1

$EQ_{CFG} = \{\langle G, H\rangle | G$ and H are CFGs and $L(G) = L(H)\}$
Claim: EQ_{CFG} is undecidable.
Proof: Suppose EQ_{CFG} is decidable, and let R be its decider. Let S be a decider for $ALL_{CFG} = \{\langle J\rangle | J$ is a CFG and $L(J) = \Sigma^*\}$ defined as:
$S =$ "On input $\langle G\rangle$, where G is a CFG

1. Simulate R on input $\langle G, H_1\rangle$ where H_1 is a CFG whose language $L(H_1) = \Sigma^*$.
2. If R accepts, then *accept*. If R rejects, *reject*."

Because Theorem 5.13 says that ALL_{CFG} is undecidable, this results in a contradiction. Therefore EQ_{CFG} can not be decidable.

2

Claim: EQ_{CFG} is co-Turing-recognizable.
Proof: EQ_{CFG} is co-Turing-recognizable if $\overline{EQ_{CFG}}$ is Turing-recognizable. $\overline{EQ_{CFG}} = \{\langle G, H\rangle | G$ is not a CFG, H is not a CFG, or both are CFGs and $L(G) \neq L(H)\}$. Define S, the recognizer for $\overline{EQ_{CFG}}$ as follows:
$S =$ "On input $\langle G, H\rangle$:

1. If G is not a CFG, *accept*.
2. If H is not a CFG, *accept*.
3. Iterate through all strings in lexicographical order. If G and H both generate the string or neither generate the string, repeat this step. Otherwise, *accept*."

3

Let $T = \{\langle M\rangle | M$ is a TM that accepts w^R whenever it accepts $w\}$.
Claim: T is undecidable.
Proof: Suppose T is decidable, and let R be its decider. Let S be a decider for $A_{TM} = \{M | M$ is a TM and M accepts $w\}$ defined as:
$S =$ "On input $\langle M, w\rangle$ where M is a Turing machine and w is a string:

1. Construct the following Turing machine M':
   $M' =$ "On input x:
   (a) If x is 10, *accept*.
   (b) If x is 01, run M on input w and *accept* if M accepts.
   (c) *Reject* all else."
2. Run R on input $\langle M'\rangle$. If R accepts, *accept*. If R rejects, *reject*."

R only accepts when M accepts w, so S is a decider for A_{TM}. Because A_{TM} is undecidable, this results in a contradiction. Therefore T can not be decidable.

I affirm that I have upheld the highest principles of honesty and integrity in my academic work and have not witnessed a violation of the honor code.