Mid Semester Examination, Spring–2019

Full marks: **60** Exam duration: **2 Hours**

Answer **all** questions. Figures next to each question in square bracket indicate marks.

All Parts of a question should be answered at one place.

This question paper contains TWO pages.

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1. Solve the following system of linear equations by reducing to triangular form and then using back substitution [10]

   (a) $5x_1 + 3x_2 - x_3 = 9$
   $3x_1 + 2x_2 - x_3 = 5$
   $x_1 + x_2 + x_3 = -1$

   (b) $x + z - 2w = -3$
   $2x - y + 2z - w = -5$
   $-6y - 4z + 2w = 2$
   $x + 3y + 2z - w = 1$

   (c) $2u - v + 2w = 2$
   $-u - v + 3w = 1$
   $3u - 2w = 1$

   (d) $3x_1 + x_2 = 1$
   $x_1 + 3x_2 + x_3 = 1$
   $x_2 + 3x_3 + x_4 = 1$
   $x_3 + 3x_4 = 1$

2. How should the coefficients a, b, and c be chosen so that the system $ax+by+cz = 3$, $ax - y + cz = 1$, $x + by - cz = 2$, has the solution $x = 1$, $y = 2$, and $z = -1$ [5]

3. Find LU factorization of following matrices and solve the linear system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b}$ is the column vector with all elements equal to 1 [10]

   (a) $\begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & -1 & -1 \\ -1 & 3 & 0 & 2 \\ 0 & -1 & 2 & 1 \end{pmatrix}$

   (c) $\begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 3 & 2 \end{pmatrix}$

   (d) $\begin{pmatrix} 2 & 1 & 3 & 1 \\ 1 & 4 & 0 & 1 \\ 3 & 0 & 2 & 2 \\ 1 & 1 & 2 & 2 \end{pmatrix}$

4. Find the equation $z = ax + by + c$ for the plane passing through the points $\mathbf{p_1} = (0, 2, -1)$, $\mathbf{p_2} = (-2, 4, 3)$, and $\mathbf{p_3} = (2, -1, -3)$ [10]

5. Write $(0, -26, -9)$ as a linear combination of $(5, 3, 7)$ and $(2, -4, 1)$. Show that $(1, 3, 5)$ cannot be written as a linear combination of these two vectors. [$2\frac{1}{2}$]

6. For what value(s) of a are the following vectors linearly independent —
$(1, 5, -2), (0, 6, a), (3, 13, -3)$ [$2\frac{1}{2}$]

7. Determine which of the following lists of vectors are linearly independent — [10]

   (a) $(5, 7)$

   (b) $(3, 1, 4), (-2, 2, 5), (3, 0, 4), (2, -1, -2)$

   (c) $(1, 0, 0, 0), (1, 1, 0, 0), (1, 1, 1, 0), (1, 1, 1, 1)$

   (d) $(1, 2, 3), (3, 2, 1), (2, 1, 3)$

   (e) $(1, 2, 0, -1, 5), (0, 0, 0, 0, 0), (15, 6, 2, -17, 0)$

8. Find a basis for the subspace $U = \{(x, y, z, t) \in \mathbb{R}^4 \mid 3x + y - 7t = 0\}$ and write the dimension of U. [5]

9. Find a basis for, and write down the dimension of, the subspace
$W = span\{(1, \frac{2}{3}, 0, -5), (-3, -2, 0, 15), (3, 0, -1, \frac{1}{2}), (\frac{7}{2}, \frac{1}{3}, -1, -2)\}$. [5]

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