P3

Wednesday, March 6, 2024 4:09 PM

$Y = f(X) + \varepsilon$, ε is independent noise

Minimize

$$E\left[(Y - \hat{f}(x))^2\right]$$

$$E\left[\left(\log(x) + \varepsilon - \hat{f}(x)\right)^2\right]$$

$\hat{Y} = \log X + c$

$$E\left[\left(\log(x) + \varepsilon - (\log(x) + c)\right)\right]$$

$$E\left[(\varepsilon - c)^2\right]$$

What value of c best describes random noise of ε?

As proved in P2, the number that minimizes the squared error loss between c and ε is the mean of ε, $\bar{\varepsilon}$.

Therefore, the function, $\hat{f}(X)$, that best predicts the output is

$$\hat{f}(x) = \log(x) + \bar{\varepsilon}$$

or

$$\boxed{\hat{f}(x) = \log(x) + 10}$$