End Semester Examination, Spring–2019

Full marks: **50** Exam duration: **3 Hours**

Answer **all** questions. Figures next to each question in square bracket indicate marks.
All Parts of a question should be answered at one place.
This question paper contains TWO pages.

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1. Two fair dice are tossed. Find the probability of each of the following events — [4]
   (a) the sum of the outcomes of the two dice is equal to 7.
   (b) the sum of the outcomes of the two dice is equal to 7 or 11.
   (c) the outcome of the second die is greater than the outcome of the first die.
   (d) both dice come up with even numbers.

2. A bag contains eight red balls, four green balls, and eight yellow balls. A ball is drawn at random from the bag, and it is not a red ball.
   What is the probability that it is a green ball? [3]

3. A fair coin was tossed two times. Given that the first toss resulted in heads, what is the probability that booth tosses resulted in heads? [3]

4. NIT Rourkela buys 1000 ICs from supplier A, 2000 ICs from B, and 3000 ICs from C. Upon testing it was found that the conditional probability of an IC being defective depends on the supplier from whom it was bought. Given that an IC bought from supplier A, the probability that it is defective is 0.05; given that an IC bought from B, the probability that it is defective is 0.10; and given that it is bought from C, the probability that it is defective is 0.10. If the ICs from all the three suppliers are mixed together and then one is selected at random, what is the probability that it is defective. [4]

5. NIT Rourkela has twice as many undergraduate students as graduate students. 25% of the graduate students live on campus, and 10% of the undergraduate student live on campus. [4]
   (a) if a student is chosen at random, what is the probability that the student is an undergraduate student living on campus.
   (b) if a student living on campus is chosen at random, what is the probability that the student is a graduate student.

6. If A and B are independent events, then so are events A and $\bar{B}$, events $\bar{A}$ and B, and events $\bar{A}$ and $\bar{B}$. Prove it. [3]

7. A batch of 100 components is checked by an inspector who examines 10 components selected at random. If none of the 10 components is defective, the whole batch is accepted; Else the batch is further subjected to inspection. What is the probability that a batch containing 10 defective components will be accepted? [4]

8. Find the probability distribution for the number of threes obtained when two dice are thrown. Is it discrete random variable? Answer why. [3]

9. Find the expectation of sum of the numbers when two dice are thrown. [3]

10. A fair die is rolled repeatedly until a number larger than 4 is observed. If N is the total number of times that the the die is rolled, find $P(N = k)$, for $k = 1, 2, 3, \ldots$ [3]

11. A discrete random variable X has the probability function — [4]

$$P(X = x) = \begin{cases} kx & \text{if } x = 2, 4, 6 \\ k(x-2) & \text{if } x = 8 \\ 0 & \text{otherwise} \end{cases}$$

where k is constant.

(a) what is the value of k?

(b) find CDF $F(5)$.

(c) find expectation $E(X)$.

(d) find $E(X^2)$

12. Let X be a discrete random variable with the following PMF [3]

x	0.2	0.4	0.5	0.8	1	otherwise
$P(X = x)$	0.1	0.1	0.2	0.3	0.3	0

(a) find the range of the random variable X.

(b) find $P(X \leq 0.5)$.

(c) find $P(0.25 < X < 0.75)$.

13. You take an exam that contains 20 multiple-choice questions. Each question has 4 possible options. You know the answer to 10 questions, but you have no idea about the other 10 questions so you choose answers randomly. Your score X on the exam is the total number of correct answers. Find the PMF of X. What is $P(X > 15)$? [3]

14. The probability distribution of a discrete random variable X is $P(X = x) = kx^2$, for $x = 1, 2, 3$ and k being a positive constant.
Find the value of k.
Find $P(X \geq 2)$ and $E(X)$. [3]

15. Let X be a discrete random variable with range $\{1, 2, 3, \ldots\}$.
Let the PMF of X be $P(X = k) = \frac{1}{2^k}$ for $k = 1, 2, 3, \ldots$.
Find and plot CDF for X.
Find $P(2 < X \leq 5)$.
Find $P(X > 4)$ [3]

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