

Mid Semester Examination, Spring-2021-22

Full marks: 30

Exam duration: 2 Hours

Answer **all** questions. Figures next to each question in square bracket indicate marks.

All Parts of a question should be answered at one place.

This question paper contains TWO pages.

1. Given a vector $\vec{v} = (1, -2, \sqrt{3}, 4, 5)$. Find its length. [1]
2. Given a vector $\vec{u} = (\frac{1}{2}, -\frac{1}{4}, p, \frac{1}{3})$ and $|\vec{u}| = 1$.
How many values p may have and what are they? [1]
3. Given two vectors $\vec{a} = (1, -2, 0, \frac{1}{2})$ and $\vec{b} = (-3, 0, 5, 7)$.
Find the angle between them. [1]
4. Test the following subsets and determine whether they are subspace. [6]
 - (a) $U_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + 2x_2 + 3x_3 = 0\}$
 - (b) $U_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + 2x_2 + 3x_3 = 4\}$
 - (c) $U_3 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1x_2x_3 = 0\}$
 - (d) $U_4 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 = 5x_3\}$
 - (e) $U_5 = \{(x, y) \in \mathbb{R}^2, \quad m, c \in \mathbb{R} \mid y = mx + c\}$
 - (f) $U_6 = \{(x, y) \in \mathbb{R}^2 \mid (x, y) \geq 0\}$
5. What value of k would make the following list of vectors are dependent? [3]
 - (a) $(2, 3, 1), (1, -1, 2), (7, 3, k)$
 - (b) $(3, 1, 4), (5, 9, k), (2, -3, 5)$
 - (c) $(k, 0, 1), (1, -1, 3), (1, 2, -2)$
6. Test whether the following list of vectors are dependent or independent. [6]
 - (a) $(1, 0, -4, \frac{1}{2}), (3, -\frac{1}{2}, 0, 0), (0, 2, 3, 4), (11, -\frac{1}{2}, \frac{13}{2}, 3)$
 - (b) $(1, 3, 2), (2, 1, 3), (3, 2, 1)$
 - (c) $(1, -3, 2), (2, 1, -3), (-3, 2, 1)$
 - (d) $(1, 0, 0), (1, 1, 0), (1, 1, 1), (2, 3, 4)$
 - (e) $(1, 1, 0, 0), (1, 0, 1, 0), (0, 0, 1, 1), (0, 1, 0, 1)$
 - (f) $(4, 0, 0), (5, 1, 0), (-3, 2, 1)$

7. Describe the subspaces spanned by:[4]
 - (a) $(1, 1, -1), (-1, -1, 1)$
 - (b) $(0, 1, 1), (1, 1, 0), (0, 0, 0)$
 - (c) all columns of a 3×5 echelon matrix with 2 pivots
 - (d) all vectors with positive components
8. Give a suitable explanations for the following:[4]
 - (a) Why no list of six polynomials is linearly independent in $\mathcal{P}_4(\mathbb{R})$?
 - (b) Why no list of four polynomials spans $\mathcal{P}_4(\mathbb{R})$?
9. Answer the following with suitable explanations/examples.[4]
 - (a) Find all vector space with exactly one basis.
 - (b) Does the list of vectors $(1, 2, -4), (7, -5, 6)$ form a basis of \mathbb{R}^3 ?
 - (c) Does the list of vectors $(1, -1, 0), (1, 0, -1)$ form a basis of $\{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$
 - (d) Find the basis of $\{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 \mid x_1 = 3x_2 \text{ and } x_3 = 7x_4\}$