Spatial Epidemics Dynamics: Synchronization

Mathematics 4MB3/6MB3
Mathematical Biology

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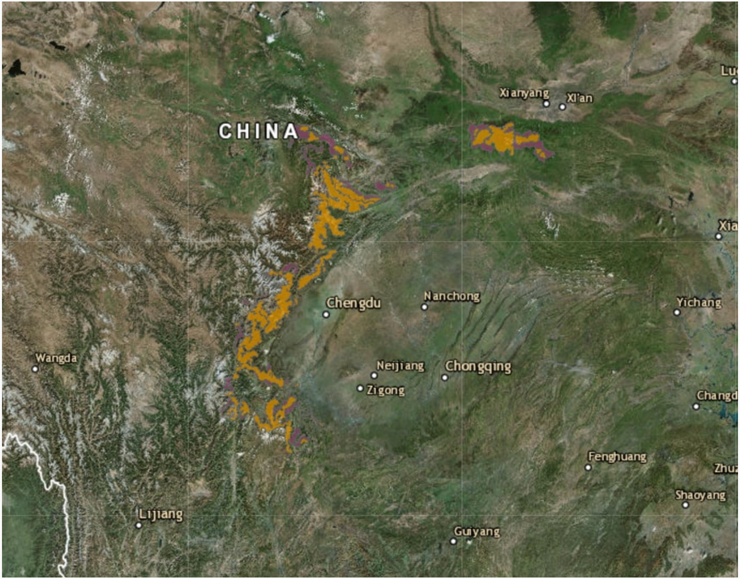
Introduction

Ailuropoda melanoleuca

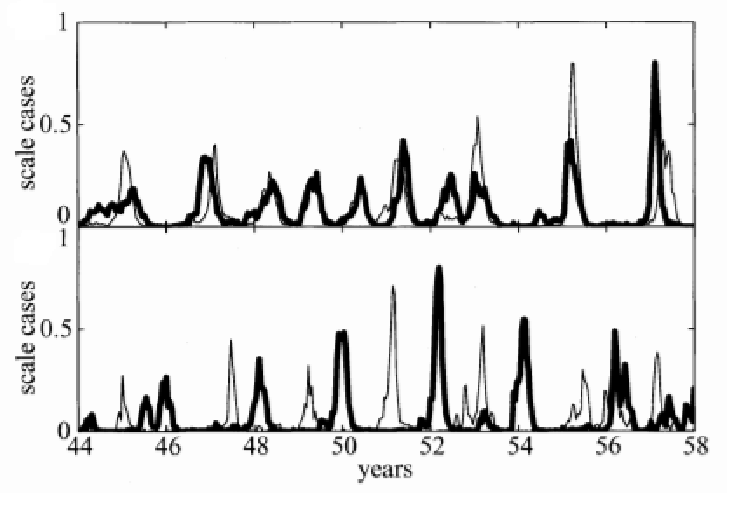
Current Habitat



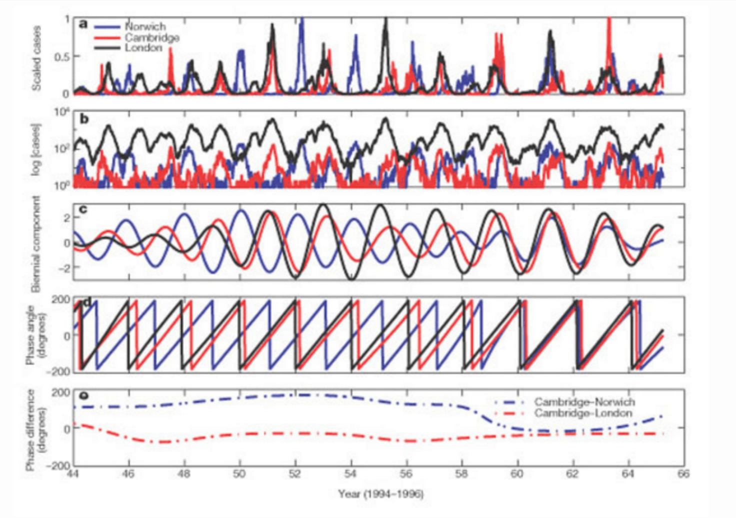
Synchronization

- First observed in nonlinear systems by Christian Huygens in 1665.

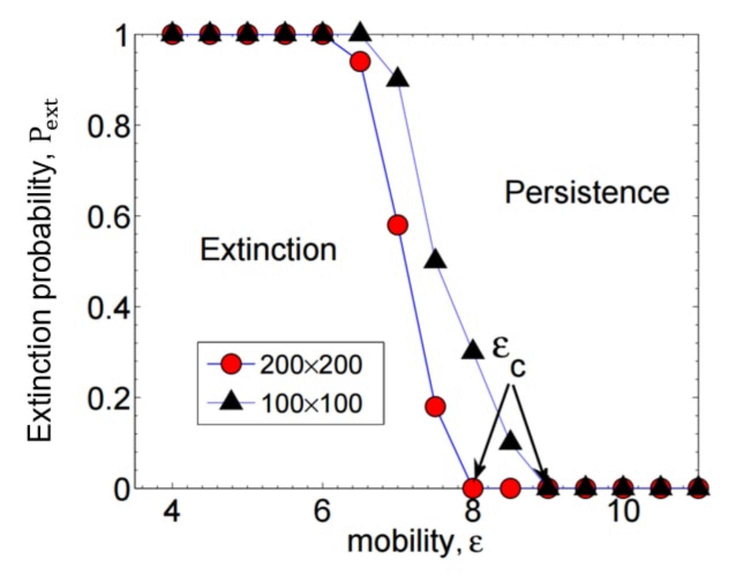
Weekly measles case reports for Birmingham, Newcastle, Cambridge and Norwich between 1944 and 1958



Wavelet phase analysis of weekly measles reports for Cambridge, Norwich and London between 1994 and 1996

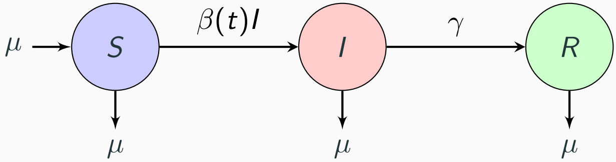


The effects of mobility on the persistence of a population



Methods

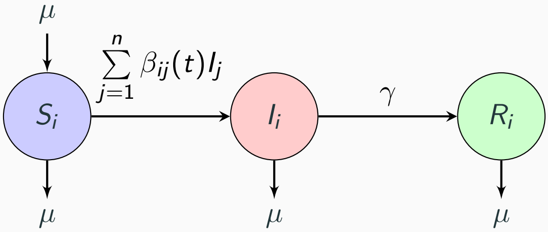
Forced SIR Model for a Single Patch



$$\frac{dS}{dt} = \mu - \beta(t)SI - \mu S$$
$$\frac{dI}{dt} = \beta(t)SI - \gamma I - \mu I$$
$$\frac{dR}{dt} = \gamma I - \mu R$$

$$\beta(t) = \langle\beta\rangle\left(1 + \alpha\cos(2\pi t)\right)$$
$$\langle\beta\rangle = \mathcal{R}_0(\mu + \gamma)$$

Metapatch SIR Model



$$\frac{dS_i}{dt} = \mu - S_i \sum_{j=1}^{n} \beta_{ij}(t) I_j - \mu S_i$$

$$\frac{dI_i}{dt} = S_i \sum_{j=1}^{n} \beta_{ij}(t) I_j - \gamma I_i - \mu I_i$$

$$\frac{dR_i}{dt} = \gamma I_i - \mu R_i$$

Beta Matrix

$$\beta(t) = \langle\beta\rangle\,(1 + \alpha\cos(2\pi t))M$$

Equal Coupling Matrix

$$M = \begin{bmatrix} 1-m & \frac{m}{n-1} & \frac{m}{n-1} & \frac{m}{n-1} & \cdots \\ \frac{m}{n-1} & 1-m & \frac{m}{n-1} & \frac{m}{n-1} & \\ \frac{m}{n-1} & \frac{m}{n-1} & 1-m & & \\ \vdots & & & \ddots & \end{bmatrix}$$

Beta Matrix

$$\beta(t) = \langle \beta \rangle \, (1 + \alpha \cos(2\pi t)) M$$

Nearest Neighbour Matrix

$$M = \begin{bmatrix} 1-m & \frac{m}{2} & 0 & 0 & \dots & \frac{m}{2} \\ \frac{m}{2} & 1-m & \frac{m}{2} & 0 & & \vdots \\ 0 & \frac{m}{2} & 1-m & & & \\ 0 & 0 & & \ddots & & \\ \vdots & & & & \ddots & \frac{m}{2} \\ \frac{m}{2} & & & \dots & \frac{m}{2} & 1-m \end{bmatrix}$$

Modelling Stochasticity

The effects of stochasticity are crucial in this model

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Exact Simulation: Gillespie Algorthim

- Get waiting time by sample from an exponential distribution with parameter equal to the total transition rate
- Randomly select which transition occurred with probability given by its transition rate relative to the total rate
- Unfeasible for large populations

Modelling Stochasticity

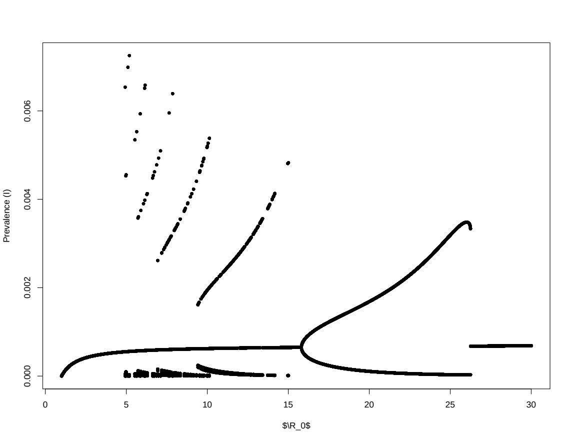
Approximate Simulation: Adaptive Tau Algorthim

- The `adaptivetau` package
- Find time periods where transition rates are approximately constant
- Skip this time and use a Possion distribution to approximate which transitions occured

Results

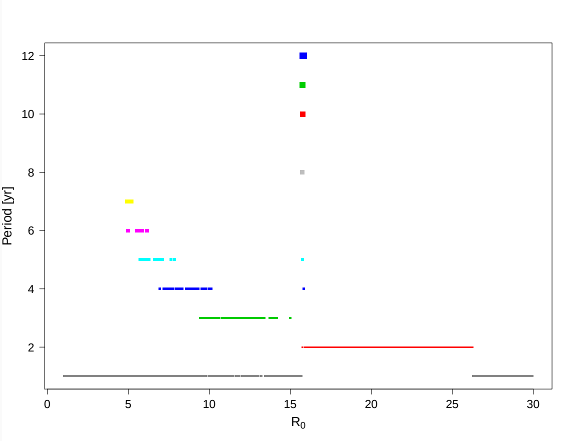
Bifurcation Diagram

Single Patch SIR model with sinusoidal seasonal forcing



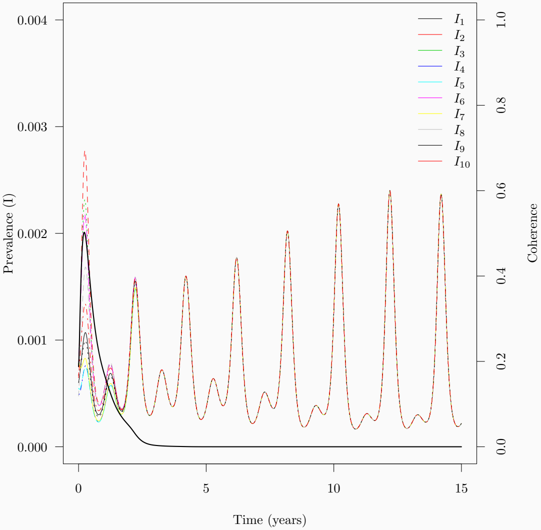
Period Diagram

Period of the oscillations



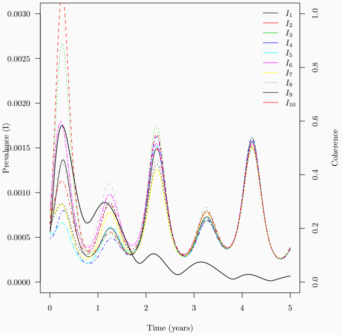
Deterministic Model

Equal Coupling, $\mathcal{R}_0 = 17$, $m = 0.2$

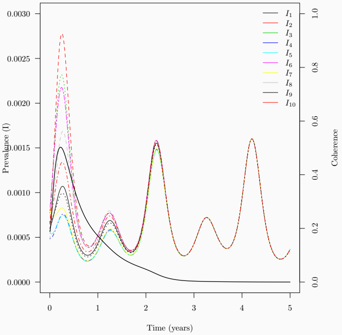


Deterministic Model: EC vs NN

$$\mathcal{R}_0 = 17,\ m = 0.2$$



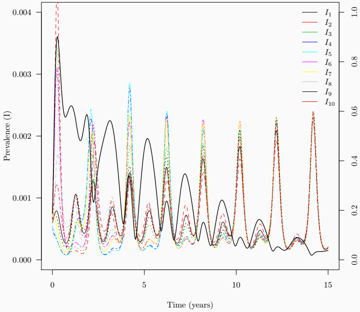
Nearest Neighbour Coupling

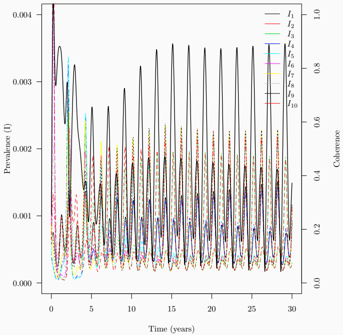


Equal Coupling

Deterministic Model

Nearest Neighbour Coupling, $\mathcal{R}_0 = 17$, $m = 0.01$





Initial Conditions

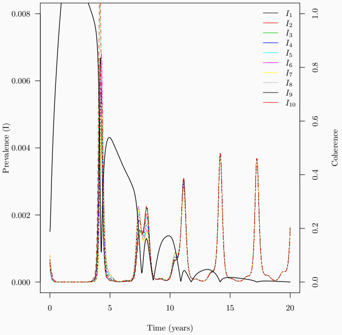
Within 30% percent of endemic equilibrium

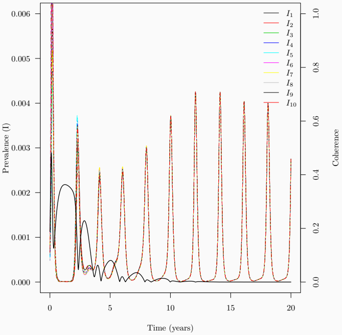
Initial Conditions

Within 40% percent of endemic equilibrium

Deterministic Model

Nearest Neighbour Coupling, $m = 0.2$



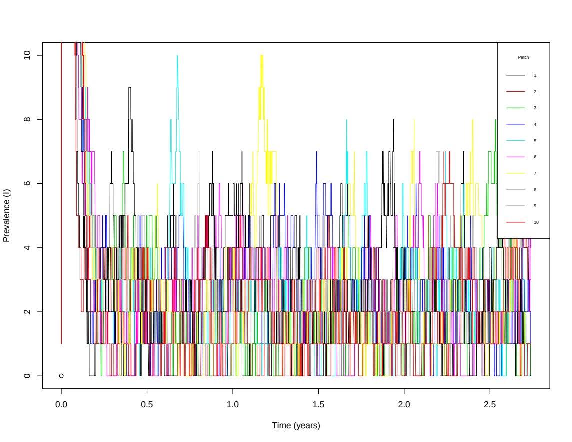


$\mathcal{R}_0 = 10$

$\mathcal{R}_0 = 25$

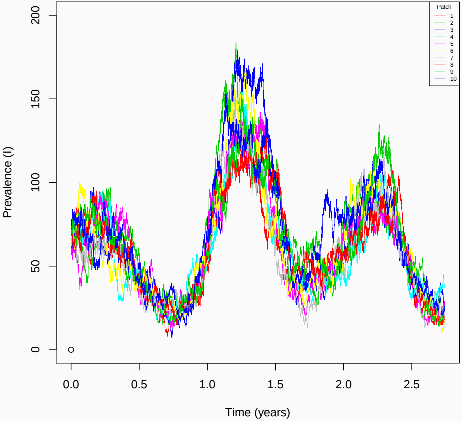
Stochastic: Gillespie Model

Equal Coupling, Population of 3000, $\mathcal{R}_0 = 17$, $m = 0.2$



Stochastic: Gillespie Model

Equal Coupling, Population of 100,000, $\mathcal{R}_0 = 17$, $m = 0.2$

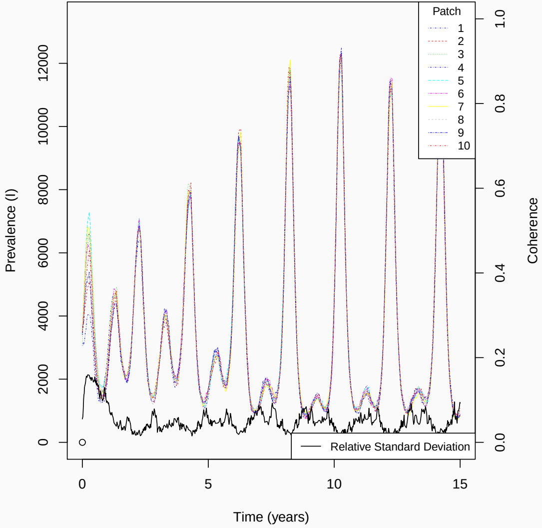


3 hour run time!

Adaptive Tau Algorthim

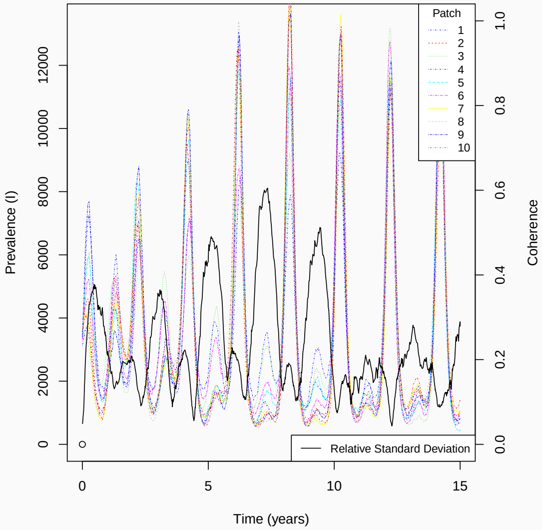
Stochastic: Adaptive Tau Algorthim

Equal Coupling, Population of 500,000, $\mathcal{R}_0 = 17$, $m = 0.2$



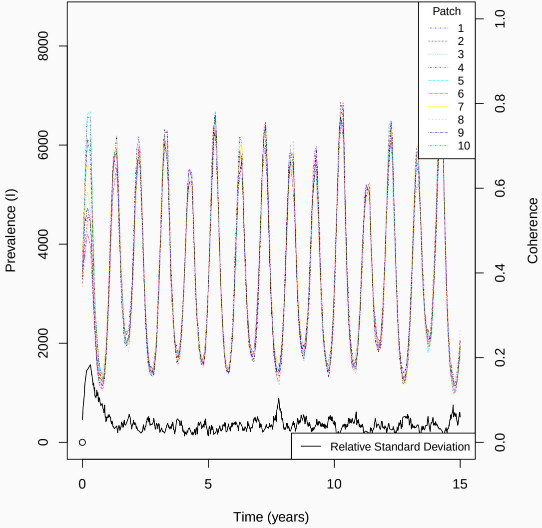
Stochastic: Adaptive Tau Algorthim

Equal Coupling, Population of 500,000, $\mathcal{R}_0 = 17$, $m = 0.01$



Stochastic: Adaptive Tau Algorthim

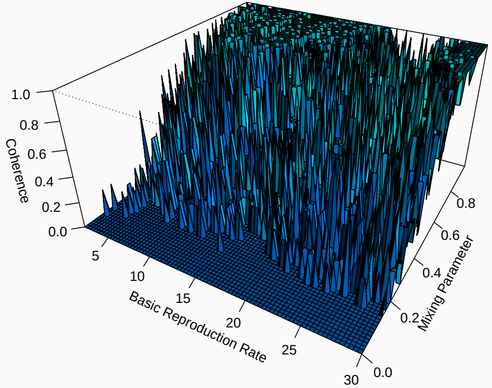
Equal Coupling, Population of 500,000, $\mathcal{R}_0 = 25$, $m = 0.2$



Coherence dependence on Parameters

Equal Coupling, Population of 250,000,

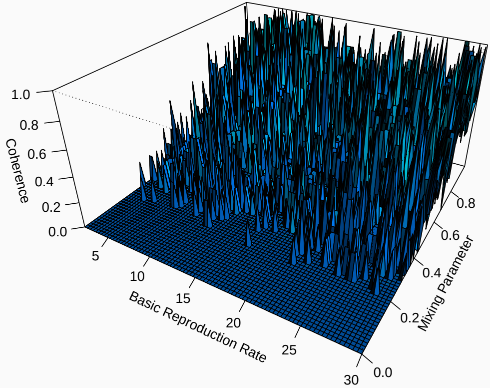
80×80 grid, 5 simulations per grid point



Coherence dependence on Parameters

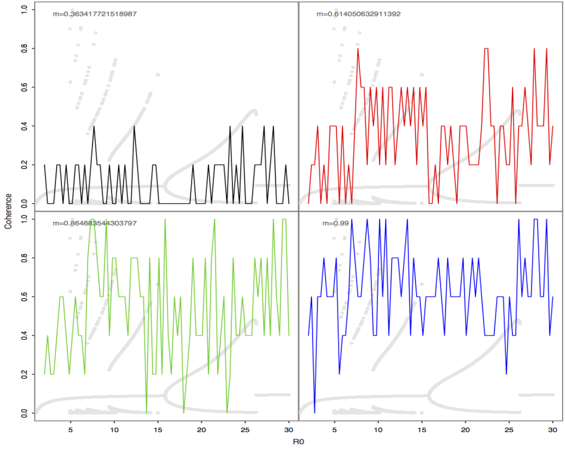
Nearest Neighbour Coupling, Population of 250,000,

80×80 grid, 5 simulations per grid point



Coherence dependence on Parameters

Nearest Neighbour Coupling, Population of 250,000, 5 simulations per grid point



Conclusion

- Importance of synchronization

- Importance of synchronization
- Coherence dependence on parameters

- Importance of synchronization
- Coherence dependence on parameters
- Further study

Questions?

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