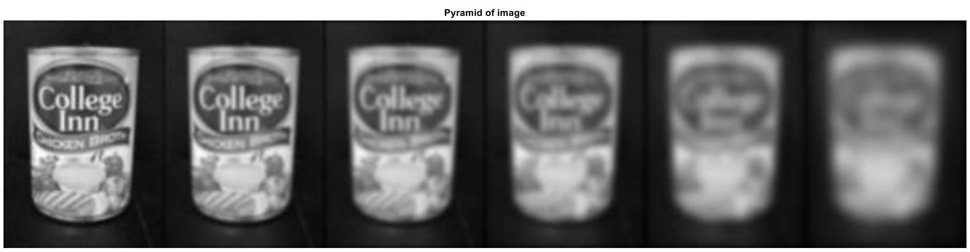
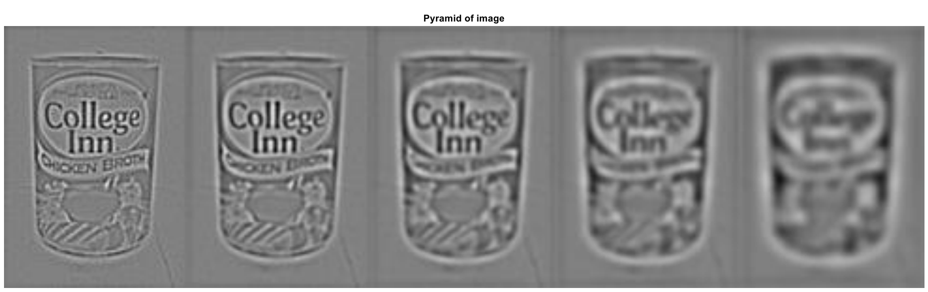
Gaussian Pyramid:



Difference of Gaussian Pyramid: (Q1.2)



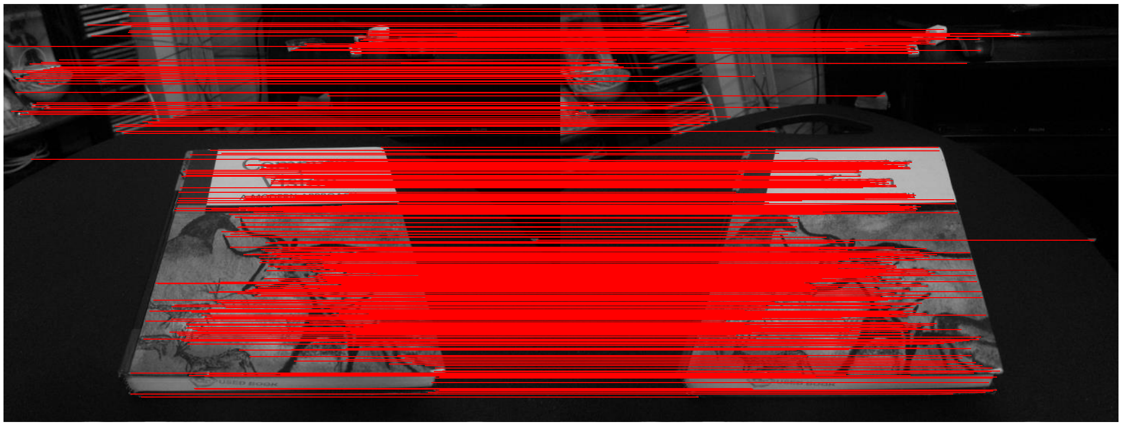
Detected Keypoints (Q1.5)

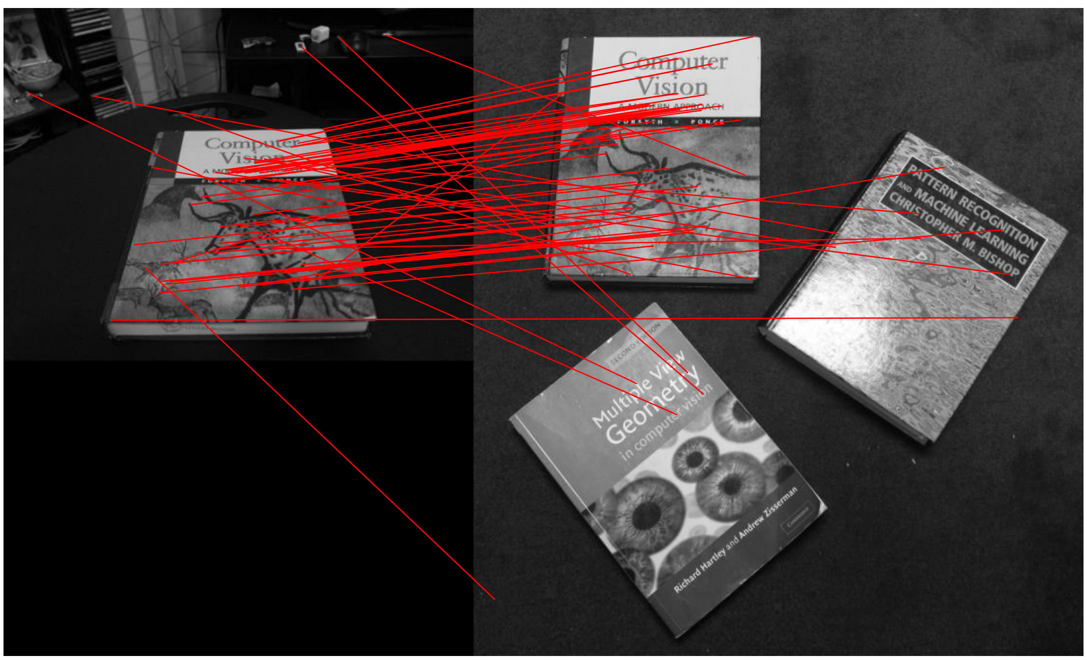
Image below shows the keypoints without dropping the edges.

Image below shows the key points detected after dropping edges.

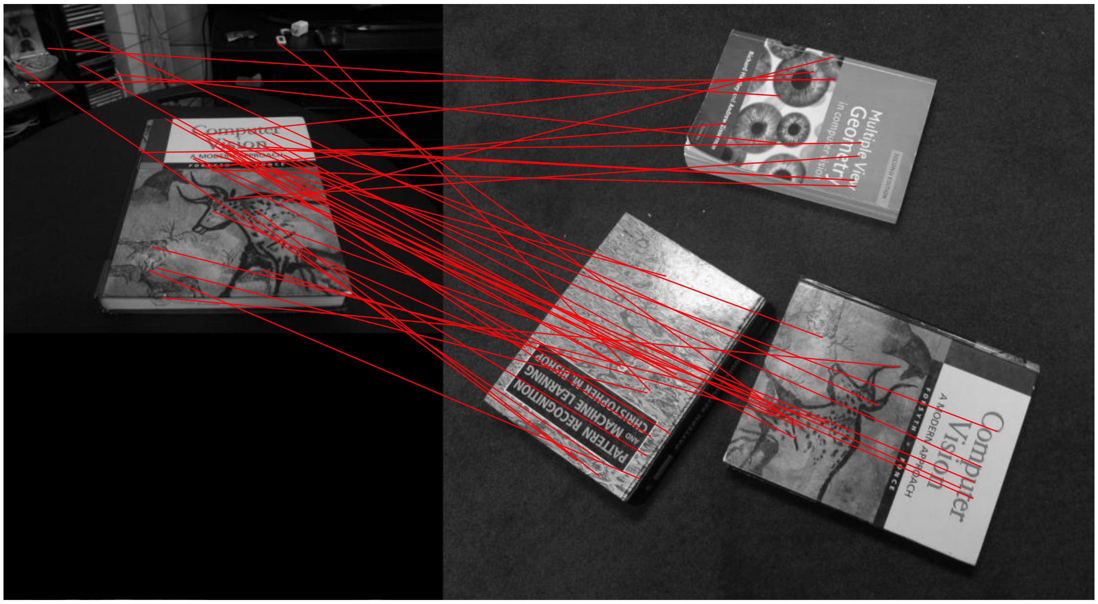
Matches (Q2.3):

As can be seen from the below images for the textbook, matches can be found well in scaled images. However, rotation images don't show good matches.

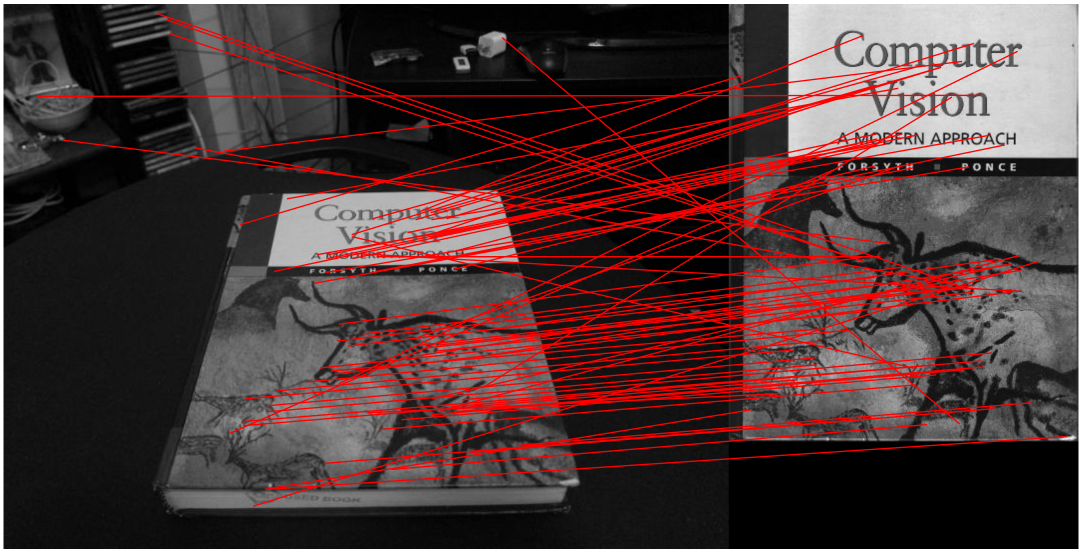


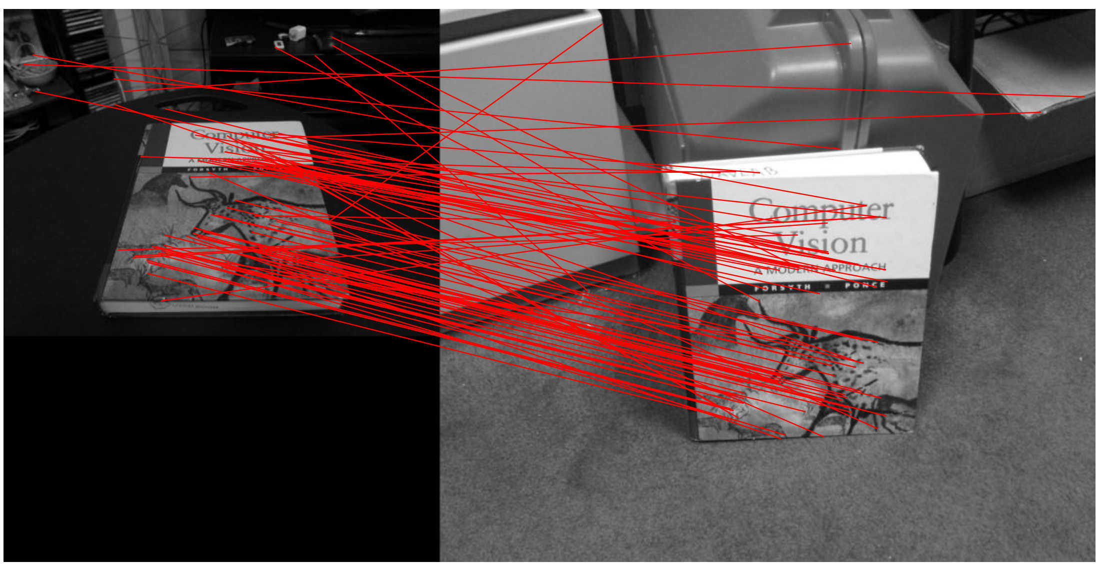


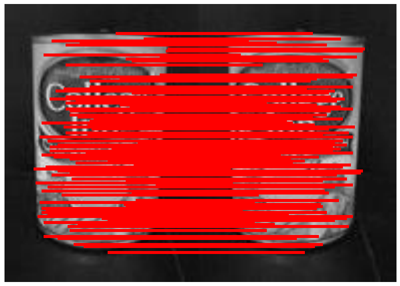
Consider the image above, since the textbook is just scaled, considerably higher matches are found as compared to the image below (where the textbook has been rotated).

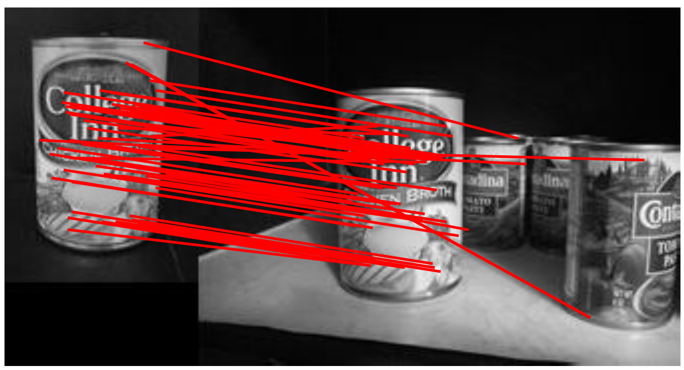


BRIEF descriptor is weak in matching under rotations, while it can perform better under scale. However, the rotation on the image below doesn't mess up the matches as much.


Computer
Vision
A MODERN APPROACH
FORSYTH PONCE


Computer
Vision
A MODERN APPROACH
FORSYTH PONCE






College
Inn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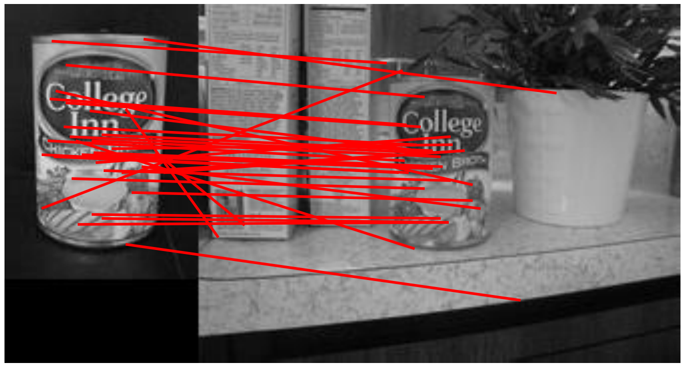
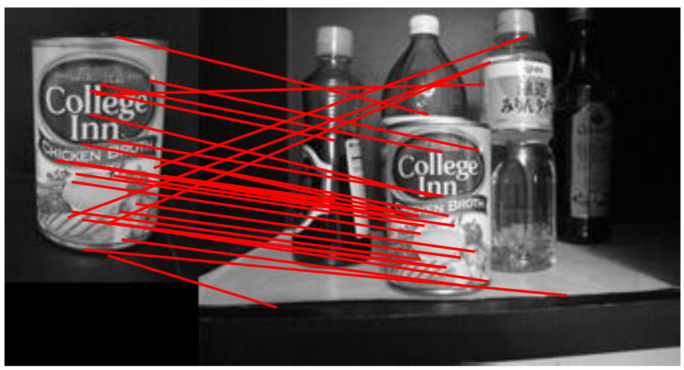
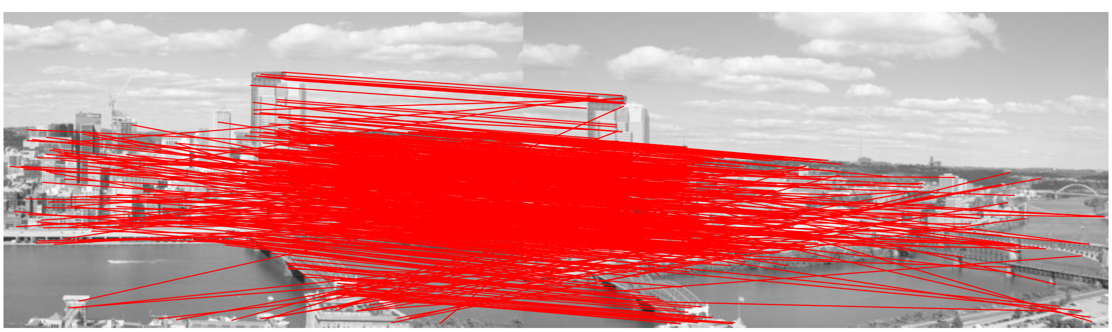
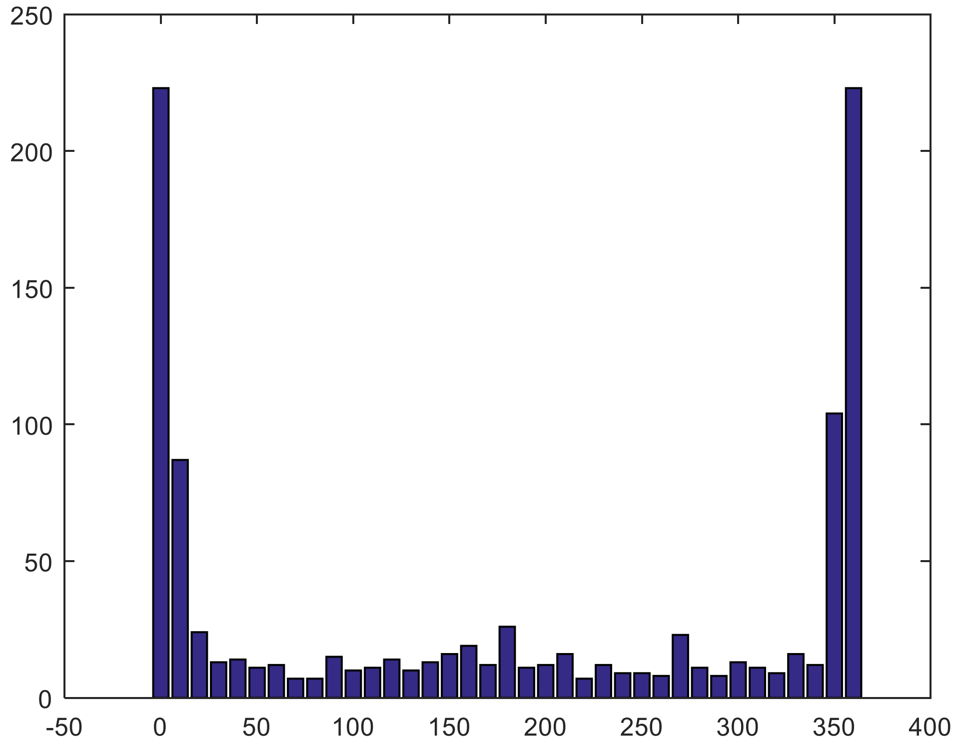
Good For You!







Rotation (Q2.5)



The bar graph shows that under rotation, the BRIEF descriptors starts showing significantly lesser matches for angles > 10 degrees. The BRIEF descriptor doesn't handle the patches' dominant rotation. By picking random pixels pairs from the patch, we are not taking into consideration the fact the same pair might be rotated. As such, no rotation information is contained by the BRIEF descriptor.

Problem 4:

Part a:

$P_2 = q$

$P_1 = P$

image   world

x', y' x, y, z

$x' = f\frac{x}{z}$ $y' = f\frac{y}{z}$

In homogenous point $P_i \to (u_i, v_i)^T$

$$P_i = \begin{bmatrix} u_i^p \\ v_i^p \end{bmatrix} \longrightarrow \begin{bmatrix} u_i^{p'} \\ v_i^{p'} \\ w_i^{p'} \end{bmatrix} \text{ s.t. } u_i^{p'} = u_i^p w_i^{p'}$$

$$\frac{u_i^{p'}}{w_i^{p'}} = u_i^p$$

for n correspondances

$$\begin{bmatrix} u_1^{p'} & & u_n^{p'} \\ v_1^{p'} & \cdots & v_n^{p'} \\ w_1^{p'} & & w_n^{p'} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_1^{q'} & & u_n^{q'} \\ v_1^{q'} & \cdots & v_n^{q'} \\ w_1^{q'} & & w_n^{q'} \end{bmatrix}$$

$$u_1^{p'} = h_{11}u_1^{q'} + h_{12}v_1^{q'} + h_{13}w_1^{q'}$$

$$\vdots$$

$$u_n^{p'} = h_{11}u_n^{q'} + h_{12}v_n^{q'} + h_{13}w_n^{q'}$$

$$v_1^{p'} = h_{21}u_1^{q'} + h_{22}v_1^{q'} + h_{23}w_1^{q'}$$

$$v_n^{p'} = h_{21}u_n^{q'} + h_{22}v_n^{q'} + h_{23}w_n^{q'}$$

$$w_1^{p'} = h_{31}u_1^{q'} + h_{32}v_1^{q'} + h_{33}w_1^{q'}$$

$$\vdots$$

$$w_n^{p'} = h_{31}u_n^{q'} + h_{32}v_n^{q'} + h_{33}w_n^{q'}$$

$$u_1^p = \frac{u_1^{p'}}{w_1^{p'}} = \frac{h_{11}u_1^{q'} + h_{12}v_1^{q'} + h_{13}w_1^{q'}}{h_{31}u_1^{q'} + h_{32}v_1^{q'} + h_{33}w_1^{q'}}$$

$$0 = u_1^p\left(h_{31}u_1^{q'} + h_{32}v_1^{q'} + h_{33}w_1^{q'}\right) - h_{11}u_1^{q'} - h_{12}v_1^{q'} + h_{13}w_1^{q'}$$

$x_{p'} * x_{q'} = x_{p'}$

$$u_1^q = \frac{u_1^{q'}}{w_1^{q'}}$$ substitute

$$\begin{bmatrix} -u_1^q & -v_1^q & -1 & 0 & 0 & 0 & u_1^p u_1^q & u_1^p v_1^q & u_1^p \\ \vdots & & & \vdots & & & \vdots & \vdots & \vdots \\ -u_n^q & -v_n^q & -1 & 0 & 0 & 0 & u_n^p u_n^q & u_n^p v_n^q & u_n^p \\ 0 & 0 & 0 & -u_1^q & -v_1^q & -1 & v_1^p u_1^q & v_1^p u_1^q & v_1^p \\ & & & \vdots & & & \vdots & \vdots & \\ 0 & 0 & 0 & -u_n^q & -v_n^q & -1 & v_n^p u_n^q & v_n^p v_n^q & v_n^p \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ \vdots \\ \vdots \\ h_{33} \end{bmatrix} = 0$$

PART B → There are 9 elements in h

PART C → 4 point correspondences are required bk in homogenous coordinates there is a hidden scaling factor. As such, there are 8 DOF instead of 9 DOFs

Part D:

PART D

$Ah = 0$ has a trivial solution $h = 0$

to find a non-trivial solution $\|h\| \neq 0$ & $\|Ah\|$ minimize

this corresponds to solving

$$\frac{h^T A^T A h}{h^T h} = 0$$

the Rayleigh quotient problem: $R(m, x) = \dfrac{x^* M x}{x^* x}$

where x^* is the hermitian transpose

in Real #'s $x^* = x^T$

As such, $\dfrac{h^T A^T A h}{h^T h} = 0$ is similar to $\dfrac{x^T M x}{x^T x}$

where $M = A^T A$ & $x = h$

The minimum of $R(m,x)$ corresponds to λ_{min} ← smallest eigenvalue of $A^T A$

when x = eigenvector of $A^T A$ corresponding λ_{min}

As such, $Ah = 0$ corresponds to minimizing $\dfrac{h^T A^T A h}{h^T h} = 0$

h is the eigenvector corresponding to λ_{min} of $A^T A$

SVD to get

$SVD(A) = U \Sigma V^T$

$\Sigma \rightarrow$ diagonals have singular values of A

$V \rightarrow$ eigenvectors of $A^T A$

row min of Σ take corresponding column from $\underline{V}$

Extra Credit

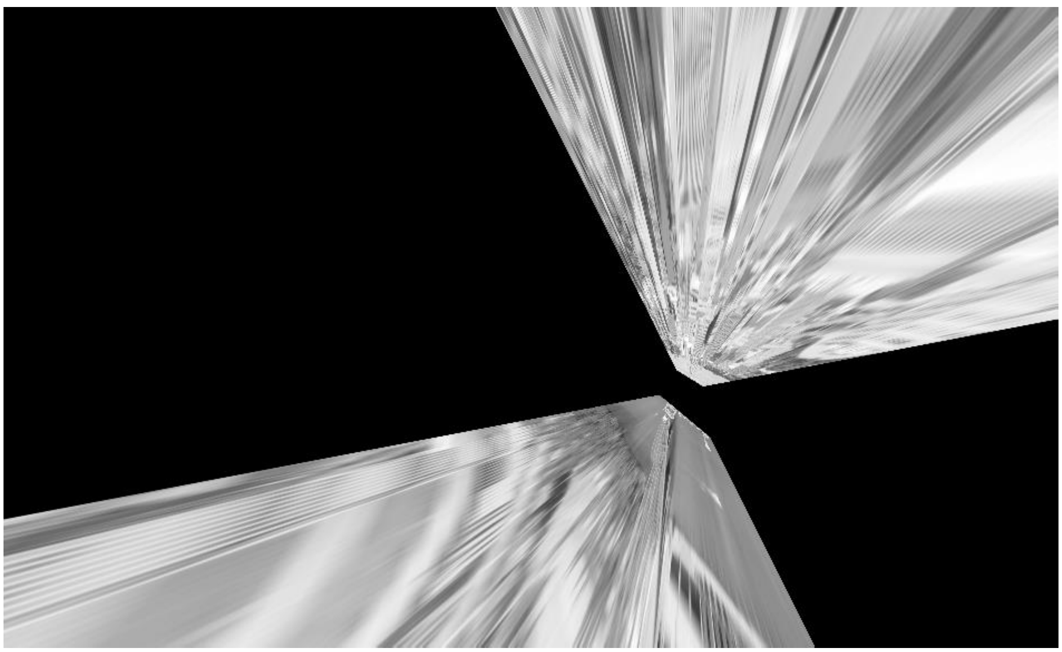
EC

$$p_2' = K_2 [R \ \ 0] P$$
$$= K_2 R [I \ \ 0] P$$
$$= K_2 R K_1^T \underbrace{K_1 [I \ \ 0] P}_{p_1} = K_2 R K_1^T p_1 \rightsquigarrow H$$

$$H = K_2 R K_1^T$$
where
$$p_2 = H p_1$$

Problem 5:

This result shows that the linear least regression has the chance of giving very bad results by picking up outliers.

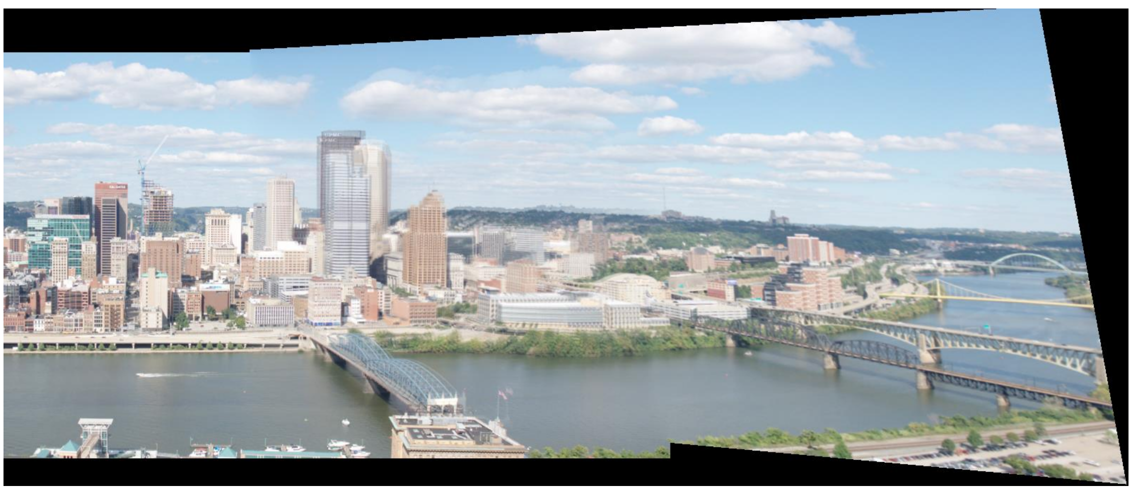


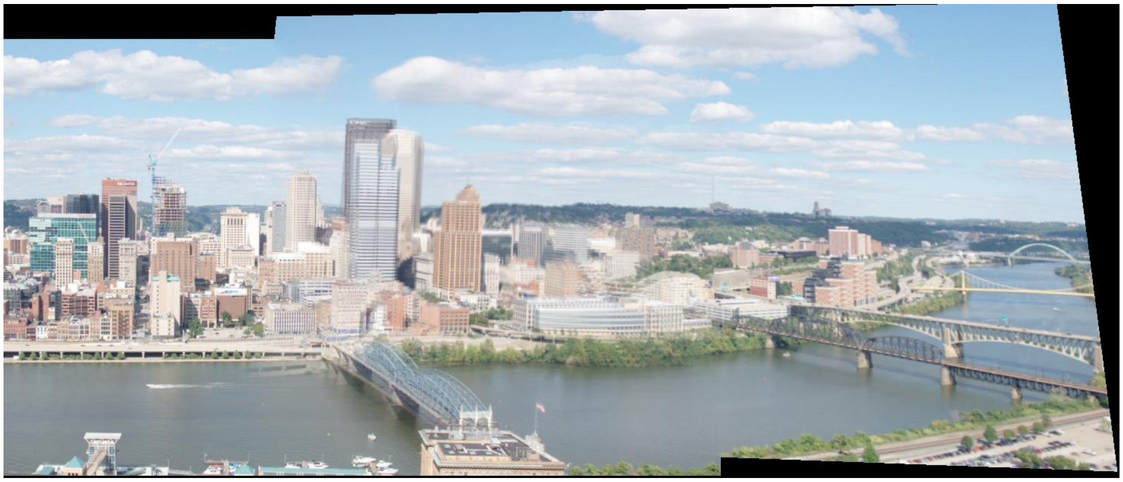
Problem 6:

The saved images were computed from RANSAC. As shown in the above image, the linear regressing method is too unreliable to post a proper image.

Problem 7:

Even with RANSAC, because the points are randomly chosen, the results have a varying degree of blurring in the middle. The below images show this varying blurriness/.





Extra Credit:

I have taken three images and merged them. The original and panorama images are given in /results folder.