

Spatial Epidemics Dynamics: Synchronization

Mathematics 4MB3/6MB3
Mathematical Biology

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Introduction

Diseases are fun

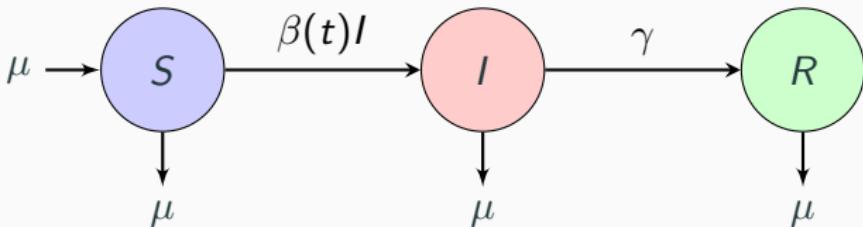
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Diseases are cool

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Methods

Forced SIR Model for a Single Patch



$$\frac{dS}{dt} = \mu - \beta(t)SI - \mu S$$

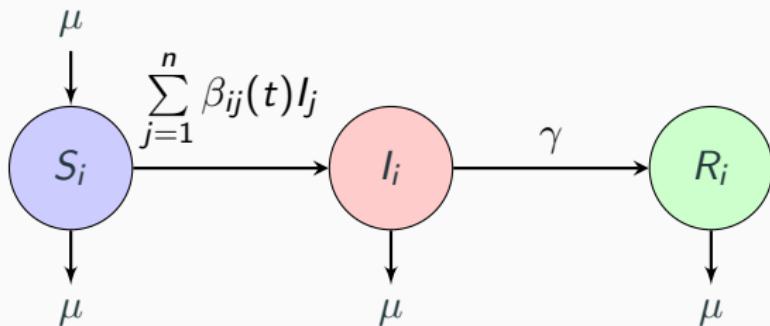
$$\frac{dI}{dt} = \beta(t)SI - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

$$\beta(t) = \langle \beta \rangle (1 + \alpha \cos(2\pi t))$$

$$\langle \beta \rangle = \mathcal{R}_0(\mu + \gamma)$$

Metapatch SIR Model



$$\frac{dS_i}{dt} = \mu - S_i \sum_{j=1}^n \beta_{ij}(t) I_j - \mu S_i$$

$$\frac{dI_i}{dt} = S_i \sum_{j=1}^n \beta_{ij}(t) I_j - \gamma I_i - \mu I_i$$

$$\frac{dR_i}{dt} = \gamma I_i - \mu R_i$$

Beta Matrix

$$\beta(t) = \langle \beta \rangle (1 + \alpha \cos(2\pi t)) M$$

Equal Coupling Matrix

$$M = \begin{bmatrix} 1-m & \frac{m}{n-1} & \frac{m}{n-1} & \frac{m}{n-1} & \dots \\ \frac{m}{n-1} & 1-m & \frac{m}{n-1} & \frac{m}{n-1} & \\ \frac{m}{n-1} & \frac{m}{n-1} & 1-m & & \\ \vdots & & & \ddots & \end{bmatrix}$$

Beta Matrix

$$\beta(t) = \langle \beta \rangle (1 + \alpha \cos(2\pi t)) M$$

Nearest Neighbour Matrix

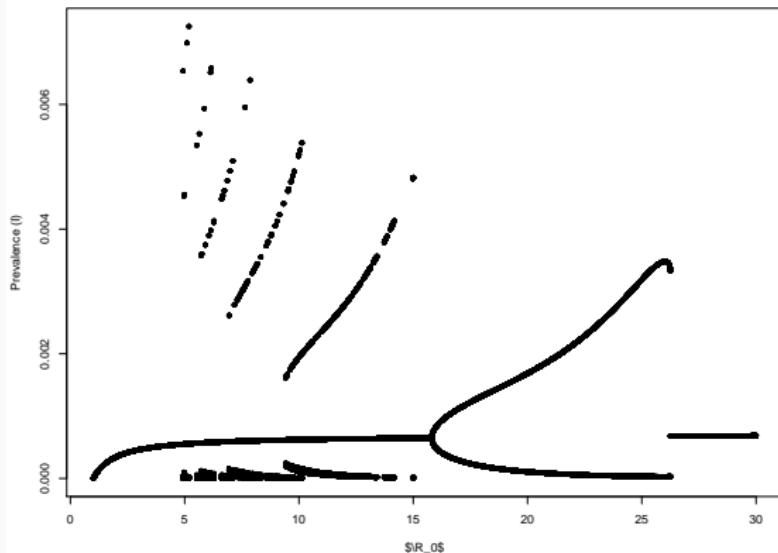
$$M = \begin{bmatrix} 1-m & \frac{m}{2} & 0 & 0 & \dots & \frac{m}{2} \\ \frac{m}{2} & 1-m & \frac{m}{2} & 0 & & \vdots \\ 0 & \frac{m}{2} & 1-m & & & \\ 0 & 0 & & \ddots & & \\ \vdots & & & & \ddots & \frac{m}{2} \\ \frac{m}{2} & & \dots & \frac{m}{2} & 1-m & \end{bmatrix}$$

Adaptive Tau Algorithim

Results

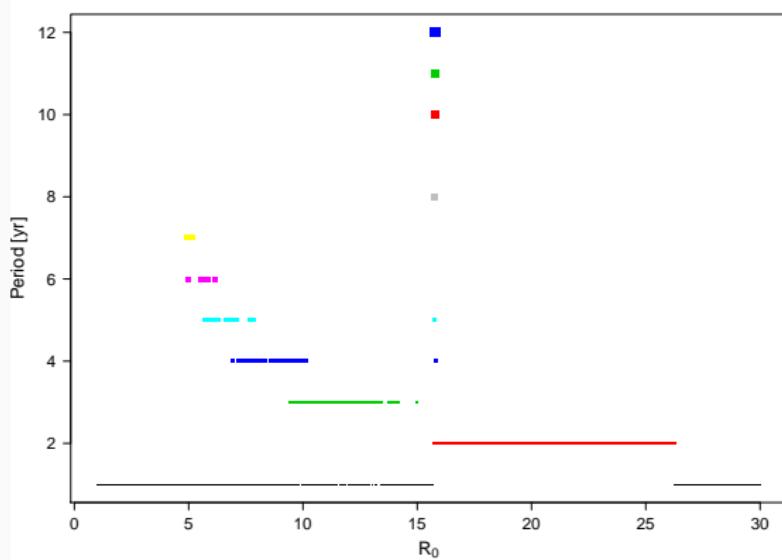
Bifurcation Diagram

Single Patch SIR model with sinusoidal seasonal forcing



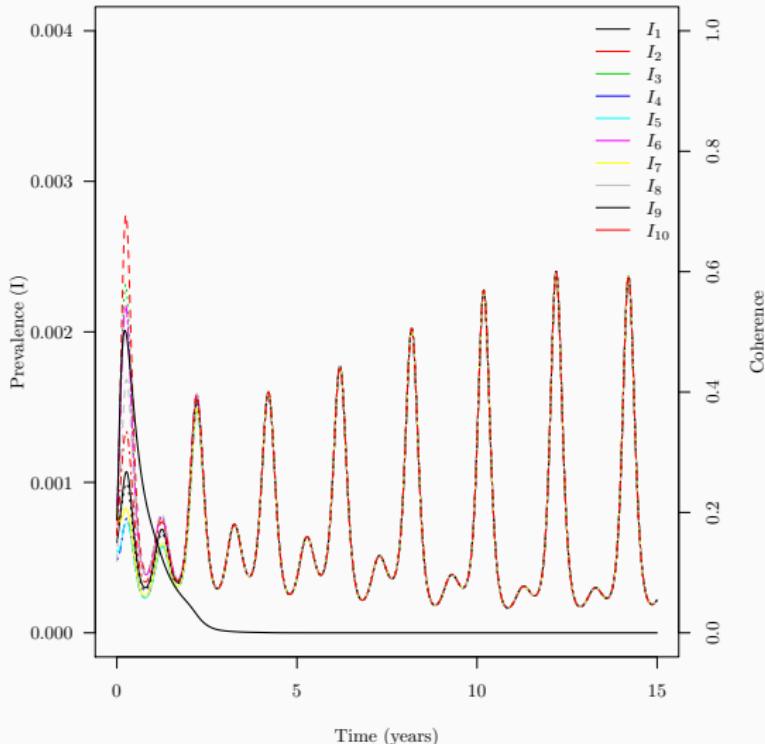
Period Diagram

Period of the oscillations



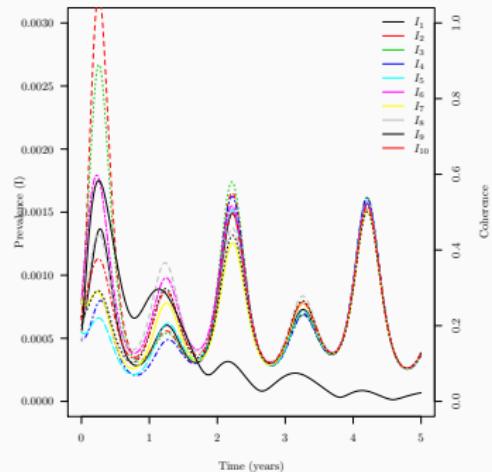
Deterministic Model

Equal Coupling, $\mathcal{R}_0 = 17$, $m = 0.2$

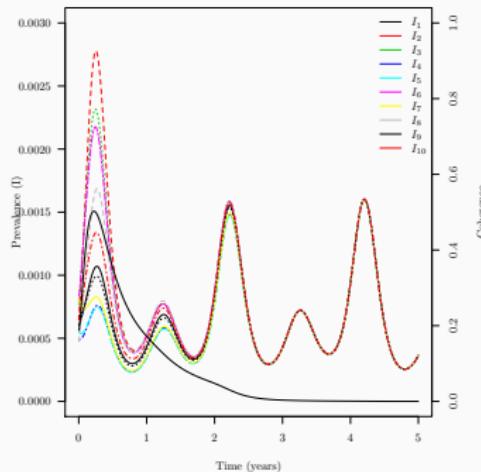


Deterministic Model: EC vs NN

$$\mathcal{R}_0 = 17, m = 0.2$$



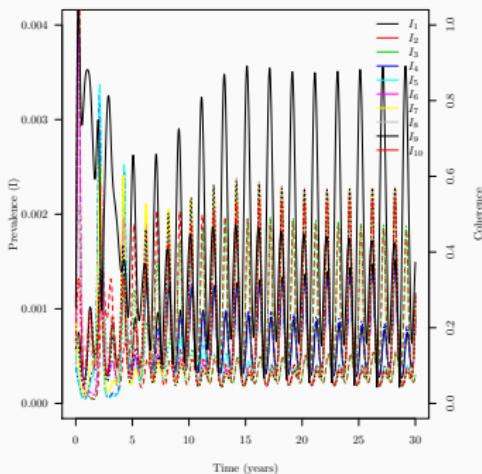
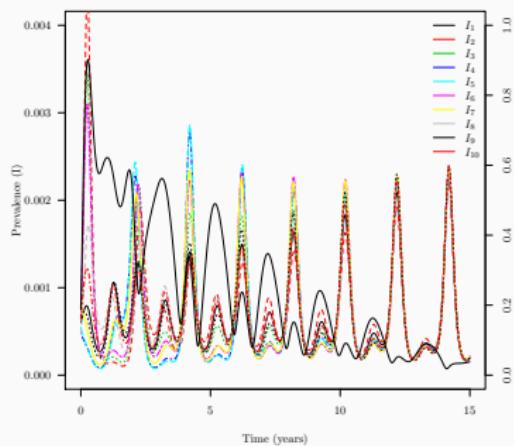
Nearest Neighbour Coupling



Equal Coupling

Deterministic Model

Nearest Neighbour Coupling, $\mathcal{R}_0 = 17$, $m = 0.01$



Initial Conditions

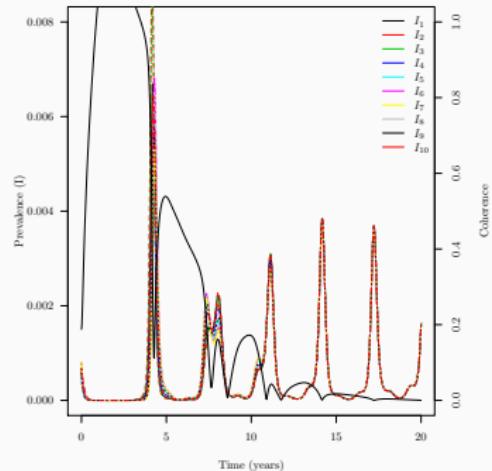
Within 30% percent of endemic equilibrium

Initial Conditions

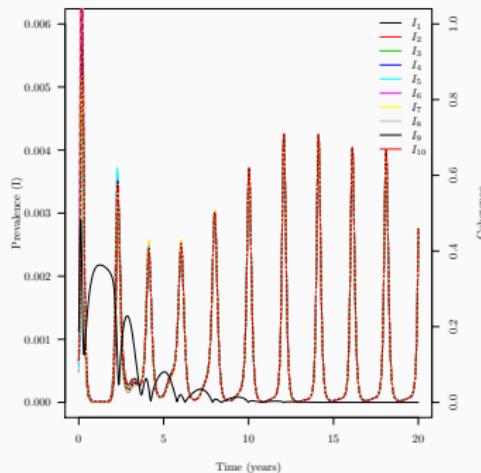
Within 40% percent of endemic equilibrium

Deterministic Model

Nearest Neighbour Coupling, $m = 0.2$



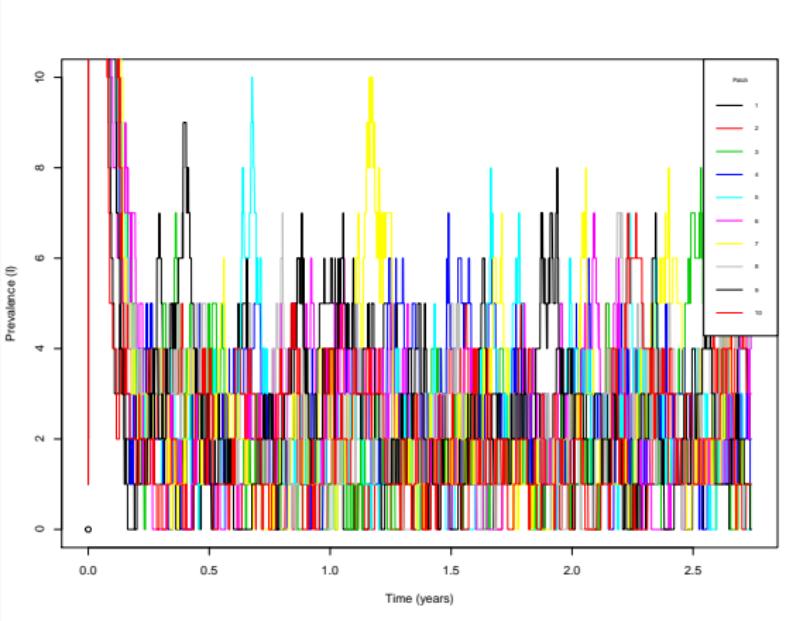
$$\mathcal{R}_0 = 10$$



$$\mathcal{R}_0 = 25$$

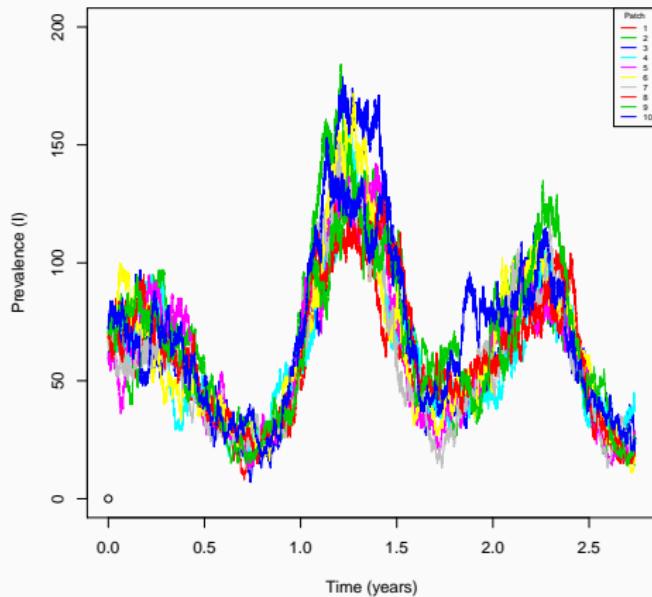
Stochastic: Gillespie Model

Equal Coupling, Population of 3000, $\mathcal{R}_0 = 17$, $m = 0.2$



Stochastic: Gillespie Model

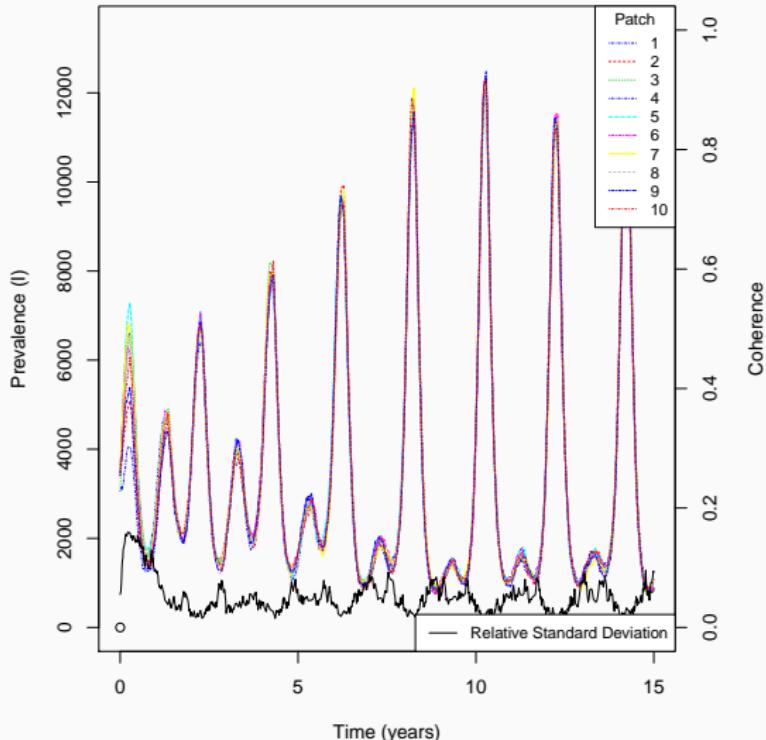
Equal Coupling, Population of , $\mathcal{R}_0 = 17$, $m = 0.2$



3 hour run time!

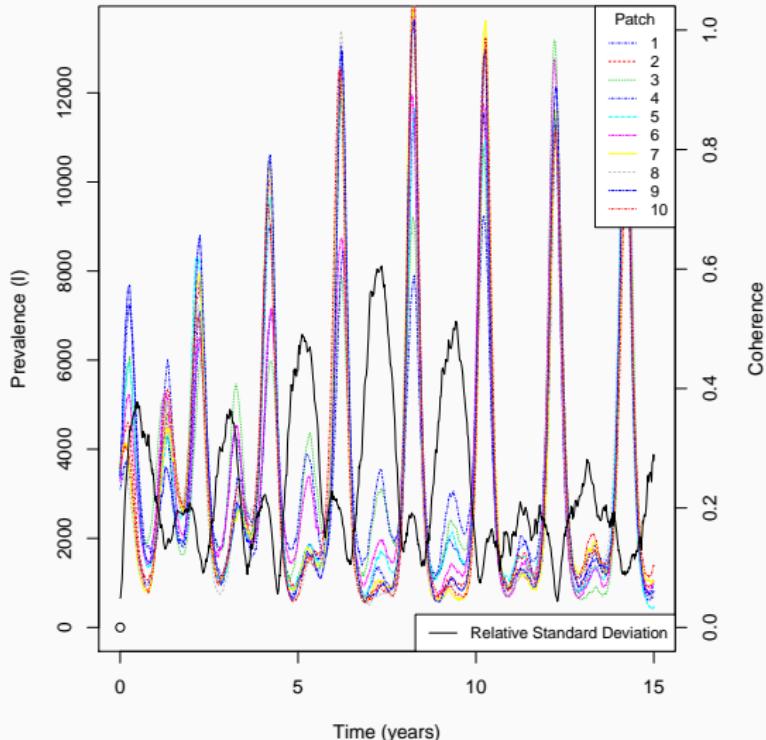
Stochastic: Adaptive Tau Algorithm

Equal Coupling, Population of 500,000, $\mathcal{R}_0 = 17$, $m = 0.2$



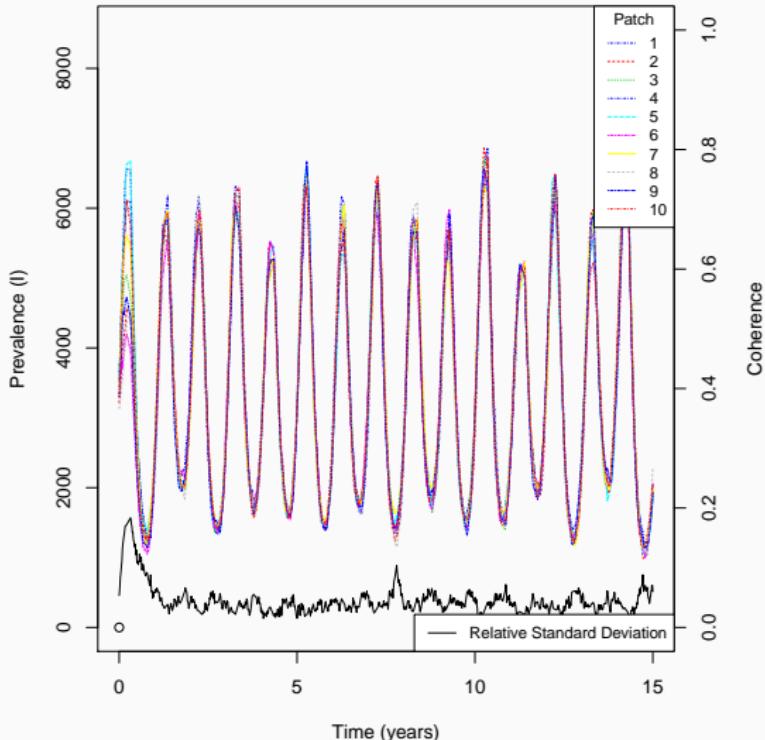
Stochastic: Adaptive Tau Algorithm

Equal Coupling, Population of 500,000, $\mathcal{R}_0 = 17$, $m = 0.01$



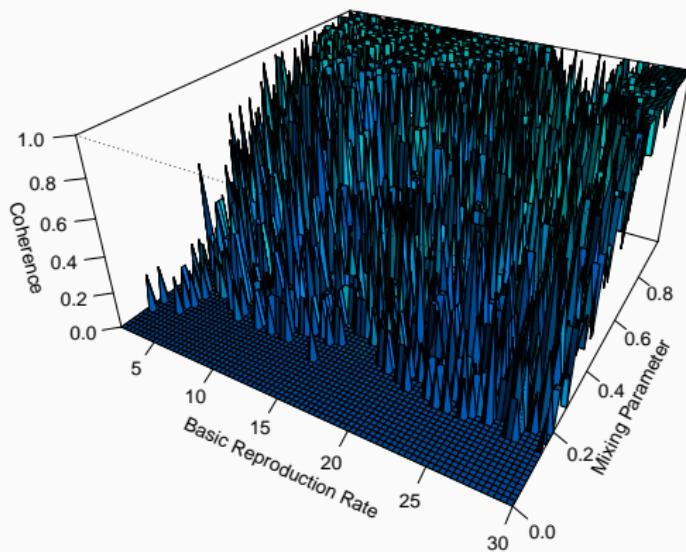
Stochastic: Adaptive Tau Algorithm

Equal Coupling, Population of 500,000, $\mathcal{R}_0 = 25$, $m = 0.2$



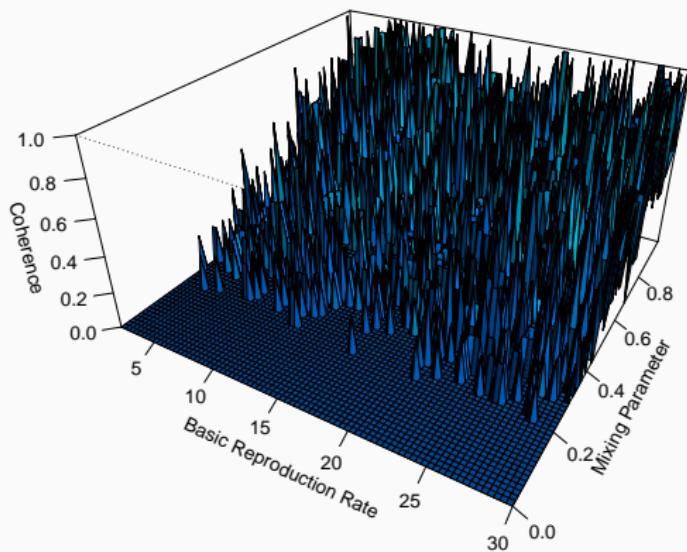
Coherence dependence on Parameters

Equal Coupling, Population of 250,000,
 80×80 grid, 5 simulations per grid point



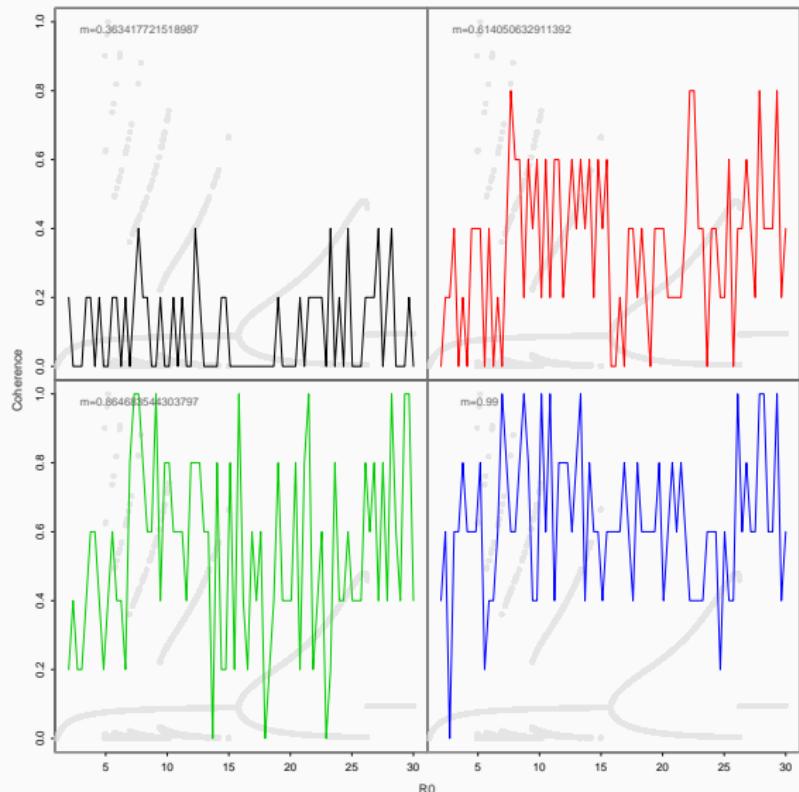
Coherence dependence on Parameters

Nearest Neighbour Coupling, Population of 250,000,
 80×80 grid, 5 simulations per grid point



Coherence dependence on Parameters

Nearest Neighbour Coupling, Population of 250,000, 5 simulations



per grid point

Conclusion

We all learned some things

References i

Questions?