Spatial Epidemics Dynamics: Synchronization

Mathematics 4MB3/6MB3
Mathematical Biology

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April 4, 2017

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Diseases are fun

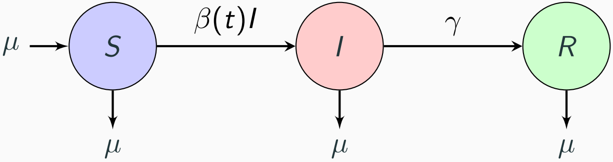
Text goes here

Diseases are cool

Text goes here

Methods

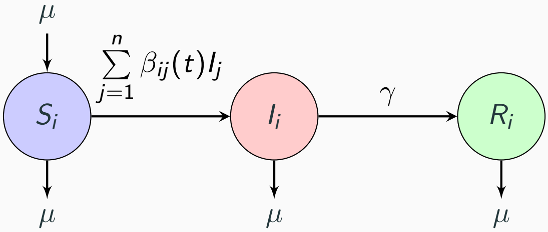
Forced SIR Model for a Single Patch



$$\frac{dS}{dt} = \mu - \beta(t)SI - \mu S$$
$$\frac{dI}{dt} = \beta(t)SI - \gamma I - \mu I$$
$$\frac{dR}{dt} = \gamma I - \mu R$$

$$\beta(t) = \langle\beta\rangle\left(1 + \alpha\cos(2\pi t)\right)$$
$$\langle\beta\rangle = \mathcal{R}_0(\mu + \gamma)$$

Metapatch SIR Model



$$\frac{dS_i}{dt} = \mu - S_i \sum_{j=1}^{n} \beta_{ij}(t) I_j - \mu S_i$$

$$\frac{dI_i}{dt} = S_i \sum_{j=1}^{n} \beta_{ij}(t) I_j - \gamma I_i - \mu I_i$$

$$\frac{dR_i}{dt} = \gamma I_i - \mu R_i$$

Beta Matrix

$$\beta(t) = \langle\beta\rangle\left(1 + \alpha\cos(2\pi t)\right)M$$

Equal Coupling Matrix:

$$M = \begin{bmatrix} 1-m & \frac{m}{n-1} & \frac{m}{n-1} & \frac{m}{n-1} & \cdots \\ \frac{m}{n-1} & 1-m & \frac{m}{n-1} & \frac{m}{n-1} & \\ \frac{m}{n-1} & \frac{m}{n-1} & 1-m & & \\ \vdots & & & \ddots & \end{bmatrix}$$

Beta Matrix

$$\beta(t) = \langle \beta \rangle \left(1 + \alpha \cos(2\pi t)\right) M$$

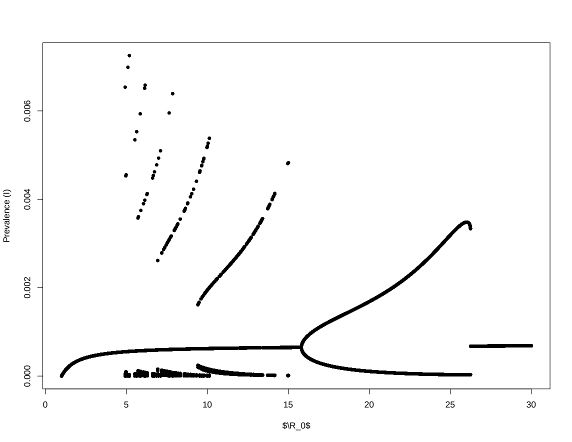
Nearest Neighbour Matrix:

$$M = \begin{bmatrix} 1-m & \frac{m}{2} & 0 & 0 & \dots & \frac{m}{2} \\ \frac{m}{2} & 1-m & \frac{m}{2} & 0 & & \vdots \\ 0 & \frac{m}{2} & 1-m & & & \\ 0 & 0 & & \ddots & & \\ \vdots & & & & \ddots & \frac{m}{2} \\ \frac{m}{2} & & & \dots & \frac{m}{2} & 1-m \end{bmatrix}$$

Results

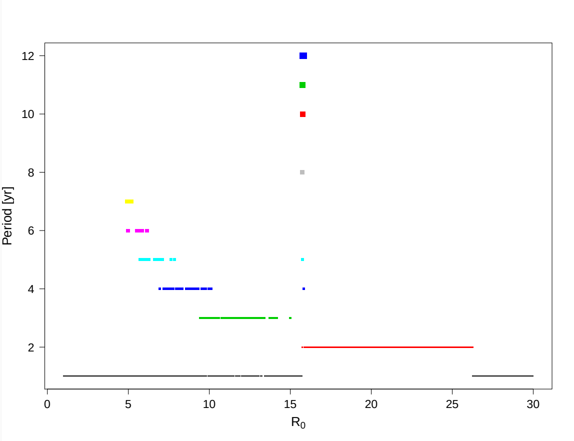
Bifurcation Diagram

Single Patch SIR model with sinusoidal seasonal forcing



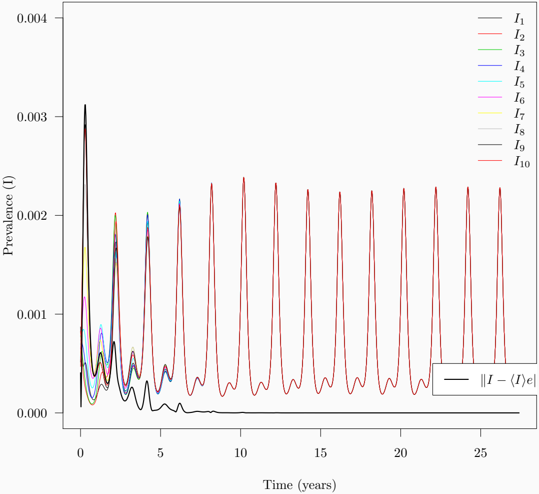
Period Diagram

Period of the oscillations



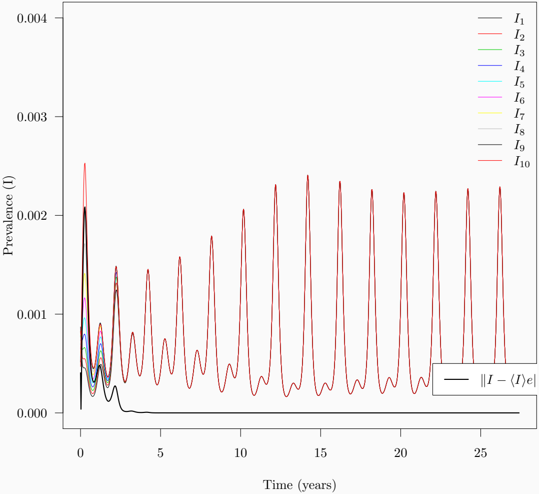
Deterministic Model

Nearest Neighbour Coupling, $\mathcal{R}_0 = 17$, $m = 0.2$



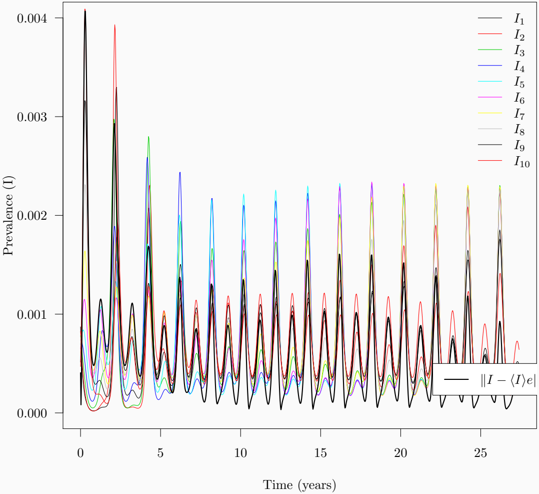
Deterministic Model

Equal Coupling, $\mathcal{R}_0 = 17$, $m = 0.2$



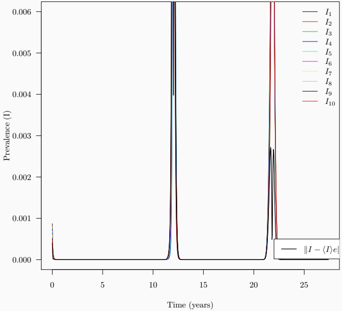
Deterministic Model

Nearest Neighbour Coupling, $\mathcal{R}_0 = 17$, $m = 0.01$

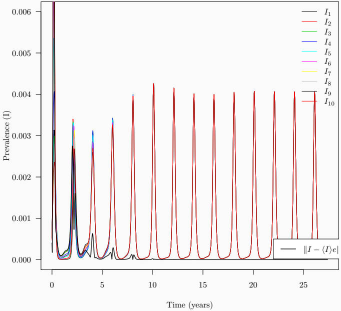


Deterministic Model

Nearest Neighbour Coupling, $m = 0.2$



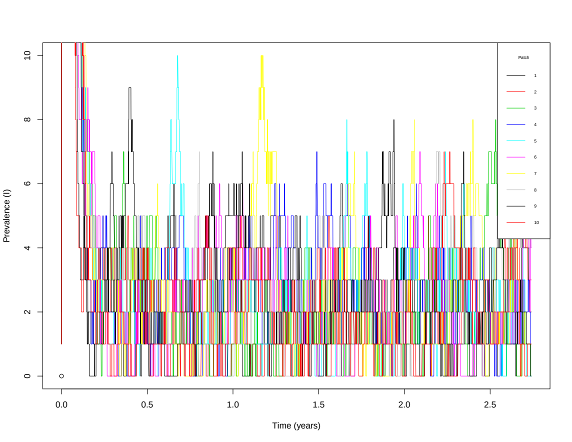
$\mathcal{R}_0 = 6$



$\mathcal{R}_0 = 25$

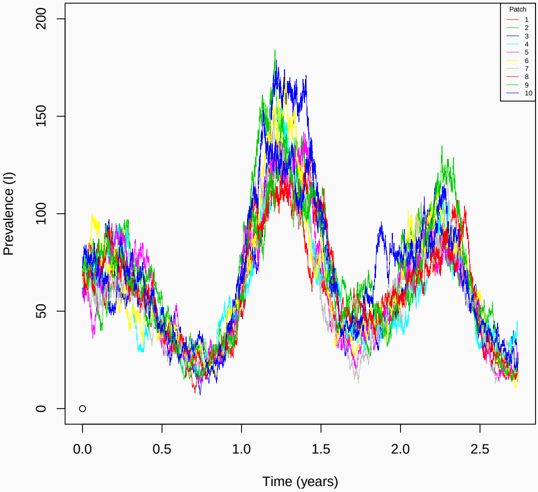
Stochastic: Gillespie Model

Equal Coupling, Population of 3000, $\mathcal{R}_0 = 17$, $m = 0.2$



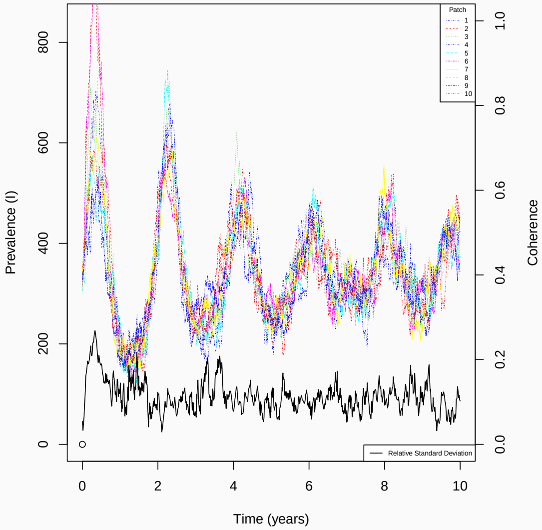
Stochastic: Gillespie Model

Equal Coupling, Population of , $\mathcal{R}_0 = 17$, $m = 0.2$



Stochastic: Adaptive Tau Algorthim

Equal Coupling, Population of 500000, $\mathcal{R}_0 = 18.69$, $m = 0.2$



Coherence dependence on Parameters

Questions?

References i