PyLogic 101: Namibians Who Code
Boolean logic

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This cheat-sheet covers the conditional statement p implies q $p \to q$ and its negation. The **truth tables** below assist with the obtaining the *truth-value* of the associated logical of statement.

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Table 1: Truth table of conditional statment $p \to q$ and its negation.

The only time the negation of the conditional statement is true is when p is true, and q is false. This means that $\sim (p \to q)$is logically equivalent to $p\wedge \sim q$,as the following truth table demonstrates.

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$p \wedge \neg q$	$\neg(p \to q) \leftrightarrow (p \wedge \neg q)$
T	T	T	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	F	T	F	T	F	T

Table 2: Demonstrate via truth truth tables that $\neg(p \to q) \leftrightarrow (p \wedge \neg q)$.

Property	Conjunction (AND)	Disjunction (OR)
Commutative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
Conditional	$\neg(p \to q) \equiv p \wedge \neg q$	$p \to q \equiv \neg p \vee q$

Table 3: Various logical constructions and their corresponding equivalences.

Remark: logically, taking the negation of statement is not logically equivalent the *contrapositive*. in speech these are often mistakenly taken to mean the same thing, but logically that is a fallacy.

Statement	$p \to q$
Converse	$q \to p$
Inverse	$\neg p \to \neg q$
Contrapositive	$\neg q \to \neg p$

Table 4: Converse, inverse and contrapositive of a conditional statement.

Exercise: Construct a truth table for the following:
a.) $p \to \neg q \equiv \neg p \vee \neg q$
b.) $\neg p \to q \equiv p \vee q$
c.) $\neg(p \to \neg q \vee \neg r) \equiv p \wedge q \wedge r$