Mid Semester Examination, Spring–2019-20

Full marks: **30** Exam duration: **2 Hours**

Answer **all** questions. Figures next to each question in square bracket indicate marks.
All Parts of a question should be answered at one place.
This question paper contains ONE page.

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1. Is A symmetric, if row space of A = column space of A, and $\mathbf{N}(A) = \mathbf{N}(A^{\mathrm{T}})$? [2]

2. The linear combinations of $\mathbf{u} = (1, 1, 0)$ and $\mathbf{v} = (0, 1, 1)$ fill a plane in $\mathbf{R}^3$. Find a vector $\mathbf{z}$ that is perpendicular to $\mathbf{u}$ and $\mathbf{v}$. Show that $\mathbf{z}$ is perpendicular to every vector $c\mathbf{u} + d\mathbf{v}$ on the plane. Find a vector $\mathbf{w}$ that is not on the plane. [4]

3. Find the matrices C_1 and C_2 containing independent columns of A_1 and A_2.
$$A_1 = \begin{pmatrix} 1 & 3 & -2 \\ 3 & 9 & -6 \\ 2 & 6 & -4 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$
Factor each of the above matrices into $A = CR$.
Produce a basis for the column spaces of A_1 and A_2. What are the dimensions of those column spaces — the number of independent vectors? What are the ranks of A_1 and A_2? How many independent rows in A_1 and A_2? [4]

4. How is the null space of C related to the null spaces of A and B, if $C = \begin{pmatrix} A \\ B \end{pmatrix}$? [4]

5. Four possibilities for the rank r and size m, n match four possibilities for $Ax = b$. Find four matrices A_1 to A_4 that demonstrate those possibilities. [4]
   (a) $r = m = n$ $A_1x = b$ has 1 solution for every b
   (b) $r = m < n$ $A_2x = b$ has 1 or ∞ solutions
   (c) $r = n < m$ $A_3x = b$ has 0 or 1 solution
   (d) $r < m, r < n$ $A_4x = b$ has 0 or ∞ solutions

6. Show that AA^{T} has the same null space as that of A. [4]

7. If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal unit vectors, show that $\mathbf{u} + \mathbf{v}$ is orthogonal to $\mathbf{u} - \mathbf{v}$. What are the lengths of those vectors? [2]

8. If $\mathbf{u}$ and $\mathbf{v}$ are not orthogonal, state the status of orthogonality between $\mathbf{w} = \mathbf{v} - \mathbf{u}(\mathbf{u}^{\mathrm{T}}\mathbf{v})$ and $\mathbf{u}$ and $\mathbf{v}$. [2]

9. Find LU factorization of following matrices and solve the linear system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b}$ is the column vector with all elements equal to 1 [4]
   (a) $\begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 3 & 2 \end{pmatrix}$

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