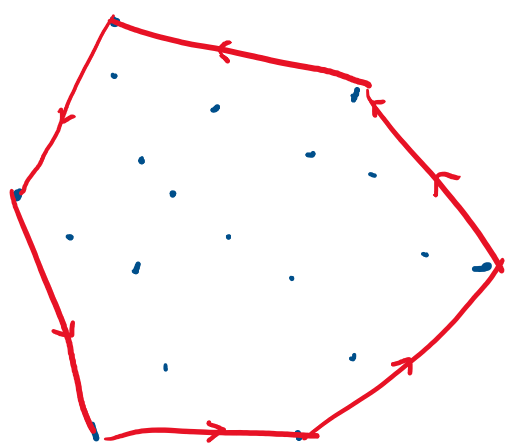
P2

Monday, February 2, 2026 4:42 PM

Part 1



ConvexHull(P), by definition, is the smallest set of points of P that contains all points in P.

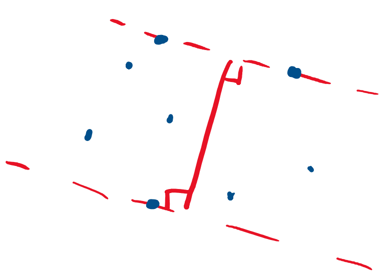
If w(P) is the smallest slab that contains all of P, then at least 1 point of P lies on each parallel line of slab(P).

If slab(P) - ConvexHull(P), then there lies a point on then there lies a point on slab(P) that is not contained by ConvexHull(P) that is in the set of P. This contradicts the definition of a Convex Hull, thus slab(P) $\subseteq$ ConvexHull(P).

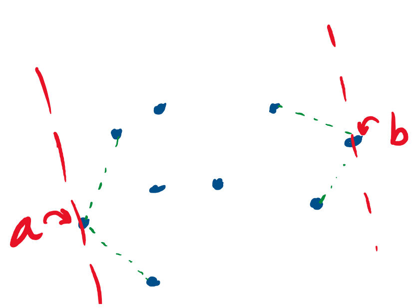
Part 2:

Citation:
https://www.nlc-bnc.ca/obj/s4/f2/dsk1/tape7/PQDD_0027/MQ50856.pdf?oclc_number=1006918422
https://www.cs.swarthmore.edu/~adanner/cs97/s08/pdf/calipers.pdf
https://www-cgrl.cs.mcgill.ca/~godfried/research/calipers.html

One line of the slab must always contain 2 point of P, creating an edge.



Let's say this wasn't the case:



$a, b \in P.$

Assume $a, b \in \text{slab}(P)$ and a and b are anti-podal, and have the furthest distance from each other.

There is some distance between the two parallel lines, by constraining them to contain a and b.

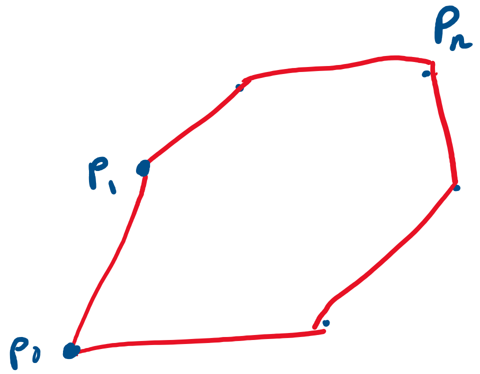
But, we must rotate slab(P) to minimize the distance in order the get the smallest width.

Because we will need to rotate, we can rotate the slab until at least one additional point is collinear. If we go further it will not contain the point, breaking the rule established in Part 1.

Now, we need to find the furthest point from any starting point.

P_n

This can be extended to find all antipodal pairs in linear time by stepping

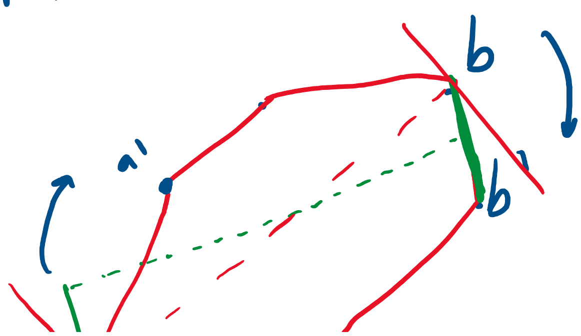


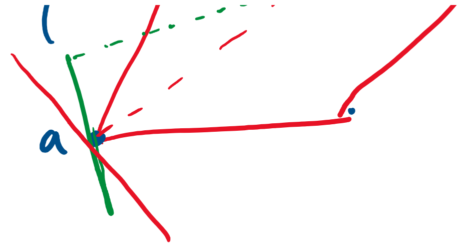
in linear time by stepping forward p_0 to p_1 and checking the $dist(p_1, p_n)$ and $dist(p_1, p_{n+1})$

We can find the antipodal pair by searching for the maximum distance between starting point and a antipodal point, p_n, in linear time, $O(n)$, since we just 'scan' the whole list of ConvexHull(P).

We have antipodal pairs a, b.

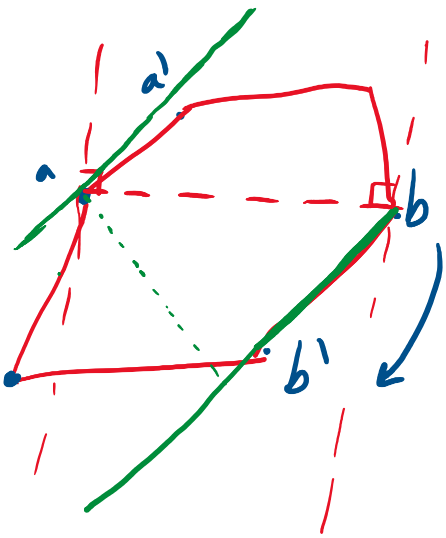
Because we know one more point must be collinear, we will calculate the distance between an edge and a point. We chose the next edge by checking if $\overline{aa'}$ or bb' creates a "flatter" line relative to the parallel, perpendicular line from the $\overline{ab}$ pair. It must be flatter to contain all points.





Here we step clockwise to b' and check $dist(a, \overline{bb'})$ and save the distance.

Now, we have the minimum slab(P) that uses a. From here, we now need to check a'. Let $a \leftarrow a'$.



Because $\overline{bb'}$ creates the 'flatter' line than $\overline{aa'}$, we use this line again.

We can repeat this walk until all $w(slab(P))$ are calculated, and track the smallest width.

The walk starting on antipodal pairs to finish is $O(n)$.

Finding the initial antipodal pair is $O(\frac{3n}{2})$,
or $O(n)$.

$$O(n) + O(\tfrac{3n}{2})$$

$$O(n) + O(n)$$

$$O(2n)$$

$$\boxed{O(n)}$$