
(if\_{}bigger\_{}then\_{}else HT CASK CASK+1 Pass)
```

In the first example, to decide how much to bid, the buyer simply looks at the minimum bid on the previous day (PMinBid) and his current reservation price

(HT), and bids at the minimum of the two. In the second example, the buyer first checks whether his reservation price (HT) is bigger than the lowest ask (CASK) in the previous round. If this condition is met, he will bid by adding one dollar to the current ask; otherwise, he will simply pass. Not all bidding strategies are that simple. A little knowledge of combinatorics or context-free grammar will lead us to see that the formed bargaining algorithm can potentially become complex, like the next one, and something beyond the scope of this paper.

```
((min (if\_{}bigger\_{}then\_{}else PMinBid PAvgBid CASK PAvgBid)
(if\_{}bigger\_{}then\_{}else HT PAvgBid PAvgBid CASK))
```

Generally speaking, the terminal set and the function set together are the set of primitive ingredients to enable genetic programming to get started, and the universe of the allowable compositions of these ingredients defines the search space for a run of genetic programming. The degree of autonomy not only depends on the size of the search space, but also on the efforts required to get the primitive ingredients. On the one hand (the output side), we hope that the search space is large enough to accommodate many hidden novelties; but, on the other hand (the input side), we want to achieve this goal with minimal effort. Intuitively speaking, the supply of primitive ingredients should involve human efforts which are relatively much smaller in comparison to the scale of the resultant automated operation.

Like Axelrod (1987), Andrews and Prager (1994) also automated the SFDA tournament as an evolutionary open tournament (Figure 5.2), in which autonomous agents are matched with a fixed set of human-written programs (Figures 5.3 and 9.6). The human-written programs were also selected from the original SFDA tournament. Their model is briefly sketched in Figure 9.6. What Andrews and Prager did was to fix a trader (Seller 1 in their case) and use genetic programming to evolve the trading strategies of only that trader. In the meantime, one opponent was assigned the trading strategy Skeleton, a strategy prepared by the SFDA tournament. The trading strategies of the other six opponents were randomly chosen from a selection of the submissions to SFDA. Therefore, what Andrews and Prager did was to see whether GP can help an individual trader to evolve very competitive strategies given their opponents' strategies (see the right panel of Figure 5.3).

One advantage of the automated tournaments is that their scale grows with the power of the CPU. Compared to Axelrod (1987), Andrews and Prager (1994) increased the number of entries from 20 to 300, and lengthened the duration of the tournament from 50 times (generations) to 300 times (generations). Since what we will learn from open tournaments is generally open ended, the increasing scale of tournaments is certainly a plus for this experimental way of studying economics.

In this way, Andrews and Prager (1994) integrated both the human agents in experimental markets and the software agents in agent-based double auction markets. In Andrews and Prager (1994) software agents are *randomly generated* but use the *initial knowledge* (the *primitives*, the *building blocks*) inspired from

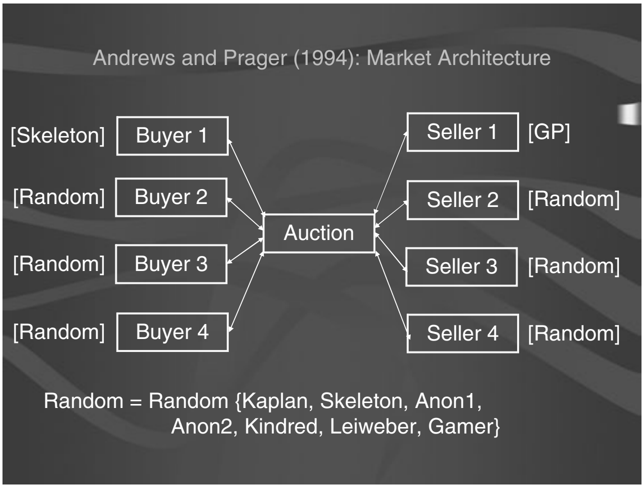

Figure 9.6 The Andrews–Prager double auction model.

human-written programs.[6] What they did was to make computers first randomly generate trading programs; in this sense, their approach is similar to Gode and Sunder's zero-intelligence agents. However, only in the very beginning are these programs truly randomly generated. After that the programs are tested by being placed in agent-based double auction markets with other software agents, e.g., software agents from SFDA. They are then reviewed and revised based on their performance, and some new programs are generated after that. Nevertheless, this further generation is no longer random, but biased toward the revision of existing well-performing programs and the deletion of ill-performing ones. This brings online learning to software agents or programmed agents and makes them become *autonomous agents* so that they can behave like human agents in the experimental markets, in terms of being spontaneous and fast reacting.

Despite this great potential to interest economists, Andrews and Prager's interpretation of their model is rather less telling and fails to draw the attention of the economists who have little background in computer science. Besides, their agent-based model is neither fully constructed nor extensively simulated. Only one of the market participants in their model is autonomous. This certainly restricts the extent of endogenous change which a genuine model of economic change may have. Besides, Andrews and Prager (1994) only attempted a few experiments and did not present their statistics well. Unfortunately, they did not return to this model. Hence, the work on the agent-based double auction market, as a genuine model of change, ceased until the late 1990s, when this model was revisited and extended by Shu-Heng Chen and his colleagues in the AIE-DA (standing for AI-ECON double auction) Tournament (Chen, 2000).

9.4 AIE-DA tournaments

In this section, we shall review the studies on agent-based double auction markets using AIE-DA, developed by the AI-ECON Research Center at National Chengchi University. The AIE-DA is probably the only agent-based double-auction market which has received extensive and systematic study. The study was developed into three different stages, which can be regarded as a comprehensive follow-up of the tournament origin of ACE as introduced in Chapter 5. The research questions can be separated into two different directions, using Alfred Marshall's term, inner nature (constitution) and outer form (see Section 5.3 for the specific quotation). The former focuses on the novelties discovered by autonomous agents which can help them stand in an advantageous position, whereas the latter refers to their observable performance. Putting the two together, we inquire into *what makes them perform well*. In fact, by observing and making sense of what our autonomous agents learned, we as onlookers are also able to learn.

However, it can be hard to tackle these two research questions simultaneously in one single step. This is mainly because we still do not quite know how to efficiently comprehend the "knowledge" generated by genetic programming. This problem has also been well documented in the GP literature (Chen, Kuo, and Hsu, 2008). This "invisibility" is not unique in GP; it is quite generally shared in other similar or more expressive representations, such as artificial neural networks (Chapter 12). This indicates a fundamental difficulty of using autonomous agents. When agents become more autonomous, they can become less tractable while evolving into more complex syntactic forms for which the associated semantics are less straightforward.

Therefore, if our focus is on the analysis of the inner nature of autonomous agents, then it is desirable to have a less complex environment, in other words, less sophisticated opponents. Of course, that means we are not really testing our autonomous agents. Alternatively, if we put our autonomous agents in a more complex environment with more sophisticated opponents, then it will become much harder to trace how they beat these opponents if they indeed behave so. The research strategy is then to kill each bird with one stone and to generate two separate studies. Chen and Yu (2011) was devoted to the analysis of what our autonomous agents discover when they outperform their opponents (Section 9.4.3), whereas Chen and Tai (2010) was devoted to the second direction (Section 9.4.2). However, before examining the behavior of the autonomous agents in the evolutionary tournament, we shall start with a full-fledged version of the tournament, i.e., the coevolutionary tournament in which all agents are autonomous (Section 9.4.1).

9.4.1 AIE-DA coevolutionary tournament

Chen and Tai (2003) first extended the Andrews–Prager model by making all market participants autonomous. The architecture applied here is a kind of multi-population genetic programming (MGP), also known as individual learning

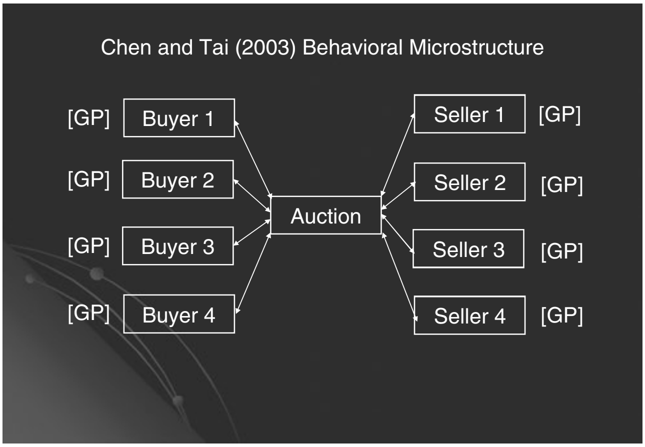

Figure 9.7 The behavioral microstructure of two coevolutionary DA tournaments (Chen and Tai, 2003).

(Figure 9.7). A similar kind of architecture has been introduced in the agent-based auction market (Section 3.3.3). In brief, we view or model each agent as *a single population of bargaining strategies*. Genetic programming is then applied to *evolve* each population of bargaining strategies. In this case, a society of bargaining agents consists of many populations of programs. Chen and Tai (2003) showed that a market composed of this kind of autonomous agent can exhibit behavior similar to what we learn from market experiments with human subjects (Smith, 1991a).

Figure 9.8 demonstrates a typical result observed in this agent-based double auction market. The left panel of the figure is the simulated market environment defined by the demand and supply schedule, whereas the right panel of the figure gives the price dynamics resulting from the bargaining behavior generated by MGP agents. As we can see from Figure 9.8, market prices in this case quickly move toward the equilibrium price (or price interval), and then slightly fluctuate around there. The successful replication of market experiment results by the AIE-DA model motivated them to return to the work initiated, but largely unfinished, by Andrews and Prager.

9.4.2 AIE-DA evolutionary tournament

The question yet to be addressed is: Given a set of opponents, regardless of who they are or how smart they are, as long as they are non-autonomous, can our genetic-programming agent eventually outperform them? This, we think, is the fundamental issue in a discipline which has *change* as her sole concern and has the *no-arbitrage state* as the consequence of change. Notice that when opponents are non-autonomous, their behaviors are largely certain, even in a stochastic sense.

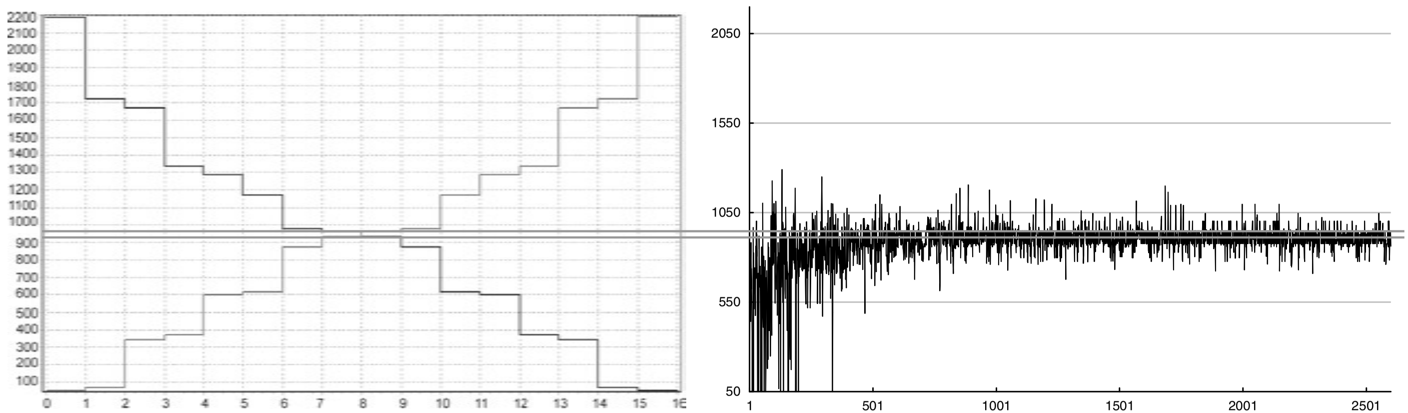

Figure 9.8 Agent-based double auction market simulation with MGP agents.

This in turn implies that, unless they are perfect, it may be possible to find a way to outperform them. Since our autonomous agents, by design, are constantly looking for chances, opportunities, and patterns, finding a way to outperform them may be just a matter of time. Andrews and Prager (1994) also had this conjecture, but they did not move far enough to document a proof. Chen and Tai (2010), therefore, went back to where Andrews and Prager started, while clothed with the legacy of Alfred Marshall or Charles Darwin, to see whether they could return the missing element, autonomous agents, to economics.[7]

Chen and Tai (2010) considered a DA tournament closer in spirit to SFDA and more complex than Andrews and Prager (1994). In their evolutionary tournament, a total of 50 entries play against 11 human-written programs, which were sampled from the literature on double auctions (Figure 9.9). This includes some well-performing strategies in the SFDA tournaments (the Kaplan strategy, the Ringuette strategy, the Skeleton strategy), the GD strategy (Gjerstad and Dickhaut, 1998), the BGAN strategy (Friedman, 1991), the EL strategy (Easley and Ledyard, 1993), the ZIC strategy (Gode and Sunder, 1993), and the ZIP strategy (Cliff and Bruten, 1997). To be an evolutionary tournament, this tournament is then run 70 times (generations). Furthermore, to ensure that the result is not sensitive to a specific environment, such as the market topology or the behavioral microstructure, the

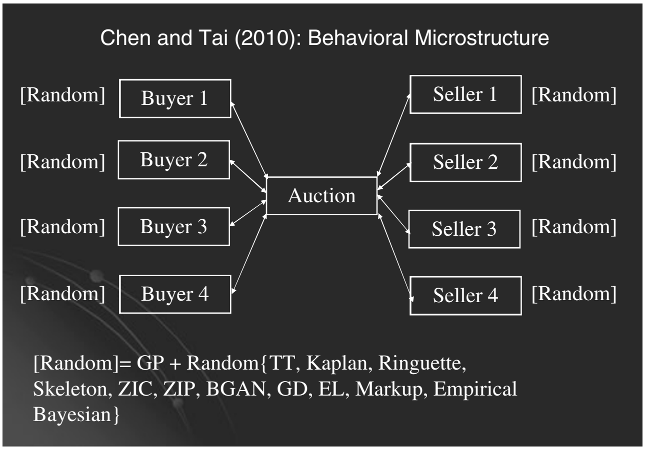

Figure 9.9 The AIE-DA evolutionary tournament.

Note: See Chen and Tai (2010). Since we have only eight traders (four buyers and four sellers) in the market while there are 12 trading strategies to be tested, we compare the strategies by randomly sampling (without replacement) those eight strategies and injecting them into the market one at a time. We did not try out all the possible combinations and permutations of strategies; instead, 300 random match-ups were created for each series of experiments. In each of these match-ups, any selected strategy will face strategies completely different from its own kind. For example, a certain type of strategy such as ZIC will never meet another ZIC trader in the same simulation. Thus, there is at most one GP trader in each simulated market, and this GP trader adjusts its bidding/asking behavior by learning from other kinds of strategies. There is no coevolution among GP traders in our experiments.

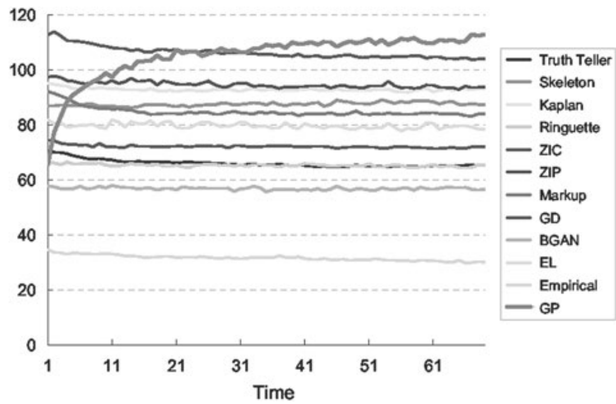

Figure 9.10 Comparison of the GP trader with program agents: time series of individual eff ciencies.

Notes: The size of entries maintained by the GP trader is 50. The horizontal axis stands for the time; the vertical axis stands for the performances of strategies in terms of individual eff ciency. The GP trader's performance series is labeled in red.

evolutionary tournament is run 300 times, each time with the market topology and behavioral microstructure being randomly formed using the token-value mechanism introduced in Section 9.2.2. With extensive trials, Chen and Tai (2010) were able to show that genetic programming can build an autonomous agent that outperforms all 11 human-written programs in a broad variety of environments.

From Figure 9.10, we can see that the performance index of the GP player increased over time, and it gradually outperformed the other opponents. It started third from the bottom by only beating the Empirical Bayesian and BGAN, and then quickly beat Ringuette, Truth-teller, ZIC, EL, Skeleton, Markup, Kaplan, and ZIP, one by one. By only taking ten generations (1000 trading periods), the GP player was already ranked number two, next to GD. Then it took about another 20 generations (2000 trading periods) before reaching the top. The rank of the other opponents, being non-autonomous, remained relatively stable.

9.4.3 Discovery using autonomous agents

To be able to analyze the strategies learned by the agents with a higher degree of autonomy, Chen and Yu (2011) simplif ed the Andrews–Prager DA market by replacing all human-written programmed agents with truth-tellers. In their proposed evolutionary tournaments, the autonomous agents (driven by genetic programming) play against a f xed set of truth-tellers in three different market topologies (demand-and-supply schedules), each associated with a different kind of market equilibrium (or equilibria), as shown in Figure 9.11. A size of 10 or 50 entries (automated programs) is maintained for each tournament, and, like Andrews and Prager (1994), the tournament is run for 300 times (generations).

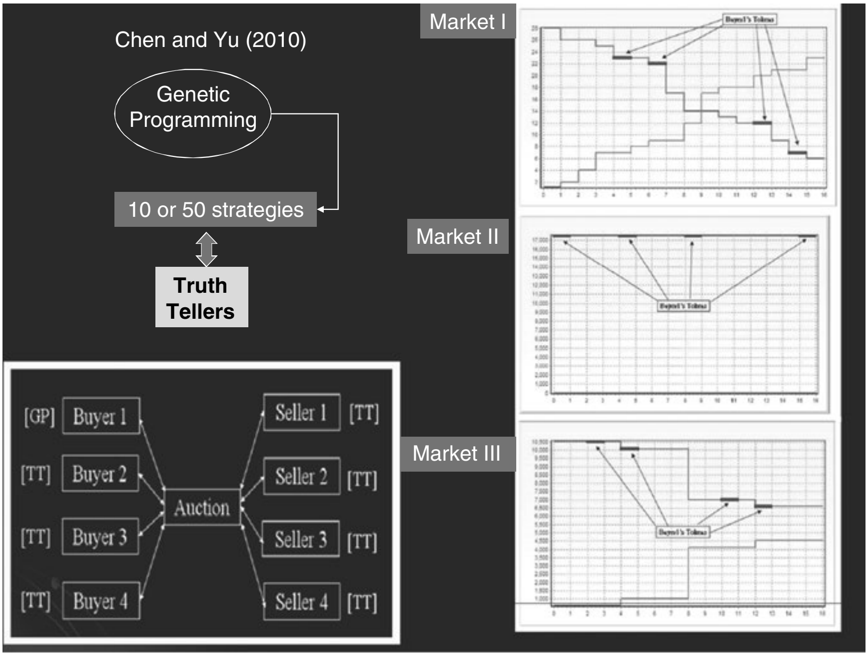

Figure 9.11 Evolutionary DAT tournament.

Notes: See Chen and Yu (2011). The lower-left panel gives the microstructure of the DA tournament. In this case, entries of automated programs (Buyer 1) play against a set of truth-tellers (TT). The automated submissions are generated by genetic programming, as indicated in the upper-left panel. The tournaments are held in three different market topologies as displayed in the right panel. The reservation prices (redemption value) of the four tokens for Buyer 1 are emphasized.

Ninety runs of this evolutionary tournament are carried out. This scale of simulation enables them to examine a set of 18,000 to 90,000 strategies for each market topology. By crossing comparisons of the results from different market topologies, a general pattern of the autonomous agents in these DA markets is observed, which is termed *optimal procrastination*.

With this rule, the autonomous agent attempts to delay his participation in the market transaction so as to avoid early high competition and become a monopsonist in the later stage. Once getting there, the agent will then fully exercise the monopsony power. However, procrastination may also cause the agent to miss some good offers; therefore, there is an opportunity cost for procrastination and the agent tries to optimize the procrastination time by balancing his monopoly profits against these costs.

Of course, the optimal procrastination strategies may become complicated or may not even be sustained when different sets of opponents are considered, but the idea of using autonomous agents as a way to inspire economic analysis remains (Figure 5.1, right panel).

9.5 U-Mart

This chapter begins with a concrete illustration of an agent-based market simulation platform, SFDA, and the later developments. There are many other market platforms which can involve both human agents and software agents; some famous ones are U-Mart (Shiozawa *et al.*, 2008), prediction markets (Williams, 2011), and trading agent competition (Wellman, Greenwald, and Stone, 2007). We have already introduced the trading agent competition in Chapter 5. The rest of the chapter will then give an overview of U-Mart (this section) and the prediction market (the next section).

U-Mart provides another novel application of agent-based double auction markets. U-Mart stands for *UnReal Market as an Artificial Research Test Bed*. U-Mart is an agent-based futures market, which serves purposes of both education and research. This research project was collaboratively initiated by many universities in Japan. Shiozawa *et al.* (2008) give a comprehensive view of its development. The usual agent-based system is entirely composed of software agents. What makes U-Mart unique is that it also introduces human agents. In fact, it is a pioneering research effort to mingle software agents with human agents. This interaction mechanism, therefore, overarches conventional agent-based computational economics and experimental economics. The latter is conventionally regarded as laboratory experiments with human subjects only.

By combining software agents and human agents, U-Mart allows us to address many new research questions that go beyond the conventional realm of both agent-based computational economics and experimental economics. Market efficiency is a case in point. One can use U-Mart to design experiments to address whether market efficiency can be enhanced when software traders are introduced to the markets that were originally composed solely of human agents. One can then go further to distinguish the case where human agents are informed of the presence of software agents from the case where human agents are not informed of their presence. Whether the human agents are well informed of the presence of the software agents can have significant impacts on market efficiency (in the form of price deviations from the fundamental price). Grossklags and Schmidt (2006) found that if human agents are well informed, then the presence of software agents triggers more efficient market prices when compared with the baseline treatment without software agents. Otherwise, the introduction of software agents results in lower market efficiency. Related issues have been pursued in the recent development of the U-Mart platform (Terano *et al.*, 2003; Sato, Kawachi, and Namatame, 2003; Koyama *et al.*, 2005). In a sense, this question can be viewed in terms of the sociopsychological impact on human behavior in the presence of interacting machine intelligence.

9.6 Wisdom of crowds

As we mentioned in Chapter 8, the zero-intelligence agent or the entropy-maximizing agent plays an important role in the design of the artificial agent. In this section, a slight detour is first made toward zero-intelligence biological agents, or ants. The point is to see the collective behavior of the agents, regardless of the intelligence of individuals, a subject which is popularly known as the wisdom of crowds.

9.6.1 Social intelligence

Ants and stigmergy

Maybe it is useful to relate zero-intelligence agents to the ants and termites studied in entomology or computational entomology. The behaviors or decisions of these insects are rather random, requiring no memory, a priori knowledge, or learning in an explicit way. Yet entomologists found that they can communicate well. The communication is, however, not necessarily direct, but more indirect, partially due to their poor visibility. Their reliance on indirect communication was noticed by the French biologist Pierre-Paul Grasse (1895–1985), and he termed this style of communication or interaction *stigmergy* (Grosan and Abraham, 2006). He defined stigmergy as: "stimulation of workers by the performance they have achieved." Stigmergy is a method of communication in which the individuals communicate with each another via modifying their local environment. The price mechanism familiar to economists is an example of stigmergy. It does not require market participants to have direct interaction, but only indirect interaction via price signals. In this case the environment is characterized as the price, which is constantly changed by market participants and hence constantly invites others to take further actions.

For social scientists, swarm intelligence is known as *social intelligence* or the *wisdom of crowds*, and has become an interdisciplinary research subject that crosses various social and technical disciplines (Bonabeau and Meyer, 2001). Inquiries into emergent social intelligence are basically concerned with the design of an interaction platform so as to expect and observe it. One example is the design of *e-participation* or, more specifically, the *prediction markets* (Wolfers and Zitzewitz, 2004).

Hayek hypothesis

In talking about social intelligence, one certainly cannot miss mentioning Friedrich Hayek's influential work (Hayek, 1945). Hayek considered the market and the associated price mechanism a reflection of the pooling or aggregate of market participants' limited knowledge about the economy. While the information owned by each market participant is rather limited, pooling it can generate efficient prices. The assertion of this article was later on coined as the *Hayek hypothesis* by Vernon Smith (Smith, 1982) in his double auction market experiments. The intensive study of the Hayek hypothesis in experimental economics has further motivated or strengthened the idea of prediction markets.

9.6.2 Prediction markets

A prediction market is essentially an artificial market environment in which forecasts of crowds can be pooled so as to generate better forecasts. Predicting election outcomes via what is known as political futures markets becomes one of the most prominent applications. Let us take the well-known *Iowa Political Stock Market* (IPSM) as an illustration (Forsythe *et al.*, 1992). In the IPSM, Forsythe and his colleagues set up a series of experimental stock markets to let people invest their own money. These markets are designed for specific political election events such as the US presidential election. Spontaneous participants can use their money to buy shares representing candidates in the elections, and the final payoffs are calculated based on the election results. Usually the vote share gained by each candidate determines the ultimate values of different "stocks." As a result, if markets can aggregate information efficiently, the final prices in these markets should be good predictions of the election outcomes. Their results, as shown by Berg *et al.* (2008), are in general very good and outperform most of the polls.

The same idea can be applied to other forecasts. For example, Hewlett Packard uses such kinds of market to predict future hardware sales, Siemens used a similar internal prediction market to help forecast the finishing of projects, and the Hollywood Stock Exchange (HSX) is used to predict the sales of movies, albums, and the outcomes of awarding Oscars (Tabarrok, 2004). The business of prediction markets is moving in the direction of testing the *limit* of what the wisdom of crowds can produce.[8] The idea of the *limit* is intriguing because in a way it can be related to a machine which can compute the future. We are then asking what this machine can compute and what it cannot compute.

Combined forecasts

The idea of using agent-based modeling or multi-agent systems to process information has a long tradition in economics. While in the early days these agent terms did not even exist, the same idea has already been implemented in other forms. Econometricians tend to pool the forecasts made by different forecasting models so as to improve the forecasting performance. In the literature, this is known as *combined forecasts* (Clement, 1989). Like prediction markets, combined forecasts tend to enhance forecasting accuracy. The difference between prediction markets and combined forecasts is that agents in the former case are heterogeneous in both the data (the information acquired) and the models (the way to process the information), whereas agents in the latter case are normally heterogeneous in the models only.

Hybrid systems

Hybrid systems in machine learning or artificial intelligence can be considered to be a further extension of combined forecasts (Kooths, Mitze, and Ringhut, 2004). Their difference lies in the way they integrate the wisdom of the crowd. Integration in the case of combined forecasts is relatively much simpler, most of the time, and involves just the weighted combination of forecasts made by different agents. This type of integration can function well because the market price under certain circumstances is just this simple weighted combination of a pool of forecasts. This latter property has been shown by the recent *agent-based financial markets* (for example, see Section 14.1.5). Nevertheless, the hybrid system is more sophisticated in its integration. It is not just the horizontal combination of the pool, but it also involves its vertical or hierarchical integration. In this way, heterogeneous agents do not just behave independently, but work together as a team.[9]

9.6.3 Agent-based prediction markets

Prediction markets, to some extent, can be considered to be a kind of experimental market, and people use laboratory research to study certain issues pertaining to prediction markets (Chen and Plott, 2002; Hanson, Oprea and Porter, 2006). It would, therefore, not be surprising to see the extension of agent-based modeling to prediction markets, given its constant interaction with experimental economics (see Chapter 6). Nonetheless, the application of agent-based modeling to the prediction markets is just in its burgeoning stage (Othman, 2008; Bothos, Apostolou, and Mentzas, 2010; Jumadinova and Dasgupta, 2010; Klingert and Meyer, 2010). The purpose of this section is to give some ideas as to why agent-based modeling can advance the theoretical study of prediction markets.

Neo-classical analysis of prediction markets

The neoclassical analysis of the operation of the prediction market is to establish a map from a distribution of beliefs to the prices observed in the prediction market (Manski, 2006; Snowberg and Wolfers, 2010). Corresponding to this map, there is also the other map from a distribution of beliefs to the actual outcome observed in the election. The neoclassical analysis then attempts to see how these two maps can be correlated and hence find an appropriate way to interpret the results obtained from the prediction market (Figure 9.12). To make the analysis tractable, one has to rely on some stringent assumptions. For example, to make the analysis easy, the risk preference is normally assumed to be risk neutral or to behave in a logarithmic form. The analysis can then be tied to the neoclassical economics by further considering maximizing behavior subject to budget constraints.

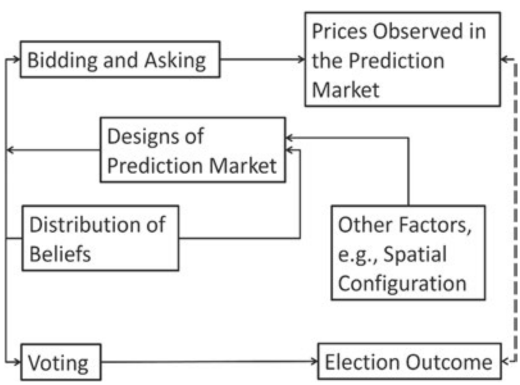

Figure 9.12 Analysis of prediction markets when applied to elections.

Alternatively, one can replace the expected-utility maximization framework with prospect theory, take into account the loss aversion behavior, and clothe the entire analysis with a behavioral flavor.

Recently, the performance of prediction markets has been examined using ACE models. Being in the vein of the Hayek hypothesis and probably going even further, most of these ACE models start with the device of zero-intelligence agents. It has been demonstrated that markets populated by these zero-intelligence traders can reach roughly informative price predictions (Othman, 2008; Klingert and Meyer, 2010; Othman and Sandholm, 2010), an outcome analogous to that of Gode and Sunder (1993).

However, what may be pertinent to the operation and the performance of the prediction market should not be just the static aggregated distribution of beliefs. If the validity of the Hayek hypothesis lies in the information aggregation, then what seems to be interesting to look at is *how much information there is in the market* and *how it is distributed among agents or situated at different spaces, initially*. Does this initial amount of information and the associated distribution over space matter?

The answer to the first question may be more intuitive since presumably there is no magic in the entire information aggregation process. Hence, the performance of the prediction market is positively related to the amount of information owned by the society as a whole; the more the better. Nevertheless, the answer to the second question is less certain, but the spatial element in the interaction and aggregation process underlying the Hayek hypothesis should not be ignored (Figure 9.12). Conceptually, what agent-based prediction markets or experimental prediction markets attempt to do, from the design viewpoint, is to first disaggregate an initial total amount of information to different agents distributed spatially (the distribution issue), and then to see how they can add up through the designed market mechanism (information aggregation).

Yu and Chen (2011) present a simple agent-based model of the political election prediction market which reflects the intrinsic feature of the prediction market as an information aggregation mechanism (Hayek, 1945). They follow the recent research trend in agent-based prediction markets to construct a spatial agent-based political futures market based on a two-dimensional cellular automaton. They consider this as a first attempt to hybridize spatial agent-based political election models and agent-based political futures markets.

They begin the study with the device of zero-intelligence agents which have been introduced into agent-based prediction markets by Othman (2008). However, instead of general-purpose prediction markets, they focus on political futures markets, which, needless to say, is one of the most active application areas of prediction markets. This focus motivates a spatial extension of Othman's model. This extension enables them to address a number of issues which cannot be easily approached by either neoclassical models of prediction markets or by agent-based prediction markets without spatial configurations.

Specifically, the question is how exactly the dissemination of information affects the information aggregation, given that agents can only form their beliefs based on the local information from their surroundings. Second, to take into account the geographical segregation phenomena, as analyzed by Thomas Schelling, they also study how clusters may affect the operational efficiency of the political futures markets. Using this Schelling model, one can study the effect of cluster size on the prediction accuracy of the political future markets.

9.7 Further explorations

The agent-based double auction market reviewed in this chapter mainly refers to discrete double auctions. For agent-based continuous double auction markets, the interested reader is referred to Ma and Leung (2008), which also has a nice review of a number of programmed agents, which we have reviewed in this chapter, as well as a number of others, which are often used in continuous double auctions and hence are not reviewed in our chapter.

After introducing each of the programmed agents, the authors singled out a list of factors that are considered by at least one, but not necessarily all, programmed agents, and then from the list pointed out the elements which are largely missing. We believe that most elements singled out in both lists have been incorporated in the SFDA programmed agents (Table 9.2), including the factor of time or the trading deadline. The authors did mention the use of evolutionary computation in generating bidding strategies, but were skeptical of its applicability in real-time market scenarios. In addition to that, the authors also applied fuzzy set theory to deal with various kinds of uncertainties which traders received from the trading environment.

The development of program agents has far-reaching implications for *electronic commerce*. Additional materials can be found in van Dinther (2006). There is a series of international conferences which are devoted to *agent-mediated electronic conmmerce*. One is the international workshop on Agent-Mediated Electronic Commerce (AMEC). There is a series of post-conference proceedings published in parallel to this series of workshops, for example, Dignum and Cortes (2001) and David *et al.* (2012). Part of the related development is the design of trading agents and trading agent competition (Wellman, Greenwald, and Stone, 2007; Wellman, 2011). As we mentioned earlier, the trading agent competition (TAC) as an annual event has been long established. This competition focuses on the design of trading strategies, i.e., more on agents' behavior. However, there is another annual event derived from TAC, known as CAT (just the opposite of TAC). CAT stands for market design competition, which focuses on the institutional aspects of the markets. A review of the market design tournament can be found in Miller *et al.* (2013).

Notes

1 As we shall see below, zero-intelligent agents or even such agents with some slight modifications cannot compete with some well-thought-out human-written programs.
2 For details, see Rust, Miller, and Palmer (1994), p. 75.
3 The Kaplan trader holds a 2 percent profit margin.
4 Based on Table 9.2, only transaction information on the previous day is provided for GP to compose bidding strategies.

5 The string representation whereby the operators in the parentheses are given before the operands is known as *prefix* representation, invented by the Polish mathematician Jan Lukasiewicz (1878–1956).

6 In brief, all randomly generated programs can be regarded as samples from the *span* of some *bases*. These bases, as list in Table 9.2, are all from human-written programs.

7 Every time when we come across a context like this, we are tempted to use a very familiar misquotation of Charles Darwin: "It is not the strongest of the species that survives, nor the most intelligent that survives. It is the one that is most adaptable to change." A possible source of this citation has been identified in Megginson (1963) as follows:

> According to Darwin's Origin of Species, it is not the most intellectual of the species that survives; it is not the strongest that survives; but the species that survives is the one that is able best to adapt and adjust to the changing environment in which it finds itself.
>
> (Megginson, 1963, p. 4)

8 A recent example is the Predictalot market developed by David Pennock and his team at Yahoo!

9 For an extensive coverage of the recent progress in various designs of collaborative agents, the interested reader is referred to Mumford and Jain (2009).

Part IV

Computational intelligence

The purpose of the next few chapters is to address a very important element of autonomous agents, i.e., *their ability to adapt to a changing environment*. The idea is to equip the artificial agents with some built-in algorithms so that they are able to develop some kinds of cognitive behavior; in particular, they are able to *learn from the past*, and hence are able to anticipate the future, and develop strategic behavior accordingly. The algorithms which can help our artificial agents achieve the goal above are initiated from many different fields and hence are interdisciplinary. Recently, they have been addressed together in the field known as *computational intelligence* (Engelbrecht, 2007).

Therefore, the next few chapters may be read as an introduction to computational intelligence from the perspective of agent-based computational economics. Computational intelligence itself could be the subject of an independent book. We by no means try to give a thorough or comprehensive review of computational intelligence; instead, the materials are presented in such a way as to help readers without technical backgrounds to gain a quick grasp of the main ideas behind the jargon so that the rest of the book and the related literature will be easier to read. Nevertheless, additional sources related to each algorithm will be given throughout the chapters.

The plan of this part is to review a number of important developments in computational intelligence, including reinforcement learning (Chapter 10), fuzzy logic and rough sets (Chapter 11), artificial neural networks (Chapter 12), and evolutionary computation (Chapter 13). While these tools have been introduced to economists on many other occasions, for example in quantitative finance, we have a different motivation to study them. Mainly, these tools allow us to discuss a number of crucial mental activities, such as attention control, memory, and pattern discovery. Therefore, even though our brief review will go through some important quantitative applications, we should remind readers at different places that our scope is broader. Let us take Chapter 12 as an example.

Chapter 12 addresses a number of different neural networks, which help us to understand how some algorithms, associated with the artificial brain, are able to conduct data compression, redundancy removal, classification, and forecasting. Let us be more specific on some of them. An important cognitive task for human agents is that, under some degree of survival pressure (or incentives), they are able to perform correct classification and react to it properly. A simple example is the salesman who needs to identify those consumers who are willing to pay a high price for a specific new product and distinguish them from more general buyers. A family of neural networks, also known as *supervised learning* (Sections 12.1, 12.2, and 12.5), are able to equip agents with this capability.

Prior to classification, one more fundamental cognitive task, is *concept formation*, i.e., to extract useful concepts from observations, and then based on these concepts to classify new observations so as to facilitate decision-making. A typical example would be a stock trader who needs to recognize some special charts to make his market-timing decisions. A family of neural networks, known as *unsupervised learning* (Section 12.6), can help agents to acquire this kind of cognitive capability.

Sometimes, it is hard to form concepts. In this case, one may directly deal with *cases*, and make decisions based on the similarity of cases. Sections 12.7 and 12.8 are devoted to the literature on *lazy learning*, i.e., learning by simply memorizing experiences and, with a little effort, developing generalized concepts on the top of these experiences.

The third important cognitive task concerns the efficient use of limited brain space. This has something to do with data compression and redundancy removal. Section 12.4 introduces a network that can perform this task. In addition, Section 12.3 gives a device to reduce data storage space by building-in loops in the "brain."

The above three cognitive tasks do not involve social interaction. They mainly describe how an individual learns from his own experience without interacting with other individuals' experiences. The latter case is referred to as social learning or population learning in the literature. Imitation (reproduction) is the most clear example of social learning: agents simply follow the behavior rules of whomever they consider the most suitable (Rendell *et al.*, 2010). Nonetheless, imitation is not enough to cover more complex patterns of social learning, such as innovations using inspiration from others. Through evolutionary computation (Chapter 13), both forms (imitation and innovation) of learning with social interactions are operated with the familiar survival-of-the-fittest principle.[1] Genetic programming is one kind of evolutionary computation. It differs from others in the sense that it gives us much more expressive power to see changes.

However, before we introduce these more complex ways of designing software agents, we shall start with the simplest one, i.e., *reinforcement learning* (Chapter 10).

Note

1 Evolutionary computation, in a sense, is a kind of bio-sociology.

10 Reinforcement learning

Learning is a multidisciplinary area which continuously draws attention from economists, psychologists, cognitive scientists, computer scientists, mathematicians, and neuroscientists. Among all proposed learning models, *reinforcement learning* seems to be the one most commonly shared by all these disciplines. Reinforcement learning was initially studied in psychology (Bush and Mosteller, 1955), then later extended to computer science by Marvin Minsky,[1] games by Arthur Samuel (1901–1990), economics by John Cross (Cross, 1973, 1983), and neuroscience by Wolfram Schultz.[2]

In psychology, Bush and Mosteller (1955) proposed the first mathematical model of reinforcement learning. The Bush–Mosteller model has later been adapted and generalized to economics in Cross (1973), Arthur (1991, 1993), Roth and Erev (1995), and Erev and Roth (1998). Based on this development, the relevance of reinforcement learning in economic experiments can be seen. In particular, given the fast growth of bounded-rationality literature, reinforcement learning stands for an important class of learning. It is a kind of *reflexive learning*, i.e., learning involving little reasoning (Bossaerts *et al.*, 2008). Because of this unique position, reinforcement learning has frequently been compared with other more sophisticated learning models in either experimental economics or agent-based computational economics (Duffy, 2006).

Reinforcement learning is a learning behavior that we may expect when the external environment is rather *simple* in the sense that it can be *repeatedly* presented to the agents with the *same finite number* of options, and the consequence of choosing each option is *stationary* with a *limited degree of noise*. This is the environment in which agents can develop a *habitual behavior*, i.e., doing the same thing every time, which is appropriate with respect to the given simple stationary environment. This ideal environment is best characterized by the movie *Groundhog Day*,[3] while real life does not offer "Groundhog Day" laboratories. Therefore, it is not surprising to see that the RL model originated from animal experiments in psychology, and, later on, were extended to human experiments both in psychology and economics.

The intuitive idea of the reinforcement learning model is that *choices that have led to good outcomes in the past are more likely to be repeated in the future*, known as the *law of effect*. To make it operational, the reinforcement learning model assigns each possible action (option) a probability of being activated, chosen, or taken. The entire probability function over the action space is based on the *propensity* of each action. The propensity of an action is its accumulated received rewards (utilities) over the past. The propensity and the activation probability of each action are constantly updated by taking into account the rewards received most recently.

In the literature, several different versions of reinforcement learning models have been proposed. They differ in how propensity is updated and how it is mapped to the activation probability. In Section 7.3, we have already seen a two-parameter version of the model proposed by Arthur (1993). The specific model which we introduce in this chapter is a version of the *Roth–Erev model* (Roth and Erev, 1995; Erev and Roth, 1998).

10.1 The three-parameter Roth–Ever model

10.1.1 Updating of propensities

At the beginning, the propensity of an action is treat equally and is given by q_1 ($q_{i,1} = q_1, \forall i = 1, \ldots, N$). Then how it is updated depends on whether it has been activated. If i is activated (chosen) in the previous period, then its propensity at time t will be updated by adding up the payoff received from the activation, i.e., Π_{t-1}; otherwise, it is the same as $q_{i,t-1}$.

$$q_{i,t} = \begin{cases} q_{i,t-1} + \Pi_{t-1} & \text{if } i \text{ is chosen at period } t-1, \\ q_{i,t-1} & \text{otherwise.} \end{cases} \tag{10.1}$$

From psychological considerations, Roth and Erev (1995) further introduced two additional elements on top of the basic model in (10.1). These two "weak psychological assumptions," as they were termed in Erev and Roth (1998), are the *recency effect* and *experimentation*.

Recency effect

The first element is to take the influence of *time* into account, and assume that propensity will decay with time. Denote the constant decay rate by φ. Then Equation (10.1) can be developed into Equation (10.2);

$$q_{i,t} = \begin{cases} (1-\varphi) * q_{i,t-1} + \Pi_{t-1} & \text{if } i \text{ is chosen at period } t-1, \\ (1-\varphi) * q_{i,t-1} & \text{otherwise.} \end{cases} \tag{10.2}$$

A reasonable range of the parameter φ is between 0 and 1. If $\varphi = 1$, the propensity of the last period $q_{i,t-1}$ will be completely ignored (forgotten), and the strength-updating only depends on the most received payoffs. On the other hand, if $\varphi = 0$, the past propensity will not decay at all, and it will roll over in its entirety to this period. Hence, φ is also known as the *forgetting parameter*.

Experimentation

The second added element had as its original motivation *experimentation* or *generalization* when it was first introduced in Roth and Erev (1995). The idea is not only to update the propensity of the activated action with its received payoff, but also use this payoff to update the propensity of *similar* actions. As interpreted by Erev and Roth (1998):

> Not only are choices which were successful in the past more likely to be employed in the future, but *similar choices* will be employed more often as well, and the player will not (quickly) become locked in to one choice in exclusion of all others.
>
> (Erev and Roth, 1998, p. 863)

or

> players will generalize their most recent experience in a way that leads to experimentation among the most similar strategies.
>
> (Erev and Roth, 1998, p. 863)

To achieve this purpose, the experimentation parameter, ε, is introduced into the model, and Equation (10.1) is expanded into Equation (10.3):

$$q_{i,t} = \begin{cases} q_{i,t-1} + (1-\varepsilon) * \Pi_{t-1} & \text{if } i = i^*_{t-1}, \\ q_{i,t-1} + \frac{\varepsilon}{(N_{i,t-1}-1)} * \Pi_{t-1} & \text{if } i \sim i^*_{t-1}, \\ q_{i,t-1} & \text{otherwise.} \end{cases} \tag{10.3}$$

Here, $N_{i,t-1}$ refers to the size of the neighborhood $\{i \mid i \sim i^*_{t-1}\}$, i.e., the collection of non-chosen actions which, however, are similar to the chosen one i^*_{t-1}. $(1-\varepsilon)$ is the reserve of the own payoff to the chosen action. The part which is not reserved is then equally shared by its neighbors (similar actions). A reasonable range of ε also lies between 0 and 1. When it is 1, there is no reserve—the current received payoff is completely shared with others; when it is 0, there is no sharing.[4]

By joining these two psychological elements, recency effect (10.2) and experimentation (10.3), the basic model (10.1) is then augmented as (10.4):

$$q_{i,t} = \begin{cases} (1-\varphi) * q_{i,t-1} + (1-\varepsilon) * \Pi_{t-1} & \text{if } i = i^*_{t-1}, \\ (1-\varphi) * q_{i,t-1} + \frac{\varepsilon}{(N_{i,t-1}-1)} * \Pi_{t-1} & \text{if } i \sim i^*_{t-1}, \\ (1-\varphi) * q_{i,t-1} & \text{otherwise.} \end{cases} \tag{10.4}$$

Equation (10.4) with the initial propensity q_1 is, therefore, the three-parameter Roth–Erev RL model. The three parameters are $(q_1, \varphi, \varepsilon)$. When $\varphi = \varepsilon = 0$, we have a version of Arthur's two-parameter RL model (Section 7.3), where the main difference is that sum of the propensities is not being renormalized to Qt^{ν} as it was in the Arthur model.

10.1.2 Stochastic choice

One essence of the reinforcement learning model is that *choice behavior is stochastic and the stochastic rule must be consistent with the law of effect.* This idea was not only shared by many earlier mathematical psychologists who studied learning behavior, but it also generally holds in the latest interdisciplinary models of learning, such as evolutionary computation. The original probabilistic choice rule considered in the Roth–Erev model is linear (10.5):

$$p_{i,t} = \frac{q_{i,t}}{\sum_i q_{i.t}}, \tag{10.5}$$

where $p_{i,t}$ is the probability of choosing action i at time t. This linear rule is consistent with the *law of effect* and the *power law of practice*. However, to give $p_{i,t}$ a probability meaning, the propensity of each action must be positive, which in turn requires that the payoff must be positive. To guarantee a positive payoff, what Roth and Erev did is to take the following truncation of the raw payoff:

$$\Pi_{i,t} = \pi_{i,t} - \pi_{\min}, \tag{10.6}$$

where $\pi_{\min}$ is the smallest possible payoff. Nonetheless, this design does not fit all possible applications, since $\pi_{\min}$ can change with time and may be difficult to figure out and fix at the beginning of the experiment. Alternatively, we may follow the device popularly used in agent-based financial models (see Chapter 14) by replacing the linear probabilistic rule with the Gibbs–Boltzmann distribution, i.e., an *exponential choice rule*:

$$p_{i,t} = \frac{\exp(\lambda * q_{i,t})}{\sum_i \exp(\lambda * q_{i.t})}. \tag{10.7}$$

The parameter λ introduced above matches the parameter known as *speed of learning* in the Roth–Erev model, while for the latter it is introduced through the initial propensity equally assigned to each action, $q_1 = q_{i,1}, \forall i$. Therefore, the parameter λ, replacing q_1, can also be a parameter of the reinforcement learning model.[5]

10.2 Generalized reinforcement learning

A generalized reinforcement learning model was proposed by Camerer and Ho (1999) as a hybridization of the two competing classes of learning models in games, namely, *reinforcement learning* and *belief learning* (Fudenberg and Levine, 1998).[6] Camerer and Ho (1999) called this generalization *experience-weighted attraction* (EWA) learning. We shall use the term generalized reinforcement learning because it generalizes what is reinforced in learning, but still keeps the fundamental formulation of reinforcement learning unchanged. More specifically, it generalizes reinforcement learning by giving a more general version of attractions (propensities) and their dynamics (updating schemes), which broadens our understanding of the original reinforcement learning.

Consider a multiple-choice environment where agent i is presented with J possible strategies. A classic example is the *multi-armed bandit problem* (see Section 7.2). Let $A_i^j(t)$ be the *attraction* of the jth strategy for individual i. Given $A_i^j(t)$ $(j = 1, 2, \ldots, J)$, agent i will make his choice in a stochastic manner by following a logit distribution:

$$P_i^j(t+1) = \frac{e^{\lambda A_i^j}(t)}{\sum_{k=1}^{J} e^{\lambda A_i^k(t)}}, \tag{10.8}$$

where $P_i^j(t)$ is agent i's probability of choosing j at time t. The equation above basically maps the attraction of each strategy monotonically to its choice probability.[7] The parameter λ appearing in the logit equation (10.8) measures the sensitivity of agent i to attractions. The usual interpretation of λ is the intensity of agent i's motivation. It can measure the uncertainty of the attractions which agent i is facing.

The next issue concerns the constituents of attractions and their dynamics:

$$A_i^j(t) = \phi A_i^j(t-1) + (\delta + (1-\delta)I(s_i^j, s_i(t))\pi(s_i^j, s_{-i}(t)). \tag{10.9}$$

$I(s_i^j, s_i(t))$ is an indicator function. It is 1 if $s_i^j = s_i(t)$; in this case, the strategy j was the chosen strategy at time t. The use of this indicator function enables agent i to update the attraction of the chosen strategy with the received payoff $\pi(s_i^j, s_{-i}(t))$, which is a function of agent i's choice (s_i^j) and his opponents' choice $(s_{-i}(t))$.

The simple reinforcement learning does not consider the *foregone payoff* (the *hypothetical payoff*), i.e., the possible payoff had agent i chosen any other strategies at t. This corresponds to a special case of Equation (10.9) when $\delta = 0$. Camerer and Ho (1999) consider δ as the most important parameter in generalized reinforcement learning, because it distinguishes two laws of effect, namely, the law of (*actual*) effect and the law of (*simulated*) effect. The latter may involve agent i's imagination capability.

ϕ is the discount factor, which is normally interpreted as *memory decay*. However, in addition to memory decay, agent i may *deliberately* discount the old experience when the environment is changing. Hence, to take both conscious and unconscious "forgetting" into account, Camerer and Ho (1999) separate physical time and *effective experience time*. For the latter, a depreciation of time is introduced as follows:

$$N(t) = \rho N(t-1) + 1. \tag{10.10}$$

They then use this experience time $N(t)$ to normalize the sum of the depreciating, experience-weighted previous attraction $A_i^j(t-1)$ plus the weighted payoff from t:

$$A_i^j(t) = \frac{\phi N(t-1)A_i^j(t-1) + (\delta + (1-\delta)I(s_i^j, s_i(t))\pi(s_i^j, s_{-i}(t))}{\rho N(t-1)+1}. \tag{10.11}$$

Equation (10.11) is the Camerer–Ho *generalized reinforcement learning* model. We shall now see how this generalized reinforcement learning model can lead to two familiar and competing learning models in game experiments, namely, reinforcement learning (Section 10.2.1) and belief learning (Section 10.2.2).

10.2.1 Reinforcement learning

There are two kinds of reinforcement learning models frequently used in the game-theoretic literature. They differ in how payoffs contribute to the level of reinforcement: one assumes that previous payoffs are accumulated (Harley, 1981; Roth and Erev, 1995), whereas the other assumes that previous payoffs are averaged (McAllister, 1991; Monkerjee and Sopher, 1994). Both of them can be framed as a special case of the Camerer–Ho generalized reinforcement learning.

The updating scheme for *cumulative* reinforcement learning is:

$$R_i^j(t) = \phi R_i^j(t-1) + I(s_i^j, s_i(t))\pi(s_i^j, s_{-i}(t)). \tag{10.12}$$

Equation (10.12) is a special case of Equation (10.11) when $\delta = \rho = 0$ and $N(0) = 1$. The updating scheme for *average reinforcement learning* is:

$$R_i^j(t) = \phi R_i^j(t-1) + (1-\phi)I(s_i^j, s_i(t))\pi(s_i^j, s_{-i}(t)) \tag{10.13}$$

Equation (10.13) is also a special case of Equation (10.11) when $\delta = 0$, $\phi = \rho$, and $N(0) = \frac{1}{1-\rho}$.

In both of the above reinforcement learning models, $\delta = 0$, which assumes that people ignore *foregone payoffs*. This assumption has been well discussed in the literature. It is certainly easier to defend this ignorance when agents are placed in a poor information environment so that they cannot possibly know what they might have lost or gained.[8] Nonetheless, ignoring possible foregone payoffs in all environments entirely does indicate that agent i's consciousness of the environment is very weak. Essentially, the agent with $\delta = 0$ acts as if he is placed in an independent individual choice situation, rather than a game-like interactive environment. What he does is basically very *reflexive*, just based on the memory that he has from past received payoffs, and he has no imagination of what he might have gained or lost. Neither makes any attempt to discover or learn about these unrealized payoffs.

10.2.2 Belief learning

As we mentioned earlier, reinforcement learning was introduced to model learning in the multi-armed bandit experiment in the 1950s (see Section 7.2). It was introduced to model learning in game experiments in the 1990s. The multi-armed bandit experiments and the normal-form game experiments are not necessarily fundamentally different, if one can express the former in the form of the latter. In this case, the machine can be considered to be the opponent who uses the mixed strategy to play and assigns a fixed choice probability to each of the strategies. Furthermore, agent i is informed about this robot and hence also *believes* that it uses a mixed strategy. Nonetheless, agent i is not given the exact details on the opponent's choice probabilities, which he has to learn from experience.

In the real normal-form game, the situation is, however, a little different. Agent i is now told explicitly that his opponent is a human, but has not been given any information on the strategies that he or she will play, be they mixed or not. In this situation, agent i needs to form a belief about how his opponent might play, and react based on this belief. This is the basic idea of belief learning.

There are many ways of forming beliefs, and some may involve a high degree of reasoning. Nonetheless, those sophisticated ways of forming beliefs are not included in the generalized reinforcement learning developed in Camerer and Ho (1999). In fact, what is modeled is just the belief formed by using a kind of average, taken over the observed repertoire (time series) of the opponent's chosen strategies, $\{s_{-i}(l)\}_{l=1}^{t}$. From there, agent i can count, all the way up to t, how frequently each possible strategy has been played by the opponent, i.e., $\{N_{-i}^{h}(t)\}_{h=1}^{J}$ and $N(t) = \sum_{h=1}^{J} N_{-i}^{h}(t)$. Agent i's estimation (belief) of the probability that his opponent will play strategy k in the next period (period $t+1$), $B_{-i}^{k}(t)$, can then be represented as follows:

$$B_{-i}^{k}(t) = \frac{\rho N_{-i}^{k}(t-1) + I(s_{-i}^{k}, s_{-i}(t))}{\sum_{h=1}^{J}(\rho N_{-i}^{h}(t-1) + I(s_{-i}^{h}, s_{-i}(t))}, \qquad k = 1, \ldots, J. \tag{10.14}$$

There is a recursive version of Equation (10.14), which updates the belief from period $t-1$ to t explicitly:

$$B_{-i}^{k}(t) = \frac{\rho N(t-1) B_{-i}^{k}(t-1) + I(s_{-i}^{k}, s_{-i}(t))}{\rho N(t-1) + 1}. \tag{10.15}$$

This form of updating the weights of observations involves multiplying those from one period ago ρ times as much as the most recent observations. When $\rho = 0$, only the most recent observation counts and one has the naive, Cournot-type learning (Cournot, 1838):[9]

$$B_{-i}^{k}(t) = I(s_{-i}^{k}, s_{-i}(t)). \tag{10.16}$$

Equation (10.16) says that agent i has a very static view of his opponents, namely, their strategies will remain unchanged. Hence, it could be naive in the sense of this passive expectation. When $\rho = 1$, all observations equally count and we have the familiar *fictitious play* (Brown, 1951):

$$B^k_{-i}(t) = \frac{N(t-1)B^k_{-i}(t-1) + I(s^k_{-i}, s_{-i}(t))}{N(t-1)+1}. \tag{10.17}$$

Based on the estimated choice probabilities of his opponent (10.17), agent i can figure out his expected payoffs for each of his possible choices, j:

$$E^j_i(t) = \sum_{k=1}^{J} B^k_{-i}(t)\pi(s^j_i, s^k_{-i}). \tag{10.18}$$

Through the belief updating equation (10.15), one can also find a recursive version of Equation (10.18):

$$E^j_i(t) = \frac{\rho N(t-1)E^j_i(t-1) + \pi(s^j_i, s_{-i}(t))}{\rho N(t-1)+1}. \tag{10.19}$$

Belief learning is a special kind of generalized reinforcement learning if one treats the expected payoffs as attractions, i.e., $A^j_i(t) = E^j_i(t)$. Then, with the three additional conditions $\delta = 1$, $\phi = \rho$, and $A^j_i(0) = E^j_i(0)$, Equation (10.19) is identical to Equation (10.11).

10.3 Level-*k* reasoning and sophisticated EWA

10.3.1 Beauty contest games

John Maynard Keynes, in Chapter 12 of his *General Theory* (Keynes, 1936), proposed a way to think of stock market investment in the context of newspaper beauty contests. In this newspaper competition, readers were asked to choose the six prettiest faces from 100 photographs. The winner was the person whose preferences were closest to the average preferences of all participants. Keynes assumed that contest participants do not choose faces that they personally find the most attractive; instead, some make their choices based on their expectations of others' expectations, and some, going one step further, make their choices based on their expectations of others' expectations of others' expectations:

> It is not the case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree, where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth, and higher degrees.
>
> (Keynes, 1936, p. 156)

Investors in financial markets have much in common with these beauty contests. Consider the market-timing decision as an example. People holding stocks in a bull market consider when the best time to sell stocks and to exit the market might be. They certainly want to do it before the majority of people start doing it. So, they have to guess when this majority movement time will happen, and sell a few days ahead of this big maneuver. Nevertheless, other investors are gauging this timing in the same way, so some smarter investors have to, being one level higher, think about what others think.

Partially because of this natural analogy between beauty contests and financial investment, in the middle of the 1990s economists started to address this issue in an experimental setup, and developed the well-known *beauty contest experiments* (Nagel, 1995, 1998). They are called *beauty contest games* in honor of Keynes. In this game, contestants are told to guess a number from 0 to 100, with the goal of making their guess as close as possible to p times the average guess. In a world where all the players are known to be fully rational, in the sense that they will form expectations about the guesses of others and can carry out as many levels of deduction as necessary, the equilibrium in this game is zero if $0 < p < 1$ and is 100 if $p > 1$. Nonetheless, this theoretical result is not well supported by human-subject experiments.

For example, Richard Thaler ran this experiment with the *Financial Times* (Thaler, 1997). He found that although many contestants did guess zero or one, the most popular guesses were 33 and 22. The former is the right guess if everyone else chooses a number at random, and 22 is the right guess if everyone else picks 33. If we consider random guessing (the behavior of zero-intelligence agents) as level-0 reasoning, then 33 corresponds to level-1 reasoning and 22 corresponds to level-2 reasoning. With these findings, Thaler suggests incorporating heterogeneity in reasoning into artificial agents:

> Although modeling how this game is actually played is not easy, some lessons are clear enough. An appropriate model would have to allow for two kinds of heterogeneity in sophistication. First, agents differ in how many levels of processing they engage in (33 is one level, 22 is two levels, and so on). Second, there is heterogeneity in how much agents think about the behavior of other agents.
>
> (Thaler, 2000, p. 135)

10.3.2 Level-k reasoning

Since the middle of the 1990s, a series of efforts have been made to develop artificial agents who are heterogeneous in their capability to reason about other agents' reasoning (Stahl and Wilson, 1994, 1995; Nagel, 1995; Ho, Camerer, and Weigelt, 1998; Costa-Gomes, Crawford, and Broseta, 2001; Bosch-Domenech *et al.*, 2002; Crawford, 2003; Camerer, Ho, and Chong, 2004; Costa-Gomes and Crawford, 2006; Ohtsubo and Rapoport, 2006; Crawford and Iriberri, 2007a, b). This literature is collectively known as *level-k reasoning*. There are several different versions of level-k reasoning, such as *theory of mind* (Stahl and Wilson, 1995), level-k reasoning (Crawford and Iriberri, 2007b), *cognitive hierarchy* (Camerer, Ho, and

Chong, 2004), and *Machiavellian intelligence* (Ohtsubo and Rapoport, 2006), but the basic structure is, roughly, that higher-level agents are able to reason what low-level agents do, and they then best respond to it.

The basic idea behind level-k reasoning is that agents are placed in a *cognitive ladder*. Along this ladder, those who are placed at higher levels are able to simulate the minds of those placed at lower levels, but not vice versa. In spirit, level-k reasoning is analogous to the *Chomsky hierarchy* in computer science. The Chomsky hierarchy is a hierarchy of formal grammars and the formal languages generated by the grammars. The hierarchy arranges four classes of grammars and the four generated languages in monotonically increasing order, so that any class of grammars (languages) appearing in this order is a *proper subset* of the one coming after it. It also implies a hierarchy of the automata which can recognize the grammars. By the same monotonic order, the automata at a higher level can simulate the automata at a lower level, but not vice versa.

10.3.3 Sophisticated EWA learning

The sophisticated EWA learning proposed by Camerer, Ho, and Chong (2002) can be regarded as an application of level-k reasoning to EWA learning. Camerer, Ho, and Chong (2002) consider two types of agents, Level-0 and Level-I, with a given distribution of each type, namely, $1-\alpha'$ and α'. Level-0 agents do not know of the existence of Level-1 agents, and behave as the EWA learning model describes. Level-I agents know of the existence of Level-0 agents and know that they behave by following the EWA learning models. In addition, Level-I agents also "know" that the percentages of Level-1 and Level-0 agents are α' and $1-\alpha'$, respectively.[10] To make their own decisions, Level-I agents form their beliefs by taking into account both Level-0 and other Level-I agents' learning. Their attractions are accordingly modified as follows:

$$\begin{cases} A^j_{i,I}(t) = \sum_{k=1}^{J}[\alpha' P^k_{-i,I}(t+1) + (1-\alpha')P^k_{-i.0}(t+1)]\pi_i(s^j_i, s^k_{-i}) \\ P^j_{i,I}(t+1) = \frac{\exp(A^j_{i,I}(t))}{\sum_{j=1}^{J}\exp(A^j_{i,I}(t))} \end{cases} . \tag{10.20}$$

The subscripts 0 and I appearing above refer to Level-0 and -I agents, respectively. Equation (10.20) can be read as a generalized and sophisticated version of Equation (10.18). The main difference lies in agents' belief formation. In the latter case it is formed exclusively based on the historical observations only, whereas in the former case it is formed by taking into account the expectations of Level-0 agents' expectations. With this sophistication, Level-I agents are able to replace the the *estimated* choice probability $B^k_{-i}(t)$ by the *true* choice probability $P^k_{-i.0}(t+1)$. In addition, Level-I agents also know of the existence of other Level-I agents, like themselves, and know that they form expectations in the same way as they did. Hence, the true choice probability of this group of agents, $P^k_{-i.0}(t+1)$, is also incorporated to replace the estimated one, $B^k_{-i}(t)$. Since, for the EWA agents, the expected payoffs are taken as the attractions, we therefore use $A^j_{i,I}(t)$

directly instead of $E^j_{i,I}(t)$. Putting these together, we obtain the expected payoffs for the Level-I agents as described in Equation (10.20).

10.4 Regime-switching agents

One kind of agent design to be introduced in the agent-based financial market (Chapter 14) is also related to reinforcement learning; we shall call them *regime-switching agents*. Regime-switching agents are agents who follow a set of behavioral regimes, and switch between them. The switching mechanism is stochastic and is used to characterize the learning behavior of agents. Among many possible regime-switching agents, the most standard one is the fundamentalist/chartist model, a two-type agent-based financial model (Kirman, 1991; Lux, 1998; Hommes, 2002).

More generally, in the H-type agent-based financial model, agents have a set of H behavioral regimes. To decide which regime to activate in each period of time, the agents choose one of these regimes in a stochastic manner, which is very similar to the generalized reinforcement learning as discussed in Section 10.2. For example, in the models initiated by Brock and Hommes (1998) and Hommes (2002), known as *adaptive belief systems* (ABS), the logit model is applied for modeling the agents' stochastic choice. Another kind, *heading-based agent-based financial models*, initiated by Kirman (1991) and Lux (1995), employ the idea of the Polya urn process as a basis for the stochastic choice, but this difference may be trivial since financial agents can be reinforced by various kinds of payoffs. In addition to monetary payoffs, they can also be reinforced by peer pressure (herding). Having said that, we notice that they can also be reinforced by risk aversion, forecasting accuracy, etc. The idea of payoffs used in agent-based financial models is, therefore, broader than what has been commonly used in game experiments.

Notes

1 Marvin Minsky, one of the founders of artificial intelligence, wrote his PhD thesis on reinforcement learning in 1954.
2 The recent progress in neuroscience indicates that humans or mammals are naturally endowed with a reinforcement learning mechanism in their brain. In fact, one of the most impressive recent results in neuroscience is the discovery of the relationship between the dopamine neural system and reinforcement learning. See Montague (2006), Chapter 4, for a vivid historical review of the research on the dopamine system and reinforcement learning.
3 See Thaler (2000) for a discussion of this connection.
4 This experimentation idea is also shared in many other learning algorithms, such as Kohonen's competitive learning—see Section 12.6.
5 There is a name for the parameter λ in the agent-based financial models: *intensity of choice*.
6 The contrasting studies of these two classes of learning even becomes a subject for neuroeconomics in the context of the *dual process model* (Charness and Levine, 2005; Fudenberg and Levine, 2006).
7 The transfer function used in Equation (10.8) is a logit function; other alternatives include the power function and probit function.

8 However, Ho, Wang, and Camerer (2008) have recently extended generalized reinforcement learning into the case where information regarding foregone payoffs is incomplete and has to be estimated.

9 Antoine Augustin Cournot (1801–1877), a French economist and mathematician, introduced the first explicit model of learning in games. He assumed that players choose the best response to what they most recently observed.

10 Camerer, Ho, and Chong (2002) actually considered the case when agents do not know the parameter α' and have to learn from the data.

11 Fuzzy logic and rough sets

11.1 Fuzzy logic

People frequently and routinely use natural language or linguistic values, such as high, low, and so on, to describe their perception, demands, anticipation, and decisions. While natural language has its ambiguities, people seem to be able to reason effectively with added vague and uncertain information and very often the decisions they make are the outcome of their approximate reasoning. For example, consider the following economic reasoning based on natural language: “The government will adopt a loose monetary policy, if the unemployment rate is continuously to be high.” “The economic prospects of the economy are not bright due to its low capital accumulation.” “If the price spread narrows, it is high time to buy.” While all these economic statements are crucial for decision-making, the words “high,” “loose,” “bright,” “low,” “narrow,” and “high time” are not accurate. Fuzzy logic (FL) is a formal approach to cope with these ambiguities. Evidence on human reasoning and human thought processes supports the hypothesis that at least some categories of human thought are definitely fuzzy, and the mathematical operations of fuzzy sets as prescribed by fuzzy set theory are a compatible and realistic description of how humans manipulate fuzzy concepts (Smithson, 1987).

After four decades of advancement and fine-tuning, the methodology of fuzzy logic has been widely used in economics, finance, and business. There are a great number of studies showing that *investment* behavior in a complex business environment is essentially fuzzy. Fuzzy logic, as it turns out, is the best fit in enhancing these difficult decisions. For example, Taylor, Abdel-Kader, and Dugdale (1998) present an operational model of decision-making about advanced manufacturing technology that incorporates the mathematics of fuzzy set theory. Peray (1999) applies fuzzy logic to mutual fund investment. Fuzzy logic is also applied to many other real-world business activities. Von Altrock (1996) and Bojadziev, Bojadziev, and Zadeh (1997) present applications for supplier evaluation, customer targeting, scheduling, evaluating leases, choosing R&D projects, and forecasting. Zopounidis, Pardalos, and Baourakis (2002) indicates the application direction toward marketing. The usefulness of FL in a spectrum of applications in economics and finance is hence well established.

Fuzzy logic is not only attractive for business practitioners, but has also been incorporated into mainstream formal economic analysis. Billot (1995) introduces fuzzy general equilibrium analysis. It presents the aggregated model of microeconomics with *fuzzy behaviors*, and presents the state of the art in the *fuzzy theory of value*. Mansur (1995) extends the application of fuzzy logic to *non-cooperative oligopoly*.

Fuzzy sets are distinct from classical sets (*crisp sets*) in the sense that the *membership* in the latter is *all or nothing*, whereas that in the former is a matter of *degree* (*more or less*). The degree is mathematically characterized by a *membership function*. Formally, a fuzzy set A can be denoted as follows.

$$A = \begin{cases} \sum_{x_i \in X} \mu_A(x_i)/x_i & \text{if } X \text{ is discrete,} \\ \int_{x \in X} \mu_A(x)/x & \text{if } X \text{ is continuous,} \end{cases} \tag{11.1}$$

where $\mu_A(x)$ is the membership function.[1] The sign $\sum$ and $\int$ stand for the union of the membership grade. / stands for a marker and does not imply division. Fuzzy mathematics has been developed to define elementary *fuzzy operations* of fuzzy sets using membership functions, such as *fuzzy OR*, *fuzzy AND*, and *fuzzy complement*. Also, via manipulation of the membership function, fuzzy mathematics can also deal with *linguistic modifiers*, such as "very," "extremely," "more or less," etc., as originally suggested by its founder Lotfi A. Zadeh.

The most frequent uses of fuzzy set methodology involves the building of *fuzzy inference systems*. A fuzzy inference system is a set of *fuzzy if–then rules* that connect one or more input sets to one output set. In the literature, there are two major styles of fuzzy if–then rules, namely the *Mamdani style* (Mamdani and Assilian, 1975) and the *Sugeno style* (Sugeno, 1985). In the Mamdani style both the antecedent and consequent of rules are fuzzy. In the Sugeno style only the antecedent of rules is fuzzy, whereas the consequent of rules is in general a function of the input variables. As an illustration, the Mamdani style of a fuzzy if–then rule is:

If x is "A" and y is "B," then z is "C,"

where the input sets "A" and "B" and the output set "C" are all fuzzy. On the other hand, the Sugeno style of a fuzzy if–then rule is:

If x is "A" and "y" is "B," then $z = f(x, y)$.

Building a fuzzy inference system consists of five main steps. First, *fuzzification*. The input variables are fuzzified by assigning a membership value with respect to different fuzzy sets. Second, a fuzzy operation, such as fuzzy AND or fuzzy OR, is applied in order to determine the degree to which each rule is applicable. Third, *implication*. Implication methods, such as *cut*, are used to decide the membership function of the consequent of fuzzy rules. Fourth, *aggregation*. The results of the applicable rules are combined to obtain an output fuzzy set. Fifth, *defuzzification*. Various methods can be used to get a number as the result of the defuzzification operation. The most common one is to use the *centroid* of the fuzzy set resulting from aggregation.

Fuzziness and complexity

One issue is whether fuzziness has helped us to harness the complex situation in the sense that it is simple in terms of linguistic description but remains complex in terms of its numerical crisp description. Maybe what we have ignored for a long time is that linguistic rules, for us humans, provide a way to achieve shorter description lengths.

11.2 Rough sets

Despite the remarkable distinction in definition, *rough set analysis*, as introduced by Zdzislaw Pawlak in 1982, is usually mentioned together with fuzzy logic. Both fuzzy logic and rough set analysis are methods for modeling uncertain, vague, or inaccurate information in a wide sense. As a matter of the fact, a "rough set" is similar to a "fuzzy set" in the sense that they are alternatives to the "crisp set," where membership in the set is a certainty. What distinguishes rough set analysis from fuzzy set analysis is that the former requires no external parameters and uses only the information presented in the given data.

A rough set is a knowledge representation system, which classifies the objects with their attributes (features) using the idea of the *approximation spaces*. Rather than attempt exact classification of objects with attributes (features), Pawlak considered an approach to solving the object classification problem by introducing *approximate descriptions* of sets of objects and considered knowledge representation systems in the context of upper and lower classifications of objects relative to their attribute values.

Rough sets define a mathematical model of vague concepts that is used to represent the relationship of dissimilarity between objects. Rough sets are not completely specified. They can be characterized by upper and lower approximations in the domain, which define the level of uncertainty of the partial specification. The lower approximations consist of objects which belong to a concept with certainty, while the upper approximations consist of those which possibly belong. On the basis of the lower and upper approximations of a rough set, the *accuracy* of approximating the rough set can be calculated as the ratio of the cardinality of its lower approximation to the cardinality of its upper approximation. It has tremendous influence when one considers the problem of the concept approximation or approximate reasoning.

$$\mathrm{IND}_{\mathrm{IS}}(B) = \{(x, x') \in U \times U \mid \forall a \in B, a(x) = a(x')\}. \tag{11.2}$$

The equivalence classes of the B-indiscernibility relation are denoted by $[x]_B$.

$$\underline{B}X = \{x \mid [x]_B \subseteq X\}, \tag{11.3}$$

$$\overline{B}X = \{x \mid [x]_B \cap X \neq \emptyset\}, \tag{11.4}$$

$$BN_B(X) = \overline{B}X - \underline{B}X. \tag{11.5}$$

A set is said to be *rough* if its boundary region is non-empty, $BN_B(X) \neq \emptyset$. From this we can develop the idea of accuracy of approximation:

$$\alpha_B(X) = \frac{|\underline{B}X|}{|\overline{B}X|}. \tag{11.6}$$

X is said to be *rough* with respect to B if $\alpha_B(X) < 1$.

Rough sets have received much attention recently with regard to interest in data mining techniques that combine automation of the rule extraction with domain expertise in economics and finance. Slowinski and Zopounidis (1995) apply the rough set approach to the evaluation of bankruptcy risk. Skalkoz (1996) uses rough set techniques to extract a set of rules for short-term trading on the OEX index based on the Hines indicator.[2] Hampton (1997, 1998a, b) provide a quick grasp of the essential elements of rough set theory for economists and finance people. Moózek and Skabek (1998) give three exemplary economic decision problems using rough sets, namely companies evaluation, the credit policy of a bank, and the marketing strategy of a company.

Notes

1 There are more than a dozen membership function forms used in the literature. Users of Matlab can find a wealth of built-in standard membership functions.

2 The OEX or S&P 100 is a capitalization-weighted index of the biggest one hundred stocks in the US market.

12 Artificial neural networks

Among all computational intelligence (CI) tools, the artificial neural network (ANN) is the most widely acceptable tool for economists and finance people, even though its history is much shorter than that of fuzzy logic so far as application to economics and finance are concerned. The earliest application of ANNs appeared in 1988 by Halbert White (White, 1988). Since then we have witnessed an exponential growth in the number of applications. ANN is probably the only CI tool which drew serious attention from econometricians and on which a lot of theoretical studies have been done. Both White (1992) and Refenes and Zapranis (1999) gave rigorous mathematical or statistical treatments of ANNs and hence they have established ANNs with a sound foundation in the econometric field. Nowadays, it has already become an essential chapter of any textbook in econometrics and, in particular, financial econometrics and financial time series. A great number of textbooks or volumes specially edited for economists and finance people are available, to name a few: Trippi and Turban (1993), Azoff (1994), Baestaens, Van Den Bergh, and Wood (1994), Refenes (1995), Gately (1996), Zirilli (1996), and Shadbolt and Taylor (2002). Its significance to finance people can also be seen from the establishment of the journal *Neurove$t* (now *Computational Intelligence in Finance*) in 1993.

It has been shown in a great number of studies that artificial neural nets, as representative of a more general class of nonlinear models, can outperform many linear models, and can sometimes also outperform some other nonlinear models.[1]

Three classes of artificial neural nets have been most frequently used in economics and finance. These are *multilayer perceptron neural networks*, *radial basis neural networks*, and *recurrent neural networks*. The first two classes will be introduced in Sections 12.1 and 12.2, and the last one in Section 12.3.

12.1 Multilayer perceptron neural networks

Let us consider the following general issue. We observe a time series of an economic or financial variable, such as the foreign exchange rate, $\{x_t\}$. We are interested in knowing its future, $x_{t+1}, x_{t+2}, \ldots$ For that purpose, we need to search for a function relation f, such that when a vector $\mathbf{x}_t$ is input into the function, a prediction on $x_{t+1}, \ldots$ can be made. The question then is how to construct such a function. Tools included in this chapter provide two with different approaches, distinguishable by different modeling philosophies. The first is based on the *universal modeling approach*, and the second on the *local modeling approach*. Alternatively, we can say that the first one is to build the function in the *time domain*, whereas the second is to work in the *feature domain* or in the *trajectory domain*.[2] We shall start with the first approach, and the canonical artificial neural network can be considered to be a representative of this paradigm.

The reason why economists can embrace the ANN without any difficulties is due to the fact that ANN can be regarded as a generalization of their already household time-series model, *ARMA* (autoregressive moving average). Formally, an ARMA(p, q) model is presented as follows:

$$\Phi(L)x_t = \Theta(L)\epsilon_t, \tag{12.1}$$

where $\Phi(L)$ and $\Theta(L)$ are polynomials of order p and q,

$$\Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p, \tag{12.2}$$

$$\Theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_q L^q, \tag{12.3}$$

$\{\epsilon_t\}$ is the white noise, and L is the lag operator. ANNs can be regarded as a nonlinear generalizations of these ARMA processes. In fact, more concretely, *multilayer perceptron neural networks* are nonlinear generalizations of the so-called autoregressive (AR) process,

$$x_t = f(x_{t-1}, \ldots, x_{t-p}) + \epsilon_t, \tag{12.4}$$

whereas *recurrent neural networks* are nonlinear generalizations of ARMA processes,

$$x_t = f(x_{t-1}, \ldots, x_{t-p}, \epsilon_{t-1}, \ldots \epsilon_{t-q}) + \epsilon_t. \tag{12.5}$$

In terms of a multilayer perceptron neural network, Equation (12.4) can then be represented as

$$x_t = h_2(w_0 + \sum_{j=1}^{l} w_j h_1(w_{0j} + \sum_{i=1}^{p} w_{ij} x_{t-i})) + \epsilon_t. \tag{12.6}$$

Equation (12.6) is thus a three-layer neural net (Figure 12.1). The input layer has p inputs: $x_{t-1}, \ldots, x_{t-p}$. The hidden layer has l hidden nodes, and there is a single output for the output layer, $\hat{x}_t$. Layers are fully connected by *weights*: w_{ij} is the weight assigned to the ith input for the jth node in the hidden layer, whereas w_j is the weight assigned to the jth node (in the hidden layer) for the output. w_0 and w_{0j} are constants, also called *biases*. h_1 and h_2 are *transfer functions*.

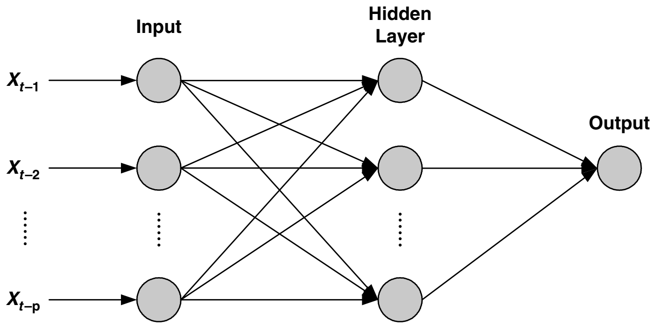

Figure 12.1 The multilayer perceptron neural network of a nonlinear AR process.

There is a rich choice of transfer functions. According to Chen and Chen (1995), a multilayer perceptron network with any *Tauber–Wiener functions* as the transfer function of the hidden units can be qualified as a *universal approximator*. Also, a necessary and sufficient condition for being a Tauber–Wiener function is that *it is a non-polynomial*. In practice, a differentiable transfer function is desirable. Commonly used transfer functions for multilayer perceptron networks are the *sigmoid function*,

$$h_s(x) = \frac{1}{1 + e^{-x}}, \tag{12.7}$$

or the *hyperbolic tangent function*,

$$h_t(x) = \frac{2}{1 + e^{-2x}} - 1. \tag{12.8}$$

Clearly, $0 < h_s(x) < 1$, and $-1 < h_t(x) < 1$.

12.2 Radial basis network

Next to the multilayer perceptron neural network is the *radial basis network* (RBN), which was also popularly used in economics and finance. Radial basis function (RBF) networks are basically feedforward neural networks with a *single hidden layer*,

$$f(x) = \sum_{i}^{k} w_i \varphi(\|x - c_i\|), \tag{12.9}$$

where $\varphi(\quad)$ is a *radial basis function*, c_i is the ith center, and k is the number of the center. Both w_i, c_i, and k are to be determined by the dataset of x. Typical choices of radial basis functions are:

– the thin plate spline function,

$$\varphi(x) = x^2 \log^x; \tag{12.10}$$

– the Gaussian function,

$$\varphi(x) = \exp\left(-\frac{x^2}{\beta}\right); \tag{12.11}$$

– the multiquadratic function,

$$\varphi(x) = (x^2 + \beta^2)^{\frac{1}{2}}; \tag{12.12}$$

– the inverse multiquadratic function,

$$\varphi(x) = \frac{1}{(x^2 + \beta^2)^{\frac{1}{2}}}. \tag{12.13}$$

Theoretical investigation and practical results seem to show that the choice of radial basis function is not crucial to the performance of an RBF network.

It has been proved that an RBF network can indeed approximate arbitrarily well any continuous function if a sufficient number of radial basis function units are given, the network structure is large enough, and the parameters in the network are carefully chosen. RBN also has the best approximation property in the sense of having the minimum distance from any given function under approximation.

12.3 Recurrent neural networks

In Section 12.1, we discussed the relation between time series models and artificial neural networks. Information transmission in the usual multilayer perceptron neural net is *feedforward* in the sense that information is transmitted *forward* from the input layer to the output layer, via all the hidden layers in between, as shown in Figure 12.1. The reverse direction between any two layers is not allowed.

This specific architecture makes the multilayer perceptron neural net unable to deal with a moving average series, MA(q), effectively. To see this, consider an MA(1) series as follows:

$$x_t = \epsilon_t - \theta_1 \epsilon_{t-1}. \tag{12.14}$$

It is well known that if $|\theta_1| < 1$, then the above MA(1) series can also be written as an AR(∞) series:

$$x_t = -\sum_{i=1}^{\infty} \theta^i x_{t-i} + \epsilon_t. \tag{12.15}$$

In using the multilayer perceptron neural network to represent Equation (12.15), one needs to have an input layer with an infinite number of neurons (*infinite memory of the past*), namely, $x_{t-1}, x_{t-2}, \ldots$, which in practice is impossible. Although, from the viewpoint of approximation, an exact representation is not required and a compromise with a finite number of neurons (*finite memory*) is acceptable, in general quite a few inputs are still required, which inevitably increases the complexity of the network, leads to an unnecessarily large number of parameters, and hence slows down the estimation and training process (Mandic and Chambers, 2001).

This explains why the multilayer perceptron neural net can only be regarded as the nonlinear extension of autoregressive (AR) time series models (12.4), but not the nonlinear extension of autoregressive moving average models (12.16):

$$\begin{aligned} x_t &= f(x_{t-1}, \ldots, x_{t-p}, \epsilon_{t-1}, \ldots \epsilon_{t-q}) + \epsilon_t \\ &= f(x_{t-1}, \ldots, x_{t-p}, x_{t-p-1}, \ldots) + \epsilon_t. \end{aligned} \tag{12.16}$$

The finite memory problem of the multilayer perceptron neural net has long been noted by ANN researchers. In his celebrated article, Jeffrey Elman stated:

> the question of how to represent time in connection models is very important. One approach is to represent time *implicitly* by its effects on processing rather than *explicitly* (as in a spatial representation).
>
> (Elman, 1990, p. 179; emphasis added)

The multilayer perceptron neural net tries to model time by giving it a spatial representation, i.e., an explicit representation. What Elman suggests is to let time have an effect on the network response rather than to represent time by an additional input dimension. Using an idea initiated by Michael Jordan (Jordan, 1986), Elman proposes an internal representation of memory by allowing the hidden unit patterns to be fed back to themselves. In this way, the network becomes *recurrent*.

The difference between the multilayer perceptron neural net (the feedforward neural net) and the recurrent neural net can be shown as follows. In terms of a multilayer perceptron neural network, Equation (12.4) can be represented as Equation (12.6). Equation (12.6) is a three-layer neural net (Figure 12.1).

In terms of a recurrent neural net, Equation (12.5) can then be represented as

$$x_t = h_2(w_0 + \sum_{j=1}^{l} w_j h_1(w_{0j} + \sum_{i=1}^{p} w_{ij} x_{t-i} + \sum_{m=1}^{l} \varpi_{mj} z_{m,t-1})) + \epsilon_t, \tag{12.17}$$

where

$$z_{m,t} = w_{0m} + \sum_{i=1}^{p} w_{im} x_{t-i} + \sum_{k=1}^{l} \varpi_{kj} z_{k,t-1}, \quad m = 1, \ldots, l. \tag{12.18}$$

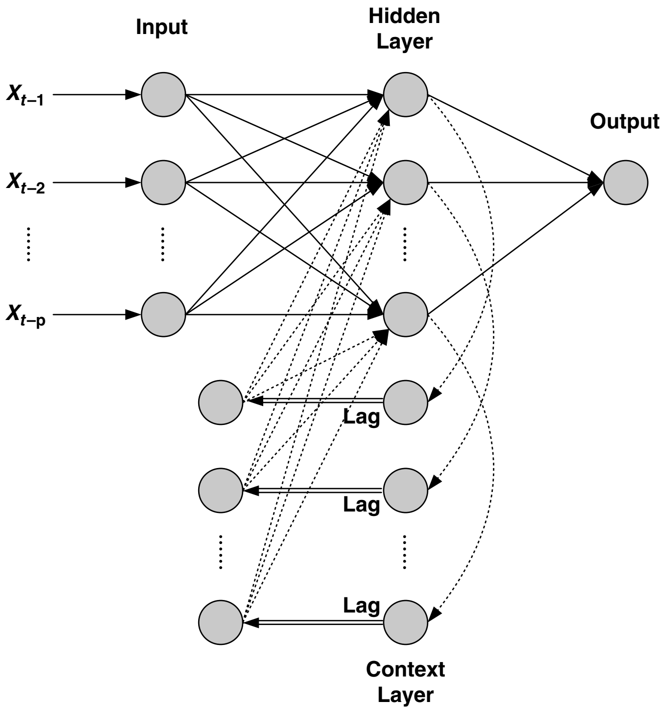

Figure 12.2 A recurrent neural network.

In the recurrent neural net, positive feedback is used to construct memory in the network, as shown in Figure 12.2. Special units called *context units* save previous output values of hidden-layer neurons (Equation 12.18). Context unit values are then fed back fully connected to hidden-layer neurons and serve as additional inputs in the network (Equation 12.17).

Compared to the multilayer perceptron neural net and the radial basis function neural net, the recurrent neural net is much less explored in the economic and financial domain.[3] This is, indeed, a little surprising, considering the great exposure of its linear counterpart ARMA to economists.

12.4 Auto-associative neural networks

While most economic and financial applications of the neural network consider its capability to develop nonlinear forecasting models, there is one important branch using artificial neural networks to engage in dimension reduction or feature extraction. In this application, ANN can provide a nonlinear generalization of conventional *principal component analysis* (PCA). The specific kind of ANN for this application is referred to as an *auto-associative neural network* (AANN).

The fundamental idea of principal component analysis is dimensional reduction, which is a quite general problem when we are presented with a large number of correlated attributes, and hence a large number of redundancies. It is, therefore, a natural attempt to compress or store this original large dataset into a more economical space by getting rid of these redundancies. So, on the one hand, we want to have a reduced space that is as small as possible; on the other hand, we still want to keep the original information. These two objectives are, however, in conflict when attributes with complicated relations are presented. Therefore, techniques to make the least compromise between the two become important.

To introduce AANN and its relationship with PCA, let us consider the following two mappings,

$$\mathcal{G}:\mathbf{R}^m \to \mathbf{R}^f \tag{12.19}$$

and

$$\mathcal{H}:\mathbf{R}^f \to \mathbf{R}^m, \tag{12.20}$$

where $\mathcal{G}$ and $\mathcal{H}$ are, in general, nonlinear vector functions with the components indicated as $\mathcal{G} = \{G_1, G_2, \ldots, G_f\}$ and $\mathcal{H} = \{H_1, H_2, \ldots, H_m\}$. To represent these functions with multilayer perceptron neural nets, let us rewrite Equation (12.6) as follows:

$$\begin{aligned} y_k &= G_k(x_1, \ldots, x_m) \\ &= h_2(w_{0k} + \sum_{j=1}^{l_1} w_{jk} h_1(w_{0j}^e + \sum_{i=1}^{m} w_{ij}^e x_i)), \quad k = 1, 2, \ldots, f, \end{aligned} \tag{12.21}$$

and

$$\begin{aligned} \hat{x}_i &= H_i(y_1, \ldots, y_f) \\ &= h_4(w_{0i} + \sum_{j=1}^{l_2} w_{ji} h_3(w_{0j}^d + \sum_{k=1}^{f} w_{kj}^d y_k)), \quad i = 1, 2, \ldots, m. \end{aligned} \tag{12.22}$$

All the notations used in Equations (12.21) and (12.22) share the same interpretation as those in Equation (12.6), except for the superscripts e and d standing for the encoding and decoding maps, respectively. By combining the two mappings together, we have a mapping from $\mathbf{X} = \{x_1, \ldots, x_m\}$ to its own reconstruction $\hat{\mathbf{X}} = \{\hat{x}_1, \ldots, \hat{x}_m\}$. Let X_n be the nth observation of X, and

$$\mathbf{X}_n = \{x_{n,1}, \ldots, x_{n,m}\}.$$

Accordingly,

$$\hat{\mathbf{X}}_n = \{\hat{x}_{n,1}, \ldots, \hat{x}_{n,m}\}.$$

Then, minimizing the difference between the observation $\mathbf{X}_n$ and its reconstruction $\hat{\mathbf{X}}_n$ over the entire set of N observations, or

$$\min E = \sum_{n=1}^{N} \sum_{i=1}^{m} (x_{n,i} - \hat{x}_{n,i})^2, \tag{12.23}$$

by searching over the space of the connection weights and biases, defines what is known as auto-association neural networks. Briefly, auto-associative neural networks are feedforward nets, with *three hidden layers* as shown in Figure 12.3, trained to produce an approximation of the *identity mapping* between network inputs and outputs using back-propagation or similar learning procedures.

The third hidden layer, i.e., the output layer of the multilayer perceptron network, Equation (12.21), is also called the *bottleneck layer*. If the transfer functions h_i $(i = 1, 2, 3, 4)$ are all identical mappings, and we remove all the bias terms, then Equation (12.21) can be written as:

$$\begin{aligned} y_k &= G_k(x_1, \ldots, x_m) \\ &= \sum_{j=1}^{l_1} w_{jk} \left(\sum_{i=1}^{m} w_{ij}^{e} x_i \right) = \sum_{j=1}^{l_1} \sum_{i=1}^{m} w_{jk} w_{ij}^{e} x_i, \\ &= \sum_{i=1}^{m} \sum_{j=1}^{l_1} w_{jk} w_{ij}^{e} x_i, = \sum_{i=1}^{m} \beta_{i,k} x_i \quad k = 1, 2, \ldots, f, \end{aligned} \tag{12.24}$$

where

$$\beta_{i,k} = \sum_{j=1}^{l_1} w_{jk} w_{ij}^{e}.$$

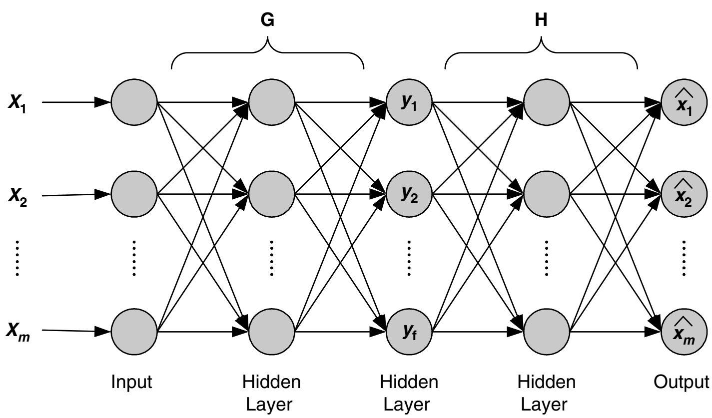

Figure 12.3 An auto-associative neural network.

In matrix notation, Equation (12.24) can be written as

$$\begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} & \dots & \beta_{1f} \\ \beta_{21} & \beta_{22} & \dots & \beta_{2f} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1} & \beta_{m2} & \dots & \beta_{mf} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1f} \\ y_{21} & y_{22} & \dots & y_{2f} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \dots & y_{nf} \end{bmatrix}, \quad (12.25)$$

or simply

$$\mathbf{XB} = \mathbf{Y}. \quad (12.26)$$

X, **B**, and **Y** correspond to the $n \times m$, $m \times f$, and $n \times f$ matrices in Equation (12.25), respectively. Likewise, Equation (12.22) can be simplified as

$$\mathbf{YB}^* = \hat{\mathbf{X}}. \quad (12.27)$$

$\mathbf{B}^*$ is the reconstruction mapping and is an $f \times m$ matrix, and $\hat{\mathbf{X}}$ is the reconstruction of **X**, and hence is an $n \times m$ matrix.

Equations (12.26) and (12.27) with the objective function (12.23) define the familiar *linear* principal components analysis. To see this, we can decompose **X** as follows:

$$\mathbf{X} = \mathbf{YB}^* + \mathbf{E} = \mathbf{XBB}^* + \mathbf{E} = \mathbf{XP} + \mathbf{E}, \quad (12.28)$$

where $\mathbf{P} = \mathbf{BB}^*$, and **E** is the reconstruction error. Then the PCA frequently presented to us takes the form of the following minimization problem:

$$\min_{\mathbf{P}} \|\mathbf{E}\|. \quad (12.29)$$

It is known that the optimal solution to the problem (12.29) has the rows of **P** being the eigenvectors corresponding to the f largest eigenvalues of the covariance matrix of **X**. Therefore, we have shown how the self-associative neural network can be a nonlinear generalization of the familiar linear PCA, and how the linear PCA can be extended to the nonlinear PCA through a feedforward neural network with three hidden layers.

The concept of using a neural network with a bottleneck to concentrate information has previously been discussed in the context of *encoder/decoder* problems.[4] McNelis (2005) indicates some directions for financial applications of nonlinear PCA.

12.5 Support vector machines

In the 1990s, based on results from *statistical learning theory* (Vapnik, 1998a), an alternative to the artificial neural network was developed, i.e. the *support vector* *machine* (SVM). SVM was founded primarily by Vladimir Vapnik, who contributed to the development of a general theory for minimizing the expected risk of losses using empirical data. Brief introductory material on the SVM can be found in Vapnik (1998b), whereas Cristianini and Shawe-Taylor (2000) is a textbook devoted to SVM.[5]

Support vector machines nonlinearly map an n-dimensional input space into a high-dimensional feature space:

$$\phi : V^n \to V^m, \tag{12.30}$$

where V^n is an n-dimensional input vector space, and V^m is an m-dimensional feature vector space. Given a series of l historical observations,

$$(y_1, x_1), \ldots, (y_l, x_l),$$

where $y_i \in V^1$ and $x_i \in V^n$, we approximate and estimate the functional relation between y_i and x_i by

$$y = f(x) = \langle w, \phi(x) \rangle + b = \sum_{i=1}^{m} w_i \phi(x)_i + b, \tag{12.31}$$

where $\langle \ldots \rangle$ denotes the inner product. The vector w and the constant b are to be determined by following the *structural risk minimization principle*, borrowed from statistical learning theory. It is interesting to note some similarities between RBN and SVM, namely, Equations (12.9) and (12.31). However, there is also a noticeable difference. Consider an input x_i as a vector of three dimensions: $(x_{i,1}, x_{i,2}, x_{i,3})$. Then for each neuron in the hidden layer of the RBN, they all share the same form as

$$(\varphi(x_{i,1}, x_{i,2}, x_{i,3}, c_1), \varphi(x_{i,1}, x_{i,2}, x_{i,3}, c_2), \ldots),$$

while being associated with different centers. However, each neuron in the hidden layer of the SVM may actually take different inputs. For example, the first neuron takes the first two inputs, but the second takes the last two as

$$(\phi_1(x_{i,1}, x_{i,2}), \phi_2(x_{i,2}, x_{i,3}), \ldots).$$

Also, notice that the transfer functions, $\varphi(\ )$, are the same for each neuron in the RBN, but in general are different for the SVM as $\phi_1, \phi_2, \ldots$

In the case where the y_i are categorical, such as $y_i \in \{-1, 1\}$, the minimization process also determines a subset of $\{x_i\}_{i=1}^{l}$, called *support vectors*, and the SVM when constructed has the form

$$f(x) = \sum_s y_i \alpha_i^* < \phi(x_s), \phi(x) > +b^*, \tag{12.32}$$

where α_i^* and b^* are coefficients satisfying the structural risk minimization principle, and s is the set of all support vectors. The category assigned to the observation x, 1 or -1, will then be determined by the sign of $f(x)$:

$$y = \begin{cases} 1 & \text{if } f(x) > 0, \\ -1 & \text{if } f(x) < 0. \end{cases} \tag{12.33}$$

Equations (12.32) and (12.33) are the SVM for the classification problem. A central concept of the SVM is that one does not need to consider the feature space in explicit form; instead, based on the Hilbert–Schmidt theory, one can use the *kernel function*, $K(x_s, x)$, where

$$K(x_s, x) = \langle \phi(x_s), \phi(x) \rangle. \tag{12.34}$$

Therefore, the SVM is also called the *kernel machine*. Equation (12.32) can then be rewritten as

$$f(x) = \sum_s y_i \alpha_i^* K(x_s, x) + b^*. \tag{12.35}$$

Following a similar procedure, one can construct the SVM for regression problems as follows:

$$f(x) = \sum_{i=1}^{l} (\alpha_i^* - \beta_i^*) K(x, x_i) + b^*, \tag{12.36}$$

where α_i^*, β_i^*, and b^* are the coefficients minimizing the corresponding objective functions.

In addition to the functional form, $f(\mathbf{x})$, the second important issue is the set of variables $\mathbf{x}$ itself, and one has to deal naturally with the problem known as *variable selection* or *feature selection*. The involvement of irrelevant variables or features may lead to poor generalization capability.

12.6 Self-organizing maps and k-means

In social and behavioral sciences, the ability to recognize patterns is an essential aspect of human heuristic intelligence. Herbert Simon, a Nobel Prize Laureate in Economics, 1978, considered pattern recognition to be critical and advocated the need to pay much more explicit attention to the teaching of pattern recognition principles. In the financial market, chartists appear to have been good at doing pattern recognition for many decades, yet little academic research has been devoted to a systematic study of these kinds of activities. On the contrary, sometimes it has been treated as nothing more than astrology, and hardly to be regarded as a rigorous science.

The self-organizing map (SOM) was invented by Teuvo Kohonen in 1982 (Kohonen, 1982). It has been applied with great success to many different engineering problems and to many other technical fields. Deboeck and Kohonen (1998) is the first volume to demonstrate the use of SOMs in finance.

SOMs solve the pattern recognition problem which deals with a class of *unsupervised neural networks*. Basically, the SOM itself is a *two-layer neural network*. The input layer is composed of p cells, one for each system input variable. The output layer is composed of neurons which are placed on n-dimensional lattices (the value of n is usually 1 or 2). The SOM adopts so-called *competitive learning* among all neurons. Through competitive learning, the neurons are tuned to represent a group of input vectors in an organized manner.

K-means clustering, developed by J.B. MacQueen in 1967 (MacQueen, 1967), is one of the widely used *non-hierarchical clustering algorithms* that groups data with similar characteristics or features together. K-means and SOMs resemble each other. They both involve minimizing some measure of dissimilarity, called the cost function, in the samples within each cluster. The difference between k-means and SOMs lies in their associated cost function, to which we now turn. Consider a series of n observations, each of which has m numeric attributes:

$$\mathbf{X}_1^m, \mathbf{X}_2^m, \ldots, \mathbf{X}_n^m, \quad \mathbf{X}_i^m \in \mathbf{R}^m, \quad \forall\, i = 1, 2, \ldots, n, \tag{12.37}$$

where

$$\mathbf{X}_i^m \equiv \{x_{i,1}, x_{i,2}, \ldots, x_{i,m}\}, \quad x_{i,l} \in \mathbf{R}, \quad \forall\, l = 1, 2, \ldots, m. \tag{12.38}$$

K-means clustering finds a series of k clusters, the centroids of which are denoted, respectively, by

$$\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_k, \quad \mathbf{M}_j \in \mathbf{R}^m, \quad \forall j = 1, 2, \ldots, k, \tag{12.39}$$

such that each of the observations is assigned to one and only one of the clusters with a minimal cost, and the cost function is defined as follows:

$$C_{k-\text{means}} = \sum_{i=1}^{n} \sum_{j=1}^{k} d(\mathbf{X}_i^m, \mathbf{M}_j) \cdot \delta_{i,j}, \tag{12.40}$$

where $d(\mathbf{X}_i^m, \mathbf{M}_j)$ is the standard Euclidean distance between $\mathbf{X}_i^m$ and $\mathbf{M}_j$,[6] and $\delta_{i,j}$ is the delta function:

$$\delta_{i,j} = \begin{cases} 1 & \text{if } \mathbf{X}_i^m \in \mathbf{Cluster}_j, \\ 0 & \text{if } \mathbf{X}_i^m \notin \mathbf{Cluster}_j. \end{cases} \tag{12.41}$$

To minimize the cost function (12.40), one can begin by initializing a set of k cluster centroids. The positions of these centroids are then adjusted iteratively by first assigning the data samples to the nearest clusters and then recomputing the centroids.

Corresponding to (12.40), the cost function associated with SOMs can be roughly treated as follows:[7]

$$C_{\text{SOM}} = \sum_{i=1}^{n} \sum_{j=1}^{k} d(\mathbf{X}_i^m, \mathbf{M}_j) \cdot h_{w(\mathbf{X}_i^m),j}, \tag{12.42}$$

where $h_{w(\mathbf{X}_i^m),j}$ is the neighborhood function or the neighborhood kernel, and $w_{(\mathbf{X}_i^m)}$, the winner function, outputs the cluster whose centroid is nearest to the input $\mathbf{X}_i^m$. In practice, the neighborhood kernel is chosen to be wide at the beginning of the learning process to guarantee the global ordering of the map, and both its width and height decrease slowly during learning. For example, the Gaussian kernel whose variance monotonically decreases with iteration time t is frequently used.[8] By comparing Equation (12.40) with (12.42), one can see in SOM the distance of each input from all of the centroids weighted by the neighborhood kernel h, instead of just the closest one being taken into account.

Despite its greater simplicity, the economic and financial applications of k-means are surprisingly much less available than those of SOM and k nearest neighbors (KNN; Section 12.7). K-means have occasionally been applied to classify hedge funds (Das, 2003), listed companies (Qian, 2006), and houses (Hollans and Munneke, 2003), but can also be applied to the classification of trajectories of financial time series. To see this, we rewrite (12.37) and (12.38) to fit the notations used in the context of time series:

$$\mathbf{X}_1^m, \mathbf{X}_2^m, \ldots, \mathbf{X}_T^m, \quad \mathbf{X}_t^m \in \mathbf{R}^m, \quad \forall\, t = 1, 2, \ldots, T, \tag{12.43}$$

$$\mathbf{X}_t^m \equiv \{x_t, x_{t-1}, \ldots, x_{t-m}\}, \quad x_{t-l} \in \mathbf{R}, \quad \forall\, l = 0, 1, \ldots, m-1. \tag{12.44}$$

$\mathbf{X}_t^m$ is a windowed series with an immediate past of m observations, also called the m-history. Equation (12.43), therefore, represents a sequence of T m-histories which are derived from the original time series, $\{x_t\}_{t=-m+1}^{T}$, by moving the m-long window consecutively, each with one step. Accordingly, the end product of applying k-means or SOMs to these windowed series is a number of centroids $\mathbf{M}_j$, which represent the specific shape of an m-long trajectory, also known as charts for technical analysts.[9]

Then the essential question pursued here is whether we can meaningfully cluster the windowed financial time series $\mathbf{X}_t^m$ by the k associated geometrical trajectories, $\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_k$. The clustering work can be meaningful if it can help us predict the future. In other words, conditional on a specific trajectory, we can predict the future better than without being provided this information, e.g.,

$$\text{Prob}(|\xi_{t+1}| > |\epsilon_{t+1}|) > 0.5,$$

where

$$\xi_{t+1} = x_{t+1} - E(x_{t+1}), \tag{12.45}$$

and

$$\epsilon_{t+1} = x_{t+1} - E(x_{t+1} \mid \mathbf{X}_t^m \in \mathbf{Cluster}_j), \quad t > T. \tag{12.46}$$

The conditional expectations above are made with the information of the trajectory (the cluster).

12.7 *K* nearest neighbors

In 1998, a time series prediction competition was held at the International Workshop on Advanced Black-Box Techniques for Nonlinear Modeling. The data to be predicted were available from November 1997 to April 1998 at Leuven. The data was generated from a generalized Chua's circuit, a well-known chaotic dynamic system. Seventeen entries had been submitted before the deadline. The winner of the competition turned out to be James McNames, and the strategy he used was the *nearest trajectory algorithm*. By using this algorithm with fast nearest neighbor algorithms, McNames was able to make an accurate prediction up to 300 points in the future of the chaotic time series. At first sight, this result may be a surprise for some, because KNN is not technically demanding compared with many other well-known tools as introduced in this chapter, but it could outperform many other familiar advanced techniques, such as neural nets, wavelets, Kohonen maps, and Kalman filters in that competition.[10]

KNN can be related to *decision trees*. What makes them different is that the latter have categories $A_1, \ldots, A_n$ to host input variables $\mathbf{X}_t^m$, while the former have $\mathbf{X}_t^m$ itself as the center of a hosting category, which will invite its own *neighbors*, $\mathbf{X}_s^m$ $(s < t)$, by ranking the *distance* $\| \mathbf{X}_t^m - \mathbf{X}_s^m \|$ over all $s < t$ from the closest to the farthest. Then the k closest $\mathbf{X}_s^m$s will constitute the neighbors of $\mathbf{X}_t^m$, $\mathcal{N}(\mathbf{X}_t^m)$. Now, for the purpose of predicting x_{t+1}, one can first study the functional relation between x_{s+1} and $\mathbf{X}_s^m$, $\forall s \in \mathcal{N}(\mathbf{X}_t^m)$, i.e.,

$$x_{s+1} = f_t(\mathbf{X}_s^m), \, s \in \mathcal{N}(\mathbf{X}_t^m). \tag{12.47}$$

One then forecasts x_{t+1} based on $\hat{f}_t$, an estimation of f_t,

$$\hat{x}_{t+1} = \hat{f}_t(\mathbf{X}_t^m). \tag{12.48}$$

Let us make a brief remark on what makes KNN different from the conventional time series modeling techniques. Conventional time series modeling, known as the Box–Jenkins approach, is a *global* model, which is concerned with the estimation of the function, be it linear or nonlinear, in the following form:

$$x_{t+1} = f(x_t, x_{t-1}, \ldots, x_{t-m}) + \epsilon_t = f(\mathbf{X}_t^m) + \epsilon_t, \tag{12.49}$$

by using all of the information up to t, i.e., $\mathbf{X}_s^m$, $\forall s \leq t$, and the estimated function $\hat{f}$ is assumed to hold for every single point in time. As a result, what will affect

x_{t+1} most is its immediate past, $x_t, x_{t-1}, \ldots$, under the law of motion estimated by all available samples. For KNN, while what affects x_{t+1} most is also its immediate past, the law of motion is estimated *only* with *similar* samples, *not all* samples. The estimated function $\hat{f}_t$ is hence assumed to only hold for that specific point in time. Both KNN and SOM challenge the conventional Box–Jenkins methodology by characterizing the hidden patterns in a different form. In their formulation, hidden patterns are not characterized by time location, but by topological trajectories.

Technical issues involved here are the choice of the distance function $d(\mathbf{X}_t^m, \mathbf{X}_s^m)$, the choice of the functional form f_t, the choice of the number of neighbors k, and the choice of the embedding dimension m.

12.8 Instance-based learning

KNN can be regarded as a special case of a broader class of algorithms, known as *instance-based learning* (IBL). To see this, let us use the notations introduced in Section 12.6, and use the time series prediction problem for an illustration.

Consider Equation (12.46). We have been given information regarding a time series up to time t, and we wish to forecast the next by using the current m-history, $\mathbf{X}_t^m$. In SOM or k-means, we will first decide to which cluster $\mathbf{X}_t^m$ belongs by checking $d(\mathbf{X}_t^m, \mathbf{M}_j)$ for all j $(j = 1, 2, \ldots, k)$, and use the forecast model associated with that cluster to forecast x_{t+1}. In other words, forecasting models are tailored to each cluster, say, $\hat{f}_j$ $(j = 1, 2, \ldots, k)$.[11] Then

$$\hat{x}_{t+1} = \hat{f}_{j^*}(\mathbf{X}_t^m), \text{ if } j^* = \arg\min_j d(\mathbf{X}_t^m, \mathbf{M}_j), \quad j = 1, 2, \ldots, k. \tag{12.50}$$

KNN, however, does not have such established clusters $\mathbf{M}_j$. Instead, it forms a cluster based on each $\mathbf{X}_t^m$, $\mathcal{N}(\mathbf{X}_t^m)$, as follows:

$$\mathcal{N}(\mathbf{X}_t^m) = \{s \mid \text{Rank}(d(\mathbf{X}_t^m, \mathbf{X}_s^m)) \leq k, \forall s < t\}. \tag{12.51}$$

In other words, $\mathbf{X}_t^m$ itself serves as the centroid of a cluster, called the *neighborhood* of $\mathbf{X}_t^m$, $\mathcal{N}(\mathbf{X}_t^m)$. It then invites its k *nearest neighbors* to be the members of $\mathcal{N}(\mathbf{X}_t^m)$ by ranking the distance $d(\mathbf{X}_t^m, \mathbf{X}_s^m)$ over the entire community,

$$\{\mathbf{X}_s^m \mid s < t\}, \tag{12.52}$$

from the closest to the farthest. Then, by assuming a functional relation, f, between x_{s+1} and $\mathbf{X}_s^m$ and using only the observations associated with $\mathcal{N}(\mathbf{X}_t^m)$ to estimate this function f_t,[12] one can construct the tailor-made forecast for each x_t,

$$\hat{x}_{t+1} = \hat{f}_t(\mathbf{X}_t^m). \tag{12.53}$$

In practice, the function f used in (12.53) can be very simple, either taking the *unconditional mean* or the *conditional mean*. In the case of the latter, the mean is usually assumed to be linear. In the case of the unconditional mean, one can simply use the simple average in the forecast,

$$\hat{x}_{t+1} = \frac{\sum_{s \in \mathcal{N}(\mathbf{X}_t^m)} x_{s+1}}{k}, \tag{12.54}$$

but one can also take the weighted average based on the distance of each member. The same idea can be applied to deal with the linear conditional mean (linear regression model): we can either take the ordinal least squares or the weighted least squares.[13]

From the above description, we can find that KNN is different from k-means and SOM in the sense that not just the forecasting function but also the cluster for KNN is tailor made. This style of tailor-made learning is known as *lazy learning* in the literature (Aha, 1997). It is called *lazy* because learning takes place when the time comes to classify a new instance, say $\mathbf{X}_{T+t}^m$, rather than when the *training set*, (12.43), is processed, say T.[14]

To make this clear, consider two types of agents: the k-means agent and the KNN agent. The k-means agent learns from the history before new instances come, and the resultant knowledge from learning is represented by a set of clusters, which is *extracted* from a set of historical instances. Based on these clusters, some *generalization pictures* are already produced before the advent of new instances, say $\mathbf{X}_{T+t}^m$. The KNN agent, however, is not eager to learn. While he does store every instance observed, he never tries to extract knowledge (general rules) from them. In other words, he has the simplest form of "learning," i.e., rote learning (plain memorization). When the time $T + t$ comes and a new instance $\mathbf{X}_{T+t}^m$ is encountered, his memory is then searched for the historical instances that most strongly resemble $\mathbf{X}_{T+t}^m$.

As said, KNN, as a style of rote learning, stores all the historical instances, as shown in (12.52). Therefore, amounts of storage increase with time. This may make the nearest-neighbor calculation unbearably slow. In addition, some instances may be regarded as redundant with regard to the information gained. This can be particularly the case when KNN is applied to *classification* rather than regression or time series forecasting. For example, if we are interested in not x_{t+1} itself, but in whether x_{t+1} will be greater than x_t, i.e., whether x_t will go up or go down, then some regions of the instance space may be very stable with regard to class, e.g., up (1) or down (0), and just a few exemplars are needed inside stable regions. In other words, we do not have to keep all historical instances or training instances. The *storage reduction algorithm* is then used to decide which instances in (12.52) to save and which to discard. This KNN with the storage reduction algorithm is IBL as initiated by Aha, Kibler, and Marc (1991).[15]

The addition of a storage reduction algorithm to KNN is also interesting from the perspectives of both neuroscience and economics. Considering the brain with its limited capacity for memory, an essential question to ask is how the brain deals with increasing information by not memorizing all of it or by forgetting some of it. How does it do pruning? This is still a nontrivial issue pursued by neural scientists today. The same issue can interest economists as well, because it concerns the efficient use of limited space. A recent study on reward-motivated memory formation by neural scientists may provide an economic foundation for memory formation (Adcock *et al.*, 2006).[16]

In this vein, the *marginal productivity* of the new instance in IBL can be considered as the reward. The marginal productivity of an instance can be defined by its contribution to enhance the capability to perform a correct classification. Those instances which have low marginal productivity will be discarded (not be remembered), and for those already stored instances, if their classification performances are poor, they too will be discarded (be forgotten). In this way, one can interpret the mechanism of the pruning algorithms or the storage reduction algorithms used in computational intelligence in the fashion of neural economics.

12.9 Finite state automata

To motivate the tool *finite state automata*, let us start from conventional time series modeling, and consider an m-history time series:

$$\mathbf{X}_t^m \equiv \{x_t, x_{t-1}, \ldots, x_{t-m}\}.$$

Suppose that we are interested in the forecasting of x_{t+1}; then conventional time series modeling would basically start by constructing a function, be it linear or nonlinear, like

$$f(x_t, x_{t-1}, \ldots, x_{t-m}).$$

This approach basically assumes that there exists a relationship between x_{t+1} and $\mathbf{X}_t^m$. Since the time series will repeat this relationship constantly, the pattern is defined in terms of f, and the task of data mining is to discover this f by having an estimation of it, $\hat{f}$, be it parametric or nonparametric. Moreover, the parameter m is usually exogenously given and is fixed.

Two questions arise from this modeling procedure. First, this approach implicitly assumes that if the parameter m is chosen appropriately, then

$$x_{t+1} \sim x_{t+j+1}, \quad \text{if} \quad \mathbf{X}_t^m = \mathbf{X}_{t+j}^m \quad \text{pointwise.} \tag{12.55}$$

"$\sim$" here could mean that the two variables share the same distribution. Or, putting it in a more broad way, it may indicate that

$$|x_{t+1} - x_{t+j+1}| < \delta, \quad \text{if} \quad |\mathbf{X}_t^m - \mathbf{X}_{t+j}^m| < \epsilon. \tag{12.56}$$

In plain English, it simply says that if $\mathbf{X}_t^m$ and $\mathbf{X}_{t+j}^m$ are close, then x_{t+1} and x_{t+j+1} should also be close.

The "ϵ–δ" argument can also be generalized with *categorization*: If $\mathbf{X}_t^m$ and $\mathbf{X}_{t+j}^m$ are *close* in the sense that they belong to *type A*, then x_{t+1} and x_{t+j+1} will also be close in the sense that they are related to $\mathbf{X}_t^m$ and $\mathbf{X}_{t+j}^m$ respectively by the same mapping f_A, where A is an index from a finite alphabet Λ, i.e. $A \in \Lambda$.[17] However, in conventional time series modeling, there exists only one global f. The existence of topological patterns, Λ, and their significance for forecasting, $\{f_A; A \in \Lambda\}$, are hence ignored.

12.9.1 Threshold models

A slight departure from the severe restriction is that one can find the famous *threshold* model, e.g., the threshold autoregressive (TAR) model. A general form would be

$$x_{t+1} = \begin{cases} f_A(\mathbf{X}_t^m) & \text{if } \mathbf{X}_t^m \in A, \\ f_{\bar{A}}(\mathbf{X}_t^m) & \text{if } \mathbf{X}_t^m \notin A. \end{cases} \tag{12.57}$$

In this case, there are two features in the set Λ, namely, A and its complement $\bar{A}$. A specific example of the TAR model is illustrated in Equation (12.58):

$$x_{t+1} = \begin{cases} \alpha_1 + \sum_{i=0}^m \beta_{1,i} x_{t-i} & \text{if } x_{t-k} \le \theta, \\ \alpha_2 + \sum_{i=0}^m \beta_{2,i} x_{t-i} & \text{if } x_{t-k} > \theta. \end{cases} \tag{12.58}$$

Notice that in Equation (12.58), when x_{t-k} is slightly above or below the threshold θ, there is a dramatic change in the corresponding law of motion for x_t. This discontinuity can be tackled by replacing the original dichotomous transition with a continuous smooth transition, as in Equation (12.57):

$$x_{t+1} = w_1(\mathbf{X}_t^m, A) f_A(\mathbf{X}_t^m) + w_2(\mathbf{X}_t^m, A) f_{\bar{A}}(\mathbf{X}_t^m). \tag{12.59}$$

Here, the function $w_1(\mathbf{X}_t^m, A)$ and $w_2(\mathbf{X}_t^m, A)$ is no longer a Heaviside function, being zero or one, but a continuous function with a range between 0 and 1. With appropriate normalization, Equation (12.59) can be regarded as a weighted average of the two regimes, f_A and $f_{\bar{A}}$, and the weight assigned to each regime depends on the strength of the membership of $\mathbf{X}_t^m$ in A. Hence, an alternative treatment for the weight function $w_1(\mathbf{X}_t^m, A)$ is to consider it as a membership function in fuzzy logic (Section 11.1). This smooth generalization of the TAR model is also known as the smooth transition threshold autoregressive (STAR) model in econometrics. An illustration of a generalization of the TAR model (12.58) is given in Equation (12.60), where a logistic equation is employed to serve as the smooth transition function or the membership function:

$$x_{t+1} = \frac{1}{1+e^{(x_{t-k}-\theta)}} \left(\alpha_1 + \sum_{i=0}^{m} \beta_{1,i} x_{t-i} \right) + \frac{1}{1+e^{-(x_{t-k}-\theta)}} \left(\alpha_2 + \sum_{i=0}^{m} \beta_{2,i} x_{t-i} \right). \tag{12.60}$$

While Equation (12.57) is an important step for moving forward, it is still very restrictive. First, the type of the feature A is not extracted from data, but is pre-specified. Second, the number of features is very small and also exogenously given.

Some recent progress in computational intelligence can be viewed as a series of efforts made toward automatic procedures to construct the feature set Λ and the mappings $\{f_A;\, A \in \Lambda\}$. Earlier, we already saw how this task could be done with self-organizing maps. However, there is a restriction: all features are constructed based on a fixed window size, i.e. a fixed number of observations. This restriction can have nontrivial effects because topological similarities usually do not depend on the window size of data.[18]

To have an approach that does not restrict the "relevant past" or the reaction time of the system to time windows of fixed lengths, a language called the *pattern description language* (PDL) was proposed to find all *frequent* subsequences, given a set of data sequences.[19] The work is an extension of an early study by Norman Packard (Packard, 1990), one of the founders of Prediction Company established in 1991 at Santa Fe, New Mexico. Those readers who have some background in the early development of financial applications of *genetic algorithms* or *genetic programming* to trading would agree that one basic difficulty for developing and evolving automated trading strategies is the *representation* and *coding* of the current state of the financial time series on which the trading decision is based.[20] Back to our previous discussion, it is about an effective language with rich expression power to deal with the complexity of the set Λ.

12.9.2 Regular languages

Using a pattern description language as a solution to this problem is motivated by the theoretical computer scientists, who define the patterns in terms of *regular languages*, specifically, *nondeterministic finite automata* (NFA). Abstractly, a finite state automaton (or a finite state machine) $\mathcal{M}$ is an ordered pair $(\mathcal{S}, \mathcal{A})$ of finite sets, with an action of $\mathcal{A}$ on $\mathcal{S}$. The elements of $\mathcal{S}$ are called the *states* of the automaton, $\mathcal{A}$ is called the *alphabet* of the automaton, and the elements of A are called *input symbols*. A transition table $\mathcal{T}$,

$$\mathcal{T} : \mathcal{S} \times \mathcal{A} \rightarrow \mathcal{S}, \tag{12.61}$$

associates with every $(s, a) \in \mathcal{S} \times \mathcal{A}$ a state $s' \in \mathcal{S}$. Therefore, the transition table $\mathcal{T}$ shows how the input s may change the state of the automaton from s to a state $s' = \mathcal{T}(s, a)$, also usually denoted just as sa.

If we allow ourselves to consider successive actions of inputs from $\mathcal{A}$ on states s then in effect we extend the input set to $\mathcal{A}^*$, which is the set of all finite strings formed by concatenating input symbols from $\mathcal{A}$. For example, if $\mathcal{A} = a, b, c$, then

$$\mathcal{A}^* = \{\aleph, a, b, c, aa, ab, ac, ba, bb, bc, ca, cb, cc, aaa, bbb, ccc, \ldots\},$$

where $\aleph$ is the empty string, i.e. the string of no letters.

To see what we can do with such a machine, we designate a certain state in S to be the *initial state*, and a subset Ξ which is the *final state*. The machine then works by being presented with an input string of symbols, say

$$w = a_1 a_2 \ldots a_n,$$

that it reads one by one starting at the left-most symbol. Beginning at the initial state, s_0, the symbols determine a sequence of states,

$$s_0 a_1, (s_0 a_1) a_2, \ldots, (s_0 a_1 a_2 \ldots a_{n-1}) a_n.$$

The sequence ends when the last input symbol has been read. Furthermore, if the ending state is

$$(s_0 a_1 a_2 \ldots a_{n-1}) a_n \in \Xi,$$

we say that the automaton *recognizes* (or *accepts*) the string w. The *behavior* of $\mathcal{M}$ (or the *language* of $\mathcal{M}$) is defined as the set of strings in $\mathcal{A}^*$ that can be recognized by $\mathcal{M}$. The famous *Kleene's theorem* in computation theory shows that every language that can be defined by a *regular expression* can be accepted by some finite state automata.

In 1959, Michael Oser Rabin and Dana Scott provided a more general definition of an automaton by introducing the following notion of *nondeterminism* for language-recognizing finite automata:

$$\mathcal{T} : \mathcal{S} \times \mathcal{A} \rightarrow \mathrm{P}(\mathcal{A}), \tag{12.62}$$

where $\mathrm{P}(\mathcal{A})$ is the set of all subsets of $\mathcal{A}$. With Equation (12.62), $\mathcal{T}(s, a)$ may contain more than one element of $\mathcal{A}$; therefore, the ultimate path through the machine is not determined by the input symbol alone. Human choice becomes a factor in selecting the transition path; the machine does not make all its own determinations. That is why we say this machine is nondeterministic. A string is accepted by the machine if *at least one* sequence of choices leads to a path that ends at a final state. It is important to know that nondeterminism does not increase the power of a finite state automaton (FSA). In fact, an *equal power* theorem says that any language definable by a nondeterministic finite automaton is also definable by a deterministic finite automaton, and vice versa.

The use of the finite automata in economics has a long history (Kramer, 1968; Futia, 1977; Rubinstein, 1986). In economics, the finite automaton was first applied as a formalization of rational economic agents.[21]

> [A] decision-maker will be considered as some sort of finite information-processing device, or *automaton*. A formal theory of such devices—the theory of finite automata—will then be used to explore the question of whether such a device is capable of behaving rationally.
>
> (Kramer, 1968, p. 41; emphasis original)

The finite automaton formalization was then proved to be useful in game theory.[22] As Rubinstein states: "In the absence of a more established tool to model a decision maker I believe Moore's machine to be a reasonable tool for formalizing a player's behavior in a supergame" (Rubinstein, 1986, p. 84).

12.10 Decision trees

Decision trees can be studied from two aspects: one is from machine learning (computer science), and the other is from decision marking (psychology). In this section, we shall start with the first aspect, and then see how it can be examined from the other angle.

The decision tree has become a canonical tool in machine learning. It is a *classification* procedure with a tree-like graph structure (Figure 12.4). The data S $(= \{x_i\}_{i=1}^{n})$ presented to the decision tree is of a common type, namely, m *attributes* and p *decision classes*. Each attribute A_j $(j = 1, \ldots, m)$ partitions the n inputs into s_j distinct classes based on the attribute value,

$$A_j : S \rightarrow (a_{j,1}, a_{j,2}, \ldots, a_{j,s_j}). \tag{12.63}$$

When the input x_i is presented to the tree, at each node of the tree a decision is made based on a test on the value of an *attribute*, a_j. According to the result of the test, the interpretation of the tree proceeds to one of the subtrees of the node. The path will continue leading x_i to the next test until it goes through all of them, and hence reaches a leaf of the tree. It is expected that all paths of the decision tree will inform us better on how different decisions are made.[23]

A decision tree is constructed based on a *top-down greedy algorithm*, known as ID3 in machine learning (Quinlan, 1986). The key idea is fairly straightforward. One first finds the attribute that *best* classifies the training data, and then uses this attribute as the *root* of the decision tree. Then the process is repeated for each subtree. The main issue involved in this greedy algorithm is the criterion regarding

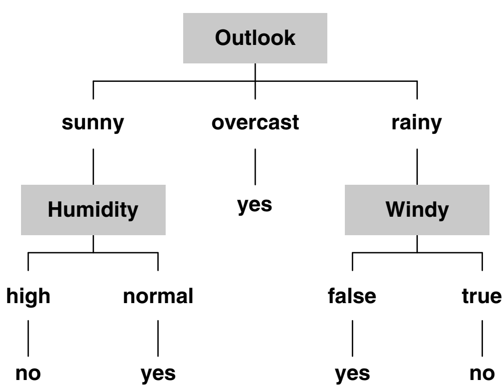

Figure 12.4 The decision tree for *Play Tennis*.

the choice of the best classifying attribute. A common solution to this problem is to select the attribute with the *highest information gain*, $G(S, A)$, which is defined as the expected reduction in the *entropy* of the dataset S caused by knowing the value of the attribute A.

A decision tree can be converted to *if–then* rules. The decision tree shown in Figure 12.4 can be represented as a decision rule as follows:

IF ([(Outlook = sunny) AND (Humidity = normal)] OR
[(Outlook = overcast)] OR
[(Outlook = rain) AND (Wind = weak)])
THEN YES (Play Tennis)

An important issue pertaining to growing decision trees is *when to stop*. Would it be desirable to grow the decision tree until it perfectly matches the data? To avoid *overfitting*, the answer is generally no. However, in practice, the greedy algorithm will grow the full tree first, for it to be pruned later. There are two different types of pruning. The first one is to prune the tree directly, known as *reduced error pruning* (Quinlan, 1987). The second is first to convert the tree into rules, and then to prune (generalize) each rule independently by removing preconditions that increase classification accuracy. This can be done by the famous C4.5 algorithm (Quinlan, 1993). In addition to pruning, one can also use a complexity measure such as the *minimum description length* (MDL) to halt tree growth when the MDL is found.

In contrast to artificial neural nets, decision trees, among all CI tools, are the least exploited, so to speak, in economics and finance.[24] Publications on the economic and financial applications of decision trees are nearly nonexistent. This is indeed a surprise to us, in particular considering that learning decision rules is no more difficult than many of the alternatives, e.g., rough sets.

Chen and Kuo (2003) introduce a feature-based (rule-based) time series model based on the algorithm *Cubist*. Cubist provides an approximation to a nonlinear time series by partitioning the series into many subsets, and then constructing a linear regression model over each subset. By converting the decision tree into rules, one can have the time series model represented by Equation (12.64):

$$x_{t+1} = \begin{cases} f_1(X_t^m) & \text{if } X_t^m \in A_1, \\ f_2(X_t^m) & \text{if } X_t^m \in A_2, \\ \vdots & \\ f_n(X_t^m) & \text{if } X_t^m \in A_n. \end{cases} \tag{12.64}$$

This way of representing time series can be related to many other approaches introduced in this part of the book. First, it is a generalization of the threshold regression model (see Section 12.9). What makes Cubist different from the threshold regression model is that the number of thresholds or gates is not fixed, and the functional form (the variables included) can change from one local model to another local model. This certainly endows Cubist with greater powers of expression than the threshold regression model. Secondly, Cubist can be used for comparisons with other feature-based regression models whose features are discovered by other tools, such as finite state automata, context-free language, self-organizing maps, and many others as surveyed in Armano, Murru, and Marchesi (2002). Finally, the classification does not have to be crisp, it can be fuzzy as well, and that leads us to the intention of the fuzzy classification and regression model methodology (Takagi and Sugeno, 1985; Sugeno and Yasukawa, 1993). This idea can be further enriched by the fusion of the evolutionary algorithms, and extended into the well-known *genetic fuzzy classifier* model (Cordon *et al.*, 2001). Tay and Linn (2001) is one of the earliest financial applications of genetic fuzzy classifiers.

12.10.1 Fast-and-frugal heuristics

The decision tree introduced above is also related to *fast-and-frugal heuristics* as advocated by the behavioral economist Gerd Gigerenzer (Gigerenzer and Todd, 1999; Gigerenzer, 2007). In this regard, the decision tree endows decision-making with a specific structure having four distinguishing features.

First, it explicitly reveals the hierarchical structure of a decision problem. Factors affecting the decision problems are ranked, starting from the most influential ones to the marginal ones. Second, the preference of the decision-maker associated with this hierarchy is clearly *lexicographical*. Agents will then only examine one factor at a time, and will then proceed to the next factor only if the decision cannot be made by the previous factor; otherwise, the following factors play no role in this decision-making. Because of this sequential structure, this second feature is also known as *lexicographic heuristics* and *noncompensatory heuristics*. Third, the stopping rule for the decision, in particular with regards to information retrieval, is then endogenously built in. Fourth, the time required for decision-making is finite and can, in effect, be substantially short (in one shot).

Gigerenzer provides many illustrations of fast-and-frugal heuristics; some notable examples are the recognition heuristic, the take-the-best heuristic, and the one-good-reason heuristic. These heuristics are easy to implement; they enable agents to make decisions with minimal search under the exposure to a massive information environment. Many studies have shown that these heuristics are, however, rather effective when compared to some sophisticated decision rules. Nonetheless, the research on under what situations agents are actually users of fast-and-frugal heuristics and whether these heuristics are actually outperforming other alternatives is still ongoing.

12.11 Further study

Bibliographic note: inductive reasoning by analogies

In economics, k nearest neighbors or instance-based learning are also known in the form of case-based decisions, due to the series of work done by Itzhak Gilboa and David Schmeidler (Gilboa and Schmeidler, 1995, 2001), who distinguish three kinds of decision-making under uncertainty: expected utility maximization under the Bayesian expectation (probabilistic and statistical reasoning); rule-based deduction; and inductive reasoning by analogy. They argued that there are many formal models for the first two kinds, but none for the last. Hence, their purpose is to fill this gap, i.e., to find formal models for inductive reasoning by analogy.

Research question: inductive reasoning by analogies

David Hume, in his book *An Enquiry Concerning Human Understanding*, has the following remarks on *experience* and *similarity*:

> In reality all arguments from *experience* are founded on the *similarity* which we discover among natural objects, and by which we are induced to expect effects similar to those which we have found to follow from such objects. And though none but a fool or madman will ever pretend to dispute the authority of experience, or to reject that great guide of human life, it may surely be allowed a philosopher to have so much curiosity at least as to examine the principle of human nature, which gives this mighty authority to experience, and makes us draw advantage from that similarity which nature has placed among different objects. *From causes which appear similar we expect similar effects*. This is the sum of all our experimental conclusions.
>
> (Ibid., Section IV; emphasis added)

Experience and similarity are the pivotal concepts in many learning algorithms which we surveyed here. Briefly discuss how Hume's remark is operated in each of the following computational intelligence tools, i.e., how the idea of *inductive reasoning by analogy* is implemented computationally. You don't have to give the technical details of each algorithm; instead, you should focus on how each of them operates the idea of similarity and how *learning with experience* can happen in terms of *similarity*, or *inductive reasoning* can proceed in terms of *analogies*.

1 K nearest neighbors
2 Self-organizing maps
3 K-means clustering
4 Decision tress
5 Reinforcement learning.

Research question

Alan Greenspan once stated:

> [H]ow ... the economy might respond to a monetary policy initiative may need to be drawn from evidence about past behavior during a period only roughly comparable to the current situation.
>
> (Greenspan, 2004, p. 38)

Discuss the possibility or the limitation of applying instance-based learning to Greenspan's statement above.

Research question: k nearest neighbors and artificial financial agents

Chan *et al.* (1999) applied the k nearest neighbors algorithm to agent-based financial markets. Briefly describe how case-based reasoning has been applied to the agent-based financial market, and what the major finding of that study is (see also Section 15.4.1).

Notes

1 This is not a good place to provide a long list, but interested readers can find some examples from Weigend (1992), Wei (1995), Wu (1995), Episopos and Davis (1996), Hann and Steurer (1996), Shi, Xu, and Liu (1999), and Alvarez-Diaz and Alvares (2005).

2 There are also other names for the local modeling approach. For example, another popular name in the literature is *guarded experts*—see Armano, Murru, and Marchesi (2002).

3 Some early applications can be found in Kuan and Liu (1995) and Chen and Xu (1998).

4 See Kramer (1990) for a brief review.

5 The financial applications have kept on expanding, and the interested reader can find some useful references directly from the SVM website: http://www.svms.org/.

6 Standard Euclidean distance assumes that the attributes are normalized and are of equal importance. However, this assumption may not hold in many application domains. In fact, one of the main problems in learning is to determine which are the important features.

7 The rigorous mathematical treatment of the SOM algorithm is extremely difficult in general; see Kohonen (1995).

8 For details, see Chen and Wang (2003), Chapter 8, p. 205.

9 For example, see the charts presented in (Chen and He, 2003, pp. 206–7).

10 For details of the competition report, see Suykens and Vandewalle (1998).

11 The notation $\hat{f}$ is used, instead of f, to reserve f for the true relation, if it exists, and in that case, $\hat{f}$ is the estimation of f. In addition, there are variations when constructing (12.50). See Chen and He (2003).

12 Even though the functional form is the same, the coefficients can vary depending on $\mathbf{X}_t^m$ and its resultant $\mathcal{N}(\mathbf{X}_t^m)$. So, we add a subscript t as f_t to make this time-variant property clear.

13 All details can be found in Fernández-Rodríguez, Sosvilla-Rivero, and Andrada-Félix (2003).

14 Note that a fixed T in (12.43) implies a fixed training set without increments. A non-incremental training set can be typical for using k-means or SOM. However, KNN learning, also known as *rote learning*, memorizes everything that happens up to the present; therefore, the "training set" (memory) for KNN grows with time.

15 As a matter of fact, the storage reduction algorithms are not just to deal with the *redundancy* issue, but also the *noise tolerance* issue. Aha, Kibler, and Marc (1991) distinguish the two by calling the former *memory updating functions* and the latter *noise-tolerant algorithms*.

16 Adcock *et al.* (2006) report brain-scanning studies in humans that reveal how specific *reward-related brain regions* trigger the brain's learning and memory regions to promote memory formation.

17 The "ϵ–δ" argument can be fuzzified: if $\mathbf{X}_t^m$ and $\mathbf{X}_{t+j}^m$ are *close*, then $\hat{x}_{t+1}$ and $\hat{x}_{t+j+1}$ will also be *close*.

18 Consider time series data with self-similarity.
19 For example, Polanski (2003) applied PDL as a language to describe financial patterns. He gives a few examples of some useful patterns. In particular, a popular pattern frequently used by chartists, namely the *golden ratio rule* (Fischer, 2001), is expressed in terms of regular language (Section 12.9.2), and the finite state machine is implemented to search for this pattern (to recognize this pattern) in financial time series. He used the NYSE index as an illustrative example, and the pattern was found to be there several times by *golden ratio matches* with lengths ranging between 6 observations and 19 observations. From this example, it is crucial to observe that *the same pattern may have different lengths, and fixing window size can then become a serious restriction*.
20 For a historical review of the progress of this work, see Chen (2002b).
21 See also (Velupillai, 2000, pp. 21–2).
22 See Fogel, Chellapilla, and Angeline (2002), pp. 256–7 for a concrete example. Also, see Marks (2002) for a brief review of the applications of finite automata to game theory.
23 In an ideal case, all inputs reaching the same branch belong to the same decision class. Of course, this is not necessarily so.
24 One of the earliest applications is Messier and Hansen (1988).

13 Evolutionary computation

13.1 Tools for evolutionary economics

In this chapter, our purpose is to give a general idea of evolutionary computation, particularly with a sharp focus on genetic programming, because we believe that GP is a powerful tool to make agent-based models capable of demonstrating essential features of evolutionary economics. Of course, having said that, we are aware of the ongoing debates on what the essential features of evolutionary economics really are and how they differ from neoclassical economics in a nontrivial way.[1] Nonetheless, there is a fundamental vision which, we believe, is largely shared by economists who take evolution seriously. That is, *constantly changing* (more will be discussed in Part VIII). While different economists may have different ways to handle constant change in their models, we consider autonomous agents or novelty-discovering agents as key. In Chapter 8, examples of this kind of agent were already introduced to agent-based modeling. There, we have seen the application of genetic programming to modeling the constant search for better trading strategies.

The third important pillar of computational intelligence is so-called *evolutionary computation* (EC). EC uses *nature* as an inspiration. While it also has a long history of utilization in economics and finance, it is, relatively speaking, a new kid on the block, as compared with neural networks, and even more so as compared to fuzzy logic. It has also drawn less attention from economists and financial analysts than the other two approaches. To gauge the comparison, there are already about a dozen books or volumes on economic and financial applications using fuzzy logic and neural nets. In the area of EC, there are a number of volumes edited for economists and financiers (Bauer, 1994; Chen, 2002a, b). Evolutionary computation is generally considered to be a consortium of *genetic algorithms* (GA), *genetic programming* (GP), *evolutionary programming* (EP) and *evolutionary strategies* (ES).

The history of evolutionary computation can be traced back to the mid 1960s, with evolutionary strategies originated by Ingo Rechenberg (Rechenberg, 1965), Hans-Paul Schwefel (Schwefel, 1965), and Peter Bienert at the Technical University of Berlin, the development of genetic algorithms by John Holland of the University of Michigan, and evolutionary programming originated by Lawrence

```
begin
  t :=0;
  Initialize P(t);
  evaluate P(t);
  while not terminating do
  begin
    M(t) :=select-mates (P(t));
    O(t) :=alternation (M(t));
    evaluate (O(t));
    P(t+1):=select(O(t)∪P(t));
    t :=t+1;
  end
end
```

Figure 13.1 A pseudo-program of evolutionary computation.

Fogel (Fogel, 1964) at the University of California at Los Angeles.[2] Despite their nontrivial differences, they share a common structure, as shown in Figure 13.1.

Evolutionary computation starts with an initialization of a population of individuals (solution candidates), called $P(0)$, with a *population size* to be supplied by the users. These solutions will then be evaluated based on an *objective function* or a *fitness function* determined by the problem we encounter. The continuation of the procedure will hinge on the *termination criteria* supplied by the users. If these criteria are not met, then we shall move to the next stage or *generation* by adding 1 to the time counter, say from t to $t + 1$. Two major operators are invoked to form the new generation, which can be regarded as a *correspondence* as follows:

$$F_{s_2} \circ F_a \circ F_{s_1}(P(t)) = P(t + 1), \tag{13.1}$$

where F_{s_1} and F_{s_2} denote *selection*, and F_a denotes *alteration*. The main purpose of the first-stage selection, F_{s_1}, is to make a mating pool (a collection of parents), $M(t)$, which can in turn be used to breed the new generation:

$$F_{s_1}(P(t)) = M(t). \tag{13.2}$$

Once the mating pool is formed, F_a is applied to generate *offspring*, $O(t)$, from these parents. Two major steps (*genetic operators*) are involved here, namely, *recombination* (*crossover*), denoted by F_r, and *mutation*, denoted by F_m, which shall be detailed later.

$$F_a(M(t)) = F_m \circ F_r(M(t)) = O(t). \tag{13.3}$$

These offspring will be evaluated first, and they then enter the second-stage selection *with* or *without* their parents $P(t)$. Finally, the new generation $P(t + 1)$ is formed as a result of the second-stage selection:

$$F_{s_2}(O(t) \cup P(t)) = P((t + 1)). \tag{13.4}$$

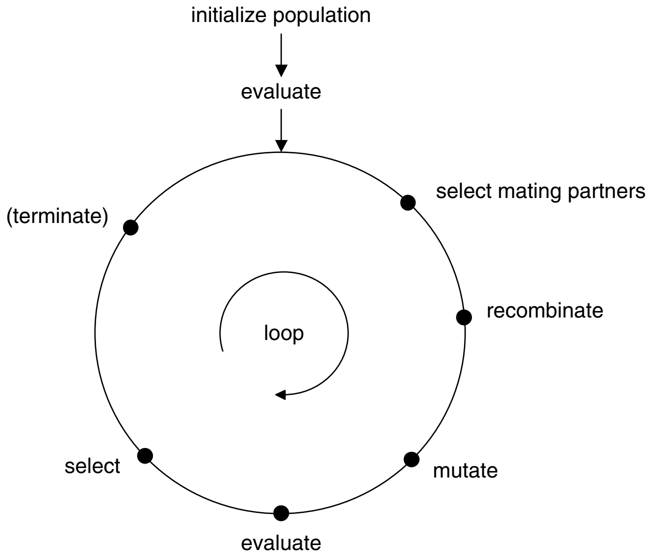

Figure 13.2 The evolutionary loop.

After that, we go back to the beginning of the loop, and then check the termination criteria to see whether we shall stop or start another generation of runs. See Figure 13.2 for the evolution loop.

Based on the description above, it is perhaps beneficial to have the seven major components of evolutionary algorithms listed for quick reference:

- individuals and their representations;
- initialization;
- fitness evaluation;
- selection;
- mutation;
- recombination;
- replacement.

13.2 Evolutionary strategies

We shall illustrate each of these components mainly within the context of *evolutionary strategies*. Individuals are also called *chromosomes*. The individual in ES is represented as a pair of real-valued vectors $v = (x, \sigma)$, where x represents a point in the solution space, and σ is a standard deviation vector that determines the mutation step size. Generally, σ is also called the *strategy parameter* in ES, and x is called the *object variable*.

The population size of ES is usually characterized by two parameters μ and λ. The former is the population size of $P(t)$, whereas the later is the population size of $O(t)$. Selection F_{s_1} is much more straightforward in ES than in GA. Usually, it takes the whole $P(t)$ as the mating pool and parents are randomly selected from this mating pool. However, selection F_{s_2} in ES can be more intriguing. There are two schemes for F_{s_2} in ES, known as the $(\mu+\lambda)$ scheme (the Plus scheme) and the (μ, λ) scheme (the Comma scheme). In the $(\mu+\lambda)$ scheme, μ individuals produce λ offspring, and a new population is formed by selecting μ individuals from the $\mu+\lambda$ individuals. In the (μ, λ) scheme, μ individuals produce λ offspring, and a new population is formed by selecting μ individuals from the λ offspring. There is generally no constraint for μ and λ for the $(\mu+\lambda)$ scheme, but for the (μ, λ) scheme, to make selection meaningful, μ has to be strictly less than λ; moreover, $\lambda/\mu \approx 7$ is an ideal ratio.

Mutation is considered the major ES operator for altering the chromosomes. Mutation is applied to each individual to perturb real-valued parameters. If we let v be the parent randomly selected from $P(t)$, then mutation on v can be described as follows:

$$v' = (x', \sigma') = (f_{m_x}(x), f_{m_\sigma}(\sigma)), \tag{13.5}$$

where

$$f_{m_x}(x) = x + N(0, (\sigma')^2), \tag{13.6}$$

and

$$f_{m_\sigma}(\sigma) = \sigma \exp(\tau N(0, 1)). \tag{13.7}$$

$N(0, \sigma^2)$ denotes the normal distribution with mean 0 and variance σ^2.[3] Notice that in implementation, Equaution (13.7) has to be computed before Equation (13.6). This is because x' is obtained by mutating x with the new standard deviation σ'.[4]

Recombination operators compose new chromosomes from corresponding parts of two or more chromosomes. For the binary case, two chromosomes $v_1 = (x_1, \sigma_1^2)$ and $v_2 = (x_2, \sigma_2^2)$ are to be recombined by an operator f_r. We can describe the composition of a new chromosome v' as follows:

$$v' = (x', \sigma') = (f_{r_x}(x_1, x_2), f_{r_\sigma}(\sigma_1^2, \sigma_2^2,)). \tag{13.8}$$

Each element of the object and strategy parameter is a recombination of the respective entries of v_1 and v_2. There are great varieties of f_{r_x} and f_{r_σ}. In the ES literature, they are differentiated by the terms *discrete* or *intermediate*, *dual* (sexual) or *global* (panmictic). With a *discrete* recombination function, one of the corresponding components is chosen at random and declared the new entry.

With an intermediate recombination, a linear combination of the corresponding components is declared the new entry. More formally, consider x' as an example:

$$x' = \begin{cases} x_1 \text{ or } x_2 & \text{discrete,} \\ \chi x_1 + (1-\chi)x_2 & \text{intermediate,} \end{cases} \tag{13.9}$$

where $\chi \in [0, 1]$ denotes a uniform random variable. So far we have only considered one-dimensional x. An n-dimensional x can further complicate the recombination function, and that is where the terms *dual* and *global* come from. *Dual* means that two parents are chosen at random for the creation of the offspring. *Global* means that one parent is chosen anew for *each component* of the offspring:

$$x_i' = \begin{cases} x_{1,i} \text{ or } x_{2,i} & \text{discrete, dual,} \\ x_{1,i} \text{ or } x_{(2),i} & \text{discrete, global,} \\ \chi x_{1,i} + (1-\chi)x_{2,i} & \text{intermediate, dual,} \\ \chi x_{1,i} + (1-\chi)x_{(2),i} & \text{intermediate, global,} \end{cases} \tag{13.10}$$

where $x_{(2),i}$ indicates that parent 2 is chosen anew for each vector component i, $(i = 1, 2, \ldots, n)$.

13.3 Evolutionary programming

While EP was proposed about the same time as evolutionary algorithms, their initial motives were quite different. Evolutionary strategies were developed as a method to solve *parametric optimization problems*, whereas evolutionary programming was developed as a method for simulated *intelligence behavior*. Lacking a capability to predict, an agent cannot adapt its behavior to meet the desired goals, and success in predicting an environment is a prerequisite for intelligent behavior:

> Intelligent behavior is a composite ability to predict one's environment coupled with a translation of each prediction into a suitable response in the light of some objective.
>
> (Fogel, Owens, and Walsh, 1966, p. 11)

During the early stage, the prediction experiment can be illustrated with a sequence of symbols taken from a finite alphabet, say a repeating sequence "(101110011101)*" from the alphabet {0, 1}. The task then is to create an algorithm that would operate on the observed indexed set of symbols and produce an output symbol that agrees with the next symbol to emerge from the environment. Lawrence Fogel took *finite state automata* as the machine to predict the sequence. A finite state automaton is a device which begins in one state and, upon receiving an input symbol, changes to another state according to its current state and the input symbol. EP was first proposed to evolve a population of finite state machines that provides successively better predictions.

13.4 Genetic programming and genetic algorithms

13.4.1 Thinking GP with context-free grammars

Consider that the population at time t, $P(t)$, is a collection of decision rules, behavioral rules, or, more generally, economic entities. Furthermore, suppose that each of these rules or entities (finite objects), in spirit of *Kolmogorov complexity* (Li and Vitanyi, 2008), can be represented by a string (word), which is a finite sequence of symbols (alphabets). In this situation, they can be studied in the context of *formal language theory*. Hence, using the *Chomsky hierarchy* of languages, we assume that they (these words) are from *Type 2*, namely, *context-free language*. Then, by the grammar of the context-free language, we know all the growth processes (the development processes) of each word, i.e., how a word grew out of a single *start* symbol by following the grammar defining the language.

This developmental process is the application of one of the production rules at one time to a single alphabet of the "developing" string. This developmental process can be illustrated using the *Backus–Naur Form* (BNF).[5] If, at any stage of this developmental process, we change one of the production outcomes from a nondeterministic production rule, then we may end up with a different string. Genetic programming can be defined as the use of the evolutionary computation paradigm in the alteration of the population of strings from one period to the next period, i.e., from $P(t)$ to $P(t+1)$. Briefly, genetic programming is to apply the evolutionary operator to a context-free language so that different strings are connected with each other as outcomes from of this evolutionary process.

While genetic programming has been applied to economic modeling for more than half a decade, its relevance to the nature of economics has not been fully acknowledged. In the most sympathetic situations, it is regarded as nothing but *alchemy*. In an unsympathetic situation, it is notorious for its *black-box* operation. Sometimes, the process and results are so complicated that economists can hardly consider it relevant or interesting. This section is intended to deliver a simple but strong message: *genetic programming is not just another fancy technique exploited by the unorthodox, but could be a faithful language to express the essence of economics*. In particular, it provides evolutionary economists with a way to *substantiate* some features which distinguish them from the mainstream economists.

13.4.2 An evolving population of decision rules

Let's start from the most fundamental issue: *why is genetic programming relevant?* Lucas (1986) provided the notion of an economic agent:

> In general terms, we view or model an individual as *a collection of decision rules* (rules that dictate the action to be taken in given situations) and *a set of preferences* used to evaluate the outcomes arising from particular situation–action combinations.
>
> (Lucas, 1986, p. 217; emphasis added)

Immediately after the *static description* of the economic agent, Lucas continued to add an *adaptive* (*evolutionary*) version of it.

> These decision rules are continuously under review and revision: new decision rules are tried and tested against experience, and rules that produce desirable outcomes supplant those that do not.
>
> (Ibid., p. 217).

So, according to Lucas, the essence of an economic agent is *a collection of decision rules which are adapting (evolving) based on a set of preferences*. In brief, it is the idea of an *evolving population*.

Suppose that an evolving population is the essence of the economic agent; then it seems important to know whether we economists know any operational procedure to substantiate this essence. Back in 1986, the answer was absolutely *no*. That certainly does not mean that we did not know anything about evolving one *decision rule*. On the contrary, since the late 1970s, the literature known as bounded rationality in macroeconomics has introduced a number of techniques to evolve a single decision rule (a single equation or a single system of equations): recursive regression, Kalman filtering, and Bayesian updating, to name a few. Sargent (1993) made an extensive survey of this subject. However, these techniques shed little light on how to build a Lucasian agent, especially since what we wanted to evolve was not a single decision rule but a population of decision rules.

In fact, it may sound a little surprising that economists in those days rarely considered an individual as a population of decision rules, not to mention attending to the details of its evolution. Therefore, all the basic issues pertaining to models of the evolving population received little, if any, attention. For example, how does the agent *initialize* a population of decision rules? Once the agent has a population of decision rules, which one should they follow? Furthermore, in what ways should this population of decision rules "be continuously under review and revision?" Should we review and revise them one by one because they are independent, or modify them together because they may correlate with each other? Moreover, if there are some "new decision rules to be tried," how do we generate (or find) these new rules? What are the relations between these new rules and the old ones? Finally, it is also not clear how "rules that produce desirable outcomes should supplant those that do not."

There is one way to explain why economists are not interested in, and hence not good at, dealing with a population of decision rules: economists used to derive the decision rule for the agent *deductively*, and the deductive approach usually leads to only one solution (decision rule), which is the *optimal* one. There was simply no need for a population of decision rules.

13.4.3 Genetic algorithms and classifier systems

We do not know exactly when or how the idea of the *evolving population of decision rules* began to attract economists, but John Holland's contribution to *genetic* *algorithms* definitely exerted a great influence. Genetic algorithms simulate the biological evolution of a society of computer programs, each of which is represented by a chromosome or, normally, a string of binary ones and zeros. Each of these computer programs can be matched to a solution to a problem. This structure provides us with an operational procedure for the Lucasian agent. First, a collection of decision rules are now represented by a society of computer programs (a society of strings of binary ones and zeros). Second, the review and revision process is implemented as a process of natural selection.

While Holland's genetic algorithms have had a great impact on computer science, mathematics, and engineering since the early 1980s, their implications for social sciences were not acknowledged until the late 1980s. In 1987, Robert Axelrod, a political scientist at the University of Michigan, published the first application of GA to the social sciences. A year later, the first PhD dissertation that applied GA to social sciences was completed by John Miller from, not surprisingly, the University of Michigan. The issue addressed by Axelrod and Miller is the well-known *repeated prisoner's dilemma*. In addition to these two early publications, perhaps the most notable event that brought GAs into economics was the invited speech by Holland at an economic conference at the Santa Fe Institute in the autumn of 1987. Among the audience were some of the most prestigious contemporary economists, including Kenneth Arrow, Thomas Sargent, Hollis Chenery, Jose Scheinkman, and Brian Arthur. In his lecture entitled "The global economy as an adaptive process," Holland introduced to the economics circle the essence of genetic algorithms: *building blocks*.

A building block refers to the specific pattern of a chromosome, i.e., an essential characteristic of a decision rule. There is a formal word for this in genetic algorithms: it is called a *schema*. In a genetic algorithm, a schema is regarded as the basic unit of learning, evolution, and adaptation. Each decision rule can be defined as a combination of some schemata. The review and revision process of decision rules is nothing more than a search for the right combination of those, possibly infinite, schemata. To rephrase Lucas's description in Holland's words, *economic agents are constantly revising and rearranging their building blocks as they gain experience*. Not only do genetic algorithms make the Lucasian economic agent implementable, but they also enrich its details.

After a gradual spread and accumulation of knowledge about GA among economists, modeling economic agents with an evolving population of decision rules finally began to increase in the 1990s. To the best of my knowledge, the first cited journal article is Marimon, McGrattan, and Sargent (1990). This paper details follow-up research to that done by Kiyotaki and Wright (1989). In a simple barter economy, Kiyotaki and Wright found that low storage costs *are not* the only reason why individuals use money. The other one is that money makes it easier to find a suitable partner. Replacing the rational agents in the Kiyotaki–Wright environment with *artificially intelligent* agents, Marimon *et al.*, however, found that goods with low storage costs play the dominant role as a medium of exchange.

The population of decision rules used to model each agent is a *classifier system*, another contribution made by Holland in the late 1970s. A classifier system is similar to a Newell–Simon-type expert system (Newell and Simon, 1972), which is a population of *if–then* or *condition–action* rules. However, the classical expert system is not adaptive. What Holland did with the classifier system was to apply the idea of competition in the market economy to a society of if–then rules. To implement market-like competition, a formal algorithm known as the *bucket brigade algorithm* credits rules generating good outcomes and debits rules generating bad outcomes. This accounting system is further used to resolve conflicts among rules. The shortcoming of the classifier system is that it cannot automatically generate or delete rules. Nonetheless, by adding the genetic algorithm on top of the bucket brigade and the rule-based system, one can come up with something similar to the Lucasian agent, which not only learns from experience, but can be *spontaneous* and *creative*.

While Holland's version of the adaptive agent is much richer and more implementable than the Lucasian economic agent, and the work was already completed before the publication of Holland's second book (Holland, Holyoak, and Nisbett, 1986), its formal introduction to economists came five years after the publication of Lucas (1986). In 1991, Holland and Miller published a sketch of the *artificial adaptive agent* in the highly influential journal *American Economic Review*. The first technique for implementing the Lucasian economic agent was finally "registered" in economic science, and genetic algorithms and classifier systems were formally added to the toolkit of economic analysis. Is five years too long? *Maybe not*, given that "[e]conomic analysis has largely avoided questions about the way in which economic agents make choices when confronted by a perpetually novel and evolving world" (Holland and Miller, 1991, p. 365).

What is next? If the Lucasian economic agent is a desirable incarnation of the economic agent in economic theory, and if Holland's artificial adaptive agent is indeed an effective implementation of it, then the follow-up research can proceed in three directions: first, the novel applications of this new technology; second, the theoretical justifications of the new technology; and finally, its technical improvement. That is exactly what we experienced during the 1990s.

For the first line of research, Jasmina Arifovic, a student of Sargent's, finished the first PhD dissertation that applied GAs to macroeconomics in 1991. It was not until 1994, however, that she published her work as a journal article. Arifovic (1994) replaced the rational representative firm in the cobweb model with Holland's adaptive firms, and demonstrated how the adaptation of firms, driven by market forces (natural selection), collectively make the market price converge to the rational-expectations equilibrium price. Since then, a series of her papers has been published in various journals with a range of new application areas, including inflation (Arifovic, 1995), exchange rates (Arifovic, 1996) and coordination games (Arifovic and Eaton, 1995).

13.4.4 SFI economics

Although Holland introduced this powerful toolkit to economists, he did not conduct any economic research with this toolkit himself, except for a joint work with

Brian Arthur. Holland and Arthur met in September 1987 at a physics and economics workshop hosted by the Santa Fe Institute. They had a great conversation on the nature of economics. The *chess* analogy proposed by Arthur led Holland to believe that the real problem with economics is, "how do we make a science out of imperfectly smart agents exploring their way into an essentially infinite space of possibilities?" (Waldrop, 1992, p. 151). On the other hand, Arthur was impressed by Holland's approach to complex adaptive systems. Holland's ideas of adaptation, emergence, and perpetual novelty, along with other notions, offered illuminating revelations to Arthur, insights he could never have gained if he had confined himself to theorizing on equilibria.

This new vision of economics turned out to be the approach of the Santa Fe Institute when it established its economics program in 1988. The essence of SFI economics was well documented by Arthur (1992). Instead of explaining genetic algorithms and classifier systems, which Holland and Miller (1991) had already done, this paper puts a great emphasis on motivation. Arthur eloquently argued why the deductive approach should give way to the inductive approach when we are dealing with a model of heterogeneous agents. His paper thus built the microfoundation of economics upon agents' cognitive processes such as pattern recognition, concept formation, and hypothesis formulation and refutation. Arthur then showed how the dynamics of these cognitive processes can be amenable to analysis with Holland's toolkit.

Maybe the best project to exemplify the SFI approach to economics is the *artificial stock market*. This research project started in 1988. Despite progress made in 1989, journal articles documenting this research were not available until 1994. Palmer *et al.* (1994) first built their stock market from a standard asset pricing model (Grossman and Stiglitz, 1980). They then replaced the rational representative agent in the model with Holland's artificial adaptive agents, and then simulated the market.

For Arthur, the relevance of genetic algorithms to economics is much more than just strengthening the rational expectations equilibrium. He would like to see how one can use this tool to simulate the evolution of a real economy, such as the emergence of barter trading, money, a central bank, labor unions, and even Communists. However, he understood that one should start with a more modest problem than building a whole artificial economy, and this led to *the artificial stock market*.

Given this different motive, it is also interesting to see how SFI economists programmed agents in their models, and, given their coding or programming, how complex their agents can evolve to be. Palmer *et al.* (1994) also used the standard trinary string to code different types of trading rules frequently used by financial market traders. Each bit of a string was randomly drawn from the trinary alphabet $\{0, 1, *\}$. Each bit corresponds to the *condition part* of a single trading rule. For example, the condition part of a *double moving average rule* could be "The 20-period moving average of price is above the 100-period moving average." The appropriate bit is 1 if the condition is true, and 0 if it is false. They typically used strings of 70–80 symbols, i.e., the same as the number of trading rules. This defines a search space with 3^{70} to 3^{80} possible nonredundant classifiers. However, each artificial trader has "only" 60 classifiers in its own classifier system. Consider a case with 100 computerized traders: there are at most 6000 different rules being evaluated in one single trading run. Compared with the size of the search space, the number of rules is infinitesimal.

This rather large search space is certainly beyond what Arthur (1992) called the *problem complex boundary*, a boundary beyond which arriving at the deductive solution and calculating it are unlikely or impossible for human agents, and this is where the SFI stock market comes into play. It provides the right place to use genetic algorithms and a great opportunity to watch evolution:

> We find no evidence that market behavior ever settles down; the population of predictors continually coevolves. One way to test this is to take agents out of the system and inject them in again later on. If market behavior is stationary they should be able to do as well in the future as they are doing today. But we find that when we "freeze" a successful agent's predictors early on and inject the agent into the system much later, the formerly successful agent is now a dinosaur. His predictions are unadapted and perform poorly. The system has changed. From our vantage point looking in, the market—the "only game in town" on our computer—looks much the same. But internally it coevolves and changes and transforms. It never settles.
>
> (Arthur, 1992, p. 24)

Maybe the real issue is not whether GA are used to strengthen the idea of rational expectations equilibrium, or to simulate artificial life, but *how we program adaptive agents*. This is crucial because different programming schemes may lead to different results. As Frank Hahn pointed out, while there is only one way to be perfectly rational, there are an infinite number of ways to be partially rational (Waldrop, 1992, pp. 250–51). This unlimited "degree of freedom" of programming adaptive agents was also noticed by Sargent: "This area is wilderness because the researcher faces so many choices after he decides to forgo the discipline provided by equilibrium theorizing" (Sargent, 1993, p. 2)). Arthur would consider letting the agents start off *perfectly stupid*, and get *smarter and smarter* as they learn from experience. Now comes the core of the issue: *how to program agents so that they can be initialized as perfectly stupid individuals, but can potentially get very smart?* To answer this question, let us go back to the origin of genetic algorithms.

13.4.5 List programming

It is interesting to note that the binary strings initiated by Holland were originally motivated by an analogy to machine codes. After decoding, they can be computer programs written in a specific language, say LISP or FORTRAN. Therefore, when a GA is used to evolve a population of binary strings, it behaves as if it is used to evolve a population of computer programs. If a decision rule is explicit enough not to cause any confusion in implementation, then one should be able to write it in a computer program. It is the *population of computer programs* (or their machine codes) which provides the most general representation of the *population of decision rules*. However, the equivalence between computer programs and machine codes *breaks down* when what is coded are the parameters of decision rules rather than decision rules (programs) themselves, as we often see in economic applications with GAs. The original meaning of evolving binary strings as evolving computer programs is lost.

The gradual loss of the original function of GAs has finally been noticed by John Koza. He chose the language LISP as the medium for the programs created by genetic programming because the syntax of LISP allows computer programs to be manipulated easily, like the bit-strings in GAs, so that the same genetic operations used on bit-strings in GAs can also be applied to GP.

LISP S-expressions consist of either *atoms* or *lists*. Atoms are either members of a *terminal set*, that comprise the data (e.g., constants and variables) to be used in the computer program, or they are members of a *function set* that consists of a number of prespecified functions or operators that are capable of processing any data value from the terminal set *and* any data value that results from the application of any function or operator in the function set. Lists are collections of atoms or lists, grouped within parentheses. In the LISP language, everything is expressed in terms of operators operating on some operands. The operator appears as the left-most element in the parentheses and is followed by its operands and a closing (right) parenthesis. For example, the S-expression $(+X3)$ consists of three atoms: from the left-most to right-most they are the function "+," the variable X, and the constant 3. As another example, $(\times X(-Y3))$ consists of two atoms and a list. The two atoms are the function "$\times$" and the variable "X," which is then followed by the list $(-Y3)$.

LISP stands for List Processing, which is a high-level computer language invented by John McCarthy (1927–2011) in 1958 at MIT as a formalism for reasoning about the use of certain kinds of logical expressions, called recursion equations. This language is strongly motivated as a practical implementation of the λ calculus or the recursive function theory developed in the 1930s by Alonzo Church (1903–1995) and Alan Turing.[6] LISP possesses unique features that make it an excellent medium for complex compositions of functions of various types, handling hierarchies, recursion, logical functions, self-modifying computer programs, self-executing computer programs, iterations, and structures whose size and shape are dynamically determined. The most significant of these features is the fact that LISP descriptions of processes (routines) can themselves be represented and manipulated as LISP data (subroutines). As Koza (1992a) demonstrated, LISP's flexibility in handling procedures as data makes it one of the most convenient languages in existence for exploring the idea of evolving computer programs genetically; however, Koza and others have noted that the use of LISP is not necessary for genetic programming—what is important for genetic programming is the implementation of a LISP-like environment, where individual expressions can be manipulated like data, and are immediately executable.

13.4.6 Symbolic regression

The distinguishing feature of GP is manifested by its first type of application in economics, known as *symbolic regression*. In symbolic regression, GP is used to discover the underlying data-generation process of a series of observations. While this type of application is well known to econometricians, the perspective from GP is novel:

> An important problem in economics is finding the mathematical relationship between the empirically observed variables measuring a system. In many conventional modeling techniques, one necessarily begins by selecting the size and shape of the model. After making this choice, one usually then tries to find the values of certain coefficients required by the particular model so as to achieve the best fit between the observed data and the model. But, in many cases, *the most important issue is the size and shape of the model itself.*
>
> (Koza, 1992b, p. 57; emphasis added)

Econometricians offer no general solution to the determination of size and shape (the functional form), but for Koza, finding the functional form of the model can be viewed as *searching a space of possible computer programs* for the particular computer program which produces the desired output for given inputs.

Koza employed GP to rediscover some basic physical laws from experimental data, for example, Kepler's third law and Ohm's law (Koza, 1992a). He then also applied it to eliciting a fundamental economic law, namely, the *quantity theory of money* or the *exchange equation* (Koza, 1992b). Genetic programming was thus formally demonstrated as a *knowledge discovery* tool. This was probably the closest step ever made toward the original motivation of John Holland's invention: "Instead of trying to write your programs to perform a task you don't quite know how to do, *evolve them.*" Indeed, Koza did not evolve the parameters of an arbitrary chosen equation; instead, he evolved the whole equation from scratch. This style of application provides an evolutionary determination of bounded rationality.

Koza (1992b) motivated a series of economic applications of genetic programming in the mid 1990s. Chen and Yeh (1996a) applied genetic programming to rediscovering *the efficient market hypothesis* in a financial time series. Chen and Yeh (1997a) then moved one step forward to propose an alternative formulation of the efficient market hypothesis in the spirit of the *Kolmogorov complexity* of algorithms for pattern extraction from asset price data. Chen and Yeh (1997a) and Szpiro (1997b) employed GP to discover the underlying chaotic laws of motion of time series data. Neely, Weller, and Ditmar (1997) and Allen and Karjalainen (1999) also adopted a GP approach to discover profitable technical trading rules for the foreign exchange market and the stock market, respectively. Another area in which GP was actively applied is *option pricing*. Chen, Lee, and Yeh (1999) used GP for hedging derivative securities. Keber (1999) showed that genetically determined formulas outperformed most frequently quoted analytical approximations in calculating the implied volatility based on the Black–Scholes model.

Chidambaran, Lee, and Trigueros (2000) and Keber (2000) derived approximations for calculating option prices and showed that GP models outperformed various other models presented in the literature.

Needless to say, one can expect many more applications of GP to the automatic discovery of economic and financial knowledge (automatic generation of economic and financial knowledge in terms of their computer-programmed representations). However, its significant contribution to economics should not be mistaken for a perfect solution to knowledge discovery, data mining, or, more generally, *function optimization*. In a nutshell, genetic programming should be used to grow *evolving hierarchies* of building blocks (subroutines), the basic units of learning and information, from an immense space of subroutines. All evolution can do is look for improvements, not perfection. John Holland believed that these evolving hierarchies are generic in adaptation, and can play a key role in understanding human learning and adaptive processes.

Notes

1 For example, what evolutionary economics is about can differ among scholars who consider themselves successors of Alfred Marshall, Thorstein Veblen, Joseph Schumpeter, etc.
2 For a description of the origin of EC, see Schwefel (1995), Fogel (1995), and Eberhart, Simpson, and Dobbins (1996).
3 Here, for simplicity, we assume that x is a real-valued number. In a more general setting, the variable x can be a vector. In that case, σ should be replaced by the variance-covariance matrix Σ.
4 In Equation (13.7), $(\sigma')^2$ is determined randomly. There is, however, some way to make it adaptive. For example, in the $(1 + 1)$-ES case, one has the famous 1/5-success rule. $(\sigma')^2$ can also be determined in a self-adaptive way. In that case, the learning rate τ can be set as a function of time. For details, see Schwefel (1995).
5 BNF was invented by John Backus (1924–2007) for describing the high-level language ALGOL. Peter Naur was the editor of the report in which it appeared. The class of languages described by BNF, including recursively defined nonterminals, is equivalent to Chomsky's context-free (type 2) languages.
6 See Abelson and Sussman (1996) for details.

Part V

Agent-based financial markets

While the financial market per se is always an attractive subject and requires little promotional effort, our purposes in placing this subject here are more specific. First, we would like to use the agent-based financial market to further illustrate the connection between ACE and *evolutionary economics*, a connection which we have started in Sections 5.3 and 9.4, and continued in Chapter 13. There we have seen the idea of *autonomous agents* (novelty-discovering agents) and the tools to make such agents. In this part, we will see their use for studying issues related to evolutionary economics. Second, we would also like to use the agent-based financial market to continue the subject of constructing agent-based models *empirically*, which we have initiated in Chapter 7.

Both intentions inevitably bring us back to the very fundamental issue of ACE, i.e., the *design of software agents* (Part III). In this context, we are able to see the applications of two kinds of financial (software) agents: one using *programmed agents* (Chapter 14) and one using *autonomous agents* (Chapter 15). Roughly speaking, they constitute the two major classes of agent-based financial markets. The former, also known as *H-type agent-based financial markets*, has agents whose behavioral rules are known and, to some extent, are fixed and simple. The latter, known as *SFI* or *SFI-like agent-based financial markets*, has agents who are basically autonomous, and their behavior, in general, can be quite complex. A survey of the former can be found in Hommes (2006), whereas a survey for the latter is available in LeBaron (2006).

Advantages and disadvantages exist in each of the two classes. The H-type class is normally well structured and hence is analytically tractable, whereas the SFI-like class mostly can only be approached by computer simulation. While both classes use microstructure dynamics to account for the generic phenomena observed in financial markets, the former does not endow its agents with a novelty-discovering capability, whereas the latter does grant such permission. Therefore, the latter is in spirit more rich with respect to the idea of evolutionary economics.

Another way to see their difference is that the H-type model takes a *mesoscopic* approach, in which the micro details of individuals are not considered important, whereas the SFI-like approach takes a *microscopic* approach, by which the mesoscopic level is not assumed but emerges. These two classes of models are, however, interestingly connected by *randomly behaved agents*, also known as zero-intelligence agent models (Section 8.3), which physicists like most.

The reason that many physicists prefer the zero-intelligence approach is simply because the strategic behaviors of financial agents (autonomous agents), including their learning processes, are poorly known and are difficult to model. Therefore, in the vein of the law of large numbers, they simply assume that these complexifications will cancel each other out so that altogether their mesoscopic behaviors are observationally equivalent to the randomly behaved agents (zero-intelligence agents). In a sense, this is an application of the *maximum entropy principle* to agent-based financial modeling, i.e, maximizing the entropy of any entity for which we have no information.[1] This maximum entropy principle is not the same as the original motivation for zero-intelligence agents as proposed by Gode and Sunder (1993).

For the former design, we are also going to see how these agents are constructed empirically (Chapter 16), an advancement extending Arthur (1991, 1993). For the latter design, empirical work has also begun but it is more difficult, and those difficulties in the end drive us to reflect on the possible limitations of the idea of empirical-based agent-based models.

For a survey of agent-based models of financial markets, the interested reader is referred to Samanidou *et al.* (2007).

Note

1 One, however, has to distinguish the randomly behaved agents from the random-programmed agents. The former simply behave randomly with no rule as guidance, whereas the latter behave by systematically following a rule which is, nonetheless randomly generated.

14 Artificial financial markets with programmed agents

There are three major H-type models, namely Kirman's ANT model (Kirman, 1993), Lux's IAH model (Lux, 1995, 1998), and Brock and Hommes' ABS model (Brock and Hommes, 1998). Among the three, the ANT model and the IAH model are similar in the sense that they rely heavily on *herding*; therefore, they can be considered together as *herding-based agent-based financial models*. Imitation or social learning is a key driving mechanism of the model. On the other hand, the ABS model is a typical *performance-based model*. Selection (evolution) is the key mechanism.

Not surprisingly, the H-type model starts with two-types, namely *fundamentalists* and *chartists*. Later on, *contrarians* are also added on to make it three-types. The two-type or three-type models are *empirically inspired*. They are highly motivated by observing how real financial agents behave. Empirical evidence accumulated since the late 1980s and early 1990s has shed new light on the forecasting behavior of financial agents. This empirical evidence was obtained through different kinds of surveys, such as questionnaires and telephone interviews, with financial specialists, bankers, currency traders, dealers, etc. (Frankel and Froot, 1990; Allen and Taylor, 1990).

The general findings from these abundantly established empirical data are twofold. First, the data indicate that, by and large, there are two kinds of expectations existing in the market. The one which is characterized as a stabilizing force of the market is associated with a type of financial agent called the *fundamentalist*. The one which is characterized as a destabilizing force is associated with another type of financial agent, called the *chartist*, *technical analyst*, or *trend extrapolator*. Second, the proportion (*microstructure*) of fundamentalists and chartists, also called the *market fraction*, changes over time, which indicates that financial agents are adaptive. These empirical findings provide the initial direction for the early development of financial agent engineering. First, they suggest what rules to look at; second, they point out the significance of learning and adaptation.

Fundamentalists and chartists are concerned with two very different beliefs regarding stock price dynamics. In a simple setting, they differ in terms of the mean-reverting speed of the stock price when it is mispriced (undervalued or overvalued). Fundamentalists tend to believe that the mispriced situation will soon be corrected, whereas chartists tend to believe that in the short run it will continue.

14.1 Few-type design

14.1.1 Two-type design

To make what we say more precise, we generally denote the forecasting rule of a type-h agent as follows:

$$E_{h,t}[p_{t+1}] = f_{h,t}(p_t, p_{t-1}, \ldots), \tag{14.1}$$

where $E_{h,t}$ refers to the expectations of the type-h agent at time t. Equation (14.1) indicates the one-step-ahead forecast. At the beginning, we start with a very general forecasting function $f_{h,t}$, which uses all the historical data on price up to the present. In addition, by considering that agents are adaptive, we allow the function to change over time and hence denote it by the subscript t.

For the fundamentalists ($h = f$) and chartists ($h = c$), their forecast rules, in a very simple setting, can be written as:

$$E_{f,t}[p_{t+1}] = p_t + \alpha_f(p_t^f - p_t), \quad 0 \leq \alpha_f \leq 1, \tag{14.2}$$

$$E_{c,t}(p_{t+1}) = p_t + \alpha_c(p_t - p_{t-1}), \quad 0 \leq \alpha_c. \tag{14.3}$$

The idea of these two behavioral rules is that the fundamentalist has a *mean-reverting* belief, and his belief is characterized by a reverting coefficient (α_f), whereas the chartist has a trend-continuing belief, and his belief is characterized by an extrapolating coefficient (α_c). The magnitude of the reverting coefficient (α_f) measures the speed at which the fundamentalists expect the price to return to the fundamental one (p_t^f), whereas the magnitude of the extrapolating coefficient (α_c) expresses the degree to which chartists expect the past to change into the future.

14.1.2 Three-type design

There is little doubt that the behavior of financial agents can be more complex than the two-type design. One obvious way to scale up this design is to add more types of agents to the model so as to take into account a finer degree of heterogeneity of financial agents. This type of expansion is called the *H-type design*. For example, in a three-type design, one can further distinguish two kinds of chartists, namely *momentum traders* and *contrarian traders*, or simply *contrarians*. Like momentum traders, contrarians extrapolate past movements of the price into the future, but they follow the opposite of the trend. More precisely, their forecasting rule is as follows:

$$E_{co,t}(p_{t+1}) = p_t + \alpha_{co}(p_t - p_{t-1}), \quad \alpha_{co} \leq 0. \tag{14.4}$$

Contrarians consider that the price trend will finish soon, and will start to reverse. However, unlike fundamentalists, contrarians do not base their forecasts on the fundamental price, which they either do not know, or they do not care about.

The recent availability of more proprietary data has enhanced the transparency of the trading behavior of financial agents, including both individual and institutional investors. Empirical studies using such data have shown that individuals and institutions differ systematically in their reaction to past price performance and the degree to which they follow momentum and contrarian strategies. On average, individual investors are contrarian investors: they tend to buy stocks that have recently underperformed the market and sell stocks that have performed well in recent weeks (Barber and Odean, 2000). With this empirical basis, financial agent engineering has already added the contrarians to the fundamentalist–chartist model, and popularized this three-type design.

14.1.3 Generalization of two- and three-type designs

Financial agent engineering can also be advanced by enriching the behavioral rules associated with each type of financial agent. This alteration may make financial agents more interdisciplinary. Considerations from different fields, including neural sciences, cognitive psychology, and statistics, can be incorporated into designs. For example, in behavioral finance, there is a psychological bias known as the *law of small numbers*, which basically says that people under-weight long-term averages, and tend to put too much weight on recent experiences (the recency effect). When equity returns have been high for many years, financial agents with this bias may believe that high equity returns are "normal." By design, we can take such bias into account. One way to do so is to add a *memory parameter* to the behavioral rules of our financial agents. This more general rule for chartists is specified as follows:

$$E_{c,t}(p_{t+1}) = p_t + \alpha_c(1-\beta_c)\sum_{i=0}^{T}\left(\frac{(\beta_c)^i}{\sum_{i=0}^{T}(\beta_c)^i}\right)(p_{t-i} - p_{t-i-1}),$$
$$0 \le \alpha_c, \quad 0 \le \beta_c \le 1, \tag{14.5}$$

and for contrarians:

$$E_{co,t}(p_{t+1}) = p_t + \alpha_{co}(1-\beta_{co})\sum_{i=0}^{T}\frac{(\beta_{co})^i}{\sum_{i=0}^{T}(\beta_{co})^i}(p_{t-i} - p_{t-i-1}),$$
$$0 \ge \alpha_{co}, \quad 0 \le \beta_{co} \le 1. \tag{14.6}$$

The momentum traders and contrarians now compute a moving average of the past changes in the stock price and they extrapolate these changes into the future of the stock price. However, we assume that there is an exponential decay in the weights given to the past changes in the stock price. The parameters β_c and β_{co} can be interpreted as reflecting the memories of momentum traders and contrarians, respectively. If $\beta_c = \beta_{co} = 0$, momentum traders and contrarians remember only the last period's price change and they extrapolate this into the future. When β_c and β_{co} increase, the weight given to older price changes increases. In other words, the chartists' memory becomes longer.

The psychological bias mentioned earlier, therefore, corresponds to a small value of this memory parameter, and this "hypothesis" can actually be tested. In fact, by using data from the S&P 500 index, one of the three major US stock market indices, from January 1980 to December 2000, Amilon (2008) actually estimated a three-type agent-based financial market model, and found that contrarians have a longer memory than momentum traders when they form their forecast of the future price. Of course, this is just the beginning in terms of seeing how agent-based financial market models can be quantified so as to communicate with behavioral finance.

14.1.4 Adaptive behavior

In the original fundamentalist–chartist model, learning does not exist. Agents who initially happen to be fundamentalists will continue to be fundamentalists and will never change this role, and likewise for chartists. As a result, the proportion (market fraction) of fundamentalists and chartists remains fixed. This simplification underestimates the uncertainty faced by each trader. In general, traders, be they fundamentalists or chartists, can never be certain about the duration of the biased trend, since the trend can finish in weeks, months, or years. This uncertainty causes the alerted traders to review and revise their beliefs constantly. In other words, traders are *adaptive*.

Therefore, a further development of financial agent engineering is to consider an evolving microstructure of market participants. In this extension, the idea of adaptive agents or learning agents is introduced into the model. Hence, an agent who was a fundamentalist (chartist) may now switch to being a chartist (fundamentalist) if he considers this switch to be more promising. Since, in the two-type model, agents can only choose to be either a fundamentalist or a chartist, modeling their learning behavior becomes quite simple, and is typically done using a *binary choice model*, specifically, the *logit model* or the *Gibbs–Boltzmann distribution*.

The logit model, also known as the *Luce model*, is the main model used in the psychological theory of choice, and was proposed by Duncan Luce in 1959 in his seminal book, *Individual Choice Behavior: A Theoretical Analysis*. Consider two alternatives, f (fundamentalist) and c (chartist). Each will produce some gains to the agent. However, since the gain is random, the choice made by the agent is random as well. The logit model assumes that the probability of the agent choosing f is the probability that the profits or utilities gained from choosing f are greater than those gained from choosing c. Under a certain assumption for the random component of the utility, one can derive the following *binary logit model*:[1]

$$\text{Prob}(X = f, t) = \frac{\exp(\lambda V_{f,t-1})}{\exp(\lambda V_{f,t-1}) + \exp(\lambda V_{c,t-1})}, \tag{14.7}$$

where $V_{f,t}$ and $V_{c,t}$ are the deterministic components of the gains from the alternatives f and c at time t. The parameter λ is a parameter carried over from the assumed random component. The logit model says that the probability of choosing the alternative f depends on its *absolute deterministic advantages*, as we can see from the following reformulation:

$$\text{Prob}(X = f, t) = \frac{1}{1 + \exp[-\lambda(V_{f,t-1} - V_{c,t-1})]}. \tag{14.8}$$

When applied to agent-based financial models, these deterministic components are usually related to the temporal realized profits associated with different forecasting rules. So, in the two-type model, if V_f can be the temporal realized profits from being a fundamentalist, then V_c can be the temporal realized profits from being a chartist. In addition, there is a new interpretation for the parameter λ, namely, the *intensity of choice*, because it basically measures the extent to which agents are sensitive to the additional profits gained from choosing f instead of c.

14.1.5 Market-maker equation

The market fractions above then determine the market fraction of each type of agent in the market. For example, if $\text{Prob}(X = F) = 0.8$, it means that 80 percent of the market participants are fundamentalists and the remaining 20 percent are chartists. The asset price will be determined by this market fraction via the *market-maker equation*:

$$p_t = p_{t-1} + \mu_0 + \mu_1 D_t, \tag{14.9}$$

where

$$D_t = \sum_h w_{h,t} d_{h,t} = \sum_h \text{Prob}(X = h, t) d_{h,t}. \tag{14.10}$$

Equation (14.9) is the market-maker equation, which assumes that the price is adjusted by the *market-maker*, whose decision is in turn determined by the excess demand normalized by the number of market participants, D_t. D_t in Equation (14.10) is a weighted average of the individual demand of each type of trader, weighted by the market fractions (14.7).

14.1.6 Risk preference and portfolio

The demand for assets of each type of trader is derived in a standard expected-utility maximization manner, which depends on the *risk preference* of the type-h agent. Risk preference is important because it is the main determinant of agents' portfolios, i.e., how agents' wealth is distributed among different assets. The classical *Markowitz mean–variance portfolio selection model* offered the first systematic treatment of asset allocation. Harry Markowitz, who later received the 1990 Nobel Prize in Economics for this contribution, assumes that investors are concerned only with the mean and variance of returns. This *mean–variance preference* has been extensively applied to modeling agents' risk preference since the variance of returns is normally accepted as a measure of risk.

In addition to the mean–variance preference, there are two other classes of risk preferences that are widely accepted in the standard theory of finance. These two correspond to two different attitudes toward risk aversion. One is called *constant absolute risk aversion* (CARA), and the other is called *constant relative risk aversion* (CRRA). When an agent's preference exhibits CARA, his demand for the risky asset (or stock) is independent of his changes in wealth. When an agent's preference exhibits CRRA, his demand for risky assets will increase with wealth in a linear way. Using a Taylor expansion, one can connect the mean–variance preference to CARA preferences and CRRA preferences. In fact, when the returns on the risky assets follow a normal distribution, the demand for risky assets under the mean–variance preference is the same as that under the CARA preference, and is determined by the *subjective-risk-adjusted expected return*:

$$d_{h,t} = \frac{E_{h,t}(\Delta p_{t+1})}{a_{h,t} V_{h,t}(\Delta p_{t+1})} = \frac{E_{h,t}(p_{t+1}) - p_t}{a_{h,t} V_{h,t}(\Delta p_{t+1})}, \tag{14.11}$$

where $\Delta p_{t+1} = p_{t+1} - p_t$, and $a_{h,t}$ is a risk aversion coefficient. The $E_{h,t}(p_{t+1})$ in the numerator of Equation (14.11) is given by Equations (14.2), (14.3), and (14.4), and $V_{h,t}$ in the denominator represents the perceived risk of the type-h agents. Further details of the formation of this subjective perceived risk can be found in the agent-based finance literature (De Grauwe and Grimaldi, 2006; Hommes, 2006).

14.2 Many-type designs

14.2.1 Adaptive belief systems

Brock and Hommes (1997, 1998) initiated an H-type design called the *adaptive belief system*. This system can be considered as an extension of the two-type or three-type design that we observed earlier. In this case, there are more than two or three kinds of beliefs in the market, and the number is denoted by H. Like the two- or three-type designs, each of these beliefs is known in advance and is fixed. Brock and Hommes (1998) further assumed that they are all linear forecasting rules, as follows:

$$E_{h,t}[p_{t+1}] = \alpha_{h,0} + \alpha_{h,1} p_t + \alpha_{h,2} p_{t-1} + \cdots + \alpha_{h,L+1} p_{t-L}, \quad h = 1, 2, \ldots, H. \tag{14.12}$$

In this simplification, each belief is characterized by a set of coefficients $(\alpha_{h,0}, \alpha_{h,1}, \ldots, \alpha_{h,L+1})$, denoted by $\boldsymbol{\alpha}_h$. The adaptation part or the switching mechanism can be easily extended from the original logit model (Equation 14.7) into the the *multinomial logit* model:

$$\text{Prob}(X = h, t) = \frac{\exp(\lambda V_{h,t-1})}{\sum_{j=1}^{N} \exp(\lambda V_{j,t-1})}, \quad h = 1, 2, \ldots, H. \tag{14.13}$$

The rest of the ABS is essentially the same as the few-type designs.

14.2.2 Large type limit and continuous belief systems

One further extension of the H-type design is to consider not just a finite H but an infinite H, i.e., $H \to \infty$. In this line, Brock, Hommes, and Wagener (2005) introduce the notion of *large type limit*, whereas Diks and van der Weide (2005) propose a *continuous belief system* (CBS). They both rely on the idea of *distribution of beliefs*. The distribution of beliefs is a distribution over a belief space, from which the observed beliefs are sampled. When the belief space is a real space, one has infinite-type design. The earlier finite H-type design can be considered as a *sample* with size H from this distribution. Once it is generated, the sample is fixed, and the later adaptation or switching mechanisms are just operated within this fixed sample. However, the infinite H-type design works directly on the *population* of beliefs, instead of a sample of beliefs. Since the distribution of beliefs can be captured by a few parameters, one can effectively use a small number of parameters to represent a large number of different types of traders. The discrete choice problem is now extended into a continuous choice problem, but still with the Gibbs–Boltzmann distribution:

$$\begin{aligned} E_{h,t}(x_{t+1}) &= f_t(x_{t-1}, \ldots, x_{t-L}) \\ &= \theta_{h,0} + \theta_{h,1}x_{t-1} + \theta_{h,2}x_{t-2} + \cdots + \theta_{h,L}x_{t-L} \\ &= \Theta_h \cdot \mathbf{x}_L, \quad h = 1, 2, \ldots, H, \end{aligned} \tag{14.14}$$

where $\Theta_h \sim F_{\mathbf{u}}$, where $F_{\mathbf{u}}$ is a general multivariate distribution.

$$\text{Prob}(X = \mathbf{h}^*, t) = \frac{\exp[\lambda V(\mathbf{h}^*, t-1)]}{\int \exp[\lambda V(\mathbf{h}, t-1)] d\mathbf{h}}. \tag{14.15}$$

Diks and van der Weide (2005) further make the above function of beliefs also adapt over time; hence, $\Theta_h \sim F_{\mathbf{u},\mathbf{t}}$.

14.2.3 "Type" in few-type and many-type designs

While the large-type design can be considered as a natural mathematical extension of the few-type design, one should notice that they do have dramatically different notions of *type*. The notion of type in the few-type designs is in fact a *cluster*, i.e., a cluster of *similar* trading strategies or beliefs. Therefore, while there is a large number of traders in the world, their behavior is limited to just a few clusters or a few types.[2] The early literature on fundamentalists and chartists is developed in this fashion. In other words, they are two different types, not because they are different, but because they are *fundamentally* different. Two fundamentalists may still have different beliefs, but compared to those of the chartists, their differences are negligible. The large-type designs no longer keep this notion of cluster; instead, they consider traders with even with ϵ difference as different types.

14.3 Illustrations

In the literature, there are three major H-type ACF models; in chronological order, they are the Kirman model, the Lux model, and the Brock–Hommes model. The entire structure of Section 14.1 is written by largely following the Brock–Hommes ABS model. Section 14.2 also gives the extension of the ABS model. Therefore, only the Kirman and Lux models will be highlighted here.

14.3.1 Kirman's ANT model

This model was first proposed by Kirman (1991, 1993). It is a two-type model, the two types being fundamentalists and chartists. The model, however, differs from the previous description of the two-type model in its switching mechanism. The switching is not driven by the Gibbs–Boltzmann distribution (Equation 14.7), but by a *herding mechanism*. Within this herding mechanism, the main determinant of the respective binary choice is not the financial success (the $V_{f,t}$ and $V_{c,t}$ appearing in Equation 14.7) but the fraction of the majority. Therefore, the decision is more psychologically driven rather than economically driven.

The herding mechanism was inspired by an observation in entomology. "Ants, faced with two identical food sources, were observed to concentrate more on one of these, but after a period they would turn their attention to the other" (Kirman, 1993, p. 137). Inspired by observing the behavior of ants, Kirman characterizes the switching potential of each individual by two parameters, namely, *a probability of self-conversion* and *a probability of being persuaded*. The self-conversion probability gives the probability that an individual will switch to other types of agents without external influence, whereas the probability of being converted gives the probability that the individual will be *persuaded* by the other individual to whom he is randomly matched. This switching process is discrete and can be studied in the general theoretical framework of Polya urn processes.

In his ANT model, Kirman shows that when the persuasiveness (herding) is strong enough, compared to the self-conversion rate and the group size (number of traders in the market), then we shall see the large switching phenomenon of the two groups, for example, chartists and fundamentalists:

$$\epsilon < \frac{1-\delta}{N-1}. \tag{14.16}$$

Since in this type of model the market fraction is the key variable explaining the price fluctuation, different dynamics of market fractions may imply different price dynamics. Hence, in Kirman's model, the price dynamics rest upon the strength of herding. This result is very similar to what other H-type models, or even the SFI market, observe.

14.3.2 Lux's interactive agent hypothesis (IAH) model

The Lux model was initiated by Lux (1995, 1997, 1998). It is a hierarchical two-type model. It is hierarchical in the sense that there are fundamentalists and chartists, but chartists are further divided into optimists and pessimists. Like the Kirman model, the Lux model also has the herding element. However, the original *discrete-time* discrete-choice behavior in the Kirman model is now replaced by a *continuous-time* discrete-choice model. This change causes the Lux model to have a very different mathematical structure, which is a *jump Markov process*. In this continuous-time mathematics, the switching mechanism among the three groups of agents is captured by the *transition rate function*.

The transition rate specifies the change rate of the conditional probability of shifting to another state after an infinitesimal time interval, conditional upon a specific current state. The state refers to each possible distribution (each set of fractions) over the types (clusters). For example, in a two-type model, the distribution at a point in time can be characterized by one parameter, i.e., the fraction of type one, say $w_1(t)$, since subtracting it from one gives the fraction of type two, i.e., $w_2(t) = 1 - w_1(t)$.

Lux considers two determinants in the transition rate function. One is the *profit differential*, i.e., the difference in trading profits between the fundamentalists and chartists. The transition rate function is negative in the profit differentials between own profits and alternative profits. In this way, as expected, when fundamentalists earn a higher profit than chartists, it will become less likely for a fundamentalist to switch, but more likely for a chartist to switch. Hence, in this regard, it is very similar to the *Gibbs–Boltzmann distribution* used by Brock and Hommes' ABS. The other one is *herding*. In this case, traders' decisions to switch between different clusters are affected by the numbers of traders in each cluster. The majority cluster may find it easier to attract traders from the minority cluster, but this is less likely the other way round. In the Lux model, herding is only applied to the switch between optimists and pessimists.

14.3.3 Minority games (MG)

Since trend-chasers and contrarians in the financial market can be positioned as the majority and minority, *a mixed minority and majority game*, also known as the $-game, has been applied to agent-based financial markets (Challet and Zhang, 1997). There are some designs of agents that are particularly suitable for MG models. One example is the *threshold model* proposed by Ghoulmie, Cont, and Nadal (2005) and Cross *et al.* (2007).

14.4 A mesoscopic approach to complex dynamics

The complex dynamics of the H-type model have been shown in Kirman (1991, 1993), Brock and Hommes (1998), and Lux and Marchesi (1999, 2000). Together, these studies show that the long-run behavior (steady state) of the market dynamics can be complex.

In physics, during the late nineteenth century, a fundamentally new approach referred to as *statistical mechanics* had been advanced by James Maxwell (1831–1879), Ludwig Boltzmann (1844–1906), Wilard Gibbs (1839–1903), and others. This approach, which had significantly contributed to the study of molecular dynamics, was also formally introduced to the study of economics and the social sciences in the 1990s.[3] This new field is broadly known as *econophysics* or *sociophysics*. Weidlich and Haag (1983) and Aoki (1996) contributed to the dissemination of the ideas at the early stage to social scientists.

This new approach is *mesoscopic*. The details of each individual are considered to be irrelevant and are not taken as the starting point of the model. Instead, the mesoscopic approach considers *clusters* of individuals. Individuals who follow similar rules are considered to be in the same cluster of agents. Each cluster is defined by the associated behavioral rules. The microstructure is characterized by the distribution (fractions) of individuals over different clusters.

The next step is then to derive the aggregates from this distribution: different distributions over the clusters may have different impacts on the aggregates. For example, an economy composed of a high fraction of optimists may behave dramatically differently from one composed of a high fraction of pessimists in terms of both consumption and investment. Furthermore, the distribution is normally not static and will change over time. Again, take the *sentiment index* (a survey of consumers' or investors' confidence in the economy) as an example. This index may be different for each of the announcements, and is clearly evolving over time. What makes things even more intriguing is that the evolution of the distribution is also affected by the aggregates as a feedback effect. The mesoscopic model, therefore, has to also spell out this feedback mechanism, and a complete picture is that both the microstructure and the aggregates are evolving with feedbacks to each other.

14.4.1 Master equation

Fortunately, statistical physics can help economists and other social scientists to build this kind of mesoscopic model. One of the toolkits which is now often taught in graduate programs is the *master equation*, also known as the *Chapman–Kolmogorov equation* (Aoki, 1996).

The master equation describes the time evolution of the probability distribution. In our case, the state refers to each possible distribution (each set of fractions) over the clusters. For example, in a two-cluster model, the distribution at a point in time can be characterized by one parameter, i.e., the fraction of cluster one, say $w_1(t)$, since subtracting it from one gives the fraction of cluster two, i.e., $w_2(t) = 1 - w_1(t)$. Then the master equation will give us the distribution of $w_1(t)$. Theoretically, one can then ask what the probability is that 95 percent of consumers will be optimistic on 1 January 2010. Or, what is the probability that 90 percent of consumers will be pessimistic on 31 December 2015? This kind of question can interest political scientists. For example, given that 58 percent of voters supported the candidate from one political party in the year 2008, what is the probability that the majority of voters will still support the same party in the year 2012?

While the master equation makes each of the questions raised above legitimate, one may have difficulties answering them, because the master equation, depending on its construction, can be difficult to solve. To construct a master equation, one needs to have one key element, namely the *probability transition rates*. Once we have the transition rates, the answers to the questions above can be derived by solving a system of differential equations, if it is solvable.

The transition rate specifies the change rate of the conditional probability of shifting to another state after an infinitesimal time interval, conditional upon a specific current state. In reality, it is not always clear what the determinants of the transition rate function are. In addition, if we invite too many variables into the transition rate function, it will make the resultant master equation (the system of differential equations) too complex to solve, which in turn will immediately render the model obsolete.

Therefore, it would be desirable to have some problems which are not so complicated in order that the above-mentioned mesoscopic model can be applied. Aoki (1996, 2002a) comprehensively covers various economic applications with this mesoscopic approach. Aoki and Yoshikawa (2006) go further to use this framework to propose a new macroeconomics, which overthrows a number of long-held views in macroeconomics, among which the most striking one is that *aggregate demand matters*. On this issue, a microeconomic foundation for Keynes's *principle of effective demand* is established. The conventional view that long-run economic growth is mainly supply driven rather than demand driven is misleading. According to Aoki and Yoshikawa (2006), a demand deficiency or a saturation of demand can do harm to the long-run economic growth, and *demand-creating technological progress* is the ultimate factor for generating economic growth. A consequence of this result is that the potential GDP and the natural rate of unemployment, two pillars used to characterize the long-term state of an economy, are also ill-defined. We shall leave interested readers to find these results on their own. We will also elaborate on one application to H-type agent-based financial markets below.

14.4.2 Transition rates

To apply the mesoscopic approach to financial markets, we need to first cluster financial agents. The N-type model has already prepared us. Taking the simplest one, we have at least two major clusters of financial agents, namely, *fundamentalists* and *chartists*. Field studies also indicate that the proportion (microstructure) of fundamentalists and chartists, also called the *market fraction*, is changing over time. That is, in reality, as time goes by, some fundamentalists may switch to become chartists, and vice versa. Modeling this behavior becomes an important part of the transition rate function. One key determinant in this function is the *profit differential*, i.e., the difference in trading profits between the two clusters (types) of agents. Normally, the transition rate function should be negative in the profit differentials between own profits and alternative profits. In this way, as expected, when fundamentalists earn a higher profit than chartists, it will become less likely for a fundamentalist to switch, but more likely for a chartist to switch. This idea has been well encapsulated in the famous *Gibbs–Boltzmann distribution*, which has become the foundation of the discrete choice model in economics.

There are other determinants which may also affect the transition rate. The one which is important in the literature is referred to as *herding*. In this case, traders' decisions to switch between different clusters are affected by the number of traders in each cluster. The majority cluster may find it easier to attract traders from the minority cluster, but this is less likely the other way round. Herding is mainly a psychological effect, which may not apply to the switch between fundamentalists and chartists. However, it can be useful for the switch between optimists and pessimists (Kirman, 1993; Lux and Marchesi, 1999).

14.4.3 Master equation and Fokker–Planck equation

Once the transition rate function is specified, the master equation is defined. However, as we mentioned before, it may not be easy to solve. Therefore, different approximations have been proposed. The frequently used one involving the idea of the Taylor expansion is known as the *Fokker–Planck equation*. Instead of deriving the entire evolution of the distribution, by using the Fokker–Planck equation one actually works with the evolution of the first or the second moment of the distribution, which leads us to have the mean dynamics or the diffusion dynamics.

Returning to our two-cluster model, given the transition rate function, one may have difficulties in solving the corresponding master equation. However, by using the Fokker–Planck equation, one can approximately derive the mean dynamics, which is the dynamics of the mean fraction of either fundamentalists or chartists, or, in the optimist–pessimist model, the dynamics of the mean fraction of optimists or pessimists.

Given the approximated mean dynamics, which is generally represented by a differential equation, one can then ask whether there is a steady state, and if so, where it is. The answer will give us an approximated equilibrium, which is equivalent to a version of the complex dynamics simplified by disregarding its fluctuation or diffusion.

14.4.4 Bifurcation and chaos

Nevertheless, the long-run behavior (steady state) of the mean dynamics can be complex. Depending on various conditions, the types of the steady states can be fixed points, periodic cycles, and strange attractors. Using the mathematics of bifurcation and chaos, economists can delineate the domain of attraction for each steady state, which sheds light on the property of *path dependence*. So, for example, different initial distributions associated with different initial means may converge to different places. However, this mean dynamics is only the first-order Taylor expansion. If we further take the fluctuation into account, the path can be constantly disturbed, and, to some degree of perturbations, it may cross the boundary and lead to a different destination. In general, the path, thanks to the constant perturbations and the coexistence of multiple equilibria, can persistently visit many different steady states at different points in time. The main contribution of agent-based finance is to use this wandering-path property to explain many *stylized facts* observed in the financial market. Let us take *volatility clustering* as an example.

14.4.5 Volatility clustering

Stock prices are not just highly volatile. The large changes in price tend to be followed by further large changes, and small changes tend to be followed by small changes. If we look at the time series of asset returns, we tend to easily identify periods of elevated volatility interspersed among more tranquil periods. This temporal concentration of volatility is commonly referred to as *volatility clustering*, and is one of the most important stylized facts in the financial market.

Volatility clustering plays a pivotal role in the recent development of financial econometrics. There are a number of econometric models being established to provide a rigorous treatment of this phenomenon. The earliest and hence the most basic one is the ARCH (autoregressive conditional heteroskedastic) model. It was first proposed in 1982 by Robert Engel, the 2003 Nobel Laureate in Economics. This model was then extended by Tim Bollerslev in 1986 to the generalized ARCH model, abbreviated as the GARCH model, which has become the model most widely used by econometricians.

However, ARCH is only a statistical description of volatility clustering. To explain the emergence of the volatility clustering phenomenon, one needs to have a theory. Unfortunately, neoclassical finance, built upon the device of representative agents with rational expectations, has difficulty accommodating this stylized fact.

In agent-based finance, this stylized fact can be explained as follows. First, a different degree of volatility, be it small or large, corresponds to a different steady state. In each steady state, the price dynamics are coupled with their own fraction dynamics. As stated above, each steady state has its own domain of attraction. The domain of attraction is determined by both exogenously given parameters (specifying the transition rate function) and endogenous variables, such as market fractions. Then, the constant perturbations to the fractions may suddenly move the steady state from one to the other, and shift the regime of a small (large) volatility to the regime of a large (small) volatility. Volatility clustering can then be explained in this manner. Of course, the rich complex dynamics driven by the master equation may provide other similar explanations, for example using the switches from a stable steady state to an unstable steady state, or from a stable steady state to a strange attractor.

14.4.6 Further reading

The limited description provided in this short chapter shows how statistical physics has helped the development of agent-based economics. Thomas Lux, an economist at Kiel university, is a pioneer in this area. His model, referred to as the Lux model, is one of the most popular agent-based financial models in the literature. Others are the ANT model invented by Alan Kirman, the adaptive belief systems model invented by Carl Hommes, and the SFI model by John Holland and Brian Arthur. For a quick overview of these models, we recommend Hommes (2006) and

LeBaron (2006). However, one must notice that only the Lux model maintains a close connection with statistical physics; the other three do not, although their dynamics may share some considerable commonalities. In particular, the working of the SFI model does not follow a mesoscopic approach; instead, it is built up from the molecular level and is more biologically inspired.

14.5 Market fraction hypothesis

It comes as no surprise to economists that there is no single strategy that can persistenly dominate all other strategies in the market. The idea of the best strategy is simply inconsistent with the intuitive notion of the efficient market hypothesis. While this feature is well expected among economists, the result shown by De Bondt and Thaler (1985), generally known as the *overreaction hypothesis*, is still very appealing. They have found that successive portfolios formed by the previous five years' 50 most extreme winners considerably underperform the market average, while portfolios of the previous five years' 50 worst losers perform better than the market average.

Recently, a similar phenomenon has been rigorously analyzed and replicated in the agent-based finance literature, in particular, in the H-type model. In this literature, markets at any point in time are composed of different clusters (types) of agents. Agents who follow similar rules are considered to be in the same cluster. Each cluster is defined by the associated behavioral rules. The market microstructure is characterized by the *fractions* (distribution) of individuals over different clusters. Different distributions (microstructure) over the clusters may have different impacts on the aggregates, and both the microstructure and the aggregates are evolving with feedbacks to each other.

Complex dynamic analysis of these models indicates two interesting properties. First, in the short run, it is *likely* that the market fractions are constantly changing. In particular, for each cluster, the market fraction can swing from very low to very high, i.e., switching between the majority and the minority. Second, in the long run, no single strategy can dominate the other, i.e., the market fraction converges to $1/H$ for each cluster. These two properties provide us with a basis to study the complex dynamics of microstructure, which we refer to together as the *market fraction hypothesis*, or, as an abbreviation, the MFH. In fact, a number of empirical studies have already attempted to estimate the parameters associated with the MFH.

14.5.1 Departure

This section, however, differs from the H-type models in two regards. First, we do not assume any pre-fixed behavioral rule (functional form) for any cluster (type) of agents; second, we do not assume that agents of the same type are homogeneous, though they can be *similar*. We consider that this departure will lead us to a more general and *realistic implication* of the MFH. Consider the three-type model as an example. In the fundamentalist–chartist–contrarian model, traders of the same type at any point in time behave in *exactly the same way*, and their functional forms of behavioral rules, in this case their forecasts of the price in the next period, $\{E_{f,t}(p_{t+1})\}$, $\{E_{c,t}(p_{t+1})\}$, and $\{E_{co,t}(p_{t+1})\}$, are all known [see Equations (14.2) to (14.4)]. Nevertheless, in the real world, the behavioral rules of each trader are expected to be heterogeneous, and even if they can be clustered into types, the representative behavior of each type is normally unknown.[4]

14.5.2 Rule inference with GP

In this section, we assume that traders' behavior, including price expectations and trading strategies, is either not observable or not available. Instead, their behavioral rules have to be *estimated* by the observable market price. Using macro data to estimate micro behavior is not new, as many H-type empirical agent-based models have already performed such estimations (see Chapter 16). However, as mentioned above, such estimations are based on very strict assumptions upon which a formal econometric model can be built. Since we no longer keep these assumptions, an alternative must be developed, and in this chapter we recommend *genetic programming*.

The use of GP as an alternative is motivated by considering the market as an evolutionary and selective process.[5] In this process, traders with different behavioral rules participate in the markets. Those behavioral rules which help traders gain lucrative profits will attract more traders to *imitate*, and rules which result in losses will attract fewer traders. This evolutionary argument in fact is, intuitively, the same as the evolution process considered by the H-type agent-based financial models. For example, their use of the Gibbs–Boltzman distribution is a formalization of this process. Genetic programming is another formalization which, unlike the former, does not rest upon any prespecified class of behavioral rules. Instead, in GP, a population of behavioral rules is randomly initiated, and the survival-of-the-fittest principle drives the entire population to become fitter and fitter in relation to the environment. In other words, given the nontrivial financial incentive from trading, traders are aggressively searching for the most profitable trading rules. Therefore, the rules that are outperformed will be replaced, and only those very competitive rules will be sustained in this highly competitive search process.[6]

Hence, even though we are not informed of the behavioral rules followed by traders at any specific time horizon, GP can help us infer what these rules are *approximately* by simulating the evolution of the microstructure of the market. Without imposing tight restrictions on the inferred behavioral rules, GP enables us to go beyond the simple but also unrealistic behavioral rules used in the H-type agent-based financial models. Traders can then be clustered based on more realistic, and possibly more complex, behavioral rules.[7]

14.5.3 Rule clustering with self-organizing maps

Once a population of rules is inferred from GP, it is desirable to cluster them based on a chosen similarity criterion so as to provide a concise representation of the microstructure. The similarity criterion we choose is based on the *observed* *trading behavior*. Based on this criterion, two rules are similar if they are *observationally equivalent* or *similar*, or, alternatively put, they are similar if they generate the same or similar market timing behavior.

Given the criterion above, the behavior of each trading rule can be represented by its series of market timing decisions over the entire trading horizon, for example six months. Therefore, if we denote the decision "enter the market" by "1" and "leave the market" by "0," then the behavior of each rule is a binary string or a binary vector. The length of these strings or the dimensionality of the vectors is then determined by the length of the trading horizon. For example, if the trading horizon is 125 days long, then the dimension of the market timing vector is 125. Once each trading rule is concretized into its market timing vector, we can then easily cluster these rules by applying Kohonen's *self-organizing maps* (Section 12.6) to the associated clusters.

The main advantage of SOMs over other clustering techniques such as k-means is that the former can present the result in a *visualizable* manner so that we can not only identify these types of traders but also locate their two-dimensional position on a map, i.e., a distribution of traders over a map. Furthermore, if we suppose that we do not have dramatic crustal plate movement, so that the map is fixed over time, then the distribution of traders over the map can, in effect, be comparable over time. This provides us with a rather convenient grasp of the dynamics of the microstructure directly, as if we were watching the population density on a map over time.

However, the assumption of crustal stability does not hold in general; therefore, *maps over time are not directly comparable*. To make them comparable, some adjustments are needed. The idea of adjustment is also very intuitive. If the dominant strategy remains unchanged from period A to period B, then when we apply the dominant trading strategy derived from period A to another period B, the strategies should behave in a way that is similar to the dominant strategy derived from period B, if it is not exactly the same. This motivates us to *emigrate* all trading strategies from one map (the home map) to the other (the host map) in such a way that each emigrant shall find its new cluster on the host map based on the same similarity metric. In this manner, we can reconstruct a time-invariant version of the map, and comparison can be made upon this reconstruction.

14.5.4 An illustration from the Taiwan stock market

Figure 14.1 gives a concrete illustration of the idea presented above (Sections 14.5.2 and 14.5.3). Here, 500 artificial traders are grouped into nine clusters. The parameter value "500" refers to the *population size* used in genetic programming, i.e., the rule inference stage, whereas the parameter value "9" is due to a 3×3 two-dimensional SOM employed in the rule clustering stage. In a sense, this could be perceived as a snapshot of nine-type agent-based financial market dynamics. Traders of the same type indicate that their market timing behavior is very similar. The market fraction or the size of each cluster can be seen from the number of traders belonging to that cluster. Not surprisingly, they are not evenly distributed. Figure 14.1 shows that the largest cluster has a market share

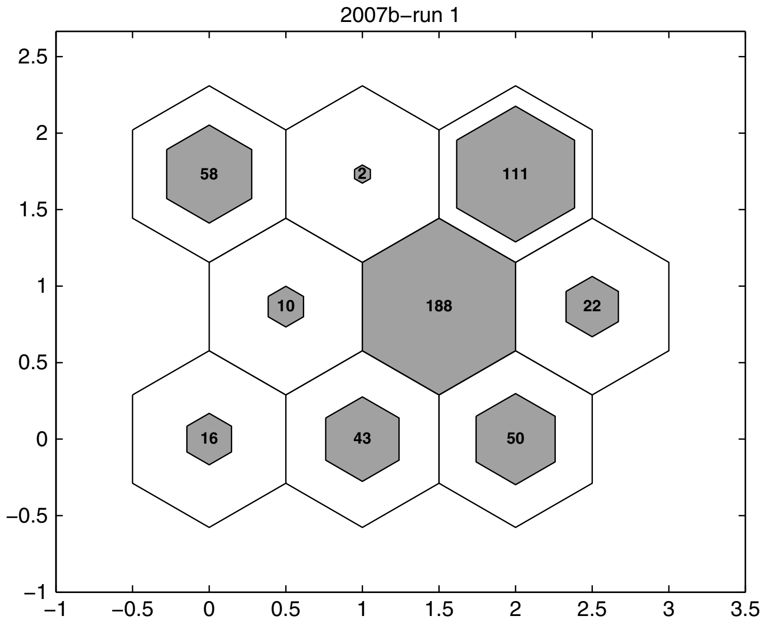

Figure 14.1 3 × 3 self-organizing feature map.

Note: This SOM was constructed by using the daily data of TAIEX from July 2007 to December 2007.

of 37.6 percent (188/500), whereas the smallest cluster has a market share of only 0.4 percent (2/500).

Once we can have a snapshot of the market fraction, we can go further over a series of snapshots so as to have a picture of the dynamics of the market fraction or the dynamics of the market microstructure. However, as we mentioned before, the SOMs constructed from different periods are not directly comparable; therefore, to make them all comparable, we have to first choose a base period and fix the map, i.e., to take the centroid of each cluster as given. In this particular example, we choose the second half of the year 2007 as the base. Once the centroids are given, all points (vectors) in other maps shall *immigrate* into this fixed map, and they are re-clustered based on their similarity to these fixed centroids. Figure 14.2 shows the reconstruction of these maps in this manner.

Figure 14.2 has the *market fraction maps* from the year 2006 to the year 2007, crossing four different periods. These maps were constructed by using the second half of 2007 as the base period. This figure gives a clear picture of what we mean by *market fraction dynamics*. First of all, we notice that the distribution over the clusters is uneven over time. In each period of time, some clusters obviously dominate others, but that dominance changes over time. This can be seen from the constant renewing of the major blocks. This visual inspection motivates us to

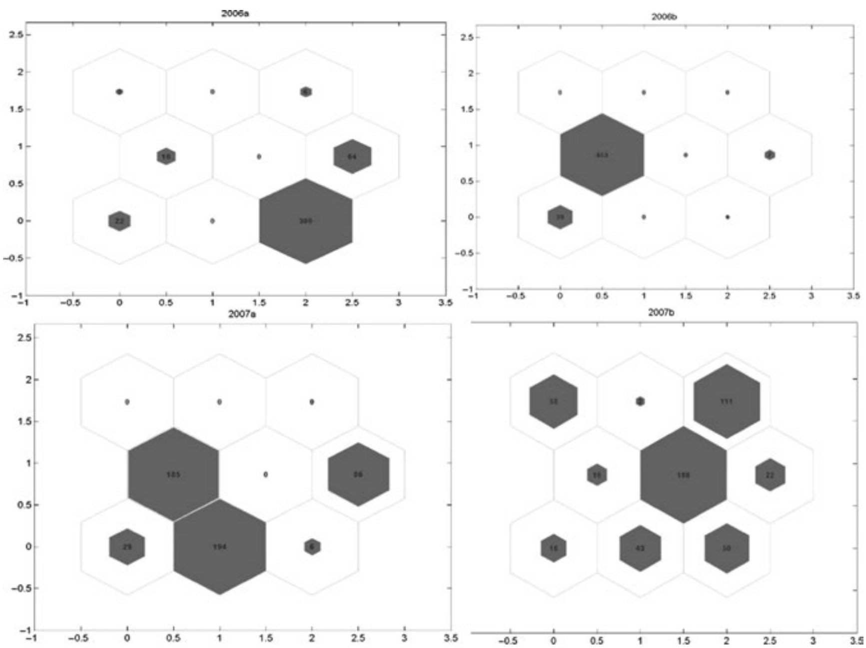

Figure 14.2 Market fraction dynamics: map dynamics.

Notes: The four SOMs are constructed using the daily data of TAIEX from 2006 to 2007. From the top-left panel to the bottom-right panel, they correspond to the first half and second half of year 2006 (2006a, b) and the first half and second half of the year 2007 (2007a, b). The first three are reconstructed by using 2007b as the base (see Section 14.5.3).

formulate two hypotheses which we already experienced from the dynamics of H-type agent-based financial models.

14.5.5 *Short-term dynamics*

The first hypothesis regards the *short-run dynamics of market fraction*. Each type of trader can be a dominant group (majority) for some of the time, but the duration of its dominance can only be temporal. The quick turnover of the dominant cluster or its short duration is consistent with the impression of the *swinging dynamics* as we saw in the two-type agent-based financial models, e.g., Kirman (1993). However, in addition to seeing the swing, it is desirable to have an objective measure of *how persistent a dominant cluster can be*. To do so, we need an operational meaning of dominance. Even though there is no unique way of doing this, we find the following threshold to be quite general and useful:

$$\bar{q} = \frac{1+p}{H+p}, \tag{14.17}$$

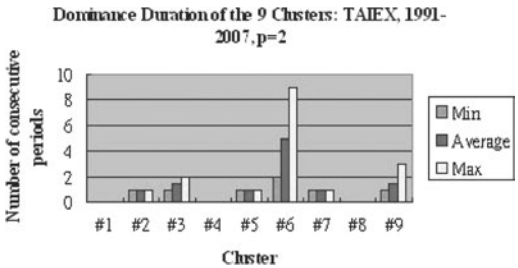

Figure 14.3 Duration of dominance ($p = 2$).

where H is the number of clusters, and p, a nonnegative integer, is a control parameter for the *degree of dominance*. Hence, a cluster is dominant if its market fraction exceeds this threshold. By varying the parameter p, one can therefore have an operational meaning that is consistent with our intuition regarding dominance. For example, if $H = 2$ (a two-type model) and $p = 2$, a cluster can be dominant only if its market fraction is greater than a $\bar{q}$ of 75 percent, a standard much higher than just breaking the tie (one half). Of course, the higher the p, the higher the threshold.

Figure 14.3 presents the *dominance-duration statistics* of each type of trader. Basically, we keep track of the persistent time of each dominance. After a type of trader becomes dominant, we count how many periods in a row it can remain the dominant cluster. Figure 14.3 gives three statistics regarding duration, namely minimum, average, and maximum. For example, for Cluster 6, these three statistics are 1, 3, and 9, respectively. In other words, the maximum duration of dominance for Cluster 6 is about nine periods, i.e., four and a half years. For other clusters, the longest duration is no more than three periods, i.e, one and a half years. So, for most of the time, dominant clusters can hardly continue for long. Hence, we reach the conclusion that, regardless of the types of traders, we rarely see consecutive dominance. In this sense, our data lend support to the market fraction hypothesis in a weak sense.

14.5.6 Long-term distribution

The second hypothesis we can form regarding the market fraction behavior is its *long-term distribution*. Many H-type agent-based f nancial models can show us that, under some proper parameter values, the long-term market fraction is *even*. In other words, if we have H types of traders, their long-term frequency of appearance should be close to $\frac{1}{H}$. Let $Card_{i,t}$ be the number (cardinality) of traders in Cluster i in time period t:

$$\sum_{i=1}^{H} Card_{i,t} = N, \quad \forall t. \tag{14.18}$$

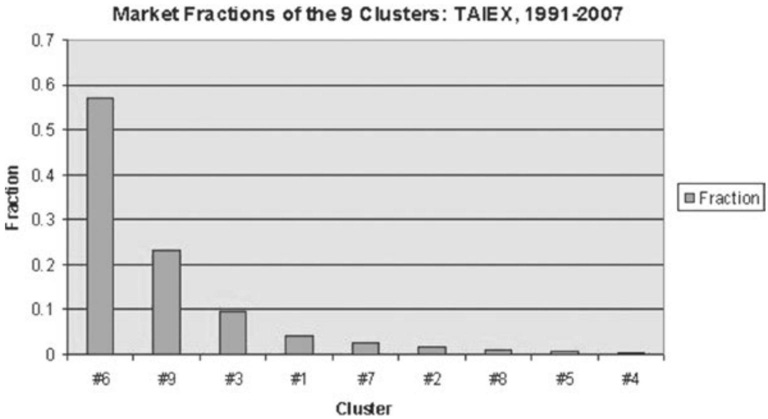

Figure 14.4 Long-term histogram.

In our current setting, N, the total number of traders, is 500. The long-term histogram can be derived by simply summing the number of traders over all periods and dividing it by a total of $N \times T$ (# of periods):

$$w_i = \frac{\sum_{t=1}^{T} Card_{i,t}}{N \times T}. \tag{14.19}$$

Figure 14.4 gives the long-term histogram of these clusters, $\{w_i\}$. Obviously, they are not equal, so we present them in descending order from the left to the right. Cluster 6 has the largest market fraction, up to almost 60 percent, whereas Cluster 4 has the lowest market fraction, not even up to 1 percent.

14.5.7 Does the number of types matter?

The illustration presented above is based on a 3×3 SOM, which automatically generates nine clusters. This analysis has its limitations mainly because we do not know how many types of agents are really there in the market. In a rather theoretical analysis, Aoki (2002b) showed that it would be enough to characterize the market behavior by a few types, say two to three. Others are rather marginal. Therefore, it would be interesting to investigate the microstructure dynamics based on a smaller SOM corresponding to the few-type agent-based financial models.[8]

In this section, we therefore repeat the above experiments by using a rather small 3×1 SOM. We then examine both its short-term dynamics and the long-term histogram. Figure 14.5 presents the duration statistics of each of the three clusters. The pattern is very similar to what we have observed in the case of nine clusters (Figure 14.3). Here, on average, no single cluster can take a duration of dominance of more than two years, but a single cluster (in this case, Cluster 3) can sometimes take a rather long duration of dominance of up to 4.5 years (9 periods).

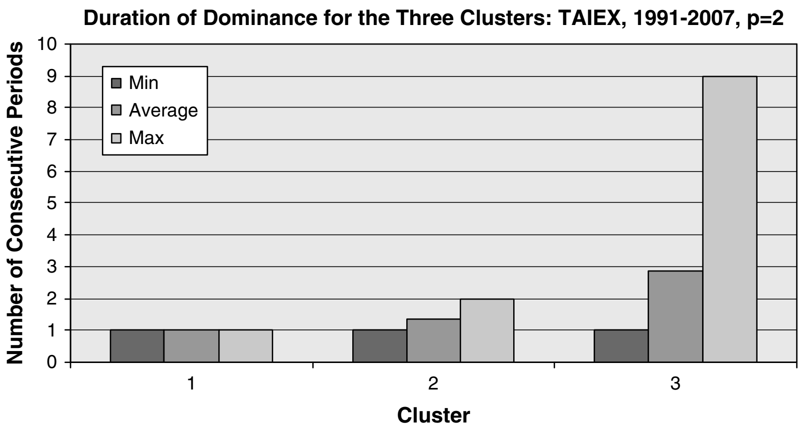

Figure 14.5 Market fraction: dominance duration ($p = 2$).

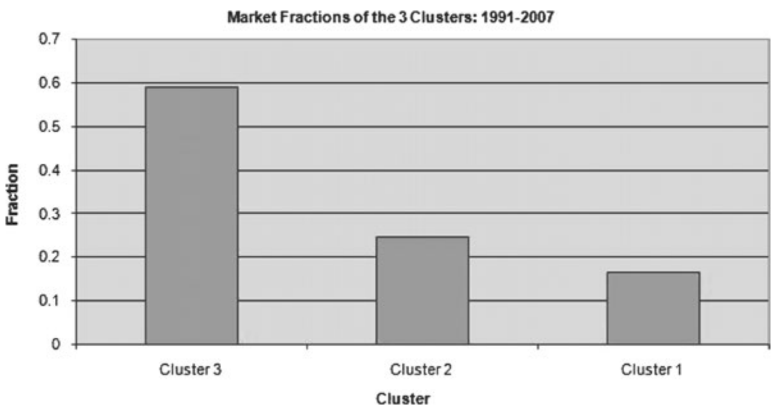

Figure 14.6 Market fraction: long-term histogram ($N = 3$).

Finally, the long-term histogram is shown in Figure 14.6. The long-term histograms of three clusters are obviously not uniform. Cluster 3 takes the largest long-term share, almost up to 60 percent, which overwhelmingly dominates the other two clusters.

The market fraction hypothesis that we have sketched in this section has been extensively explored in Chen, Kampouridis, and Tsang (2011) and Kampouridis, Chen, and Tsang (2012). Here, they have studied the dominance of duration and the long-term distribution among ten financial markets with different numbers of clusters. The results can be briefly summarized as follows. First, while most of the types, most of the time, can only have a short duration of dominance, there is always one or few types that can occasionally dominate the market for a long duration, say even up to four years. Second, even in the long run, different clusters would not be equally attractive. Population mainly concentrates on a few clusters, a property featured by Aoki (2002b). Nevertheless, the minimum number of clusters required to cover a large proportion of the behavior of market participants varies. If the target is to cover 90 percent of the market participants, then most markets need to consider four to five types, and if the target is even higher, up to 95 percent, then most markets need to consider five to six types; 95 percent is the parameter used in Aoki (2002b). Hence, our finding about the minimum number required is also larger than his suggested two or three. Two or three types can be sufficient if we have a target somewhat lower than 90 percent. Nevertheless, the property that a large number of clusters only contribute to the tail distribution is well supported by their findings.

In light of the Santa Fe artificial stock market, Kampouridis, Chen, and Tsang (2012) further examine the *dinosaurs hypothesis* by taking into account the possibility of the emergence of novel types of financial agents. Like the earlier proposed market fraction hypothesis, the dinosaurs hypothesis is also tested with various settings, such as its sensitivity to the rule inference engine (Kampouridis, Chen, and Tsang, 2011a), and is extended from a single market to multiple markets (Kampouridis, Chen, and Tsang, 2011b). Generally, they do not find strong evidence for the emergence of novel types of trading behavior over time (the dinosaurs hypothesis). Hence, the long-term microstructure dynamics can be largely restricted to the same set of trading behavior. This evidence may support the long-standing fundamentalist–chartist model as a good approximation to the reality.

Notes

1 The extension into the multinomial logit model is straightforward.

2 Aoki (2002b) is probably the only paper known to us that deals with the number of types of agents in multi-agent systems. Using the Ewens–Pitman–Zabell induction method, Aoki applies a result from the evolution of biological species and population genetics to determine the minimum number of types of behavior required to capture multi-agent economic systems. He shows that when agent behaviors are positively correlated, two large clusters are likely to develop to which most agents belong. This result, therefore, provides some justification for studying economic models composed of many interacting agents of two or three strategy types.

3 What may interest both economists and physicists is that the early study of molecules in physics was motivated by observing interactions among humans (Ball, 2006).

4 While the ideas of fundamentalists and chartists are the results of field work, abstracting the general observed behavior into a very specific mathematical model is a big leap.

5 See Lo (2005) for his eloquent presentation of the *adaptive market hypothesis*.

6 It does not necessarily mean that the types of traders surviving must be smart and sophisticated. They can be dumb, naive, randomly behaved, or zero-intelligent. Obviously, the notion of rationality or bounded rationality applying here is *ecological* (Simon, 1956; Gigerenzer and Todd, 1999).

7 Duffy and Engle-Warnick (2002) provide the first illustration of using genetic programming to infer the behavioral rules of human agents in the context of ultimatum game experiments. Similarly, Izumi and Ueda (2002) use genetic algorithms to infer behavioral rules of agents from market data.

8 As we shall see in Chapter 16, two-type or three-type agent-based financial models are still the most popular classes in the literature.

15 Artificial financial markets with autonomous agents

So far, all the types and rules of financial agents are given at the beginning of the design, and what financial agents can do is to choose among these different types and rules based on their past experiences. The H-type design has characterized a major class of agent-based financial markets. However, this way of doing things also severely restricts the degree of autonomy available for financial agents. First, they can only choose how to behave based on what has been offered; secondly, as a consequence, there will be no new rules available unless they are added from outside by the designers. If we want our artificial financial agents to behave more like real financial agents, then we will certainly expect that they learn and discover *on their own*. Therefore, as time goes by, new rules which have never been used before and have not been supplied by the designer may be discovered by these artificial agents inside the artificial world.

This chapter introduces the use of autonomous agents in artificial financial markets. Specifically, we review the development of the famous SFI artificial stock markets and other related developments.

15.1 Genetic algorithms

Designing artificial agents who are able to design on their own is an idea similar to John von Neumann's *self reproducing automata*, i.e., a machine which can reproduce itself. This theory had a deep impact on John Holland, the father of the *genetic algorithm*. Under von Neumann's influence, Holland had devoted himself to the study of a general-purpose computational device that could serve as the basis for a general theory of automata. In the 1970s, he introduced the genetic algorithm, which was intended to replace the ad hoc learning modules in contemporary mainstream AI. Using genetic algorithms, Holland could make an adaptive agent that not only learned from experience but could also be spontaneous and creative. The latter property is crucial for the design of artificial financial agents. In 1991, Holland and John Miller, an economist, published a sketch of an artificial adaptive agent in the highly influential *American Economic Review*. This blueprint was actually carried out in an artificial stock project in 1988 at the Santa Fe Institute (Palmer *et al.*, 1994; Arthur *et al.*, 1997).

15.1.1 Santa Fe Institute artificial stock market

Armed with GAs, the *Santa Fe artificial stock market* (SFI-ASM) considers a novel design for financial agents. First, like many H-type designs, it mainly focuses on the forecasting behavior of financial agents. Their trading behavior, as depicted in Equation (14.11), will depend on their forecasts of the price in the next period. Second, however, unlike the H-type designs, these agents are not divided into a fixed number of different types. Instead, the forecasting behavior of each agent is "customized" via a GA. We shall be more specific regarding its design because it provides us with a good opportunity to see how economists take advantage of the increasing computational power to endow artificial decision-makers with a larger and larger degree of autonomy.

In the SFI-ASM, each financial agent h uses a linear forecasting rule as follows:

$$E_{h,t}(p_{t+1}) = \alpha_{h,t} + \beta_{h,t} p_t. \tag{15.1}$$

However, the coefficients $\alpha_{h,t}$ and $\beta_{h,t}$ not only change over time (time dependent), but are also state dependent. That is, the value of these two coefficients at time t will depend on the state of the economy (market) at time t. For example, the recent price dynamics can be an indicator, so, say, if the price has risen in the last three periods, the financial agent may consider lower values of both α and β than otherwise. The price dividend ratio can be another indicator. If the price dividend ratio is lower than 50 percent, then the financial agent may want to take a higher value of β than if it is not. This state-dependent idea is very similar to what is known as *classification and regression trees* or *decision trees*, a very dominant approach in machine learning.

Therefore, one simple way to think of the artificial agents in the SFI-ASM is that they each behave as machine-learning people who use *regression trees* to forecast the stock price. At each point in time, the agent has a set of indicators that help him to decompose the state of the economy into m distinct classes, $(A^1_{h,t}, A^2_{h,t}, \ldots, A^m_{h,t})$, and corresponding to each of the classes there is an associated linear forecasting model. Which model will be activated depends on the state of the market at time t, denoted by S_t. Altogether, the behavior of the financial agent can be summarized as follows:

$$E_{h,t}(p_{t+1}) = \begin{cases} \alpha^1_{h,t} + \beta^1_{h,t} p_t & \text{if } S_t \in A^1_{h,t}, \\ \alpha^2_{h,t} + \beta^2_{h,t} p_t & \text{if } S_t \in A^2_{h,t}, \\ \vdots & \vdots \\ \alpha^m_{h,t} + \beta^m_{h,t} p_t & \text{if } S_t \in A^m_{h,t}. \end{cases} \tag{15.2}$$

A few remarks can be made here. First, the forecasting rule summarized above is updated as time goes by, as we keep the subscript t there. So, agents in this system are learning over time with a regression tree, or they are using a time-variant regression tree, in which all the regression coefficients and classes may change accordingly with the agents' learning. Second, agents are completely heterogeneous as we also keep the subscript h above. Therefore, if there are N financial agents in the markets at each point in time, we may observe N regression trees, each of which is owned and maintained by one individual agent. Third, however, the forecasting rules introduced in the SFI-ASM are not exactly regression trees. They are, in fact, *classifier systems*.

15.2 Classifier system

A classifier system is another of John Holland's inventions in the late 1970s. This system is similar to the Newell–Simon type of expert system, which is a population of if–then or condition–action rules. Conventional expert systems are not able to learn by themselves. To introduce adaptation into the system, Holland applied the idea of market competition to a society of if–then rules. A formal algorithm, known as the *bucket brigade algorithm*, credits rules generating good outcomes and debits rules generating bad outcomes. This accounting system is further used to resolve conflicts among rules. The shortcoming of the classifier system is that it cannot automatically generate or delete rules. Therefore, GA is applied to evolve them and to discover new rules.

This autonomous agent design has been further adopted in many later studies. While most studies continuously carried out this task using genetic algorithms, a few studies also used other population-based learning models, such as evolutionary programming and genetic programming.

15.3 Genetic programming and autonomous agents

The development from the few-type designs to the many-type designs and further to autonomous agent designs can be considered to be part of a continuous effort to increase the collective search space of the forecasting function $E_{h,t}$, from finite to infinite space, and from parametric to semi-parametric functions. The contribution of genetic programming to this development is to further extend the search space to an infinite space of nonparametric functions, whose *size* (e.g., the dimensionality, the cardinality, or the number of variables used) and *shape* (for example, linearity or nonlinearity, continuity or discontinuity) have to be determined, via search, simultaneously. This way of increasing the degree of autonomy may not contribute much to the price dynamics, but can enrich other aggregate dynamics as well as the behavior at the individual level. As we shall see below, the endogenous determination of the size and shape of $E_{h,t}$ provides us with great opportunities to see some aspects of market dynamics that are not easily available in the H-type designs or other autonomous agent designs.

The first example concerns *the sophistication of agents* in market dynamics. The definition and operation of GP rely on a specific language environment, LISP (see Section 13.4.5). For each LISP program, there is a tree representation. The number of nodes (leaves) or the number of depths in the LISP tree provides one measure of complexity in the vein of the *program length*. This additional observation enables us to study not just the heterogeneity in $E_{h,t}$, but also the associated complexity of $E_{h,t}$. In other words, genetic programming can not only distinguish agents by their forecasts, as the H-type designs did, but can further delineate the differentiation according to the agents' sophistication (complexity). Must the survival agents be sophisticated, or can simple agents prosper as well?

One interesting hypothesis related to the above inquiry is the *monotone hypothesis*: the degree of traders' sophistication is an increasing function of time. In other words, traders will evolve to be more and more sophisticated as time goes on. However, this hypothesis is rejected in Chen and Yeh (2001). They found that, based on the statistics on the node complexity or the depth complexity, traders can evolve toward a higher degree of sophistication, and at some point in time they can be simple as well.

The second example concerns the capability to distinguish information from noise. As we mentioned earlier, the variables recruited in the agents' forecasting function are also endogenously determined. This variable selection function allows us to examine whether the smart picking of these variables is crucial for survival. In particular, the hypothesis of the extinction of noisy traders says that traders who are unable to distinguish information from noise will become extinct. Chen, Liao, and Chou (2008) tested this hypothesis. In an agent-based artificial market, they supplied traders with both informative and noisy variables. The former included prices, dividends, and trading volumes, whereas the latter were just series of pseudo-random numbers. Their simulation showed, as time goes on, that traders who are unable to distinguish information from noise do have a tendency to decline and even become extinct.

15.4 Artificial stock markets

Among all applications of the agent-based approach to macroeconomic modeling, the most exciting one is the *artificial stock market*. By all standards, the stock market is qualified to be a complex adaptive system. However, conventional financial models are not capable of demonstrating this feature. On the contrary, the famous no-trade theorem shows in equilibrium how inactive this market can be (Tirole, 1982). It was therefore invigorating when John Holland and Brian Arthur established an economics program at the Santa Fe Institute in 1988 and chose artificial stock markets as their initial research project. The SFI artificial stock market is built upon the standard asset pricing model of Grossman and Stiglitz (1980). What one can possibly learn from this novel approach was well summarized in Palmer *et al.* (1994), which is in fact the first journal publication on an agent-based artificial stock market. A series of follow-up studies uncovered the content of this new fascinating frontier in finance.

15.4.1 Agent engineering and trading mechanisms

Agent-based artificial stock markets have two main aspects: agent engineering and institution (trading mechanism) designs. Agent engineering mainly concerns the construction of the financial agents. Tayler (1995) showed how to use genetic algorithms to encode trading strategies of traders. A genetic fuzzy approach to modeling traders' behavior was shown in Tay and Linn (2001), whereas the genetic neural approach was taken by LeBaron (2001). To simulate the agent-based artificial stock market based on the standard asset pricing model, the AI-ECON Research Center at National Chengchi University developed software known as the *AI-ECON artificial stock market* (AIE-ASM), which differs from the SFI stock market in the computational tool that is employed. The former applies genetic programming, while the latter has genetic algorithms. In AIE-ASM, genetic programming is used to model agents' expectations of the price and dividends. A menu-like introduction to AIE-ASM Version 2 can be found in Chen, Yeh, and Liao (2002).

In Chan *et al.* (1999) and Yang (2002) we see a perfect example of bringing different learning schemes into the model. The learning schemes incorporated into Chan *et al.* (1999) include empirical Bayesian traders, momentum traders, and nearest-neighbor traders, whereas those included in Yang (2002) are neural network traders and momentum traders. LeBaron (1999) gave a more thorough and general discussion of the construction of artificial financial agents. In addition to models, data is another dimension of agent engineering. What can be addressed here is the issue of stationarity that the series traders are looking at. Is the entire time series representative of the same dynamic process, or have things changed in the recent past? LeBaron (2001) studied traders who are initially heterogeneous in perception with different time horizons, which characterize their interpretation of how much of the past is relevant to the current decision-making.

Chen and Yeh (2001) contributed to agent engineering by proposing a modified version of social learning. The idea is to include a new mechanism, called the *business school*. Knowledge in the business school is open for everyone. Traders can visit the business school when they are under great survival pressure. The social learning version of genetic programming is applied to model the evolution of the business school rather than directly on traders. Doing it this way, one can avoid making an implausible assumption that trading strategies, as business secrets, are directly imitable. Yeh and Chen (2001a) further combined this modified social learning scheme with the conventional individual learning scheme in an integrated model. In this integrated model a more realistic description of traders' learning behavior is accomplished: the traders can choose to visit the business school (learning socially), to learn exclusively from their experience (learning individually), or both. In their experiments, based on the effectiveness of different learning schemes, traders will switch between social learning and individual learning. Allowing such a competition between these two learning styles, their experiment showed that it is the individual learning style which won the trust of the majority. To the best of our knowledge, this is the only study which leaves the choice of the two learning styles to be endogenously determined.

The second component of agent-based stock markets is the institutional design. An institutional design should answer the following five questions: Who can trade? When and how can orders be submitted? Who may see or handle the orders? How are orders processed? How are prices eventually set? Trading institutional designs in the conventional SFI artificial stock market either follow the Walrasian tâtonnement scheme or the rationing scheme. Chan *et al.* (1999) and Yang (2002), however, considered a double auction mechanism. This design narrows the gap between artificial markets and the real market, and hence makes it possible to compare the simulation results with the behavior of real data, e.g., tick-by-tick data. Since stock market experiments with human subjects were also conducted within the double auction framework (Smith, Suchanek, and Williams, 1988), this also facilitates the conversation between the experimental stock market and the agent-based artificial stock market.

Based on agent engineering and trading mechanism designs, agent-based artificial stock markets can generate various market dynamics, including price, trading volumes, the heterogeneity and complexity of traders' behavior, and wealth distribution. Among them, price dynamics is the one under the most intensive study. This is not surprising, because ever since the 1960s price dynamics have been the focus of studies on random walks, the efficient market hypothesis, and market rationality (the rational expectations hypothesis). With the advancement of econometrics, it further became the focus of the study of nonlinear dynamics in the 1980s.

15.4.2 Mispricing

Agent-based artificial stock markets make two important contributions to our understanding of the behavior of stock prices. First, they enable us to understand what may cause the price to deviate from the rational equilibrium price or the so-called fundamental value.

Both Yang (2002) and Chan *et al.* (1999) discussed the effect of momentum traders on price deviation. Yang (2002) found that the presence of momentum traders can drive the market price away from the homogeneous rational equilibrium price. Chan *et al.* (1999) had a similar finding: adding momentum traders to a population of empirical Bayesian traders has an adverse impact on market performance, although price deviation decreased as time went on. LeBaron (2001) inquired whether agents with a long-horizon perception can learn to effectively use their information to generate a relatively stable trading environment. The experimental results indicated that while a simple model structure with fixed long-horizon agents replicates the usual efficient market results, the route to evolving a population of short-horizon agents to long horizons may be difficult. Arthur *et al.* (1997) and LeBaron, Arthur, and Palmer (1999) found that when the speed of learning (the length of a genetic updating cycle) decreased (which forces agents to look at longer horizon features), the market approached the rational expectations equilibrium.

Chen and Liao (2002) is another study devoted to price deviation. They examined how well a population of financial agents can track the equilibrium price. By simulating the artificial stock market with different dividend processes, interest rates, risk attitudes, and market sizes, they found that the market price is not an unbiased estimator of the equilibrium price. Except in a few extremely bad cases, the market price deviates from the equilibrium price moderately from −4 percent to 16 percent. The pricing errors are in fact not patternless. They are actually negatively related to market sizes: a thinner market size tends to have a larger pricing error, and a thicker market tends to have a smaller one. For the thickest market that they have simulated, the mean pricing error is only 2.17 percent. This figure suggests that the new classical simplification of a complex world may still provide a useful approximation if some conditions are met, such as, in this case, the market size.

15.4.3 Complex dynamics

As to the second contribution, agent-based artificial stock markets also enhance our understanding of several stylized features well documented in financial econometrics, such as fat tails, volatility clusters, and nonlinear dependence. LeBaron, Arthur, and Palmer (1999) showed that the appearance of the ARCH effect and the nonlinear dependence can be related to the speed of learning. Yang (2002) found that the inclusion of momentum traders generates a lot of stylized features, such as excess volatility, excess kurtosis (leptokurtotic), lack of serial independence of return, and high trading volume.

Another interesting line is the study of emergent properties within the context of artificial stock markets. Emergence is about "how large interacting ensembles exhibit a collective behavior that is very different from anything one may have expected from simply scaling up the behavior of the individual units" (Krugman, 1996, p. 3). Consider the efficient market hypothesis (EMH) as an example. If none of the traders believe in the EMH, then this property will not be expected to be a feature of their collective behavior. Thus, if the collective behavior of these traders indeed satisfies the EMH as tested by standard econometric procedures, then we would consider the EMH as an emergent property. As another example, consider the rational expectations hypothesis (REH). It would be an emergent property if all our traders are boundedly rational, with their collective behavior satisfying the REH as tested by econometrics.

Chen and Yeh (2002) applied a series of econometric tests to show that the EMH and the REH can be satisfied with some portions of the artificial time series. However, by analyzing traders' behavior, they showed that these aggregate results cannot be interpreted as a simple scaling-up of individual behavior. The main feature that produces the emergent results may be attributed to the use of genetic programming, which allows us to generate a very large search space. This large space can potentially support many forecasting models in capturing short-term predictability, which makes simple beliefs (such as that where the dividend is an iid series, or that when the price follows a random walk) difficult to be accepted by traders. In addition to preventing traders from easily accepting simple beliefs, another consequence of a huge search space is the generation of sunspot-like signals through mutually reinforcing expectations. Traders provided with a huge search space may look for something that is originally irrelevant to price forecasts.

However, there is a chance that such kinds of attempts may mutually become reinforced and validated. The generation of sunspot-like signals will then drive traders further away from accepting simple beliefs.

Using Granger causality tests, Chen and Yeh (2002) found that dividends can indeed help forecast returns. By their experimental design, the dividend does not contain information on future returns. What happens is a typical case of mutually supportive expectations that make the dividend eventually contain the information on future returns.

As demonstrated in Chen and Yeh (2001, 2002), one of the advantages of agent-based computational economics (the bottom-up approach) is that it allows us to observe what traders are actually thinking and doing. Are they martingale believers? Are they sunspot believers? Do they believe that trading volume can help predict returns? By counting the number of traders who actually use sunspots or trading volumes to forecast returns, one can examine whether sunspot effects and the causal relation between stock returns and trading volume can be two other emergent properties (Chen and Liao, 2005; Chen, Liao, and Chou, 2008).

15.4.4 Market diversity and market efficiency

Yeh and Chen (2001b) examined another important aspect of agent engineering, i.e., *market size* (number of market participants). Few studies have addressed the significance of market size on the performance of agent-based artificial markets. One good exception is Bell and Beare (2002), whose simulation results showed that the simple tradeable emission permit scheme (an auction scheme) can be the most effective means for pollution control when the number of participants is small. However, as the number of participants increases, its performance declines dramatically and becomes inferior to that of the uniform tax scheme. Another exception is Bullard and Duffy (1999). In most studies, the number of market participants is usually determined in an arbitrary way, mainly constrained by the computational load. Arifovic (1994), however, justified the number of participants from the viewpoint of search efficiency. She mentioned that the minimal number of strings (agents) for an effective search is usually taken to be 30 according to artificial intelligence literature. Nonetheless, agent-based artificial markets have different purposes and concerns.

Related to market size is *population size*. In the case of social learning (single-population GA or GP), market size is the same as population size. However, in the case of individual learning (multi-population GA or GP), population size refers to something different, namely the number of solution candidates each trader has. Like market size, population size is also arbitrarily determined in practice.

Yeh and Chen (2001b) studied the effect of market size and population size upon market efficiency and market diversity under social and individual learning styles. Their experimental results can be summarized as two effects on market efficiency (price predictability), namely the *size effect* and the *learning effect*. The size effect says that the market will become efficient when the number of traders (market size) and/or the number of models (GP trees) processed by each trader (population size) increases. The learning effect says that the price will become more efficient if traders' adaptive behavior becomes more independent and private. Taking a look at market diversity, we observe very similar effects except for population size: market diversity does not go up with population size. These findings motivate us to search for a linkage between market diversity and market efficiency. A "theorem" may go as follows: a larger market size and a more independent learning style will increase the diversity of traders' expectations, which in turn make the market become more active (high trading volume) and hence more efficient (less predictable). Their simulation results on trading volumes also supported this "theorem." They further applied this "theorem" to explain why the US stock market behaves more efficiently than Taiwan's stock market.

15.4.5 Sunspot equilibria

Chen, Liao, and Chou (2008) studied the plausibility of sunspot equilibria within the context of agent-based financial models. Sunspot equilibria concern the real impacts of purely extrinsic uncertainty. The typical question, when coming to stock markets, is whether the stock price dynamics can be affected by any nonfundamental causes simply because investors believe so. It is not surprising to see that there is mapping between sunspot beliefs and sunspot equilibria, which has already been shown in the literature as an *existential proof*. However, how agents initially with heterogeneous beliefs can coordinate themselves to converge to such equilibria is largely unknown. In fact, the few studies using experiments with human subjects all find that it is difficult to coordinate such agents' beliefs to sunspot equilibria (Marimon, Spear, and Sunder, 1993; Duffy and Fisher, 2005).

Through agent-based simulation, Chen, Liao, and Chou (2008) extended those human-subject experiments to financial markets. In this special setting, agents do not know which inputs provided to them are fundamentals and which are simply sunspots. They have to build their own expectations, and, through time, review and revise their expectations so that, hopefully, irrelevant variables will be excluded. Genetic programming enables them to learn and discover the relevancy of each variable on their own, which also provides a natural correspondence to human decision-making in the real world. The subtlety of this work is the following. Sunspots should not have real impacts on the stock price, and agents are expected to eventually learn this; however, their trial-and-error process may mistakenly bring sunspots into their early formation of expectations and, in a self-fulfilling manner, successfully coordinate the emergence of a sunspot equilibrium before they can *collectively* see the irrelevance of sunspots. We believe that this is the essence of the study of sunspot equilibria, which is nonetheless missing in early literature. Chen, Liao, and Chou (2008) went further to estimate the possibility of having this "successful" coordination as an emergent property.

Their simulation results show that the chance of coordinating sunspot equilibria is almost nil. They found that the estimated probability of observing sunspot equilibria in this kind of artificial stock market is only 10 percent. This is consistent with what we generally learned from the experiments with human subjects.

However, what is missing in the human-subject experiments is that the absence of sunspot equilibria at the macro level does not automatically imply the extinction of sunspot beliefs at the micro level. Chen, Liao, and Chou (2008) found that the fraction of sunspot believers can be high—up to 60 to 80 percent of the entire population.

The inconsistency between the macro level and micro level requires an explanation. Following LeBaron (2001), they manipulated the parameter *time horizon*, and found that, given that the underlying data generation process is *stationary*, sunspot believers become extinct when the time horizon is lengthened. Therefore, how far agents look back while forming their expectations is essential for the determination of the market fraction of sunspot believers.

16 Empirically based agent-based models

Earlier, we have mentioned the relation between ACE and experimental economics with specific reference to the *mirroring function* (Chapter 6).[1] This goal generally applies to other ACE models which deal directly with actual field data instead of experimental data. While there are still a lot of ACE researchers considering their model exclusively for thought experiments, there is an increasing interest in building "empirically based, agent-based models" (Janssen and Ostrom, 2006). This requires ACE researchers to validate their models with real data, and has further developed ACE into econometric models which may be estimated by standard econometrics or other less standard estimation approaches. Maybe the most mature area to see the connection between ACE and econometrics is once again agent-based financial models.

This chapter reviews the development of agent-based computational economics from an econometrics viewpoint. The review comprises of three stages, characterizing the past, the present, and the future of this development. The first two stages can be interpreted as an attempt to build the econometric foundation of ACE, and, through that, enrich its empirical content. The second stage may then invoke a reverse reflection on the possible agent-based foundations of econometrics. While ACE modeling has been applied to different branches of economics, the one, and probably the only one, which is able to show this three-stage development is finance or financial economics. We will therefore focus our review only on the literature of agent-based financial markets.

16.1 Financial stylized facts

To some extent, the rise of agent-based computational finance is largely attributed to the advances in financial econometrics since the 1980s. From the 1980s to the present, financial econometrics has successfully identified a series of stylized facts. These stylized facts provide the statistical properties of price, return, trading volume, trading frequency, transactions, and bid-ask spread, which cover financial market dynamics to a quite extensive degree. However, the mainstream finance theory, which is built upon the assumption of the neoclassical rational expectations hypothesis, has difficulty in giving good explanations to many of these stylized facts. Therefore, this motivates the need for an alternative paradigm, and agent-based computational finance is developed as an attempt to account for these stylized facts. In the following, we shall provide a review of these stylized facts before we proceed to discuss what agent-based computational finance intends to accomplish.

Stylized facts refer to some properties that are common to a wide range of instruments, markets, and time periods. In other words, these properties are not restricted to a few specific stock markets. They can, for instance, be applied to the Tokyo stock market or the New York stock market. They are not restricted to a few specific financial instruments. They can, for example, be applied to stocks, foreign currencies, and commodities. Finally, they are not restricted to a certain period of time. They could be applied to the 1930s, 1960s, 2000s, . . . These stylized facts can be categorized into five groups, namely, returns, volumes, transactions, duration, and bid-and-ask spreads. This chapter will focus only on stock returns, since only the stylized facts of returns have been extensively addressed by the studies on agent-based computational financial markets and financial engineering. Volumes, transactions, duration, and spreads will certainly be research subjects in the near future, in particular as the accessibility of high-frequency financial data increases. Therefore, as a compromise, we have a small section below to serve as a reference guide for readers who are interested in knowing about financial stylized facts beyond returns.

Asset return tells us how much profit an investment generates for each dollar of the asset. Let P_t denote the price of a financial asset in time period t. Then the asset return can be written as follows:

$$r_{t,\Delta t} = \ln P_{t+\Delta t} - \ln P_t. \tag{16.1}$$

The expression above states the asset return in terms of the associated investment horizon Δt. It is called the daily return if Δt equals to 24 hours or one day.

16.1.1 Absence of autocorrelations

The first stylized fact, which also concerns most financial engineering people, has to do with whether one can forecast future returns using historical observations. The stylized fact known as the *absence of autocorrelations* gives a negative answer to this question. This stylized fact says that, by sampling any two points in time, the correlation of the two respective returns is largely insignificant. This property generally holds for any investment horizon longer than 20 minutes, i.e. $\Delta t > 20$ minutes (Cont, 2001).

$$\mathrm{Corr}(r_{t,\Delta t}, r_{s,\Delta t}) \approx 0 \quad \text{if} \quad t \neq s \quad \text{and} \quad \Delta t \geq 20 \text{ seconds}. \tag{16.2}$$

This property can also be read as part of the familiar efficient market hypothesis, which is the cornerstone of modern finance theory.

16.1.2 Calendar effect

While there is a lack of pattern in terms of linear correlation, some other patterns may still exist; the most famous and the most extensively studied is the so-called *calendar effect*, the second stylized fact. Basically, the calendar effect says that the performance of stocks can differ and is correlated with different times. So, for example, certain days of the week, weeks of the month, or months of the year are subject to above-average price changes in market indexes and can therefore represent good or bad times to invest. Famous calendar effects are the *Monday effect*, the *Halloween effect*, the *January effect*, and the *October effect* (Taylor, 2005).

16.1.3 Fat-tail distribution

The third stylized fact concerns the distribution of the return, in particular, the probability (risk) of observing extreme returns. The measurement of this probability or risk plays an extremely important role in modern risk management. While, in the conventional literature, the return is assumed to follow a normal distribution, recent empirical finance has quite successfully rejected this assumption. Instead, it is found that an asset has a higher probability of extreme returns than predicted by the normal distribution. In statistics, this property is referred to as *excess kurtosis*, and a distribution with excess kurtosis is called a *fat-tail*, *long-tail*, or *heavy-tail distribution* because the probability of extreme values decreases much more slowly than with other distributions such as the exponential or the normal. The third stylized fact is that returns are *leptokurtic* (Cont, 2001).

16.1.4 Aggregational Gaussianity

However, the fat-tail distribution will gradually converge toward a Gaussian distribution (normal distribution) when the investment horizon (Δt) increases. This property, as the fourth stylized fact, is referred to as aggregational Gaussianity.

16.1.5 Excess volatility

In addition to the distribution of returns, another important measure of the financial time series is volatility. Volatility refers to the amount of uncertainty or risk with regard to the size of changes in a stock's price. Intuitively speaking, higher volatility means that a stock's price can potentially increase or decrease dramatically over a short time period, whereas lower volatility means that a stock's price does not fluctuate dramatically, but changes in value at a steady pace over a period of time. Shiller (1981) shows that stock prices are too volatile in the sense that they fluctuate more than can be explained by the movement of the fundamentals, e.g., dividends. This famous work coins the fifth stylized fact, known as excess volatility. Whether stock price volatility systematically exceeds that justified by the fundamentals is currently still an unsettled debate. The feature that prices of financial assets do not behave as the dominant finance theory predicts leaves us with the so-called price volatility puzzle in modern financial economics.

16.1.6 Volatility clustering

Stock prices are not just highly volatile. Large changes in price tend to be followed by further large changes, and small changes tend to be followed by small changes. If we look at the time series of asset returns, we tend to easily identify periods of elevated volatility interspersed among more tranquil periods. This temporal concentration of volatility is commonly referred to as *volatility clustering*, and is the sixth stylized fact. Volatility clustering plays a pivotal role in the late development of financial econometrics. There are a number of econometric models being established to provide a rigorous treatment of this phenomenon. The earliest, and hence the most basic, is the *ARCH* model. It was first proposed in 1982 by Robert Engel, the 2003 Nobel Laureate in Economics. This model was later on extended by Tim Bollerslev in 1986 to the *GARCH* model, which has become the model most widely used by financial engineers.

16.1.7 Conditional heavy tails

It has been suspected that the fat-tail distribution may come from volatility clustering. Nonetheless, it turns out that even after correcting returns for volatility clustering, e.g., via the GARCH models, the residual time series still exhibit heavy tails Cont (2001). This latter property is referred to as *conditional heavy tails*, which can be considered as the seventh stylized fact.

16.1.8 Long memory

Earlier, we mentioned the quick decline in autocorrelation to zero as the first stylized fact of asset returns. While returns that are remote in time are uncorrelated, their absolute values $|r_{t,\Delta t}|$ or squares $|r_{t,\Delta t}|^2$ are not. In econometrics, the existence of a strong dependence between distant events is referred to as *long-range dependence* or *long memory*. Technically speaking, long memory refers to the very slow decay of the autocorrelation function: the decay in the autocorrelation function is *hyperbolic* and decays more slowly than exponentially, as shown below:

$$\text{Corr}(|r_{t,\Delta t}|, |r_{t+s,\Delta t}|) \approx s^{-\lambda}. \tag{16.3}$$

Long memory (long-term dependence) seems to be widespread in financial time series, and is regarded as the eighth stylized fact. Therefore, even though returns are not predictable, absolute returns and squared returns are still predictable. The most standard econometric model used to describe the long-memory property of stock returns is the *ARFIMA* (*autoregressive fractionally integrated moving average*) model, initially proposed by Clive Granger, winner of the 2003 Nobel Prize in Economics. Granger in 1980 introduced a process known as $I(d)$ to time series analysis. $I(d)$, where $0 < d < 1$, is a class of time series lying between $I(0)$ and $I(1)$. In the $I(0)$ time series, shocks to the system die out at an exponential rate, which implies rapid *mean reversion*. In the $I(1)$ time series, shocks to the system remain forever and do not dissipate at all; therefore, there is no mean reversion. Between the two extremes, shocks in the $I(d)$ series die out at a slow hyperbolic rate, which implies slow mean reversion.

It is interesting to note that $I(d)$ was derived by Granger as a particular limit of an infinite sum of a first-order autoregressive process, usually referred to as $AR(1)$, that has different parameters. This property can be closely connected to agent-based financial models. As we have seen before, the price change can be modeled as the sum of many heterogeneous agents' beliefs. Hence, the agent-based financial model provides a candid explanation for the emergence of the long-memory property of stock returns.

In addition to returns, it is found that volatility also exhibits long memory. As a result, the idea of an $I(1)$ process is applied to modify the GARCH process to capture a finite persistence of volatility shocks. This new class of volatility process models, known as *FIGARCH* (*fractionally integrated GARCH*), was first proposed by Richard Baillie in 1996.

16.1.9 Equity premium puzzle

The following three stylized facts all concern the relation between the stock return and stock volatility (risk). We start with the most prominent one, i.e., the *equity premium puzzle*, the ninth stylized fact. In economics and finance, normal investors are assumed to be risk averse, i.e., given the same target return, they prefer less risk. Hence, there must be an extra return to entice investors to hold more risky assets. This extra return is called the *risk premium*. The risk premium of the equity is defined as equity returns less bond returns, when it is considered that holding stocks is more risky than holding government bonds. The puzzle comes from this difference. Mehra and Precscott (1985) show that the equity premium has been 6 percent on average for the past century. This difference is unexpectedly large because it corresponds to an anomalously high level of risk aversion among investors implied by standard economic models.

16.1.10 Gain–loss asymmetry

Given the highly volatile nature of the stock price, the immediate relevant issue for an investor is market timing, i.e. when to enter and when to exit the market. Or, if we are already in the market, what is the optimal investment duration or the optimal waiting time before we can exit the market. A natural way to address this question is to set a predefined level of return, say 5 percent, and ask what waiting time is required to achieve this target. In principle, this waiting time can vary and is stochastic. Therefore, it is useful to know the waiting-time distribution. Such a distribution typically goes through a maximum at a time called the *optimal investment horizon*, since this defines the most likely waiting time for obtaining a given return. One can figure out the optimal investment horizon for 5 percent, and likewise the optimal investment horizon for -5 percent. The tenth stylized fact is the asymmetry between these two horizons. The one for the loss is faster than the one for the gain. This is known as the *gain–loss asymmetry*, which signifies that there are large drawdowns in stock prices and stock index values but not equally large upward movements (Cont, 2001).

16.1.11 Leverage effect

The eleventh stylized fact concerns the negative correlation between the stock return and stock volatility. Market knowledge has long been that the volatility of an asset is negatively correlated with the rate of return of the asset; a higher volatility is usually accompanied by a lower return. This common knowledge was first formally established in Christie (1982), which also explained the negative correlation between stock returns and stock volatility based on the change in the leverage of the underlying firm. This explanation was later named the *leverage effect*, and broadly refers to all kinds of negative correlations between asset returns and asset volatility, even though there are alternative explanations for this negative correlation.

16.1.12 Other stylized facts

Chapters 4 and 12 of Taylor (2005) serve as a good starting point to systematically learn more regarding the stylized facts above and some others. Chapter 12 is uniquely written for the stylized facts observed in intra-day high-frequency datasets, which have only been available over the last decade. The stylized facts for high-frequency returns are, however, similar to many of those for daily returns which we surveyed above, such as the absence of autocorrelation, fat-tail distributions, and long memory.

16.1.13 Stylized facts other than financial returns

Pacurar (2006) documents the stylized facts regarding trading duration, i.e., the time interval between two transactions. Some properties of returns, such as clustering and long memory, are also found in trading duration. Gabaix *et al.* (2003) study the stylized facts from the perspective of physics. They use the power-law distribution to describe financial returns, trading volumes, and transaction sizes. Muranaga and Ohsawa (1997) and Tsay (2002) provide some stylized facts pertaining to the bid-and-ask spread. These stylized facts other than returns will not be further discussed.

16.1.14 Summary and explanation of stylized facts

The first contact between ACE and econometrics was motivated by examining whether ACE models are able to provide explanations for some stylized features which the existing economic theories have difficulties accounting for. The most notable examples can be found in the modern financial literature, with a list of stylized facts that cannot be easily accounted for by the long-standing standard asset pricing models built upon the device of representative agents. Table 16.1 provides a comprehensive list of the stylized facts reviewed in this section.

Table 16.1 Stylized facts

No.	*Code*	*Stylized facts*	*Reference*
1	AA	Absence of autocorrelation	Cont (2001)
2	AG	Aggregational Gaussianity	Cont (2001)
3	BC	Bubbles and crashes	Rosser (1997)
4	CE	Calendar effect	Taylor (2005)
5	CHT	Conditional heavy tails	Cont (2001)
6	EPP	Equity premium puzzle	Kocherlakota (1996)
7	EV	Excess volatility	Cont (2005)
8	FT	Fat tails	Cont (2001)
9	GLA	Gain–loss asymmetry	Cont (2001)
10	LE	Leverage effect	Cont (2001)
11	LM	Long memory	Cont (2001)
12	PLBR	Power-law behavior of return	Gabaix *et al.* (2003)
13	PLBV	Power-law behavior of volatility	Lux (2009)
14	VC	Volatility clustering	Cont (2001)
15	VVC	Volatility volume correlations	Cont (2005)
16	PLBTV	Power-law behavior of trading volume	Gabaix *et al.* (2003)
17	VLM	Long memory of volume	Engle and Russell (2007)
18	AA-H	Absence of autocorrelation	Taylor (2005)
19	FT-H	Fat tails of return distribution	Taylor (2005)
20	LM-H	Long memory	Taylor (2005)
21	PE	Periodic effect	Taylor (2005)
22	BU	Bursts	Taylor (2005)
23	CTD	Clustering of trade duration	Pacurar (2006)
24	DLM	Long memory	Pacurar (2006)
25	DO	Overdispersed	Pacurar (2006)
26	PLBT	Power-law behavior of trades	Gabaix *et al.* (2003)
27	US	U shape	Tsay (2002)
28	SCPC	Spread correlated with price change	Tsay (2002)
29	TLS	Thinness and large spread	Muranaga and Ohsawa (1997)
30	TD	Turn-of-the-year declining	Muranaga and Ohsawa (1997)

Note: The stylized facts are separated into six blocks in the table. The first two refer to the stylized facts pertaining to *return* and *trading volume*, using low-frequency data. The next four refer to the stylized facts of *return*, *trading duration*, *transaction size*, and *bid-ask spread*, using high-frequency data.

There is a total of 30 stylized facts listed in this table, separated into six blocks. The first two blocks refer to the stylized facts of low-frequency financial time series, whereas the next four blocks refer to the stylized facts of high-frequency ones. The first two blocks give the stylized facts of returns and trading volume. The next four blocks refer to the stylized facts of returns, trading duration, transaction size, and bid-ask spread.

Stylized facts explained

This section gives summary tables (Tables 16.2 to 16.5) of each of 50 ACE models surveyed in Chen, Chang, and Du (2012), including the stylized facts replicated or explained by each of the models. In the second column of each table, we also indicate the intellectual origin or ingredients of each model. The most often seen ones are those which have been surveyed in Chapter 14, but there are also a few that are new here. The names of the origin are given in an acronym. Here, we list these acronyms with their full names. Some of the names given here are very suggestive.

- ABS: Adaptive belief systems
- ANT: ANT
- IM: Ising model
- IAH: Interacting agent hypothesis
- GT: Game theory
- MG: Minority games
- MS: Microscopic simulation
- PT: Prospect-theory-based model
- TM: Threshold model

16.2 Presenting agent-based economics with econometrics

Can the above stylized facts be generated from a collection of interacting software agents? To see the bigger picture, a few tables have been prepared to answer this question. We first categorize all agent-based computational finance (ACF) models based on the taxonomy given in Chapters 14 and 15. Some selected ACF models are separately listed in Tables 16.2, 16.3, and 16.4 (see Section 16.1.14), based

Table 16.2 Two-type designs and the stylized facts explained

Models	*Origin*	*Facts explained*	*Switch*
Alfarano, Lux, and Wagner (2005)	IAH	AA, FT, LM, PLBR, VC	O
Alfarano, Lux, and Wagner (2006)	IAH	FT, VC	O
Amilon (2008)	ABS	FT, VC	O
Boswijk, Hommes, and Manzan (2007)	ABS	BC	O
Chiarella, Dieci, and Gardini (2002)	ABS	FT, VC	X
Chiarella, He, and Hommes (2006)	ABS	AA, FT, LM, VC	O
De Grauwe and Grimaldi (2005a)	ABS	EV, FT, PLBR	X
De Grauwe and Grimaldi (2005b)	ABS	AA, AG, FT, PLBR, VC	O
de Jong, Verschoor, and Zwinkels (2009)	ABS	BC	O
Gaunersdorfer and Hommes (2007)	ABS	AA, EV, FT, VC	O
Gilli and Winker (2003)	ANT	AA, FT, VC	O
He and Li (2007)	ABS	AA, FT, LM, VC	X
Hommes (2002)	ABS	AA, FT, LM, VC	O
Kirman and Teyssière (2002)	ANT	AA, BC, LM, VC	O
Levy, Levy, and Solomon (2000)	MS	BC	X
Li and Rosser (2004)	ABS	AA, FT, LM, PLBR, VC	O
Manzan and Westerhoff (2005)	ABS	AA, FT, LM, VC	X
Winker and Gilli (2001)	ANT	AA, FT, VC	O

Table 16.3 Three-type designs and the stylized facts explained

Models	*Origin*	*Facts explained*	*Switch*
Amilon (2008)	ABS	FT, VC	O
Föllmer, Horst, and Kirman (2005)	ABS	BC, FT	O
Kaizoji (2003)	ABS	BC, FT	O
Lux (1998)	IAH	AG, BC, FT	O
Lux and Marchesi (1999)	IAH	AG, FT, PLBR, VC	O
Lux and Marchesi (2000)	IAH	AA, FT, LM, PLBR, VC	O
Parke (2007)	ABS	AA, FT, VC	O
Sansone and Garofalo (2007)	ABS	FT, RLM, VC	O
Suominen (2001)	GT	VC, VVC	O

Table 16.4 Many-type models: stylized facts explained

Models	*Origin*	*Facts explained*	*Switch*
Bovier, Cerny, and Hryniv (2006)	TM	BC, VC	X
Challet and Galla (2005)	MG	AA	X
Cont and Bouchaud (2000)	IM	AG, FT	X
Cross *et al.* (2007)	MG	AA, FT, LM, VC	X
Diks and van der Weide (2005)	ABS	AA, FT, VC	O
Ferreira *et al.* (2005)	MG	AA, FT, LM, VC	X
Ghoulmie, Cont, and Nadal (2005)	TM	AA, EV, FT, LM, VC	X
Iori (2002)	IM	AA, AG, LM, PLBR, VC, VVC	O
Pollard (2006)	TM	AA, EV, FT, VC, VVC	X
Sallans *et al.* (2003)	ABS	AA, FT, LM, VC, VVC	O
Shimokawa, Suzuki, and Misawa (2007)	PT	AA, EPP, EV, FT, GLA, VC, VVC	X

Table 16.5 Autonomous agent designs: stylized facts explained

Models	*Origin*	*Facts explained*	*Switch*
Arifovic and Gencay (2000)	SFI	AA, FT, VC	X
Chen and Yeh (2001)	SFI	FT	X
Derveeuw (2005)	SFI	BC, FT	X
Lawrenz and Westerhoff (2001)	SFI	AG, FT, LM, PLBR, VC, VLM	X
LeBaron, Arthur, and Palmer (1999)	SFI	AA, FT, VLM, VC, VVC	X
LeBaron (2006)	SFI	FT, VC	X
LeBaron and Yamamoto (2007)	SFI	LM, VLM	X
Martinez-Jaramillo and Tsang (2009)	SFI	AA, FT, LM, PLBR, VC	X
Neuberg and Bertels (2003)	SFI	BC, FT	X
Neuberg, Louargant, and Protin (2004)	SFI	FT, LM, VC	X
Raberto *et al.* (2001)	SFI	FT, LM, VC	X
Reimann and Tupak (2007)	SFI	AA, FT, LM, VC	X
Tay and Linn (2001)	SFI	AA, FT, VC	X

on their designs being two-type, three-type, or many-type, whereas another group of ACF models using the autonomous-agent (AA) design is listed in Table 16.5 (also in Section 16.1.14). The four tables together include 50 ACF models, among which there are 38 N-type models and 12 autonomous-agent models; the ratio of the two is 3 : 1. The constituents of the N-type models are 18 (47 percent) two-type models, 9 (24 percent) three-type models, and 11 (29 percent) many-type models.

16.2.1 Demographic structure

These four tables are by no means exhaustive, but comprise just a sample of a large group of existing studies. Nonetheless, we believe that they well represent some basic characteristics of the underlying literature. First, the largest class of ACF models is the few-type design. The sum of the two-type and three-type models account for about 50 percent of the entire sample. The few-type design models not only have support from empirical studies, but their simplicity also facilitates analytical work. This is probably the most important reason for seeing the dominance of this few-type design. Nonetheless, if a large degree of heterogeneity can be represented by a parametric distribution, then one can directly work with this distribution so as to extend the analytical work to the many-type designs. Using the continuous-choice model to replace the discrete-choice model is one case in point. This helps N-type design models gain an additional market share of 20 to 25 percent. The residual goes to the autonomous-agent design models, which only take up one-fourth of the sample. This minority position may indicate that economists are not yet ready to accept the complex heterogeneous agents which may easily defy analytical feasibility, but the more important question is: can more complex designs provide us with better explanatory power?

16.2.2 General performance

In Tables 16.2 to 16.5, each ACF model is shown with the stylized facts which *can* be replicated by the designated model. We have to be careful as to what we mean by *can*. First, we have not verified the model, and hence are not in a position to provide a second check as to whether the reported results are correct. In this regard, we assume that the verification of each model has been confirmed during the referral process. Second, we do, however, make a minimal effort to see whether proper statistics have been provided to support the claimed replication. A study that does not satisfy this criterion will not be taken seriously.

In addition, each ACF model is associated with an acronym which stands for the origin of the respective ACF model. Those origins have been briefly mentioned in Section 16.1.14. In addition to that, the last column serves to show whether the ACF model has a switching mechanism to allow agents to change their type. Switching or *evolving fractions* have been considered to be a fundamental cause of many stylized facts (Kirman, 1991, 1993; Hommes, 2002; Lux and Marchesi, 1999, 2000). Therefore, it is useful to have this column to examine whether the evolving fraction is a necessary condition for replicating stylized facts.

Table 16.6 Summary of stylized facts explained

No.	*Stylized facts*	*Two-type*	*Three-type*	*Many-type*	*AA*	*# studies*
1	AA	11	2	9	5	27
2	AG	1	2	2	1	6
3	BC	3	2	2	2	9
4	EPP	0	0	1	0	1
5	EV	2	0	3	0	5
6	FT	14	7	9	11	41
7	GLA	0	0	1	0	1
8	LM	7	2	5	6	20
9	PLBR	4	3	1	2	10
10	VC	14	6	9	8	37
11	VLM	0	0	0	3	3
12	VVC	0	1	4	1	6
# facts		8	8	11	9	

Note: The last row "# facts" is the total number of stylized facts that are replicated by the corresponding class of ACF models. So, for example, the numbers 8, 8, 11, and 9 appearing in the last row indicate that there are a total of 8 stylized facts replicated by two-type models, 8 stylized facts replicated by three-type models, and so on. It is *not* the total sum of the number appearing in the respective column.

We shall start with an overview table, Table 16.6. The last column of the table is a frequency count of the replications of the respective stylized facts shown in the first column. While there are 30 stylized facts shown in Table 16.1, only 12 appear in Table 16.6. The stylized facts which do not appear in this table are the ones which have not been replicated in any of the 50 papers in our survey. As a result, all stylized facts pertaining to high-frequency data, or the so-called intraday data, are left unexplained by the ACF models, which include high-frequency returns (18–22, Table 16.1), trading duration (23–25), transaction size (26–27), and the bid-ask spread (28–30). In fact, even for those stylized facts appearing in the table, the counts associated with them are quite uneven.

First, there are four stylized facts which obviously receive more intensive attention than the others. These four are fat tails (41 counts), volatility clustering (37), absence of autocorrelation (27), and long memory of returns (20). Fat tails and volatility clustering received the highest attention. This, in fact, could be anticipated given the close connection between the two and the dominance of various GARCH-type models in financial econometrics. Long memory ranks fourth. This again is closely related to volatility clustering since long memory reflects long-run dependencies between stock market returns, and volatility clustering describes the tendency of large changes in asset prices to follow large changes and small changes to follow small changes.

Second, we also notice that all the stylized facts pertain exclusively to *asset prices*; in particular, all these efforts are made to tackle *low-frequency* financial time series. This sharp bias is also not difficult to understand. Although most ACF models have their own artificial clock, it is not clear what the interval $[t, t+1]$ can best mean in real time. Does it refer to a tick, a minute, an hour, a day, or a week? There is no easy way in which we can tell the difference from the corresponding models. The difference can only be made via the empirical part of the models, which requires a consideration of which dataset is involved. Since the high-frequency dataset is rarely used as the empirical correspondence of these ACF models, it remains a challenge to extend these ACF models to the high-frequency domain. The other reason for the observed bias is that most ACF models use either the Walrasian tâtonnement scheme or the market-maker scheme to determine the market price. The continuous-time double auction mechanism or the order-driven mechanism is rarely used. Time, therefore, becomes less sensible with the simplified mechanism.

16.2.3 The role of heterogeneity and learning

Our taxonomy of the ACF models enables us to address two specific questions. First, would the degree of heterogeneity matter, and, second, would learning matter? Of course, these two aspects are not completely disentangled because complex learning models may also be the ones with complex heterogeneity. Therefore, our answer below may be very tentative or rough, and is best used to motivate further studies.

Do many-type models gain additional explanatory power compared to few-type models? The last row of Table 16.6 shows that the few-type models (two-type and three-type) can together replicate nine stylized facts, whereas many-type models can add two more. These two are the equity premium puzzle (EPP) and gain–loss asymmetry (GLA). Nevertheless, from Table 16.4, these two are the results of the same study (Shimokawa, Suzuki, and Misawa, 2007), which focuses more on the prospect theory feature of financial agents. The loss-averse feature of financial agents is largely not shared by other ACF models; therefore, it is not clear whether the credit should go to the many-type design. If we conservatively remove this "outlier," then the many-type models do not perform significantly better than the few-type models.

Would more complex learning behavior help? Table 16.6 shows that the only additional stylized fact captured by the autonomous-agent design is the long memory of volume (VLM), and there are studies devoted to this replication (LeBaron, Arthur, and Palmer, 1999; Lawrenz and Westerhoff, 2001; LeBaron and Yamamoto, 2007; see also Table 16.5). In all these three studies, autonomous agents are modeled with genetic algorithms, but GAs alone should not be the main cause of long memory in trading volume, and there is no particular reason why N-type models are unable to replicate long memory in volume. Data on trading volume existed in many N-type ACF models, but they were not well exploited. Hence, this advantage of AA designs may not be that absolute, and it is likely that N-type models or even the few-type models just perform equally as well as the autonomous-agent models.

In sum, we do not find significant evidence in support of the complex design of ACF models, including many-type designs and autonomous-agent designs. If our interests are just restricted to the stylized facts listed in Table 16.6, then we can confine ourselves to just the few-type designs. This can have very important implications for the next stage of the development of ACF models, namely, building econometric models of agent-based financial markets. As we shall see in Section 16.3, few-type designs have another advantage when we want to treat the ACF model as an econometric model and estimate it.

16.3 Building ACE with econometrics

The use of econometric techniques to validate or estimate the agent-based models surveyed in Section 16.2 started in the late 1990s and early 2000s (Miller, 1998; Winker and Gilli, 2001). As we see in Section 16.2, the stylized facts can be replicated qualitatively by many different classes of agent-based models, ranging from low-dimensional parametric models to high-dimensional parametric ones. Moving one step further, the second stage of the development of agent-based models is not just satisfied with its capabilities to grow stylized facts in a qualitative sense, but is more concerned with the appropriate parameter values used in the model. This development is keenly anticipated. Supposing that we are given the significance of the *intensity of choice* (λ in Equation 14.7) in generating some stylized facts, then the next legitimate question will be: can this intensity be empirically determined, and if so, how big or how small is it?

16.4 How to estimate?

16.4.1 Direct estimation

The general idea of estimation is to derive the *aggregation equation* in which all these parameters are encapsulated. Then by applying appropriate statistical techniques to these equations one can derive the estimates of these parameters. Hence, if the aggregate equation happens to be a likelihood function, then the method involved is naturally the maximum likelihood method (Alfarano, Lux, and Wagner, 2005; Alfarano, Lux, and Wagner, 2006, 2007). If the aggregate equation happens to be a regression, then the employed method is least squares (de Jong, Verschoor, and Zwinkels, 2006; Boswijk, Hommes, and Manzan, 2007; Manzan and Westerhoff, 2007).

16.4.2 Indirect estimation

However, for some classes of agent-based financial models, particularly those complex agent-based models reviewed in Chapter 15, it may not be easy to derive the aggregation equation in an analytical way, so that the direct application of econometric techniques is not feasible. Hence, an alternative approach is that, instead of deriving the aggregation analytically, the aggregation is derived via simulation, and the econometric steps are applied based on these simulated aggregations. Some estimation work shown in Table 16.7 belongs to this kind, which is also known as *indirect estimation*, to be distinguished from the above-mentioned direct estimation. Examples of indirect estimation are Winker and Gilli (2001), Gilli and Winker (2003), Winker, Gilli, and Jeleskovic (2007), and Amilon (2008).

Table 16.7 Estimation methods of ACF models

Models	*Origin*	*Methods*	*Parameters estimated*
Alfarano, Lux, and Wagner (2005)	IAH	ML	Herding tendency
Alfarano, Lux, and Wagner (2006)	IAH	ML	Herding tendency
Alfarano, Lux, and Wagner (2007)	IAH	ML	Herding tendency
Amilon (2008)	ABS	EMM/ML	Intensity of choice
Boswijk, Hommes, and Manzan (2007)	ABS	NLS	Belief coefficients/intensity of choice
de Jong, Verschoor, and Zwinkels (2006)	ABS	NLS	Belief coefficients/intensity of choice
de Jong, Verschoor, and Zwinkels (2009)	ABS	NLS	Belief coefficients/intensity of choice
Diks and van der Weide (2005)	ABS	ML	ARCH and GARCH relations/sign of MA
Ecemis, Bonabeau, and Ashburn (2005)	AA	IEC	Market fractions/behavioral rules
Gilli and Winker (2003)	ANT	MSM	Mutation/conviction rate
Manzan and Westerhoff (2007)	ABS	OLS	Reaction coefficients/switching threshold
Midgley, Marks, and Kunchamwar (2007)	A-Life	EC	NA
Miller (1998)	A-Life	EC	NA
Reitz and Westerhoff (2007)	ABS	Quasi ML	Behavioral rules/intensity of choice
Westerhoff and Reitz (2003)	ABS	Quasi ML	Behavioral rules/intensity of choice
Winker and Gilli (2001)	ANT	MSM	Mutation/conviction rate
Winker, Gilli, and Jeleskovic (2007)	ANT	MSM	Mutation/conviction rate

Note: The full meaning of the acronyms under "Origin" are available in Section 16.1.14. Here, we only provide the full name of those under "Methods": ML stands for maximum likelihood, EMM for efficient method of moments, NLS for nonlinear least squares, OLS for ordinal least squares, IEC for interactive evolutionary computation, and MSM for method of simulated moments.

Since this approach is relatively new for agent-based economists, and can potentially be applied to a larger class of ACE models, we will, therefore, give a brief review of this approach in this section.

16.4.3 Simulated-based estimation

Despite the progress observed in Section 16.4.3, agent-based models in general are very hard to estimate due to the lack of tractable criterion functions (likelihood functions, moments). Nevertheless, this problem has been largely shared by many other economic or econometric models,[2] and has received in-depth treatment during the last two decades, which has also inspired the development of entirely new procedures based on simulation, referred to as *simulation-based econometric methods*.[3] As we shall see later, these new proposed methods are very applicable and may dramatically open the empirical accessibility of agent-based models in the future.

Simulation-based econometric methods include the *method of simulated moments* (MSM), *simulated maximum likelihood* (SML), *methods of simulated scores* (MSS), *efficient method of moments* (EMM), and *indirect inference*. Some of these methods have already been applied to estimate agent-based models. For example, the MSM is used in Winker and Gilli (2001), Gilli and Winker (2003), and Winker, Gilli, and Jeleskovic (2007), whereas the EMM is used in Amilon (2008).

16.4.4 Methods of simulated moments

The basic idea of simulation-based inference is to calibrate the parameter vector so that the properties of the simulated series *resemble* those of the observed data. Take MSM as an example. We first choose a vector of parameter values to generate the simulated time series by running the agent-based model with this chosen set of parameter values. We then compare some statistics (moments) of this simulated time series, the *simulated moments*, with those using real data, the *sample moments*. The difference between the two is used to form a distance function (the objective function). The MSM is purported to minimize the distance by searching over the entire parameter space.

Formally speaking, let $\mathbf{X}$ be a set of chosen statistics derived from the real data, $\mathbf{X} = (X_1, X_2, \ldots, X_m)$, and $\mathbf{Y}$ be $\mathbf{X}$'s counterpart in the artificial data, $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_m)$. In addition, let $\mathcal{L}$ be a distance function between $\mathbf{X}$ and $\mathbf{Y}$. Then the indirect estimation of a set of parameters θ involves finding out the θ^* such that

$$\theta^* = \arg\{\min_{\theta \in \Theta} \mathcal{L}(\mathbf{X}, \mathbf{Y}; \theta)\}. \tag{16.4}$$

The indirect method, as claimed by Winker and Gilli (2001), is general enough to be applied to other agent-based financial models, including SFI-like models. However, one of the main difficulties is solving Equation (16.4). Of course, an analytical solution in general is rarely available, and one has to rely on some numerical approaches.

Example: Winker and Gilli (2001)

Winker and Gilli (2001) attempted to estimate a version of the Kirman model (Kirman, 1991, 1993). There are two major parameters in this model, the self-conversion rate and the conviction rate, denoted by θ_1 and θ_2, respectively. To replicate the two characteristic features of financial time series, namely, fat tails and volatility clustering, their choice of $\mathbf{X}$ includes the *kurtosis* and the *first-order ARCH coefficient*, which are denoted by X_1 and X_2. Then, given a specific choice of the distance function as shown below, the indirect estimator of θ is:

$$\theta^* = (\hat{\theta}_1, \hat{\theta}_2) = \arg \min_{(\theta_1,\theta_2)\in\Theta} |Y_1 - X_1| + |Y_2 - X_2|. \tag{16.5}$$

Y_1 and Y_2 are the counterparts of X_1 and X_2 in the artificial data. They are not based on a single run of the Kirman model $\mathcal{M}(\theta_1, \theta_2)$, but on 10,000 runs. In other words, Y_1 and Y_2 are the averages taken over the 10,000 sample statistics derived from the corresponding 10,000 artificial time series. To solve the optimization problem posed in Equation (16.5), Winker and Gilli (2001) and Gilli and Winker (2003) suggested the use of the simplex method combined with threshold accepting.

Practical issues of MSM

It can be seen from the previous illustration that the indirect estimation results, $\hat{\theta}$, can depend on a number of choices, which include:

1 the dimensionality of the target vector ($\mathbf{X}$);
2 the statistics included in the target vector ($X_1, X_2, \ldots$);
3 the parameters considered (θ);
4 the distance function ($\mathcal{L}$);
5 the search algorithm used to solve global optimization;
6 the number of runs of the model $\mathcal{M}(\theta)$, or the sample size upon which $\mathbf{Y}$ is derived.

This sensitivity may introduce a number of issues involved in the use of the indirect estimation approach. The first issue has to do with the selection of adequate statistics $\mathbf{X}$. Winker, Gilli, and Jeleskovic (2007) proposed two criteria: *robustness* and *powerfulness*. The robustness criterion is involved because θ may be sensitive to the actual realization of $\mathbf{X}$, say, $\mathbf{x}$. If $\mathbf{x}$ is not stable over the entire sample, it would be less meaningful to anchor a specific $\hat{\theta}$ to the whole series. The powerfulness criterion requires the statistics involved to exhibit the potential to discriminate between alternative models and/or parameter constellations.

The second issue is the selection of the parameter vector (θ). While, in principle, all parameters involved in the agent-based model under study should be included, it is very difficult to do this in practice. This is mainly because the search space grows exponentially with the dimension of θ. Hence, this practical restriction makes us able to estimate only a subset of $\theta = (\theta^1, \theta^2)$, say θ^1, and the rest, θ^2, has to be fixed a priori.

16.4.5 Genetic algorithms

What seems to be interesting is that the estimation of agent-based models actually starts with the one involving complex objective functions (Miller, 1998; Ecemis, Bonabeau, and Ashburn, 2005; Midgley, Marks, and Kunchamwar, 2007). A common feature shared by these papers is the use of *genetic algorithms*. Midgley, Marks, and Kunchamwar (2007) provide a general framework for this approach.[4] The idea is to use genetic algorithms to evolve the agent-based models so as to optimize the objective function.

Among all econometric agent-based models, Ecemis, Bonabeau, and Ashburn (2005) is probably the one with the most general objective function. They consider the possibility that the objective function for a model cannot be practically formulated mathematically, and is highly qualitative. In general, how well the model reproduces the data qualitatively can be no different from the appraisal of a performing art. A few statistics or moments may not be able to capture all we want. When it is hard to articulate the objective function that reveals what we really want, then an automatic evaluation of a model becomes unfeasible. An alternative solution is to allow financial experts to directly evaluate the models based on their expertise.

16.5 What to estimate?

The next issue concerns *what is estimated*. This issue is as important as the issue of how to estimate, because it is what was estimated that helps us to gain more insights from the agent-based financial models. In particular, many agent-based financial models emphasize the contribution of learning or social interaction to the asset price dynamics. It would then be desirable to know whether these effects are *empirically significant*. In the last column of Table 16.7, we list the key parameters estimated by each model.

16.5.1 ANT model

We start with the ANT model, proposed by Kirman (1991, 1993), since this is the first agent-based financial model being estimated. The ANT model is a two-type model consisting of fundamentalists and chartists. Let q_1 and q_2 be the fraction of fundamentalists and chartists, respectively, $q_1 = 1 - q_2$. There are two key parameters in the ANT model. Let θ_1 and θ_2 be the mutation rate (the self-conversion rate) and the conviction rate, respectively.[5] It is found that when the following inequality, Equation (16.6), holds, the market fraction $q_{1,t}$ spends little time around the value of one-half but a great deal of time in the extremes, i.e., nearly zero or one:

$$\theta_1 < \frac{\theta_2}{N-1}, \tag{16.6}$$

where N is the number of agents. The ANT model has been estimated three times in the literature (Winker and Gilli, 2001; Gilli and Winker, 2003; Winker, Gilli, and Jeleskovic, 2007). Using the indirect estimation mentioned in Section 16.4.4, Winker and Gilli (2001) found

$$\hat{\theta}_1 < \frac{\hat{\theta}_2}{N-1}. \tag{16.7}$$

Gilli and Winker (2003) considered a three-parameter ANT model; in addition to the two parameters above, they also included a parameter related to noise distribution.[6] Equation (16.7) is again sustained. Based on Kirman (1991), this finding indicates that there is significant switching between the fundamentalists and chartists; sometimes the market is dominated by fundamentalists, and sometimes by chartists.[7] Winker, Gilli, and Jeleskovic (2007) moved back to the two-parameter ANT model, but by using an objective function, which is closer in spirit to the simulated method of moments. This time they also gave a significance test of the model being estimated via a bootstrap method.[8] Whether the relation (16.7) holds was no longer discussed explicitly, even though the model with the parameter vectors considered was rejected.

16.5.2 Lux's IAH model

As discussed in Section 14.3, there are two determinants that govern the switching behavior in Lux's IAH model, i.e., *herding* and *the profit differential*. When it comes to the estimation of the Lux model, only the simple version with herding was estimated in Alfarano, Lux, and Wagner (2005); Alfarano, Lux, and Wagner (2006, 2007), whereas the more general version with both determinants was estimated in Winker, Gilli, and Jeleskovic (2007). It should be noted that although herding is a common switching setting in the ANT model and the Lux model, the magnitudes of the parameters play different roles in these two models. In the ANT model, the switching parameters must satisfy Equation (16.6) in order to generate a bimodal distribution which replicates the original observation in entomology, and this is also an indication of significant switching behavior. In the Lux model, herding is presented only in the sense of following the majority (micro level) but is not necessarily a strongly dominated group all the time (macro level). In addition, the significance of switching behavior can be estimated and inferred directly (Alfarano, Lux, and Wagner, 2005; Alfarano, Lux, and Wagner, 2006, 2007) or indirectly (Winker, Gilli, and Jeleskovic, 2007).

Alfarano, Lux, and Wagner (2005); Alfarano, Lux, and Wagner (2006, 2007)

A two-type Lux model with only a herding mechanism in the transition rate function was estimated by Alfarano, Lux, and Wagner (2005); Alfarano, Lux, and Wagner (2006, 2007).[9] Similar to Kirman (1991, 1993), they introduced the parameters of an idiosyncratic propensity a and herding tendency b. The difference is that the idiosyncratic propensity to switch to the other strategy is *asymmetric*, and is denoted as a_1 and a_2, which is different from the common probability of self-conversion implying $a_1 = a_2$ in Kirman's setting. The major parameters to be estimated in this model are ε_1 and ε_2, where $\varepsilon_{1,2} \equiv \frac{a_{1,2}}{b}$. Let z be the fraction of noise traders. It can be shown that ε_1 and ε_2 determine the average percentage of noise traders in the market:

$$E(z) = \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}. \tag{16.8}$$

They can characterize the market by a dominance of fundamentalists' or noise traders' activity through estimating the magnitude of switching parameters. Although there is an exception in the Australian stock market (Alfarano, Lux, and Wagner, 2006), the noise traders' dominance in stock markets, i.e., $\hat{\varepsilon}_1 > \hat{\varepsilon}_2$, seems a robust result over almost every individual stock in Japan and Germany (Alfarano, Lux, and Wagner, 2005; Alfarano, Lux, and Wagner, 2007). Furthermore, the model with an extreme switching tendency, $\varepsilon_1 \gg \varepsilon_2$, implying extreme dominance of noise traders, cannot be rejected for a large number of cases in Japan.

Winker, Gilli, and Jeleskovic (2007)

Another empirical estimation of the Lux model was performed by Winker, Gilli, and Jeleskovic (2007), based on the indirect estimation mentioned in Section 16.4.4. They estimated a three-type Lux model including fundamentalists and two types of chartists, i.e., optimists and pessimists. Unlike Alfarano, Lux, and Wagner (2005); Alfarano, Lux, and Wagner (2006, 2007), the complexity of this version of the Lux model makes the selection of the parameters being estimated no longer so trivial. They only chose three parameters: the general frequency of the revision of opinion between optimists and pessimists, the general frequency of the revision of opinion between fundamentalists and chartists, and the reaction strength of the fundamentalists' excess demand. A significance test of the model being estimated was also performed via the bootstrap method. As it turns out, the Lux model was rejected similar to the rejection of the ANT model, and the estimates of the parameters were not discussed in this work.

16.5.3 ABS model

Unlike Kirman (1991), the ABS model (Brock and Hommes, 1998) leaves the forecasting rules, in parametric forms, that are employed by agents to be determined by the data. In some settings, the estimation of these coefficients can be interesting. For example, in the simple fundamentalist–chartist model, the expectations of fundamentalists and chartists can be described as in Equations (14.2) and (14.3). The parameters to estimate are the reverting coefficients (α_f) and (α_c). As discussed in Section 14.1, the magnitude of α_f measures the speed with which the fundamentalists expect the price to return to the fundamental one, whereas the magnitude of α_c expresses the degree to which chartists expect the past to change in the future. Together with the intensity of choice (λ) in the switching model (Equation 14.7), there are at least three economically meaningful parameters to be estimated, namely:

$$(\theta_1, \theta_2, \theta_3) = (\alpha_f, \alpha_c, \lambda). \tag{16.9}$$

There are at least two empirical estimations of the ABS model (Boswijk, Hommes, and Manzan, 2007; Amilon, 2008).

Boswijk, Hommes, and Manzan (2007)

Boswijk, Hommes, and Manzan (2007) estimated a three-parameter ABS model using yearly S&P 500 data from 1871–2003. The three parameters are those described in (16.9).[10] Their estimated θ_1 and θ_2 are significantly different from zero, and are also correct in terms of their magnitude, which justifies the coexistence of the mean-reverting belief and the trend-following belief. Their estimation also provides us with a time series of the market fraction, which again shows that $\hat{q}_1$ (the fraction of fundamentalists) can swing from zero to one, with a mean of around 0.5. This result is similar to the empirical results from the ANT model (Winker and Gilli, 2001; Gilli and Winker, 2003), and lends support to the *market fraction hypothesis* or *evolving fraction hypothesis*. What may differ from the ANT model is the learning behavior of agents. In the ABS model, the learning behavior is mainly captured by the parameter *intensity of choice*, which, however, is found not to be significant in Boswijk, Hommes, and Manzan (2007).

Market fraction hypothesis

The market fraction hypothesis basically says that the financial time series observed can be described in terms of switching between different types of agents, each with either different beliefs or different trading rules. In sum, the market dynamics are well explained by the evolving fractions of different types of financial agents. This is one of the most fundamental implications obtained from ACF models. Using yearly S&P 500 data from 1871 to 2003, Boswijk, Hommes, and Manzan (2007) found that the two types of investors, fundamentalists and chartists, coexist and their fractions exhibit considerable fluctuations over time. Before the 1990s, only occasionally did chartists (trend-chasers) dominate, and this dominance never persisted for more than two consecutive years. However, in the late 1990s the fraction increased to close to one and persisted for a number of years. The persistently high fraction of chartists in the market contributed to an explosive growth of the stock market, and resulted in annual returns of more than 20 percent for four consecutive years. Similar findings regarding the considerable fluctuations in the fractions are also obtained in other different agent-based financial markets, including foreign exchange markets.

Amilon (2008)

Amilon (2008) estimated both a two-type and a three-type ABS model. For the latter, in addition to the behavioral rules (14.2) and (14.3), he also introduced *contrarians* into the model with their behavioral rule described in Equation (14.4). Furthermore, both the behavorial rules (14.3) and (14.4) were extended to incorporate a *memory* parameter, β_c and β_{co} respectively, as in Equations (14.5) and (14.6). Therefore, in addition to the mean-reverting and extrapolation coefficients, the memory of both chartists and contrarians becomes another parameter to estimate. This, with other extensions, actually leads to a 9-parameter two-type ABS model and a 14-parameter three-type ABS model.[11]

It was found that all the major behavioral coefficients in the two-type model are insignificant, whereas they, including the intensity of choice, are all significant in the three-type model.[12] Nevertheless, most of the parameters are right in terms of magnitude. In particular, in the three-type model, all three heterogeneous beliefs are successfully identified. What, however, is interesting is that the swing between different types of agents is restricted only to the chartists (momentum traders) and contrarians. The fraction of fundamentalists is effectively zero. In other words, the inclusion of contrarians makes it hard for fundamentalists to survive. It, therefore, makes us wonder whether these three types of agents are redundant and either a fundamentalist–chartist model or a trend-chaser and contrarian model would suffice to do the job.

SFI-like models: AGEDASI TOF

AGEDASI TOF, standing for *A GEnetic-algorithmic Double Auction SImulation in the TOkyo Foreign exchange market*, was initially proposed by Izumi and Okatsu (1996) and Izumi and Ueda (1999). It follows an autonomous-agent design. Like the SFI-ASM, agents (artificial dealers) make their investment decisions (buying or selling foreign currency) based on their forecasts of the investment return and risk. To forecast investment return, each artificial dealer will refer to a number of economic and political variables, such as gross domestic product, the consumer price index, interest rates, money supply, etc. However, the exact relationships between the foreign exchange rate and these variables are unknown, and have to be exploited.

Therefore, in the vein of the SFI-ASM, AGEDASI TOF also used genetic algorithms to model agents' learning behavior (forecasting). GA helps agents to decide the magnitudes and signs assigned to each variable. Up to this point, there is nothing significantly different from the SFI model. However, AGEDASI TOF allows its artificial dealers to get access to external data, i.e., data from the real world, to forecast the exchange rate, which means the data, such as GDP, CPI, etc. are all real. This modeling strategy is essentially equivalent to a direct estimation of the behavioral rules (forecasting models) of agents, and then uses the estimated rules of each agent to generate the aggregate behavior (exchange rate).

The system has been shown to have a better forecasting performance than benchmarks, such as the random walk. In particular, it has a good performance in long-term forecasting; it can also be used to forecast the probability of a particular event, such as bubbles and crashes. For example, the model gives a probability of 42 percent of observing a bubble in the Tokyo foreign exchange market in the years 1989–1991.

From an estimation viewpoint, the econometric approach exemplified by AGEDASI TOF is not a real econometric approach but a computational intelligence approach in a bottom-up style. This direct bottom-up estimation approach is more intuitive and applicable to the SF-type models; nonetheless, whether it can satisfy the econometrics standard requires further study.

16.6 Forecasts with agent-based financial models

It has been long asked, instead of just replicating or growing the stylized facts, whether the agent-based model can be a useful tool for forecasting. In other words, in addition to providing a bottom-up mechanism to explain the stylized fact as an emergent outcome, interest is further drawn to the prediction power of the same mechanism. The recent research trend seems to indicate that one can be cautiously optimistic about this possibility. This is particularly so given the recent contributions of de Jong, Verschoor, and Zwinkels (2006) and Manzan and Westerhoff (2007).[13]

16.6.1 De Jong, Verschoor, and Zwinkels (2006)

Following standard econometric forecasting procedures, first of all, de Jong, Verschoor, and Zwinkels (2006) estimated two versions of a three-type ABS model, one with a switching mechanism and one without. The first two types of agent are the usual fundamentalists and chartists (Equations 14.2 and 14.3, or, in general, Equation 14.5), but, for the third type, they considered a different kind of chartist, called the *MA-chartist*, whose forecasting rule is based on the difference between a long-term moving average and a short-term moving average:

$$E_{ma,t}[p_{t+1}] = p_t + \alpha_{ma}(MA_t^L - MA_t^S), \tag{16.10}$$

where MA_t^L refers to the moving averages with a window length of L and MA_t^S refers to the one with S ($L > S$). One can consider the MA-chartist as a kind of fundamentalist if the fundamental price p_t^f in Equation (14.2) is replaced by the long-term moving average MA_t^L.

De Jong, Verschoor, and Zwinkels (2006) estimated this three-type model using data from eight foreign exchange markets. They found evidence supporting heterogeneity in the behavior of agents, but not supporting a significant intensity of choice. This, with similar findings in Boswijk, Hommes, and Manzan (2007) and Amilon (2008) (the three-type case), really casts doubt on the essence of the adaptive belief system, namely the sensitivity to profit or performance differentials. The estimated model is then applied to forecast the future exchange rate with different horizons. Using the Diebold–Mariano test (Diebold and Mariano, 1995), they were able to demonstrate that the three-type model can outperform the random walk in most markets with various settings of different horizons or criteria. Nevertheless, this superiority in forecasting may not be taken seriously since the usual distinction between the training sample and test (unseen) sample is not made in their study.

16.6.2 Manzan and Westerhoff (2007)

In a similar vein, Manzan and Westerhoff (2007) estimated two-type models with and without switching using monthly exchange rate data. The adaptive behavior in this paper is not driven by the *profit differential* or *herding*, but it is an exogenous *threshold value* that governs the sentiment of the chartist. From the econometrics perspective, they actually incorporated a *threshold autoregressive* structure, hence the nonlinearity, into the model. On the other hand, since this kind of threshold mechanism itself implies an immediate switch once the state variable passes the threshold, it is equivalent to having an infinitely large *intensity of choice* in the binary choice model (Equation 14.8). The estimation results and nonlinearity tests are in favor of the model with a switch. They then dynamically estimated the proposed two-type model with a rolling window and made the one-step-ahead forecast accordingly. Using the Diebold–Mariano test, they showed that the two-type model can outperform the random walk model for two of six currencies.

In not using the Diebold–Mariano test just as a predictability test, Manzan and Westerhoff (2007) also raised a question about its test power. They treated their agent-based model as a real data-generating process that contains both a linear adjustment process (the fundamentalists) and a nonlinear switching mechanism (the chartists). These two components can be captured by the estimation methods proposed by Mark (1995) and Diebold and Nason (1990). It should be expected that the nice in-sample estimation results could carry over into the out-of-sample forecasting for simulated time series and lead to better forecasting accuracy over the random walk. However, the Diebold–Mariano test can not really endorse such an improvement, which means that its test power is quite low.

16.6.3 Prospectives

Even though the forecasting capability of the agent-based model has not been consolidated, there are reasons to be optimistic. First, the bottom-up mechanism in the agent-based model behaves as a pool of many agents' forecasting, which is very similar to what is known as *prediction markets* or information markets. Second, the emergent behavior (the aggregate behavior) also behaves as a combination of different forecasts, which is, therefore, a kind of *combined forecast*. In the literature, evidence of the superiority of the prediction markets over the poll and other forecasts and evidence of the superiority of the combined forecast over simple forecasts already exists, while not necessarily overwhelmingly. Therefore, it is anticipated that the agent-based model under a suitable construction process may lead to some desirable outcomes.

16.7 ACE as a foundation of econometrics

Most empirically based ACE models only consider how econometrics can help to build or validate the ACE models. Few have explored that the other way around may be equally interesting. In this section, we shall present some thoughts on the reverse direction, i.e., instead of an econometric foundation for agent-based economics, an agent-based foundation for econometrics. In this regard, ACE can present challenges to econometrics.

16.7.1 Aggregation problems

Intuitively, ACE can help econometrics because it is a kind of *micro–macro approach*. This micro–macro approach has been reviewed by Stoker (1993) as an approach to address the *aggregation problem*. The ACE model, as a computational model, provides us with greater flexibility to deal with various levels of aggregation over individuals. Unlike many other micro–macro models, it does not have to make very stringent assumptions regarding individual behavior in order to have a tractable aggregation. This advantage makes us able to include more realistic behavioral aspects of individuals into the aggregation, such as learning and interactions. In the following, by using an *agent-based consumption asset pricing model* (Chen, Huang, and Wang, 2010), we shall demonstrate how the ACE model can help solve the aggregation problem.

16.7.2 Estimation using the individual data

Chen, Huang, and Wang (2010) start with an individual i's consumption Euler equation:

$$\Delta c_t^i = \tau^i + \psi^i r_{t-1}^i + \xi_t^i, \quad i = 1, 2, \ldots, I, \tag{16.11}$$

where Δc_t^i is the consumption growth at time t,

$$\Delta c_t^i = \log\left(\frac{c_t^i}{c_{t-1}^i}\right). \tag{16.12}$$

In a consumption capital asset pricing model (CAPM), ψ^i, under suitable assumptions, is also known as the *elasticity of intertemporal substitution* (EIS). r_t^i is the real return on the asset at t, τ^i is a constant, and ξ_t^i is the residual variable. Notice that the heterogeneity of individuals in terms of ψ^i makes Equation (16.11) also heterogeneous among agents. Furthermore, the heterogeneity in terms of investment behavior also makes the rates of return r_t of each agent heterogeneous, which are denoted by r_t^i in Equation (16.11).

The return facing each individual is determined by his or her chosen portfolio $\alpha_t^i = \{\alpha_{m,t}\}_{m=1}^M$, and can be calculated as:

$$r_t^i = \log(R_t^i), \tag{16.13}$$

where

$$R_t^i = \sum_{m=1}^{M} \alpha_{m,t}^i R_{m,t} \tag{16.14}$$

and

$$R_{m,t} \equiv \frac{p_{m,t} + w_{m,t}}{p_{m,t-1}}. \tag{16.15}$$

$p_{m,t}$ is the price of the asset m at time t, and $w_{m,t}$ is the dividend paid for the asset m at time t.

To estimate the coefficients ψ_i, we may consider the set of l individual equations as one giant equation, and estimate the l ψ_is altogether. This is the familiar *seemingly unrelated regression estimation* (SURE). SURE can be useful when the error terms (ξ_t^i) of each equation in (16.11) are related. In this case, the shock affecting the consumption of one agent may spill over and affect the consumption of the other agents. Hence, estimating these equations as a set, using a single large equation, should improve efficiency. To apply SURE, they rewrite the set of equations (16.11) into a single equation as (16.16):

$$\Delta\mathbf{c} = \Gamma + \mathbf{r}\Psi + \Xi, \tag{16.16}$$

where

$$\Gamma = \begin{pmatrix} \tau^1 \\ \tau^2 \\ \vdots \\ \tau^{30} \end{pmatrix}, \Delta\mathbf{c} = \begin{pmatrix} \Delta c^1 \\ \Delta c^2 \\ \vdots \\ \Delta c^{30} \end{pmatrix}, \mathbf{r} = \begin{pmatrix} r^1 & 0 & \dots & 0 \\ 0 & r^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & \dots & r^{30} \end{pmatrix}, \Psi = \begin{pmatrix} \psi^1 \\ \psi^2 \\ \vdots \\ \psi^{30} \end{pmatrix}, \Xi = \begin{pmatrix} \xi^1 \\ \xi^2 \\ \vdots \\ \xi^{30} \end{pmatrix}.$$

16.7.3 Estimation using the aggregate data

The early econometric work on the estimation of the elasticity of intertemporal substitution mainly used only aggregate data (Hall, 1988). In their paper, Chen, Huang, and Wang (2010) also estimate the ψ based on the aggregate data:

$$\Delta c_t = \tau + \psi r_{t-1} + \xi_t. \tag{16.17}$$

Equation (16.17) is the Euler consumption equation that is based on the assumption of treating the whole economy as a single representative agent. This version is the one frequently used in macroeconometrics. Δc_t and r_t are the corresponding aggregate variables of consumption and returns.

16.7.4 Micro–macro relation

One way to formulate the aggregation problem is the relation between $\hat{\Psi}$ (the individual estimates) based on Equation (16.16) and $\hat{\psi}$ (the aggregate estimate) based on Equation (16.17). For example, if ψ^i are all identical, say $\psi^i = 1$, $\forall i$, can we have $E(\hat{\psi}) = 1$? Furthermore, if we are informed of the distribution of ψ^i, what can we further say about $E(\hat{\psi})$? This is the kind of aggregation problem that has been generally surveyed in Stoker (1993), which detailed a solution called the *micro–macro approach*. This approach models the comparability of individual behavioral patterns and aggregate data patterns, removing any mystery induced by the one-sided focus of studying aggregate data alone or individual data alone. The approach has been further pursued recently by Gallegati *et al.* (2006b).

Chen, Huang, and Wang (2010) used an agent-based consumption CAPM to simulate all artificial time series required by the individual Euler equation, and they estimated the individuals' EIS coefficients through Equation (16.16). Then they also ran the aggregate Euler equation (16.17) by using the aggregate data. What surprises us is that the two estimates can be quite distinct. Given the assumption that $\psi^i = 1$ $(\forall i)$, none of the individual and aggregate Euler equation can recover this behavioral parameter in a reasonable close range. Most estimated individual ψ^is are around 0.3, whereas the estimated aggregate ψ is close to zero.

Elasticity puzzle: real or spurious?

If we further assume EIS to be the reciprocal of the coefficient of risk aversion, then this result implies an unreasonably high risk aversion. The latter conundrum is also known as the *elasticity puzzle* (Neely, Roy, and Whiteman, 2001). What, therefore, shows in Chen, Huang, and Wang (2010) is that if we follow the standard econometric practice with the representative agent framework, then we can easily have this purely spurious result simply because of the negligence of aggregation over interacting heterogeneous bounded rational agents. This phenomenon has also been noticed by Delli Gatti *et al.* (2007):

> If agents are heterogeneous, some standard procedures (e.g. cointegration, Granger-causality, impulse-response functions of structural VARs) lose their significance. Moreover, neglecting heterogeneity in aggregate equations generates *spurious evidence* of dynamic structure.
>
> (Delli Gatti *et al.*, 2007, p. 62; emphasis added)

ACE as a data generation mechanism

Hence, by using ACE in the model as a data generation mechanism, we can examine the behavior of various econometric tests, particularly, those macroeconometric tests, to see whether they actually behave well when the data are indeed the aggregation over interacting heterogeneous bounded rational individuals. Since the ACE model can easily accommodate different kinds of learning algorithms and interaction mechanisms, this prompts us to perform a sensitivity analysis to see whether the ideal behavior of the standard econometrics can be robust to different micro-underpinnings.

16.7.5 Imperfect data

ACE can help to test some hypotheses that are very difficult to test on the real data. This can happen when the respective real data are not easily available. One example concerns agents' expectations. In the literature on *sunspot equilibria* in macroeconomics, there is a concern among macroeconomists as to whether something that is totally extrinsic to a system can eventually have effects on the operation of the system, simply because agents "naively" *believe* so. Nevertheless, it is widely shared among macroeconometricians that a direct empirical test of the existence of sunspot beliefs and sunspot equilibria can be rather difficult due to the lack of observed data. Some recent progress in this area has been made using the experimental approach (Duffy and Fisher, 2005), but ACE can provide an alternative way out of the empirical difficulty (Chen, Liao, and Chou, 2008).

ACE models can directly control the interactions among agents as well as their learning processes, and indirectly define the transmission process of sunspots. Of course, it is hard to know whether the real counterpart is the same or similar to the one used in a specific ACE model, but at least we know what the setting is and we can change the setting. This specific setting is equivalent to the assumption of a theorem, and we can then use standard econometric tests to examine whether belief matters with respect to different assumptions.

16.8 Concluding remarks

This chapter provides a first review of the ACE models from an econometric viewpoint. Given the fast growth of both financial econometrics and agent-based finance, this chapter, in many ways, serves only as a beginning and is by no means exhaustive.

First, the list of stylized facts is not exhaustive. We believe that the exploration of various financial data which were hardly available before will soon contribute to the lengthening of the list of stylized facts. Actually, one area which we do not cover well concerns the data on the *order book* and the recent empirical findings that have triggered another wave of development of ACF models. This is definitely a space which we should fill in a future study.

Second, the standpoints by which we examine and differentiate various ACF models are also limited. We focus mainly on the behavioral rules of financial agents. This very suggestive taxonomy of the literature makes us able to see the minimum condition required to replicate the stylized facts in terms of the heterogeneity and complexity of behavioral rules. Needless to say, there are other ingredients that deserve further distinctions. The design of a network of interactions is an example. While most ACF models do implicitly assume a network for interactions (Kirman, 1991; Cont and Bouchaud, 2000; Iori, 2002), work on the explicit modeling of interactions (Zovko and Farmer, 2007; Hein, Schwind, and Spiwoks, 2008) is the one area neglected in this survey. Therefore, the extent to which social networks can contribute to the understanding of stylized facts is also a direction for the next. Other examples include risk preferences (Shimokawa, Suzuki, and Misawa, 2007), asynchronous updating (Boswijk, Hommes, and Manzan, 2007), information asymmetry (Suominen, 2001), and, finally, institutional constraints (Farmer, Patelli, and Zovko, 2005).

Despite this incompleteness, it is found that the models with simple heterogeneity and simple rules (few-type models), in particular the variations of the fundamentalist–chartist model, are sufficient to replicate a number of stylized facts. A complex extension of this model may gain additional explanatory power, but so far this power has not been well exploited. In addition, the simple model makes later econometric estimation much more feasible.

The econometric estimation of agent-based financial market models is an ambitious task. In principle one can identify various details of agents, such as their beliefs, memory, intensity of choice, risk perceptions, risk aversions, etc. It may also help us to infer the underlying fitness function, e.g., profits versus risk-adjusted profits, from the data. For the few-type model, one can further discover the evolving market fractions so as to track the mood of the market. Nevertheless, before we reap the fruits of these models, there is still a long way to go. So far, we have not been assured that these agent-based financial market models are econometrically significant, and the estimates are stable. One of the difficulties in front of us is how to effectively tackle the numerical aspects of the estimation of a complex objective function involving a large number of parameters.

However, the value of agent-based models should not be very much restricted to just replication or validation. This is particularly so when the econometrics to support quality validation is still lacking. Section 16.7 suggests that treating agent-based models as theoretical models that behave as many other stochastic simulation models can help us to examine the power of the established econometric tests. This function can be most valuable when the environment is filled with various degrees of polluted or missing data. The chapter therefore ends with the message that the agent-based foundation of econometrics is the next stage to move towards.

Notes

1 Also recall the calibration work done by Arthur on the reinforcement learning model (Chapter 7).

2 Examples include nonlinear dynamic models, models with latent (or unobserved) variables, and models with missing or incomplete data.

3 A review of the development of simulation-based econometrics is beyond the scope of this chapter. The interested reader is referred to Gourieroux and Monfort (1996). In addition, the *Journal of Applied Econometrics* has a special issue on this subject; see its Vol. 8 (1993).

4 See Midgley, Marks, and Kunchamwar (2007), p. 890, Figure 1.

5 Kirman himself used ϵ and $1-\delta$.

6 In Kirman (1991), each agent tries to assess what the majority opinion is. Each agent observes $q_{1,t}$ but with some noise. The noise follows a normal distribution $\mathcal{N}(0,\sigma^2)$. The third parameter considered by Gilli and Winker (2003) is σ^2.

7 Unfortunately, a unique series $\hat{q}_{1,t}$ is not available from this estimation. The estimation only gives us $\hat{\theta}_1$ and $\hat{\theta}_2$, which allows us to simulate many equally likely series $q_{1,t}$. Hence, we are not able to answer the question: between 1991 and 2000, *when* is the market dominated by fundamentalists and when is it dominated by chartists?

8 Notice that, both in Winker and Gilli (2001) and Gilli and Winker (2003), whether Equation (16.6) holds was only examined numerically as in (16.7), rather than statistically. This is because a formal test had not been proposed then.

9 In Alfarano, Lux, and Wagner (2005); Alfarano, Lux, and Wagner (2006, 2007), the two clusters of traders are defined as fundamentalists and *noise traders* instead of fundamentalists and chartists in Kirman (1991, 1993).

10 Actually, Boswijk, Hommes, and Manzan (2007) studied a modified version of the standard ABS model. Instead of forecasting price, (14.2) and (14.3), agents are assumed to forecast the *price to cash flow ratios*, but counterparts of the reverting coefficient

(α_f) and the extrapolating coefficient (α_c) remain. However, with this modification, the reasonable ranges for these two parameters are $0 < \alpha_f < 1$ and $\alpha_c > 1$.

11 Amilon (2008), in fact, modified the standard ABS models in many ways, including the agents' perceived risk of investment, risk preference, fitness measure, and, most importantly, the noise structure. The additional numbers of parameters actually come from this extension.

12 Of course, the two models are associated with different noise structures; hence, they are estimated with different methods. The two-type model is estimated by the maximum likelihood method, and the three-type model is estimated by the efficient method of moments.

13 In fact, the earliest application of the agent-based financial model to forecasting is Izumi and Okatsu (1996). See also Izumi and Ueda (1999).

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Part VI

Cognitive and psychological agent-based modeling

Make machines more human-like

Behavioral economists care about what people actually do and why they do it, instead of what people ought to do in light of pure logic. Behavioral economists are not satisfied with the act-as-if methodology, because it hides the real process by which the actual decision is made. This tendency drives them to learn more about the "hardware" by which the agent's decisions are made, such as their cognitive capacity, personality, cultural background, and even down to neurophysiological details. Eventually, this leads to an interdisciplinary study overarching economics, psychology, and neuroscience.

This book started with the connection between ACE and experimental economics (Chapter 6); it is now back to this connection. Obviously, over the last decade, we have seen a rapid development of so-called *behavioral experiments*, which take cognitive capacity, intelligence, personality, emotion, risk attitude, and culture as control attributes of experiments with human subjects. Table 17.1 gives a summary of all the studies, classified by cognitive capacity, personality, cultural backgrounds, and neural systems. Some main results from this table will be reviewed in Sections 17.2, 17.6, and 17.7.

Needless to say, this development will soon be further rooted in the brain and its underlying DNA (Chapter 18). While ACE is very sympathetic to the idea of bounded rationality, most ACE models developed so far have not taken this development into account, even though they may have the potential to do so. Given the relationship between ACE and experimental economics discussed earlier, it is to be expected that ACE can develop various autonomous agents or software agents such that this cognitive capacity, personality, and cultural background can be incorporated. This is certainly an important step toward successful models of *Homo sapiens*.

Heterogeneity: other kinds

Furthermore, human agents are *heterogeneous* in intelligence, cognitive ability, and personality.[1] In general, these differences are expected to influence their judgment, decision-making, and their social and economic status.[2] For example, a lot of studies support the view that cognitive capacity matters for the well-being of individuals, and cognitive capacity is heterogeneous among individuals (Chapter 17). Based on these findings, one may wonder whether it would be useful to bring these "empirical facts" (heterogeneities) into agent-based economic modeling. This extension can be useful if the societies with different distributions of cognitive capacities result in different aggregate (emergent) phenomena. Some empirical studies using field data and experimental data are now moving in this direction (Chapter 17). So, the remaining question is *how* to make artificial agents heterogeneous in their cognitive capacity (Chapter 19). Alternatively, what are the mechanisms that we should explicitly take into account so that cognitive capacity can become transparent?

In this book, we provide three examples of cognitive and psychological agent-based modeling. That which follows the idea of Brian Arthur (see also Chapter 7) and uses a three-parameter Roth–Erev reinforcement learning model to fit human subject behavior has been given in Chapter 10. In this part of the book, we provide two more examples and a brief literature review of some similar work. We have to admit that this part is still developing and what is presented here are just illustrations of the direction in which we may expect more work to come in the future. After all, we believe that what is ahead of us is the issue to which a century's work has been devoted, but we are still feeling that it is just the beginning, i.e., *how our brains compute*.

The two examples provided are given in the context of agent-based double auction markets (Chapter 19) and agent-based lottery markets (Chapter 21).

Notes

1 Here is a quotation from the Italian sociologist and economist Vilfredo Pareto (1848–1923):

> Human society is not homogeneous; it is made up of elements which differ more or less, not only according to the very obvious characteristics such as sex, age, physical strength, health, etc., but also according to less observable, but no less important, characteristics such as intellectual qualities, morals, diligence, courage, etc. The assertion that men are objectively equal is so absurd that it does not even merit being refuted. On the other hand, the subjective idea of the equality of men is a fact of great importance, and one which operates powerfully to determine the changes which society undergoes.
> (Pareto, 1971, Chapter II, 102)

2 Again, we quote Pareto (1971) for its brevity and powerfulness:

> Social Heterogeneity. As we have already indicated in II, 102, society is not homogeneous, and those who do not deliberately close their eyes have to recognize that men differ greatly from one another from the physical, moral, and intellectual viewpoints. To these inequalities of human beings per se correspond economic and social inequalities, which we observe among all peoples, from the most ancient times to the present, everywhere in the world, and such that this characteristic is always present. Human society may be defined as a hierarchical collectivity.
> (Pareto, 1971, Chapter VII, 2)

17 Economic significance of personal traits

17.1 Intelligence, income, and prosperity

We are aware of the measurement problems pertaining to rationality or intelligence. Certainly, so far, there is no formal measure of rationality, and whether it can be positively related to the *intelligence quotient* (IQ) is also unclear, even though the latter is frequently used as a proxy for the former. In addition, our experience that smart people are not immune from doing dumb things further casts doubt on the connection between the two. That is why we frequently see books like Feinberg (1995). Needless to say, the study of human intelligence is still an open-ended ongoing body of research. The current research trend in empirical economics, however, has attempted to make the behavior of bounded-rational agents transparent or observable using real-world data. In addition, bounded rationality is frequently used as an input in models since it may generate different predictions or outcomes.

Despite the incurring criticisms leveled against them, some empirical studies support a positive correlation between IQ and income. While the correlation coefficient is often found to be less than 0.5, it may increase with age to some extent (Herrnstein and Murray, 1996; Jensen, 1998). Labor econometricians have shown that a higher IQ causes higher wages (Cawley *et al.*, 1997; Zax and Rees, 2002; Gould, 2005).

Lynn and Vanhanen (2002, 2006) and Lynn (2006) have further provided rich resources on the comparative studies of IQ among different countries and races, and have indicated that IQ's significance can even come to the social or country level. The significant contribution of IQ to economic growth has been further confirmed in other studies (Weede and Kampf, 2002; Jones and Schneider, 2006; Ram, 2007). Some of these studies even suggest that IQ is a good proxy for *human capital*.

17.2 Intelligence in experimental economics

Intelligence, measured in one way or another, provides the most fundamental notion of bounded rationality. Herrnstein and Murray (1996) define intelligence as a person's capacity for complex mental work. It limits agents' capability to process information and hence to make decisions. Yet it is only very recently that the significance of this human factor has been formally used as a variable and parameterized in experimental economics. In addition, these limited studies indicate that there is great heterogeneity in human intelligence (Ohtsubo and Rapoport, 2006), but few of them have looked at the possible implications of this diversity.

In this section, we shall briefly review the literature that connects *parameterized intelligence* with behavioral experiments in economics. We shall categorize the literature based on how intelligence is introduced as a key parameter in the respective experiments, or, alternatively, which aspect of intelligence is involved in the decisions of the associated experiments. A number of cognitive tasks stand out, in the realms of the depths of *reasoning*, *judgments*, and *cooperation*.

17.2.1 Depth of reasoning

Among all aspects of decision-making, *complexity* seems to be the most natural connection to human intelligence. Obviously, complex problem-solving requires intelligence. However, not all experiments can give rise to a natural measure of complexity with regard to the elicited decision-making. Exceptions exist only in a few experiments, particularly those based on the notion of *iterated dominance*, such as the dirty face game (Littlewood, 1953), and the beauty contest game (Nagel, 1999). The interesting feature of these games is that they allow us to develop a *step-by-step* computation in the acquisition of a dominant strategy, and hence develop a complexity measure in the spirit of computational theory.

In the iterated dominance game, the number of steps required to eliminate all dominated strategies is also referred to as the *depth of reasoning*. How deliberate a strategic behavior is can then be connected to this depth. Differentiating the complexity of games based on the depth of reasoning or steps of iterated reasoning is not new (Camerer, 2003); however, the involvement of the intelligence variable in experimental games was absent until Devetag and Warglien (2003) broke this silence. Devetag and Warglien (2003) chose *short-term memory* (STM) as the intelligence variable, and by using evidence from experimental games, they showed that the subjects' depth of reasoning or steps of iterated thinking can be affected by the STM constraints.

The research line on this issue continued to be pursued by experimental economists. In a *beauty contest game* (Nagel, 1999), Ohtsubo and Rapoport (2006) also found a positive relationship between the depth of reasoning and the intelligence measure known as the Imposing Memory Task. Devetag and Warglien (2008) analyzed the complexity of the payoff matrix in two-person games in light of the notion of relation complexity. By means of this measure, they ranked the complexity of four two-person games. Among the four, the cooperation game is the simplest one, whereas the prisoner's dilemma game is the most difficult one. The chicken game and the conflict game are in the middle. Their study shows that there is a significant positive relationship between the subjects' performance and their short-term memory in the chicken game and the conflict game, whereas such a relationship is not significant in the other two games.[1]

17.2.2 Judgments and learning

Of course, not all games have an iterated-dominance structure, yet intelligence remains important in other contexts. *Judgmental forecasting* can be an example. Whether the ability to make a good judgment can be related to subjects' cognitive capacity becomes another issue of interest for experimental economists. Casari, Ham, and Kagel (2007) introduced intelligence variables into the *common value auction* experiment (Kagel and Levin, 2002), and found that cognitive ability as measured by SAT/ACT scores matters in terms of avoiding the *winner's curse* (Kagel and Levin, 1986). It was also found that bidders with below median composite SAT/ACT scores, on average, suffer from winner's curse even as experienced bidders.

To better understand how cognitive ability influences the judgments or decisions made by human subjects, Burks *et al.* (2009) introduced three different tests of cognitive skills: Raven's Standard Progressive Matrices, a numeracy test from Educational Testing Services, and a game which tests subjects' planning abilities called Hit 15. In analyzing truck driver trainees' exit rates from the contracted training and post-training service, they found that poor ability to plan is a key link between subjects' cognitive ability and exits. Considering the cost of training fees required to be paid to the truck driver training firm in the case of an early exit, subjects with higher planning ability survive longer, which means that they are more capable of correctly anticipating their own performance in a new environment.

Similarly, Christelis, Jappelli, and Padula (2006) investigated the investment behavior of senior European citizens by using the EU's Survey of Health, Aging and Retirement in Europe (SHARE) database. To clarify the confounded possible channels of cognitive ability on financial market participation through risk preference, the ability to process information, and overconfidence, they looked into the relationship between subjects' portfolio choices and their cognitive skills. They found that the difference in people's cognitive skills is less important in the case of low cognitive-required assets, such as mutual funds and saving accounts, than in the case of assets needing more cognitive processing, such as stocks. This shows that the information barrier, rather than preference heterogeneity, is also an important factor in explaining the correlation between cognitive ability and stock market participation.

17.2.3 Cooperation, generosity, and fairness

Intelligence involved in the depth of reasoning or judgmental forecasting mainly concerns the correlation between intelligence and *individual performance*. What may be equally, or even more, important is the correlation between intelligence and *social performance*. The study of this correlation is important because there are already many studies pointing to the positive impact of intelligence on a country's economic performance, and another group of studies showing that trust and social capital are essential elements of economic development (Landes, 2000; Francois and Zabojnik, 2005). Therefore, to connect these two collections of empirical studies, it is natural to ask: would intelligence facilitate the formation of cooperation, social capital, and trust? Alternatively, as posed by Jones (2008): *are smart groups more cooperative?*

Segal and Hershberger (1999) is the first study to examine how IQ can affect cooperation. They report how pairs of identical and fraternal twins play a repeated prisoner's dilemma game. Their results indicate that pairs scoring higher in terms of IQ are more likely to be *mutually cooperative* than pairs scoring lower in terms of IQ. Jones (2008) corroborates this result using university students as experimental subjects. It is found that students cooperate 5 to 8 percent more often for every 100-point increase in the school's average SAT score. Following Axelrod (1984), he argued that *patience* and *perceptiveness* are two important personal traits that promote cooperation, and smarter people, as many other empirical studies have verified, are more patient and more perceptive (Benjamin and Shapiro, 2005; Frederick, 2005).

In addition to the test based on parametric intelligence, there are a number of studies that examine the effects of intelligence by manipulating cognitive loading through taxing short-term memory capacity. In a *dictator game*, Cornelissen, Dewitte, and Warlop (2007) found that under a higher cognitive load the "dictator" tends to offer more. However, not all games support the existence of an intelligence effect. In an *ultimatum game*, Cappelletti, Guth, and Ploner (2008) found that short-term memory has no effect on the behavior of either proposers or respondents. In the above two studies, short-term memory is manipulated by asking the subjects to memorize a string of eight numbers. In the case of Cappelletti, Guth, and Ploner (2008), an incentive scheme was even included to reward the subjects who could correctly recall the number.

17.2.4 Machiavellian intelligence

Depth of reasoning requires not only the agent's own *intelligence* but also *mentalizing ability* which can access other players' reasoning ability (Ohtsubo and Rapoport, 2006). As we have mentioned in Section 17.2.1, the complexity of decisions can be characterized by computational ability for step-by-step solvable games with iterated dominated strategies. Intelligence may matter in this case. However, when it involves the expectation about what other agents will do, the decision problem is more complex than just a mathematical one. For example, if a player is smart enough to realize that one strategy is dominated by another one, the next question is whether other players also grasp this fact.

Game theorists have recognized the importance of *theory of mind*. Stahl and Wilson (1995) assumed that each player keeps a certain type of mental model about other players. The hierarchical structure of mental models manifests the players' level of reasoning. They actually found experimental evidence to support *level-k thinking* (Crawford and Iriberri, 2007b), which implies *heterogeneity* or *individual differences* in the players' mental models. It should be noted that players with high-level thinking do not necessarily win the game. The winner should have the mental model that best fits the real one.

Ohtsubo (2002) introduced *Machiavellian personality* and implements the Imposing Memory Task (IMT) as a measure of mentalizing ability. He found a positive correlation between choice in the beauty contest game and the score on the IMT. Ohtsubo and Rapoport (2006) viewed Machiavellian personality as a cognitive ability and use the term *Machiavellian intelligence*.[2]

17.3 Intelligence and risk preference

In modeling a human's decisions under risk, expected utility theory (von Neumann and Morgenstern, 1944), together with risk attitudes, has long been a standard descriptive and normative model to follow (Keeney and Raiffa, 1976; Arrow, 1971; Friedman and Savage, 1948). However, experimental results in the laboratory exhibit deviations from what the theory predicts (Kahneman and Tversky, 1979). Behavioral economics therefore rises to provide alternative theories to account for these deviations.

Although contemporary behavioral economic theories try to incorporate cognitive and emotional factors to better describe human economic decisions, they still cannot fully capture the heterogeneous nature of human behavior. For example, what prospect theory from Kahneman and Tversky (1979) describes is the pattern observed from the majority of their subjects, giving no explanation concerning the behavior of the minority (14–35 percent of their subjects).[3]

To put it in a different way, heterogeneous decisions observed in the lab are usually either viewed as representative behavior contaminated by anomalies or as decisions that have been used to build up models with various types of agents. For either case, it is argued that such heterogeneous behavior is worth further investigation.[4] Here we are going to unveil the potential impact of heterogeneous behavior correlated with cognitive ability by introducing a series of experimental evidence.

17.3.1 Reflection effect

Recently there have been a number of studies trying to discover the relationship between individual cognitive abilities and risk preference. Frederick (2005) introduced a three-item Cognitive Reflection Test (CRT) to measure cognitive ability. He offered the subjects lotteries of gains as well as losses and recorded their choices. Interestingly, Frederick (2005) found that subjects with higher CRT scores were more willing to gamble in the domain of gains and preferred a sure loss when making choices involving losses. On the contrary, subjects with low CRT scores were more risk averse when facing lotteries involving gains, and were more risk seeking when provided with chances to gamble. Frederick's results manifest the importance of cognitive ability to decisions under risk. Therefore, he regarded this phenomenon of "cognitive reflection" as being analogous to the "reflection effect" from Kahneman and Tversky (1979).

Similar results can be observed from a series of recent experiments. Benjamin and Shapiro (2005) conducted experiments with Harvard undergraduates and Chilean high school students. They found that students with higher SAT math scores (or the Chilean equivalent) were less risk averse over small-stakes gambles.

To test further, Benjamin, Brown, and Shapiro (2006) conducted a new experiment and replicated the result that small-stakes risk aversion is less common among those students with higher standardized test scores. In addition, they imposed cognitive load manipulation to decrease the subjects' working memory. These results showed that this manipulation exacerbated small-stakes risk aversion.

Other studies such as Dohmen *et al.* (2008) found that individuals with higher cognitive ability are significantly more willing to take risks in the lottery experiments; Andersson and Svensson (2006) found that respondents with higher cognitive ability are less flawed by scale bias which refers to the insensitivity and non-near-proportionality of the respondents' willingness to pay to the size of the risk reduction.

17.4 Intelligence and time preference

Cognitive capacity can also be related to subjects' *time preference*. Accordingly, there has been a recent trend to incorporate this connection observed from lab experiments into the expected utility framework. Intertemporal utility maximization under a constant discount rate (exponential discounting) predicts a consistent intertemporal tradeoff. However, this seemingly rational model had been severely challenged, first by Strotz's (1955) introspection, and then by a series of experimental results (Thaler, 1981; Benzion, Rapoport, and Yagil, 1989; Ainslie, 1992). This evidence has shown that people are less patient when facing choices involving payment in the near future, that is, people have *diminishing impatience* (Halevy, 2008).[5]

Evidence from the laboratory has forced economists to rethink the adoption of a constant discount rate, and hence to embrace a hyperbolic discount function as a replacement (Phelps and Pollak, 1986; Laibson, 1997). While hyperbolic discounting suggests that the discount rates are greater in the short run than in the long run, there are nevertheless counterexamples observed in the lab. Rubinstein (2003) conducted a series of online experiments and gained results which contradict the predictions of hyperbolic discounting behavior. He therefore proposed a procedural approach to better explain the experimental results.

Likewise, although the finding in Rubinstein (2003) can serve as a challenge to hyperbolic discounting, it is itself based on observations from only part of the subjects. This casts doubt on the validity of its universal description of human time preference. Recently, a number of studies have started to investigate the relationship between an individual's characteristics and his/her choice of behavior. This provides a new way to rethink this problem, and *differences in cognitive abilities might be the key to better understanding the heterogeneous intertemporal choices made by human decision-makers*.

It is noticeable that almost all experiments which showed a relationship between cognitive abilities and risk preference also demonstrated a similar relationship between cognitive abilities and time preference (or patience). Frederick (2005) provided his subjects with choices involving instant payoffs or losses and larger future payoffs/losses. He found that those who scored higher in CRT were generally more patient (and therefore implicitly had a lower discount rate). However, this relationship weakened as the time horizons became longer. Benjamin and Shapiro (2005) found that the subjects in the experiments in Chilean high schools, were more patient over short-term tradeoffs. To test the influence of cognitive resources, Benjamin, Brown, and Shapiro (2006) imposed additional work on subjects, and they observed that subjects became more impatient. Such a positive relationship between cognitive abilities and patience over the lottery problem can also be seen in Dohmen *et al.* (2008), as well as in Burks *et al.* (2009).

To summarize, recent experimental studies strongly suggest that it is no longer appropriate to assume homogeneity in either risk preference or time preference. Furthermore, heterogeneity in intelligence can partially account for the diversity in both risk and time preference. Economic theory can benefit by taking such heterogeneity and the corresponding relationship into account. For example, both Benjamin and Shapiro (2005) and Christelis, Jappelli, and Padula (2006) showed that people with higher cognitive abilities participate more in financial markets. If this is the case, together with the finding by Frederick (2005) that the risk attitudes of the subjects with higher cognitive ability are exactly the opposite of the prospect theory from Kahneman and Tversky (1979), the foundation of modeling financial market participants according to prospect theory might not be well grounded.

The studies on the influences of cognitive abilities on risk as well as time preferences have just begun. There exists a need to probe this issue with more systematically designed experiments, and it may need more advanced methods coming from psychology and neuroscience. Thaler (2000) once stated that

> After all, analyses of market interactions between agents of various types is exactly what differentiates economics from other social sciences. Psychologists, sociologists and anthropologists might help us improve our characterizations of economic behavior, but economists are the only social scientists with the tools to analyze what happens in market contexts.
>
> (Thaler, 2000, p. 136)

17.5 Personality and earning capacity

Personality refers to the sets of predictable behaviors by which humans are recognized and identified. In psychology, *cognitive ability* and *personality traits* are distinguished, while they are not independent of each other and their distinctions are not easy.

Labor economists have started to explore the relevance of personality to economics (Borghans *et al.*, 2008). For example, Heckman, Stixrud, and Urzua (2006) and Mueller and Plug (2006) have shown that personality traits can affect an employee's earning capacity.

17.6 Personality in experimental economics

In addition to cognitive ability, human agents are also heterogeneous in various attributes of personality, which is partially due to genetic and environmental causes. Many behavioral game experiments have repeatedly shown that *heterogeneity in personality matters* and isolating this factor can help accommodate not-otherwise-explained differences in experiments.

17.6.1 Social preference, social norms and cooperation

Economists have recognized the importance of *social norms*, the rules based on widely shared beliefs about how individual group members ought to behave in a given situation. Social norms accompanied by sanctions help norm enforcement and *human cooperation* (Fehr and Fischbacher, 2004). Moreover, different social norms which can be represented by multiple equilibria may lead to different economic growth rates (Cole, Mailath, and Postlewaite, 1992). How can a society reach desirable social norms? Camerer and Fehr (2004) stated that *reciprocity*, *inequality aversion*, and *altruism* can have large effects on the regularities of social life and, in particular, on the enforcement of social norms. These issues have been studied in the context of experimental games, such as the ultimatum game, dictator game, trust game, and the prisoner's dilemma game. Subjects participating in these experiments exhibit great individual differences. One legitimate question that follows from a psychological perspective is *whether personality traits are sources of heterogeneity in human behavior*.

Introducing personality traits into the analysis of human behavior in game-theoretic experiments dates back to the 1960s (Lutzker, 1960; Deutsch, 1960; Wrightsman, 1966). Wrightsman (1966) investigated a wide range of personality and attitudinal variables and concluded that scores on the *philosophies of the human nature scale* were related to trusting behavior in the prisoner's dilemma game.

Subsequently, it seems that personality traits have played a minor role in game-theoretic experiments up to the late 1990s (Boone, De Brabander, and van Witteloostuijn, 1999; Brandstätter and Güth, 2002; Gunnthorsdottir, McCabe, and Smith, 2002). Recently, findings regarding the correlation between personality and behavior in game-theoretic experiments have been mixed. For example, both Gunnthorsdottir, McCabe, and Smith (2002) and Burks, Carpenter, and Verhoogen (2003) investigate the significance of the *Machiavellian personality* effect on trust and reciprocal behavior in a trust game, but their conclusions are in sharp contrast to each other. Another example relates to research on the *Big Five*. It seems that the Big Five has no prediction ability in the trust game (Ben-Ner and Halldorsson, 2007), but it affects behavior in the prisoner's dilemma game (Hirsh and Peterson, 2009) and the dictator game (Ben-Ner, Kong, and Putterman, 2004; Ben-Ner *et al.*, 2004).

17.6.2 Risk preference

Personality traits are likely to prove useful in economic models of decision-making under uncertainty. People who are *open to experience*, i.e., more intellectually curious and motivated to learn, may acquire information more cheaply and process information more efficiently. That in turn will lead to a reduction in the intrinsic uncertainty in their environments, and make them have different observed risk attitudes from other people. Therefore, risk preference may not be independent of personality.[6]

Statman and Wood (2004) have designed a financial personality test. It builds upon a tried-and-true personality profiling system called the *Keirsey Temperament Sorter*, as well as a series of studies used to isolate the characteristics of each personality type. Four categories emerge: *Guardians*, who tend to be cautious with their money; *Artisans*, who are freewheeling and daring; *Idealists*, who care less about money than other goals; and *Rationals*, who make most decisions by the numbers. Statman and Wood's research shows that most of us will find a strong affinity with one or at most two of these categories.

Lo, Repin, and Steenbarger (2005) use daily emotional-state surveys as well as personality inventory surveys to construct measures of personality traits and emotional states for a group of 80 day-traders and correlate these measures with daily normalized profit-and-loss records. They find that subjects whose emotional reaction to monetary gains and losses was more intense on both the positive and negative side exhibited significantly worse trading performance, and large sudden swings in emotional states seem especially detrimental to cumulative profit-and-loss.

17.6.3 Asset markets

Individual performance

Biais *et al.* (2005) and van Witteloostuijn and Muehlfeld (2008) both show a correlation between personality traits and investors' behavior or performance. In particular, Biais *et al.* (2005) claims that the two personality traits, namely miscalibration and self-monitoring, can explain whether investors are able to avoid the winner's curse in experimental asset markets (see also Section 17.2.2 for the effect of intelligence on the common-value auction).

Emergent complexity

In addition to its effect on individual behavior, risk attitude or risk aversion also matters for market behavior. Ang and Schwarz (1985) adopt two psychological tests on risk preference and risk-taking behavior: the *Jackson Personality Inventory* and the *Jackson, Hournay, and Vidmar tests* to separate the market with speculative investors only from the market with conservative investors only. They found that the market for speculators exhibits greater price volatility. However, it also exhibits several desirable properties in terms of convergence speed, allocation efficiency, and price discovering ability. Fellner and Maciejovsky (2007) use binary lottery choice to measure risk aversion and, by that, to classify traders. They found that the higher the degree of risk aversion, the lower the observed market activity.

17.7 Culture in experimental economics

"Does culture matter in economic behavior?" is the title of a paper by Joseph Henrich, in which he states that

> economic decisions and economic reasoning may be heavily influenced by cultural differences—that is, by socially transmitted rules about how to

> behave in certain circumstances (economic or otherwise) that may vary from group to group as a consequence of different cultural evolutionary trajectories.
>
> (Henrich, 2000, p. 973)

17.7.1 Culture and ultimatum games

At the beginning of the 1990s, economists started to address this issue using the controlled laboratory approach, the so-called *cross-cultural experiments*. Roth *et al.* (1991) is the pioneering work that considers the possible cultural influence in two main categories of economic behavior, namely, *bargaining* and *market behavior*. The bargaining behavior is examined in the context of the standard ultimatum game (bargaining game), whereas the market behavior is examined using an extended ultimatum game (market game).

In the standard ultimatum game (bargaining game), there is one proposer, who makes the splitting offer to a responder, who will decide to accept or to reject. In the extended version (market game), there are nine proposers, and each of them has to make an offer to the only responder, who will decide to accept or reject the highest bid. These two games, therefore, differ in the degree of competition. What was found is that cultural differences only matter in the standard ultimatum game, where competition is rather lacking.[7] When competition is properly presented, as in the market game, cultural differences are no longer important. Kachelmeier and Shehata (1992) continued this line of research and also failed to detect cultural differences in market behavior experiments in North America (the United States and Canada) and China.

Following Roth *et al.* (1991), Joseph Henrich led a group of researchers to see how ultimatum bargaining behavior differed across 15 small-scale societies in Latin America and Africa (Henrich, 2000; Henrich *et al.*, 2004). They found enormous variations in behavior across communities. They found that there was a wide variation in the offers across locations: in two societies mean offers were 30 to 40 percent, while mean offers exceeded 50 percent in other societies. They were able to relate this difference to interactional patterns of everyday life and the social norms operating in these various communities.

As a specific example, Henrich (2000) conducted the ultimatum game on the Machiguenga of the Peruvian Amazon and UCLA graduate students and compared his experimental results with previous works, including Roth *et al.* (1991).[8] The Machiguenga data differed substantially from the data of other groups of subjects. Moreover, in post-game interviews, the Machiguenga often had difficulty explaining why they were willing to accept lower-than-average offers and some even made clear that they would always accept whatever splitting rule they were offered. It is obvious that the Machiguenga do not share similar senses of fairness with people from the industrial world.

Recent studies have cut into the cultural issue in more subtle ways. Chuah *et al.* (2007) conducted the ultimatum game in Malaysia and the UK on Malaysian Chinese and British subjects recruited from these two countries. They found that the location where the experiment was conducted and whether the proposer and the responder were of the same nationality did make a difference in the experimental results. For example, they found that the Malaysian proposer's splitting offers made to a British responder were sensitive to the experiment's location, while the British proposer's splitting offers made to Malaysian responder were not. The Malaysian proposer's splitting offers were slightly higher than the British proposer's splitting offers. Malaysian responders' rejections of British proposers' splitting offers in the UK are relatively high, while British responders' rejections of Malaysian proposers' splitting offers in the UK were less frequent, and British responders rejected offers more frequently than Malaysian responders.

17.7.2 Culture in other experiments

The absence of cultural effects in market games (Roth *et al.*, 1991; Kachelmeier and Shehata, 1992) may shape the later development of cultural economic experiments in a direction leaning toward bargaining experiments in the form of two-person games, as we have seen in Section 17.7.1. These games normally explicitly involve elements such as cooperation, trust, sympathy, and reciprocity. Therefore, this invites other experiments to be investigated in the light of cultural differences. These include the prisoner's dilemma game, public good games, and investment games (trust games; see Table 17.1 for this expansion in a chronological order).

17.7.3 Evolution of cross-cultural experiments

The way in which we think of and perform cross-cultural experiments also evolves. In the initial stage, the cross-cultural experiment is basically *location-based*. It is the same experiment implemented in different locations (countries, cities) by recruiting location-specific people as the human subjects. The location-specific experiments generally do not involve *interactions* among human subjects with different cultural backgrounds. Consider Roth *et al.* (1991) as an example. Their ultimatum games are all local people playing with local people: American versus American, Japanese versus Japanese, etc. Group-crossing interactions among human subjects do not take place; in other words, *cultural clash* is not part of the design (Chuah *et al.*, 2007). We shall call this design the *static cross-cultural experiment*, static in the sense that cultures do not collide, and to be distinguished from the dynamic ones to which we now turn.

The incessant globalization process has brought people with different cultural backgrounds to the same location. This enables us to explore cross-cultural experiments in a different form, namely, to cause subjects belonging to different ethnic groups to interact, say, Chinese from Mainland China now residing in New York interacting with local Americans. Fershtman and Gneezy (2001) is probably the first study to use this design.[9] The dynamic cross-cultural experiments proposed above enable us to observe the interaction patterns of subjects from different cultures. However, if location in this design is not explicitly taken into account, then we may neglect the effect ofthe "home-court advantage." The research question here is: *What is the impact of cultural differences on the interactions of individuals*

Table 17.1 Human factors in experiments

Experiments	*Cognitive capacity*	*Personality*	*Cultural background*	*Neural evidence*
Common value auction	Casari, Ham, and Kagel (2007)			
Acquiring company game	Charness and Levin (2007)			
Bargaining market			Roth *et al.* (1991); Kachelmeier and Shehata (1992)	
Asset market (portfolio choice)	Christelis, Jappelli, and Padula (2006)	van Witteloostuijn and Muehfield (2008); Biais, Hilton, and Pouget (2002)		
Prisoner's dilemma games	Jones (2008); Segal and Hershberger (1999); Devetag and Warglien (2008); Hirsh and Peterson (2009)	Ueijo and Wrightsman (1967); Wrightsman (1966); Boone, De Brabander, and van Witteloostujin (1999); Kuhlman and Marshello (1975); Hirsh and Peterson (2009); Swope *et al.* (2008)	Hemesath and Pomponio (1998); Carpenter Daniere, and Takahashi (2004)	Rilling *et al.* (2007); Rilling *et al.* (2004); Rilling *et al.* (2002); Singer *et al.* (2004); Wood *et al.* (2006); Tse and Bond (2002a); Tse and Bond (2002b)

Table 17.1 Continued

Experiments	*Cognitive capacity*	*Personality*	*Cultural background*	*Neural evidence*
Ultimatum games	Cappelletti, Guth, and Ploner (2008)	Meyer (1992); Swope *et al.* (2008); Brandstätter and Güthe (2002); Spitzer *et al.* (2007)	Roth *et al.* (1991); Okada and Riedl (1999); Henrich (2000); Botelho, Hirsch, and Rustrom (2000); Fershtman and Gneezy (2001); Hoffmann and Tee (2003); Tanner, Lusk, and Tyner (2005); Chuah *et al.* (2007); Gutiérrez *et al.* (2007)	Rilling *et al.* (2004); Sanfey *et al.* (2003); Knoch *et al.* (2006); Xiao and Houser (2005); Koenigs and Tranel (2007); Spitzer *et al.* (2007); Tabibnia, Satpute, and Lieberman (2008)
Dictator games	Cornelissen, Dewitte, and Warlop (2007)	Ben-Ner, Kong, and Putterman (2004); Ben-Ner *et al.* (2004); Swope *et al.* (2008); Brandstätter and Güth (2002); Spitzer *et al.* (2007)	Fershtman and Gneezy (2001); Cardenas and Carpenter (2003)	Harbaugh, Mayr, and Burghart (2007); Spitzer *et al.* (2007); Knafo *et al.* (2008)
Chicken games	Devetag and Warglien (2008)	Marin (1973)		
Public goods game			Parks and Vu (1994); Weimann (1994); Burlando and Hey (1997); Ockenfels and Weimann (1999); Tanner (2005)	

(*Continued*)

Table 17.1 Continued

Experiments	*Cognitive capacity*	*Personality*	*Cultural background*	*Neural evidence*
Common pool resource game		Koole *et al.* (2001)	Cardenas and Carpenter (2003)	
Dominance-solvable game	Devetag and Warglien (2003)			Bhatt and Camerer (2005); Kuo *et al.* (2005)
The dirty faces game	Devetag and Warglien (2003)			
Backward induction game	Devetag and Warglien (2003)			
A coordination game	Devetag and Warglien (2008)			Kuo *et al.* (2005)
A game of conflict	Devetag and Warglien (2008)			
Beauty contest game		Ohtsubo and Rapoport (2006)		
Investment game (trust game)	Ben-Ner and Halldorsson (2007); Burks *et al.* (2009)	Gunnthorsdottir, McCabe, and Smith (2002); Ohtsubo and Rapoport (2006); Ben-Ner and Halldorsson (2007); Swope *et al.* (2008); Burks, Carpenter, and Verhoogen (2003)	Croson and Buchan (1999); Fershtman and Gneezy (2001); Holm and Danielson (2005); Ashraf, Bohnet, and Piankov (2006); Cronk (2007)	King-Casas *et al.* (2005); McCabe *et al.* (2001); Tomlin *et al.* (2006); Zak, Kurzband, and Matzner (2005); de Quervain *et al.* (2004); Kosfeld *et al.* (2005); Zak, Stanton, and Ahmadi (2007); Krueger (2007); Chiu *et al.* (2008); Delgado, Frank, and Phelps (2005)

Table 17.1 Continued

Experiments	*Cognitive capacity*	*Personality*	*Cultural background*	*Neural evidence*
Normal form pairwise interactions		Christie, Gergen, and Marlowe (1970)		
Risk preference	Frederick (2005); Benjamin and Shapiro (2005); Benjamin, Brown, and Shapiro (2006); Dohmen *et al.* (2008); Andersson and Svensson (2006); Benjamin, Brown, and Shapiro (2006)			
Time preference	Frederick (2005); Benjamin and Shapiro (2005)			
Induced budget experiment	Banerjee and Murphy (2007)			
Decisions in domains of life		Blais (2002)		

from different countries? This question was first raised and addressed in Chuah *et al.* (2007), as we have briefly summarized in Section 17.7.1.

Notes

1 In addition to the complexity measure, neuroeconomics has recently also tried to differentiate games based on their involved cognitive activities. For example, Kuo *et al.* (2005) applied the dual-system theory in psychology to differentiate between dominance-solvable games and coordination games. Using functional magnetic resonance imaging, they found that the middle frontal gyrus, the inferior parietal lobule, and precuneus were more active in dominance-solvable games than in coordination games. The insula and anterior cingulate cortex showed the opposite pattern. We shall come back to this point in Section 18.6.

2 On the other hand, Burks, Carpenter, and Verhoogen (2003) and Gunnthorsdottir, McCabe, and Smith (2002) regard it as a personality trait, called "Machiavellian personality." Given the difficulty in disentangling cognitive psychology from personality psychology, we simply accept the gray line here (Sections 17.3 and 17.4).

3 In this seminal paper (Kahneman and Tversky, 1979), the reported pattern is based on observations of the majority's decision. Here the majority ranges from 65 to 86 percent, which means that the behavior of the remaining 14 to 35 percent of the subjects cannot be characterized by their theory.

4 For example, Roth (1995) stated that experimental evidence (with counterexamples) is of the upmost importance to know where approximations break down.

5 In fact, Halevy (2008) transformed the problem of intertemporal choices into choices under risk by introducing the subject's concern about mortality risk.

6 Recent experimental economics also indicates that risk preference can differ among people with different cognitive abilities (see Sections 17.3 and 17.4 for a review).

7 Among the four countries on which their comparisons are based, except for the pair comprising the United States and Slovenia, they find statistically significant differences in behavior between countries. These differences apply to proposers as well as responders. The highest offers are made in Slovenia and the United States followed by Japan and Israel. Responders accept disproportionate offers more frequently in Israel and in Japan than in Slovenia and the United States. In particular they find quite pronounced behavioral differences between Slovenian and Japanese subjects. The modal offer in Slovenia was 50 percent but only 40 percent in Japan. Furthermore, Slovenian responders accepted disproportionate offers at a lower rate than Japanese responders. In addition, they also observe that the differences increase as subjects gain experience.

8 The Machiguenga are a primitive people living in mobile single-family units and subsisting on a combination of hunting, fishing, gathering, and manioc-based slash-and-burn horticulture.

9 Fershtman and Gneezy (2001) report experiments with members of the Sephardic (i.e., Asian and African) as well as Ashkenazic (i.e., European-American) Jewish communities in Israel that play trust, dictator, and ultimatum games.

18 Neuroeconomic agents

> As a discipline, psychology has undergone a revolution over the past few years because of the confluence of new technologies such as brain imaging and interdisciplinary collaborations between neuroscientists and psychologists. The two disciplines are now often considered one, sometimes called brain sciences, and more often called cognitive neurosciences. Any serious student of behavioral economics and finance cannot afford to ignore this literature.
>
> (Lo, 2005, p. 39)

18.1 Neuroeconomics: an ACE viewpoint

From the perspective of agent-based computational economics, our interest in neuroeconomics is different from that of general psychologists and neural scientists. Agent-based computational economics advocates a *bottom-up research paradigm* for economics. This paradigm does not treat micro and macro as two separate entities and work with each of them separately; instead, it studies the relationship between the two in a coherent framework. Therefore, given the bottom-up manner, we pay more attention to the micro details, and always start the modeling at the level of agents. This *methodological individualism* drives us to incorporate the psychological, cognitive, and neural attributes of human beings into the study of economics. What causes ACE to differ from these behavioral sciences is the scope of the research questions; therefore, while ACE cares about the fundamental cause (the neural cause) of the cognitive biases, it is more concerned with the implications of these cognitive biases for any possible emergent mesoscopic or macroscopic phenomena. Furthermore, ACE researchers do not regard the behavioral factors as given (exogenous); they also study the feedback from the aggregate level (social outcome) to the bottom level (individual behavior).[1]

Given what has been said above, we believe that unless neuroeconomics can provide some important lessons for agent-based computational economists, its significance may hardly go far beyond neural science, and would not draw much attention from economists. This, therefore, motivates us to ask: *Does neuroeconomics provide some important lessons for agent-based economic modeling?* It is this question that this chapter would like to address.

In the following, we will review recent progress in neuroeconomics in light of its contributions to different aspects of agent engineering. We start from the most fundamental part of agents, i.e., *preferences* (Section 18.2), which points to two foundational issues in economics, namely, the *measurement* or *representation* of preference and the *formation* of preference. Some recent advances in the study of these two issues may lead to new insights in the future of agent engineering with regard to *preference development*. We then move to the immediate issue after preferences, i.e., *choices*, or, more precisely, value-based choices (Section 18.3), and further specify the *intertemporal choice* (Section 18.3.1), where we can see how the *discount rate* should be more carefully designed. We then focus more on two behavioral aspects pertaining to the design of financial agents, namely *risk perception* (Section 18.4.1) and *risk preference* (Section 18.4.2). The neural mechanism regarding learning or adaptation is given in Section 18.5. Finally, the paper ends with a final remark that connects the relationships among behavioral economics, neural economics, and agent-based economics, which is a continuation of the points made earlier (Chen, 2008b).

18.2 Preference

> The nature of wealth and value is explained by the consideration of an infinitely small amount of *pleasure* and *pain*, just as the theory of statics is made to rest upon the equality of indefinitely small amounts of energy.
>
> (Jevons, 1879, p. 44; emphasis added)

Standard economic theory takes individual preferences as given and fixed over the course of an individual's lifetime. It would be hard to imagine how economic models can stand still by giving up preferences or utility functions. They serve as the very foundation of economics, just as we quoted above from William Stanley Jevons (1835–1882). Without preference or utility, it will no longer be clear what we mean by welfare, and hence we make welfare-enhancing policy ill-defined. Nevertheless, preference is now in a troubling moment in the development of economics. Even though its existence has been questioned, the development of neuroeconomics may further deepen this turbulent situation.

18.2.1 The brain as a multi-agent system

The recent progress in neural science provides economists with some foundational issues of economic theory. Some of its findings may lend support to many heated discussions which are unfortunately neglected by mainstream economics. The most important series of questions is that pertaining to *preference*. While its existence, formalization (construction), measurement, consistency, and stability has long been discussed outside mainstream economics, particularly in the realm of behavioral economics, neuroeconomics provides us with solid ground to tackle these issues.[2]

To see how neuroscience can inform economists, it is important to perceive that *the brain is a multi-agent system*. For example, consider the *triune brain model* proposed by Maclean (1990). The brain is composed of three major parts: the reptilian brain (the brainstem), the mammalian brain (the limbic system), and the hominid brain (the cerebral cortex). Each of the three is associated with different cognitive functions, while receiving and processing different signals. The three parts also have various interactions (competition or cooperation) with the embedded network. The three "agents" and their interactions, therefore, constitute the very basis of this multi-agent system.

This multi-agent system (MAS) approach to the brain compels us to think hard on what would be a *neural representation of preference*. Preference is unlikely to be represented by a signal neuron or a single part of the brain, but by an emergent phenomenon from the interactions of many agents. Hence, many agents of the brain can contribute to part of the representation. So, when asked what the preference for commodity A is and its relative comparison to B, many agents of the brain work together either in a synchronous or asynchronous manner to generate a representation, the utility of A and B, say $U(A)$ and $U(B)$.

During the process, some agents retrieve the past experiences (memory) of consuming A and B, and some agents aggregate this information. These processes can be collaborative or competitive; it is likely that some agents inhibit the function of others. As a result, the memory can be partial, which, depending on the external elicitation and other conditions, can vary from time to time. This rough but simple picture of multi-agent neurodynamics may indicate why the steady preference conventionally assumed in economics may not be there. The alternative is that people do not have given unchanging preferences, but rather their preferences are constructed to fit the situations they face. Herbert Simon is one of the precursors of the idea of preference construction (Simon, 1955a, 1956).

18.2.2 Preference construction

> On the contrary, we approach choice within specific, quite *narrow frames of reference* that continually shift with the circumstances in which we find ourselves and with the thoughts that are evoked in our minds by these *particular circumstances*. Thus, in any given choice situation, we evoke and make use of only a small part even of the limited information, knowledge and reasoning skills that we have stored in our memory, and these memory contents, even if fully evoked, would give us only a pale and highly inexact picture of the world in which we live.
>
> (Simon, 2005, p. 93; emphasis added)

The MAS approach to the study of the brain may connect us to the literature on preference construction for real human beings (Fischhoff, 1991; Slovic, 1995; Lichtenstein and Slovic, 2006), and, in particular, the role of *experiences* and *imagination* in preference formation. In the following, we would like to exemplify a few psychological studies which shed light on *experience-based* or *imagination-based preferences*.

Adaptive decision makers

The *effort–accuracy* framework proposed by Payne, Bettman, and Johnson (1993) represents an attempt to shift the research agenda from demonstrations of irrationality in the form of heuristics and biases to an understanding of the causal mechanisms underlying the behavior. It has considerable merit as a model of how decision-makers cope with cognitive limitations. The *adaptive decision-maker* is a person whose repertoire of strategies may depend upon many factors, such as cognitive development, experience, and more formal training and education. Payne, Bettman, and Johnson (1993) suggest that decision-making behavior is a *highly contingent form* of information processing and is highly sensitive to task factors and context factors. They consider that the cognitive effort required to make a decision can be usefully measured in terms of the total number of basic information processes needed to solve a particular problem using a specific decision strategy. In addition, they state that individual differences in decision behavior may be related to differences in how much effort the various elementary information processes require the individuals to make.

Hedonic psychology

Hedonic psychology is the study of what makes experiences and life pleasant or unpleasant (Kahneman, 2003). It is concerned with feelings of pleasure and pain, of interest and boredom, of joy and sorrow, and of satisfaction and dissatisfaction. All decisions involve *predictions of future tastes or feelings*. Getting married involves a prediction of one's long-term feelings towards one's spouse; returning to school for an advanced degree involves predictions about how it will feel to be a student as well as predictions of long-term career preferences; buying a car involves a prediction of how it would feel to drive around in different cars. In each of these examples, the quality of the decision depends critically on the accuracy of the prediction; errors in predicting feelings are measured in units of divorce, dropout, career burnout, and consumer dissatisfaction (Loewenstein and Schkade, 2003).

Empathy gaps

People are often incorrect about what determines happiness, leading to prediction errors. In particular, the well-known *empathy gaps*, i.e., the inability to imagine opposite feelings when experiencing heightened emotion, be it happy or sad, lead to errors in predicting both feelings and behavior (Loewenstein, 2005). So, people seem to think that if disaster strikes it will take longer to recover emotionally than it actually does. Conversely, if a happy event occurs, people overestimate how long they will emotionally benefit from it.

Psychological immune system

The cognitive bias above also indicates that agents may underestimate the proper function of their psychological immune systems. The psychological immune system is a system which helps fight off bad feelings that result from unpleasant situations (Kagan, 2006). This system is activated when humans are faced with potential or actual negative events in their life. The system functions to assist in protecting humans from extreme reactions to those negative events.

Sharot, De Martino, and Dolan (2009) studied how hedonic psychology affects our choices from a neural perspective. They combined participants' estimations of the pleasure they will derive from future events with fMRI data recorded *while they imagined those events*, both before and after making choices. It was found that activity in the *caudate nucleus* predicted the choice agents made when forced to choose between two alternatives they had previously rated equally. Moreover, post choice the selected alternatives were valued more strongly than pre choice, while discarded ones were valued less. This *post-choice preference change* was mirrored in the caudate nucleus response. The choice-sensitive preference observed above is similar to behavior driven by reinforcement learning.

18.3 Value and choice

> Neuroeconomics is a relatively new discipline that studies the computations that the brain carries out in order to make value-based decisions, as well as the neural implementation of those computations. It seeks to build a biologically sound theory of how humans make decisions that can be applied in both the natural and the social sciences.
>
> (Rangel, Camerer, and Montague, 2008, p. 545)

> In a choice situation, we usually look at a few alternatives, sometimes including a small number that we generate for the purpose but more often limiting ourselves to those that are already known and available. These alternatives are generated or evoked in response to specific goals or drives (i.e. specific components of the utility function), so that different alternatives are generated when we are hungry from when we are thirsty; when we are thinking about our science from when we are thinking about our children.
>
> (Simon, 2005, p. 93)

The very basic economics starts with value assignment and choice-making. However, traditional economics makes little effort to understand the cognitive and computation loading involved in this very fundamental economic activity. A number of recent studies have challenged the view that what we used to be taught may be misplaced when we take into account the value assignment problem more seriously (Iyengar and Lepper, 2000; Schwartz, 2003). These studies lead us to question the impact of the dimensionality of the choice space upon our behavior of value assignment and choice-making. It seems that when the number of choices increases, the ability to make the best choice becomes problematic.

Going one step further, Louie, Grattan, and Glimcher (2011) attempt to theorize this *paradox of choice* by exploring the neural mechanism underlying *value representation* during decision-making and how such a mechanism influences choice behavior in the presence of alternative options. In their analysis, value assignment is relatively normalized when new alternatives are presented. The linear proportionate normalization is a simple example. Because value is relatively coded rather than absolutely coded, the value differences between two alternatives may become narrow when more alternatives are presented.

18.3.1 Intertemporal choice

Agent-based economic models are dynamic. Time is an inevitable element, and *time preference* becomes another important setting for agents in agent-based models. In mainstream economic theory, time preference has been largely standardized as an exponential discounting with a time-invariant discount rate. However, recent studies have found that people discount future outcomes more steeply when they have the opportunity for immediate gratification than when all outcomes occur in the future. This has led to the modification of the declining discount rates or *hyperbolic discounting* (Laibson, 1997). Frederick, Loewenstein, and O'Donoghue (2002) provided an extensive survey of the empirical studies showing that the observed discount rates are not constant over time, but appear to decline.

Loewenstein (1988) has further demonstrated that discount rates can be dramatically affected by whether the change in delivery time of an outcome is framed as an *acceleration* or a *delay* from some temporal reference point. So, when asked whether they would be willing to wait for a month to receive $110 instead of receiving $100 today, most people choose $100 today. By contrast, when asked whether they would prefer to speed up the receipt of $110 in a month by receiving $100 today instead, most people exhibit patience and take the $110 in a month. This phenomenon has been used as evidence for the gain–loss asymmetry or the prospect theory. It has also been connected to the *endowment effect*, which predicts that people tend to value objects more highly after they come to feel that they own them (Kahneman, Knetsch, and Thaler, 1990, 1991). The endowment effect explains the reluctance of people to part with assets that belong to their endowment. Nonetheless, Lerner, Small, and Loewenstein (2004) show that the agents' *mood*, sad or neutral, can affect the appearance of this effect.

Query theory

Recently, *query theory*, proposed by Johnson, Haeubl, and Keinan (2007), has been used to explain this and other similar choice inconsistencies. Query theory assumes that preferences, like all knowledge, are subject to the processes and dynamics of memory encoding and retrieval, and explores whether *memory and attentional processes* can explain observed anomalies in evaluation and choice. Weber *et al.* (2007) showed that the directional asymmetry in discounting is caused by the different order in which memory is queried for reasons favoring immediate versus future consumption, with earlier queries resulting in a richer set of responses, and reasons favoring immediate consumption being generated earlier for delay versus acceleration decisions.

Neural representation of hyperbolic discounting

McClure *et al.* (2004) investigate the neural systems that underlie discounting the value of rewards based on the delay until the time of delivery. They test the theory that hyperbolic discounting results from the combined function of *two separate brain systems*. The β system is hypothesized to place special weight on immediate outcomes, while the δ system is hypothesized to exert a more consistent weighting across time. They further hypothesize that β is mediated by limbic structures and δ by the lateral prefrontal cortex and associated structures supporting higher cognitive functions. Extending McClure *et al.* (2004), Finger *et al.* (2008) conducted an fMRI study investigating participants' neural activation underlying acceleration versus delay decisions. They found hyperbolic discounting only in the delay, but not the acceleration, function.

18.4 Risk

Risk preference plays an important role in many agent-based economic models, in particular agent-based financial models. The frequently used assumptions are CARA, CRRA, HARA (hyperbolic absolute risk aversion), and mean–variance, but, so far, few have ever justified the use of any of these with a neural foundation. This question can be particularly hard because, with the recent development of neuroscience, we are inevitably pushed to ask a deeper question: what the risk is. How does the agent recognize the risk involved in his or her decision-making? What may cause the perceived risk to deviate from the real risk? Is there any particular region in our brain that corresponds to a different order of *moments*, the statistics used to summarize the probabilistic uncertainty?

18.4.1 Neural representation of risk

One of the main issues currently discussed in neuroeconomics is the neural representation of risk. Through a large variety of risk experiments, it can be shown that many different parts of the brain are involved in decisions under risk, and they vary with experimental designs. Based on the activated areas of the brain, one may define a neural representation of the risk associated with a given experiment. Different kinds of risks may be differentiated by their different neural representations, and different risk-related concepts may also be distinguished in this way. For example, the famous Knight's distinction between uncertainty and risk can now be, through delicate experimental designs, actually distinguished from their associated neural representations. Using the famous Iowa gambling task, Lin *et al.* (2008) show that uncertainty is represented by the brain areas closely pertaining to emotion, whereas risk is associated with the prefrontal cortex. In this vein, Pushkarskaya *et al.* (2010) distinguish ambiguity from conflicts, and Mohr *et al.* (2008) separate behavioral risk from reward risk.

Identifying the neural representations of different risks may also shed light on the observed deviations of human behavior based on probability-based predictions. For example, a number of experiments, such as Feldman's experiment (Feldman, 1962) or the Iowa gambling task (Lin *et al.*, 2008), have indicated that even though subjects are given a risk environment, they may still behave as if they are in an uncertain environment. It is left for further study as to what the neural processes are behind this pattern recognition test which may inhibit or enhance the discovery of the underlying well-defined probabilistic environment.

18.4.2 Risk preference

Different assumptions of risk preference, such as mean–variance, CARA, CRRA, or HARA, are used in economic theory, usually in an arbitrary way. While agent-based modeling relies heavily on the idea of heterogeneity, preference or risk preference in most studies is normally assumed to be homogeneous. Little has been explored of the aggregate dynamics generated by a society of agents with heterogeneous risk preference.[3] Nevertheless, it seems to be quite normal to see agents with heterogeneous risk preferences in neuroeconomic experiments (Paulsen *et al.*, 2011).

Genetics have contributed in accounting for the difference in risk preference. Kuhnen and Chiao (2009) showed that several genes previously linked to emotional behavior and addiction are also found to be correlated with risk-taking investment decisions. They found that 5HTLPR ss allele carriers are more risk averse than those carrying the sl or ll alleles of the gene. D4DR 7-repeat allele carriers are more risk seeking than individuals without the 7-repeat allele. Individuals with the D2DR A1/A1 genotype have more stable risk preferences than those with the A1/A2 or A2/A2 genotype, while those with D4DR 4-repeat alleles have less stable preferences than people who do not have the 4-repeat allele.

One of the essential developments in neuroeconomics is to provide neural foundations for risk preferences. It is assumed that the human brain actually follows the finance approach, encoding the various statistical inputs needed for the effective evaluation of the desirability of risky gambles. In particular, neurons in parts of the brain respond immediately (with minimal delay) to changes in expected rewards and with a short delay (about one to two seconds) to risk, as measured by the payoff variance (Preuschoff, Bossaerts, and Quartz, 2006). Whether one can find evidence of higher-order risk (skewness aversion, for instance) remains an interesting issue.

Some initial studies indicate that risk preference may be *context dependent* or *event driven*, which, to some extent, can be triggered by how the risky environment is presented. D'Acremont and Bossaerts (2008) show that the dominance of mean–variance preference over the expected utility depends on the number of states. When the number of states increases, it is more likely that the mean–variance preference may fit the data better than the expected utility.

18.5 Learning

One essential element of agent-based computational economics is the notion of *autonomous agents*, i.e, agents who are able to learn and adapt on their own. It would have been a big surprise to us if neuroscience had not cared about learning. However, it will also be a surprise to us if the learning algorithms that we commonly use for software agents can actually have neural representations. Nonetheless, a few recent studies have pointed in this direction.

Dopaminergic reward prediction error hypothesis

Studies start with how the brain encodes the prediction error, and how other neural modules react to these errors. The most famous hypothesis in this area is the *dopaminergic reward prediction error* (DRPE) hypothesis. This hypothesis states that neurons that contain the neurotransmitter release dopamine in proportion to the difference between the *predicted reward* and the *experienced reward* of a particular event. Recent theoretical and experimental work on dopamine release has focused on the role that this neurotransmitter plays in learning and the resulting choice behavior. Neuroscientists have hypothesized that the role of dopamine is to update the *value* that humans and animals attach to different actions and stimuli, which in turn affects the probability that such an action will be chosen. If true, this theory suggests that a deeper understanding of dopamine will expand economists' understanding of how beliefs and preferences are formed, how they evolve, and how they play out in the act of choice.

Caplin and Dean (2008) formulate the DRPE hypothesis in axiomatic terms. Their treatment has precisely the *revealed preference* characteristic of identifying any possible reward function directly from the observables. They discuss the potential for measured dopamine release to provide insight into belief formation in repeated games and to learning theory, e.g., reinforcement learning. Their axiomatic model specifies three easily testable conditions for the entire class of reward prediction error (RPE) models. Briefly, the axioms will be satisfied if activity is (1) increasing with prize magnitude, (2) decreasing with lottery expected value, and (3) equivalent for outcomes from all lotteries with a single possible outcome. These three conditions are both necessary and sufficient for any RPE signal. If they hold, there is a way of defining experienced and predicted reward such that the signal encodes RPE with respect to those definitions. Rutledge *et al.* (2010) used blood oxygen level dependent responses at the outcome time to test whether activity in the nucleus accumbens (NAc) satisfies the axioms of the RPE model.

Klucharev *et al.* (2009) show that a deviation from group opinion is detected by neural activity in the rostral cingular zone (RCZ) and ventral striatum. These regions produce a neural signal similar to the prediction error signal in reinforcement learning that indicates a need for social conformity: a strong conflict-related signal in the RCZ and NAc trigger adjustment of judgments in line with group opinion. Using an olfactory categorization task performed by rats, Kepecs, Uchida, and Mainen (2008) attempt to obtain evidence for quantitative measurements of learning increments and test the hypothesis implied by reinforcement learning, i.e., one should learn more when uncertain and less when certain.

Studies also try to find the neural representation of different learning algorithms. The commonly used reinforcement learning and Bayesian learning is compared in Bossaerts *et al.* (2008), where they address the existence of the dual system.[4] They consider the reflective system and the reflexive system as the neural representation of Bayesian learning and reinforcement learning, respectively. Using the trust game, they were able to stratify subjects into two groups. One group used well-adapted strategies. EEG recordings revealed activation of a reflective (conflict-resolution) system, evidently to inhibit impulsive emotional reactions after disappointing outcomes. Pearson, Hayden, and Platt (2011) initiated another interesting line of research, i.e., the neural representations which distinguish *exploration* from *exploitation*, the two fundamental search strategies frequently used in various intelligent algorithms, for example genetic algorithms.

18.6 Software agents with neurocognitive dual system

18.6.1 Dual system conjecture

The dual system conjecture generally refers to the hypothesis that human thinking and decision-making are governed by two different but interacting systems. This conjecture has been increasingly recognized as being influential in psychology (Kahneman, 2003), neural science (McClure *et al.*, 2004), and economics. The two systems are an *affective system* and a *deliberative system* (Loewenstein and O'Donoghue, 2005), or a *reflexive system* and a *reflective system* (Lieberman, 2003). The affective system is considered to be myopic, activated by environmental stimuli, and primarily driven by affective states. The deliberative system is generally described as being goal oriented and forward looking. The former is associated with the areas of the brain that we have labeled the ventral striatum (nucleus accumbens, ventral caudate, and ventral putamen), the right striatum, neostriatum, and amygdala, among others, whereas the latter is associated with the areas of the brain that we have labeled the ventromedial and dorsolateral prefrontal and anterior cingulate, among others.

The dual system of the brain has become the neuroeconomic area which economic theorists take the most seriously. This has also helped with the formation of the new field known as *neuroeconomic theory*. A number of dual-process models have been proposed in economics with applications to *intertemporal choice* (Loewenstein and O'Donoghue, 2005; Fudenberg and Levine, 2006; Brocas and Carrillo, 2008a), *risk preferences* (Loewenstein and O'Donoghue, 2005), and *social preferences* (Loewenstein and O'Donoghue, 2005). All these models view economic behavior as being determined by the interaction between two different systems.

The application of the dual system conjecture to learning is just the beginning. Earlier, we mentioned the cognitive loading between different learning algorithms, such as reinforcement learning versus Bayesian learning (Section 18.5). This issue has recently been discussed in experimental economics (Charness and Levine, 2005), and now also in neuroeconomics (Bossaerts *et al.*, 2008).

While agents with dual systems have been considered to be a new research direction in neuroeconomic theory (Brocas and Carrillo, 2008a, b), software agents or autonomous agents in agent-based modeling mostly follow a single system. However, the dual system interpretation exists for many agent-based economic models. Consider the fundamentalist–chartist model as an example, where the fundamentalist's and chartist's behavior can be differentiated by the associated neural systems, say, assuming the former is associated with a deliberative system while the latter is associated with the affective system.

Another example is *individual learning* versus *social learning*. These two learning schemes have frequently been applied to model learning behavior in experiments and their fit to the experimental data are different (Hanaki, 2005). Agent-based simulation has also shown that their emergent patterns are different. For example, in the context of an artificial stock market, Yeh and Chen (2001b) show that agents using individual learning behave differently from agents using social learning in terms of market efficiency, price dynamics, and trading volume. If individual learning can be associated with, say, the deliberative system, and social learning can be connected to the affective system, then the dual system can also be applied to agent-based modeling. This issue opens the future to collaboration between agent-based economics and neuroeconomics.

18.7 Agent-based or brain-based?

Can we relate agent-based economics to brain-based economics (neuroeconomics)? Can we use the knowledge that we obtain from neuroeconomics to design software agents? One of the features of agent-based economics is the emphasis on the *heterogeneity* of agents. This heterogeneity may come from behavioral genetics. Research has shown that genetics has an effect on our risk preference. Kuhnen and Chiao (2009), Jamison *et al.* (2008), and Weber *et al.* (2008) show that preferences are affected by genes and/or education (environment). With the knowledge of genetics and neuroeconomics, the question is: How much more heterogeneity do we want to include in agent-based modeling? Does it really matter?

Heterogeneity may also result from age. The neuroeconomics evidence shows that certain functions of the brain will age. The consequence is that elderly people will make some systematic errors more often than young people, and age will affect financial decisions as well (Samanez Larkin, Kuhnen, and Knutson, 2008). Thus the same question arises: when engaging in agent-based modeling, should we take age heterogeneity into account? So, when a society ages, should we constantly adjust our agent-based model so that it can match the empirical age distribution of the society? So far we have not seen any agent-based modeling that features the aspect of aging.

Neuroeconomics does encourage the modular design of agents, because our brain is a modular structure. Many different modules in the brain have been identified. Some modules are related to emotion, some are related to cognition, and some are related to self-control. When human agents are presented with different experimental settings, we often see different combinations of these modules.

Notes

1 See also Baldassarre (2007). While it has a sharp focus on the *economics of happiness*, the idea of building economic agents upon the empirical findings of psychology and neuroscience and placing these agents in an agent-based computational framework is the same as what we argue here. From Baldassarre (2007), the reader may also find a historical development of *cardinal utility* and *ordinal utility* in economics. It has been a while since economists first considered that utility is a very subjective thing that cannot be measured in a scientific way, so that interpersonal comparison of utility is impossible, which further causes any redistribution policy to lose its ground.

2 It is not clear where preferences come from, i.e., their formation and development process, nor by when in time they come to their steady state and become fixed. Some recent behavioral studies have even asserted that people do not have preferences, in the sense in which that term is used in economic theory (Kahneman, Ritov, and Schkade, 1999).

3 For an exception, see Chen and Huang (2008).

4 See Section 18.6 for the dual system conjecture.

19 Cognitive agents

19.1 Introduction: heterogeneity and hierarchy

Despite the increasing tendency to make intelligence an explicit control variable in experimental economics and to explore its emergent outcomes, agent-based computational economics, which is normally claimed as the *software counterpart* of experimental economics, has paid almost no attention to this development. On the contrary, from a pure engineering viewpoint, there is a tendency to make software agents as smart as possible, and usually *equally smart*. This design principle, therefore, obviously contradicts our understanding of human agents. If the society of software agents cannot reasonably reflect the dispersion of human intelligence, then any resultant social simulation would be of little help to us in gaining insights into the emergent complexities, such as the perplexing relationship between IQ and social development (Lynn and Vanhanen, 2002; Lynn, 2006). Therefore, designing software agents with heterogeneous intelligence is the next step in exploring the emergent complexities of agent-based computational economics.

In this chapter, we shall review the development of cognitive agents in this light. We shall proceed in two directions. Agents with different cognitive capabilities can be endowed with either the same learning algorithms, associated with different parameters (Section 19.2.1), or with totally different learning algorithms (Section 18.5). Either way, it seems that there exists a connection between cognitive psychology and these learning algorithms, such that the latter can be placed in a cognitive hierarchy well structured by the former (the rest of this chapter).

Starting from Section 19.3.1, we shall also compare the development of the artificial agents in two popular kinds of agent-based computational economic models. One is agent-based models guided by *game experiments*. Artificial agents built in these models normally follow *reinforcement learning* or *generalized reinforcement learning* to make decisions (Chapter 10). The other is *agent-based financial markets*. As generally reviewed in Chapter 15, there are two different kinds of agent-based financial markets, namely the H-type model and the SFI model. Artificial agents built in these two models are different. The one extensively used in the H-type model, *regime-switching agents* (Brock and Hommes, 1998), is also a kind of reinforcement learning agent.

Despite the possible connections through the idea of generalized reinforcement learning, the conversation between these two classes of agent-based model is rather infrequent. Figure 19.1 lists all the main artificial agents used in these two classes of agent-based model, and shows how well each of them has been shared by the two camps. The artificial agents mentioned in this figure are zero-intelligence agents, reinforcement learning agents, belief learning agents, experience-weighted attraction learning agents, level-k reasoning agents, regime-switching agents, and novelty-discovering agents. From the position where they stand, one can see that a gap exists between the two. Few of these are simultaneously shared by both camps. Reinforcement learning agents seem to be the only ones that can connect the two, but, even for this one, the share is not equal between the two. They are much more inclined toward game experiments than financial simulations.[1] Further down the figure, we can see that some of the agents are exclusively employed in only one camp: level-k reasoning agents have only been applied in game experiments; on the other hand, regime-switching agents and novelty-discovering agents have been only active in financial modeling.

One has to wonder if the gap is a natural consequence of the limited applicability of the agents developed in each of the camps, or whether there is a potential to explore them in different camps so that the gap can be possibly narrowed. In any case, a conversation between the two holds the key for the answer. In this chapter, we attempt to make the two talk, and this talk is relevant for the theme of this part of the book, i.e., cognitive and psychological agent-based modeling. This is because the artificial agents in game experiments have been well developed into a *hierarchical framework* such that cognitive capacity can be incrementally added to the artificial agents from a low-level one, such as zero-intelligence agents (Gode and Sunder, 1993), to a high-level one, such as belief learning agents (Fudenberg

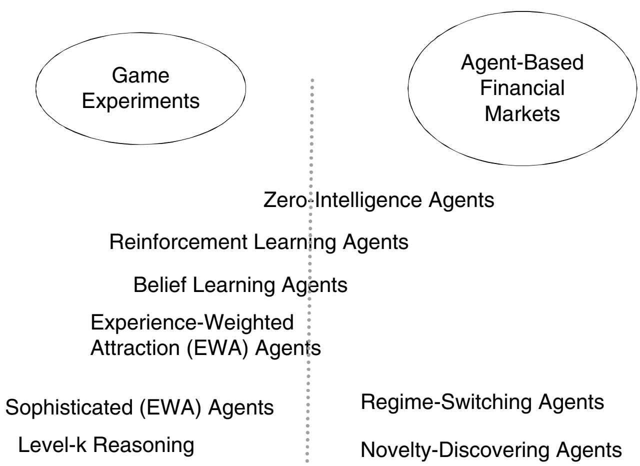


Figure 19.1 Artificial agents in game experiments and financial markets.

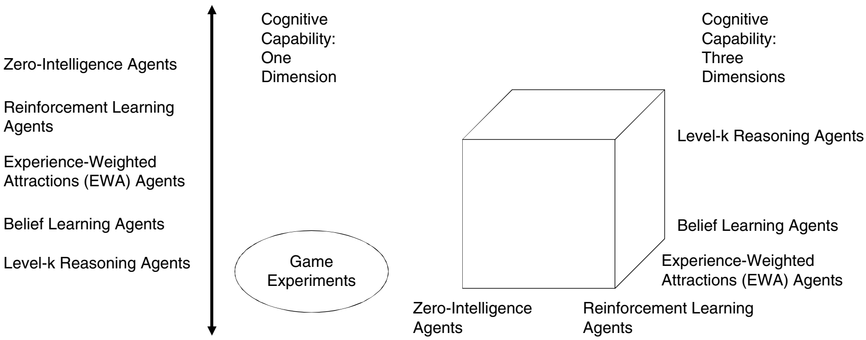


Figure 19.2 Cognitive capacity of artificial agents.

and Levine, 1998) or level-*k* reasoning agents (Camerer, Ho, and Chong, 2004; Crawford and Iriberri, 2007a, b), as we demonstrate in Figure 19.2 (left panel).

This conversation is beneficial for reflecting upon artificial financial agents, since a similar hierarchy as shown in Figure 19.2 has not been found in agent-based financial markets. Therefore, bridging the two classes of the agent-based model through artificial agents can help us to sort out the cognitive hierarchy underlying various financial agents. This step is crucial for the cognitive foundation of agent-based financial markets and is related to the recently more numerous empirical studies of *cognitive finance*.[2]

From Section 19.3.1 to 19.3.4, we shall argue that generalized reinforcement learning already incorporates three elements related to cognitive capacity, namely *memory*, *consciousness*, and *reasoning*, which may give a different cognitive map of the artificial agents, as shown in Figure 19.2 (right panel). With these three attributes, we indicate how the artificial agents of SFI can be considered as an extension of the artificial agents of the H-type model in one or more of these three elements. A usual criticism of the application of genetic programming to artificial agents is its lack of a cognitive foundation. With this bridge and the related extension, one can see that this criticism may underestimate the significance of genetic programming in the development of cognitive financial agents. Cognitive financial agents can, therefore, be firmly developed using computational intelligence.

19.2 Heterogeneity

19.2.1 Parameterizing heterogeneous working memory capacities

Chen, Zeng, and Yu (2008) have probably developed the first agent-based model to tackle this issue. In the context of the agent-based double auction market, they used genetic programming to model agents' adaptive behavior. This way of modeling is not new; however, they no longer assume that agents are equally smart. Instead, following the series of experiments which provided evidence of the importance of heterogeneity in subjects' short-term memory capacity (Devetag and

Warglien, 2003, 2008; Cornelissen, Dewitte, and Warlop, 2007), they manipulated one control parameter of GP so that the agents' "cognitive capacity" can be "born" differently. The parameter that they manipulated was the *population size*.

Genetic programming is a population-based algorithm that can implement parallel processing. Hence, on the one hand, the size of the population will directly determine the capability of parallel processing. On the other hand, the human's working memory capacity is frequently tested based on the number of cognitive tasks humans can simultaneously process (Cappelletti, Guth, and Ploner, 2008). Dual tasks have been used in hundreds of psychological experiments to measure the attentional demands of different mental activities (Pashler, 1998). Hence, *the population size seems to be an appropriate choice with regard to mimicking the working memory capacity of human agents*.[3] A smaller population size, therefore, corresponds to a smaller working memory capacity, whereas a larger population size corresponds to a larger working memory capacity. In this way, a market composed of agents with different working memory capacity is introduced.

Chen, Zeng, and Yu (2008) then simulated this agent-based double auction market, and examined the emergent properties at both the macro level (market performance) and the micro level (individual performance). At the macro level, they ran a regression of market efficiency on the population size (a proxy for the working memory capacity). It was found that working memory capacity has a positive and significant impact on the market efficiency, which is measured by the percentage of the realized consumer's and producer's surpluses.[4] Hence, the same institutional arrangement when applied to a population of agents with different intelligence may have different results in terms of market efficiency. The more intelligent group will perform better than the less intelligent one. Of course, this result can still be crude, and require more extensive tests, but the point here is how agent-based simulation, by incorporating the intelligence variable, can be more communicative with field data (Weede and Kampf, 2002; Jones and Schneider, 2006; Ram, 2007).

In addition to the aggregate outcome, they also compared the strategies learned from agents with different working memory capacity. Since all the strategies learned from GP have their LISP structure, they can be depicted as parse trees. This tree structure gives us a simple measure of the complexity for any acquired strategies based on the sizes of the trees. Chen, Zeng, and Yu (2008) then analyze the relation between the complexity of profitable strategies learned by the agents and their associated working memory capacity. They find that some strategies which are more complex but also more profitable had never been found by agents with a smaller capacity, say 10, but could quite frequently be found by agents with a larger capacity, say 50. Further analysis of these strategies shows that additional capacity facilitates the combinatorical operation that agents need to cook up with more complex and profitable strategies. In a sense, this result extends the findings of Devetag and Warglien (2003, 2008) and Ohtsubo and Rapoport (2006) to a more complex situation, namely, a double auction game.

A more challenging part of their work is to examine the coevolutionary dynamics when competing agents become equally smarter. This brings us closer to the

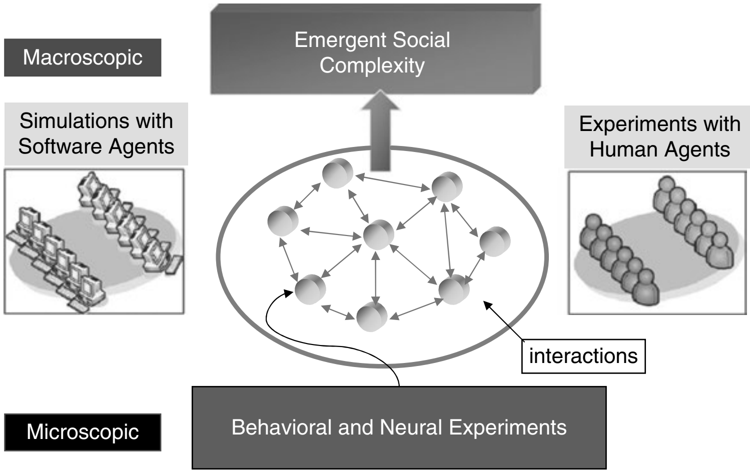


Figure 19.3 Agent-based computational economics and experimental economics: building a bridge by human-like designs of software agents.

situation discussed in Section 17.2.3. Chen, Zeng, and Yu (2008) show that, even in a competing situation like the double auction game, pairs of smarter agents can figure out a way to cooperate so as to create a win–win situation, whereas this collaboration is not shared by pairs of less smart agents.

Altogether, agent-based modeling with proper incorporation of the essential characteristics of human agents can make itself a proper toolbox to enhance our understanding of the emergent outcomes from human experiments or from field data. The entire picture is provided in Figure 19.3.

19.2.2 Intelligence and learning algorithms

We have mentioned the recent experimental economics that has focused on the intelligence effect. By parameterizing intelligence using short-term memory, Casari, Ham, and Kagel (2007) introduced intelligence variables into the *common value auction* experiment (Kagel and Levin, 2002), and found that cognitive ability as measured by SAT/ACT scores matters in terms of avoiding the *winner's curse* (Kagel and Levin, 1986). Their result has important implications for agent-based economic modeling and agent engineering, because they show that intelligence not only influences agents' static performance, but also their *learning dynamics*. So, even though the experiments are repeatedly conducted with the same subjects, their performance may not converge or may only converge at a slow rate.

In agent-based modeling, this phenomenon was first addressed in Feltovich (2005), which shows that if decision-makers learn via a specific version of reinforcement learning, their behavior typically changes only *very slowly*, and persistent mistakes are likely. Feltovich (2005) pointed out the difference between slow learning and no learning. While Feltovich (2005) did not make explicit reference to short-term memory capacity, his manipulation of reinforcement learning can, in a sense, be interpreted as a search for software agents with lower short-term memory capacity.

In the case of Feltovich (2005), the cognitive capacity of software agents is manipulated with the same learning algorithm, namely, reinforcement learning. Nevertheless, reinforcement learning has been frequently compared with other learning algorithms in the agent-based modeling of games, for example, belief-based learning and genetic algorithms (Duffy, 2006). Therefore, here comes another issue, i.e., instead of manipulating the control parameters of the same algorithm, be it genetic programming (Chen, Zeng, and Yu, 2008) or reinforcement learning (Feltovich, 2005), *how can one choose or compare different learning algorithms in light of the heterogeneity of various forms of parameterized intelligence?* This is the second kind of issue facing agent-based economic modeling, and is slightly different and more advanced than the one discussed in Section 19.2.1. To make this clear, we shall use Charness and Levin (2007) as an illustration.

Charness and Levin (2007) further pursued the issue between intelligence and learning behavior. Their finding shows that the failure to perform Bayesian updating can be a cause of the winner's curse, and the ability to perform Bayesian updating is dependent upon the agents' cognitive capacity. In this case, obviously, two learning algorithms are involved, and they are assumed to be associated with different types of cognitive loading.

Intuitively, intelligence can affect the way in which agents learn from the environment's feedback, because different learning algorithms, as decision-making, have different degrees of complexity. In computational learning theory, there is even a formal treatment on the complexity of machine learning (Kearns, 1990; Hutter, 2000). While these complexity measures are not necessarily computable, it is conceivable that some learning algorithms may be more complex than others. For example, Bayesian learning can generally be more complex than reinforcement learning.

Due to the general negligence of intelligence effects in agent-based computational economics, there is also no effort being made to consider a mixture of various learning algorithms, which can reasonably reflect the empirical lessons drawn from market or game experiments, and to explore its consequence. To the best of our knowledge, Chan *et al.* (1999) is probably the only study of this kind. In a context of agent-based artificial stock markets, they consider three different types of agents, namely, momentum traders (chartists), empirical Bayesian traders, and k-nearest-neighbor traders.

The efficient markets hypothesis implies that there are no profitable strategies, and hence learning, regardless of its formalism, does not matter. As a result, the three types of traders, momentum traders, empirical Bayesian, and traders, should behave equally well, at least in the long run. However, when the market is not efficient, and learning may matter, it is expected that smarter agents can take advantage of dumber agents. In their experiments, Chan *et al.* (1999) found that momentum traders, who never learn, performed the worst during the transition period when the market is not efficient. Furthermore, the empirical Bayesian traders were also outperformed by the KNN traders. While the two types of traders started learning at the same time and competed against each other to discover the true price, the KNN traders were evidently able to exploit predictability more quickly than the empirical Bayesian traders.

Like Feltovich (2005), Chan *et al.* (1999) did not make any explicit reference to agents' heterogeneity in intelligence; therefore, the way in which they introduced the three types of agents is not empirically driven. One of the next steps of ACE should be to attempt to empirically ground the mixture of different learning algorithms in either field data or experimental data so as to ensure the empirical relevance of the then-established agent-based models of heterogeneously intelligent agents.

19.3 Cognitive hierarchy

19.3.1 Cognitive capability of generalized reinforcement learning

> While the super-rational theories require unrealistic strong assumptions, the naive reinforcement models suffer from the opposite sin of assuming players are too limited in their reasoning powers.
>
> (Stahl, 2000, p. 106)

Reinforcement learning agents are often considered to be agents with minimal intellect because they demonstrate no strategic behavior. They act as if they were in the multi-armed bandit environment, and are ignorant or unconscious of the payoffs that they might have received from the slot machines had they tried them. *Imagination* is the word used in the literature to indicate the required cognitive efforts involved in this situation:

> The parameter δ measures the relative weight given to foregone payoffs, compared to the actual payoffs, in updating attractions. It can be interpreted as kind of "imagination" of foregone payoffs, in updating attractions, "simulation" of outcomes under alternative competitive scenarios ("counterfactual thinking" to psychologists), or responsiveness to foregone payoffs.
>
> (Camerer, Ho, and Chong, 2002, p. 141)

In a more general social interaction (not necessarily in a well-articulated normal-form game) the reinforcement-learning artificial agent may not even be aware of the presence of the opponents and mentally still position himself in a multi-armed bandit game, a game that just has himself and the machine.

Strategic thinking is developed in belief learning. Agents with belief learning form beliefs regarding their opponents. In addition to the choice history of himself, $\{s_i(l)\}_{l=1}^t$ (using the same notation as in Section 10.2.2), the agent also observes the choice history of his opponents, $\{s_{-i}(l)\}_{l=1}^t$. The latter is ignored by the reinforcement-learning agents. Despite this advance, what the belief-learning agent can do for his memory of the opponents' choice history $\{s_{-i}(l)\}_{l=1}^t$ is simply to memorize it, count it, and update the count. Sophisticated analysis of the data, such as examining the statistical dependence of the time series of $\{s_i(l), s_{-i}(l)\}_{l=1}^{t}$, is not seen in the usual weighted fictitious play. Serious reasoning about what opponents might do is not available in belief learning. However, by using a kind of level-k reasoning, Camerer, Ho, and Chong (2002) add the reasoning capability to generalized reinforcement learning, which is referred to as the *sophisticated* EWA model.

19.3.2 Cognitive hierarchy proposed

Table 19.1 proposes a summary of how cognitive capacity can be incrementally added from the zero-intelligence agent all the way up to the sophisticated EWA agents (Camerer, Ho, and Chong, 2002). The key cognitive elements used to distinguish these learning models are *memory*, *consciousness*, and *reasoning*. Zero-intelligence agents have none of these. They have no memory of the past and even no memory of the present. They basically make decisions unconsciously. Hence, when making decisions, all human factors such as cognitive capacity, mental states, neurophysiological elements (dopamine), and so on play no role. Reinforcement learning agents are endowed with some power of memory, which can help them to recall their past experiences of feeling in each of the same situations. However, they are not given sufficient cognitive capacity for them to be aware of the entire environment within which their decisions are embedded. Essentially, they are not aware of what their players have done and would not try to outguess what they will do. Without enough consciousness and reasoning, decisions made with reinforcement learning can be very *reflexive*, as if being automatically driven by the dopamine system.[5]

With the given memory, be it short or long, the belief learning agents are given additional cognitive capacity to be aware of the presence of their opponents. They are conscious of the effect of the opponents' actions on their payoffs. Their decisions are, therefore, based on such reasoning by taking this feedback into account. In particular, they are able to figure out, in an ex post manner, what would happen had they chosen different actions, and to take these so-called *foregone payoffs* into account. However, they are not advanced enough to develop a sophisticated model to predict the behavior of their opponents' actions, apart from counting

Table 19.1 Artificial agents with incremental cognitive capacity

Model	*Memory*	*Consciousness*	*Reasoning*
Zero intelligence	None	None	None
Reinforcement learning	Short to long	None	None
Belief learning	Short to long	Strong	Weak
EWA learning	Short to long	Weak to strong	Weak
Sophisticated EWA	Short to long	Weak to strong	Weak to strong
Regime switching	Short to long	Weak	None
Novelty-discovering agents (autonomous agents)	Short to long	Weak to strong	Weak to strong

the frequencies of their past actions. The EWA agents are a hybridization of the reinforcement learning agents and belief learning agents in the sense that their cognitive capacity to "imagine" all foregone payoffs is only partial. Their consciousness of the environment and the mental capability to engage in counter-factual thinking lies between those of the reinforcement learning agents and belief learning agents. The sophisticated EWA involves the incorporation of level-k reasoning into EWA learning. Therefore, agents with this design are clothed with the additional power to reason about what other agents, agents with a lower level of cognitive hierarchy, might think.

Cognitive hierarchy implies heterogeneity; it refers to different degrees of sophistication of agents. Evidence of the heterogeneity of human subjects in game experiments was first found in Cheung and Friedman (1997), where the artificial players are presented with a binary choice in the context of normal-form games. Their binary choice is stochastically determined by the familiar logit model [Equation (10.8)]. The main input of the logit model is related to the differences in the expected payoffs of the associated choices, derived with respect to the player's belief (subjective probability function). Cheung and Friedman (1997) then parameterized the decision rule (the logit model) and the learning rule (the belief updating scheme) of agents with two and one parameters, respectively. The three-parameter belief learning agent is then estimated based on four kinds of normal-form game experiments, involving a total of 393 human subjects. Cheung and Friedman (1997) then proposed a log-likelihood ratio test to test the null of a homogeneous agent (representative agent) with the alternative of complete heterogeneous agents (all agents are different). The null was rejected, indicating that heterogeneity across subjects in terms of learning and decision parameters is an important feature of their observations. In particular, the observed heterogeneity in the learning parameter allows us to further classify the majority of players, 257 out of 393, into three groups of agents with distinct memory lengths, from the shortest to the longest, corresponding to the *Cournot agents* and the *fictitious-playing agents*, respectively (Section 10.2.2).

Evidence on the heterogeneity of human subjects was also found in other studies (Camerer and Ho, 1998; Stahl, 2000; Ho, Wang, and Camerer, 2008; Chen and Hsieh, 2011).[6] However, few have ever addressed what may cause the observed heterogeneities of these human agents. In light of the recent trend in cognitive and psychological economic experiments, for example, one may inquire whether these observed heterogeneities are related to the idiosyncratic differences in subjects' cognitive or psychological attributes. As an empirical foundation of ACE, this query can be quite crucial because the learning models developed in game experiments enable us to think of the heterogeneities of agents from a bounded-rationality perspective, and hence provide a justification for the development of heterogeneous agents in ACE. What is particularly intriguing is that these models may possibly be assembled into a hierarchical framework, from the ones with a lower cognitive capacity to the ones with a higher cognitive capacity. It demonstrates a way to model artificial agents with *incremental* cognitive capacity.

19.3.3 Cognitive capacity of regime-switching agents

In this section, we move to the agent-based financial markets, and examine the cognitive capacity of the artificial financial agents built in these markets in light of what we have learned from Section 19.3. Before proceeding further, we would like to point out that generalized reinforcement learning has been applied to agent-based economic modeling, including electricity markets (Bunn and Oliveria, 2001) and financial markets. Therefore, they are not just limited to behavioral games. Nevertheless, these applications are relatively rare compared to others. Therefore, in the following, we shall focus more on the two dominant classes of artificial financial agents, namely *regime-switching agents* and *novelty-discovering agents*. The former is standard in the H-type agent-based financial markets, and the latter is widely used in the SFI-like agent-based financial markets.

The cognitive capacity of regime-switching agents can be examined using Table 19.1. First of all, most regime-switching agents do take into account the foregone payoffs. For example, in the Brock–Hommes model (Brock and Hommes, 1998), the agents who chose to behave like fundamentalists (chartists) would also update the payoffs which they might have received had they behaved like a chartist (fundamentalist). Hence, as with the usual distinction made between reinforcement learning and belief learning, i.e., the law of actual effect and the law of simulated effect, they may be positioned closer to the latter. However, belief learning agents will form beliefs on what other agents may tend to do and react upon these expectations. This degree of consciousness is simply absent in these regime-switching agents. For example, they will not form expectations of the possible microstructure (percentages of fundamentalists or chartists) in the next period, and make their choice accordingly. This missing awareness is probably because of the overwhelming reliance on the use of simple programmed agents and the pursuit of simplicity in this type of agent-based financial model.

Second, without sufficient consciousness, reasoning capability as characterized by various cognitive hierarchies is also not presented in these regime-switching agents. While level-k reasoning was partially motivated by Keynes's beauty contest as an influential metaphor of stock markets, its application to agent-based financial models has not yet been realized. Accordingly, they are not heterogeneous in their cognitive capacity. As a matter of fact, while frequently termed as models of heterogeneous agents, most regime-switching agents in financial markets are *homogeneous ex ante*, i.e., their heterogeneous behavior is just a different realization drawn from the same choice probabilities. The only source of agent heterogeneity is this random component.

The features of the cognitive capacity of these regime-switching agents are summarized in Table 19.1 as a comparison with those agents developed in game experiments. The artificial agents built in game experiments (Sections 10.2 and 10.3) have two characteristics, namely *incremental cognitive capacity* and *cognitive heterogeneity*. Neither of these two exists in the H-type agent-based financial models. Hence, the question is whether agent-based financial markets need artificial agents with cognitive hierarchy and cognitive heterogeneity. What are the additional insights one may gain when we allow our financial agents to be heterogeneous in their reasoning capability?

We believe that these questions have not been seriously taken into account in current agent-based financial literature, while some initial attempts exist. These attempts usually tried to identify different intelligent algorithms with different levels of intelligence (Yeh, 2007, 2008). In other words, a cognitive hierarchy as suggested in Figure 19.2 is assumed implicitly. More generally, doing this is equivalent to assuming that all intelligence algorithms, such as genetic algorithms, neural networks, decision trees, nearest neighbors, support vector machines, etc., can be placed in a cognitive ladder like Figure 19.2.

However, the difference is that all artificial agents in Figure 19.2 are constructed with incremental cognitive capacity, whereas these intelligence algorithms, each motivated by different disciplines, are not constructed incrementally in this way. Therefore, one may abuse much power to assign their position in the hierarchy, not to mention the psychological ground to support this assignment. While this does not mean that these algorithms are equally smart and cannot be comparable, a theoretical foundation to make these tools have a psychologically equivalent intelligence quotient has to be laid first. Of course, in principle, it is not impossible to do so. In fact, from the perspective of computational neuroscience, if brains with different cognitive capacities can be shown computationally, then all intelligent algorithms may also be examined in this vein.

19.3.4 Cognitive capability of novelty-discovering agents

Through the tournament origin (Chapter 5), we can answer where the reasoning capabilities of financial agents are, if we do not find them in the H-type agent-based financial models. Our answer is that novelty-discovering agents are the reasoning agents missing in the H-type agent-based financial models. That is because they demonstrate the essence of level-k reasoning, i.e., *iterative reasoning*, even in a broader sense and are not limited to iterated dominance games.[7]

To see the novelty-discovering agents in relation to level-k reasoning agents, let us recall the findings from Axelrod's IPD open evolutionary tournament (Section 5.3). In the former case, the novelty-discovering agent was able to outsmart the TIT-FOR-TAT agent by being able to simulate what he, and other agents too, may do. In other words, this novelty-discovering agent stood at a higher level on the cognitive ladder mentioned in Section 10.3.2. Notice that he did not stand at a high level at the outset, but learned to be there through repetition.

Similarly, Rust, Miller, and Palmer (1993) were concerned with the robustness of Kaplan's "wait in the background" strategy, the best-performing strategy in the Santa Fe DA tournament, just like what had surprised Axelrod on the beatableness of the TIT-FOR-TAT strategy (Section 5.2). Even though John Rust and his colleagues did not run another DA tournament, almost two decades later Chen and Tai (2010) re-ran the tournament in an automated fashion. They found that the Kaplan agent could indeed be beaten by the novelty-discovering agent driven by genetic programming.

These two examples, the beatableness of TIT-FOR-TAT in the IPD tournament and that of Kaplan in the DA tournament, indicate that from the very beginning our designed novelty-discovering agents were there to learn who their opponents were, and tried to find their Achilles' heel and reacted to it. What they did is essentially the same as what the level-k reasoning agents would have done in a similar environment. Hence, unlike regime-switching agents, novelty-discovering agents are equipped with a reasoning capability (Table 19.1).

19.3.5 Cognitive capacity hypothesis

Empirical studies of level-k reasoning based on human-subject experiments have constantly shown that there is a distribution for k. Most frequently, k is one or two. That is, for human subjects, few of them are level-0, and also few of them are beyond level-3 (Stahl and Wilson, 1994; Duffy and Nagel, 1997; Ho, Camerer, and Weigelt, 1998; Bosch-Domenech *et al.*, 2002). The *cognitive economic experiments* that began in the early 2000s attempt to see whether there is a link between *cognitive capacity* and this *cognitive hierarchy* (level-k; Devetag and Warglien, 2003, 2008; Burnham *et al.*, 2009; Georganas, Healy, and Weber, 2010). There are two possibilities that make this research intriguing, namely *nature* or *nurture*. In the former case, k is exogenously given, for example, possibly from genetic inheritance; in the latter case, k is endogenous, evolving with experience and learning.

The *cognitive capacity hypothesis* basically says that k is exogenously given and can be positively related to cognitive capacity (Burnham *et al.*, 2009). Therefore, in every novel situation, human subjects tend to perform at the same cognitive hierarchical level. On the other hand, the *literacy hypothesis* predicts the opposite. The more the human subjects experience, the higher their cognitive hierarchies are. Hence, if we have an experiment by pooling differently experienced subjects, we tend to find that k is positively related to experience. While cognitive economic experiments seem to lend some support to the cognitive capacity hypothesis, the research is still too limited and premature to draw any strong conclusion.[8]

What, however, interests us is the possible implications for the design of artificial agents. Earlier, we mentioned that the agent community had already tried to identify different intelligence algorithms with their IQ, while in a somewhat causal manner. What we would like to suggest here is that this can be done more rigorously if we can ground this identification within cognitive psychology or cognitive neuroscience. Saying so is not too far-fetched considering that many intelligence algorithms are already naturally inspired, and also now that computational neuroscientists are working on genuine brain networks (Sporns, 2010).

Notice that artificial agents in both Casari (2004) and Chen, Zeng, and Yu (2008) are still novelty-discovering agents (Section 19.2.1). They are able to learn, to discover, and to adapt. They just perform these tasks with different cognitive capacities (population sizes). Therefore, their design enables us to answer questions regarding nature or nurture under environments with different complexities.

19.4 Concluding remarks

This chapter provides a review of the development of cognitive agents in agent-based computational economics. We focus on two major domains of ACE: one is behavioral game experiments with human subjects, and the second is financial markets. The artificial agents reviewed in this chapter are distributed over these two different domains, although not evenly (Figure 19.1). Some artificial agents are quite extensively used in one domain, but not the other. Nevertheless, in this chapter we try to align them in such a way that the cognitive capacity of these artificial agents can be sorted in monotonic order. By adding the zero-intelligence agent to the list of artificial agents in game experiments, we are able to construct a cognitive spectrum for the artificial agents (Figure 19.2). The salient feature of using artificial agents in laboratory games to construct this spectrum is that this spectrum may have real counterparts for human beings. Hence, the term "agents" actually covers both artificial automata as well as natural automata. Our perspective for the cognitive agents is, therefore, not just purely from an engineering viewpoint, but is also inspired and guided by the recent progress in cognitive psychology, computational neuroscience, and economics experiments.

19.5 Further study

Research question: the role of cognitive capacity in inductive reasoning with analogies

In Section 15.4.1, we mentioned that Chan *et al.* (1999) has applied KNN to the agent-based financial markets (see also the related research question in Section 12.11). If learning with experiences is so much dependent upon similarity to what we have experienced, and that itself depends on our memory, then memory capacity should play a role in inference reasoning with analogies. Hence, subjects endowed with different memory capacities may come up with different perceptions of similarities and hence different decisions, even though they share the same experiences.

As we have seen in this chapter, there are a couple of agent-based artificial markets already taking into account the role of memory capacity (Casari, 2004; Chen, Zeng, and Yu, 2008; Chen and Yu, 2011; Chen *et al.*, 2011). In these markets, either genetic algorithms or genetic programming has been applied to model artificial agents. Explain how these GA- or GP-driven artificial agents can be further differentiated by memory capacity. Can you then combine these two lines of study to propose an agent-based artificial stock market with artificial traders who are reasoning with analogies but are further constrained by their memory capacities? What are the additional insights that we can gain from these artificial markets.

Notes

1 In agent-based financial markets, we see very few formal applications of generalized reinforcement learning. One exception is Pouget (2007).

2 There is growing evidence that cognitive ability is an important predictor of financial outcomes. See, for example, Agarwal and Mazumder (2010); Finke (2010).

3 The idea of using population size as a proxy variable for working memory is first proposed in Casari (2004), who literally treated the population size used in the genetic algorithm as equivalent to the number of chunks that humans can process at a time. According to the famous "7 ± 2" rule proposed by George Miller (Miller, 1956), the capacity lies between five and nine. Casari (2004) then set the population size of genetic algorithms to six, "which implies that decision-makers have a hardwired limitation in processing information at six strategies at a time" (Casari, 2004, p. 261). This is probably the earliest article to connect the *population size* used in evolutionary computation to *working memory capacity* in cognitive psychology. However, in Casari (2004), agents are still treated as homogeneous.

4 What Chen, Zeng, and Yu (2008) did is to fit their simulation results to the following regression equation:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon, \tag{19.1}$$

where Y is the market efficiency index, X_1 is the working memory capacity, and X_2 is the number of agents (buyers) with that working memory capacity in the market. In Chen, Zeng, and Yu (2008), the working memory capacity is only applicable for the GP traders since the rest of the traders are all truth-tellers. They have the following result:

$$\begin{array}{lll} Y = 99+ & 0.028X_1 & -1.224X_2. \\ & (3.43) & (-11.56) \end{array} \tag{19.2}$$

The numbers inside the bracket are the associated t value of the estimated coefficients. Equation (19.2) indicates that β_1 is positively significant, meaning that increasing the working memory capacity of the GP traders in the market will result in better market efficiency in terms of realized market efficiency. On the other hand, keeping the working memory capacity fixed, simply increasing the number of GP traders with that working memory capacity has a negative effect, as indicated by the statistically significant β_2.

5 For more details, see Montague (2006) and Glimcher (2010).

6 It is not always the case that the null of homogeneous agents can be rejected by the laboratory data. For example, Camerer and Ho (1999) tested the null of homogeneous agents by assuming two types of agents as the alternative, and they found that the null cannot be rejected.

7 Iterated dominance games, such as the beauty contest (Section 10.3.1), allow us to develop a step-by-step computation in the acquisition of a dominant strategy, and hence develop a complexity measure in the spirit of computational theory. The number of steps required to eliminate all dominated strategies is also called the depth of reasoning. See Camerer (2003) for details.

8 The fundamental question is whether we can possibly examine the possible influence of cognitive capacity using laboratory games with human subjects. This question, as a whole, is what cognitive economic experiments have to answer.

20 Culturally sensitive agents

As we have seen in earlier cross-culture economic experiments (Section 17.7), culture can matter for human decision-making and behavior. Broadly interpreted, culture is a set of routines, rules, norms, routines, and beliefs that will govern the behavioral mode of human agents. Geert Hofstede, well known for his proposed *five dimensions of culture*, defines culture as "the *collective programming* of the mind that distinguishes the members of one group of people from another" (Hofstede, 2001, p. 9; emphasis added). With this understanding, it is not surprising to expect research efforts attempting to integrate culture into models of software agents at the micro level, i.e., to build *culturally sensitive agents*. The aim is to map each identified culture to a set of behavioral rules, for example, decision tress (Section 12.10). Hence, different cultures are associated with different behavioral rules. Culture here is taken as exogenously given. A further attempt is, therefore, to *endogenize* it so as to examine how culture may arise, develop, and evolve through time. To differentiate these two different attempts, we shall call the former models *exogenous* agent-based cultural models and the latter *endogenous*.

Work on both classes of models is just at the initial stages and further advancements are expected in the near future. Due to the size of this book, we intend to make this chapter very short. The purpose is simply to make readers aware of the existence of this research line, and its potential relevance to ACE. Readers may quickly get a flavor and see the significance and relevance of these studies in a broader context.

We shall start with the endogenous models first (Section 20.1), simply for chronological reasons. Work on endogenous models of culture had already appeared in the late 1990s, whereas work on exogenous models of culture was not available until the mid 2000s. One may wonder why exogenous models (Section 20.2) happened after the endogenous ones, since the former should naturally be considered as the continuing step of the latter. However, as we shall see below, the culture dealt with by the endogenous models are formal culture (like formal language), whereas the culture dealt with by exogenous models is natural culture (like natural language); hence, they each have their own difficulties, and the latter may be even more challenging.

20.1 Evolutionary models of cultures

Robert Axelrod proposed the first agent-based model of culture dissemination using cellular automata (Axelrod, 1997b). This model has been further extended by replacing the cellular automata models with general social networks (Centola *et al.*, 2007). This more general framework would enable us to think of the evolution of culture in terms of the evolution of both social space and physical space.

One use of this model is to investigate the effect of *globalization* on culture diversity. Globalization is technically represented as *migration* (physical mobility) and *social connection* (social mobility, to take into account the revolution of communication technology). The idea is to built a stochastic interaction model based on agents' physical and social distance. The stochastic interaction will then further lead to consequent behavior on culture exchanges, social networking, and migration. The developed system is generally complex enough. Hence, the only way to see the globalization effect upon culture diversity or homogenization is through computer simulation.

20.2 Culturally based behavioral rules

Gert Hofstede, Catholijn Jonker, and Tim Verwaart, as a team, have collaborated, in their series of studies to construct culturally sensitive software agents.[1] They have incorporated four of the five dimensions of culture into agent-based models of trade negotiation. The four are *power distance* (Hofstede, Jonker, and Verwaart, 2009), *individualism* (Hofstede, Jonker, and Verwaart, 2008a), *uncertainty avoidance* (Hofstede, Jonker, and Verwaart, 2008b), and *long-term orientation* (Hofstede, Jonker, and Verwaart, 2008c).

Hofstede, Jonker, and Verwaart (2008a) programmed individualistic and collectivistic trading agents with eight behavior rules. These rules attempt to differentiate these agents based on the characterizing behaviors of *fairness*, *trustworthiness*, and *benevolence*. For collectivistic agents, their perception of fairness and hence the tolerance of unfair offers will sensitively depend on whether their trading partners are from the same "family." It was modeled in such a way that they can be more tolerant with or patient with the "unfair" offer, be more benevolent to and easier to have trust with trading partners with the same "family." In a more general framework, we can consider that each agent has a distance measure between him/her and the interacting partners. Their decision or action will then be based on this "distance." In the computational model of Hofstede, Jonker, and Verwaart (2008a), this distance measure differs between individualistic and collectivistic agents. Roughly speaking, either this distance measure does not exist for the individualistic agents, or their decisions are largely distance-independent. Experimental economics may be able to give quite intensive examinations of the validity of these rules. In particular, recently, a number of experiments have been designed to examine the role of *identity* in strategic behavior.[2]

20.3 Culturally based preference

There are many different ways in which cultural factors can be built into software agents. *Social preference*, for example, is a key attribute. Conventional neoclassical economics is mainly concerned with *economic man* with his own *personal preference*. An agent's preference is personal if it depends only on his own consumption or received income; otherwise, it is social. Fehr and Schmidt (1999) have a simple operational model for agents with social preferences. Consider the following utility function for player i in a two-person game:

$$u_i(\pi_i, \pi_j) = \begin{cases} \pi_i - \alpha_i(\pi_j - \pi_i) & \text{if } \pi_i \leq \pi_j, \\ \pi_i - \beta_i(\pi_i - \pi_j) & \text{if } \pi_i \geq \pi_j, \end{cases} \tag{20.1}$$

where π_i and π_j are payoffs received by player i and j respectively, and $0 \leq \beta_i \leq \alpha_i$. The special case of $\alpha = \beta = 0$ provides personal preferences, whereas $\alpha \geq \beta > 0$ provides social preferences. For the latter, player i still likes high monetary payoffs, but is now also averse to inequality, being especially uneasy with inequality for which he receives lower payoffs ($\pi_i \leq \pi_j$). Cultures which support cooperation or sharing, like Ache hunters in Paraguay (see Henrich *et al.*, 2004), can be considered to be agents with a sufficiently large value of β, whereas the Machiguenga in Peru are associated with a low value of β. Then many familiar games that are studied using CI, such as the prisoner's dilemma game, bargaining game, double auction game, trust game, etc., can be extended to agents with social preferences so as to further simulate the possible influence of culture on human behavior.

Kuznar (2007) provides another illustration of cultural agents. Using anthropological data on individual men's wealth and political affiliations from Kapauku, a New Guinea tribal village, he tested 24 software agents (decision algorithms) from 7 different paradigms by simulating men's decisions with theorized decision rules and examining which rules produce Kapauku-like alliances. The attempt is made, therefore, to match software agents with real human agents in Kapauku, and, through the exploratory modeling, to shed light on the cultural characteristics of the Kapauku people.

While in ACE there are already many studies comparing different software algorithms in terms of their ability to fit outcomes of experiments with human subjects, cultural elements have generally been ignored. This is because most experiments which the ACE researchers considered mirroring have no cultural elements. For example, these experiments are repeatedly performed using university students, and, most likely, in the United States. Therefore, this makes the cultural aspects of these different algorithms rather implicit. For example, reinforcement learning is individual learning, which may apply well for societies whose people have little experience regarding sharing; on the other hand, the genetic algorithm is a form of social learning, which may apply to the opposite situation. So, for the same experiments, the software agents that are fitting for Machiguenga hunters may be inapplicable to Lamelara whale-hunters Henrich *et al.* (2004).

Notes

1 Gert Hofstede is the son of the Dutch organizational sociologist Geert Hofstede, who is famous for his *five dimensions theory of culture* (Hofstede, 2001). The significance of his work in cross-culture studies has been compared to the work of Darwin in evolutionary theory. Of course, controversies concerning this work also exist; see, for example, McSweeney (2002).

2 This development has been well summarized by Yan Chen in her keynote speech, entitled *The Potential of Social Identity*, delivered at the 2008 International Economic Science Association Conference. One year later, something similar also became the topic of George Akerlof's keynote speech at IAREP/SABE 2009.

21 Agent-based lottery market

This part reflects the recent trend in grounding agents in behavioral foundations. The four preceding chapters, Chapters 17 to 20, indicate the economic relevance of the psychological, cognitive, neural, and cultural attributes of human decision-making, and highlight some possible directions for advancement in modeling. We shall conclude this part with a concrete demonstration on a psychological agent-based model, namely the agent-based lottery market.

21.1 Behavioral institutional economics

The lottery market is an area where gambling psychology also plays quite an active role. Chen and Chie (2008) develop an agent-based lottery market, whose autonomous agents are grounded largely in this gambling psychology. Their artificial agents have the potential to develop three psychology characteristics: the *halo effect* (Lottomania), *conscious selection* (neglect of probability), and *regret aversion* (interdependent preference). What differs in their model from typical behavioral models is that these three characteristics are not imposed exogenously, but can only emerge as an evolutionary outcome. This exemplifies how ACE can work with behavioral economics and make the implicit selection process explicit, and provides a stability test for these behavioral patterns.

This model is also used to answer the question of the optimal lottery tax rate. In this regard, we shall illustrate the idea of gauging the uncertainty associated with policy designs, in this case, the determination of the tax rate, particularly when the surroundings on which the new policy will operate are not very well understood or not well explored. This can often happen when doing policy experiments is either infeasible or very expensive, and hence it is always a challenge for decision-makers to “visualize” *what may happen after change*. One fundamental difficulty pertaining to this challenge is the unpredictable behavior of stakeholders. Unless we can reasonably track their behavioral modes, the risks of policy changes can hardly be estimated.

The integration of computational intelligence into agent-based modeling has, however, proposed a possible solution to this conundrum. Computational intelligence, as a tool, can help us to give our artificial stakeholders some features of real ones, and hence policy simulation based on this bottom-up design may reduce the otherwise greater uncertainty due to inappropriate behavioral assumptions of agents. This we shall call *behavioral institutional economics*.[1] In this chapter, a concrete demonstration based on Chen and Chie (2008) is given to illustrate the idea above. We show how an agent-based lottery market can be built and used to gauge the potential uncertainty of tax revenue after either raising or reducing lottery tax rate.

The distinguishing feature of agent-based simulation is that the model-building begins with individuals. Given that, a lot of micro details can be brought to the models. These micro details can include all behavioral (cognitive, psychological, cultural) aspects of agents, the materials substantially reviewed in this part of the book. In general, agent-based simulation results can depend on the design of agents, their behavioral rules, and parameters.

21.2 Lottery market designs

We now introduce the agent-based model of the lottery market proposed by Chen and Chie (2008). Each agent-based model is largely composed of two parts, the embeddedness, i.e. the environment, and the software agent design. We shall first introduce their embeddedness (Section 21.2.1), and then move to the software agent design (Section 21.2.2).

21.2.1 Embeddedness

Generally speaking, a lottery game can be parameterized by two parameters (x, X). In an "x/X" lottery game, we required both a gambler and the lottery agency to pick x numbers out of a total of X numbers, and then different prizes are set for different numbers matched. Let y denote the numbers matched. Clearly, $y = 0, 1, \ldots, x$. Let S_y be the *prize pool* reserved for the winners who matched the y numbers. A special term is given to the largest pool, S_x, namely, the *jackpot*. Each prize pool, S_y, is to be shared by the number of players who match y numbers, say N_y. In the event that $N_y = 0$, S_y is added to the next draw. A particularly interesting case is $N_x = 0$. A common feature of lotteries is that if there are no winners in a given draw, the jackpot prize pool from that draw is added to the pool for the next draw, referred to as a *rollover*. Rollovers usually enhance the attractiveness of the next draw, called the *rollover draw*.

A lottery game normally has a fixed time interval from one draw to another. For example, in the case of Taiwan, it is drawn twice a week, once on Wednesday and once on Saturday. The drawing interval is supposed to be a parameter of the lottery game.

Lottery design also depends on the potential market size, which can be roughly measured by the population size and the income per capita. Here, in Chen and Chie (2008), both of these variables are fixed. They choose a relatively small population size of 5000. Obviously, they can make it larger but it will be more time-consuming in simulation. Because of this relative small population size, they have to design the winning odds in such a way that it can match a real market. In addition, income is also assumed to be fixed and to be identical among agents.

21.2.2 Software agent designs

There are more fundamental issues: how to build software agents. What is the guideline? This issue is generally known as agent engineering. As we will see in the example below, there are two rudimentary elements involved in agent engineering: first, the functions of the agent; second, the tools which can deliver such functions. Given the functions, it may not be difficult to find suitable tools, but the question is how to find the functions. One answer is that they must be empirically grounded (Chapter 16).

Halo effect

That the lottery participation level is positively related to the size of the jackpot prize seems to be one of the most important empirical observations. The phenomenon that sales following a rollover are higher than sales prior to the rollover is known as the *halo effect* in the industry. The halo effect is partially due to the considerable media attention paid to rollovers, which in turn creates a bout of lottomania. Chen and Chie (2008), therefore, build the software agent with this halo effect. Basically, the amount of money to invest depends on the *size of jackpot*. They start building artificial agents from a *participation function* that relates the participation level to the jackpot size,

$$\mu = \rho(J), \tag{21.1}$$

where ρ is the participation function, μ is a measure of the participation level, and J is the size of the jackpot. The exact functional form of ρ depends on the framework within which the problem is formulated. In the standard rational analysis, μ is related to J via a change in the *expected value* or, more generally, the *expected utility*, of the lottery ticket (Hartley and Lanot, 2003). However, here, they take a heuristic approach, and assume that gamblers base their decisions on some heuristics rather than the possibly quite demanding work on expectations computation.

Based on the heuristic approach, Equation (21.1) can be approximated by a few simple *if–then* rules. For example, "if the jackpot is unusually high, then I will spend 10 percent of my income to buy lottery tickets," or "if there is no rollover, I will spend only a little." Notice that the antecedent or consequent of the rule contains the use of natural language that may not have concrete numerical meanings, such as the linguistic terms "high" and "only a little" in the above example. While natural language has its ambiguities, people seem to be able to reason effectively with vague and uncertain information, and very often the decisions they make are the outcome of their approximate reasoning. Over the last four decades, we have seen the development of *fuzzy logic* as a formal approach to deal with these ambiguities (Chapter 11). Chen and Chie (2008) propose representing the function ρ

by a set of *fuzzy if–then* rules that are manipulated by the standard mathematical operations of fuzzy sets as prescribed by fuzzy set theory.

The implementation of the fuzzy rules (21.2) proceeds as follows. First, let J_{t_r} be the jackpot prize updated on the rth day of the tth issue, where $r = 1, 2, \ldots, w$; w denotes the gap between two draws. If we suppose that the lottery draw takes place weekly, then $w = 7$. Furthermore, let the set $\{J\}_{t_r}$ be the time series of the jackpot prize up to the time of t_r. Second, given the historical data, the *attractiveness* of the lottery game can be measured by how unusual the J_{t_r} is as compared to $\{J\}_{t_{(r-1)}}$, if $r > 1$, or $\{J\}_{(t-1)_w}$, if $r = 1$. The agent will then act upon the degree of attraction. For example, if the jackpot is "huge," the agent may react more energetically by betting greatly. Alternatively, if the jackpot is perceived as "low," the agent may be not interested in spending anything.

Technically, each agent gambles with his/her own *fuzzy rule-based system*, which comprises a number of fuzzy if–then rules. Each fuzzy if–then rule within the system can be represented as follows:

$$\text{If } J_{t_r} \text{ is } A_i, \quad \text{then } a_i. \tag{21.2}$$

The A_i $(i = 1, \ldots, k)$ are *fuzzy sets* representing k different states of the jackpot prize. For example, consider the case $k = 4$. Then $A_1, \ldots, A_4$ can denote the following four linguistic descriptions of the size of the jackpot: "low," "medium," "high," and "huge." a_i is the level at which the agent decides to participate given that the current state is A_i. The participation level can be measured by the *proportion of income* that agents would spend to purchase lottery tickets. Call the vector $\boldsymbol{a}$ $[= (a_1, \ldots, a_k)]$ the *participation vector*. Then different heuristics can be captured by different $\boldsymbol{a}$s. For example, $\boldsymbol{a} = (0.1\%, 1\%, 5\%, 10\%)$ characterizes an agent whose betting stake increases with the size of the jackpot prize. On the other hand, $\boldsymbol{a} = (0.1\%, 0.1\%, 0.1\%, 0.1\%)$ indicates that the agent's betting stake is independent of the size of J.

Based on our description above, only the input set, A_i, of (21.2) is fuzzy, and the output set, a_i, is a crisp numerical value. This type of fuzzy rule is known as the Sugeno style of fuzzy rules, as distinguished from the Mamdani style of fuzzy rules, in which the input and output sets are both fuzzy. Fuzzy sets are distinct from the classical sets (crisp sets) in the sense that the *membership* in the latter is *all or nothing*, whereas that in the former is a matter of *degree* (*more or less*). The degree is mathematically characterized by a *membership function*.

There is a wealth of membership functions. For simplicity, we choose the frequently used *triangular-shaped fuzzy membership function* shown in Figure 21.1, where the domain of J is partitioned into four overlapping intervals by a sequence of base points Q_0, Q_1, Q_2, Q_3: $[Q_0, Q_1)$, (Q_0, Q_2), (Q_1, Q_3), and (Q_2, ∞). Let us denote them by $I_1, \ldots, I_4$ respectively. For each fuzzy set A_i, $\mu_{A_i}(J) > 0$ if $J \in I_i$; otherwise $\mu_{A_i}(J) = 0$, where $\mu_{A_i}(J)$ is the membership function, $\mu_{A_i} : R^+ \to [0, 1]$. However, unlike the usual fuzzy membership functions, the base points upon which the membership functions are defined are *not fixed*. This is because all the linguistic terms have no absolute meaning. What is perceived

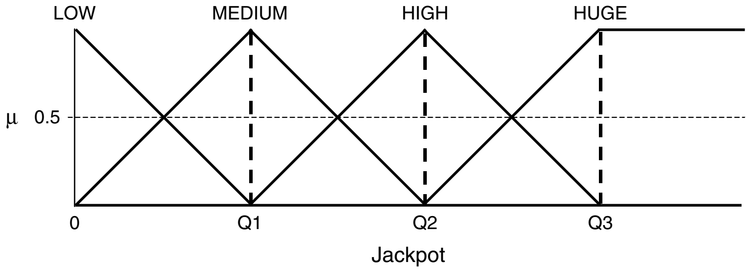


Figure 21.1 The membership function.

as *high* or *low* by agents will depend on what has happened before. It is the *frequency* that determines how we describe the event perceived so "*huge*" should refer to some events that happen more infrequently than what the term "*medium*" may refer to. This justifies the use of *sample statistics* as the base points, such as *quartiles*, and Q_1, Q_2, and Q_3 are the first, second, and third quartiles of the sample $\{J\}_{t_r}/0$. The sample quartiles may converge if $\{J\}_{t_r}$ turns out to follow a stationary distribution; otherwise, they will change over time.

The implementation of the fuzzy rules (21.2) proceeds as follows. For each period of time t_r, the agents observe the time series of the jackpot up to the beginning (the first second) of t_r, $\{J\}_{t_r}$. All Q statistics can be determined accordingly, as with the membership function $\mu_A(J)/J$. Given J_{t_r} (i.e. the jackpot at the beginning of this period), the agent can then figure out the membership degree of each possible state (each fuzzy set), for example, $\mu_{A_i}(J_{t_r})$ $(i = 1, \ldots, k)$. In the Sugeno fuzzy model, each corresponding rule is activated to a degree $\mu_{A_i}(J_{t_r})$, and the output is a weighted average of all consequent actions a_i, weighted by the membership degree:

$$\alpha_{t_r} \equiv \alpha_{t_r}(J_{t_r}) = \sum_{i=1}^{k} \mu_{A_i}(J_{t_r}) a_i. \tag{21.3}$$

The agents' involvement in the lottery is defined by the fuzzy if–then rules (21.2) associated with the participation vectors $\boldsymbol{a}$. Adaptive behavior can be characterized by changes made in $\boldsymbol{a}$. $\boldsymbol{a}$ can then be encoded as a bit string and made to evolve via genetic algorithms.

Conscious selection

The second important empirical observation of the lottery market is a general ignorance of the way probability operates. While all methods of selecting lottery numbers presumably have an equal probability, some of the general public do not seem to believe that the probability of some numbers, say 1, 2, 3, 4, 5, and 6, being picked is equally as likely as any other sequence of six numbers.

Conscious selection behavior is also programmed into the software agents, as follows. Let $\boldsymbol{b}$ be an X-dimensional vector whose entities take either "0" or "1." Consider a number z, where $1 \leq z \leq X$. If "0" appears in the respective zth dimension, the number z will not be consciously selected by the agent, while "1" indicates the opposite. Therefore, $\boldsymbol{b}$ shows a list of numbers that may be consciously selected by the agent. If $\boldsymbol{b}$ has exactly x 1s, then one and only one combination is defined and the agent would select only that combination while purchasing the lottery ticket(s). If $\boldsymbol{b}$ has more than x 1s, then many more combinations can be defined. The agent will then randomly select from these combinations while purchasing the ticket(s). Finally, if $\boldsymbol{b}$ has less than x 1s, then those designated numbers will appear in each ticket bought by the agent, whereas the rest will be randomly selected from the non-designated numbers. For example, if the agents have some lucky numbers in mind, say 01, 03, 05, 06, 08, 12, 14, then use these seven as a package—they can choose any combination from this package, which has 21 possibilities. In this case, they will not let the computer randomly pick the numbers for them. If, in another case, the agent has only four lucky numbers in mind, then he will always choose these four in each of his lottery tickets, but the last one will be randomly determined by the computer.

The agent's betting heuristic, h, is fully characterized by the vector

$$h = (\boldsymbol{a}, \boldsymbol{b}).$$

To make it apparent that the h are different over time (evolving) and are different over space (heterogeneity), we shall denote the heuristic used by agent ι at time t by $h_{\iota,t}$.

Aversion to regret

The last important empirical observation is the non-negligible influence of the mass media on betting momentum. When someone wins the jackpot, the mass media will exert their great influence to let everybody know, and that can make some of those people who did not buy tickets feel regret. Lottery promoters capitalize on the aversion to regret when they encourage lottery buyers to keep on buying.

To model this other-regarding preference, Chen and Chie (2008) assume that agent ι has a simple one-period linear utility function of consumption:

$$u(c) = c, \tag{21.4}$$

with the budget constraint

$$c \leq e - \alpha(\boldsymbol{a})e + \pi, \tag{21.5}$$

where e is her initial income, α is the proportion of her income spent on the lottery, and π is the lottery prize. For those agents whose α is zero, their utility depends on whether there is a jackpot winner. The utility function (21.4) has to be modified as follows:

$$u(c) = \begin{cases} (1-\theta)c & \text{if } \alpha = 0 \text{ and } N_x > 0, \\ c & \text{otherwise.} \end{cases} \tag{21.6}$$

The θ in the utility function (21.6) measures how regretful the non-gambler would be if the jackpot were drawn. On the other hand, opposite to regret, non-gamblers may also derive pleasure from gamblers' misfortunes, in particular when the jackpot is not drawn ($N_x = 0$). As a result, the utility function (21.6) can be extended as follows:

$$u(c) = \begin{cases} (1-\theta)c & \text{if } \alpha = 0 \text{ and } N_x > 0, \\ (1+\theta)c & \text{if } \alpha = 0 \text{ and } N_x = 0, \\ c & \text{otherwise.} \end{cases} \tag{21.7}$$

Obviously, the larger the θ, the less independent will agent ι's utility be. While we can treat θ as an exogenous variable, from the viewpoint of psychology, it would be interesting to see how θ is determined endogenously. In this way, θ is treated as a personal *trait* that indicates how agents experience things and how they feel about them.

To sum up, agents in the artificial lottery markets are fully characterized by the vector

$$(h_{\iota,t}, \theta_{\iota,t}) = (\boldsymbol{a}_{\iota,t}, \boldsymbol{b}_{\iota,t}, \theta_{\iota,t}), \tag{21.8}$$

where $\theta_{\iota,t}$ is the preference parameter of agent ι at time period t. The vector $(h_{\iota,t}, \theta_{\iota,t})$ will be encoded as a bit string, and then genetic algorithms will be applied to evolve a population of $(h_{\iota,t}, \theta_{\iota,t})$. Therefore, the three behavioral aspects of lottery players, lottery participation decision, conscious selection, and aversion to regret, will all evolve over time. An agent's decision is based on heuristics (including fuzzy reasoning) and is influenced by his personality trait (aversion to regret). So, *the cognitive and psychological parts of agents are both encapsulated in the software agents*. Moreover, both of these parts are evolving, which means agents are learning socially. The essence of the above software model is that we don't fix the agent's decision rule. Instead, we allow them to learn and to explore the embedding environment without further intervention. Software agents are, therefore, modeled as autonomous agents. We shall not go into further details on the genetic algorithms that were used to evolve the psychologically oriented artificial agents. The interested reader is referred to Chen and Chie (2008) directly.

21.3 Lottery tax rates

A lottery is mainly a source for public revenue, which is later used for public good, such as education. Therefore, not all money collected from lottery sales will be used for the lottery prize. The money which is reserved the public good is called the lottery tax. The reason for studying lottery tax rate is simple. As we can see from the third column of Table 21.1, there are great differences in the lottery tax rate among different lottery markets. It ranges from the lowest in Taiwan at a rate of 40 percent to the highest in Brazil at a rate of 70 percent. The histogram in Figure 21.2 makes this range even clearer.

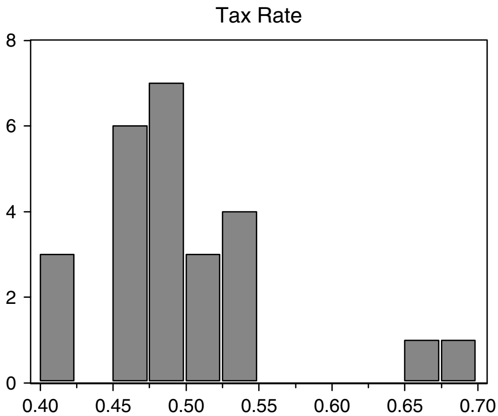
Now, assuming that maximizing lottery tax revenue is a goal commonly shared by all governments, then they are actually facing the same optimization problem, i.e., how to maximize the lottery revenue by finding the optimal lottery tax rate. And if so, the answer should be similar from market to market, not with the 30 percent range shown in Table 21.1.

One possible way to account for this divergence is that there are a lot of uncertainties involved in the lottery market and the optimal lottery tax rate is not unique but has multiple values. The distribution (Figure 21.2) may reflect this multiplicity of optimality. However, we have to prove whether this is indeed the case. So

Table 21.1 Lottery tax rates

Nation	*Official issuer*	*Tax rate (%)*	*Commission rate (%)*	*Net tax rate (%)*
Austria	Austrian Lotteries	54.6	9.30	45.3
Belgium	Lotterie Natlonale	48.4	6.60	41.8
Brazil	Caixa Econômica Federal Bank	68.4	8.20	60.2
Canada	Loto-Quebec	48.7	6.80	41.9
Canada	Ontario Lottery Corp.	51.2	7.40	43.8
France	La Francaise	42.3	5.00	37.3
Germany	Westdeutsche	53.0	8.30	44.7
Italy	Lottmatica S.P.A.	48.4	10.00	38.4
Italy	Sisal Sport Italia	65.4	7.90	57.5
Japan	Dai-Ichi Kangyo Bank	54.2	7.40	46.8
Spain	ONCE	50.4	16.50	33.9
Sweden	Svenska Spel	48.8	9.60	39.2
Taiwan	Taipei Bank	40.0	8.40	31.6
UK	UK National Lottery	53.4	5.10	48.3
USA	Ohio State	40.3	6.40	33.9
USA	Michigan State	45.4	7.00	38.4
USA	Georgia State	45.9	7.00	38.9
USA	Maryland State	46.1	5.70	40.4
USA	Illinois State	45.9	5.10	40.8
USA	Texas State	46.4	5.20	41.2
USA	New Jersey State	47.2	5.40	41.8
USA	California State	49.3	6.70	42.6
USA	New York State	49.4	6.00	43.4
USA	Florida State	50.0	5.60	44.4
USA	Pennsylvania State	49.1	4.70	44.4

Data sources: *U.S. Lotteries' Unaudited FY00 Sales by Game*, La Fleurs Lottery World (http://www.lafleurs.com); Taiwan Lotto (http://www.roclotto.com.tw). (The data for Taiwan are from the year 2002, whereas the data for other markets are for the year 2000.)



	Tax Rate
Mean	49.7%
Median	48.8%
Maximum	68.7%
3rd Quartile	50.8%
1st Quartile	45.9%
Minimum	40.0%
Std. Dev.	0.064538
Skewness	1.358239
Kurtosis	5.332021

Figure 21.2 The distribution of tax rates.

Note: Data source: see Table 21.1.

far, there is no formal mathematical model to help us figure out the answer, and agent-based simulation provides us with an alternative.

Before seeing our agent-based simulation results, it is worth mentioning some similar studies using econometrics to analyze this issue and make their prediction. In one case, the prediction made for the UK lottery market indicated that a lower lottery tax rate would increase tax revenue. The UK lottery agency did reduce the lottery tax rate in October 2001, but the result was not consistent with the prediction based on econometric models, and lottery sales continued to decline, as did lottery tax revenue (Panton, Siegel, and Vaughan, 2002; Hartley and Lanot, 2003).

21.4 Simulation

21.4.1 Emergent Laffer curve

The simulation of Chen and Chie (2008) is based on different lottery tax rates, from 10 percent to 90 percent. For each simulation, they ran 25 runs, each run lasting for 5000 draws.

Figure 21.3 shows the impact of different lottery tax rates upon tax revenue, normalized by income as effective tax rate. As we can see, when the lottery tax rate increases, tax revenue also increase, but when the lottery tax rate goes up further, tax revenue starts to decline. This is the famous *Laffer curve* known in public finance. So, they first replicate the familial law associated with taxing.

The second thing to notice is that it is hard to decide where the peak is. This is because different runs have different results—if we use the median statistics (the blue line), then 40 percent seems to be the highest. Nonetheless, given the large dispersion, we find that actually lottery tax rates from 30 percent to 70 percent are

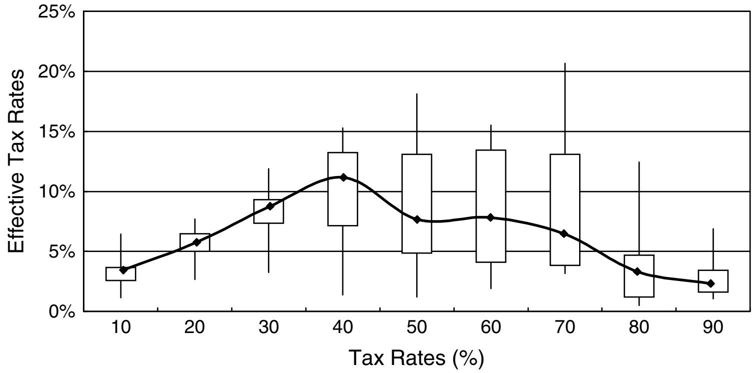


Figure 21.3 Tax revenue curve and the associated box–whisker plot.

not statistically significantly different in their resulting tax revenue, which does suggest a flat curve instead of a unique peak. This range is also roughly consistent with the empirical data shown in Table 21.1.

Finally, the agent-based simulation does show us the risk associated with policy change. For example, if one wanted to decrease the lottery tax rate from 50 percent to 40 percent, what would be the result? While Figure 21.3 does suggest a high likelihood of increasing tax revenue, it also shows that there is still a 25 percent chance of it getting worse if the current tax revenue is represented by the median. So, decision-makers may not like to take this risk, and then the lottery tax rate gets stuck at 50 percent.

21.4.2 Negligence of probability

Figure 21.4 shows the degree of conscious selection—the higher the index, the lower the degree. As we can see, at the beginning, conscious selection (the negligence of probability) is quite prevalent in the market, but as times goes on agents behave more rationally and the degree of conscious selection declines. However, it does not converge to one, and seems to settle down around a level between 0.6 and 0.7. This means, even taking the evolutionary force into account, conscious selection behavior can still survive, although to a rather weak degree. What is interesting about this is that empirical data on conscious selection is very difficult to obtain, and we can get a feeling of its dynamics only through agent-based simulation.

21.5 Agent-based modeling and policy design

What is demonstrated above shows the value of building agent-based modeling. Running policy experiments in real life may be difficult if not impossible, but without so doing it is difficult to get data to build econometric models. So, there is a limitation of using the conventional econometric approach. Agent-based modeling

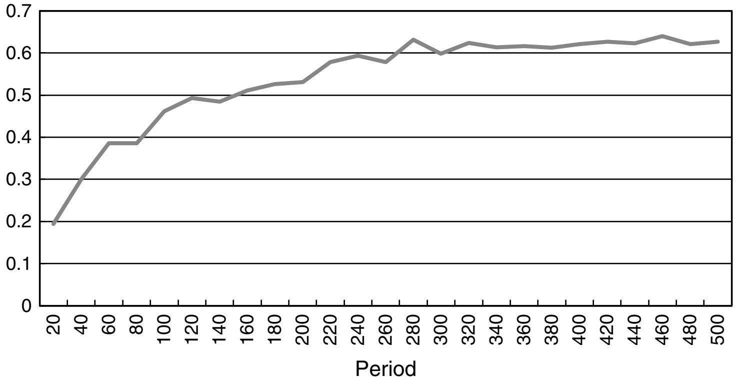


Figure 21.4 The measure of belief in a fair game.

provides an alternative. Also, empirical study of economic behavior is infeasible when data is not available. In this case, agent-based modeling may help as to get around this problem and move forward.

21.6 Further exploration

Exercise

Assume that you are working with a government agency on an agent-based model of markets. The purpose is to use your agent-based model to simulate and evaluate some regulations or policies to be implemented later on. As a first step, you need to give a good description of the market which interests you. However, to facilitate the later modeling work, the way in which you describe the market, in addition to being verbally clear, must also have all the essences of agent-based modeling. Choose one of any markets which interest you and give such a description, or a rough blueprint.

Note

1 This new terminology may be extra because behavioral economics and institutional economics are frequently mentioned together in many contexts.

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Part VII
Networks

Starting from Chapter 4 and in many of the following chapters, we have seen the *network-based decision models* as an essential component of agent-based modeling. In these models, agents' decisions are made mainly based on their "neighbors." These neighbors, however, are not necessarily physically determined. By replacing the physical distance with *social distance*, we can have a more relevant group of "neighbors," who are socially determined. This replacement and generalization leads us from more rigid physical cellular automata to a more flexible social network.

In this part of the book, we will examine the relationship between social networks and agent-based modeling. Agent-based modeling is built upon the interaction of agents. These interactions are known as human relations or social relations in sociology, psychology, or anthropology, and social network analysis is a formal analysis of these social relations. It, therefore, will not surprise us if we find that agent-based modeling is, in effect, synonymous with social network analysis in many regards and that they have many intellectual origins and backgrounds in common.

First, they both share the "system" idea and consider that parts or individuals cannot be understood well unless their embeddedness is taken into account. This style of thought, for example, can be found in both Jacob Moreno (1889–1974), who was under the influence of the Gestalt psychology, and Kurt Lewin, who invented field theory in social psychology. Second, this system should be regarded as a complex system composed of many social relations or as a society that is composed of a group of mutually interacting individuals, a view that has been long held since the time of social anthropologist Alfred Radcliffe-Brown (1881–1955) and Lloyd Werner (1898–1970) in the early 1940s. Third, the micro–macro relation or bottom-up emergence can also be found in early social network analysis; for example, the use of the triad census starting from the 1950s has been viewed as linking the micro–macro structure to global ones since the time of George Simmel (1858–1918).

There are two large bodies of literature related to the subject of this part of the book. One relates to *network-based agent models*, and the other to *agent-based modeling of (social) networks*. The former refers to agent-based models which explicitly involve networks, mainly for the purposes of *interaction*, *decision-making*, and *exchange* (Chen, 1997b; Wilhite, 2001; Vriend, 2002). The *network-based discrete choice model*, such as the models reviewed in Chapter 4, is an example. Agent-based models of networks use agent-based models as the formation algorithms for networks, as distinct from sociological models, sociophysical models, and game-theoretic models.

22 Graphs and social networks

The idea of cellular automata can be read as a special case of *graphs* or *networks*. Therefore, the property that agents following simple rules can collectively generate complex functions or intelligent behavior, as demonstrated in cellular automata, can also be featured by more general network topologies.[1] Before we formally introduce graphs as a generalization of cellular automata and as a more powerful representation of networks, it remains a curiosity as to when the idea of *using graphs as a manifestation of networks* was first used in the social sciences. So, we start with the subject by tracing its origin, which will take us back to the 1950s to meet John Barnes, who simultaneously introduced the idea of social networks and suggested the use of a graph to represent a social network. We will briefly review his work first in Section 22.1.1.

We then move forward to the 1970s (Section 22.1.2), and introduce the work by the great sociologist Mark Granovetter, who is also generally regarded as the founder of economic sociology. His work on the strength of weak ties (Granovetter, 1973) is one of the pioneering works that depict the social network as an important exogenous economic variable, which may impact the operation of the labor market. What is even more powerful is the more general idea pertaining to *information flow* and hence market efficiency. Furthermore, it also has a strong implication for individuals' rationality, namely, how to get connected to the society. This second implication contains the idea of the formation of economic networks and the endogeneity of the network.[2]

The idea of the strength of weak ties can be related more generally to what has been studied as *small-world phenomena*. In Section 22.1.3, we provide a brief historical review of these small-world phenomena, which eventually leads to the development of the celebrated *small-world network*. The small-world phenomenon starts with an informal prediction made by a Hungarian writer Frigyes Karinthy in his short story "Chains." It has attracted a lot of interest from scientists. We shall mention two important works here.

While the awareness of networks began in sociology, it is economists and physicists who provided formal modeling of network formation or network dynamics. These two disciplines, economics and physics, provide us with two ways to think of network formation. The one provided by mathematicians and physicists is more intuitive; they simply proposed heuristic algorithms to generate graphs which have interesting features (Section 22.3), whereas the one provided by economists is game theoretic. Network formation under this framework thus becomes a kind of game, known as a network game.

22.1 Origin in sociology

22.1.1 John Barnes and the social network

Since the sociometric work pioneered by Jacob Moreno in the 1930s, social network analysis, as a research paradigm, has developed for more than half a century. Psychology, anthropology, and sociology have each contributed to its burgeoning stage. They constitute the three so-called trajectories to social network analysis. A good historical review of these three trajectories can be found in Prell (2012).

From the 1930s to the 1950s, various formalizations of social relations through mathematics have been attempted. They include the use of matrices, matrix algebra, graphs, and topology to deal with various concepts or issues related to social relations, such as cliques, clique discovery, centrality, the centrality index, centralization, the efficiency of communication networks, balance, dyads, triads, and connectedness. Among many pioneers, social anthropologist John Barnes was the first person to use the term "social network" in a field study, and suggested representing this social network by a graph:

> The third social field has no units or boundaries; it has no coordinating organization. It is made up of the ties of friendship and acquaintance, which everyone growing up in Bremnes society partly inherits and largely builds up for himself . . . Each person is, as it were, in touch with a number of people, some of whom are directly in touch with each other and some of whom are not . . . I find it convenient to talk of a social field of this kind as a *network*. The image I have is of *a set of points* some of which are joined by *lines*. The points of the image are people, or sometimes groups, and the lines indicate which people interact with each other.
>
> (Barnes, 1954, pp. 42–3; emphasis added)

22.1.2 Mark Granovetter and weak ties

Mark Granovetter, a pioneer and the founder of economic sociology, helped show the relevance of the social network to economics. In his Boston study (Granovetter, 1973) of male professional, technical, or managerial workers who made job changes, he discovered that most workers found their jobs not from people close to them, but from *casual acquaintances* or what he called "weak ties." In this very influential work, he hypothesized the economic implication of network topologies from a perspective of information diffusion.

> Strong ties tend to be clustered and more transitive, as are ties among those within the same clique, who are likely to have the same information about

> jobs and less likely to have new information passed along from distant parts of the network. Bridges between clusters tend to be weak ties, and strong ties are less likely to be bridges. Hence acquaintances are more likely to pass job information than close friends, and the acquaintances of strategic importance are those whose ties serve as bridges in the network.
>
> (White, 2003)

The idea of weak ties can be read as *the curse of homogeneity*, and the lesson goes well, in spirit, with the very standard portfolio theory in economics, namely, not putting all one's eggs in one basket. Granovetter's result, now also known as the *old brother network*, has drawn economists' attention to the role of social networks in economic theory; in particular, it makes economists realize how different network topologies can impact individuals' received information, and hence their perceived alternatives.

22.1.3 Small-world phenomeon

Networks presenting both the strong-tie and the weak-tie property were later studied more formally in the literature on the small-world network. The strong tie means that the presence of a connection between vertices A and B, and another between B and C, makes it likely that there will also be a connection between A and C. Put alternatively, the network with strong ties is highly clustered or has high transitivity. To formalize this idea, a *clustering coefficient* is introduced by Watts and Strogatz (1998).

The *small-world phenomenon*, also known as the *six degrees of separation*, was first mentioned in 1929 by the Hungarian writer Frigyes Karinthy in his world could be connected to anyone else through a chain consisting of no more than five intermediaries. Because the last person in the chain does not count as an intermediary, five intermediaries is equivalent to *six degrees of separation*.

The first scientific exploration of the *small-world problem* came almost three decades later in the work of Manfred Kochen (a mathematician) and Ithiel de Sola Pool (a political scientist), who proposed a mathematical explanation of the problem (Pool and Kochen, 1978).[3] They asked a hard question: What is the probability that two strangers will have a mutual friend? And an even harder one: If there is no mutual friend, how long would the chain of intermediaries be? Assuming that individuals choose 1000 friends at random from a population as large as 100 million, Kochen and Pool showed that no more than two or three intermediaries (hence three or four degrees of separation) would be required to connect any two people. People, however, do not choose friends at random, which implies that the real answer should be higher. Kochen and Pool realized this, but were unable to solve the more difficult problem.

Stimulated by Pool and Kochen's work, the social psychologist Stanley Milgram devised an ingenious experiment in the late 1960s to test the hypothesis (Milgram, 1967; Travers and Milgram, 1969). Milgram and his graduate student Jeffrey Travers started with a slightly different question: How many intermediaries are needed to move a letter from person A to person B through a chain of acquaintances? They gave 300 letters to subjects in Boston and Omaha, with instructions to deliver them to a single target person (a stockbroker from Sharon, MA) by mailing the letter to an acquaintance whom the subject deemed to be closer to the target. The acquaintance then got the same set of instructions, thus setting up a chain of intermediaries. Milgram found that the average length of the chains that were completed (64 of them) was about six—quite remarkable in light of Karinthy's prediction 40 years earlier.[4]

The small-world experiment continued to be attempted in the era of internet (Dodds, Muhamad, and Watts, 2003). Instead of snail mail, emails were used in this experiment. They provided 13 targets for 24,000 subjects distributed over 18 countries. Only 384 subjects reached their intended targets. While their obtained distance within the successful sample was just over four, after statistically processing the unfinished chains the estimated result was largely the same as in Travers and Milgram (1969).

22.2 Origin in mathematics: graphs

As mentioned earlier, until now, the most powerful mathematical treatment of social networks has been mathematical graph theory. Probably the first paper on graph theory was that by Leonhard Euler. In 1736 he wrote, translated into English, "The solution of a problem relating to the geometry of position." In the paper Euler discusses what was then a trendy local topic, that is whether or not it is possible to stroll around Konigsberg (now called Kaliningrad) crossing each of its bridges across the Pregel River exactly once. However, the birth of graph theory occurred almost two centuries later, when Kenneth Appel and Wolfgang Haken solved the famous four-color problem in 1976 by developing many fundamental graph theoretical terms and concepts.

Following the standard notation in graph theory, one can represent a network by $G(V, E)$, where G is the name for the network in question, and V and E denote sets of *vertices* and *edges* respectively, which are the mainstays of a network. $V = \{1, 2, \ldots, N\}$ represents all N constituents of V, and the number N also refers to the *size* of the network. In many economic and social applications, each node corresponds to a single agent (decision-maker), and V denotes the set of all the agents considered in the economy. $E = \{b_{i,j} : i, j \in V\}$ encodes the relationship between any two vertices in the net. Normally, in a binary representation, $b_{ij} = 1$ if there exists an edge (connection, relation) between i and j; otherwise it is zero. In the special case where $b_{ij} = b_{ji}$, direction or flow between i and j is irrelevant, which is also known as a *non-directed network*.

22.3 Network formation: sociophysical models

In its simple version, network formation can be considered to be a computer program specifying how links can develop among a fixed set of points or, more generally, an increasing set of points. In this section, we will give a few examples of these programs and it is then not hard to extend this idea further to more examples. The examples to be presented below are a random graph (Section 22.3.1), regular network (Section 22.3.2), small-world network (Section 22.3.3), and scale-free network (Section 22.3.4).

22.3.1 The Erdős–Reyni model

In the 1950s graph theory was used to describe large networks, with no particular distributions of nodes and links, whose organizational principles were not easily definable. These networks were first studied by Paul Erdős (1913–1996) and Alfred Reyni (1921–1970) and were called *random graphs* (Erdős and Renyi, 1959, 1960), due to their generating method: we start with N nodes and connect every pair of them with probability p.

22.3.2 Regular network

The fully connected network has the feature that agents are completely connected with each other. In other words, each agent has $(N-1)$ links: $b_{ij}=1$ for all $i \neq j$. In the fully connected network, all interactions are *global*; however, in many realistic settings, interactions are rather local and are confined by geographical constraints. There are a number of spatial networks, such as *cellular automata* (Chapter 4), which may be a better representation of these constraints. There is, however, an alternative with a similar virtue, known as the *regular network*. In a regular network, N agents are placed on a circle (a one-dimensional, periodic lattice) and each agent is linked with his $2k$ nearest neighbors, k on the left and k on the right; k is a constant.

As we have seen in Section 3.2, the regular network (the ring network) is the second kind of network, next to random graphs, that came to economists' minds when they attempted to model the non-tâtonnement decentralized market processes.

22.3.3 The small-world network: the WS model

The regular network focuses only on local interactions. It captures a kind of clustering activity, but does not allow for interactions crossing clusters. Nevertheless, inter-cluster interactions or bridges are important in reality. Sociologist Mark Granovetter first noticed their significance in the labor market and proposed the so-called *weak-tie connection* (Granovetter, 1973). A network which allows for both local and bridging interaction was first proposed by Watts and Strogatz (1998) and is known as the small-world network.

The small-world network combines the ideas of random networks and regular networks. These two kinds of networks can be interestingly compared by the two essential characterizations of network topologies, namely, the *clustering coefficient* and the *average path length*. The clustering coefficient is a formal measurement of the extent to which *friends of mine are also friends of each other*, also known as *transitivity*. Average path length, defined as the average length of the shortest path connecting two vertices, is used to measure the average distance between two nodes, which corresponds to the degree of separation in a social network.

Given $G(V, E)$, let $d(i, j)$ be the length of the shortest path between the vertices i and j. Then the mean shortest length of $G(V, E)$ is simply the mean of all $d(i, j)$,

$$L = \frac{1}{\frac{1}{2}N(N-1)} \sum_{i \geq j} d(i, j). \tag{22.1}$$

The above definition may be problematic if there is an isolated vertex which actually has no edge on any other vertices. So, $G(V, E)$ with isolated vertices are not considered here.

Given a vertex i in a graph G, we would first like to measure how well its neighbors get connected to each other. Specifically, if j is connected to i, and k is also connected to i, is j also connected to k? Formally, we define the set of neighbors of i as

$$\vartheta_i = \{j : b_{ij} = 1, j \in G\}. \tag{22.2}$$

Then the cluster coefficient in terms of i is

$$C_i = \frac{\#\{(h, j) : b_{hj} = 1, h, j \in \vartheta_i, h < j\}}{\#\{j : j \in \vartheta_i\}}, \tag{22.3}$$

and the cluster coefficient of G is the average of all C_i,

$$C = \frac{1}{n} \sum_{i=1}^{n} C_i. \tag{22.4}$$

Watts and Strogatz (1998) show that regular networks tend to have a larger clustering coefficient and also a larger average path length (the large-world phenomenon); random networks of equivalent size tend to have a smaller diameter and also a smaller clustering coefficient.

To have a network with a large clustering coefficient but also a small diameter, Watts and Strogatz proposed a network generation algorithm as follows. First, it generates a regular network with N nodes, each with $2k$ neighbors. Secondly, a rewiring probability, p, is applied to each link of each agent. If rewiring happens, then that link will be disconnected and rewired to a randomly selected agent. By fine-tuning the probability parameter, p, a spectrum of networks, known as the small-world network, which has the random network and the regular network as two extremes, can be generated. In fact, when $p = 0$, we have the regular network and, when $p = 1$, we have the random network. Small-world networks, as compared to other random graphs with the same number of nodes and edges, are characterized by clustering coefficients significantly larger than expected and a mean shortest-path length smaller than expected.

The possible economic efficiency of the small-world network is first shown in Wilhite (2001)—see also Section 3.2.

22.3.4 The scale-free network: the BA model

Now, we have networks with a mixture of local and bridging connections, each up to different degrees. In addition, due to the randomness introduced by the rewiring parameter, nodes (agents) can have different numbers of connections, a property which is not shared by the regular networks. This phenomenon, known as the *degree distribution* (see the definition below), corresponds well with what we experience in real social settings: some agents have many more connections than others. However, the way in which the degree distribution is introduced by the rewiring parameter is basically *random* rather than being based on a certain social mechanism. Therefore, one cannot directly control the degree distribution in a manner that mimics the empirical degree distribution observed in real social contexts, such as the *power-law distribution*.[5]

The power-law distribution of the degree has been found in many social contexts, such as the citation network of scientific publications (Redner, 1998), the World Wide Web and the Internet (Albert, Jeong, and Barabási, 1999; Faloutsos, Faloutsos, and Faloutsos, 1999), telephone call and email graphs (Aiello, Chung, and Lu, 2002; Ebel, Mielsch, and Bornholdt, 2002), and in the network of human sexual contacts (Liljeros *et al.*, 2001). A class of networks which is strongly motivated by the observed prevalence of the power-law distribution is known as the *scale-free network*. It is called a scale-free network because it is able to feature a power-law distribution of degree (to be discussed later), and the power-law distribution is scale free.

The *scale-free network* was first proposed by Barabasi and Albert (1999), and hence is also known as the *BA model* (the Barabási–Albert model). The BA model is based on two mechanisms: (1) networks grow incrementally, by the addition of new vertices; and (2) new vertices attach *preferentially* to vertices that are already well connected. Let us assume that initially the network is composed of m_0 vertices, and that each is connected to m other vertices ($m < m_0$). Then, at each point in time, a number of new vertices, m_T, are added to the network, each of which is again connected to m vertices of the net by *preferential linking*. This idea of *preferential attachment* is similar to the classical "rich get richer" model originally proposed by Simon (1955b).[6]

It is implemented as follows. At time T, each of the new m_T vertices is randomly connected to a node $i \in V_T$ according to the distribution

$$\pi_i = \frac{k_i}{\sum_{j \in V_T} k_j}, i \in V_T, \tag{22.5}$$

where $V_T = \{1, 2, \ldots, \sum_{t=0}^{T-1} m_t\}$. That is, the probability of becoming attached to a node of degree k is proportional to k, π_k, and nodes with high degrees attract new connections with a high probability. To avoid redundancy, the random attachment with (22.5) is done by sampling *without* replacement.

Needless to say, all interactions between agents must be conducted through networks. The question is: do we need to explicitly isolate the importance of the network topology and treat it as a relevant exogenous economic variable to determine the outcomes of the model, or, even further, do we need to model it endogenously as it is codetermined simultaneously with other endogenous variables?

22.3.5 Generalized preferential attachment

Physicists have constantly worked on things which always start from simple entities but can become complex in their development, referred to as *simple-to-complex modeling*. In network science, this principle remains, and that is how we have *preferential attachment* as a "universal" formation principle of networks. However, like the device of the *zero-intelligence agent*, we can always ask: *is it sufficient?* The answer is *no*. The networks formed by preferential attachment encounter two difficulties. First, preferential attachment cannot anticipate well or explain why the nodes which are attached to the network in the later stages can eventually take the lead, and become the most connected nodes during the growth of the network. Google and Facebook are cases in point. Second, the use of this formation mechanism requires an infinitely growing network with an unboundedly increasing number of nodes. It does not answer why a network with a large but finite number of nodes can have a power-law distribution of its degree.

For the first issue, the simple preferential attachment mechanism is augmented with a fitness function, known as *preferential attachment with fitness* (Bianoconi and Barabasi, 2001; Borgs *et al.*, 2008). The idea is to give each node a fitness. In their original design (Bianoconi and Barabasi, 2001), this fitness is sampled from a distribution called the fitness distribution. Then the competition for connections does not just depend on the degree of nodes but also on their fitness. The resulting model provides a much more accurate description of many real-world networks (Bianoconi and Barabasi, 2001).

In Bianoconi and Barabasi (2001), this fitness function is exogenously given and fixed, but in more realistic economic models, this fitness function can be endogenously formed and constantly evolving. In fact, broadly speaking, the degree of the node can also be part of the fitness function, and when it is the only determinant of the fitness function, we have a type of *herding model*. In our agent-based financial models, we have already seen a kind of herding model, though without using the term "network" (Section 14.3). More naturally and generally, there will be some performance measure associated with each node, such as profits, utility, and so on. Hence, in this more general setting, preferential attachment with fitness can be related to the evolutionary learning model, such as the adaptive belief system, as reviewed in Sections 14.1 and 14.2.

The above discussion leads to a more generalized version of preferential attachment. In this generalization, the degree of a node is replaced by the intrinsic fitness (importance) of a node. The measure of the intrinsic fitness can be multidimensional, and the degree can be one of them; nevertheless, in many situations, it does not have to be there.

> [I]t is reasonable that two vertices are connected when the link creates a *mutual benefit* (here we restrict ourselves to bidirectional links) depending on some of their intrinsic properties (authoritativeness, friendship, social success, scientific relevance, interaction strength, etc.).
>
> (Caldarelli *et al.*, 2002, p. 1; emphasis added)

This *generalized preferential attachment rule*, as first proposed by Caldarelli *et al.* (2002), directly assumes that the probability of having a link between any two nodes depends on the intrinsic fitness of the two nodes. Hence, for each pair of nodes, $(i^*, j^*) \in V$,

$$\text{Prob}(b_{i^*,j^*} = 1) = f(\pi_{i^*}, \pi_{j^*}), \tag{22.6}$$

where π_{i^*} and π_{j^*} are the intrinsic fitness of nodes i^* and j^*. Some examples are

$$f(\pi_{i^*}, \pi_{j^*}) = \frac{\pi_{i^*}\pi_{j^*}}{\pi_{\max}^2}, \tag{22.7}$$

where $\pi_{\max} = \max_{i \in V}\{\pi_i\}$ (Caldarelli *et al.*, 2002), and

$$f(\pi_{i^*}, \pi_{j^*}) = \frac{(\pi_{i^*} + \pi_{j^*})}{\sum_{i,j>i}(\pi_i + \pi_j)} = \frac{\pi_{i^*} + \pi_{j^*}}{(N-1)\upsilon_{\text{tot}}}, \tag{22.8}$$

where $\upsilon_{\text{tot}} = \sum_{i \in V} \pi_i$ (Garlaschelli *et al.*, 2005; De Masi, Iori, and Caldarelli, 2006). It can be shown that this generalized preferential attachment rule can also lead to the emergence of various scale-free networks (Servedio, Caldarelli, and Butta, 2004).[7]

22.3.6 Controllability of networks

Recently, the studies on networks have been integrated with the early studies of complex systems and optimal control. This integration leads to a version of *network controllability*. The work starts with a dynamic system representation of a network:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t). \tag{22.9}$$

In the above equation, $\mathbf{x}(t) = [x_1(t), \ldots, x_N(t)]$ corresponds to the state of the N nodes of the network V at time t. $\mathbf{A}$, an $N \times N$ matrix, is the matrix representation of the network of V. In more general applications, the coefficients in matrix $\mathbf{A}$ are not necessarily zero or one, but can be real scalar; hence, it is also applicable to weighted networks.

The definition of controllability which has been used in optimal control theory can then be applied to this case. A network is said to be *controllable* if by starting with any initial state, $\mathbf{x}(0)$, one can drive it to any desired state in a designated time T, say, $\mathbf{x}^*(T)$. A well-known *full-rank condition* can be applied to check whether the network is controllable.

However, checking the full-rank condition can generally be very computationally demanding. An alternative analysis based on structural controllability has been proposed in Liu, Slotine, and Barabasi (2011). This analysis enables us to know *the minimal set of driver nodes* required to control the network. The minimal set of driver nodes then further leads to two interesting characterizations, namely, its size (how big it is) and its members (who are there).

The first characterization directly provides us with a measure of the controllability of the network. For example, by letting N_D be the cardinality of the minimal set of driver nodes, then the ratio $n_d = \frac{N_D}{N}$ can be a measure of the easiness or hardness of full control of V. The larger the N_D or n_d, the less easy it is to control the network. In an extreme case, when $N_D = 1$, to have direct control of the network one only needs to grasp one node. On the other hand, when $n_d \approx 1$, one then needs to have a full grasp of all nodes to be able to fully control the network.

With this measure, Liu, Slotine, and Barabasi (2011) examine a great diversity of familiar networks from gene regulatory networks, the World Wide Web, and citation networks to social networks. The results show a large variation in this ratio n_d, ranging from a low of 0.013 to a high of 0.965. Among them, gene regulatory networks have the highest ratio of driver nodes, ranging from 0.751 to 0.965. Hence, while the operation of the cellular process is highly efficient, this self-regulated process is not easily susceptible to external control. This analysis provides us with another perspective to see the essential spirit of the Austrian School of Economics led by Ludwig von Mises (1881–1973) and Friedrich Hayek (1899–1992), with respect to the controllability of the economy, in particular, the famous socialist calculation debate. In this context, when the whole of the economy is viewed as a network, it is not easy to control, presumably having a very high n_d.[8]

The second characterization is equally interesting. Which nodes are more likely to play the role of a driving force? Our intuition might suggest that nodes with large numbers of connections (hubs) should take this position, and should be more likely to be chosen to play the role of driver nodes. In other words, to control the hubs is essential to control the network. However, what was found in Liu, Slotine, and Barabasi (2011) is that nodes with lower degrees actually have a higher fraction of driver nodes than the nodes with higher degrees. Hence, *driver nodes tend to avoid the hub*.

22.4 Networks in economics

While it is only very recently that the social network has drawn the attention of economists, network thinking has a long history in economics. The idea of providing a network representation of the whole economy started with Quesnay's

Tableau Economique in 1758, which depicted the circular flow of funds in an economy as a network. Quesnay's work later on inspired the celebrated input–output analysis founded by Wassily Leontief (1905–1999) in the 1950s (Leontief, 1951), which was further generalized into social accounting matrices by Richard Stone (1913–1991) (Stone, 1961) in the 1960s. This series of developments forms the backbone of computable general equilibrium analysis, a kind of applied microfounded macroeconomic model, pioneered by Herbert Scarf in the early 1970s (Scarf and Hansen, 1973). These earlier "network representations" of economic activities enable us to see the interconnections and interdependence of various economic participants. This "visualization" helps us to address the fundamental issue in macroeconomics, i.e., how disruption propagates itself from one participant (sector) to others through the network.

The era of globalization provides us with a new drive to study and explore economic networks in a global context, which is important for addressing the timely issue of financial security and stability. Hence, tremendous efforts have been made over the last decade to construct networks of various flows within the global economy, offering alternative approaches to the network representation of the real economy. These networks range from the flow of commodities (exports and imports, *world trade web*) (Serrano and Boguna, 2003) to the flow of capital (direct and indirect foreign investment, *investment networks* or *financial networks*) (Battiston and Rodrigues, 2007; Hochberg, Ljungqvist, and Lu, 2007; Kogut, Urso, and Walker, 2007; Song, Jiang, and Zhou, 2009). An international economic network can also be built upon the correlations of macroeconomic fluctuations using the techniques introduced below (*GDP network*) (Ausloos and Lambiotte, 2007). In addition to the macroeconomic networks, various industrial networks have also been established. These include networks of companies, firms, and banks (Uzzi, 1996; Souma, 2007; Aoyama *et al.*, 2010).

The economists' interest in networks is motivated by different sources, but maybe the most prominent one is *network effects*. Network effects were first studied in the context of consumption networks, such as the bandwagon effect (Leibenstein, 1950) or preference interdependence (Rohlfs, 1974). However, the large exploration of the network effect and network externality did not happen until the year 1985, when a series of seminal papers devoted to the economics of standardization appeared (David, 1985; Farrell and Saloner, 1985; Katz and Shapiro, 1985). *Network effects* occur when a number of consumers of a good influence the utility that an individual derives from a good. *Network externalities* occur when markets characterized by network effects fail to allocate resources properly.[9]

In the early literature on network effects or network externalities, what interested economists is the idea of preference interdependence and its possible economic consequences, where network topologies (such as weak ties or strong ties) play a negligible role, not to mention the use of graphs. In this literature, an important issue currently known as *the formation of networks* or *the evolution of networks* seems to be irrelevant in this context. The term network is just a different way of expressing the idea of preference interdependence by noticing the nontrivial implication of a presumably global network. This literature, therefore, has little connection to the social network literature as reviewed earlier in Section 22.1. In particular, the central ingredient of Granovetter's economic sociology that economic action is based on the networks of relations between people is completely missing here.

However, in the 1990s, some economists started to study interdependent demand using embedded social networks. They integrated local interactions into the conventional discrete choice models. In this class of models, network topologies serve as exogenous variables, and economists can study how changes in network topologies can impact the economic performance or efficiency. Nonetheless, the network topologies chosen by them are mostly from social networks developed by sociologists or sociophysicists. The formation of these social networks, such as those reviewed in Section 22.3, may be intuitive, but the underlying economic forces are not clearly presented.

When economists start to work on the networks, they are obviously not satisfied with various kinds of random networks or stochastic networks. They are looking for the *rationale* behind the networks. If the network has value, as Granovetter (1973) articulates, then economists have the impulse to make these values explicit and to examine how they can affect rational people to form the network in a desirable way. Therefore, the network formation rule is changed from a probabilistic or ad hoc approach to a rational (or bounded rational) approach. In the next section, we shall give two illustrations of the agent-based approach to the network formation. Of course, these two are by no means exhaustive, but they are the earliest (pioneering) work and they give the general scope of agent-based models of network formation.

22.5 Network formation: agent-based models

Agent-based modeling of network formation can be regarded as the applications of games, conventionally studied in small groups (two persons, three persons), to a grand large society with various social constructs (social norms, reputations, social punishment, taxes, laws), and then to see how these games played in a decentralized fashion can constantly reshape the topologies of social networks, which may further cause a change in agents' strategies and behavior. Hence, while quite distinct from the game-theoretic approach to network formation—as reviewed in Jackson (2008) and Goyal (2012))—agent-based modeling of network formation does rely heavily on the use of various games, such as the prisoner's dilemma game, stag hunt game, ultimatum game (bargaining game), trust game, public good game, or various social dilemmas, to provide mechanisms (algorithms) for the evolution (dynamics) of social networks. The primary goal of these studies, in a nutshell, is to have a view of the coevolution of individuals and social networks and to create models that are more true to a life fraught with structural changes.

The idea of playing games in a lattice or in a graph already existed in the 1980s. Axelrod (1984), Albin (1992), and Nowak and May (1992) placed the prisoner's dilemma (PD) game on a checkerboard and studied the PD games from the perspective of cellular automata (Chapter 4). These studies have become pioneering work in the field known as the *spatial prisoner's dilemma game*. The spatial prisoner's dilemma game provides a possible route to sustain cooperation. A survey of various models of spatial games can be found in Nowak and Sigmund (2000). The essential message of spatial prisoner's dilemma games is to show that by imposing a social structure (network topology) one can accommodate the existence of both cooperators and defectors. Therefore, the social structure (network topology) is sufficient to support the emergence of cooperative behavior. The upshot of this research line initiated by Axelrod (1984) and extended by others has been well described by Nowak and Sigmund (2000).

> The main message so far is that neighborhood structure seems to offer a promising way out of the Prisoner's Dilemma toward the emergence of cooperation. There are many alternative explanations of the prevalence of cooperation, but, arguably, none require less sophistication on the part of the individual agents than those with spatial structure. The latter need no foresight, no memory, and no family structure. *Viscosity suffices*.
>
> (Nowak and Sigmund, 2000, p. 145; emphasis added)

22.5.1 The Skyrms–Pemantle Model

The spatial game has later on been generalized into the network game. Hence, the games are played in an exogenously given social network (Abramson and Kuperman, 2001). When the literature has advanced to this stage, it is natural to expect that the game played in endogenous social networks is the next step to take. Skyrms and Pemantle (2000) is one of the earliest studies in this direction. At the time when their article was written, "the idea of simultaneous evolution of strategy and social network appears to be almost completely unexplored" (Ibid., p. 9340).

The social network that Skyrms and Pemantle (2000) started to work with is a weighted directed network. The general idea is that each link variable b_{ij} is not binary but a probability, which indicates the probability of the event that agent i will contact j when playing a two-person game. To not get confused with the notation, let us use $p_{i,j}$ ($0 \leq p_{i,j} \leq 1$) as an alternative expression for the real b_{ij}. Let the vector of the connecting probabilities be given as

$$p_i(t) = \{p_{i,j}(t)\}_{j=1}^{N}, \tag{22.10}$$

where $p_{i,i}(t) = 0, \forall t$. These choice probabilities are dynamically adjusted based on agent i's experience of playing against agent j. Reinforcement learning is applied to the dynamic adjustment of these choice probabilities as it is applied to the $(N-1)$-armed bandit problem (Chapter 7). For example, if agent i and agent j play a two-person prisoner's dilemma game, and agent j always cooperates, then the intensity of playing the same game with agent j will be reinforced. The general question addressed by Skyrms and Pemantle (2000) is the dynamics of the social network, characterized by the connection probabilities of all agents, $\mathbf{P}(t)$,

$$\mathbf{P}(t) = (p_1(t), p_2(t), \ldots, p_N(t))'. \tag{22.11}$$

Skyrms and Pemantle (2000) show that their model is rich enough to generate a great variety of structure. Depending on the payoff function of the game, the memory decay rate of reinforcement learning, and noise associated with learning, different topologies of social networks can emerge. The limit of $\mathbf{P}(t)$ can be symmetric or asymmetric; the limit vector $p_i(t)$ can be a random vector, a uniform vector, or a degenerate vector with only one entry being non-zero (one). There are other interesting topologies such as the pairing component where i only connects with j and vice versa, or star networks where a center player appears. Pemantle and Skyrms (2004) extended their model with two-person network games into one with multi-person games, and a three-person *stag hunt game* is studied.

The stag hunt game is a story that became a game. The story is briefly told by Rousseau (1984). This game has been further well illustrated in Skyrms (2004). Consider a two-person stag hunt game. Denote the action "hunt a stag" by "1," and denote the action "hunt a hare" by "0." In its normal form, the stag hunt game has the payoff matrix

$$\begin{bmatrix} & 1 & 0 \\ 1 & \pi_{11} & \pi_{10} \\ 0 & \pi_{01} & \pi_{00} \end{bmatrix} \tag{22.12}$$

where $\pi_{11} > \pi_{01} = \pi_{00} > \pi_{10}$. Assume that hunting a stag requires good teamwork, i.e., requiring both players to choose the action "1." If they both do so, they will successfully hunt a stag with a payoff π_{11} for each. However, if one player is not faithful to this commitment and changes to hunt a hare, then he will obtain a hare characterized by a payoff π_{01}. Nonetheless, in this situation, the one who is still hunting a stag will be doomed to fail and will receive a payoff π_{01}. If both decide to hunt for a hare, they will both obtain a hare with the payoff π_{00}. Hunting a stag is more valuable ($\pi_{11} > \pi_{01} = \pi_{00}$), but is risky because its success depends on the cooperation of the other player. On the other hand, hunting a hare is riskless because the job can be accomplished alone, but the payoff is just a hare. In general, consider an H-person game, where $2 \leq H \leq N$. The decision for i to make in each of his iterations is to form a team by finding the other $H-1$ players. Pemantle and Skyrms (2004) assume that agent i's choice is team-based, instead of individual-based. For i, there are a total of $\binom{N-1}{H-1}$ possible ways to form a team, and the reinforcement is made on these $\binom{N-1}{H-1}$ possible options.

Reinforcement learning with a decay factor ϕ can result in the so-called *trapping states*. This has been shown in the theory of urn processes.[10] In the stag hunt network game, starting with an equal number of stag hunters and hare hunters, initially they randomly make attempts to connect to each other in forming a three-person stag hunt game, and then, in the limit, *cliques* of different sizes are formed.[11] Agents (hunters) will then only have a positive probability of hunting with the agents in the same clique. Typically, stag hunters will only hunt with stag hunters, and their choice probability for hunting with hare hunters is zero, even though hare hunters may still choose to connect with stag hunters.

The essential purpose of the Skyrms–Pemantle model is to study the evolution of individual behavior and social structure together. Hence, something has to be said on the evolution of strategies. Review and revision of strategies take place through a kind of social learning, popularly known as imitation, or simply, "copy the best." At each point in time, there is a small probability, say q, that agent i will be granted a chance to review and revise his incumbent strategy. When this time comes, he will change his strategy (action) to whichever strategy (action) was most successful in the previous round. Hence, the Skyrms–Pemantle model simultaneously applies both individual reinforcement learning and social learning to the adaptive behavior of agents, reinforcement learning for the interaction structure and imitation for strategy revision, which is also called the *reinforcement imitation* model.

The parameter q controls the relative speed of the evolution of the social structure and the evolution of individual behavior. This feature is the essence of the study of the endogenous formation of social networks. When q is small, the evolution of the social structure toward one steady state can proceed with less perturbation, whereas when q is large the evolution of the social structure may be constantly disturbed by the change in individual behavior. For example, in their two-person stag hunt game, by simulating the stag hunt game for 1000 time steps, they found that:

> When $q = 0.1$, we found that in 22% of the cases all hunters ended up hunting stag, while in 78% of the cases, all hunters hunted rabbit. Thus there was perfect coordination, but usually not to the most efficient equilibrium. On the other hand, when $q = 0.01$, the majority (71%) of the cases ended in the optimal state of all hunting stag, while 29% ended up all hunting rabbit.
>
> (Skyrms and Pemantle, 2000, p. 9346)

Generally speaking, the essential question here is that, when an agent is not satisfied with his current payoffs, will he change his interaction structure (network) first or change his strategy first? For example, consider an agent playing a network prisoner's dilemma game; when he is not satisfied with his payoffs, will he attribute his frustration to the partners, or to the strategy that he applied, or to both? If he has habitual inertia in networking structure, then he may change his strategy first or more frequently, and vice versa. Of course, to address the question above, the models need to be further modified. It will be more realistic by allowing for agents' heterogeneity in their degree of habitual inertia, either in networking behavior or strategy-playing behavior.

22.5.2 The Zimmermann–Eguiluz model

Needless to say, the number $\binom{N-1}{H-1}$ can increase very fast and makes the choice become increasingly overloading. In fact, the famous *social brain hypothesis* provides a nontrivial cognitive constraint for the agent when he is placed in a large crowd.[12] Of course, one can argue that this parameter may be captured by the linkage cost; however, in agent-based modeling, an alternative way is to directly place a ceiling on the number of connections that agents can build. For that purpose, we introduce the second agent-based model of social networks, which has exactly this feature and can effectively address network formation in a large society. This model, called the *Zimmermann–Eguiluz model*, was introduced by Zimmermann, Eguiluz, and San Miguel (2004), Zimmermann and Eguiluz (2005), and Eguiluz, Zimmermann, and Cela-Conde (2005).

The Zimmermann–Eguiluz model is the earliest prominent example showing how a social network can be developed through a prisoner's dilemma game. Each agent i ($i \in V$) is randomly assigned a set of connections, say V_i ($V_i \subset V$). The number of connections for i, i.e., the degree of V_i, is randomly determined by a Poisson distribution. The relationship with each neighbor j stands for an undirected link $(i, j) \in E$, $b_{i,j} = 1$. Agent i will then play a *two-person game* with each of his neighbors. Let $\pi_{i,j}$ be the payoffs that agent i obtained from the game, and the total payoff of agent i's game playing with all his neighbors is

$$\pi_i(t) = \sum_{j \in V_i(t)} \pi_{i,j}(t). \tag{22.13}$$

Notice that we have indexed both the payoffs and the set of neighbors by time t because both of these two sets may change over time. For the former, player i and/or j may change his or her strategies (actions) in the course of the game, whereas for the latter agent i may change some of his neighbors. To make this point explicit, we rewrite $\pi_{i,j}(t)$ as a functional relationship with players' employed strategies,

$$\pi_{i,j}(t) = \pi(a_i(t), a_j(t)), \tag{22.14}$$

where $a_i(t) \in A_i$ and $a_j(t) \in A_j$. A_i is the action (strategy) space of agent i. At each time all agents play with all of their neighbors simultaneously. After that, agents learn and adapt. Learning and adaptation include two parts: first, the strategy, and second, the neighborhood.

Eguiluz, Zimmermann, and Cela-Conde (2005) complement the game-theoretic approach to social network formation. The network topologies studied in game theory and experimental network formation games are not exciting in that they are small networks, and not really large complex networks. On the other hand, the theoretical underpinnings of the laboratory work actually contribute to this smallness, since they seem to be interested in some specific network topologies, such as star or ring networks, which occupy only a small portion of the empirical complex network literature (see Section 22.4). The small-world network, the scale-free network, and the complex core–peripheral network which receive much attention in empirical observations are not in the center of the game theory of network formation. The general question posed by social scientists who are interested in how these networks can be formed socially remains unanswered.

Using the network PD game with adaptive agents, Eguiluz, Zimmermann, and Cela-Conde (2005) can generate social networks with a number of properties which have empirical relevance. First, the model is able to generate a social network that can demonstrate an interesting social hierarchy, a leader–follower hierarchy. There are only a few leaders but many followers standing at different levels of the hierarchy. Each agent in this hierarchy is followed by some agents at his immediate lower level, unless he is the one at the bottom level. These leaders and followers are not given initially, but emerge from their own adaptation process. Psychologists may argue that leaders have some personal traits which may distinguish them from the rest. However, this agent-based model shows that this property is not a necessary condition. As experimental economics has shown, personal traits may play a determining role in accounting for the results of PD games (Chapter 17), but none of these personal traits have been included in the Zimmermann–Eguiluz model. A direction for further research is to augment this model with agents' personal traits and individualize agents' behavioral rules.

Even without the inclusion of personal traits, the system has already generated a leader–follower network. This shows that the agent-based model has the potential to replicate the self-organization property of social structure, sometimes called *a spontaneous division of labor*. Second, it generates a distribution of social capital (number of connections) as well as a distribution of economic gains. While the society initially starts with a Poisson distribution of the number of connections, the end distribution can be exponential, depending on the payoff structures and rewiring rules.

While the basic idea that a group of agents play games in an evolving social network is shared by both the Zimmermann–Eguiluz (ZE) model and the Skyrms–Pemantle (SP) model, these two models differ in several interesting ways, which actually offer two different kinds of agent-based modeling of social networks. First, the game played in the network is different. For the ZE model, it is the prisoner's dilemma game, whereas for the SP model, it is the stag hunt game. Both games are important for our general understanding of the cooperative behavior emerging in the society and, technically, they differ only in the parameters of the payoff matrix; hence this difference may be secondary.

Second, nevertheless, there is a nontrivial difference between the two. The games played in the SP model are network games (multi-person games), such as the three-person stag hunt as demonstrated in Skyrms and Pemantle (2010), but the games played in the ZE model are two-person games. In other words, in the SP model, agents play with their neighbors *altogether in one game*, while in the ZE model agents play with each of their neighbors individually in multiple games.

Third, in addition to the payoffs, this difference also affects the concept of the social structure. In economics what matters is the team production captured by the value function in cooperative game theory. This idea has been well presented in the SP model, but is absent in the ZE model. This contrast prompts us to think more deeply about the purpose of networking for individuals. Clearly, simultaneously having many bilateral relations has a value, such as having a co-author relationship in many papers, but it is not up to the same level as "hunting a stag."

The later development in agent-based models of network formation can be regarded as a synthesis of these two approaches. In a context of network formation, Chen, Chie, and Zhang (2015) extend the conventional one-shot two-person trust game (investment game) into an N-person multi-period trust game, and allow agents to play a dual role, being both trustor and trustee simultaneously. In the network setting, each agent has to decide his set of trustees and makes a portfolio decision to invest in each of his trustees. In this sense, the connection $b_{i,j}$ is not binary, but a weight between zero and one, very much similar to the Skyrms–Pemantle model. In addition to investing in his trustees, each agent can also be a trustor and receive investments; therefore, as in the usual trust game, he also has to decide the kickbacks returned to his trustors. Hence, an investment network is well formulated here. The network formation process is characterized by the constant adjustment of each agent's set of trustors and trustees. This adjustment can be modelled through various learning algorithms. Chen, Chie, and Zhang (2015) consider both the myopic stochastic choice model and the reinforcement learning model. As one may expect, different learning algorithms can affect the network formation. In addition to the standard operation of the trust game, Chen, Chie, and Zhang (2015) also replace constant productivity of investment by a state-dependent one, which depends on the cohesiveness of network embeddedness. In this way, "hunt a stag" as joint production is brought back to this multi-person trust game. They then study the network formation both at the aggregate level and at the individual level, and examine how this network formation process contributes to wealth creation and wealth distribution.

22.6 Further reading

Other network formation algorithms introduced by physicists involve the use of *minimum spanning trees* by Rosario Mantegna (Mantegna, 1999) and the *thresholding approach* by Jukka-Pekka Onnela (Onnela, Kaski, and Kertesz, 2004). These techniques allow us to provide a network representation of correlation matrices, known as *correlation networks*. When applied to financial data, these networks provide investors with a new way of examining financial information or making investment decisions. Correlation networks have been applied to examine networks of different assets, such as equities (Mantegna, 1999; Onnela, Kaski, and Kertesz, 2004) and currencies (Mizunoa, Takayasub, and Takayasu, 2006). Additional techniques have been introduced to build cross-correlation networks; in this way, the network is associated with a law of motion and is endowed with a dynamic interpretation (Aste and Di Matteo, 2006; Ausloos and Lambiotte, 2007).

Correlation networks can be considered to be an approach to a more general attempt to map time series data into networks. There are other approaches being developed for this more general attempt, such as the *visibility graph* (Lacasa *et al.*, 2008). Some features of time series, such as periodicity and their being fractal, can then be inherited and manifested through different network topologies, such as regular networks and scale-free networks.

The other important development is the more flexible and rich representation of networks. The conventional binary network has been extended to the weighted network, such as the correlation network. In addition, the single graph has been expanded to multigraphs (Souma, 2007), i.e., there can be multiple links between nodes. The heterogeneity of nodes is also taken into account and the characteristics of nodes are then incorporated as part of the network construction through hidden variable mechanisms (Caldarelli *et al.*, 2002; Garlaschelli and Loffredo, 2004).

Finally, various properties of economic networks have been identified, including the small-world and scale-free characterization of economic networks (Aoyama *et al.*, 2010), the scaling laws (Battiston and Rodrigues, 2007; Duan, 2007), giant components (Kogut, Urso, and Walker, 2007), clustered structures (Mizunoa, Takayasub, and Takayasu, 2006), and weak and strong ties (Fagiolo, Reyes, and Schiavo, 2010), etc. These findings may have far-reaching implications for survivability (Uzzi, 1996; Hochberg, Ljungqvist, and Lu, 2007), security (Nagurney and Qiang, 2009), efficiency, and many other issues. However, the causes and consequences of various network topologies in general remain a challenge.

Notes

1 In fact, the recent study of swarm intelligence, such as *ant colony optimization*, is an example.

2 Nowadays, tremendous efforts are being devoted to exploring both the significance of the exogeneity and endogeneity of economic and social networks (Jackson, 2004; Durlauf and Young, 2004).

3 Ithiel Pool is also a pioneer in the computer simulation of social processes. His work included the first computer simulation of decision-making in international crises—the outbreak of World War I (*The Kaiser, the Tsar, and the Computer*)—and the first major computer simulation of the American electorate based on public opinion data, which was used to advise President Kennedy's campaign for the Presidency in 1960.

4 The small-world experiment cited here is based on Travers and Milgram (1969), which is different from the earliest one (Milgram, 1967). Milgram (1967) had a sample of 96 subjects, but only 18 successfully reached the target. However, within the successful sample, the result is very similar to the one obtained by Travers and Milgram (1969).

5 A power-law distribution is a density function which is proportional to a power function, i.e.,

$$y = f(x) = \text{Prob}(X = x) \sim x^{-\alpha}, \tag{22.15}$$

where X is a random variable. A nice feature of the power distribution is that it is *scale free*. A random variable X is called scale free or said to have a scale-free distribution if

$$f(bx) = g(b) f(x). \tag{22.16}$$

Intuitively, the shape of the distribution in an interval $[x_1, x_2]$ is the same as that of $[bx_1, bx_2]$ except for a multiplicative constant. The definition above obviously applies to the power-law distribution since

$$f(bx) = (bx)^{-\alpha} = b^{-\alpha} x^{-\alpha}. \tag{22.17}$$

The power-law distribution has gained quite significant popularity these days in the sciences. It has often been cited by scientists, while sometimes being given different names. For example, when the exponent α is equal to 2, it is also known as *Zipf's law*, in memory of Harvard linguistics professor George Zipf (1902–1950). Alternatively, it has also been cited as *Pareto's Law* when what interests us is the tail distribution of Equation (22.15), i.e.,

$$\mathrm{Prob}(X \geq x) \sim x^{-\beta}, \tag{22.18}$$

where $\beta = \alpha - 1$.

6 In fact, the BA model which leads to power-law degree distributions is an independent rediscovery of earlier work by Simon (1955b) on systems with skewed distributions. It can be interpreted as an application of Simon's growth model in the context of networks, readily explaining the emergent scaling in the degree distribution.

7 Servedio, Caldarelli, and Butta (2004) show that for any fitness distribution there exists a factorized connecting probability function such that a scale-free network with an arbitrary real exponent can be formed.

8 There are a number of social networks examined by Liu, Slotine, and Barabasi (2011), but they generally tend to be lower than the gene regulatory network. For the details, see their Table One.

9 Lewin (2002) provides an excellent critical review of the literature on network externalities.

10 For the mathematical background of reinforcement learning with a decay factor, please see the review in Pemantle and Skyrms (2004).

11 See Pemantle and Skyrms (2004), Theorem 4.1, p. 320.

12 Based on the social brain hypothesis (Dunbar, 1998; Dunbar and Shultz, 2007), humans' brains are capable of managing a maximum of just 150 friendships. A related argument is given in Chen and Du (2014), who question whether reinforcement learning can be effectively applied to a situation with a large number of options.

Part VIII

Economics of changes

Economics is about change, and that subject has been very clearly stated in Alfred Marshall's famous quotation:

> Economics, like biology, deals with a matter, of which the inner nature and constitution, as well as outer form, are constantly changing.
>
> (Marshall, 1924, p. 772)

While "constantly changing" is highlighted frequently in various documents of daily life, it seems that economists have not yet been sure whether they do have a capable model for this subject. In fact, the recent book by Frydman and Goldberg (2007) has just affirmed the lack of an adequate economic model for changes, which was also pointed out by Herbert Simon many years ago (Simon *et al.*, 1992).

For Simon, what matters is the *process* which leads to constant changes and novelty discovering.

> [I]f we want to have a theory of technological change, it will have to be a theory of the processes that bring about change rather than a theory of the specific nature of the changes.

To have these features, one needs to have a model that can constantly generate new opportunities (potentials to changes), and agents, as part of the model, who are able to constantly exploit these opportunities (potential to discover novelties).[1] What may or may not come as a surprise to us is that infinitely smart agents, the *Homo economicus*, are not qualified to be constituents of this kind of model. Neither can most adaptive agents used or studied in economics serve the purpose. Mainly this is because most of these adaptive agents are equipped with tools that can only handle *well-structured* problems, not *ill-structured* ones.[2]

Genetic programming is one way, though not the only one, to equip agents with these capabilities.[3] Using the terms of Simon (Simon *et al.*, 1992), genetic programming is a *chunk*-based search algorithm; based on Simon, these chunks provide the basis by which human agents can recognize patterns and develop intelligent behavior. These chunks may also be known as *building blocks* (Holland, 1975) or *modules* (Simon, 1962, 1965). Simon considers 50,000 chunks as one of the requirements for being an expert, in addition to ten years' experience.[4] These two magic numbers nicely match two parameters in GP, namely, *population size* and the *number of evolving generations*.

Hence, an agent, endowed with a population size of 50,000 "chunks" (chromosome, building blocks, LISP trees, parse trees), after iterations equivalent to ten years (learning, evolution), can become an expert. This kind of adaptive agent, called GP-based agents for convenience, provide us a starting point for modeling change and novelty discovery. To us, one of the best demonstrations is the use of GP in agent-based double auction markets (Chapter 8). The second demonstration of this is agent-based models of innovation, to be presented in the coming chapter (Chapter 23). The following is a preview of the chapter.

One of the issues that agent-based economists frequently encountered is to what extent agent-based economics can be considered as a truly alternative paradigm to mainstream economics (neoclassical economics or the new classical economics), instead of just doing some interior renovation, by and large, within the same tarnished building. This criticism also applies to other possible alternative paradigms to economics, such as behavioral economics.

In this book, we have seen a number of possible "perturbations," including the replacement of optimization schemes (rational expectations, global optima) with heuristics based on limited search over a finite set of alternatives, the replacement of representative agents with heterogeneous agents, and hence nontrivial aggregation over these replacements. Nevertheless, many studies surveyed in this book also show that this replacement is not grand, but partial. Hence, for example, the Cobb–Douglas utility function or production function, the expected utility, the Walrasian auctioneer, those backbones of neoclassical economics, may remain and coexist with some renovation. Whether this "fixing one at a time" process can be coherently organized is yet to be seen. In the next chapter, by using some tools introduced early in this book, we review a work which, to some extent, has deviated further from the familiar economy which we have studied. It is not just replacing heterogeneous agents, inserting bounded-rational behavior, but something far beyond that.

Modularity

Modularity is still not a part of agent-based economics modeling. This absence is a little disappointing since ACE is regarded as a complement to mainstream economics in terms of articulating the mechanisms of evolution and automatic discovery. One way to make progress is to enable autonomous agents to discover the modular structure of their surroundings, and hence to adapt by using modules. This is almost equivalent to causing their "brain" or "mind" to be designed in a modular way as well.

The only available work in agent-based economic modeling which incorporates the idea of modularity are the *agent-based models of innovation* initiated by Chen and Chie (2004a, b). They proposed a *modular economy* whose demand side and supply side both have a *decomposable* structure. While the decomposability of the supply side, i.e., production, has already received intensive attention in the literature, the demand side has not. Inspired by the study of *neurocognitive modularity*, Chen and Chie (2004a, b) assume that *the preference of consumers can be decomposable*.[5] In this way, the demand side of the modular economy corresponds to a market composed of a set of consumers with *modular preference*.

In the modular economy, the assumption of modular preference is made as a dual relation to the assumption of modular production. Nevertheless, whether in reality the two can have a nice mapping, e.g., a one-to-one relation, is an issue related to the distinction between *structural modularity* and *functional modularity*. While in the literature, this distinction has been well noticed and discussed, "recent progress in developmental genetics has led to remarkable insights into the molecular mechanisms of morphogenesis, but has at the same time blurred the clear distinction between structure and function" (Callebaut and Rasskin-Gutman, 2005, p. 10).

The modular economy considered by Chen and Chie (2004b) does not distinguish between the two kinds of modularity, and they are assumed to be identical. One may argue that the notion of modularity that is suitable for preference is structural, i.e., *what it is*, whereas the one that is suitable for production is process, i.e., *what is does*. However, this understanding may be partial. Using the LISP parse-tree representation, Chen and Chie (2004b) have actually integrated the two kinds of modularity. Therefore, consider drinking coffee with sugar as an example. Coffee and sugar are modules for both production and consumption. Nevertheless, for the former, producers *add* sugar to coffee to deliver the final product, whereas for the latter, the consumers drink the mixture knowing of the existence of both components or by "seeing" the development of the product.

The last section (Section 23.6) of the chapter deals with the competitive advantage of firms which incorporate modular designs in their production and innovation. To demonstrate the significance of modularity, Chen and Chie (2007) further introduce an augmented version of genetic programming that, technically, uses automatically defined terminals (ADTs), which provide an *encapsulated* version of modules. In this way, modules, as normally perceived, will not easily become fragmentized when they are further applied to more hierarchical products. This technical device is then applied to the oligopolistic competition between two rival firms: one firm uses ADTs in their production, and the other does not. These two different designs match well with Simon's famous metaphor on of the competition of two watchmakers: Hora and Tempus (Simon, 1965). The modular preferences of consumers not only define the search space for firms, but also a search space with different hierarchies. While it is easier to meet consumers' needs with very low-end products, the resulting profits are negligible. To gain higher profits, firms have to satisfy consumers up to higher hierarchies. However, consumers become more and more heterogeneous when their preferences are compared at higher and higher hierarchies, which calls for a greater diversity of products.[6] It can then be shown that the firm using a modular design performs better than the firm not using a modular design, as Simon predicted.

Notes

1 This later property can be related to what is known as *second-order emergence* (Trajkovski and Collins, 2009), where agents are *conscious* of the emergent outcomes and react accordingly.
2 See Simon *et al.* (1992), pp. 28–30.
3 Genetic algorithms and learning classifier systems can be other alternatives. However, to the best of our knowledge, most agent-based economic applications of genetic algorithms do not manifest this capability, and, for some reason not exactly known, there are almost no agent-based economic applications of learning classifier systems.
4 However, one has also to notice that Simon was speaking about cognitive heterogeneity when giving these necessary conditions for experts, and did not himself go further to expound on this kind of heterogeneity. Hence, by taking these heterogeneities into account, one may not naively assume that all agents have a size of 50,000 chunks as they differ in their *long-term memory capacity*.
5 Whether one can build preference modules upon the brain/mind modules is of course an issue deserving further pursuit.
6 If the consumers' preferences are randomly generated, then it is easy to see this property through the combinatoric mathematics. On the other hand, in the parlance of economics, moving along the hierarchical preferences means traveling through different regimes, from a primitive manufacturing economy to a quality service economy, from the mass production of homogeneous goods to the limited production of massive quantities of heterogeneous customized products.

23 Agent-based modular economy

In this chapter, we introduce a non-standard approach to modeling economic dynamics, which, however, still keeps the essential ingredients of the conventional economic model, such as preferences, utility, production, and technology. We shall call this approach the *modularity approach*, and the economic model built upon modularity the *modular economy*. The attempt is to bring *modularity*, particular *hierarchical* modularity, into economic modeling and see whether we can gain new insights from the study of economic dynamics, such as the evolution of technology (Basalla, 1988), innovation, and growth.

Modularity refers to the structural relation between a system as a whole and its constituent components which can function as independent entities. It is a key to harnessing a possibly unbounded complex system. Herbert Simon is probably one of the most influential pioneers, who inspired many follow-up works appearing in many different fields (Callebaut and Rasskin-Gutman, 2005).[1] This chapter can be positioned as a continuation of Simon's work on complex system, and specifically his addressing of *hierarchical complexity*.[2]

Modularity is becoming more important today because of the increased complexity of modern technology. Using the computer industry as an example, Baldwin and Clark (2000) show that the industry has experienced previously unimaginable levels of innovation and growth because it embraced the concept of modularity. Kamrani (2002) also asserts that embracing the principle of modular design can enable organizations to respond rapidly to market needs and allow the changes to take place in a cost-effective manner. Other references include Garud, Kumaraswamy, and Langlois (2002) and Langlois (2002).

The departure of this chapter comes from the use of *genetic programming* as a tool to manifest hierarchical modularity. This manifestation is then applied to reconstruct the foundation of an economy, namely, *preference* and *technology*. However, this reconstruction is not immediately sensible without first assuming something that we cannot just take for granted. We shall call this the *twin assumptions*. One side of the twin assumptions is that *preferences are hierarchical modular*; the other side of the twin assumption is that *products and production processes are also hierarchical modular*. In fact, they are called a *twin* because preferences and products (and processes) are all unified in the same hierarchical modular structure. Direct defense of these assumptions may not be necessary for an abstract model like this; however, immediate rejection of them may also require more careful thought than it might seem. Nonetheless, what one should do at least is to specify the backgrounds under which these two assumptions can be well motivated or even justified. We shall start with production (Section 23.1.1) because its hierarchical modular structure is more evident than preference (Section 23.1.2).

23.1 Twin assumptions of the modular economy

23.1.1 Modular production

The hierarchical modular nature of production can be described by tracing a production process, from its primitive stage (raw material), through its intermediate stage (intermediate) to its final stage (finished product). Let us take Chinese macaroni as a simple illustration. Figure 23.1 shows the final product, and Figure 23.2 gives a highly sketched process for making the macaroni. The point of these figures is to show:

1 A product (commodity) is a realization of a production process, such as cooking and frying.
2 A product (commodity) can therefore be represented as the associated production process.
3 A production process can be described by a sequence of processors employed to process raw materials or ingredients, such as flour, water, oil, and salt.

Two sets of elements stand out in the above description, which become the building blocks of any product or production process. The first one is a set of *primitive processors*, and the second one is a set of *raw materials*. Abstraction of these two sets are given as follows:

$$\text{Processor Set:} \quad \Xi = \{F_1, F_2, \ldots, F_k\}, \tag{23.1}$$

Figure 23.1 Chinese macaroni.

Note: The two styles of Chinese macaroni are presented. On the left is the one with soup, whereas on the right is the fried one.

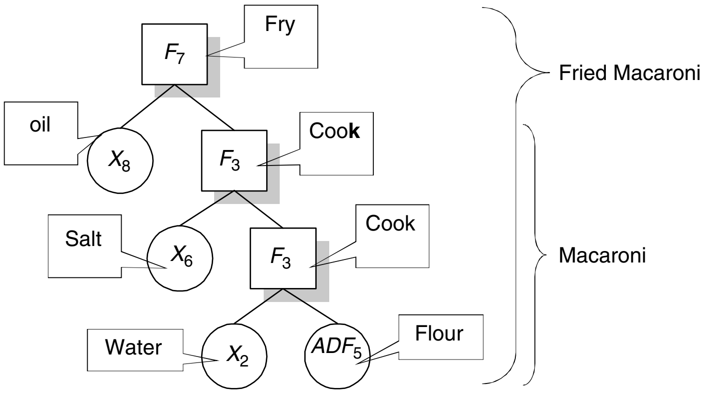


Figure 23.2 A naive recipe for Chinese macaroni.

Note: A naive recipe for Chinese macaroni is presented. It starts with cooking water mixed with flour, then adding salt while cooking. This results in Chinese macaroni soup. Fried Chinese macaroni can be made without soup.

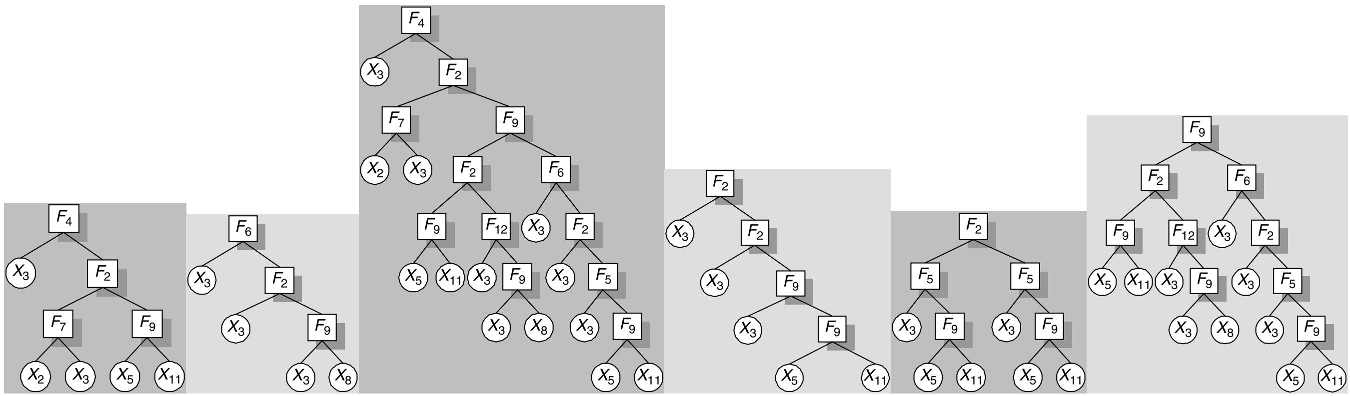
$$\text{Raw Material Set:} \quad \Sigma = \{X_1, X_2, \ldots, X_\kappa\}. \tag{23.2}$$

We shall call the elements in these two sets *primitives*. Taking the example of Chinese macaroni, the processor set includes cook and fry, whereas the raw material set include flour, water, oil, and salt.

Given a set of primitives, a product or a production process as shown in Figure 23.2 can then be represented by a *LISP S-expression* or, simply, *a parse tree*. In Figure 23.3, a product is represented as a *parse tree* (see also Section 13.4.5). Each parse tree corresponds to a LISP program. The very bottom of the tree, i.e., the leaves, corresponds to the raw inputs (materials) X_1, X_2, ..., whereas the root and all intermediate nodes represent the processors, F_1, F_2, . . . , applied to these raw materials in a bottom-up order—the usual behavior of a LISP program. The whole parse tree can, therefore, be viewed as a production process associated with the product.

The idea of modularity in production is then explicitly shown in the *parse tree*. The essence is that *each parse tree has its autonomy*. First, it can independently function as a process or product; second, it can be taken as part of a higher level of production or product. Hence, the parse-tree representation clearly shows the hierarchical modular nature of product and production.

Going one step further one can also see the role of modularity in a sequence of products or product dynamics. The upper panel of Figure 23.4 shows the constituent modules of a mobile phone. The battery (power model), keypad (input model), plastic case, communication IC and LED screen (the display module), shown at the bottom of the upper panel of Figure 23.4, are considered to be the encapsulated (internal) modules, and the base station (connecting or networking module) to be an external module. One form of the evolution of the mobile phone



Commodity 1 Commodity 2 Commodity 3 Commodity 4 Commodity 5 Commodity 6 ...

Figure 23.3 A functional–modularity representation of commodities.

Note: Commodities are associated with their respective production processes which, when written in LISP, can be depicted as parse trees, as shown here.

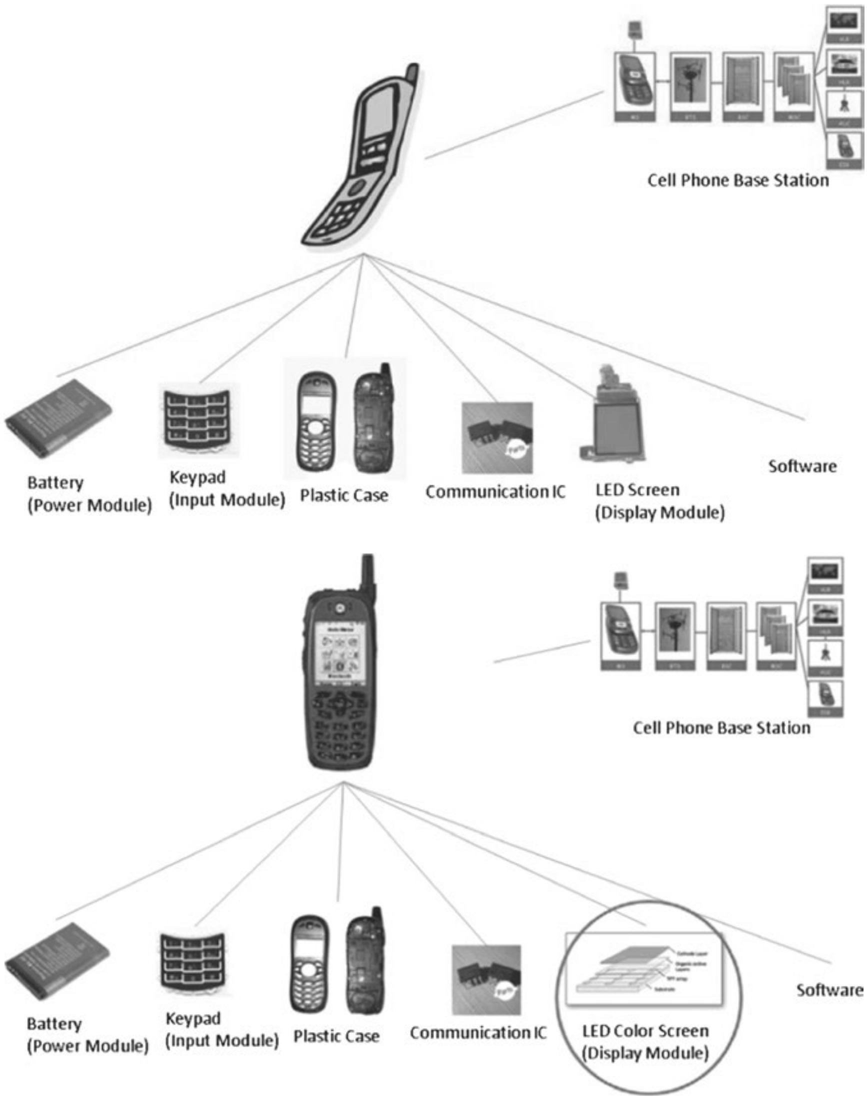


Figure 23.4 Mobile phone: black and white (upper panel) and color (lower panel).

is that one or many of its modules undergo a "mutation." For example, in the lower panel of Figure 23.4, a "mutation" occurs in the display module; the original monochrome display screen is replaced by a color screen. Then, a new product emerges. At this point, the mobile phone is still just a phone.

As time goes by, some modules evolve into independent modules of a two-level hierarchy and work with another independent module (camera). This synergy

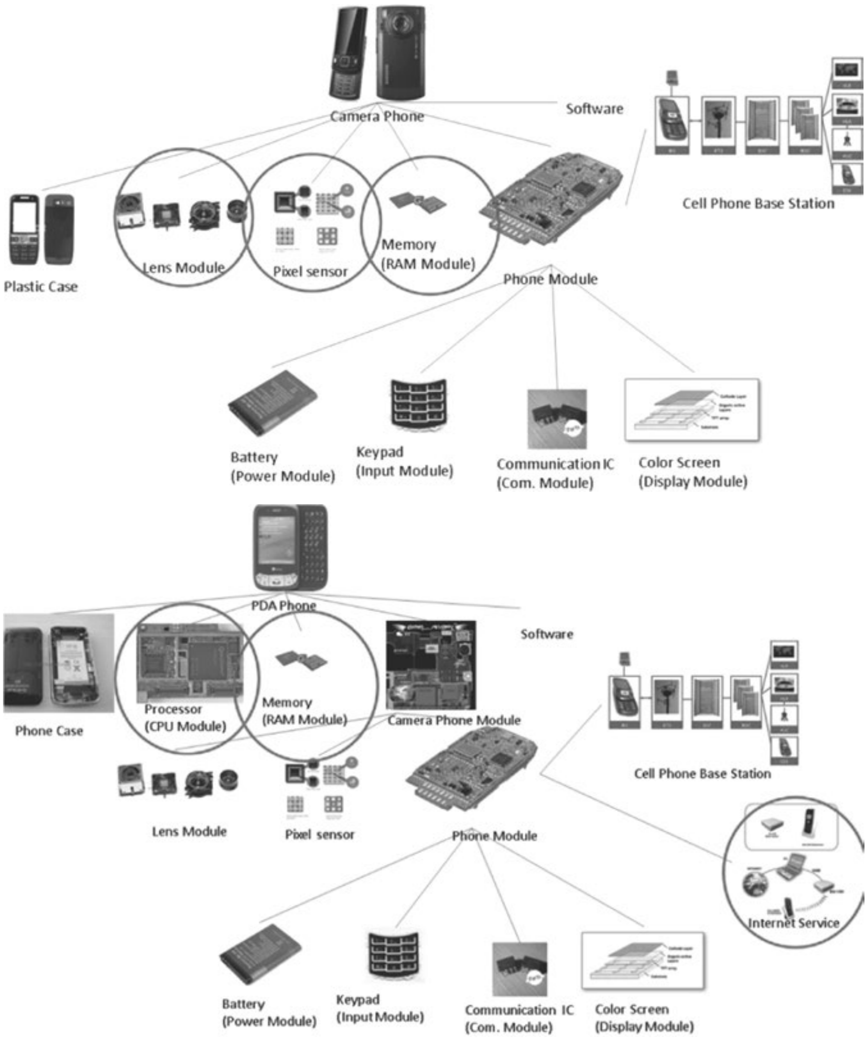


Figure 23.5 Camera phone (upper panel) and smart phone (lower panel).

takes place by incorporating into the mobile phone a number of conventional camera modules, including a lens, pixel sensor, and memory, as shown in the upper panel of Figure 23.5. Now we have a new product, the camera phone, a product with three levels of hierarchy, as shown in the upper panel of Figure 23.5. Next, this camera phone serves as another independent module and is integrated with a module known as the “assistant” or “secretary” module, which has the submodules of, for example, CPU and RAM (the lower panel of Figure 23.5). Then, this integration is accompanied by another indispensable independent module, namely, the

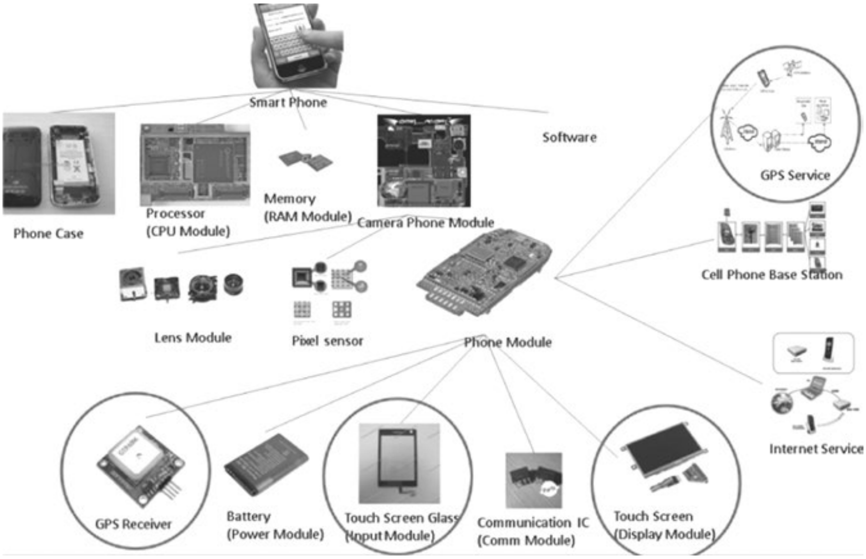


Figure 23.6 Smart phone.

Internet service. As a result, a new product with four levels of hierarchy, known as the PDA phone or the feature phone, emerges.

In the f nal stage of this example, a number of the constituent modules of the feature phone are enhanced. The input module is upgraded to touch screen glass and the display module is upgraded to a touch screen. The further additions of a GPS receiver as an internal module and the GPS service as an external module make the feature phone become a smart phone (Figure 23.6).

Figure 23.6 gives an overview of the evolution of the mobile phone, specif cally expressed in terms of hierarchical modularity. They together show a real evolution of products, in this case, mobile phones. These real dynamics can in principle be abstracted into their corresponding parse-tree dynamics so long as we can trace the associate parse trees of each of these products. Figure 23.7 gives such an abstract demonstration.

As expected, Figure 23.6 with the corresponding parse tree can also be regarded as an innovation process because it features two common properties shared by many innovation processes. First of all, as regards the innovation process, we consider it to be a *continuous process* (*evolution*), rather than a *discontinuous process* (*revolution*). According to the continuity hypothesis, novel artifacts can only arise from antecedent artifacts. Second, the evolution can be regarded as a *growing process* by combining low-level parse trees or modules to achieve certain kinds of high-level parse trees or modules. In plain English, new ideas come from the use (the combination) of old ideas (building blocks). New ideas, once invented, will become modules for other more advanced new ideas. Both of these features are

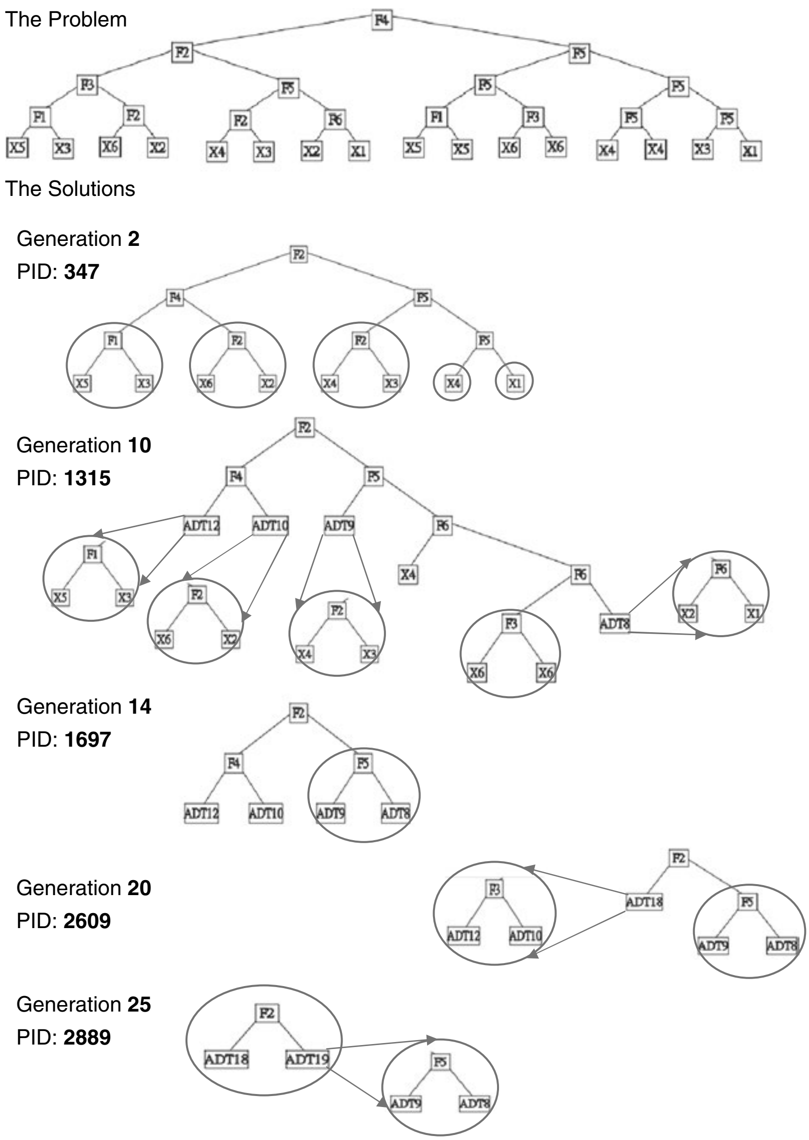


Figure 23.7 An illustration of a process of product innovation.

clearly seen from Figure 23.7 as how parse trees at the early stage lead to the parse trees at later stage.

Hence, the parse tree dynamics are not random, but time dependent. Moreover, they may be purpose driven. In other words, there may be an underlying market force which causes the appearance of one parse-tree dynamics more likely than the appearance of another. In the following, we would like to examine the influence of the demand side on the product dynamics. To do so, we have to first introduce a new framework of consumer behavior which is compatible with the modular economy.

23.1.2 Modular preference

The second assumption that preferences also have hierarchical modular structure is less evident than product. While modularity in brain and mind has been extensively studied these days (Fodor, 1983, 2000; Churchland and Sejnowski, 1992; Brocas and Carrillo, 2008a), whether one can claim that the modular brain or mind actually implies a modular preference is still an issue yet to be addressed. Without empirical grounds, Chen and Chie (2005) proposed a kind of modular preference. By assuming three regular conditions, this modular preference can lead to a utility function which is able to deal with product with parse-tree representation. The three regular conditions are *monotonicity*, *synergy*, and *consistency* (to be detailed in Section 23.2.3). By satisfying these three conditions, a module-matching algorithm can be constructed to perform utility computation (see Section 23.2.4). With the derived utilities, much of the economic analysis can still be carried out in a conventional way.

Despite this limited change, the modular preference does make us think about preference in a rather non-standard way. Preference in conventional economic analysis mainly focuses on *quantity* and does not deal with quality except by quantifying it. In addition to treading the utility of a bundle of commodities, the conventional analysis does not address the structure of commodities. The distinguishing feature of the modular preference is its direct processing of product quality by using the *module-matching algorithm* (to be detailed in Section 23.2). Hence, in a modern dynamic economy which is service oriented and quality driven, the modular preference has its potential application value.

With a given modular preference, the conventional economic analysis easily carries over, and the product dynamics become purpose driven. As an illustration, Figure 23.7 traces such a purpose-driven process. Consider a target consumer whose preference is depicted in the first row of Figure 23.7, which can be regarded as the solution to the problem. Firms do not know this design, and have to figure out the best design by themselves. The five products listed below are the designs discovered in generations 2, 10, 14, 20, and 25. These products match the consumer's needs to a higher and higher level. For example, the product PID 2889, i.e., the 2889th new product designed by the firm, has completely answered the target's need to the entire first half at level four.

Nonetheless, this product does not come out all of a sudden; all it has done is to combine two commodities which were already known before, namely, modular commodities ADT 18 and ADT 19, both of which were already known to the firm before generation 25.[3] The "marginal" effort here is to assemble them in the right way, i.e., using processor F2.[4]

23.2 Preference and utility function

The twin assumptions of modularity gives us a very different representation of preference. To make sense of this representation, it would be useful to study its behavior in light of the conventional microeconomic analysis of a consumer. In particular, we would like to know whether, from a given preference, we can derive a utility function upon which the concept of the maximum willingness to pay can be based. Also, what constraints are required for making the derived utility function to satisfy our general economic intuition?

23.2.1 Commodity space

The utility function $U(.)$ in conventional economic theory is generally a mapping from nonnegative real space to real space $\mathcal{R}$:

$$U : \mathcal{R}^n_+ \to \mathcal{R}. \tag{23.3}$$

This mapping is of little help to us when what we evaluate is a sequence of processors rather than just a quantity. In our economy, what matters to consumers is not the *quantity* they consumed, but the *quality* of what they consumed. Therefore, the conventional commodity space $\mathcal{R}^n_+$ is replaced by a new commodity space which is a collection of sequences of processors. We shall denote the space $\mathcal{Y}$. The representation of the commodity space $\mathcal{Y}$ can be constructed by using the *theory of formal language*, for example, the *Backus–Naur form* of grammar (see Section 13.4.1). So $\mathcal{Y}$ is to be seen simply as the set of all expressions which can be produced from a start symbol Λ under an application of *substitution rules* (*grammar*) and a finite set of primitive processors (Σ) and materials (Ξ). That is, $\mathcal{Y}$ represents the set of all commodities which can be produced from the symbols Σ (set of raw materials) and Ξ (set of processors):

$$\mathcal{Y} = \{Y \mid \Lambda \Rightarrow Y\}. \tag{23.4}$$

While, as we saw in Figure 23.3, each Y ($Y \in \mathcal{Y}$) can be represented by the language of expression trees (ETs), a more effective representation can be established by using *gene expression programming* (GEP) developed by Ferreira (2001). In GEP, the individuals are encoded as *linear strings of fixed length* (the genome or set of chromosomes) which are afterwards expressed as nonlinear entities of different sizes and shapes, i.e. different expression trees. As Ferreira (2001) showed, the interplay of chromosomes and expression trees in GEP implies an unequivocal translation system for translating the language of chromosomes into the language of ETs. By using GEP, the commodity space can then be defined as a subset of the *Kleene star*, namely,

$$\mathcal{Y} = \{Y_n, \mid Y_n \in (\Sigma \cup \Xi)^* \cap GEP, n = 1, 2, \ldots\}, \tag{23.5}$$

where Y_n is a string of length n,

$$Y_n = y_1 y_2 \ldots y_n, \quad y_i \in (\Sigma \cup \Xi), \forall i = 1, \ldots, n. \tag{23.6}$$

We have to emphasize that, in order to satisfy syntactic validity, $\mathcal{Y}$ is only a subset of the Kleene star $(\Sigma \cup \Xi)^*$. To make this distinction, the $\mathcal{Y}$ described in (23.5) is referred to as the *strongly typed Kleene star*. Each Y_n can then be translated into the familiar parse tree by using GEP.

23.2.2 Preferences

Unlike a commodity space, a preference space cannot be a collection of finite-length strings, since they are not satisfied by the *non-saturation* assumption. Economic theory assumes that consumers always prefer more to less, i.e. the marginal utility can never be negative. Even though we emphasize the *quality* dimension instead of the *quantity* dimension, a similar vein should equally hold: *you will never do enough to satisfy any consumer*. If consumers' preferences are represented by finite-length strings, then, at a point, they may come to a state of complete happiness, known as the *bliss point* in economic theory. From there, no matter how hard the producers try to upgrade their existing commodities, it is always impossible to make consumers feel happier. This is certainly not consistent with our observation of human behavior. As a result, the idea of a commodity space cannot be directly extended to a preference space.

To satisfy the non-saturation assumption, a preference must be a string of infinite length, something like

$$\ldots u_1 u_2 \ldots u_l \ldots = \ldots U^l \ldots \tag{23.7}$$

However, by introducing the symbol ∞, we can regain the finite-length representation of the preference, i.e.

$$\infty u_1 u_2 \ldots u_l \infty = \infty U^l \infty = [U^l]. \tag{23.8}$$

First of all, as we mentioned earlier, consumers may not necessarily know what their preferences look like, and may not even care to know. However, from Samuelson's *revealed preference theory*, we know that consumers' preferences *implicitly* exist. Equation (23.8) is just another way of saying that consumers' preferences are *implicit*. It would be pointless to write down the consumers' preferences of the 30th century, even though we may know that these are much richer than what has been revealed today. To approximate the feedback relation between technology advancements and preferences, it would be good enough to work with *local-in-time* preferences (temporal preferences).

Secondly, Equation (23.8) enables us to see the possibility that preference is adaptive, evolving and growing. What will appear in those ∞ portions may crucially depend on the commodities available today, the commodities consumed by the consumer before, the consumption habits of other consumers, and other social, institutional, and scientific considerations.

23.2.3 Utility function

Given the preference $[U^l]$, let $U \mid [U^l]$ be the utility function derived from $[U^l]$. $U \mid [U^l]$ is a mapping from the *strongly typed Kleene star* to $\mathcal{R}_+$:

$$U \mid [U^l] : \mathcal{Y} \rightarrow \mathcal{R}_+. \tag{23.9}$$

Hereafter, we shall simply use U instead of $U \mid [U^l]$ as long as it causes no confusion.

The modular approach to preference regards each preference as a hierarchy of modular preferences. Each of these modular preferences is characterized by a parse tree or the so-called building block. For example, the preference shown in Figure 23.8 can be decomposed into modular preferences of different depths. They are all explicitly indicated in Figure 23.9. Consider S_i to be the set of all modular

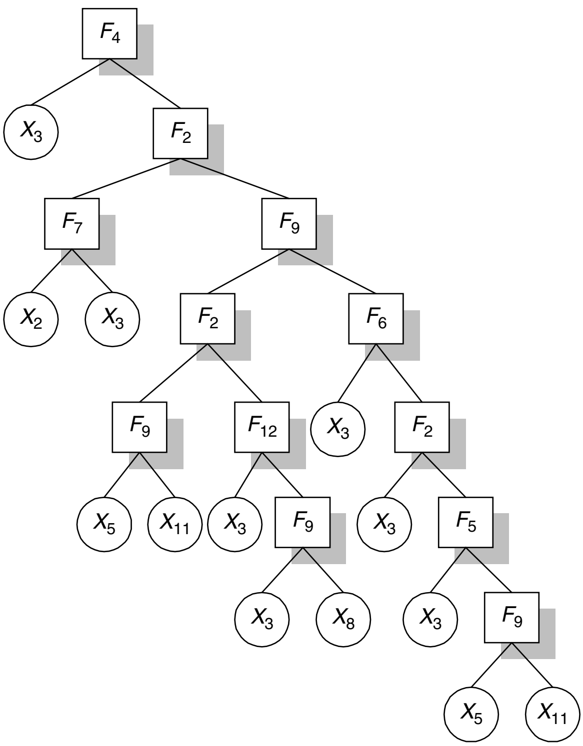


Figure 23.8 Preference: the parse tree representation.

Note: What is shown here is only part of the potentially infinitely large parse tree, i.e. only U^l of $[U^l]$.

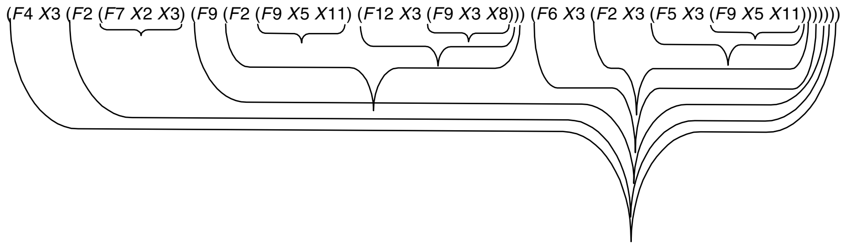


Figure 23.9 Modular preference: the LISP representation.

Table 23.1 Modular preferences sorted by depth

D (d)	*Subtrees or terminals*	
1	$X_2, X_3, X_5, X_8, X_9, X_{11}$	1 (4^0)
2	$S_{2,1} = (F_7X_2X_3)$ $S_{2,2} = (F_9X_5X_{11})$ $S_{2,3} = (F_9X_3X_8)$ $S_{2,4} = (F_9X_5X_{11})$	4 (4^1)
3	$S_{3,1} = (F_{12}X_3(F_9X_3X_8))$ $S_{3,2} = (F_5X_3(F_9X_5X_{11}))$	16 (4^2)
4	$S_{4,1} = (F_2(F_9X_5X_{11})(F_{12}X_3(F_9X_3X_8)))$ $S_{4,2} = (F_2X_3(F_5X_3(F_9X_5X_{11})))$	64 (4^3)
5	$S_5 = (F_6X_3(F_2X_3(F_5X_3(F_9X_5X_{11}))))$	256 (4^4)
6	$S_6 = (F_9(F_2(F_9X_5X_{11})(F_{12}X_3(F_9X_3X_8)))$ $(F_6X_3(F_2X_3(F_5X_3(F_9X_5X_{11})))))$	$1,024(4^5)$
7	$S_7 = (F_2(F_7X_2X_3)(F_9(F_2(F_9X_5X_{11})(F_{12}$ $X_3(F_9X_3X_8)))(F_6X_3(F_2X_3(F_5X_3$ $(F_9X_5X_{11}))))))$	$4,096$ (4^6)
8	$S_8 = (F_4X_3(F_2(F_7X_2X_3)(F_9F_2(F_9X_5X_{11})$ $(F_{12}X_3(F_9X_3X_8)))(F_6X_3(F_2X_3(F_5X_3$ $(F9X_5X_{11})))))))$	$16,384$ (4^7)

preferences of *depth* i. Then Table 23.1 lists all modular preferences by means of these S_i. From both Figure 23.9 and Table 23.1, it is clear that each subtree at a lower level, say S_j, can always find its parent tree, of which it is a part, at a higher level, say S_i where $i > j$. This subsequence relation can be represented as follows:

$$S_i \sqsupset S_j. \tag{23.10}$$

A commodity Y_n is said to *match* a modular preference S_i of U^l if they are exactly the same, i.e. they share the same the LISP expression and the same tree representation. Now, we are ready to postulate the first regularity condition regarding a well-behaved utility function, which is referred to as the *monotonicity condition*.

Monotonicity

Given a preference $[U^l]$, the associated utility function is said to satisfy the *monotonicity condition* iff

$$U(Y_{n_i}) > U(Y_{n_j}), \tag{23.11}$$

where Y_{n_i} and Y_{n_j} are the commodities matching the corresponding modular preferences S_i and S_j of U^l and S_i and S_j satisfy Equation (23.10).

The *monotonicity* condition can be restated in a more general way. Given a preference $[U^l]$ and by letting $\{h_1, h_2, \ldots, h_j\}$ be an increasing subsequence of $\mathcal{N}_+$, then the associated utility function is said to satisfy the *monotonicity condition* iff

$$U(Y_{n_j}) > U(Y_{n_{j-1}}) > \cdots > U(Y_{n2}) > U(Y_{n1}), \tag{23.12}$$

where $Y_{n1}, \ldots, Y_{n_j}$ are the commodities matching the corresponding modular preferences $S_{h1}, \ldots, S_{h_j}$ of U^l, and

$$S_{h_j} \sqsupset S_{h_{j-1}} \sqsupset \cdots \sqsupset S_{h2} \sqsupset S_{h1}. \tag{23.13}$$

Synergy

If S_k is a subtree of S_i as in Equation (23.10), then S_k is called the *largest subtree* of S_i if S_k is a *branch* (descendant) of S_i. We shall use "$S_i \lhd S_k$" to indicate this largest-member relation. Depending on the grammar which we use, the largest subtree of S_i may not be unique. For example, each modular preference in Figure 23.8 has two largest subtrees. In general, let $S_{h1}, S_{h2}, \ldots, S_{h_j}$ be all the largest subtrees of S_i, denoted as follows:

$$S_i = \sqcup_{h1}^{h_j} S_k \lhd \{S_{h1}, S_{h2}, \ldots, S_{h_j}\}, \tag{23.14}$$

where $\{h_1, h_2, \ldots, h_j\}$ is a nondecreasing subsequence of $\mathcal{N}_+$. Notice these largest trees do not have subrelationships (23.10) among each other. However, they may have different depths, and the sequence $\{h_1, h_2, \ldots, h_j\}$ ranks them by depth in an ascending order so that S_{h_1} is the largest subtree with minimum depth, and S_{h_j} is the one with maximum depth.

The second postulate of the well-behaved utility function is the property known as *synergy*. Given a preference $[U^l]$, the associated utility function is said to satisfy the *synergy condition* iff

$$U(Y_{n_i}) \geq \sum_{k=1}^{j} U(Y_{n_k}), \tag{23.15}$$

where Y_{n_i} and $\{Y_{n_k}; k = 1, \ldots, j\}$ are the commodities matching the corresponding modular preferences S_i and $\{S_{h_k}; k = 1, \ldots, j\}$ of $[U^l]$, and S_i and $\{S_{h_k}; k = 1, \ldots, j\}$ satisfy Equation (23.14).

For convenience, we shall also use the notation $\sqcup_{k=1}^{j} Y_{n_k}$ as the synergy of the set of commodities $\{Y_{n_k}; k = 1, \ldots, j\}$. Based on the *New Oxford Dictionary of English*, synergy is defined as "the interaction or cooperation of two or more organizations, substances, or other agents to produce a combined effect greater than the sum of their separate effects." "The whole is greater than the sum of the parts" is the fundamental source for *business value creation*. Successful business value creation depends on two things: *modules* and the *platform* to combine these modules. Consider the consumer characterized by Figure 23.8 as an example. To satisfy this consumer, what is needed are all of the modules listed in Table 23.1. Even though the technology has already advanced to the level S_7, knowing that using processor F_4 to combine X_3 and S_7 can still satisfy the consumer to a higher degree, and hence create a greater business value.

Consistency

A modular preference may appear many times in a preference. For example, $S_{2,4}$ in Table 23.1 appears twice in Figure 23.8. In this case, it can simultaneously be the largest subtree of more than one modular preference. For example, $S_{2,4}$ is the largest subtree of both $S_{3,2}$ and $S_{4,1}$. Let S_k be the largest subtree of $S_{h1}, S_{h2}, \ldots,$ and S_{h_j}. Denote this relation as

$$S_k = \sqcap_1^j S_{h_i} \rhd \{S_{h1}, S_{h2}, \ldots, S_{h_j}\}. \tag{23.16}$$

Given a preference $[U^l]$, the associated utility function is said to satisfy the *consistent condition* iff

$$U(Y_{n_i} \mid S_k \rhd S_{h1}) = \cdots = U(Y_{n_i} \mid S_k \rhd S_{h_j}), \tag{23.17}$$

where "$Y_{n_i} \mid S_k \rhd S_{h1}$" is the commodity which matches the corresponding modular preference S_k in the designated position, $S_k \rhd S_{h_i}$. The consistency condition reiterates the synergy effect. No matter how intensively the commodity Y_{n_i} may significantly contribute to the value creation of a synergy commodity, its value will remain identical and lower when it is served *alone*.

Given a preference $[U^l]$, the associated utility function U is said to be *well behaved* iff *it satisfies the monotone, synergy, and consistency conditions*. It generates a sequence of numbers $\{U(Y_{n_i})\}_{i=1}^{h}$ where Y_{n_i} matches the respective modular preference $S_{d,j}$. $S_{d,j}$ is the jth modular preference with depth d. The utility assigned in Table 23.1 is an illustration of a well-behaved utility function derived from the preference shown in Figure 23.8. In fact, this specific utility function is generated by the following exponential function with base 4:

$$U(S_{d,j}) = 4^{d-1}. \tag{23.18}$$

Utility function (23.18) sheds great light on the synergy effect. Thus, primitive materials or rudimentary commodities may only satisfy the consumer to a rather limited extent. However, once suitable processing or integration takes place, their value can become increasingly large to the consumer. The exponential function with base 4 simply shows how fast the utility may be scaled up, and hence may provide a great potential incentive for producers to innovate. Of course, to be a well-behaved utility function, U can have many different functional forms.

23.2.4 Module matching

Now, it is high time to answer the question: *What would be the enjoyment for a consumer with a preference* $[U^l]$ *consuming a commodity* Y_i*?* Let us start tackling this issue by a commodity, called the *simple commodity*. Given a preference $[U^l]$, a commodity Y_i is called *simple* with respect to $[U^l]$ if it matches exactly one modular preference of U^l. It is easy to evaluate the simple commodity, as discussed in the previous section and exemplified in Table 23.1.

However, not all commodities are simple. Y_i, as a whole, may match no modular preference of $[U^l]$. Nevertheless, it can still be enjoyable for the consumer if it is *similar* or *close* to consumer preference $[U^l]$ in many regards. In this section, we propose an evaluation scheme based on a idea of *similarity* or *closeness*. The evaluation scheme is called *module matching*.

The idea of modular matching is very straightforward. As should now be clear, each *commodity* is composed of many *modular commodities* with different depths. For example, the commodity represented in Figure 23.10 has a list of modular commodities as shown in Table 23.2. Let $Y_{d,j}$ be the jth modular commodity with depth d. Now let the commodity be presented to the consumer with a preference as depicted in Figure 23.8. Clearly, $Y_{4,1}$ does not match any modular preference as

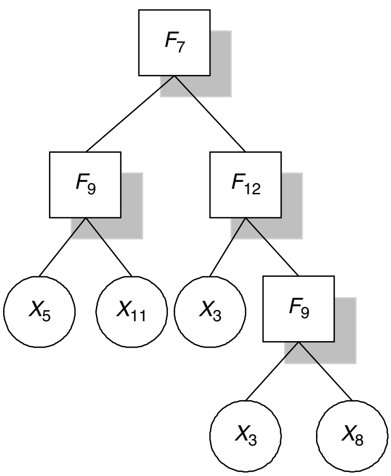


Figure 23.10 An example of commodity: the LISP representation.

Table 23.2 Modular commodities ($\sigma(Y_i)$)

d	*Subtrees or terminals*
1	X_3, X_5, X_8, X_{11}
2	$Y_{2,1} = (F_9 X_5 X_{11})$ $Y_{2,2} = (F_9 X_3 X_8)$
3	$Y_{3,1} = (F_{12} X_3 (F_9 X_3 X_8))$
4	$Y_{4,1} = (F_7 (F_9 X_5 X_{11})(F_{12} X_3 (F_9 X_3 X_8)))$

listed in Table 23.1. However, the commodity is similar to the consumer's preference since it has a major part, $Y_{3,1}$, which matches the consumer preference exactly by the modular $S_{3,1}$ (Table 23.1). Therefore, while the commodity is not the same as the consumer's preference, it is not irrelevant and can satisfy the consumer to some degree.

Next, let us take away $Y_{3,1}$ from the commodity Y_i; what is left is only the subtree corresponding to $Y_{2,1}$ (Figure 23.10). This part also matches the the consumer preference by the modular $S_{2,2}$. The value of the commodity to the consumer would, therefore, be enhanced as opposed to the case if $Y_{2,1}$ were completely useless. We then take $Y_{2,1}$ away from Y_i, and there is nothing left. So, the set of modular preferences matched by Y_i is

$$M_{Y_i:[U^l]} = \{S_{3,1}, S_{2,2}\}. \tag{23.19}$$

Therefore, the utility of the commodity Y_i with respect to the preference $[U^l]$ can be written as

$$U(Y_i) = U(S_{3,1}) + U(S_{2,2}) = 4^2 + 4^1 = 16 + 4 = 20. \tag{23.20}$$

It is crucial to make some working principles underlying this example explicitly. First, we do not start module matching from the smallest modules, such as X_3, X_5, X_8, or X_{11} (Table 23.1). Instead, we start from the biggest one, i.e., the one with the maximum depth. Doing this is referred as to the *descending principle*. Secondly, once any modular commodity is shown to match the corresponding modular preference, it is no longer usable for the rest of the matching exercise. This is called the *nonredundancy principle*. The working of these two principles excludes the consideration of the modular commodity $Y_{2,2}$, while it also matches the preference by $S_{2,3}$.

The main purpose of these two principles is to avoid *double counting* and simultaneously to derive the maximum value of the respective commodity. For example, if one started the modular matching in an *ascending* order, then after the matches of the four raw materials X_3, X_5, X_8, and X_{11}, nothing would be left. Hence the utility of Y_i would come up with only 4, which obviously fails to take the synergy effect into account.

The module-matching algorithm is summarized as follows:

- Step 1: List all modular commodities of Y_i in a collection $\sigma(Y_i)$, and group them by depth, say from $d = 1, 2, \ldots, d_{\max}$.
- Step 2: Set $d = d_{\max}$. Start the modular matching from $Y_{d_{\max}} (= Y_i)$. If there is a match, which means Y_i is a *simple commodity*, then set

$$U(Y_i) = U(S_{i,j}), \tag{23.21}$$

  where $S_{i,j}$ is the modular preference matched by $Y_{d_{\max}} = Y_i$. Go to Step 7. If there is no match, go to the next step.
- Step 3: Decrease d by 1, and do modular matching for $Y_{d,j}, \forall j$.
- Step 4: For any match, Y_{d,j^*}, delete all its modular commodities from $\sigma(Y_i)$.
- Step 5: Put all matches into the set $M_{Y_i:[U_l]}$.
- Step 6: If $d = 0$, or σ_{Y_i} becomes a null set, then

$$U(Y_i) = \sum_{S_{d,j} \in M_{Y_i:[U_l]}} U(S_{d,j}); \tag{23.22}$$

  otherwise, go back to Step 3.
- Step 7: Stop.

23.3 Firms' behavior

To gain further insights from the competitive behavior of firms in this modular economy, it is necessary to define the cost structure of modular production. The point here is not to present a realistic or empirical-based cost structure for modular production. The point is to simply see the complex competitive environment which the firms may face, and to see how this resultant complexity can make the conventional marginal cost analysis hardly applicable as a basis for firms' decisions. Moreover, we want to use this cost structure to show that the critical decisions of firms are not just quantity based, as the conventional economic analysis postulates, but really involve quality and diversity.

23.3.1 Production and costs

To start, let us assume that the economy is composed of N_f producers, each of which is initially assigned an equal operating capital, K_0:

$$K_{1,0} = K_{2,0} = \cdots = K_{N_f,0} = K_0. \tag{23.23}$$

With this initial capital, the producers are able to buy materials and processors from the input markets up to the amount that they can afford. There are two types of input markets at the initial stage, namely, the *raw material market* and the *rudimentary processor market*. For simplicity, we assume that the supply curves of the two markets are infinitely elastic with a fixed unit cost (c) for each raw material and for each rudimentary processor:

$$C_{X_1} = C_{X_2} = \cdots = C_{X_\kappa} = C_{F_1} = C_{F_2} = \cdots = C_{F_k} = c, \tag{23.24}$$

where the parameters κ and k above correspond to the cardinality of the set of processors and the set of raw materials [Equation (23.1)].

With the materials and the rudimentary processors purchased from the input market, the producer can produce a variety of commodities, defined by the associated sequence of processors. The cost of each commodity is then simply its total amount of materials and the number of processors, or, in terms of GP, the *node complexity* (number of nodes) of the parse tree.

This setup is simple and intuitive, but it is flexible enough to accommodate various considerations in conventional production analysis. For example, to allow for the *scale effect*, i.e., each additional unit of the same commodity produced by the producer should be less costly, one can introduce a monotonically decreasing function $\tau(q)$ $(0 \leq \tau(q) \leq 1)$, where q is the qth unit of the same commodity being produced. The cost of each additional unit produced is simply the cost of the first unit premultiplied by $\tau(q)$. With this description, the *capacity constraint* for a *fully specialized producer* i $(i \in [1, \ldots, N_f])$, i.e. a producer who supplies only one commodity, should be

$$K_0 \geq \sum_{q=1}^{\bar{q}} C_q, \tag{23.25}$$

where $C_q = \tau(q)C_1$ is the unit cost of the qth unit and $\tau(1) = 1$. For a *fully diversified producer*, i.e. a producer who produces a variety of commodities and one for each, the capacity constraint is

$$K_0 \geq \sum_{m=1}^{\bar{m}} C_{m,1}, \tag{23.26}$$

where $C_{m,1}$ is the cost of the first unit of commodity m. In general, the capacity constraint for the producer i is

$$K_0 \geq \sum_{m=1}^{\bar{m}} \sum_{q=1}^{\bar{q}m} C_{m,q}, \tag{23.27}$$

where $C_{m,q} = \tau_m(q)C_{m,1}$.

In Equation (23.27), the strategic parameters are $\bar{m}$, $\bar{q}_m$ $(m = 1, 2, \ldots, \bar{m})$, and C_m $(m = 1, 2, \ldots, \bar{m})$.[5] $\bar{m}$ can be be taken as a measure of the degree of *diversification*, whereas $\bar{q}$ can be taken as the degree of *specialization*. C_m, i.e. the node complexity of the commodity m, is also a strategic variable. Given the capacity constraint, the producer can choose to supply a large amount of primitive commodities (a quantity-oriented strategy), or a limited amount of highly delicate commodities (a quality-oriented strategy). Therefore, the choice of C_m can be regarded as a choice of the level of *quality*. With these three core choices, we can, step by step, take into account other aspects of firms' behavior, such as marketing and R&D.

23.3.2 Marketing strategies

The marketing strategy consists of two mainstays: *pricing* and *advertising*. On the pricing part, the producer has to decide a *mark-up* η, i.e., the expected profit rate of the commodity. Suppose that $\bar{C}_m$ is the average cost of producing the mth commodity,

$$\bar{C}_m = \frac{\sum_{q=1}^{\bar{q}_m} C_{m,q}}{\bar{q}_m}. \tag{23.28}$$

Then by the associated mark-up η_m, the label price (the *ask*) of the commodity is

$$ask_m = (1+\eta_m)\bar{C}_m. \tag{23.29}$$

Advertising can be considered as cost expenditures to subside consumers' search costs (see the discussion of Equation (23.45) below). It is used to enhance consumers' knowledge of the commodity. Without this expenditure, consumers may not be able to reach this commodity and hence would not buy it. Suppose that the advertising strategy is simply to decide a lump-sum expenditure which is used to cover a portion of consumers' search costs. Let A_m be the advertising expenditure spent for the promotion of commodity, then the capacity constraint for producer i is

$$K_0 \geq \sum_{m=1}^{\bar{m}} A_m + \sum_{m=1}^{\bar{m}} \sum_{q=1}^{\bar{q}_m} C_{m,q}. \tag{23.30}$$

23.3.3 R&D

Let $\mathcal{Y}_t$ be a subset of $\mathcal{Y}$ which is a collection of the commodities been produced at the market period t. Let

$$\widetilde{\mathcal{Y}}_t = \cup_{j \leq t} \mathcal{Y}_j, \tag{23.31}$$

which is the history of commodities produced up to the time t. A *definition* of innovation at time $t+1$ can then be stated as a commodity Y_i, where $Y_i \in \mathcal{Y}/\widetilde{\mathcal{Y}}_t$, i.e. a commodity which has never been produced before. Sometimes, one would like to assume that innovation must be successful in some sense. In that case, one can strengthen the above definition by adding some performance criteria, such as the *lifespan*, *popularity*, or *accumulated profits*. Only the commodities which can meet these criteria will be coined as *essential innovation*. Others can only be seen as a step in an enduring trial-and-error process.

While there is no guarantee that innovation will necessarily lead to profit opportunities, survival pressure may leave each firm little choice but to give it a try. This section will discuss the innovation activities of producers. In a sense, innovation activities behave similarly to knowledge discovery and data mining in a supervised-learning fashion. What interests producers are consumers' preferences $\{[U_i^l]\}_{i=1}^{N_c}$, where N_c is the number of consumers. Since preferences are implicit, producers have to learn about $\{[U_i^l]\}_{i=1}^{N_c}$ from their marketing experiences, i.e. the market responses to their commodities.

Let $\mathcal{Y}_t^j$ be the collection of commodities producer j supplied at time t, and let Ψ_t^j be the *profit profile* of $\mathcal{Y}_t^j$,

$$\Psi_t^j \equiv \{\pi_{m,t}^j\}_{m=1}^{\bar{m}_t}, \tag{23.32}$$

where $\pi_{m,t}^j$ is the *mean profit* earned by the mth commodity supplied by the producers j at time t. Let $\Pi_{m,t}^j$ be the *aggregate profit* of the respective commodity, and Π_t^j be the aggregate profits earned at time t:

$$\Pi_t^j = \sum_{m=1}^{\bar{m}_t} \Pi_{m,t}^j. \tag{23.33}$$

Taking $\pi_{m,t}^j$ as a *fitness measure*, one can simulate the evolution of commodities and technology via genetic programming. However, caution should be exercised to make sure the application of GP appropriate (economically meaningful). It is, therefore, not just a straightforward application of the standard GP.

First of all, the capacity constraint (23.34) has to be satisfied for all market days:

$$K_{t+1}^j = K_t^j + \Pi_t^j \geq \sum_{m=1}^{\bar{m}_{t+1}} A_m + \sum_{m=1}^{\bar{m}_{t+1}} \sum_{q=1}^{\bar{q}_m} C_{m,q}. \tag{23.34}$$

In a simple setting, there are no external resources to finance producers' budget deficits. Therefore, a commodity, no matter how novel it is, cannot be produced unless it is well supported by the producer's own capital.

Secondly, a run of genetic programming is usually composed of many iterations (number of generations). Nevertheless, since the profit (fitness) coming from the new commodity is unknown until it is actually sold to the market, we actually don't know how to do selection after finishing the first iteration. In this case, search intensity (R&D) would be very limited if learning is confined to a producer's own experience, usually referred to as *individual learning* in agent-based computational economics. It therefore motivates us to think of other possible ways to learn from a large pool of knowledge, and *social learning* seems to be a natural direction to move.

Social learning basically assumes that technologies are public goods, which are pooled together and are commonly shared by all prospective users (producers). This idea certainly would not work if we consider state-of-the-art technologies as *business secrets*, which can only be peered at by business espionage. There is a famous criticism of the applicability of social learning to agent-based computational economics, known as *Harrald's criticism* (Chen and Yeh, 2001). We believe that that criticism applies to the current context equally well, which implies that the social learning scheme is not directly applicable. However, a mechanism known as *school*, introduced by Chen and Yeh (2001), may be helpful.

The school scheme exists between the individual learning scheme and the social learning scheme. The school publicize some modular technologies (modular commodities) which seem to be crucial as *keys to success*. Students can come to school to learn these modules by paying for tuition. To entice students to come to register, school must somehow also be *adaptive*. Chen and Yeh (2001) showed how to build such a school in an agent-based stock market. What needs to be done here sounds similar, a school or schools in *an agent-based knowledge market*.

At any point in time, some producers may like to acquire, at a cost, technologies which have been shown to be quite successful over the past. On the other hand, some producers may also like to release their know-how in exchange for pecuniary rewards. As a result, there is room for the emergence of a *knowledge market*, which is the subject of Section 23.4.2.

Let $\mathcal{E}_t^j$ be the collection of all modular commodities (designs) producer j has paid to learn from other producers (schools) at time t. Since the producer would not pay for anything (any module) already known to him, it must be the case that

$$\mathcal{E}_t^j \cap \sigma(\mathcal{Y}_t^j) = \emptyset, \tag{23.35}$$

where $\sigma(\mathcal{Y}_t^j)$ is all the modular technologies derived from $\mathcal{Y}_t^j$.[6] After acquiring $\mathcal{E}_t^j$, the *knowledge pool* of producer j, defined as the collection of all modular technologies known to the producer j at time t, is denoted as $\sigma(\mathcal{F}_t^j)$, where

$$\mathcal{F}_t^j = \mathcal{Y}_t^j \cup \mathcal{E}_t^j. \tag{23.36}$$

The profit profile is now extended to

$$\Psi_t^j \cup \Psi_t^\vartheta, \tag{23.37}$$

where

$$\Psi_t^\vartheta \equiv \{\pi_{e,t}^j\}_{e=1}^{\bar{e}_t}. \tag{23.38}$$

$\pi_{e,t}^j$ is the eth commodity (design) the producer j acquired from the knowledge market at time t.[7] $\bar{e}_t$ is the total number of technologies acquired by producer j at period t. The producer j can then use $\mathcal{F}_t^j$ as the new *mating pool*, and apply standard genetic programming to invent a new commodity (design) by a recombination of the existing modular technologies in $\sigma(\mathcal{F}_t^j)$.

However, the capacity constraint facing producers is now different from Equation (23.34). First of all, producers have to pay to learn new designs (commodities) from the knowledge market. Let the price paid for acquiring the technology Y_e be $\tilde{P}_e$; then the total spending of producer j on knowledge acquisition, or money spent on *research and development*, is

$$RD_t^j = \sum_{e=1}^{\bar{e}_t} \tilde{P}_e. \tag{23.39}$$

On the other hand, producer j may also be a supplier for the knowledge market, and hence shall receive *royalties* from his exported technologies. Let RY_t^j be the royalties received by producer j at time t; then the new capacity constraint should be

$$K_{t+1}^j = K_t^j + \Pi_t^j + RY_t^j \geq \sum_{m=1}^{\bar{m}_{t+1}} A_m + \sum_{m=1}^{\bar{m}_{t+1}} \sum_{q=1}^{\bar{q}_m} C_{m,q} + RD_t^j. \tag{23.40}$$

At the end of each market day or the beginning of the next market day, the number of *new commodities* can be obtained.[8] This number can be used as a rough measure of *innovation*. The time series analysis of this number itself can be an interesting subject of study, and may have empirical implications.

23.4 Markets

There are many possible ways to introduce the market mechanism to the model, and they may have nontrivial implications to the economy. This issue is not specific to our model, but has been generally addressed in the economic literature of search, bargaining, matching mechanisms, trading institutions, or market designs. What we learn from this literature may be applicable to our model and its relevancy. Since these issues have already been studied generally in Chapter 3, the attempt here is only to give a few examples so as to see its easy extensions.

23.4.1 Market process

In the following, two *shopping schemes* are proposed as a way of *price determination*. The first one is based on *random matching* (*blind search*), and the second is based on *purposive search*. Not surprisingly, the second one is computationally more demanding than the first. By no means are these two exhaustive; they are simply used to demonstrate the rich variety of trading institutions as studied in Chapter 3. Readers should find no difficulty developing their own proposals, and part of the fun of doing agent-based computational economics is to experience playing the role of an engineer (Roth, 2002).

Random matching

A *market day* is composed of several *trading rounds*. In a trading round t, each consumer i ($i = 1, 2, \ldots, N_c$) is randomly matched to one producer j ($j = 1, 2, \ldots, N_f$) with one of the commodities he produced, say, $Y_{j,m}$.[9] The utility from consuming that commodity, $U_i(Y_{j,m})$, defines the maximum amount (*reservation price*), $bid_i(Y_{j,m})$, which the consumer would like to *bid* for that commodity, i.e.,

$$bid_i(Y_{j,m}) = U_i(Y_{j,m}). \tag{23.41}$$

Let $q^d(Y_{j,m})$ be the *aggregate demand* for the commodity $Y_{j,m}$ at trading round t,

$$q^d(Y_{j,m}) = \sum_{i=1}^{N_C} I_i, \tag{23.42}$$

where I_i is an indicator function. $I_i = 1$ if consumer i is connected to the commodity $Y_{j,m}$ at the trading round t and $bid_i(Y_{j,m}) \geq ask(Y_{j,m})$; otherwise, it is zero. Also, let $q^s(Y_{j,m})$ be the total number of units available at trading round t. The trading price of $Y_{j,m}$ at trading round t will then be determined as follows:

$$P(Y_{j,m}) = \begin{cases} ask(Y_{j,m}) & \text{if } q^d(Y_{j,m}) \leq q^s(Y_{j,m}), \\ bid_{i*}(Y_{j,m}) & \text{if } q^d(Y_{j,m}) > q^s(Y_{j,m}), \end{cases} \tag{23.43}$$

where i^* satisfies the equality

$$Card\{i \mid bid_i(Y_{j,m}) \geq bid_{i*}(Y_{j,m})\} = q^s(Y_{j,m}). \tag{23.44}$$

The price determination process (23.43) and (23.44) can be considered as a combination of *take it or leave it* and *English auction* (ascending-price auction). The sellers just post the price and would basically not change it on the same market day. However, if at a particular moment the demand is too high, then the seller will leave the consumers to determine where the price shall go. This finishes one trading round. All commodities sold in the trading round t shall be removed from the shops during the next trading round. The matching and trading process continues until either we come to the end of the trading day, i.e., at a maximum of T trading rounds, or all consumers have run out of their budgets.

Purposive search

A variety of trading processes exists. For example, to decide which commodities should be included in their baskets, consumers can first experiment with different commodities. They can do this by shopping around, and sampling some commodities. However, there is a *search cost* associated with this shopping activity. Let us assume that the unit search cost for experimenting with one commodity is a. The total resource spent in search should not be beyond their budget constraints.

Consumers can then evaluate the satisfaction that they have from each commodity in their sample. With this evaluation, they can determine the reservation price of each commodity. By computing the difference between the reservation price and the label price of the commodity, the net utility of a commodity (consumer surplus) can be derived, and all the commodities in the sample can be ranked accordingly. A rational consumer is expected to buy commodities starting from the top-most commodity and descending down until they have spent their last penny. Hence, the consumers' budget constraint can be written as

$$I \geq a \times N_S + \sum_{\tau} P_{\tau}, \tag{23.45}$$

where I is the budget constraint and N_S is the sample size (search intensity) of consumers, i.e., the number of commodities consumers have some knowledge of. P_{τ} is the price of the commodity which is ranked τth of consumers' experienced commodities.

If we assume that there is no search cost ($a = 0$), then it is rational to expect that the consumer will search over the commodity space at time t, $\mathcal{Y}_t$, or, depending on availability, a subset of $\mathcal{Y}_t$, and get those which can maximize her utility under her budget constraint. In this case, our consumer behaves very much in an ideal neoclassical sense.

23.4.2 Knowledge markets

Here comes one of the most intriguing parts of the agent-based model of innovation, i.e. the *knowledge market*. As mentioned in the previous section, to facilitate their innovation processes, producers have an incentive to learn from others. Of course, not all producers are equally qualified to be *mentors*, since they are not equally successful. Those commodities which help producers to earn gigantic profits are more likely to become targets to imitate. The *roulette-wheel selection scheme*, well known in *evolutionary algorithms*, can be applied to this *mentor-hunting* process.

On the other hand, not all producers would like to be a mentor. While releasing their know-how to some producers may bring authorizing producers a lucrative profit today, it may foster a group of potential competitors tomorrow. Due to this perplexing concern, some technologies may not be available to the knowledge market. It therefore needs to remove them from the knowledge market first. Let $\tilde{\mathcal{Y}}_t$ be the set of designs (commodities) after this removal. Clearly, $\tilde{\mathcal{Y}}_t \subseteq \mathcal{Y}_t$.

Furthermore, to use roulette-wheel selection, let Ψ_t be the profit file of $\mathcal{Y}_t$,

$$\Psi_t = \cup_j \Psi_t^j, \quad j = 1, 2, \ldots, N_f. \tag{23.46}$$

We assume that Ψ_t is *public* information, which is shared among all producers. In correspondence to $\tilde{\mathcal{Y}}_t$, we also restrict our attention to a subset of Ψ_t, namely, $\tilde{\Psi}_t$.

We also assume that producer j's hunting behavior is random and follows the distribution:

$$Prob_m^j = \frac{\exp(\pi_m)}{\sum_{\{m: Y_m \in \tilde{\mathcal{Y}}_t / \mathcal{Y}_t^j\}} \exp(\pi_i)}. \tag{23.47}$$

$Prob_m^j$ indicates the probability that producer j will choose to learn the technology associated with the mth commodity in the set $\tilde{\mathcal{Y}}_t/\mathcal{Y}_t^j$.[10] Equation (23.47) applies only to the first round of hunting. Since producer j may like to acquire more than one technology, he would come back to the market for a few more rounds. Therefore, assuming that he is not interested in the technologies already acquired, we need to modify Equation (23.47) in a *sampling without replacement* fashion.

The next question is the determination of *royalties*. There are several possibilities for introducing pricing mechanisms to the knowledge market. However, we just keep one simple principle: *the quality of learning is as good as the price the producer pays to the authorities*. The more you pay, the better you learn. Call it the *deserving principle*. This principle can be implemented as follows.

Suppose that Y_m is the commodity whose associated design has been chosen by producer j to acquire. Recall that $\sigma(Y_m)$ is the collection of all modular commodities (designs) derived from Y_m. These modular designs can be grouped by the associated depth d, e.g. Table (23.2), $d \in [1, d_{\max}]$. Let us say $\{Y_d\}$ is the set of the modular commodities (designs) with depth d, which the producer has learned from the authority after paying royalties $\tilde{P}$. Then, by the deserving principle, d should be positively related to $\tilde{P}$. If $\tilde{P}$ goes up, d goes up as well.

$$d(\tilde{P}_H) \geq d(\tilde{P}_L) \quad \text{if} \quad \tilde{P}_H \geq \tilde{P}_L. \tag{23.48}$$

Moreover, for the same d, the modular commodity (design) with a high profit shall be more expensive than the modular commodity (design) with a low profit:

$$d_m(\tilde{P}) \leq d_{m'}(\tilde{P}) \quad \text{if} \quad \pi_m \geq \pi_{m'}. \tag{23.49}$$

Therefore, Equations (23.48) and (23.49) together define the deserving principle. Furthermore, considering that usually the bargaining behavior may introduce some uncertainties to the price and service, we therefore propose a stochastic version of the deserving principle. Let us first normalize d from 0 to 1 by the following transformation:

$$\delta = \frac{d-1}{d^*_{\max,t} - 1}, \tag{23.50}$$

where

$$d^*_{\max,t} \equiv \max\{d_{Y_i} : Y_i \in \mathcal{Y}_t\}. \tag{23.51}$$

Then an effective δ which producer j will acquire is stochastically determined by the *beta distribution*:

$$g(\delta) = \frac{\Gamma(\tilde{P} + \pi)}{\Gamma(\tilde{P})\Gamma(\pi)} \delta^{\tilde{P}-1}(1-\delta)^{\pi-1}, \tag{23.52}$$

where $0 \le \delta \le 1$. Under the *beta* distribution, the skewness of the distribution is used to characterize the *quality of learning*, and it is determined by the interplay of the tuition ($\tilde{P}$) paid by producer j and the potential value (π) of the knowledge which producer j attempts to acquire. One concern is that the size of $\tilde{P}$ and π may not be directly comparable. If that happens, a normalization procedure should be applied to them before using Equation (23.52). Once the δ value is determined, producer j can acquire one of the modular designs whose associated commodity is $Y_{m,d'}$, where $d' = \lfloor \delta \rfloor$.

23.5 ModularEcon: a demonstration

The interaction between consumers and producers drives the evolution of a series of new products (the innovation process), as shown in Figure 23.11.

The idea presented above has been written into a computer program called ModularEcon, which stands for "Modular Economy."[11] In this section, we demonstrate a *vanilla* version of ModularEcon. What we mean by *vanilla* will become clear as we give the demo.

First, as in all computational economic models, we start with a simple description of *initialization*. The initialization in ModularEcon includes the generation of preferences. As shown in Figure 23.12, the preferences of three consumers are randomly generated. The complexity of the preferences has been severely restricted to a depth of five and is fixed throughout the entire evolution.[12] Notice that these

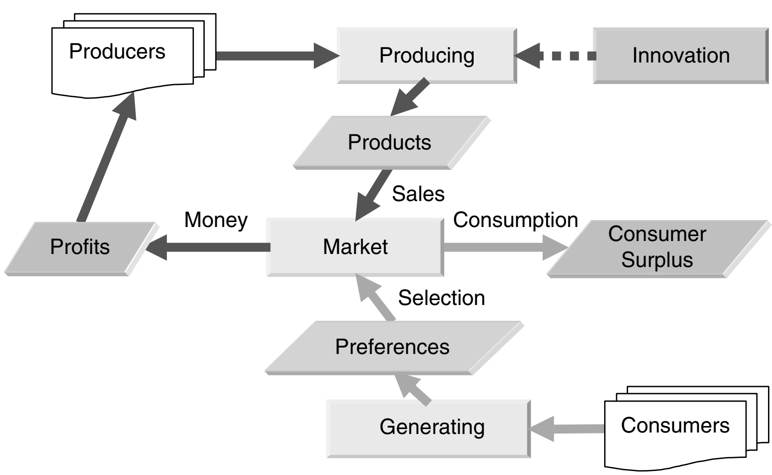


Figure 23.11 The agent-based modular economy.

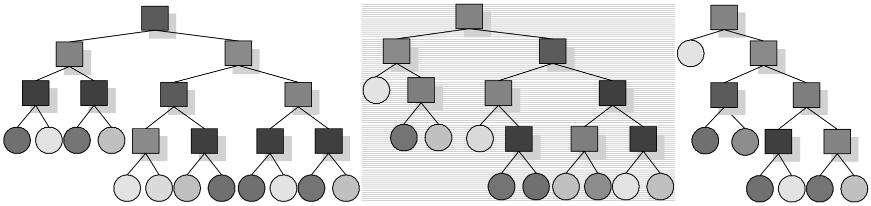


Figure 23.12 ModularEcon: Preferences initialization.

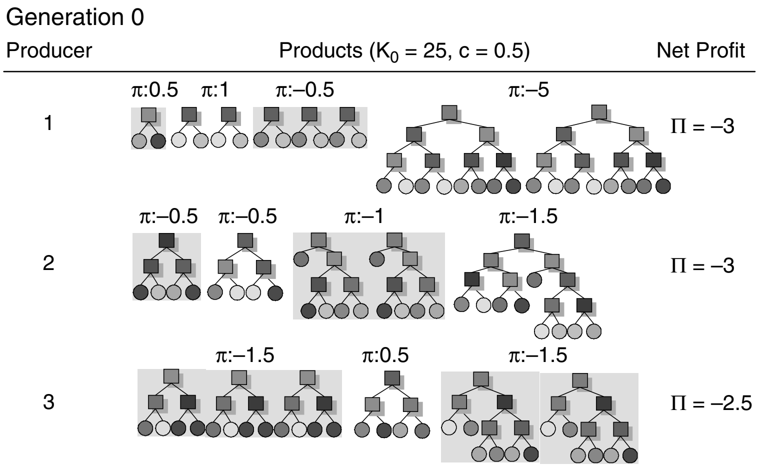


Figure 23.13 ModularEcon: Generation 0.

preferences are characterized by colorful nodes. Each different color is referenced to a different primitive, sampled from a given primitive set.

In addition to the three consumers, there are three producers in the economy. Based on the same given primitive set, commodities are also randomly generated by these producers, as shown in Figure 23.13. Notice that the three dimensions of production behavior, i.e. quantity, quality, and diversity, are all randomly determined as long as they together satisfy the capacity constraint (23.27). The initial capital capacity K_0 is set to 25, and the unit cost c is set to 0.5. The scale effect is ignored. Each of the commodities is then served to the consumers whose preferences are displayed in Figure 23.12.

Without the details about how trade actually proceeds, it is not easy to describe how the final price and hence the profit is determined. Therefore, in this vanilla version of ModularEcon we simply take the highest reservation price (Equation 23.41) as the market price. This simplification will facilitate our calculation of profits, π. In Figure 23.13, π is shown at the top of each commodity. We can see that most commodities have negative profits. This is not surprising because at this initial stage all commodities are randomly designed, and the chance of meeting consumers' needs to any significant degree is naturally low with the given combinatoric complexity.

Sophisticated commodities may be even worse than those simple designs because they induce higher production costs, and can only satisfy consumers to a very limited degree. So, as evidenced in Figure 23.13, commodities that suffer great economic losses tend to be those with sophisticated designs, i.e. trees with a large degree of node complexity. By summing the profits over all commodities, we get aggregate profits for each firm, which are shown on the right-hand side of the figure. In this specific case, all three firms make a loss.

Moving to the next market period (next generation), firms start to learn from *experience*. Their profit profiles provide them with fundamental clues on how to redesign their products for the next generation. What is shown in Figure 23.14 is the result of their adaptation. It is interesting to notice that these new-generation commodities seem to become simpler compared with those of the previous generation (Figure 23.13). This is mainly because sophisticated designs do not contribute to profits but losses. Therefore, firms tend to replace those sophisticated designs with simpler ones. The economy as a whole can be described as a *quantity-based economy*, since all firms choose to produce a large number of rudimentary commodities (i.e. they repeat doing simple standard things).

However, this strategy turns out to work well. While each simple commodity can earn a firm a tiny profit, summing them together is still quite noticeable. So, in the end, the profits of all three firms improve quite significantly. This process is then further reinforced, and in the coming generations more resources are devoted to rudimentary commodities. Sophisticated designs are almost entirely given up. However, since there are not too many rudimentary commodities to develop in the market, when all firms concentrate on producing rudimentary commodities, the limited number of rudimentary-commodity markets become highly competitive,

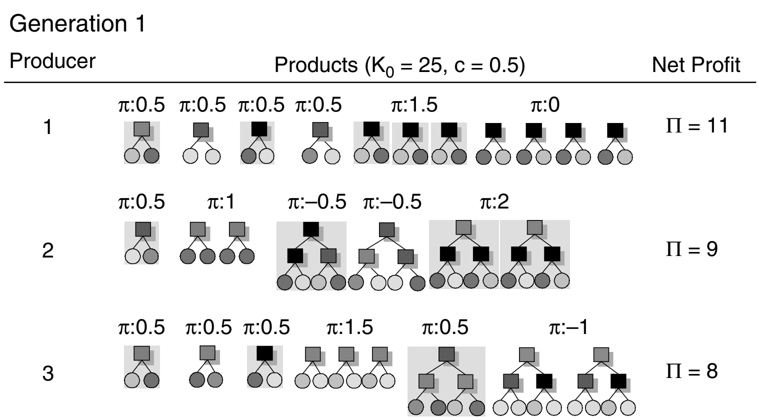


Figure 23.14 ModularEcon: Generation 1.

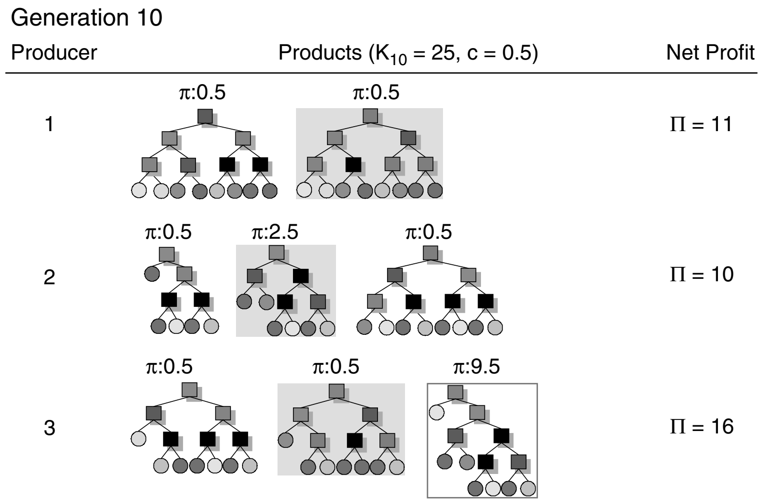


Figure 23.15 ModularEcon: Generation 10.

and the profits from producing these commodities decline as a result of the keen competition. At this point, the economy actually moves toward *an era of zero profit*.

Once producing primitive commodities is no longer profitable, the selection bias towards it also becomes weaker. Some sophisticated designs occasionally coming out of the crossover and mutation operators may find it easier to survive. That improves the chances of satisfying consumers to a higher degree. When that indeed happens, not only do firms make a breakthrough by successfully having a sophisticated (delicate) design, but the lucrative profits also attract more resources that can be devoted to quality products. While this does not always happen and the process is not always smooth, the process may be reinforcing. So, commodities with more delicate designs and higher profits may come one after the other. In the end, the economy is gradually transformed into a quality-based economy, as shown in the 10th generation of our simulation (Figure 23.15).[13]

23.6 Innovation

The modular economy differs from conventional economic analysis only in its cores: production, commodities, and preferences. The essential ingredient is a modular structure of these basics. Other fundamental concepts of conventional economic analysis remain largely unchanged but have to be adapted to be compatible with the modular economy. On the part of consumers, what is required is new thinking on the utility function, toward a quality-based utility function. The numerical algorithm, i.e., the module matching algorithm, which behaves like conventional utility functions, enables us to generate cardinal utilities in the new context. On the part of firms, we explicitly enlarge firms' dimension of decisions.

In addition to the conventional quantity-based decision, the firms have to also take into account a diversity of products as well as their qualities. Given capacity constraints, these decisions are interdependent.

Other strategies, such as advertising and research and development, also remain as usual. Nonetheless, the modular structure motivates us to a formal model of the discovery (R&D) process. Firms can discover on their own, but they can also imitate from others. For the latter purpose, a knowledge market is introduced, which not only gives knowledge a formal measure, but also explicitly addresses royalties, patents, and intellectual property rights. In fact, the modular economy is a representation of a knowledge-based economy or innovation-based economy. It offers a measure of innovation, and can simulate the dynamics of innovation. Innovation in this context can be understood as the discovery of hierarchical modules.

In this section, we move further to address innovation in the modular economy, namely, how to make firms have the capability to discover critical modules in the production and innovation process. While using a parse tree as the modular representation, simple genetic programming is not good at discovering and maintaining a modular structure. The standard crossover and mutation can easily destroy the already established structure, which may cause the whole discovery or learning process to be nonincremental and nonprogressive, and hence very inefficient. This problem is well known in the genetic programming literature, and has been extensively studied with various treatments (Angeline and Pollack, 1993; Koza, 1994; Rosca and Ballard, 1994; Hoang *et al.*, 2007). Motivated by these earlier studies, Chen and Chie (2007) propose *automatically defined terminals* as a way to enhance GP to find modular-structured products. Below we will review this work.

23.6.1 Automatically defined terminals

An automatically defined terminal, as shown in Figure 23.16, is very similar to the automatically defined function (ADF; Koza, 1994). It has a fixed structure, in this case a tree with a depth of two. The root of an ADT can be any function from the primitives (function set), while its leaves can be either a terminal from the primitives (terminal set) or can be any existing ADT. In this way, it shares the same spirit as an ADF, namely *simplification*, *reuse*, and *encapsulation*. The last

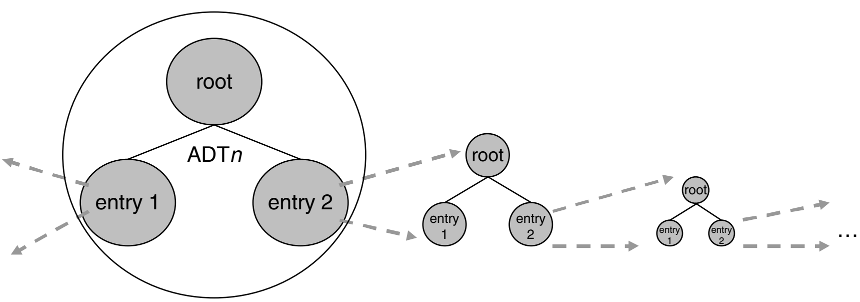


Figure 23.16 Automatically defined terminals.

item is particularly important because it means that whatever is inside an ADT will not be further interrupted by crossover and mutation. In this way, ADTs can be considered to be the part of learning in which we have accepted them as routines and will implement them automatically. Through ADTs we distinguish what is considered to be *knowledge* from what is still in a trial-and-error process. Only the former can then be taken as the building blocks (modules), but not the latter. Without ADTs or equivalents, simple genetic programming is essentially not designed to develop building blocks; therefore, it is not very good at finding the modular structure inherent in the problem.

23.6.2 Modularity and consumer satisfaction

To see the significance of modular structure in production, Chen and Chie (2007) considered an economy composed of one monopoly firm and a group of heterogeneous consumers with the usual objectives. To maximize profits, the monopoly firm has the incentive to search for products which can satisfy consumers to the highest degree and hence to induce their highest willingness to pay. In the meantime, consumers allocate their limited budget to commodities which provide them with the greatest degree of enjoyment (measured in terms of the consumer's surplus).

In their simulation, there are 100 consumers in the market. Each consumer has a preference tree with a depth of six. Viewed from the top-most level (the root level), the preference tree is composed of two modules. The one on the left, having a depth of five as shown in the first row of Figure 23.7, is identical among all consumers, whereas the one on the right, having a depth of five or less, is heterogeneous, and is randomly generated by the ramped half-and-half method, an initialization method frequently used in GP. In this way, consumers' preferences have a common part as well as an idiosyncratic part. For the idiosyncratic part, the complexity is also different.

A profit-maximizing monopoly firm will try to serve the needs of this group of consumers by producing different products with different quantities and also with different degrees of specialization or diversification (customization). The firm has to learn the consumers' preferences and hence, through R&D (driven by GP), design better products. The entire market process is summarized in Figure 23.17. The learning cycle (GP cycle) is run with a number of generations (in their case, 5000 generations or 5000 trading days). Each generation (trading day) is composed of a number of trading rounds (in their case, five). After each learning cycle, the firm has to decide what to produce, including some new products developed via production innovation, how many to produce, and how much to charge. The decision regarding production and R&D is based on the sales and profits statistics collected on the previous market days. The firm then supplies what has been produced, including those new items, during the next market days.

For further analysis, in each generation, statistics regarding consumer satisfaction are reported. Consumer satisfaction is measured by the actual utility the consumer received from consumption divided by the maximum potential utility the consumer can possibly gain given his preference. The ratio is then multiplied

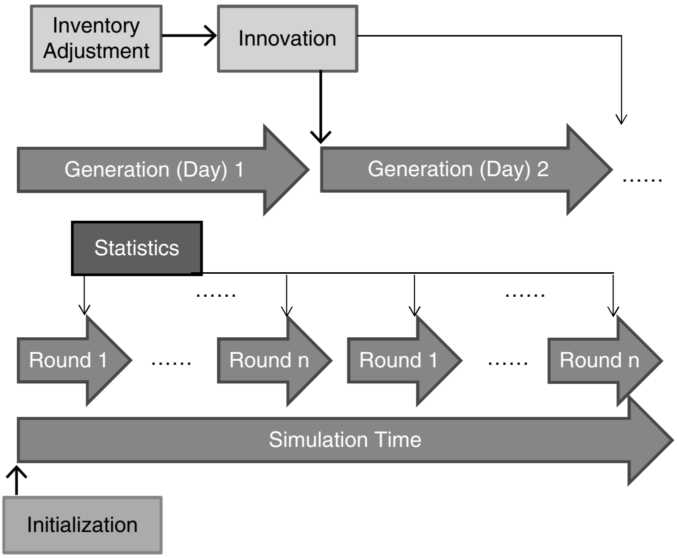


Figure 23.17 Market days and learning cycles.

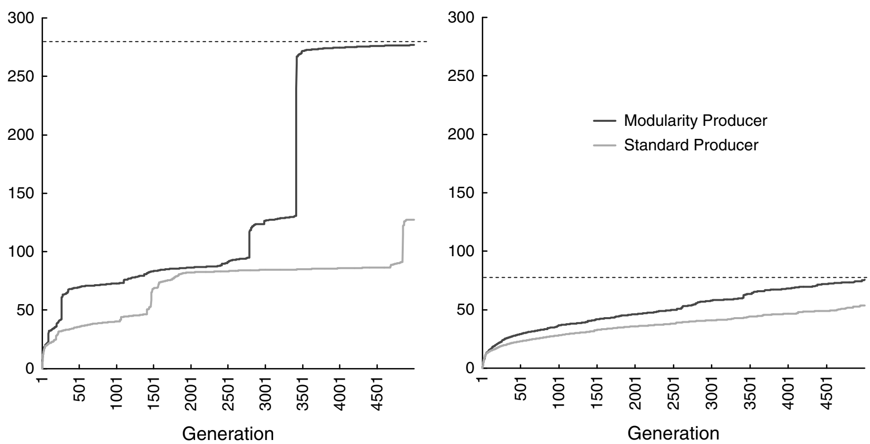


Figure 23.18 Modularity and consumer satisfaction.

Notes: Average consumer satisfaction index under modular production and standard production is shown based on 50 runs of each economy. The left panel shows the *maximum* of the average consumer satisfaction over the 50 runs, whereas the right panel shows the *average* over the 50 runs.

by 1000, and the measure lies in [0, 1000]. By averaging the consumer satisfaction over all 100 consumers, they then derived the aggregate consumer satisfaction, which also lies in the same interval. The result is shown in Figure 23.18, which shows not the result based on a single run, but on 50 runs. Accordingly, what is shown in the right panel of Figure 23.18 is the average of the 50 runs, whereas what is shown in the left panel is the maximum of the 50 runs. It can be seen quite easily that the firm whose product design uses a modular structure can satisfy the consumers to a higher degree than the firm whose product design uses a non-modular structure.

23.6.3 Modularity and competitiveness

In the previous section, under the assumption of a monopoly firm, we have seen the positive impact of using a modular structure on consumer satisfaction. In this section, we shall pursue this matter further by inquiring about the implications of a modular structure for the competitiveness of firms. In a sense, we attempt to re-examine the story given by Herbert Simon on the competition between two watchmakers: one using a modular structure and one not.[14] For that purpose, we consider a duopolistic competition in which one firm uses a modular structure in her R&D (new product designs) and the other firm does not.

Chen and Chie (2007) simulate two duopolistic firms competing with each other in a market composed of 100 consumers whose preferences are partially identical and partially idiosyncratic, as described in Section 23.6.2. They then watched the market share of the two firms, i.e., the total sales of each firm divided by the total sales of the market, and the results are displayed in Figure 23.19.[15] The results presented here are not based on a single run, but on 100 runs. The left panel of Figure 23.19 shows the mean of the 100 runs, whereas the right panel shows the median of the 100 runs. Below the separation line is the market share owned by the non-modular firm, and above the line is the market share owned by the modular firm. Clearly, their sum equals 100 percent.

Due to the existence of outliers, the time series behavior of the mean and that of the median is not quite the same, but the eventual dominance of the modular firm is evident. In the first few hundreds of generations, the non-modular firm, however, did have some competitive advantage over the modular firm. This is because establishing modules is costly. The idea of encapsulation associated with ADTs implies a fixed cost and hence a lower degree of mobility, depending on the degree

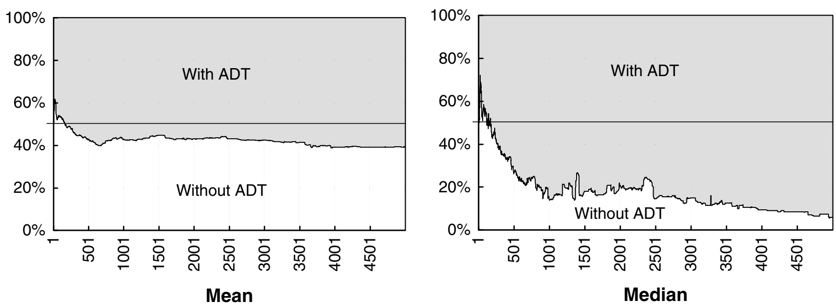


Figure 23.19 Modularity and competitiveness.

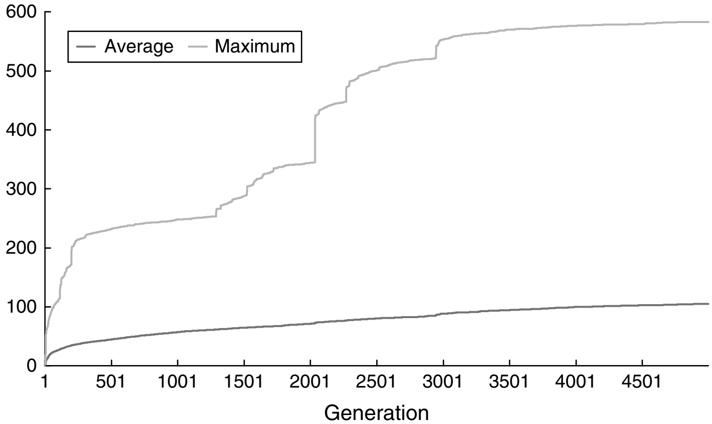


Figure 23.20 Consumer satisfaction under competition.

of encapsulation or the complexity of ADTs.[16] Hence, the modular products will generally be more expensive. Unless these products can fit the consumers' needs to a higher degree, these high-priced products will have an adverse influence on marketing. Therefore, there is no guarantee (a probability of one) that the modular firm will always beat the non-modular firm. In fact, in 39 out of their 100 runs, the non-modular firm achieved a higher market share than the modular firm in the last generation (the 5000th generation).

Finally, as one may expect, competition does bring better quality to consumers. This is reflected in Figure 23.20.

23.7 Further explorations

The agent-based modular economic model proposed in this chapter belongs to the agent-based models of innovation or technological changes. Dawid (2006) has a survey article in this area.

The agent-based modular economic model has also been applied to other issues related to competitiveness and innovation. Chie and Chen (2010) study how a firm's culture in terms of facilitating social interactions can help innovation. By manipulating the parameter "crossover rate," they were able to distinguish a high social-interaction firm from a low social-interaction firm. The two firms compete in a duopoly market in a setup very similar to that of Chen and Chie (2007). It was found that the high social-interaction firm has a high probability (up to 61 percent) of driving out the low social-interaction firm. They also went through the market share dynamics run by run, altogether 100 runs. They noticed the catastrophic nature of these market share dynamics: a sudden movement made by firms results in their quick triumph or extinction. The market share of firms can move to one or zero in a rather short span of time regardless of its current position.

The aforementioned modular economy is also applied to the study of price competition or non-price competition. Chie and Chen (2013) begin this study with a duopolistic competition market. There are two firms; each one applies a uniform markup rate to set the prices of all of the products they supply. One firm adopts a high markup rate and the other adopts a low markup rate. The research aims to see whether price competition is critical for the firms' survival in this duopolistic competition. Through multiple runs (100 runs) of the simulation, we find that the high markup firm has competitive superiority as compared to the low markup firms. Out of these 100 runs, the high markup firm actually drives out the low markup firm 50 times, while the process also takes place in the other direction 37 times. Despite the lack of overwhelming dominance, the result is counterintuitive, because, in a weak sense, it has violated the law of one price, or at least people would expect the result to be expressed the other way round.

Based on their observation of the simulations, Chie and Chen (2013) proposed one explanation for this puzzling result. Basically, in the modular economy, the competition among firms does not just proceed in a horizontal way, but more in a vertical way. The sophisticated products (the more hierarchical ones) may be substitutes for the less sophisticated products (the less hierarchical ones) if the former can match the consumers' modular preference at a higher level. In this case, the former can generate a larger synergy effect than the latter and, hence, can provide consumers with a higher utility. Nonetheless, to successfully develop this product, the firm needs a higher markup in order to generate enough revenue for R&D. Although this may not explain the whole story in the complex evolving economy, it provides the basic justification for the survival of the high markup firm in the presence of a low markup firm as its rival.

Chie and Chen (2014) extend the simulation model in Chie and Chen (2013). While the previous study shows the superiority of a high markup strategy, the simulation was performed with a fixed pair of markups. In a more realistic setting, the firm may change its markup rate in response to its competitor's strategy under the survival pressure. This may trigger further reactions of the rival firm. This chain of reactions can lead to different sorts of dynamics, and it is not clear a priori which one will emerge. Would there be a fixed point in the end, so that only one markup can exist in the market and the law of one price is saved in this sense? Or, would there be limit cycles or strange attractors, so that the differences in markups can persistently exist? Chie and Chen (2014) show the possible attractors of the coevolving markup dynamics, which shed light on the interesting patterns observed in the empirical industrial organization literature (Aguirregabiria, 1999; Pesendorfer, 2002; Atkinson, 2009; Nakamura, 2011). For that purpose, Chie and Chen (2014) also give some thought to the discovered attractors. In sum, this work extends Chie and Chen (2013) by allowing dynamically evolving markups and then examines the role of price in duopolistic competition.

Notes

1 Simon may never have used the term modularity, while his notion of near decomposability was frequently renamed modularity in the literature (Egidi and Marengo, 2004), and Simon did not seem to object to this different name (Callebaut and Rasskin-Gutman,

2005). Of course, one has to pay particular attention to the fine difference between the two. In particular, modules, as manifested in many concrete applications, may be viewed as only parts of near-decomposable systems. They serve as the elementary units, which are fully decomposable and fully encapsulated and upon which near decomposable (weakly interacting) systems can be built.

2 Herbert Simon viewed hierarchy as a general principle of complex structures. Hierarchy, he argued, emerges almost inevitably through a wide variety of evolutionary processes, for the simple reason that hierarchical structures are *stable*.

3 ADT here stands for automatically defined terminals, a technical representation of modular production. A formal introduction is to be given in Section 23.6.

4 Of course, from an ex ante view, knowing what to combine and in which way is not trivial. In fact, in this example, it took the firm five generations to learn this. In this sense, the contribution is not entirely *marginal*. However, from an ex post view, it is just a combination of what we already knew.

5 This constraint assumes that all working capital comes from the firms' own capital. One can certainly introduce a capital market and consider firms' decision on capital structure.

6 Suppose that a commodity, say Y_i, was sold with great profits in the last period (period t), and that that commodity was not supplied by producer j. However, $Y_i \in \sigma(\mathcal{Y}_t^j)$. In other words, producer j knows Y_i well, and, as a matter of a fact, Y_i was actually used as an input for at least one $Y_k \in \mathcal{Y}_t^j$, i.e. $Y_k \sqsupseteq Y_i$. Unless that Y_i is protected by a patent, there is no need for producer j to pay anything to replicate Y_i. If he still considers Y_i a profitable commodity in the next period (period $t+1$), he can simply go ahead and produce it without requiring permission. If what was just said applies to many other producers, then the commodity Y_i defines a *potentially* competitive market.

7 A technical issue may arise here. When what producer j acquired from the knowledge market is not a final good, but just a module which was involved in the production of other commodities, the profit figure of this module may therefore not be available. In this case, an estimation of the profit, $\hat{\pi}_{e,t}^j$, shall be used instead.

8 As we have already mentioned, not all new commodities can be taken as "innovation." In the real world, this new commodity may differ from the existing commodities in a *trivial* way, and it either does not help to earn lucrative profits, or dies very quickly. Also, a "new" commodity may not really be new. It can be a rediscovery of an old commodity which has already been forgotten or discarded by the market. To have a more accurate number of inventions, criteria must be carefully established.

9 At this point, we are not specific on the *date* on which $Y_{j,m}$ is finished. It may be brand new, or it was unsold before and hence is put in the shop again. Certainly, taking an *inventory* into account would give rise to a few more complications in our computer simulation of the economy, but would not hurt our understanding of the operation of this economy.

10 $\tilde{\mathcal{Y}}_t/\mathcal{Y}_t^i$ is only a convenient device for the determination of the *learning pool*. With other decision rules, producers may be interested in something different from $\tilde{\mathcal{Y}}_t/\mathcal{Y}_t^i$. For example, producers may further restrict the learning pool to only those technologies whose associated commodities have earned a profit higher than a threshold. In this case, they are looking for something more specific. At this moment, we make no attempt to explore the rich possibilities of these *shopping rules*.

11 ModularEcon is available on the website for open-source code for agent-based modeling, http://www.openabm.org/. The software is written using Java and is compiled with Borland JBuilder 6. It is implementable under the DOS environment. Parameters are specified in the file innvo.ini, and innvo.jar is the execution file. Various outputs regarding households and firms are written in Excel files. As a simulation of the evolution of an economy, the modular economy is very computation demanding. Depending on the given complexity of consumer preferences and the endowment, it may take an Intel i5 four-core processor 10 hours for a run with 5000 market days.

12 In other words, we do not consider the general preferences as discussed in Equation (23.8), nor the evolution of preferences, in this vanilla version.

13 The progress may not be smooth. Severe fluctuations can happen. The progress may not be sustained for long enough either. The economy may stagnate after a short but fast take-off, and consumers are only supplied with some "basic needs." For a more detailed demonstration of the complex variety, see Chen and Chie (2004b).

14 To demonstrate the importance of a *hierarchical structure* or *modular structure* in production, Simon offered his well-known story about a competition between Hora and Tempus, two imaginary watchmakers (Simon, 1962). In this story, Hora prospered because he used a modular structure in his design of watches, whereas Tempus failed to prosper because his design was not modular. Therefore, the story is mainly about a lesson: the advantage of using a modular structure in production.

15 Notice that firms generally produce more than one product which can be very different from each other. Therefore, it is meaningless to measure the market share based on a single product.

16 To define and measure complexity, Simon (1965) advocated the use of a hierarchical measure—the number of successive levels of hierarchical structuring in a system or, in our case, the depth of the parse tree.

24 Epilogue

In this final chapter of the book, we shall conclude our ACE journey with both a retrospect and a prospect. After almost two decades of development, agent-based modeling has still not been well accepted by mainstream economists (Lehtinen and Kuorikoski, 2007), but we can still see some major progress along the journey.

First, through agent-based modeling and social simulation, the "social" mobility of economists has increased. As we have seen in Chapter 2, social simulation has reshaped and reorganized the network topologies of social scientists. Traditionally, conversations between economists and their colleagues from sister disciplines have existed, but often in the form of economic imperialism (Maki, 2009). That is a *directed graph*. However, a new network of social scientists has emerged with the advent of agent-based modeling and simulation. Although our identity as economists remains unchanged, we have now also won back another identity, namely that of a social scientist. We are not providing a cliché. A good agent-based economic model frequently requires knowledge from our sister disciplines; in the meantime, for our "allied colleagues," it is easier to comment on the agent-based models built therein than the usual equation-based models. Hence, the network now becomes *undirected*.

Second, within the economics network, ACE is not an isolated vertex. Over the last ten years constant progress has been made on the research joining ACE and experimental economics. One of the most prominent examples is the *learning to forecast experiment* (LtFE) (Hommes, 2011). While LtFE is basically another series of examples demonstrating that ACE can be used to replicate the results observed in experimental economics, it differs from the earlier ACE replications of EE in various scales. In this LtFE, what ACE provides is not just about replication, but a systematic treatment and explanation of the whole set of LtFEs (Anufriev and Hommes, 2012a, b). With these consolidations, it has already *scaled up* the learning-to-forecast experiments and has brought us to a new research territory (Hommes and Lux, 2013). While ACEers (agent-based computational economists) have long realized the value of experimental economics to their work, relatively few experimental economists have been able to appreciate the value of simulation to their work. However, since the use of the *strategy method* and *robot players* (programmed players) in experimental economics has a long tradition

(Duffy, 2006), from those standpoints, it may be only a matter of time for them to see this potential connection.

Third, in terms of dialogues with policy-makers who are not well equipped with analytic machines, it is easier to communicate with them through an agent-based simulation model as a toy model in which they may surprisingly find their incarnations (avatars) (Lempert, 2002; Colander and Kupers, 2014). While this potential has not been realized, by advocating for universal literacy and with the growth of social simulation platforms, such as NetLogo, this is something we can expect.

With these promising features that face us, the rest of this chapter will focus on the challenges that lie ahead for ACEers. There are a number of challenges that have been well discussed in the social simulation community, in a general sense, and in agent-based computational economics specifically, such as validation. We will not reiterate these challenges, except those already mentioned in Chapter 16. These challenges are well known and we have reason to expect some substantial progress in answering them in the years to come. However, there are other challenges less known and less discussed, but they are probably more fundamental and a longer period of time may be required to make a breakthrough. We believe that if agent-based modeling can become a new page in the history of economics, it is not just because the validation problem has been successfully handled or the agent-based model can be built upon solid empirical ground. Such progress is, of course, nontrivial, but compared to the challenges to be discussed in this chapter, it reflects only warm-up preparations for the forthcoming major battles.

So, what are these challenges? From a computation-theoretical viewpoint, the challenge is: can we have an agent-based model which can demonstrate universal computation as that performed by a universal Turing machine? From a simulation viewpoint, can we have an agent-based model which can generate evolving hierarchical (multilevel) systems? These two challenges are probably the two sides of the same coin, one on modeling and one on simulation. We shall begin with simulation and present the challenge in terms of the *Simon hierarchy* (Section 24.1), and then move to modeling and present the challenge in terms of the *Chomsky hierarchy* (Section 24.2).

24.1 The Simon hierarchy

Agent-based modeling has long been considered a third way to do science in the direction suggested by Charles Peirce (Peirce, 1997) and Robert Axelrod (Axelrod, 1987)—Section 2.1.3—and probably in a novel way, if not the only way, to study complex adaptive systems as a "new kind of science" (Wolfram, 2002). However, even so, we are sometimes not fully aware of our theoretical surroundings, and many times we seem to get lost in a number-crunching jungle. Hence, while agent-based models are normally taken as manifestations of complex (adaptive) systems, an issue often ignored is: is our understanding of agent-based models consistent with what we may learn from the existing literature on complex systems?

For example, the book entitled *Complex Adaptive Systems* (Miller and Page, 2007) is a great review and collection of many early agent-based models; nonetheless, the celebrated work done by Herbert Simon on complex systems, i.e., *The Architecture of Complexity* (Simon, 1962), is not mentioned in that book. Hence, the connection between ACE as a study of a complex adaptive economic system and the pioneering work done by an early economist is not made, even though the two should not be unrelated.

John Davis (Davis, 2013) recently makes an attempt to place agent-based models in the context of Simon (1962). Using what is called the *Simon hierarchy*, his work prompts us to reflect upon agent-based models as complex adaptive systems. The essence of Davis (2013) is two-fold. First, it is to place agent-based models in Simon's framework of complex (adaptive) systems, namely *Simon's evolving hierarchy*. Second, it uses this Simon-hierarchical version of agent-based models to introduce a novel notion of stability, changes, and (financial) crises.

The second part itself can be an invitation for ACEers to think of a more challenging model of a financial crisis through the coevolution of rules, of agents as collections of rules, of groups as collections of rules and agents, of groups of groups, and so on. In brief, Davis proposed a framework of a coevolving hierarchy for analyzing a financial crisis. The idea of using the coevolving hierarchy to explain emergent macroeconomic phenomena is related to the *micro–meso–macro analysis* proposed by Dopfer and Potts (2007) and the *markomata system analysis* initiated by Mirowski (2010). In this chapter, we shall focus on the first part only. The second part seems to be more interesting to economists, but the corresponding agent-based model, to the best of our knowledge, does not exist. Therefore, it becomes part of the challenges presented in this chapter.

24.1.1 Near decomposability

In Simon (1962), near decomposability is an important characterization of a complex adaptive system. Near decomposability refers to a structural relationship between a system as a whole and the constituent components which can function as independent entities. The interactions of the elements within the same constituent component are strong; however, the interactions of elements across different constituent components are weak, but not zero. The relevance of the near-decomposable property to the agent-based model has been vividly depicted by Davis (2013):

> Suppose market economies are self-organizing, complex adaptive systems made up of many interacting self-organizing collections or groups of human individuals, including firms, groups within firms, groups of firms, and also other more complicated combinations, and with all exhibiting continually changing memberships. In Simon's terms, these groups' continually changing memberships could be said to reflect one type of *weak interactive,*

cross-subsystem, low frequency force, associated with the circulation of individuals across groups via labor markets and administrative systems. The self-organizing nature of groups of individuals themselves could be said to ref ect his *stronger within-subsystem, high frequency force*, and associated with how various kinds of groups function as groups. The system would be hierarchic in Simon's sense in that *groups of individuals contain groups of individuals, which also contain groups of individuals, and so on*, all of which are seen as self-organizing in terms of his *strong within-system force*. All would also exhibit Simon's weak interactive force associated with changing memberships, as individuals circulate simultaneously across groups on multiple levels, having and changing many kinds of sometimes linked and sometimes unlinked memberships.

(Simon, 1962, p. 240–1; emphasis added)

In light of this characterization of complex adaptive systems, the immediate issue is whether the current state of agent-based economic models can demonstrate this feature. The very general picture described by Davis (2013) may sound familiar to ACEers. To conf rm this familiar picture, Figure 24.1 provides a snapshot of the near-decomposable hierarchical system. This near-decomposable system can also be displayed in the form of networks and bears some resemblance to the core–peripheral structure frequently seen in economic and social networks.

While there are some agent-based models devoted to replicating the core–peripheral network topology, demonstrating the core–peripheral structure does not automatically imply a near-decomposable hierarchical system because the vertices

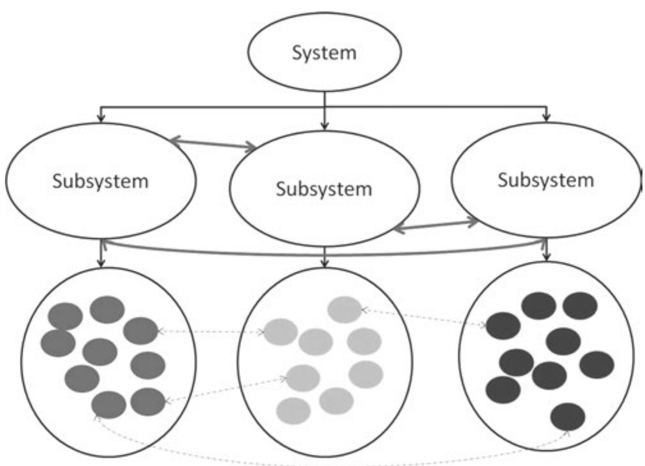


Figure 24.1 The Simon hierarchy and near decomposability.

Note: A pictorial demonstration of the Simon hierarchy and near decomposability. The relationship between subsystems at the same level means that elements of the same color (belonging to the same subsystem) have a strong-tie relation (Granovetter, 1973), whereas elements of different colors (belonging to different subsystems) have a weak-tie relation, indicated by the dotted lines.

of the former are normally at the same level (all agents or firms or banks, etc.), whereas the vertices of the latter are not (individuals, firms, unions, cartels, conglomerates, supply chains). In other words, entities such as groups (networks, societies) of groups (networks, societies) of individuals are rarely seen in the current agent-based models of economic and social networks.

In fact, a comprehensive review of the ACE literature may find that most agent-based models developed so far are confined to only two levels, such as the individual–market hierarchy and the firm–market hierarchy (Tesfatsion and Judd, 2006; Miller and Page, 2007; Railsback and Grimm, 2010). The recent development of agent-based macroeconomics (Section 3.4) does provide a three-level hierarchy, individual–market–aggregate. Nonetheless, even in this case, these levels are given exogenously. For example, firms are not endogenously formed through individuals. Hence, by and large, it may be fair to say that we have not seen agent-based economic models as being able to demonstrate evolving hierarchical near-decomposable systems as an essential characterization of complex adaptive systems.[1]

24.1.2 Is ACE near decomposable?

One can, of course, inquire whether the agent-based model interpreted in the vein of computational irreducibility (Wolfram, 2002) is consistent with the pursuit of reducibility (tractability, comprehensibility), which actually motivates the idea of near decomposability in Simon's notion of complex systems. Alternatively, one can ask whether, should the emergent dynamics generated by agent-based models be largely computationally irreducible, it is still possible for us to have a near-decomposable hierarchy to harness its operation. Are these two concepts necessarily in contradiction to each other, or can they be reconciled?

> In studying the interactions of the two nations, we do not need to study in detail the interactions of each citizen of the first with each citizen of the second.
>
> (Simon, 1962, p. 477)

Nonetheless, for ACEers, the modeling philosophy is that if we are interested in the interactions between the two nations, we will tend to model each citizen in each country, while, of course, not in great detail. One of the best illustrations is Epstein and Axtell (1996), who simulate the "big history" of two civilizations. In their model, everything between the two civilizations emerges from the interactions of individuals. Arifovic (1996) is another example (see also Section 6.3); in the context of an international economy, exchange rates are generated by behavior based on the interactions of individuals in both countries.

Simon, nonetheless, also stated:

> The hierarchic structures we have been discussing have *a high degree of redundancy*, hence can often be described in economical terms.
>
> (Simon, 1962, p. 478; emphasis added)

In the same paragraph as the above quotation, he further added: "If a complex structure is completely unredundant . . . then it is its own simplest description. We can exhibit it, but we cannot describe it by a simpler structure." In fact, it is probably the second part of the statement which is more familiar to ACEers. For example, as mentioned in Vriend (1995), "we are interested in those regularities that cannot be deduced from the built-in properties of the individual agents or some other microeconomic aspect of the model; at least not by any argument which is substantially shorter than producing that regularity by running the simulation itself" (p. 212). Then, as a footnote to this quote, Vriend added:

> Clearly, the emergent behavior and self-organization *are* a function of the underlying configuration. The relevant point is, however, the following. Given a certain model with a certain parametrization, can one reason, i.e., without running a simulation, *which* functions of the parametrization the outcomes are?
>
> (Vriend, 1995, p. 228)

At this point, maybe we can say that the agent-based models constructed by ACEers are not entirely the same as the complex adaptive systems considered by Simon. Specifically, whether the agent-based models satisfy the near-decomposable property is endogenously determined. However, this issue is not critical because, as we mention in Section 24.1.1, agent-based models which are able to simulate an endogenous hierarchical structure are rare. Hence, the challenge for ACEers is to simulate an endogenous hierarchical structure and then show whether the hierarchical structure is necessarily near decomposable.

24.1.3 The Simon–Davis criterion

Agent-based models, as a bottom-up modeling approach, should definitely start from the bottom level, but should we start from individual agents, neurons, cells, or genes? Davis (2013) makes it clear that we need a basic unit, as a starting point, for agent-based modeling. Based on Simon's hierarchical complexity theory (Simon, 1962), Davis proposes a criterion for such elementary units. This criterion, called the *Simon–Davis criterion*, requires the unit (bit) serving as the fundamental building block of agent-based models to be the system for which all subsystems, including all their subordinated elements, whether encapsulated or not, are *fully decomposable*, not even nearly decomposable, not capable of self-organizing by themselves, and not weakly interactive.

Simon does not directly indicate that the elementary unit for a hierarchical system needs to be *fully decomposable*.[2] Nonetheless, given the near decomposability which he introduced in his subsequent discussion, the following implication, as suggested by Davis, seems to be logical or compulsory.

> In Simon's terms, to say that some subsystems are basic agents and are made up of subsystems that are not self-organizing is effectively to say that the

> latter subsystems are "fully" decomposable rather than "nearly decomposable." That is, a fully decomposable subsystem is like a set of Russian dolls.
>
> (Davis, 2013, pp. 235–6)

Davis (2013) further suggests that since (behavioral) rules are fully decomposable, and hence they can be taken as the *elementary units* of agent-based modeling, everything beyond then emerges endogenously or is self-organized.

> The idea behind basic agents developed above is that they self-organize themselves around sets of behavioral rules that are fully decomposable in the sense that *these rules do not communicate across individuals* ...
>
> (Davis, 2013, p. 242; emphasis added)

By this criterion, individual agents, who are normally, even implicitly, considered to be the basis of agent-based models, do not satisfy the criterion, since agents obviously are interactive in agent-based models. Therefore, they are not taken as the basis (bits) of complex adaptive systems.

Taking rules as bits of agent-based models certainly has some attractive features, since rules provide the substantial contents of agents, both static and dynamic. Using rules to classify agents is also quite common in agent-based economic models, such as the H-type financial agents as reviewed in Chapter 14. In addition, as reviewed in Chapter 15, a class of agent-based economic models has already been built upon the idea of the evolving hierarchical rules using genetic programming. In genetic programming, rules themselves can be encapsulated as modules and can be arranged in a hierarchical order. Modularity is another important concept related to near decomposability. Simon actually introduced the significance of the idea of modularity by telling a story about the competition between two watchmakers (see Section 23.6.3). The essence of the story is as follows:

> systems that are nearly decomposable are much less vulnerable than systems that are not, as disturbances are more likely to remain confined to specific subcomponents: near-decomposable systems limit interactions and information flows among different parts of the systems and thus are better able to keep damaging events *stricto sensu*; it applies to all disturbances to the system that may feed adaptation and change.
>
> (Egidi and Marengo, 2004, p. 346)

Earlier, in Section 23.6, we have seen how the established routines (modules) will not be disturbed during the innovation process. Therefore, the elementary units of ACE are rules, but they have different forms, including collections of rules in hierarchical form.

For ACEers, it is not difficult to replace individual agents with rules as the bits. The main problem, however, is whether taking rules as bits satisfies the Simon–Davis criterion. In fact, there is little doubt that rules themselves are complex systems, as well demonstrated by Simon (1962), when he illustrated various symbolic systems, such as books, music, and other artworks, as complex systems. Nevertheless, without any theoretical underpinning, it is hard to comment on whether, and in what sense, they are fully decomposable. If all rules are expressed in terms of formal language theory, then one may assume that all grammars (production rules) already make the generated rules have some degree of interaction, unless we restrict our consideration only to the grammars per se. Even so, one still cannot disregard the possibility that these grammars will generate rules which revise these grammars and, in the end, not just self-replicate the system, but make the system evolvable.

24.2 The Chomsky hierarchy

> Before proceeding let me point out that there have been numerous successes in building simple dynamical systems capable of supporting computation universality—i.e., simulating the behaviour of a (universal) Turing machine. Modern practitioners of so-called agent-based economic models seem to be ignorant about this kind of result, which was, by the way, for the proto-typical agent-based model first developed by Ulam and von Neumann.
>
> (Velupillai, 2010, p. 339)

Let us leave the issue of the elementary unit aside, and, for convenience, assume that rules are perfectly decomposable. Then, as shown in Figure 24.2, let us use the Simon hierarchy rules to group the agents or, alternatively, define or characterize the agents by rules. For example, fundamentalists and chartists, as discussed in Chapter 14, are sets of rules. Many agents discussed in Chapter 9 are also sets of rules. Agents formed by different sets of rules may generally behave differently, and hence they are heterogeneous agents. Some of the rules will also determine agents' interactions, when they are situated in a social entity, such as games, workplaces, firms, and markets. These social entities are sets of some other rules; for example, posted pricing, auctions, and bilateral bargaining are all sets of rules which determine how the market is operated. In this way, each agent-based model can then be interpreted as a computing machine comprising a set of rules. In light of Davis (2013), we may be interested in whether a specific machine which we consider can evolve toward a Simon hierarchy; however, an alternative perspective to ponder on this machine is *what it can compute*.

Treating the observed natural phenomena as computational processes out of an abstract machine and asking what this machine can compute is not a new perspective when looking at an economy. It has been a long tradition by treating the market as a computer (algorithm). Goodwin (1951) states: "Therefore it seems entirely permissable to regard the motion of an economy as a process of computing the answers to the problem posed to it" (p. 2). Nevertheless, only in a few exceptional cases has this question been seriously taken and addressed on the grounds of the theory of computation (Velupillai, 2000, 2010; Bartholo *et al.*, 2009).

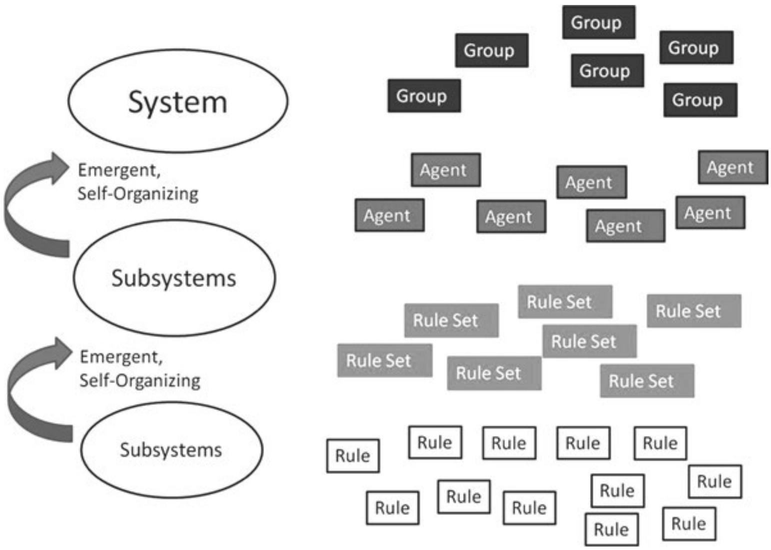


Figure 24.2 The Simon hierarchy and rules.

Note: The Simon hierarchy using rules or groups of rules as the elementary unit by assuming that rules are fully decomposable. Agents are defined as collections of rules. Agents, through interactions, self-organize into groups, such as organizations, firms, institutions, and markets. These groups are, therefore, collections of agents; in addition to the rules encapsulated within agents, they may have additional rules to define their operation.

In the theory of computation, a natural framework to taxonomize computing machines is the Chomsky hierarchy (Section 8.5.4). According to the Chomsky hierarchy, these computing machines (agent-based models), from low to high, can be taxonomized into finite state automata, push-down automata, linear-bounded automata, and Turing machines. A machine at a high level of the hierarchy can simulate one at a lower level of the hierarchy, but not vice versa. Up to the present, while there is a lot of interest in validating an agent-based model, few have asked whether the validated agent-based model is a Turing machine or just a finite state automaton. Alternatively put, to understand or to replicate economic phenomena, is it sufficient to have a finite state automaton or do we need a machine at a higher level?

The above question may be seemingly far-fetched for most pragmatists in the ACE community. Since their agent-based models as computer codes have already been implemented by computing machines, based on the Turing–Church thesis these models (functions) are of course Turing computable. However, the above quest is not just trivially about the computability of the model (the function) per se, but the nature of the model as an abstract machine. For example, in Section 4.2 we introduced Conway's Game of Life. The Game of Life is an agent-based model, but can also be taken as an abstract machine, and it can be shown that a Turing machine can be implemented by using Game of Life rules (Rendell, 2002; Abbott, 2006). This implementability then allows us to apply the results of computability to the Game of Life. For example, the property of being Turing complete (computationally universal) applies to the Game of Life. In addition, whether a Game of Life run ever reaches a stable configuration is undecidable because we know that the halting problem for the Turing machine is undecidable.

Let us consider our agent-based model (a set of rules) to be a Game of Life, also as an abstract machine, and assume that it is also Turing complete, in which case the above undecidable result applies. This result may interest ACEers, in particular, in the way they present their simulation results or "discoveries."

> Correspondingly, when viewed as a dynamical system, whether the global behaviour is an attractor, or is in a particular basin of attraction of the dynamical system, is algorithmically undecidable. Whether a set of initial conditions, for the transition function, can be algorithmically determined such that their halting state is the desired global behaviour, or such that the global behaviour is in the basin of attraction of the transition function as a dynamical system is decidable only for trivial sets.
>
> (Velupillai and Zambelli, 2011, p. 268)

However, if our "abstract machine" is not a Turing machine, then the above halting problem does not exist. Then the way in which we summarize our simulation results or discoveries may also be different.

This difference can be critical when we come to the aspect of the epistemology of agent-based simulation (Lehtinen and Kuorikoski, 2007; Grune-Yanoff and Weirich, 2010). One reason that economists shun simulation is that the simulation results are beyond the reach of their reasoning ability. They "are willing to incorporate less-than-fully-rational behavior if they are allowed to continue their mathematical theorem-building" (Lehtinen and Kuorikoski, 2007, p. 312). In addition to the cognitive harness, robustness is also an issue and has sometimes been pushed too hard. As Nannen (2009) has correctly pointed out,

> While it is possible in principle to assess the importance of any given parameter of a simulation model by running different simulations with one parameter fixed at a time, this is usually impractical because of the amount of computation required, the volume of the resulting data, and interaction between different parameter values. There is a lack of efficient statistical tools that can tell whether parameter values and simulation details are crucial for the results.
>
> (p. 18).

However, if the "abstract machine" is a universal Turing machine, then the issue becomes intrinsically difficult, and there is no guarantee of the efficiency of any statistical tools. Even though the "robustness check" becomes a routine of running an agent-based simulation, there is no formal algorithm to describe what this check actually is. Under what procedures and to what extent a "robustness check" can be considered as robust is generally unknown. For example, we generally do not know the required number of runs, the required duration of the runs, and the required number of agents for this check. If we are lucky, we obtain a few identical or similar results, and then we can draw a conclusion. That is the robustness check in practice. However, if the "abstract machine" is indeed a (universal) Turing machine, this kind of robustness check can be shaky. In general, one cannot be assured when the robustness check has been proper enough and when it has not.

Of course, that does not mean that messy data with unclear dependences are generously allowed. Experienced practitioners have accumulated valuable recipes that are available for us to learn, and which can help make the procedure neater and clearer (Helbing and Balletti, 2011). On the other hand, some "abstract machines" are just finite state machines;[3] their properties can be studied and even be proved as theorems in analytical models or "perfect models" (Lehtinen and Kuorikoski, 2007).

Hence, should we restrict our "abstract machines" only to finite state machines? A negative answer to this question introduces the second challenge. If the economic phenomena generated by nature are computationally irreducible, then we may suspect that the proper economic model corresponding to the economy should be a "machine" that is equivalent to the universal Turing machine or Class IV of Wolfram's elementary cellular automata or Conway's Game of Life. However, if the construction of the agent-based economic models is only used to support the equilibrium in analytical economic models or generate some new classes of equilibria, then it is not entirely clear how far we are actually away from the "mainstream" before sailing into the ocean.

The journey of agent-based economic modeling began with an exciting moment at the Santa Fe Institute. It had been one time when economists would have liked to have their time tunnel (agent-based model) and make the big history of the economy reappear. Hence, from a barter economy to a monetary economy, to a stock market, to a central bank, to labor unions, and to revolutions, the economy never settles down to some steady state;[4] instead, agents, markets, and institutions are constantly endogenously evolving. One may argue that was just a dream, but is the dream still alive or just wishful thinking?

The dream of the Santa Fe Institute in regard to agent-based models is one of emergent phenomena from a historical viewpoint. The steady state does not exist in the context of history. It is hard to imagine a history that can be characterized by an equilibrium. The finite state automata which generate a stationary description of history can only survive for a finite period of time. The so-called "long-term phenomena" generated by finite state machines may actually remain for some time, but eventually disappear as time moves on. Its disappearance or termination can be due to the shocks externally imposed on the finite state machine in question. However, a different perspective to look at is that the seemingly steady states may have inherent forces to bring in subsequent changes or unstable states internally, which eventually cause the change. This latter point is hard to reconcile within finite state machines.

24.3 Challenges ahead

To sum up, in this final chapter we have argued that there are theoretical and technical issues ahead of us in the agent-based modeling of economics. The notion of equilibrium familiar to economists has a property of computational reducibility. In terms of algorithmic complexity, one can eventually have a simple (brief) description to harness a long process without displaying the whole process itself. In these models, basically, there is nothing in the long run that we cannot know, at least qualitatively. This kind of thinking and modeling is certainly suitable for many small-scale economic phenomena, but when coming to large-scale economic phenomena such as the evolution of technology, institutions, cultures, economic history, and policy impacts, what we need is thinking and modeling which can generate economic processes with endlessly endogenous changes. This alternative thinking and modeling is exactly what agent-based models have the potential to accomplish but have not done enough to do so. This point should be viewed as a challenge instead of a criticism of ACE.

Notes

1 One may anticipate that agent-based models of organizations, specifically those focusing on internal organizations, may have some behavioral algorithms to form hierarchies endogenously. However, as Chang and Harrington (2006) have shown in their survey article, such work is rare, and this is still the case.

2 It should be noticed that Simon did not make any efforts to articulate a criterion to decide the elementary unit. As he said: "In most systems in nature, it is *somewhat arbitrary* as to where we leave off the partitioning, and what subsystems we take as elementary" (Simon, 1962, p. 468; emphasis added).

3 For an example, see Velupillai and Zambelli (2010). In addition, Mirowski (2007) has argued that an individual market, which he called a market automaton, can be a finite state automaton or a push-down automaton or a limited bounded automaton, but not a Turing machine, even though the emergent interactions (computation) of a network of markets may introduce the possibility of the emergence of a Turing machine. There are some heated discussions on this taxonomy; in particular, whether markomata can possess the power of a Turing machine (Zambelli, 2007).

4 See Waldrop (1992) for a vivid description.

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