**Converting Higher Order Tensors into Vectors and Matrices**

In MATLAB, we can convert a second order tensor into a (9×1) vector using the reshape( ) function where elements are ordered column-wise in each representation.



This approach allows for the computation of higher order tensor products by matrix multiplication. For example, the product becomes



Similarly, the product  becomes



In this way, the grand mobility matrix can be written as a (15×15) matrix

 .

**Inverting the Mobility Matrix**

Importantly, the grand mobility matrix  as written is singular and cannot be inverted to obtain the corresponding resistance matrix . This is due to the fact that the stresslet **S** and the strain rate **E** are symmetric and traceless; therefore, they require only 5 independent components to specify them

 and .

Therefore, we introduce the vector , which contains 5 linearly independent components derived by from the stresslet **S**

,

where **a** is a 5×9 matrix (to be determined). Additionally, the conditions imply that

,

where **b** is the following 4×9 matrix

.

Here, the coefficients are chosen such that each row of **b** is a unit vector. We can combine and to obtain the square matrix [**ab**]



Substituting this relation into equation , we obtain

,

which can be rearranged to give

.

We can maintain the symmetry of  by choosing the matrix **a** such that [**ab**] is an orthogonal matrix satisfying

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In particular, we find that the following matrix satisfies this condition

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Using this matrix, we compute the new symmetric mobility matrix as

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and retain only the first (11×11) components, which are now nonsingular.