Software Project Management

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More Examples on CPM / PERT

Algorithm for Critical Path Method (CPM)

1. List all the jobs and then draw network diagram. Each job is indicated by an arrow with the direction of the arrow showing the sequence of jobs. The length of the arrows has no significance. Place the jobs on the diagram one by one keeping in mind that what precedes and follows each job as well as what job can be done simultaneously.
2. Consider the job's time can be deterministic . Indicate them above the arrow representing the task.
3. Calculate EST, EFT, LFT and LST.

Algorithm for CPM cont …

4. Tabulate various times, i.e., activity normal times, earliest and latest times, and mark EST and LFT on the arrow diagram.
5. Determine the total float (slack) for each activity by taking difference between EST and LFT.
6. Identify the critical activities and connect them with the beginning node and the ending node in the network diagram by the double line arrows. this gives the Critical path.
7. Calculate the total project duration.

Example - CPM

A project consists of a series of tasks labelled A, B,...., H, I with the following relationships (W < X ,Y means X and Y cannot start until W is completed ;X, Y < W means W cannot start until both X and Y are completed). With these notations, construct the network diagram having the following constraints:

A < D, E ; B, D < F ; C < G ; B, G < H; F, G < I.

- Find also the minimum time completion of the project, when the time (in days) of completion of each task is as follows:

Task:	A	B	C	D	E	F	G	H	I
Time:	23	8	20	16	24	18	19	4	10

Activity Network

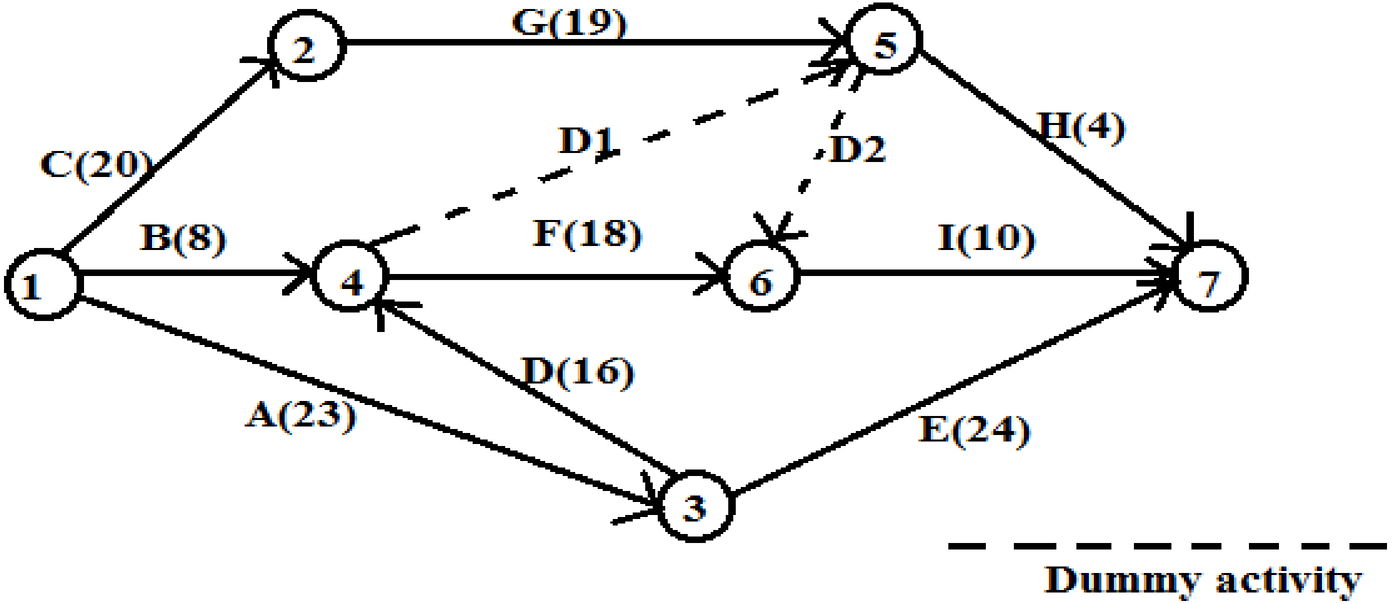


Figure 1: Resulting Activity Network

Calculation table

Task	Normal time *tij*	Earliest time		Latest time		Total float
		Start	Finish	Start	Finish	
(1,2)	20	0	20	18	38	18
(1,3)	23	0	23	0	23	0
(1,4)	8	0	8	31	39	31
(2,5)	19	20	39	38	57	18
(3,4)	16	23	39	23	39	0
(3,7)	24	23	47	43	67	20
(4,5)	0	39	39	57	57	18
(4,6)	18	39	57	39	57	0
(5,6)	0	39	39	57	57	18
(5,7)	4	39	43	63	67	24
(6,7)	10	57	67	57	67	0

Critical path

Critical events: (1,3), (3,4), (4,6), (6,7)

Critical tasks: A, D, F, I, **Critical Path**–Shown as double arrows

Total duration: 67 days

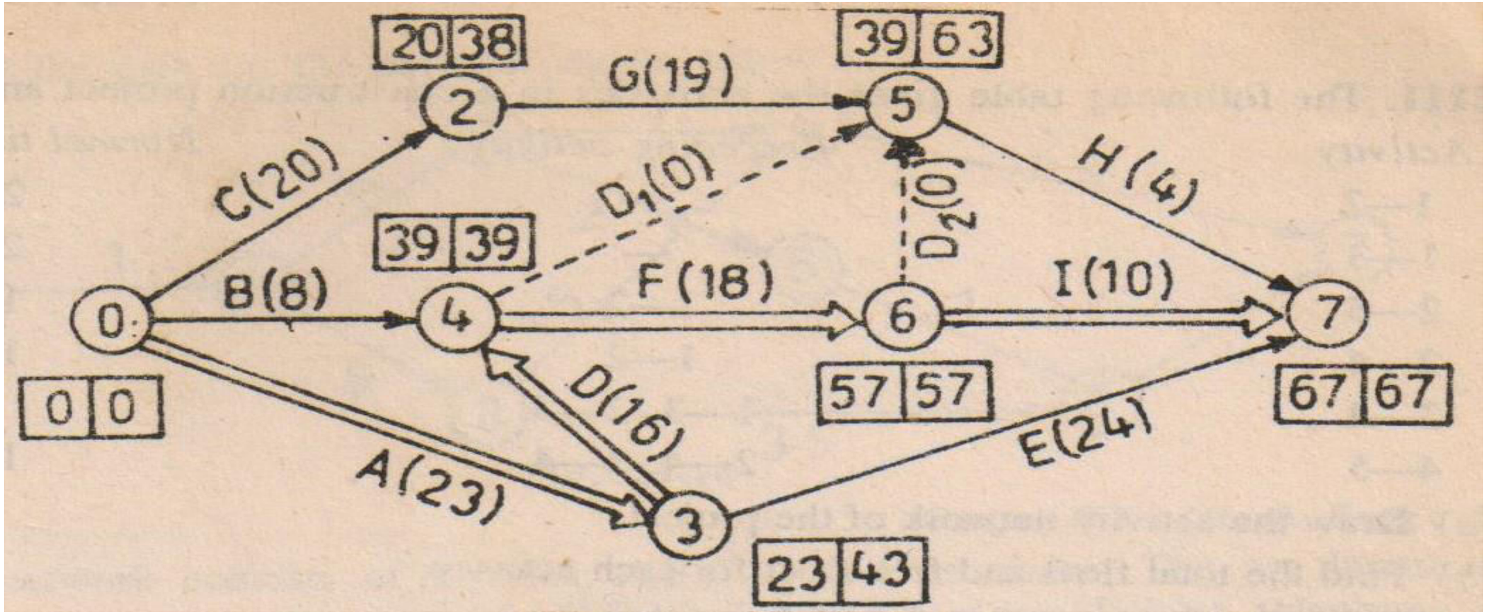


Figure 2: Resulting network with critical path

PERT Algorithm

1. Make a list of activities that make up the project including immediate predecessors.
2. Making use of step 1 sketch the required network.
3. Denote the most likely time by m the optimistic time by a and pessimistic time by b .
4. Using beta distribution for the activity duration, the expected time t_e , for each activity is computed by using the formula

$$t_e = (a + 4m + b)/6 \qquad ()$$

5. Tabulate various times, i.e., expected activity times, earliest and latest times and mark the ES and LF on the arrow diagram.

PERT Algorithm cont…

6. Determine the total float for each activity by taking the difference between EF and LF .
7. Identify the critical activities and connect them with the beginning node and the ending node, in the network diagram by double line arrows. This gives the critical path and the expected date of completion of the project.
8. Using the values of b and a , compute the variance (σ^2) of each activity's time estimates by using the formula:

$$\sigma^2 = \left(\frac{b-a}{6}\right)^2$$

PERT Algorithm cont…

9. Compute the standard normal deviate:

$$z_0 = \frac{DueDate - ExpectedDateOfCompletion}{\sqrt{Variance}}$$

- Use standard normal tables to find the probability $P(z \le z0)$ of completing the project within the scheduled time, where $z \sim N(0, 1)$.

Example - PERT

A small project is composed of seven activities whose time estimates are listed in the table as follows.

Activity		Estimated duration (weeks)		
i	j	Optimistic	Most likely	Pessimistic
1	2	1	1	7
1	3	1	4	7
1	4	2	2	8
2	5	1	1	1
3	5	2	5	14
4	6	2	5	8
5	6	3	6	15

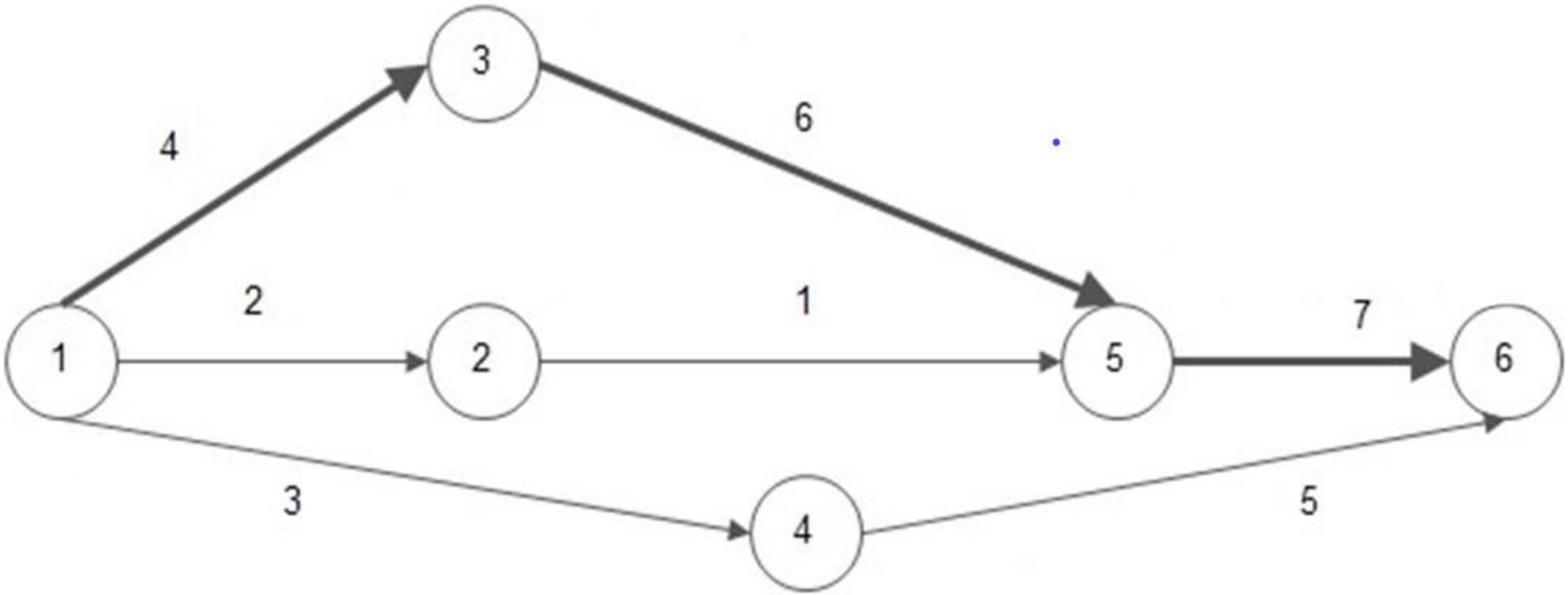
Example - PERT cont…

- Draw the project network.
- Find the expected duration and variance of each activity. Calculate early and late occurrence times for each event. What is the expected project length?
- Calculate the variance and standard deviation of project length. What is the probability that the project will be completed:
  a. at least 4 weeks earlier than expected?
  b. no more than 4 weeks later than expected?
- If the project due date is 19 weeks, what is the probability of meeting the due date?

Given

z	0.50	0.67	1.00	1.33	2.00
p	0.3085	0.2514	0.1587	0.0918	0.0228

Resultant network



Solution

The expected time and variance of each activity is computed in the table shown below.

Activity $(i–j)$	a	m	b	$t_e = \frac{a+4m+b}{6}$	$\left(\frac{b-a}{6}\right)^2$
1–2	1	1	7	2	1
1–3	1	4	7	4	1
1–4	2	2	8	3	1
2–5	1	1	1	1	0
3–5	2	5	14	6	4
4–6	2	5	8	5	1
5–6	3	6	15	7	4

Solution cont…

- Earliest expected times E {μi } for each event is obtained by taking the sum of expected times for all the activities leading to an event i .
- When more than one activity leads to event i , the greatest of E {μi } is chosen.

Thus,

- E {μ1} = 0.
- E {μ2} = 0 + 2 = 2.
- E {μ3} = 0 + 4 = 4.
- E {μ4} = 0 + 3 = 3.
- E {μ5} = max.{4 + 6, 2 + 1} = 10.
- E {μ6} = max.{10 + 7, 3 + 5} = 17.

Solution cont…

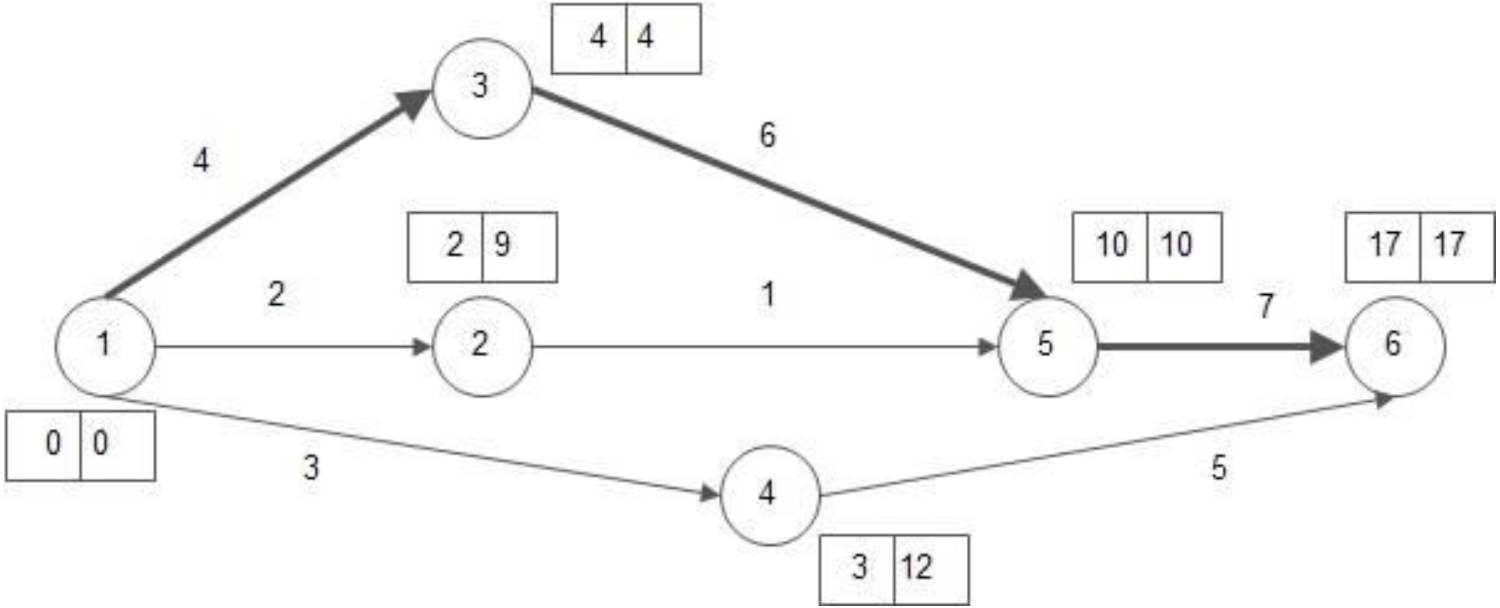
- For the latest expected times we start with E {μi } for the last event and move backwards, subtracting ” t_e ” for each activity link.

Thus,

- E {L6} = 17.
- E {L5} = 17 − 7 = 10.
- E {L4} = 17 − 5 = 12.
- E {L3} = 10 − 6 = 4.
- E {L2} = 10 − 1 = 9.
- E {L1} = min.{9 − 2, 4 − 4, 12 − 3} = 0.

Solution cont…

- Resultant network
- The critical path is shown by thick line arrows.



Solution cont…

- Critical path: 1→3→5→6.
- Other paths: 1→2→5→6 and 1→4→6.
- The expected duration of the project is 17 weeks.
- Variance of the project length is given by $\sigma^2 = 1 + 4 + 4 = 9$. The standard normal deviate is:

$$z_0 = \frac{DueDate - ExpectedDateOfCompletion}{\sqrt{Variance}}$$

Solution cont…

(a) at least 4 weeks earlier than expected?

$$z = \frac{13-17}{3} = -\frac{4}{3} = -1.33$$

Now,

$P(z \leq -1.33) = 0.5 - \Phi(1.33) = 0.5 - 0.4082 = 0.0918 \equiv 9.18\%$

Or

See normal table in next slide

$P(z \leq -1.33) = \Phi(-1.33) = 1 - \Phi(1.33) = 1 - 0.9082 = 0.0918 \equiv 9.18\%$

Interpretation: If the project is performed 100 times under the same conditions, there will be 9 occasions when this job will be completed 4 weeks earlier than expected.

Normal Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Solution cont…

(b) no more than 4 weeks later than expected?

$$z = \frac{21-17}{3} = \frac{4}{3} = 1.33$$

The probability of meeting the due date (4 weeks later than expected) is

$P(z \leq 1.33) = \Phi(1.33) = 0.9082 \equiv 90.82\%$

Solution cont…

When the due date is 19 weeks

$$z = \frac{19-17}{3} = \frac{2}{3} = 0.67$$

The probability of meeting the due date is

$P(z \leq 0.67) = \Phi(0.67) = 0.7486 \equiv 74.86\%$
Thus, the probability of not meeting the due date is 1-0.7486, i.e., 0.2514 or 25.14%.

Summary

- Discussed the algorithm for CPM.
- Discussed the algorithm for PERT.
- Solved some more problems on CPM and PERT.

References :

1. Kanti Swarup, P. K. Gupta, Man Mohan, Operations Research, Nineteenth Edition, Sultan Chand and Sons, 2018.

Thank you