Part 2: Finding and Classifying Critical Points and their Level Curves (continued from handwritten)

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
```

Getting the coefficients from:

$$0 = 8y^3 - 42y + 25$$

In [ ]:
```
coeff = [8,0, -42, 25]
roots = np.roots(coeff)
roots
```

Out[ ]:
```
array([-2.54515663,  1.89838443,  0.6467722 ])
```

Removing the negative because it will create a NaN and is not a critical point because it cannot be solved.

In [ ]:
```
roots = roots[1:]
roots
```

Out[ ]:
```
array([1.89838443, 0.6467722 ])
```

Solving for the x-coordinates using:

$x = \pm\sqrt{\frac{10y-5}{2}}$

In [ ]:
```
pos_f = np.sqrt((10*roots -5)/2)

neg_f = -1* pos_f
```

In [ ]:
```
pos_f
```

Out[ ]: array([2.6442243 , 0.85665687])

In [ ]:
```
neg_f
```

Out[ ]: array([-2.6442243 , -0.85665687])

Organizing the solved x and y coordinates

In [ ]:
```
y = np.concatenate((roots, roots, np.array([0])))
y
```

Out[ ]: array([1.89838443, 0.6467722 , 1.89838443, 0.6467722 , 0.   ])

In [ ]:
```
x = np.concatenate((pos_f, neg_f, np.array([0])))
x
```

Out[ ]: array([2.6442243 , 0.85665687, -2.6442243 , -0.85665687, 0.   ])

Inputting the critical points into:

$$f(x, y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4$$

In [ ]:
```
func = 10*x**2*y - 5*x**2 - 4*y**2 - x**4 - 2*y**4
func
```

Out[ ]: array([8.49585813, -1.48467882, 8.49585813, -1.48467882, 0.   ])

Solving for the Second Derivative test by taking

$$det(\nabla^2 f)$$

In [ ]:
```
d_x_y = (20*y - 10 - 12 * x**2)*(-24*y**2 - 8)  - (20*x)**2
d_x_y
```

Out[ ]:
```
array([2488.7172337 , -187.63626429, 2488.7172337 , -187.63626429,
         80.        ])
```

Solving for f_{xx}

In [ ]:
```
f_xx = (20*y - 10 - 12 * x**2)
f_xx
```

Out[ ]:
```
array([-55.93537725,  -5.87088796, -55.93537725,  -5.87088796,
       -10.        ])
```

In [ ]:
```
crit = pd.DataFrame()
crit["x"] = x
crit["y"] = y
crit["f(x,y)"] = func
crit["D(x,y)"] = d_x_y
crit["f_xx"] = f_xx

crit
```

Out[ ]:

	x	y	f(x,y)	D(x,y)	f_xx
0	2.644224	1.898384	8.495858	2488.717234	-55.935377
1	0.856657	0.646772	-1.484679	-187.636264	-5.870888
2	-2.644224	1.898384	8.495858	2488.717234	-55.935377
3	-0.856657	0.646772	-1.484679	-187.636264	-5.870888
4	0.000000	0.000000	0.000000	80.000000	-10.000000

Classifying the critical points found using the Second Derivative Test

In [ ]:
```
def classify(row: pd.DataFrame):
    if row["D(x,y)"] < 0:
        return "saddle"
    elif row["D(x,y)"] == 0:
        return "unknown"
    else:
        if row["f_xx"] > 0:
            return "minimum"
        else:
            return "maximum"
```

```
crit["classification"] = crit.apply(classify, axis = 1)
crit
```

Out[ ]:

	x	y	f(x,y)	D(x,y)	f_xx	classification
0	2.644224	1.898384	8.495858	2488.717234	-55.935377	maximum
1	0.856657	0.646772	-1.484679	-187.636264	-5.870888	saddle
2	-2.644224	1.898384	8.495858	2488.717234	-55.935377	maximum
3	-0.856657	0.646772	-1.484679	-187.636264	-5.870888	saddle
4	0.000000	0.000000	0.000000	80.000000	-10.000000	maximum

B) Plotting the contour and level curves of the function

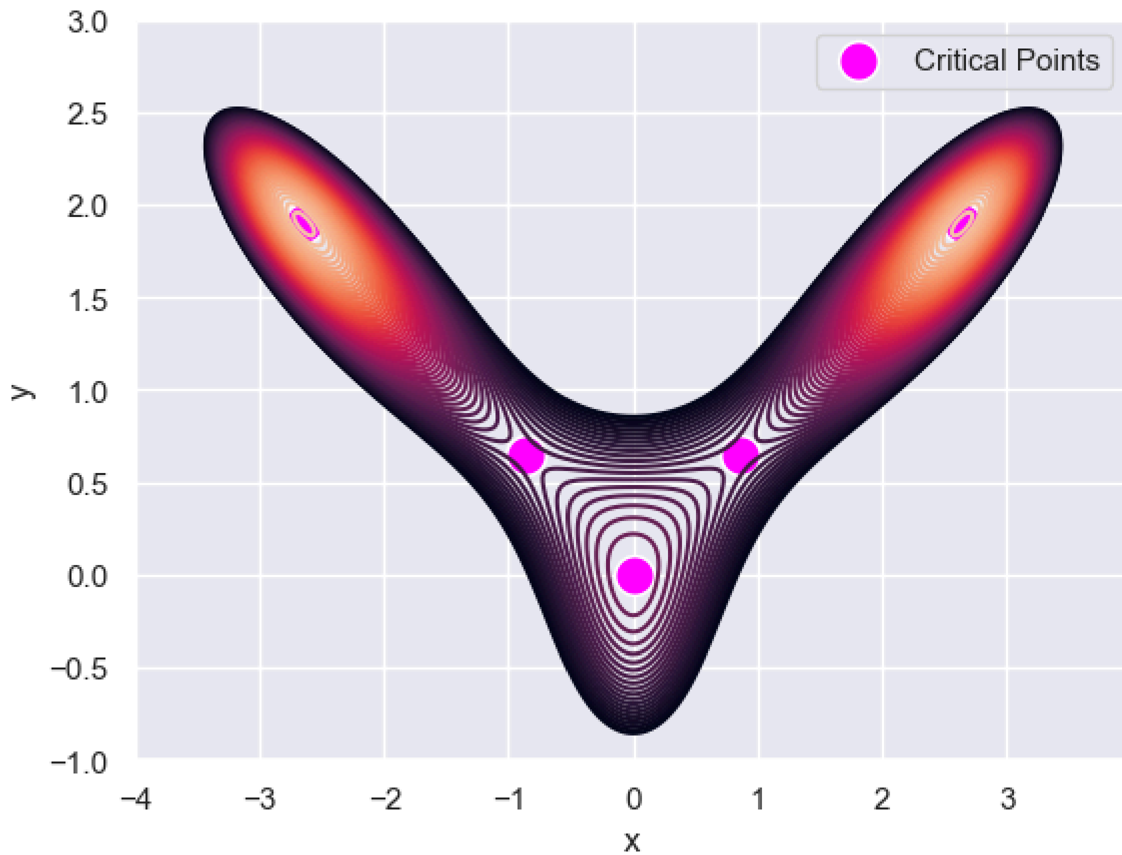
In [ ]:

```
delta = 0.025
x = np.arange(-4.0, 4.0, delta)
y = np.arange(-4.0, 4.0, delta)
X, Y = np.meshgrid(x, y)
Z = 10*X**2*Y - 5*X**2 - 4*Y**2 - X**4 - 2*Y**4

levels = np.arange(-4,10, 0.2)
```

In [ ]:

```
sns.set_theme()
plt.contour(X,Y,Z, levels = levels)
plt.ylim(-1, 3)
sns.scatterplot(data = crit, x = "x", y="y", s= 200, color = "magenta", label = "Critical Points")
plt.show()
```



Critical Points
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
y
−4
−3
−2
−1
0
1
2
3
x