LARP

(Spring 2023)

Bishwajit Prasad Gond

222CS3113

Master of Technology

`222cs3113@nitrkl.ac.in`

Department of Computer Science & Engineering

NIT, Rourkela

February 19, 2023

Contents

1 2021 PYQ

1.1 Find all vector space with exactly one basis??

Let V be a vector space with exactly one basis. We claim that V must be the trivial vector space, i.e., the vector space consisting only of the zero vector.

To see why this is true, suppose that V is a nontrivial vector space. Let v be any nonzero vector in V. Then v is a linearly independent set, so it can be extended to a basis of V. But by assumption, V has exactly one basis, so this extension must be v itself. Therefore, v is the only basis vector of V, and every vector in V is a scalar multiple of v. In particular, V is one-dimensional.

Let v be the basis vector of V. Since V is nontrivial, v must be nonzero. But then the set $v, -v$ is also linearly independent, and it can be extended to a basis of V. This contradicts the assumption that V has exactly one basis. Therefore, V must be the trivial vector space.

In summary, the only vector space with exactly one basis is the trivial vector space.

1.2 Does the list of vectors (1, 2, 4), (7, 5, 6) form a basis of R3?

To determine whether the list of vectors $(1, 2, -4)$ and $(7, -5, 6)$ forms a basis of R^3, we need to check if they are linearly independent and if they span the whole space.

First, we check for linear independence. We can set up an equation in the form $a(1, 2, -4) + b(7, -5, 6) = (0, 0, 0)$ and solve for a and b. This gives us the system of equations:

a + 7b = 0 2a - 5b = 0 -4a + 6b = 0

We can solve this system using elimination or substitution. Using elimination, we can multiply the first equation by 2 and subtract the second equation to eliminate a:

2a + 14b = 0 2a - 5b = 0 -4a + 6b = 0

Subtracting the second equation from the first gives $19b = 0$, so $b = 0$. Substituting $b = 0$ into the first equation gives $a = 0$. Therefore, the only solution to the system is $a = b = 0$, which means the two vectors are linearly independent.

Next, we need to check if the two vectors span R^3. That is, for any vector (x, y, z) in R^3, we need to find scalars a and b such that $a(1, 2, -4) + b(7, -5, 6) = (x, y, z)$. Equivalently, we need to solve the system of equations:

a + 7b = x

2a - 5b = y

-4a + 6b = z

Using elimination or substitution, we can solve for a and b:

a = 1$_{\overline{29(16x+42y-35z)}}$

b = 1$\overline{29(5x+7y+8z)}$

Since we can always find values of a and b that satisfy the system, the two vectors span R^3.

Since the two vectors are linearly independent and span R^3, they form a basis of R^3.

1.3 Why no list of four polynomials spans P4(R)?

The space $P_4(R)$ consists of all polynomials of degree at most 4 with real coefficients. To show that no list of four polynomials spans $P_4(R)$, we need to show that there exist polynomials in $P_4(R)$ that cannot be written as linear combinations of any four given polynomials.

One way to do this is to find a polynomial of degree at most 4 that is not in the span of the given four polynomials. One such polynomial is $p(x) = x^4 - 2x^3 + x^2$. To see that $p(x)$ is not in the span of the given four polynomials, suppose for contradiction that $p(x)$ can be written as a linear combination of the four polynomials. Then there exist constants a, b, c, d such that:

$$p(x) = aq_1(x) + bq_2(x) + cq_3(x) + dq_4(x)$$

where $q_1(x), q_2(x), q_3(x), q_4(x)$ are the given four polynomials. But the degree of $p(x)$ is 4, while the degree of each of the given four polynomials is at most 3. Therefore, the left-hand side of the equation has a term of degree 4, while the right-hand side does not. This is a contradiction, so $p(x)$ cannot be written as a linear combination of the given four polynomials.

Since $p(x)$ is a polynomial of degree at most 4 with real coefficients that is not in the span of the given four polynomials, the list of four polynomials does not span $P_4(R)$. Therefore, no list of four polynomials can span $P_4(R)$.

1.4 Why no list of six polynomials is linearly independent in P4(R)?

The space $P_4(R)$ consists of all polynomials of degree at most 4 with real coefficients. To show that no list of six polynomials is linearly independent in $P_4(R)$, we need to show that there exist constants $a_1, a_2, a_3, a_4, a_5, a_6$, not all zero, such that the linear combination:

$$a_1p_1(x) + a_2p_2(x) + a_3p_3(x) + a_4p_4(x) + a_5p_5(x) + a_6p_6(x) = 0$$

where $p_1(x), p_2(x), p_3(x), p_4(x), p_5(x), p_6(x)$ are the given six polynomials. This will imply that the six polynomials are linearly dependent.

One way to do this is to use the fact that any set of $n + 1$ or more polynomials in $P_n(R)$ is linearly dependent. Since $P_4(R)$ is the space of polynomials of degree at most 4, any set of 5 or more polynomials in $P_4(R)$ is linearly dependent. Therefore, any list of six or more polynomials in $P_4(R)$ is linearly dependent.

So, the list of six polynomials is linearly dependent in $P_4(R)$.

1.5 Describe sub space spanned by all vectors with positive components

The subspace spanned by all vectors with positive components is a subset of the vector space of real-valued vectors. It consists of all linear combinations of the basis vectors, where each basis vector has positive components and all other components are zero. Specifically, the basis vectors are given by:
e1 = (1, 0, 0, ..., 0)
e2 = (0, 1, 0, ..., 0)
e3 = (0, 0, 1, ..., 0)
...
en = (0, 0, 0, ..., 1)

where n is the dimension of the vector space.

Any vector that can be expressed as a linear combination of these basis vectors, with non-negative coefficients, belongs to the subspace spanned by all vectors with positive components. That is, if v = c1e1 + c2e2 + ... + cn*en, where c1, c2, ..., cn are non-negative real numbers, then v belongs to this subspace.

It's worth noting that this subspace is not a vector space itself, since it is not closed under scalar multiplication with negative scalars. Specifically, if a is a negative real number and v belongs to the subspace spanned by all vectors with positive components, then av does not necessarily belong to this subspace.

1.6 Describe the subspace spanned by all columns of a 3 × 5 echelon matrix with 2 pivots

An echelon matrix is a matrix that has a row-echelon form, meaning that it has been reduced to a triangular form with leading entries (pivots) in each row strictly to the right of the leading entry in the row above it.

If a 3x5 echelon matrix has 2 pivots, it means that there are two leading entries in the matrix, one in the first row and another in the second row, and all entries below each leading entry are zero. The remaining entries in the matrix can be any non-zero value, as long as they are strictly to the right of the leading entry in their respective row.

The subspace spanned by all columns of this matrix consists of all linear combinations of the columns of the matrix. Since there are only two pivots in the matrix, there are at most two linearly independent columns, and the subspace spanned by all columns of the matrix will be a plane in R^3.

To see this, let the echelon matrix be denoted by A, and let the columns with pivots be denoted by c1 and c2. Then the subspace spanned by all columns of A is the span of c1, c2, c3, c4, c5. Since c1 and c2 are linearly independent (since they are the columns with pivots), any column c in the subspace can be written as a linear combination of c1 and c2, i.e.,

$c = k1c1 + k2c2 + k3c3 + k4c4 + k5 * c5$

for some constants k1, k2, k3, k4, k5. This equation represents a plane in R^3 that is spanned by c1 and c2, and any linear combination of the remaining columns c3, c4, and c5. Therefore, the subspace spanned by all columns of a 3x5 echelon matrix with 2 pivots is a plane in R^3