① Prove $\hat{\alpha}_t(j) = \prod_{s=1}^{t} C_s \alpha_t(j)$

$\because C_s = \dfrac{1}{\sum_{j=1}^{N} \tilde{\alpha}_s(j)}$

$\therefore \prod_{s=1}^{t} C_s = \dfrac{1}{\prod_{s=1}^{t} [\sum_{j=1}^{N} \tilde{\alpha}_s(j)]} = \dfrac{1}{P(O(t)|\lambda)}$ ①

$O(t)$ means first t words of observation O.

$\because \alpha_t(j) = P(O(t), q_t = j \mid \lambda)$

$\therefore \prod_{s=1}^{t} C_s \cdot \alpha_t(j) = P(O(t), q_t = j \mid \lambda) \cdot \dfrac{1}{P(O(t)|\lambda)}$

$= P(q_t = j \mid O(t), \lambda) = \hat{\alpha}_t(i)$

② Prove that $\sum_j \alpha_T(j) = \dfrac{1}{\prod_{s=1}^{T} C_s}$

$\sum_{j=1}^{N} \alpha_T(j) = \sum_{j=1}^{N} P(O, q_T = j \mid \lambda) = P(O|\lambda)$

$\because \prod_{s=1}^{T} C_s = \dfrac{1}{P(O(T)|\lambda)} = \dfrac{1}{P(O|\lambda)}$ (because of ①)

$\therefore \sum_{j=1}^{N} \alpha_T(j) = \dfrac{1}{\prod_{s=1}^{T} C_s}.$