2D Steady Conduction Problem

Problem 1 – Using incomplete LU decomposition(ILU):

Consider 2D conduction problem governed by equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Take a square domain of width and height of 1 unit. The boundary conditions are:

At x = 0: T = 0

At x = 1: T = 0

At y = 0: T = 0

At y = 1: T = 1

Discretize the governing equation and solve the obtained system of linear equations by writing code for Incomplete LU decomposition (ILU). This problem has an analytical solution which is given by series solution given by-

$$T(x,y) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\sin(n\pi x)\frac{\sinh(n\pi y)}{\sinh(n\pi)}$$

In the code you also write a subroutine to calculate temperature variation using the above analytical expression given by the above equation.

Results to be shown:

1. Compare the temperature contours obtained by the code and with those obtained by using the above analytical expression.
2. Compare the temperature variation with y along mid-vertical plane x = 0.5 from the code and the above analytical expression.
3. Compare the temperature variation with y along mid-horizontal plane y = 0.5 from the code and the above analytical expression.

Problem 2 – Using strongly implicit procedure:

Consider 2D conduction governed by equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Take a domain of width L=2 and height W=2. The boundary conditions are:

At x = 0: T = 0

At x = L: T = 0

At y = 0: T = 0

At y = W: $T = T_m sin\left(\frac{\pi x}{L}\right)$

Take Tm = 2. Discretize the governing and solve the obtained system of linear equations by writing code for Strongly Implicit procedure (SIP). This problem has an analytical solution which is given by expression-

$$T(x,y) = T_m \sin\left(\frac{\pi x}{L}\right)\frac{\sinh\left(\frac{\pi y}{L}\right)}{\sinh\left(\frac{\pi W}{L}\right)}$$

In the code write a subroutine to calculate temperature variation using the above analytical expression given by above equation.

Results to be shown:

1. Compare the temperature contours obtained by the code and with those obtained by using the above analytical expression.
2. Compare the temperature variation with y along mid-vertical plane x = L/2 from the code and the above analytical expression.
3. Compare the temperature variation with y along mid-horizontal plane y = W/2 from the code and the above analytical expression.

Problem 3 – Using conjugate gradient method:

Consider 2D conduction governed by equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -8\pi^2[\sin(2\pi x)\sin(2\pi y)]$$

Take a square domain of width and height of 1 unit. The boundary conditions for all the boundaries are $T = 0$. Discretise the equation. Then solve the obtained system of linear equations by writing code for conjugate gradient method. This problem has an analytical solution which is given by

$$T(x,y) = \sin(2\pi x)\sin(2\pi y)$$

In the code write a subroutine to calculate temperature variation using the above analytical expression given by above equation.

Results to be shown:

1. Compare the temperature contours obtained by the code and with those obtained by using the above analytical expression.
2. Compare the temperature variation with y along mid-vertical plane x = 0.5 from the code and the above analytical expression.
3. Compare the temperature variation with y along mid-horizontal plane y = 0.5 from the code and the above analytical expression.