P2

Monday, April 22, 2024 1:32 AM

A)

$$f(x,y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4$$

$$\frac{\partial f}{\partial x} = 20xy - 10x - 4x^3$$

$$\frac{\partial f}{\partial y} = 10x^2 - 8y - 8y^3$$

$$\nabla f = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial x \end{bmatrix}$$

Critical points when $\nabla f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix} = \begin{bmatrix} 20xy - 10x - 4x^3 \\ 10x^2 - 8y - 8y^3 \end{bmatrix}$$

(0,0) is guaranteed critical point

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2x(2x^2 - 10y + 5) \\ 10x^2 - 8(y^3 + y) \end{bmatrix}$$

$\frac{\partial f}{\partial x}$ $\frac{\partial f}{\partial y}$

$0 = -2x(2x^2 - 10y + 5)$

$0 = 2x^2 - 10y - 5$

$2x^2 = 10y - 5$

$x^2 = \frac{10y-5}{2}$

$0 = 10x^2 - 8(y^3 + y)$

$10x^2 = 8(y^3 + y)$

$x^2 = \frac{4}{5}(y^3 + y)$

$$\frac{10y-5}{2} = \frac{4}{5}(y^3 + y)$$

$$5(10y-5) = 8(y^3 + y)$$

$$50y - 25 = 8y^3 + 8y$$

$\boxed{0 = 8y^3 - 42y + 25}$ will be used to solve for y-coordinates

$$x^2 = \frac{10y-5}{2}$$

$\boxed{x = \pm\sqrt{\frac{10y-5}{2}}}$ will be used to solve for x-coordinates

$$\nabla^2 f = \begin{bmatrix} \partial^2 f/\partial x^2 & \dfrac{\partial^2 f}{\partial x \partial y} \\ \dfrac{\partial^2 f}{\partial x \partial y} & \dfrac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

$$\nabla^2 f = \begin{bmatrix} 20y - 10 - 12x^2 & 20x \\ 20x & -24y^2 - 8 \end{bmatrix}$$

$$D(x,y) = (20y - 10 - 12x^2)(-24y^2 - 8) - (20x)^2$$

used to classify points with second derivative test.

Rest of P2 in Jupyter Notebook

From numpy:

critical points are

(x,y)	$f(x,y)$
$(-2.644, 1.898)$	8.5
$(-0.856, 0.647)$	-1.48
$(0,0)$	0
$(0.856, 0.647)$	-1.48
$(2.644, 1.848)$	8.5