P6

Tuesday, April 23, 2024 11:07 PM

P6. (5 points) Suppose A and B are $n \times n$ matrices and $x \in \mathbb{R}^n$ are the variables. What is the gradient of the dot product of Ax with Bx? What about the Hessian?

$$f(\vec{x}) = (A\vec{x}) \cdot (B\vec{x})$$

$$\nabla f(\vec{x}) = \nabla\left[(A\vec{x}) \cdot (B\vec{x})\right]$$

$$= \nabla(A\vec{x}) \cdot B\vec{x} + A\vec{x} \cdot \nabla(B\vec{x})$$

$$= A^T B\vec{x} + A\vec{x} \cdot B^T$$

$$\nabla f(\vec{x}) = A^T B\vec{x} + A\vec{x}B^T$$

$$H_f = \nabla(\nabla f)$$

$$= \nabla\left(A^T B\vec{x} + A\vec{x} \cdot B^T\right)$$

$$= \nabla(A^T B\vec{x}) + \nabla(A\vec{x} \cdot B^T)$$

$$H_f = AB^T + A^T B$$

$$H_f = AB^T + A^T B$$