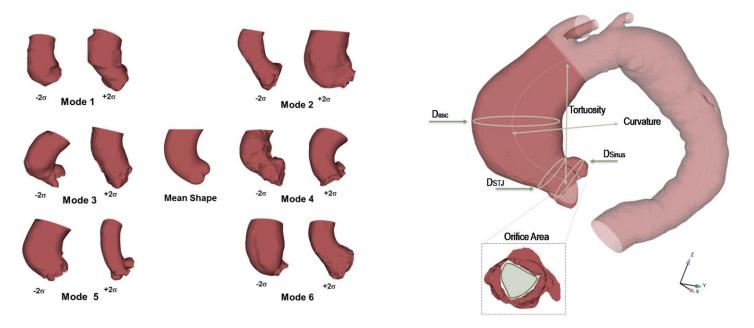




Master Ingénierie de la Santé (BMED)

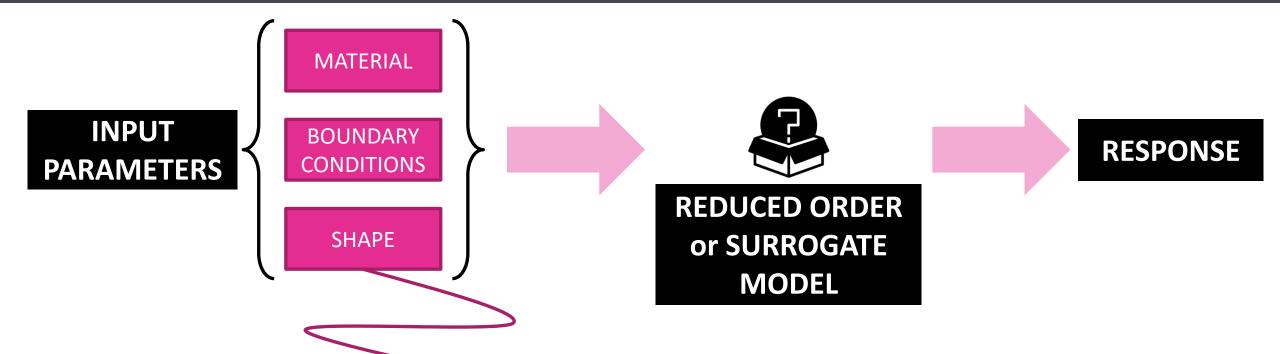


STATISTICAL SHAPE MODELING FOR BIOMECHANICS

ROM: NUMERICAL APPROACH

Beatrice Bisighini, Baptiste Pierrat

Statistical Shape Modeling in Reduced Order Modeling?



How to parametrize a set of complex and variable shapes, capturing their common geometrical features ?

Outline of the Class

Shape Modeling

Geometrical vs Statistical Shape Modeling

Mathematical Foundations of Statistical Shape Modeling

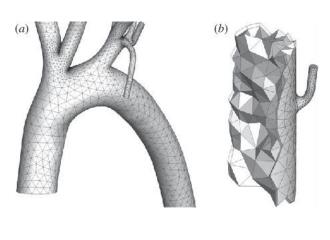
Preprocessing:

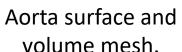
- Rigid Registration
- Iso-Topological Modeling

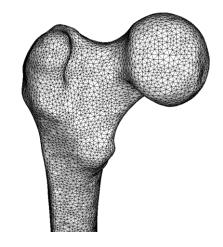
Application of Statistical Shape Modeling in Biomechanics

+ Followed by practical session

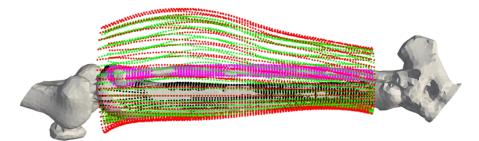
Key Concepts: Shape Representation







Femur mesh.



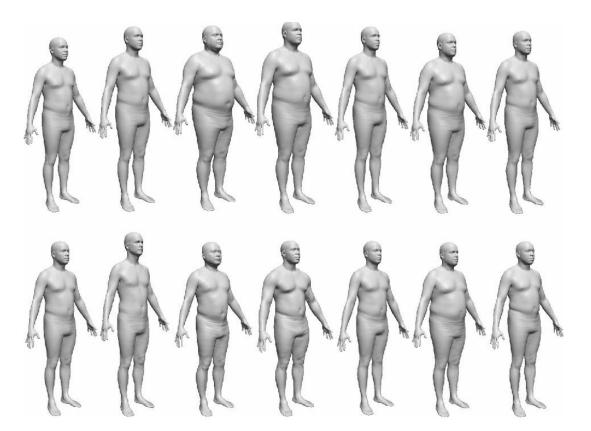
Point-cloud of the lower limb.

- Biological structures can be represented as 3D point clouds, meshes, or voxel grids.
- Each shape is described by a collection of data points that define its boundary or surface.

 $\begin{aligned} \text{SUBJECT}_{\text{i}} & \rightarrow \text{MESH}_{\text{i}} \text{ or POINT CLOUD}_{\text{i}} = \\ \begin{bmatrix} p_{x,1} \\ p_{y,1} \\ p_{z,1} \\ \dots \\ p_{x,N} \\ p_{y,N} \\ p_{z,N} \end{bmatrix} \end{aligned}$

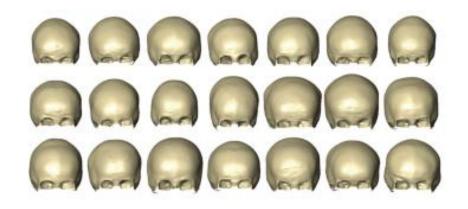
where N is the number of points/nodes.

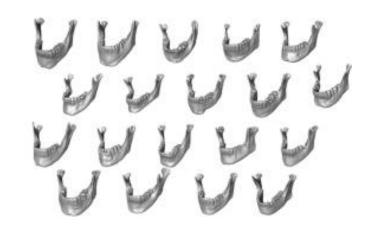
Key Concepts: Geometrical Variability



Human body shapes of different individuals

- The same anatomical structure may have different shapes across individuals or within a single subject over time.
- These variations can be due to different anatomical configurations, development patterns, disease progression, or responses to environmental factors.





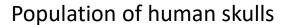
Population of human skulls

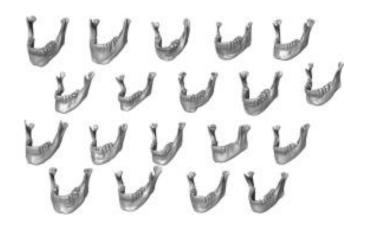
Population of human jawbones



- Analyzing Differences: Investigate how shape differences impact movement, function, performance, and health.
- **Personalized Insights**: Supports tailored treatment plans, prosthetic design, and injury prevention.







Population of human jawbones

DEFINITION

Mathematical and computational techniques to represent the **geometry** of biological objects of **semantically similar objects** (bones, organs, muscles) in a **compact** way.

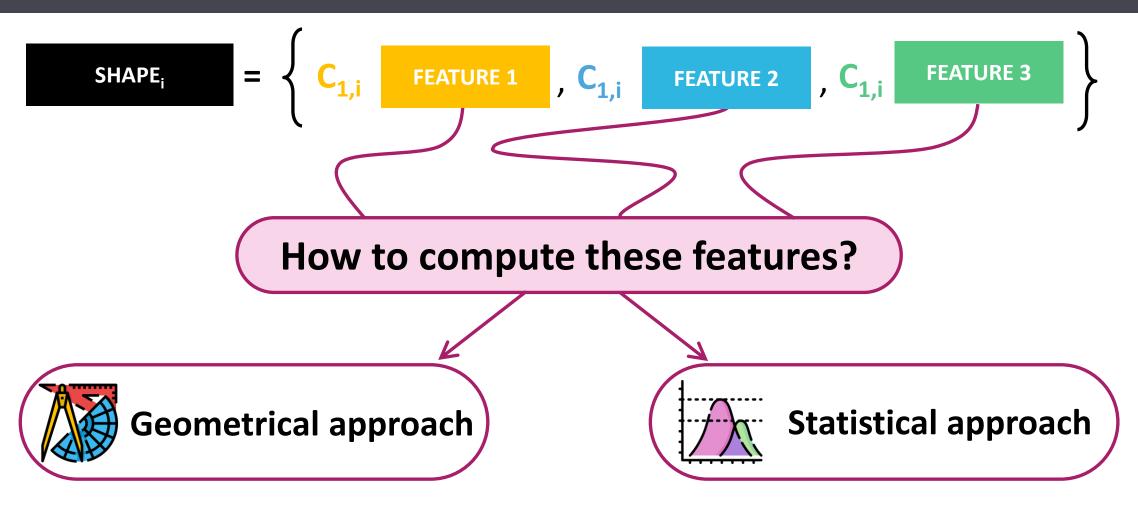
GOAL

Identify the **principal features** in shape across individuals.

$$SHAPE_{1} = \left\{ \begin{array}{cccc} C_{1,1} & FEATURE_{1} \\ \end{array} \right., C_{1,1} & FEATURE_{2} \\ \end{array} \right., C_{1,1} & FEATURE_{3} \\ \end{array}$$

$$SHAPE_{2} = \left\{ \begin{array}{cccc} C_{1,2} & FEATURE_{1} \\ \end{array} \right., C_{1,2} & FEATURE_{2} \\ \vdots & \vdots & \vdots \\ \end{array}$$

$$SHAPE_{M} = \left\{ \begin{array}{cccc} C_{1,M} & FEATURE_{1} \\ \end{array} \right., C_{1,M} & FEATURE_{2} \\ \end{array} \right., C_{1,M} & FEATURE_{3} \\ \end{array}$$



Geometrical Shape Modeling



Focuses on representing shapes and surfaces using **measurable** properties derived **directly** from the geometry.

Local variables: e.g., curvatures, distances

Global variables: e.g., surface, volume.

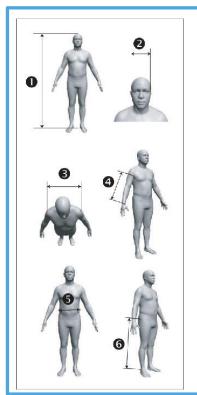


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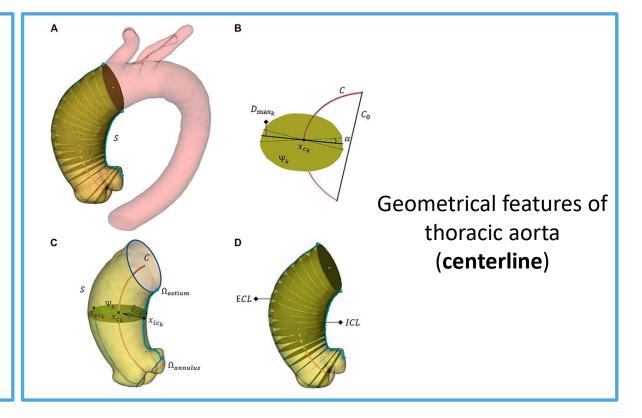
Geometrical Shape Modeling



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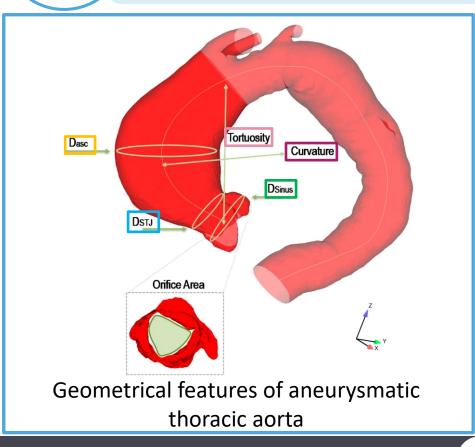
Geometrical features of human body shapes

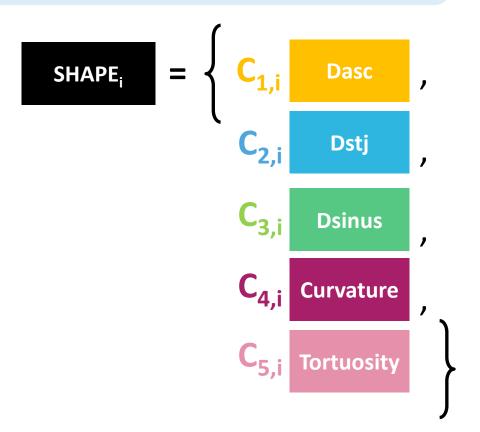


Geometrical Shape Modeling



Focuses on representing shapes and surfaces using **measurable** properties derived **directly** from the geometry.





Valsalsa sinuses, sino-tubular junction, and mid-ascending aorta

Inscribed circle



Deterministic: Measurements are calculated directly on individual geometries without relying on a population-based analysis.



Local focus: Emphasizes specific features on the surface, often neglecting broader patterns or variations.

But...



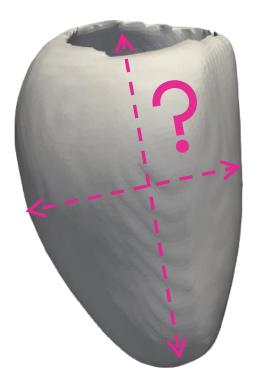
Limited Generalization: Geometry-based methods focus on local features but may **fail to capture** the **global** shape context.



Limited Generalization: Geometry-based methods focus on local features but may **fail to capture the global** shape context.



Complexity of High-Dimensional Geometry: Identifying meaningful geometric features that universally describe shape differences is challenging.





Limited Generalization: Geometry-based methods focus on local features but may fail to capture the global shape context.



Complexity of High-Dimensional Geometry:
Identifying meaningful geometric features that
universally describe shape differences is challenging.



Noise Sensitivity: Geometrical features measured directly on individual surfaces can be noisy due to artifacts in imaging or reconstruction processes.





Deterministic: Measurements are calculated directly on individual geometries without relying on a population-based analysis.



Local focus: Emphasizes specific features on the surface, often neglecting broader patterns or variations.

But...



Lack **robustness** (sensitive to noise) and **generalization** (difficult to generalize across a population).

CIT: DOI:10.4271/2004-01-2188



Statistical Shape Modeling



Uses statistical methods to analyze and represent the population-based variability of shapes in biological systems.

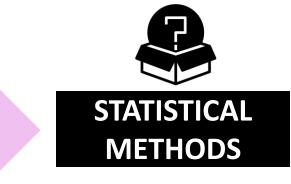
DATABASE

SUBJECT 1

SUBJECT 2

SUBJECT 3

SUBJECT N

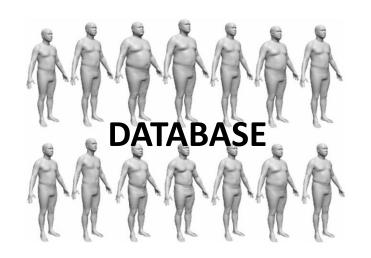




STATISTICAL SHAPE MODEL (SSM)

captures common trends, variability, and patterns in the shape data

Statistical Shape Modeling



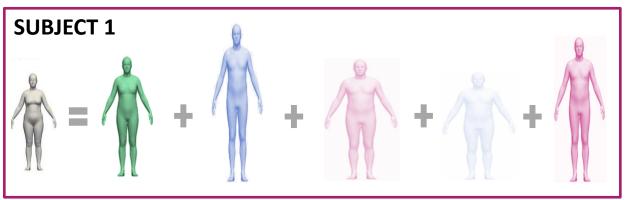
STATISTICAL SHAPE MODEL

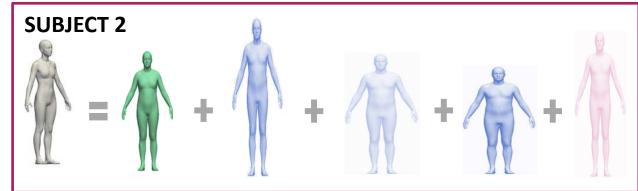
=

MEAN SHAPE

+

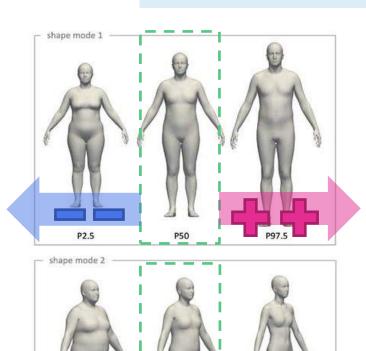
SHAPE MODES



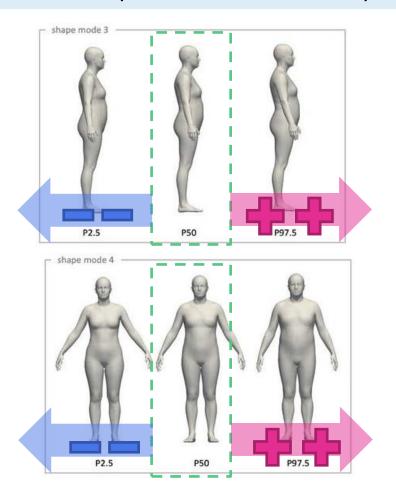


Statistical Shape Modeling

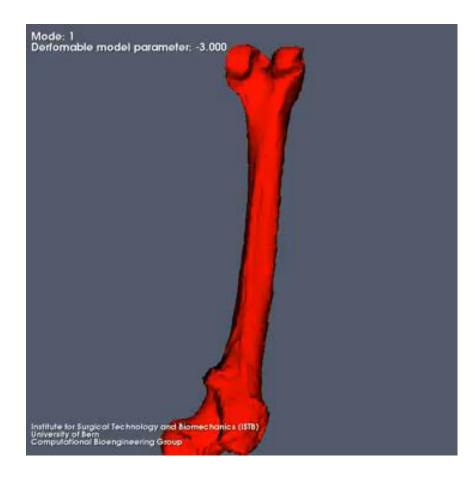
The modes of a SSM are **organized** from the most important to the least important.



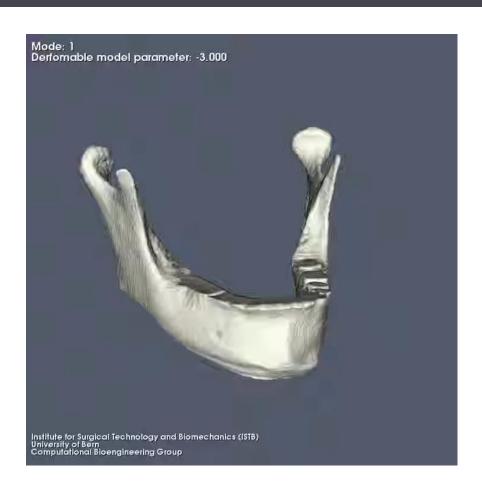
The **first modes** are **meaningful** and can be explained geometrically.



The last modes typically convey less information, often representing noise or deformations that are less significant in the dataset.



Statistical shape model of human femur



Statistical shape model of human jawbone

Statistical Shape Modeling: Pros and Cons



Global Representation: Encodes the overall structure and variability of shapes in a compact form (mean shape + deformation modes).



Robustness: Reduces the impact of noise or outliers by focusing on dominant patterns.





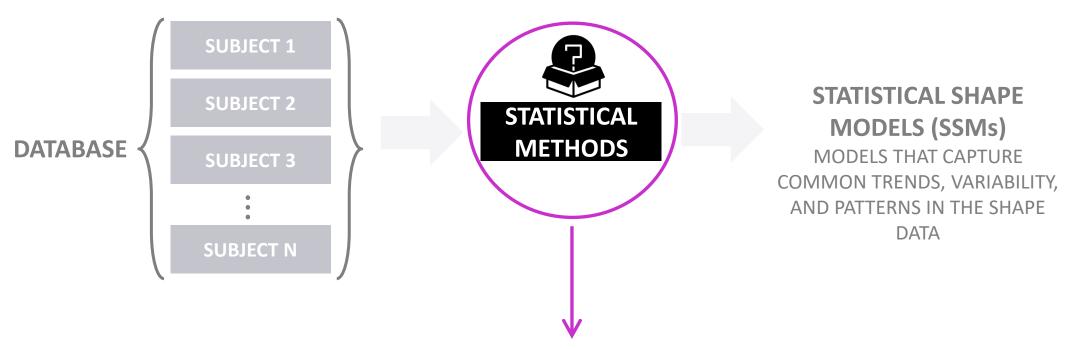
Data-driven: Requires a dataset of shapes for training.

Geometrical vs Statistical: Recap

ASPECT	GEOMETRICAL APPROACH	STATISTICAL APPROACH
BASIS	DIRECT MEASUREMENT ON INDIVIDUAL GEOMETRIES	ANALYSIS OF POPULATION-BASED SHAPE VARIATIONS
FOCUS	LOCAL OR GLOBAL FEATURES OF A SINGLE SHAPE	GLOBAL TRENDS AND VARIABILITY ACROSS SHAPES
GENERALIZATION	POOR; SHAPE-SPECIFIC	GOOD; LEARNS SHARED PATTERNS
NOISE SENSITIVITY	HIGH	LOW, DUE TO AVERAGING AND SMOOTHING
DEPENDENCE ON DATA	NONE; WORKS WITH SINGLE GEOMETRIES	REQUIRES A DATASET OF SHAPES
APPLICATIONS	FEATURE EXTRACTION, LOCALIZED ANALYSIS	RECONSTRUCTION, PREDICTION, AND INTERPOLATION



Principal Component Analysis

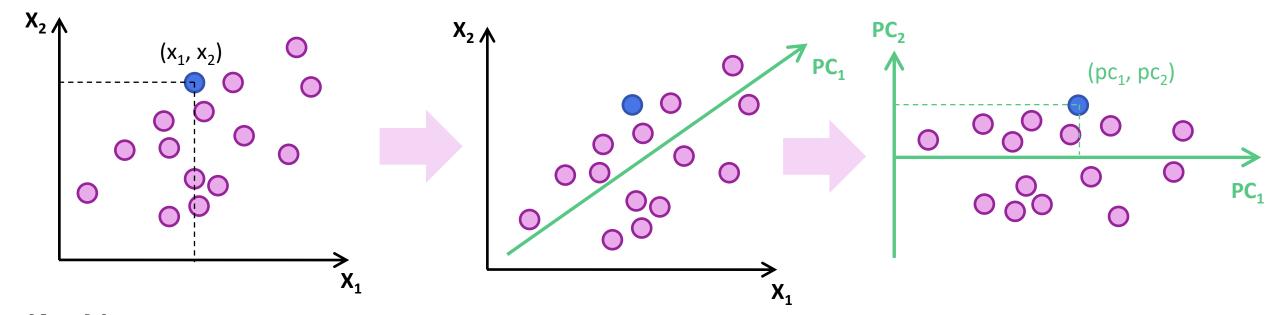


One of the most commonly used techniques:

PRINCIPAL COMPONENT ANALYSIS (PCA)

MODES = COMPONENTS

Principal Component Analysis: Key Concepts



Key Idea:

 Identify the directions (principal components/modes) that capture the most variance to create a new coordinate system.

Steps:

- Rotate data to align with the axes of maximum variance.
- Project data onto these axes (principal coefficients).



FEATURE 1

FEATURE N

One shape:

$$\boldsymbol{x}_{i} = \begin{bmatrix} p_{x,1} \\ p_{y,1} \\ p_{z,1} \\ \dots \\ p_{x,N} \\ p_{y,N} \\ p_{z,N} \end{bmatrix}$$

 $N \gg M$: the number of points per shape is much larger than than the number of shapes

matrix:

Arrange shapes from database in

 x_2 x_3 x_4 $p_{x,1} p_{x,1} p_{x,1} p_{x,1}$ $p_{x,1}$ $p_{y,1} p_{y,1} p_{y,1} p_{y,1}$ $p_{y,1}$ $p_{z,1} p_{z,1} p_{z,1} p_{z,1}$ $p_{z,1}$ $p_{x,N}p_{x,N}p_{x,N}p_{x,N}$ $p_{x,N}$ $p_{\mathsf{y},\mathsf{N}} p_{\mathsf{y},\mathsf{N}} p_{\mathsf{y},\mathsf{N}} p_{\mathsf{y},\mathsf{N}}$ $p_{y,N}$ $\lfloor p_{z,N} p_{z,N} p_{z,N} p_{z,N} \rfloor$ $p_{z,N}$

FEATURE = VARIABLES = COORDINATE

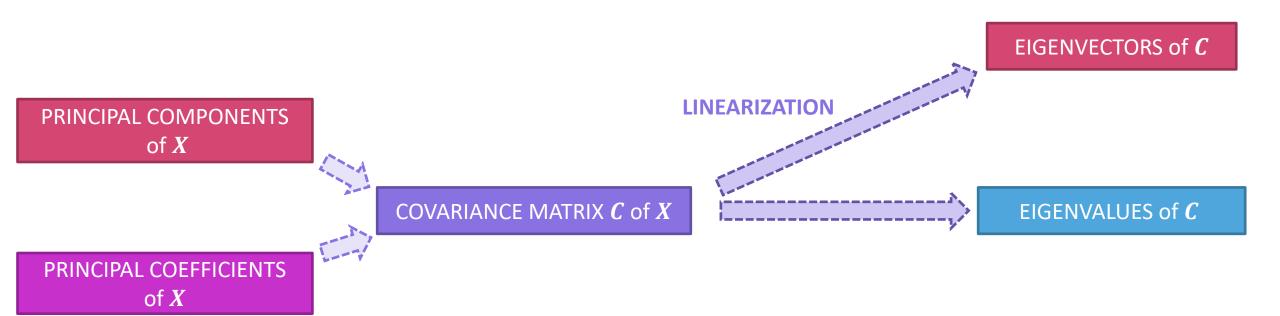
PRINCIPAL COMPONENTS of *X*

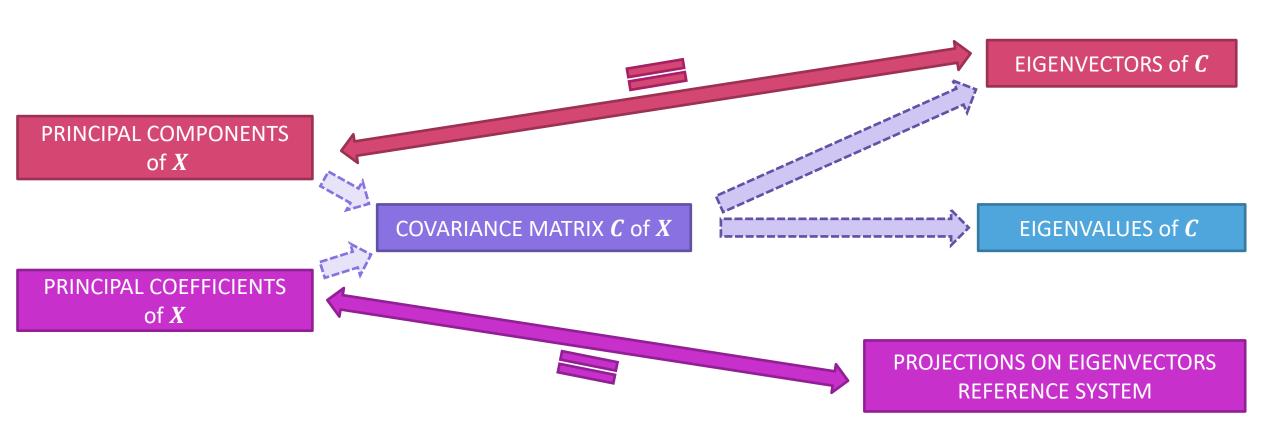
COVARIANCE MATRIX C of X

PRINCIPAL COEFFICIENTS of *X*

Quantifies the relationships between features, enabling PCA to identify directions of maximum variance in the data.

Square matrix where each element $C(x_i, x_j)$ represents the covariance between the x_i and x_j features.





Compute the Covariance

The covariance matrix (\boldsymbol{C} , dimensions: $N \times N$) captures the relationship between pairs of features:

$$C = \frac{1}{N-1} X X^{\mathrm{T}}$$

Symmetric with diagonal elements representing the variance of each feature, and off-diagonal elements representing the covariance between features.

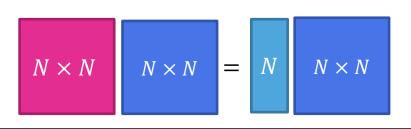
Linearization: Eigenvectors and Eigenvalues (→ Compute principal components)

The eigenvectors (V, dimensions: $N \times N$) of the covariance matrix represent the principal components and the eigenvalues (λ , dimensions: N) indicate the amount of variance captured by each component.

$$C V = \lambda V$$

 $C V = \lambda V$ eigenvalue equation

How many principal components?





The eigenvalues are sorted from the largest to the smallest: $\lambda_1 \gg \lambda_2 \gg ... \gg \lambda_N$.

5 Projection on the principal components (→ Compute principal coefficients)

The principal coefficients w are derived by projecting each vector x into the reference system defined by the principal component (V).

For one shape:
$$w = V^T x$$

$$N = N \times N$$

For all the shapes: $W = V^T X$



Principal coefficients are scalar value providing the **geometrical** influence of each shape mode (= principal component) on the final deformed shape (x).

6 Reconstruction using principal components and coefficients

$$w = V^{\mathrm{T}} x$$

$$x = V w = (w_1)v_1 + w_2(v_2) + w_3 v_3 + w_4 v_4 + \cdots + w_{N-1} v_{N-1} + w_N v_N$$



i-element of the vector w (i = 1 ... N)

i-column of the matrix
$$V$$
 $(i = 1 ... N)$

6 Reconstruction using principal components and coefficients

6 Reconstruction using principal components and coefficients

$$x = w V = w_1 v_1 + w_2 v_2 + w_3 v_3 + w_4 v_4 + \dots + w_{N-1} v_{N-1} + w_N v_N$$

The eigenvalues:

- are ordered from largest to smallest.
- indicate the amount of variance captured by each component.

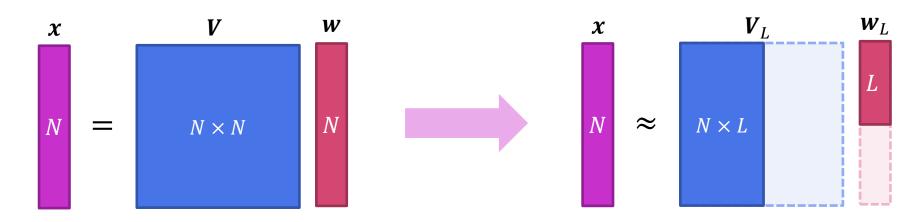
The first principal components can explain most of the variance within the original dataset.

The vector x can be accurately approximated using only the first L principal components.

6 Reconstruction using principal components and coefficients

$$\boldsymbol{x} = \boldsymbol{w} \, \boldsymbol{V} \approx \underbrace{\boldsymbol{w}_1 \, \boldsymbol{v}_1 + \boldsymbol{w}_2 \, \boldsymbol{v}_2 + \boldsymbol{w}_3 \, \boldsymbol{v}_3 + \boldsymbol{w}_4 \, \boldsymbol{v}_4 + \dots + \boldsymbol{w}_{N-1} \, \boldsymbol{v}_{N-1} + \boldsymbol{w}_N \, \boldsymbol{v}_N}_{L}$$

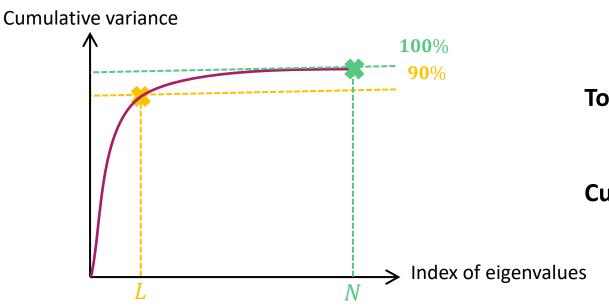
The vector x can be accurately approximated using only the first L principal components.



$$x \approx V_L w_L$$

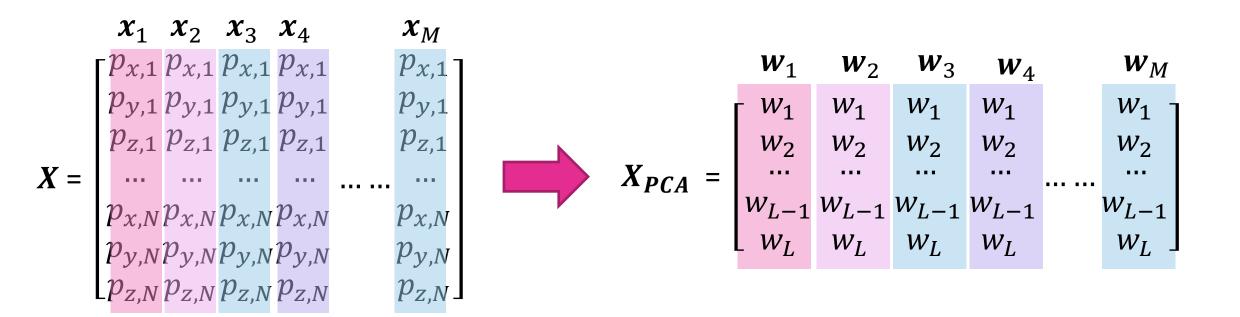
How to choose *L*?

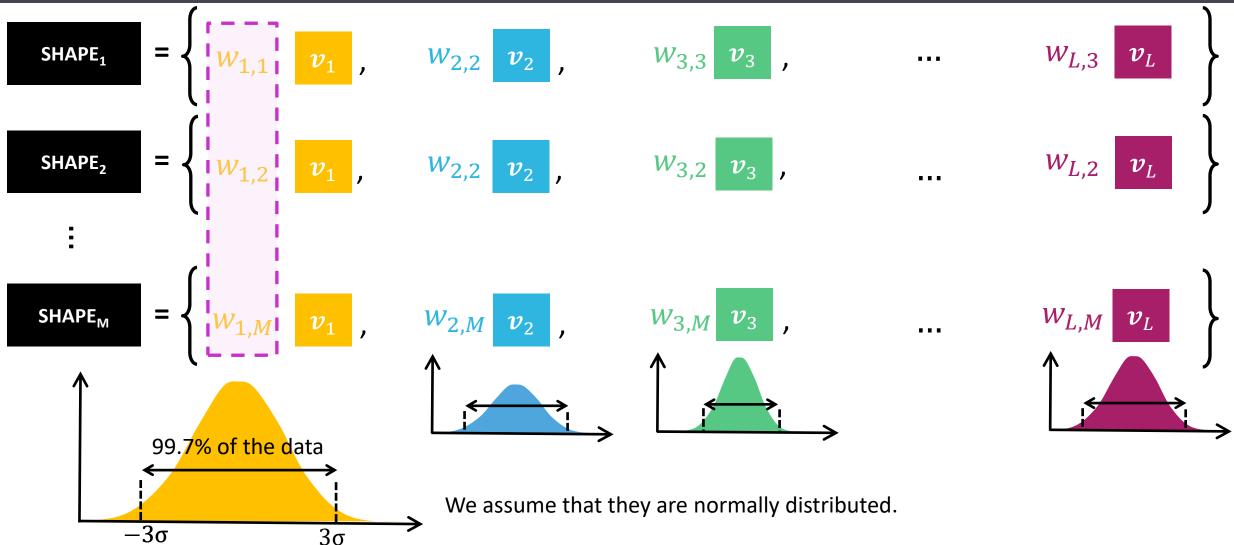
The eigenvalues are sorted from the largest to the smallest: $\lambda_1 \gg \lambda_2 \gg ... \gg \lambda_N$.

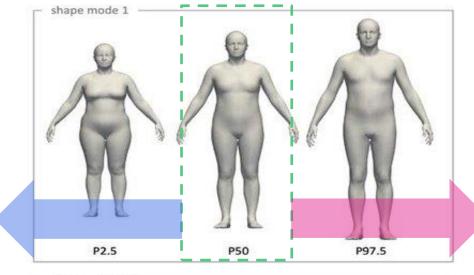


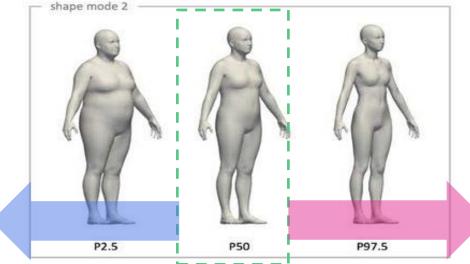
Total variance = $\sum_{i=1}^{N} \lambda_i$

Cumulative variance (compactness) = $\frac{\sum_{i=1}^{L} \lambda_i}{\sum_{i=1}^{N} \lambda_i}$





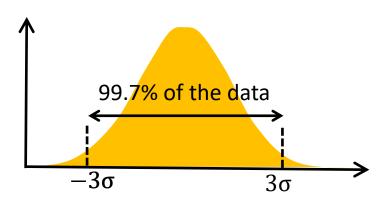




Visualization of the effect of one principal component:

$$x_k = \xi v_k$$

 $m{v}_k$ is the k-th column of V ξ bounded between \pm 3 σ of $m{w}_k$ (the principal coefficient related to $m{v}_k$ across the database)



ADVANTAGES

CHALLENGES AND LIMITATIONS

Feature Extraction: Identifies key patterns and trends in the data by capturing maximum variance.

Dimensionality Reduction: Reduces high-dimensional data into a smaller number of meaningful components, simplifying analysis and visualization.

Noise Reduction: Filters out noise by discarding components with low variance, leading to cleaner datasets.

Complementary to Augmentation: Can be applied to augment datasets.

- Assumes linear relationships.
- May lose interpretability of features.
- Sensitive to input data.

Standardization:
$$\overline{X} = \frac{X - \mu}{\sigma}$$
Scaling: $\overline{X} = X - \mu$

 σ = row-wise standard deviation

 μ = row-wise average

Why and What Preprocessing for PCA?



PCA assumes that all shapes are already aligned in a common coordinate system.



RIGID REGISTRATION



PCA assumes that all shapes have the same topological structure (e.g., same number of vertices, edges, and faces).



ISO-TOPOLOGICAL MODELING

Rigid Registration: Overview

DEFINITION

Aligns two or more shapes by applying only translation and rotation (no deformation).

GOAL

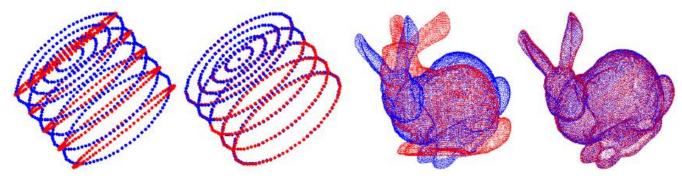
Minimize differences between shapes while maintaining original geometry.

APPLICATIONS

- · Align anatomical structures for comparison.
- Ensure consistent orientation in datasets.
- Initialize non-rigid registration.

Transformation Equation:

$$X_{aligned} = \mathbb{R} X_{original} + t$$



Examples of shapes before and after rigid alignment

Template mesh
Target mesh

Rigid Registration: Common Methods



HOW TO COMPUTE ROTATION MATRIX AND TRANSLATION VECTOR?

Several methods are possible:

(1) Iterative Closest Point (ICP):

Iteratively matches points and minimizes distances.

(2) Coherent Point Drift (CPD):

Treats shapes as probability distributions for smoother alignment.

(3) Principal Component Analysis (PCA):

Aligns principal axes of the shapes for a quick initial fit.

+ many more...

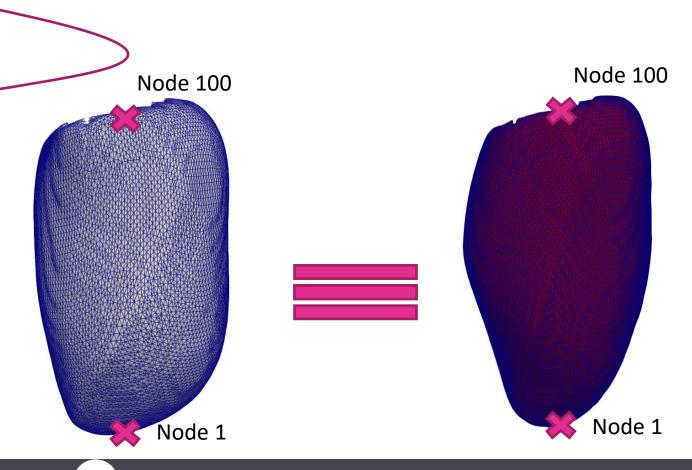
Open source implementations of these algorithms are available, e.g. Python (VTK).

Iso-Topological Modeling: Overview and Methods

DEFINITION

Represent with consistent topology across the dataset.

All shapes **share** the **same structure**: not only same number of points or vertices but also correspondence between them .



Iso-Topological Modeling: Overview and Methods

DEFINITION

Represent with **consistent** topology across the dataset.

Several methods are possible:

(1) REMESHING

Simplifies or densifies the mesh to achieve uniform resolution.

(2) PARAMETRIZATION

Maps 3D shapes to a lower-order domain for easier manipulation.

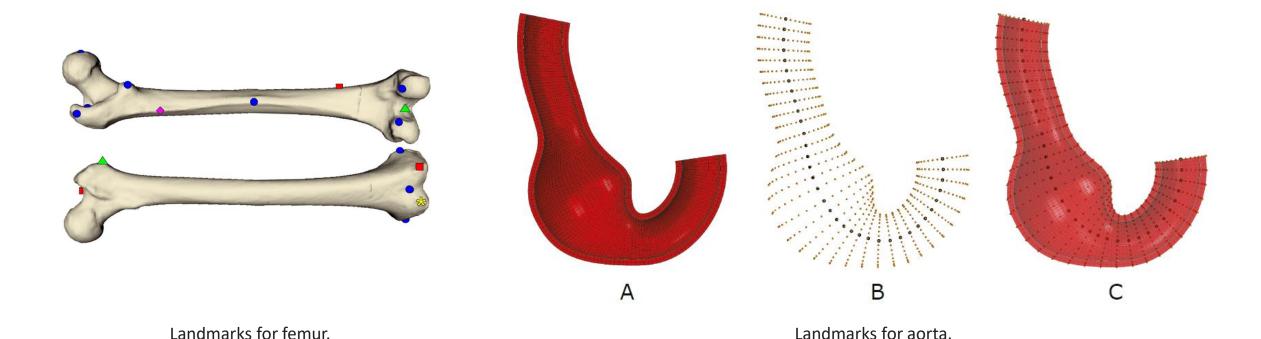
(3) TEMPLATE FITTING

Deforms a standard template shape to match individual samples.

NON-RIGID REGISTRATION

Iso-Topological Modeling: Non-Rigid Registration

LANDMARKS: specific, identifiable points or features on a shape, image, or structure that are used as references to guide the alignment process in nonrigid registration. Can be **manual** (selected by experts) or **automatic** (detected using algorithms).

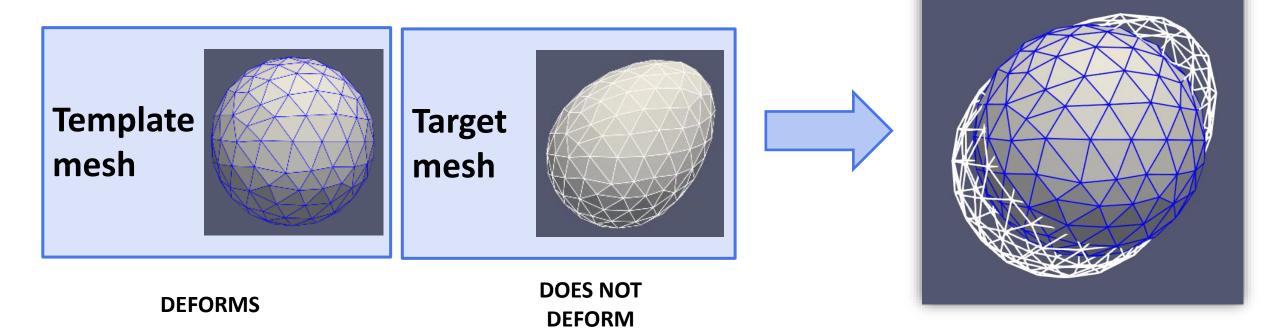




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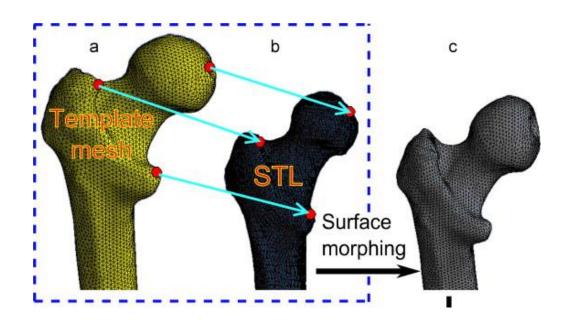
MESH MORPHING: process of smoothly deforming a mesh (a geometric representation of a surface using vertices, edges, and faces) to align with another shape or structure (template).



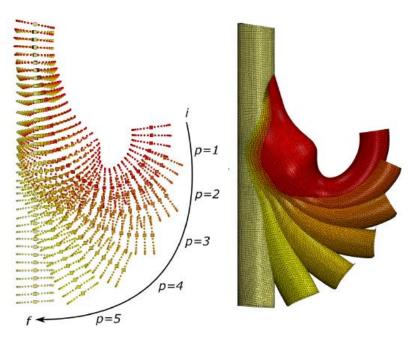
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MESH MORPHING: process of smoothly deforming a mesh (a geometric representation of a surface using vertices, edges, and faces) to align with another shape or structure (template).



Mesh morphing for femur.



Mesh morphing for aorta.

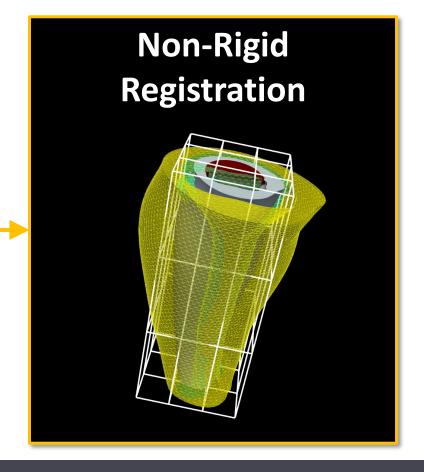
Non-Rigid Registration: An example

Example: Free form deformation applied to lower limb





Rigid
Registration:
Rotate + Trans.



Statistical Shape Modeling: Pipeline

Data Collection and Preprocessing

Gather a set of shapes (e.g., 3D scans) and extract shape as surface (e.g. meshes).

Rigid registration

Apply rigid registration methods to bring all shapes into a common coordinate frame and align them.

Isotopological Modeling

Adjust models to ensure all shapes share a consistent topological structure.

Principal Component Analysis

Extract modes of variation and reduce dimensionality.

Shape Analysis

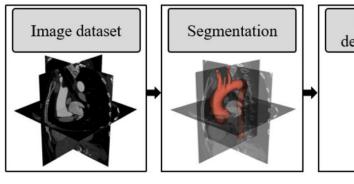
Use principal components to study and compare shapes, generate new shapes, or predict missing data.

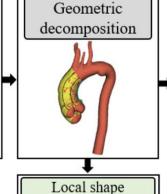


Example

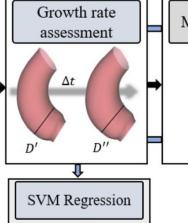
Compare local and global shape features in predicting ascending aortic aneurysm (AsAA) growth rates.

3D imaging of 70 patients with two scans over six months.

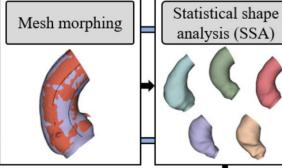


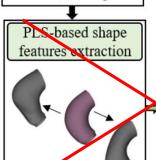


features extraction



Local shape features

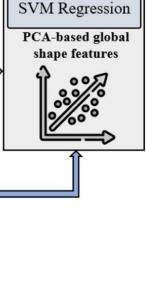




PCA-based shape

features extraction

analysis (SSA)



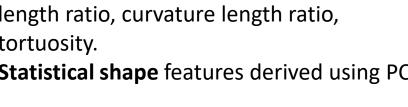
PLS Regression

PLS-based

global shape features

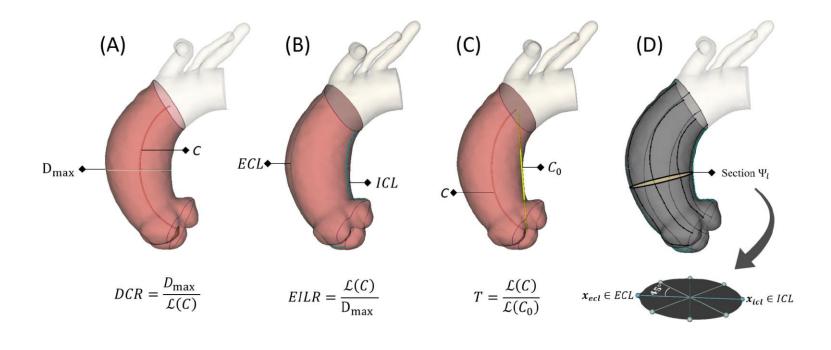
Geometrical shape features: diameter-tolength ratio, curvature length ratio, tortuosity.

Statistical shape features derived using PCA



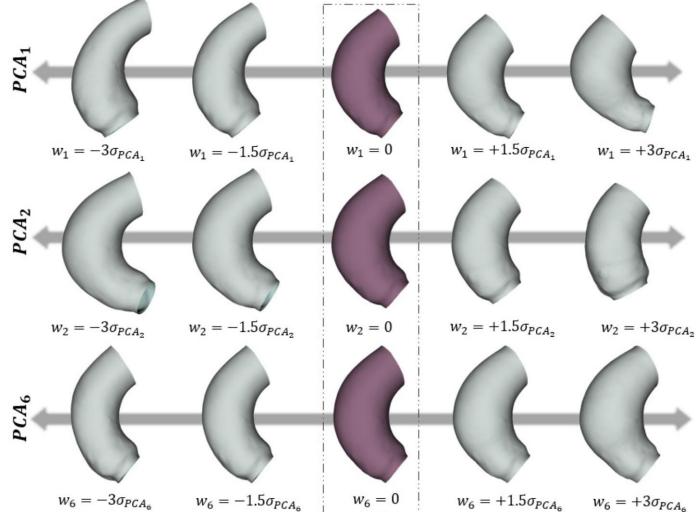


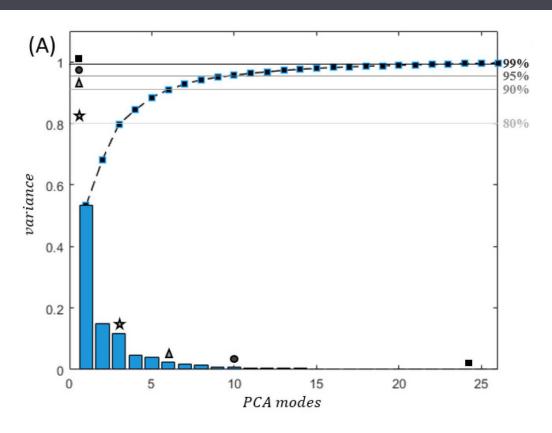
First example



Geometrical shape features

First example





Statistical shape features

First example

Which shape features were most effective in predicting aneurysm growth?

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Computer-aided shape features extraction and regression models for predicting the ascending aortic aneurysm growth rate

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https://t.ly/tVvRg

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