Uncertain of Your Uncertainties

Integrating Predictions and Distributionally Robust Optimization

RTX AI Assurance Workshop

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- **Operations**
  - *"… fully integrated information regime with interoperable sharing of information. This can be leveraged to accurately estimate the current state and confidently predict the future state of the NAS."*

- **Safety Assurance**
  - *"… With big data, the NAS assures real-time safety through continuous monitoring, modeling, and verification to detect anomalies and correct for real-time spikes in risk."*

Operations in an Info-Centric NAS

- Continuous refinement of predictive models to support decision-making
- Apply predictive model to evaluate choices for traffic management performance, efficiencies
- Manage uncertainties

Delta 2222

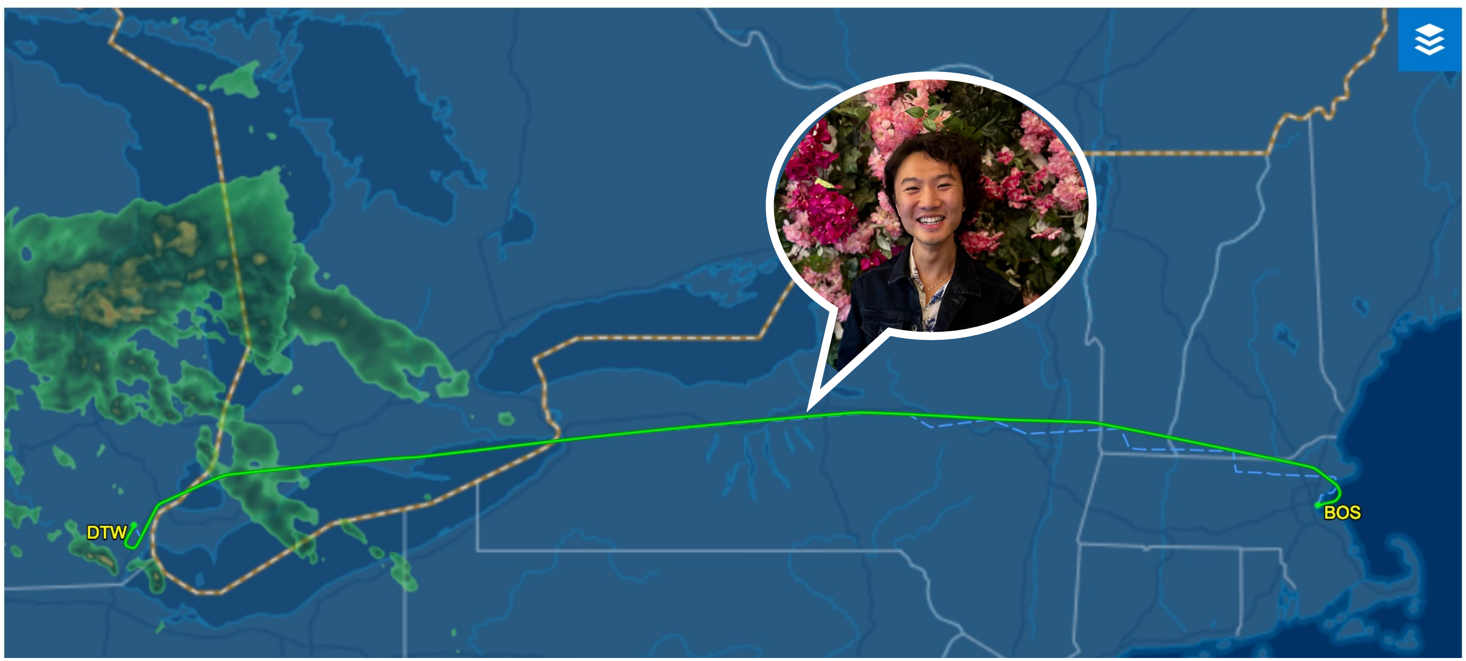
DAL2222 / DL2222
Upgrade account to see tail number

ARRIVED OVER A DAY AGO
Gate A9

BOS	DTW
BOSTON, MA	**DETROIT, MI**
left **GATE A19**	arrived at **GATE A9**
Boston Logan Intl - **BOS**	Detroit Metro Wayne Co - **DTW**
SATURDAY 05-AUG-2023	SATURDAY 05-AUG-2023
01:16PM EDT (on time)	(2 minutes early) **03:18PM EDT**

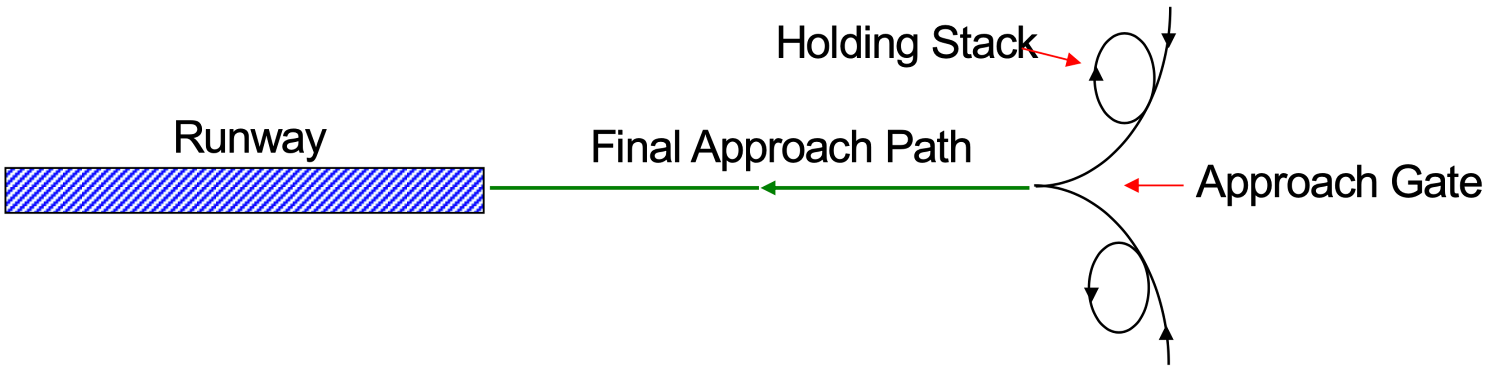
2h 2m total travel time

NOT YOUR FLIGHT? **DAL2222 flight schedule**



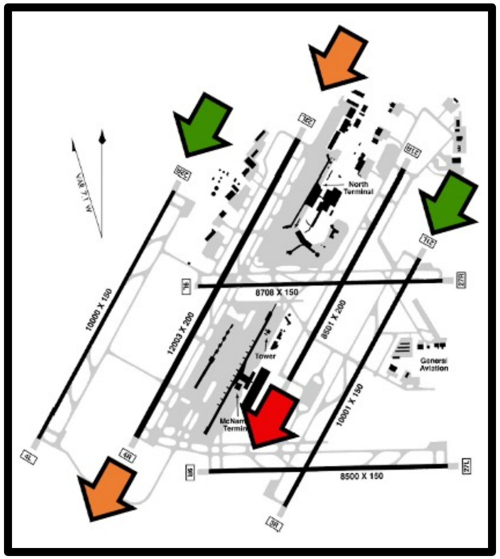
FlightAware

Supply-Side



- Runway as principal bottlenecks

- Predict:
  - Runway *configurations*
  - Runway *assignments*



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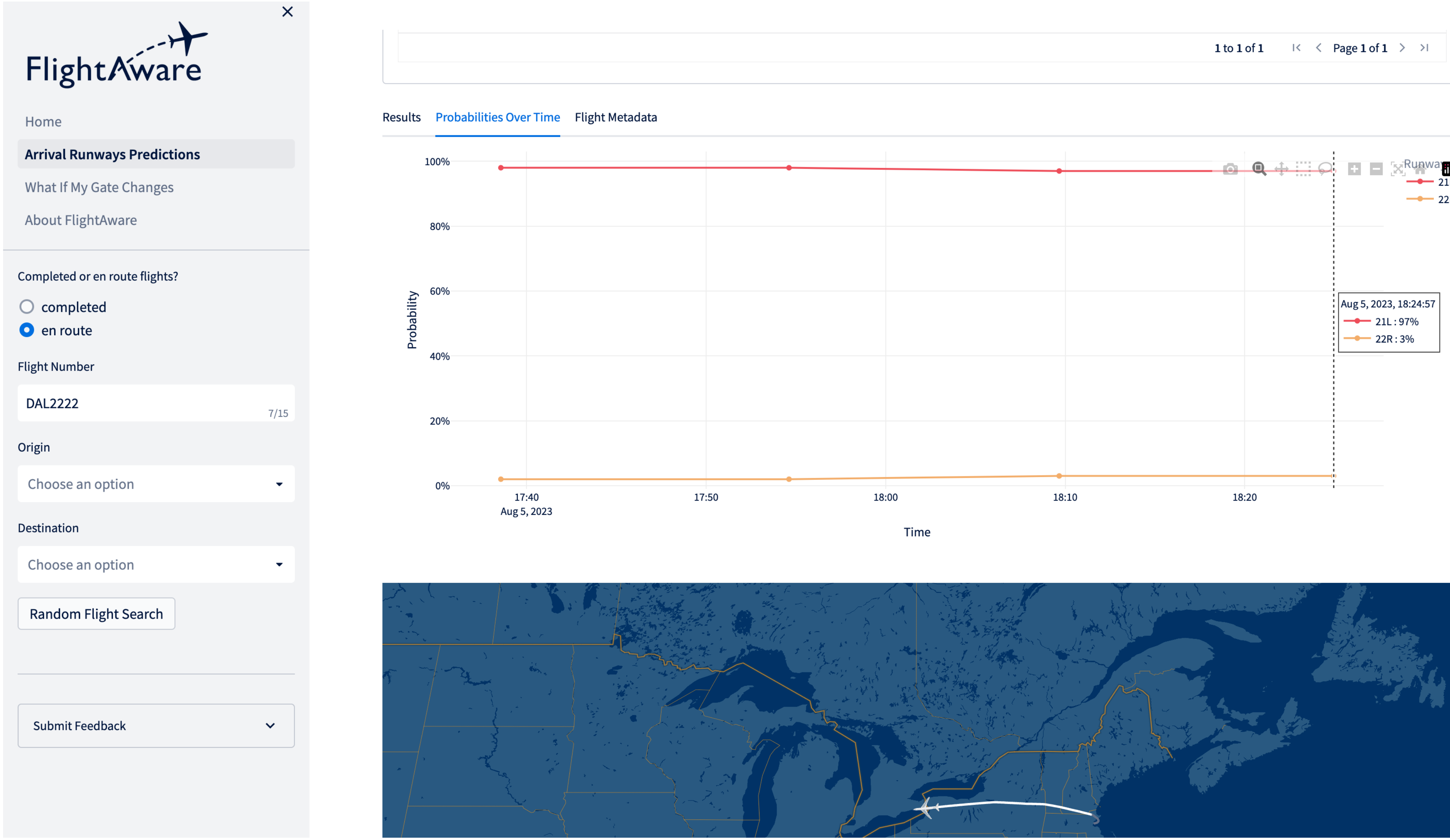
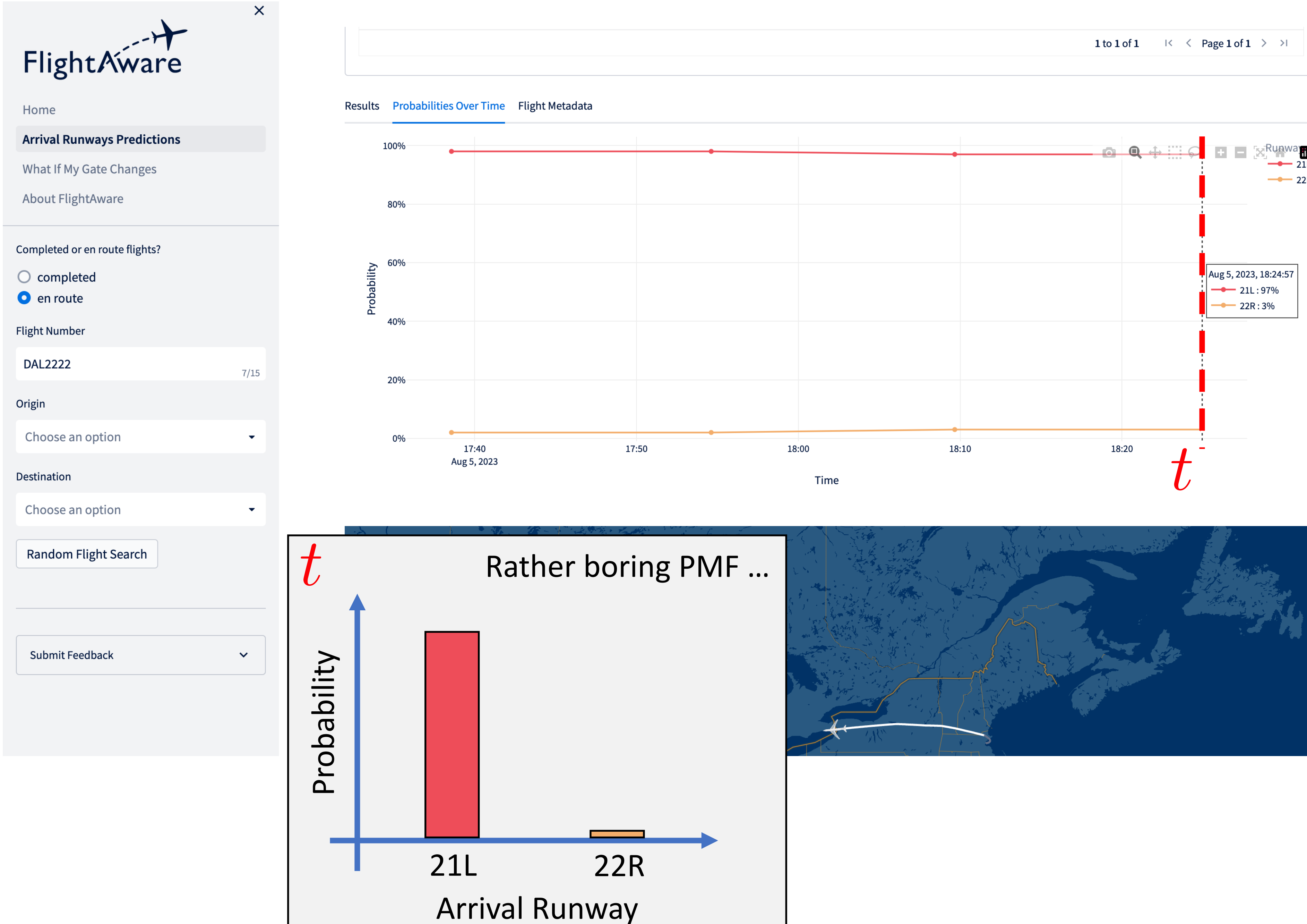
FlightAware Foresight

Collins Aerospace
An RTX Business

FlightAware


FlightAware
Home
Arrival Runways Predictions
What If My Gate Changes
About FlightAware
Completed or en route flights?
completed
en route
Flight Number
DAL2222
7/15
Origin
Choose an option
Destination
Choose an option
Random Flight Search
Submit Feedback
1 to 1 of 1
Page 1 of 1
Results
Probabilities Over Time
Flight Metadata
Probability
100%
80%
60%
40%
20%
0%
17:40
Aug 5, 2023
17:50
18:00
18:10
18:20
Time
21L
22R
Aug 5, 2023, 18:24:57
21L : 97%
22R : 3%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21L
22R
Aug 5, 2023, 18:24:57
21L : 97%
22R : 3%
t
t
Rather boring PMF …
Probability
21L
22R
Arrival Runway

21L-3R
W5

Korean Air 401

KAL401 / KE401

Seoul → Sydney

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Korean Air 401

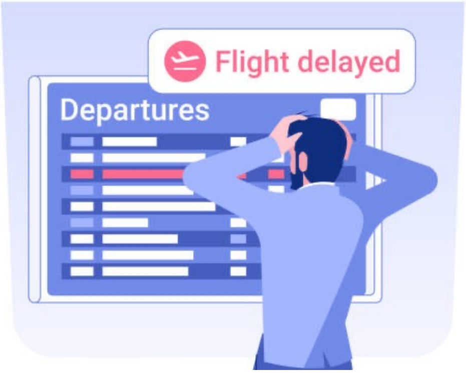
KAL401 / KE401

Seoul → Sydney

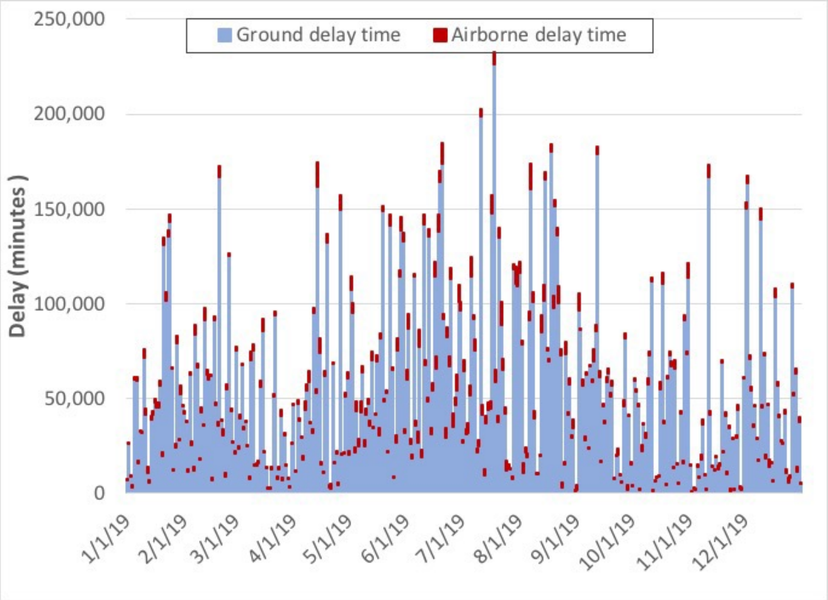
Significant distribution shifts

Air Traffic Flow Management

- Demand-capacity imbalance



- One strategy: Ground holds and ground delays
  - Rationale: Ground delays are safer and less costly than airborne delays



- Popular strategy …

iStock, FAA OPSNET

Ground Delay Programs (GDPs)

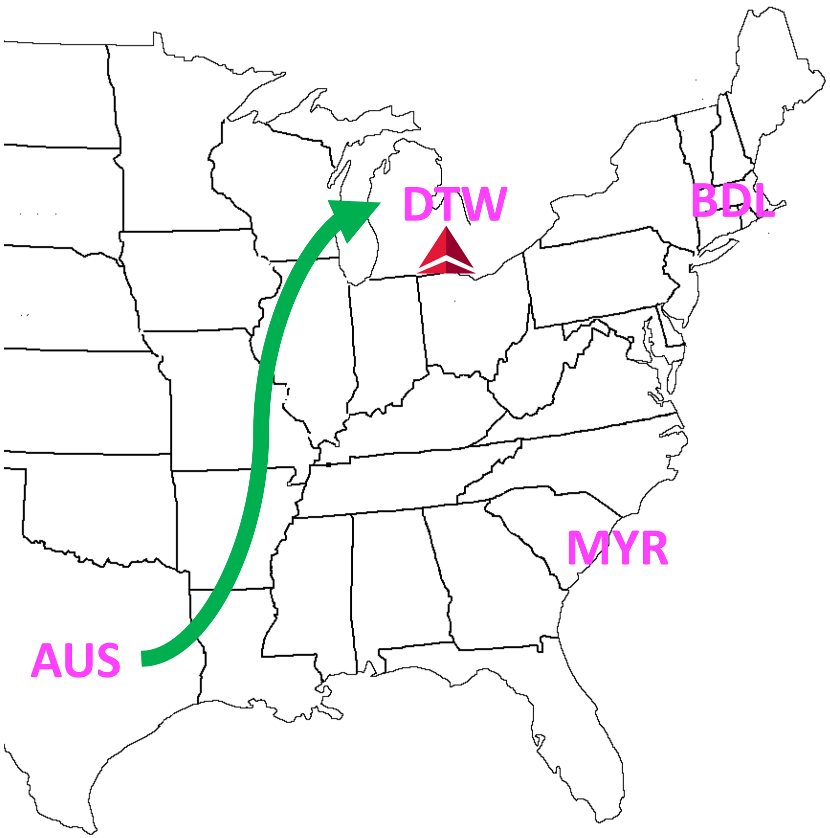
(Note: This page will refresh every 5 minutes. Last updated Fri, 01 Feb 2019 23:03:11 UTC. Provided by the FAA's Air Traffic Control System Command Center.)

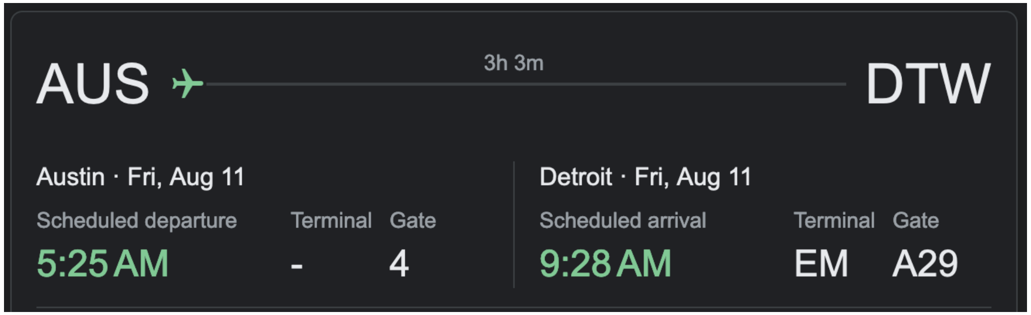
NATIONAL PROGRAMS									
PROGRAM NAME	START	END	SCOPE	REASON	AVG	AAR	PR	ADVZY	DA
DCA	2128	0159	1000 MILES + CZY	WEATHER / SNOW-ICE	66	28	28	063	DA
SFO	1808	0759	ALL+CZV_AP	WEATHER / WIND	175	28	28	053	DA

- ***All domestic and Canadian*** *inbound flights to* ***SFO*** *will be held at their origin for an average of* ***175 minutes*** *(~ 3 hr) due to unfavorable winds at* ***SFO*** *reducing arrival capacity to* ***28 aircraft per hour*** *(SFO's nominal arrival capacity is 60 aircraft per hour). This GDP is in effect from* ***1808Z*** *(1:08 PM EST) to* ***0758Z*** *(2:58 AM EST next day).*

- (Be a more informed air traveler! https://www.fly.faa.gov/ois/)

Ground Delay Programs (GDPs)
A "Live" Example …





DAL 1040

AUS → MYR

Ground Delay Programs (GDPs)
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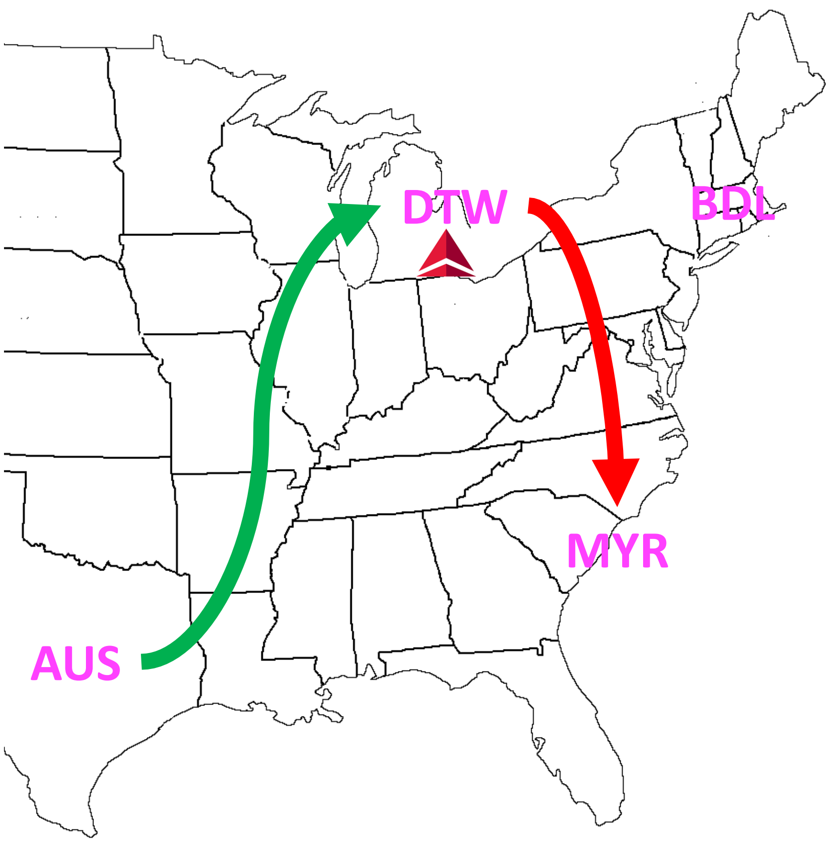

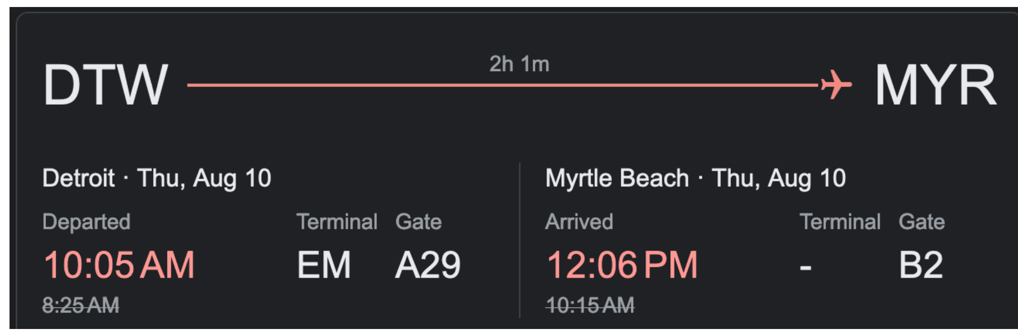

DAL 2057

DTW → MYR

Ground Delay Programs (GDPs)
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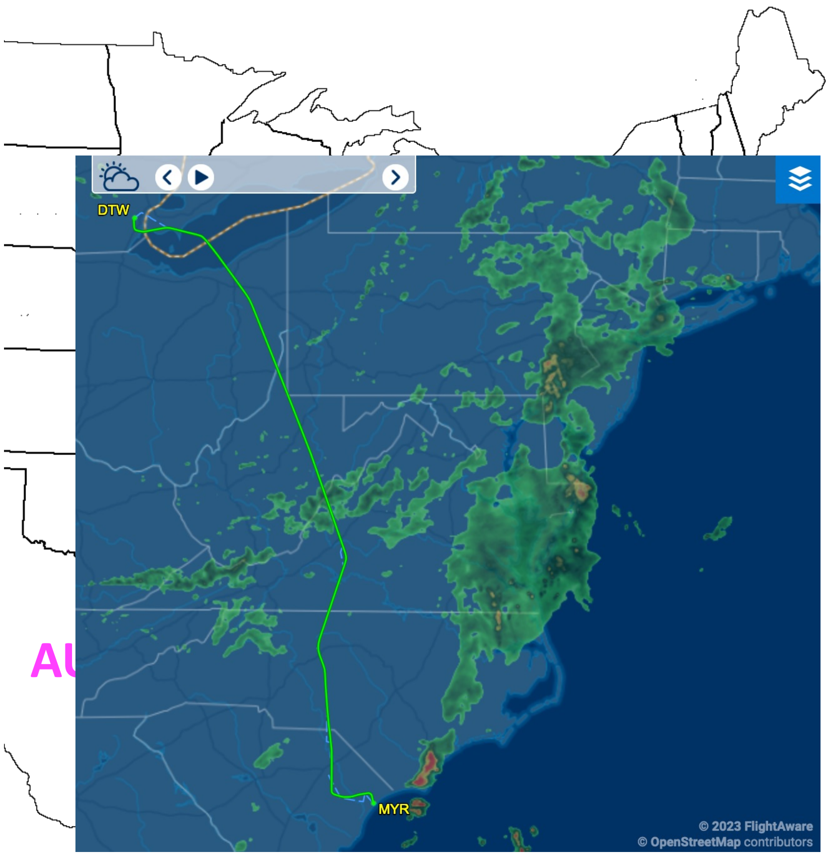

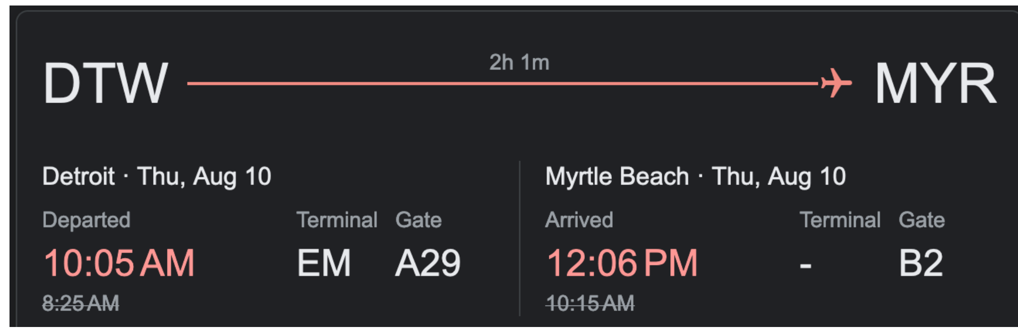

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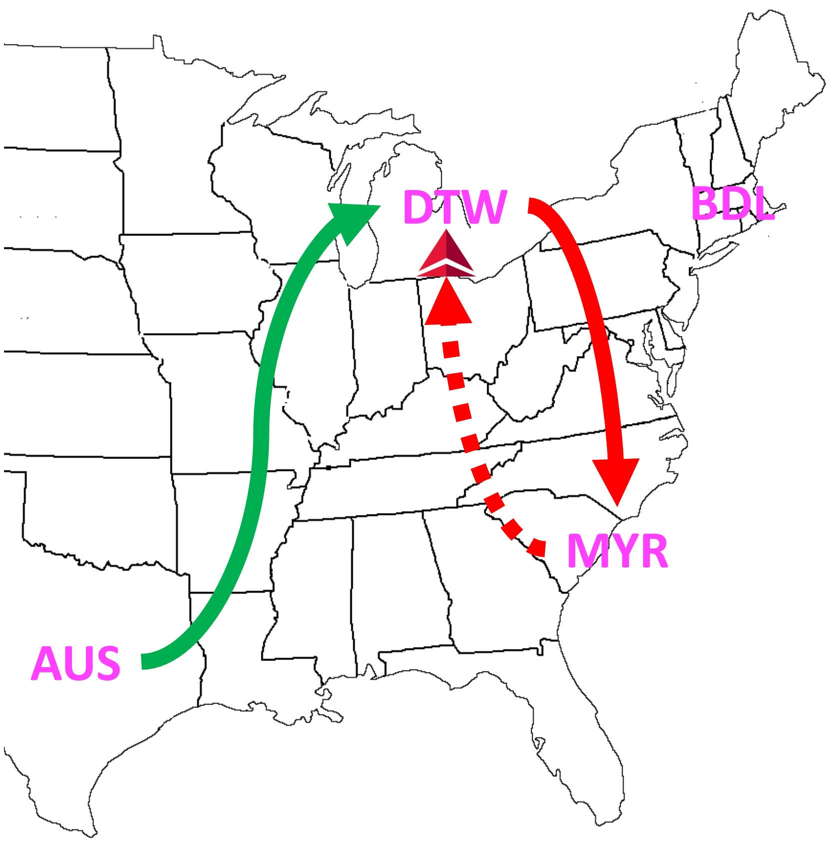

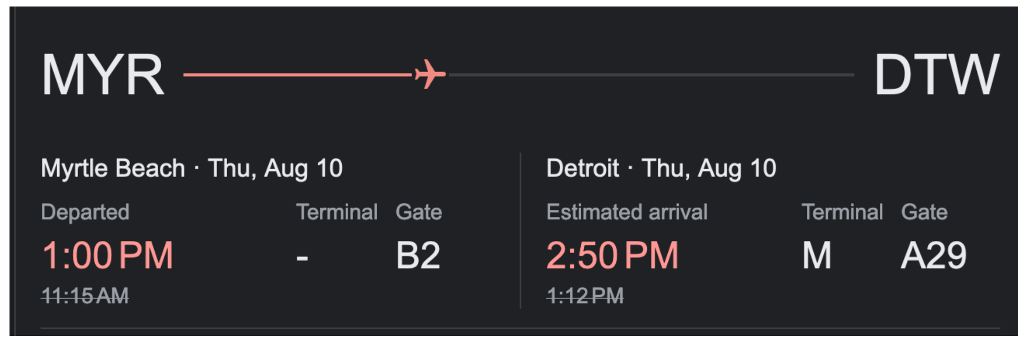

DAL 2057

MYR → DTW

Ground Delay Programs (GDPs)
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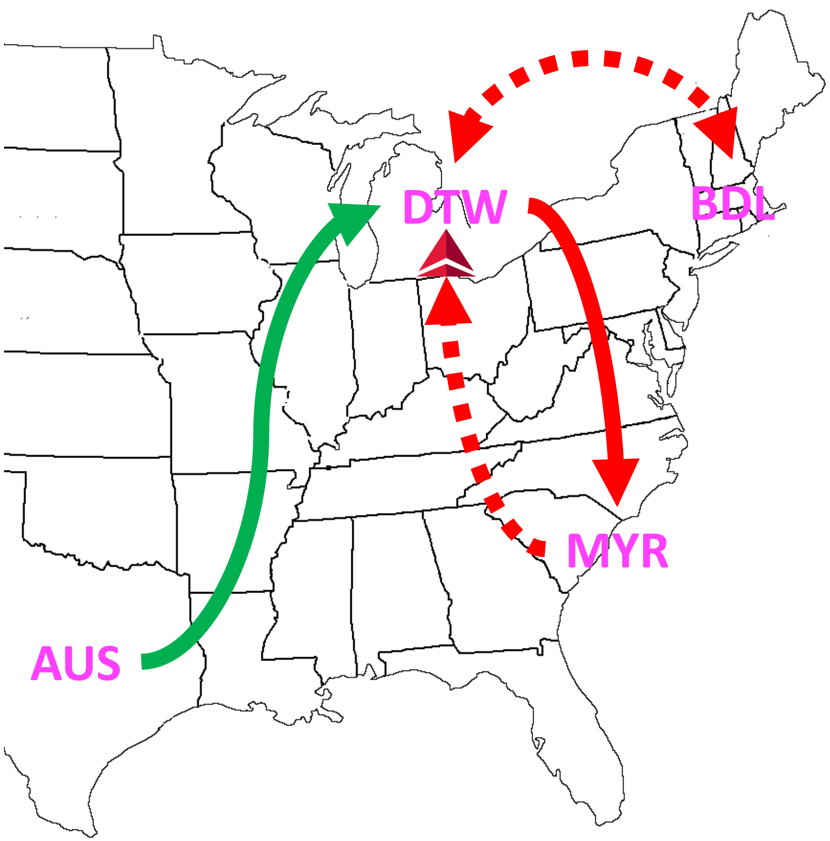

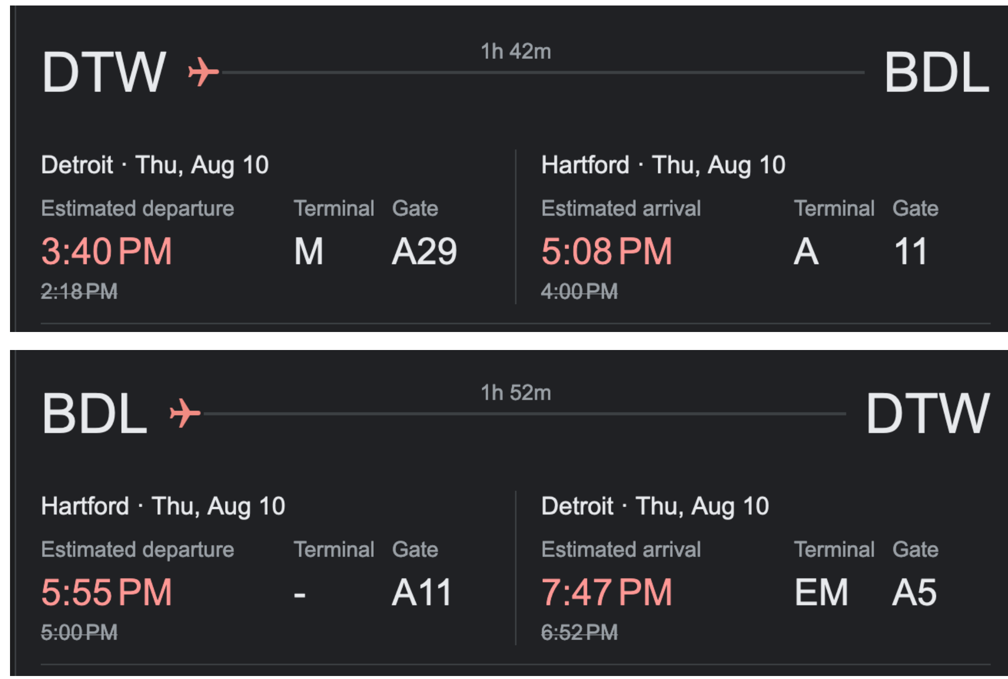

DAL 2234 (*delayed*)

DTW → BDL → DTW

Ground Holding Problems (GHPs)

- Decision variable:

$$x_{it} = \begin{cases} 1, & \text{if aircraft } i \text{ is assigned to land in time period } t, \\ 0, & \text{otherwise} \end{cases}$$

- Formulation:

$$\begin{array}{lll} \text{minimize} & \sum_{i=1}^{N} \sum_{t=t(i)}^{T+1} G_{it} x_{it} & \\ \text{subject to} & \sum_{i=1}^{N} x_{it} \leq M_t, & t = 1, \cdots, T+1 \\ & \sum_{t=t(i)}^{T+1} x_{it} = 1, & i = 1, \cdots, N \\ & x_{it} \in \{0, 1\}, & \forall i, t \end{array}$$

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ML model → Prediction of airport capacity distribution …

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… uncertain of your predicted capacity distributions
$t = 1, \ldots, T,\ T+1$

increasing uncertainty

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Ground Holding Problems (GHPs)

- Decision variable:

Robustify decisions against distributional uncertainty …

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… uncertain of your predicted capacity distributions

$t = 1, \ldots, T,\ T+1$

increasing uncertainty

Assurance? Or at least … cautious optimism?

H. Wu, **L.** “Distributionally robust airport ground holding problem under Wasserstein ambiguity sets.” http://arxiv.org/abs/2306.09836

H. Wu, Y. Zhou, X. Zhu, L. Li, **L.** “Distributionally robust airport ground delay programs with learning-driven airport capacity predictions.”

Ying Zhou

Dr. Xinting Zhu

Haochen Wu

Prof. Lishuai Li

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CityU
香港城市大學
City University of Hong Kong

Wasserstein Ambiguity Sets

- Probability distribution $\mathbb{Q}$ with support Ξ
- Space of probability distributions $M(\Xi) \ni \mathbb{Q}$
- Wasserstein distance d_w

$$d_w(\mathbb{Q}_1, \mathbb{Q}_2) = \inf_{\Pi \in \mathcal{D}_\Pi(\xi_1, \xi_2)} \int_{\Xi^2} \|\xi_1 - \xi_2\| \; \Pi(d\xi_1, d\xi_2)$$

Wasserstein Ambiguity Sets

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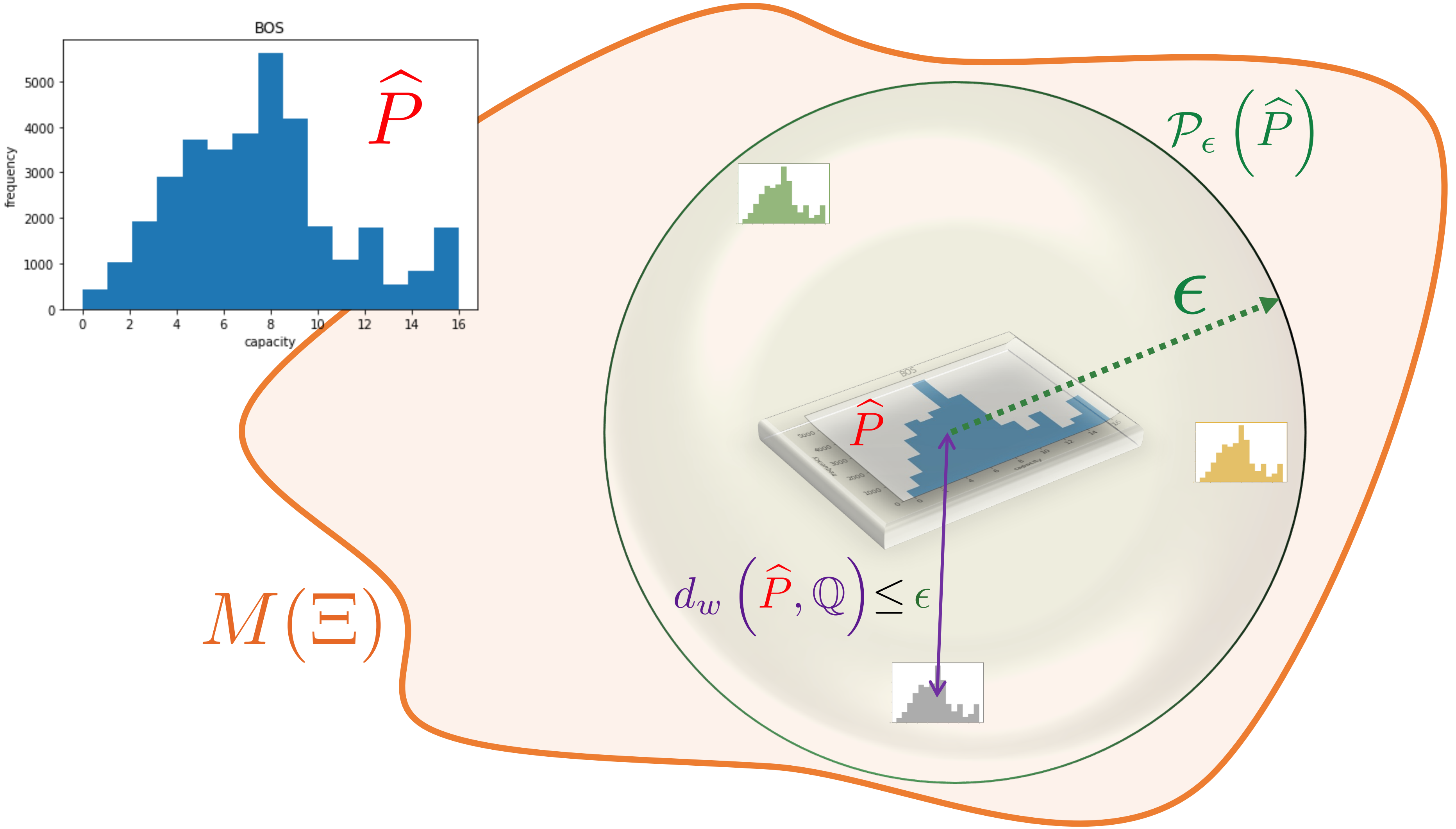
- Ambiguity set of size ϵ around empirical distribution $\widehat{P}$

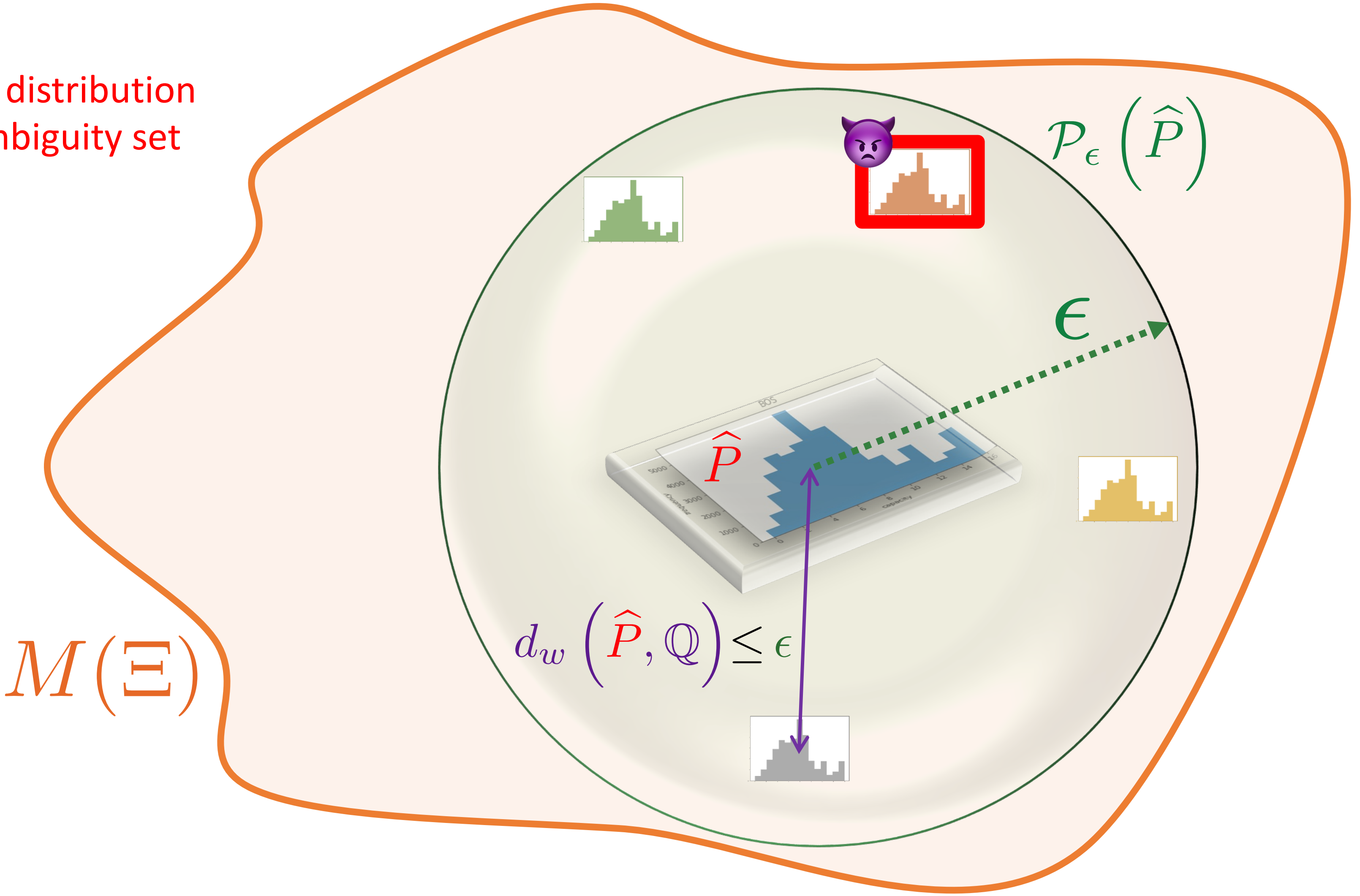
$$\mathcal{P}_\epsilon\left(\widehat{P}\right) := \left\{\mathbb{Q} \in M(\Xi) : d_w\left(\widehat{P}, \mathbb{Q}\right) \leq \epsilon\right\}$$

Wasserstein Ambiguity Sets



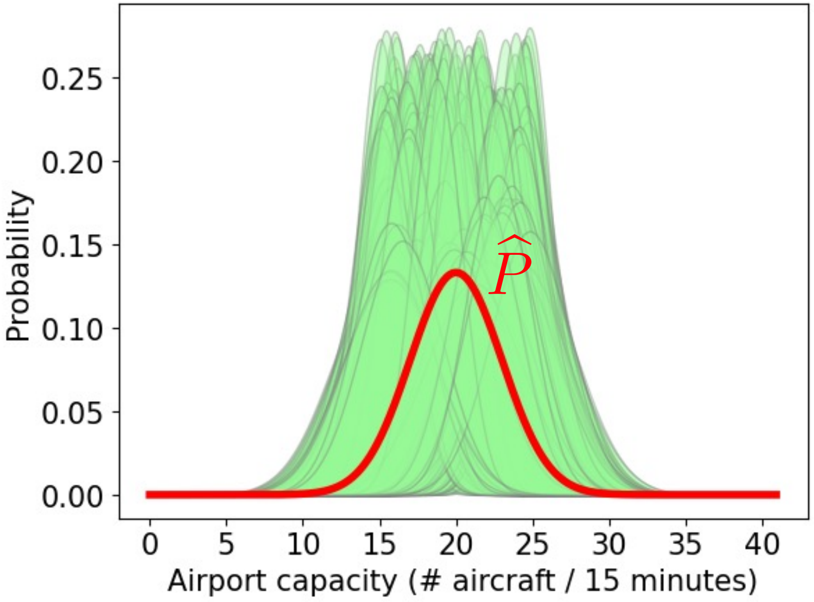
Wasserstein Ambiguity Sets

"Worst-case" distribution within the ambiguity set



Example Ambiguity Set (Gaussian)

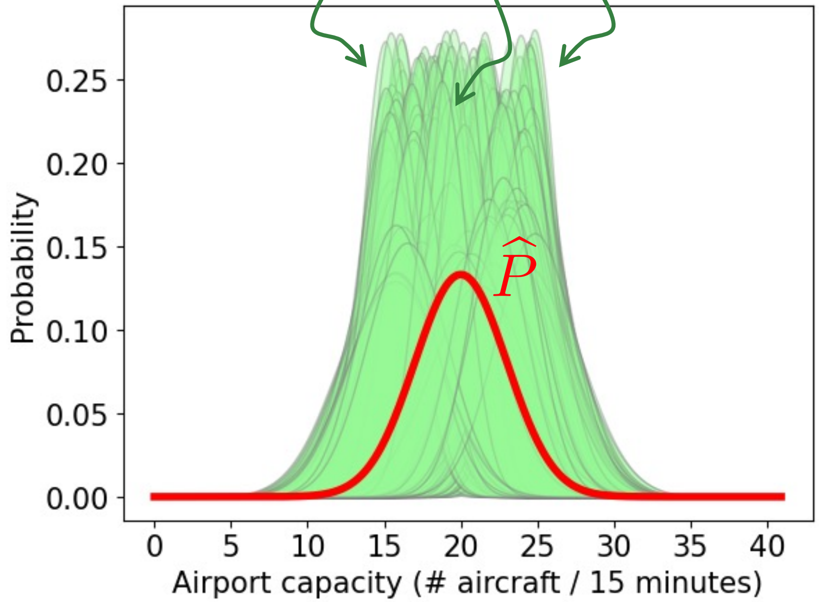
- Empirical distribution is Gaussian
- Sampled Gaussian subset of full ambiguity set $\mathcal{P}_\epsilon(\hat{P})$
- Accept/reject criteria with $\epsilon = 0.005$

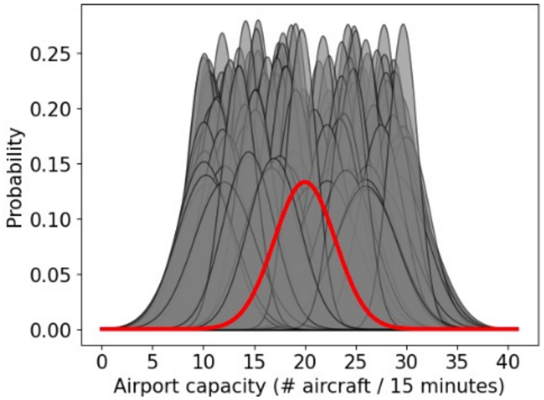


Example Ambiguity Set (Gaussian)

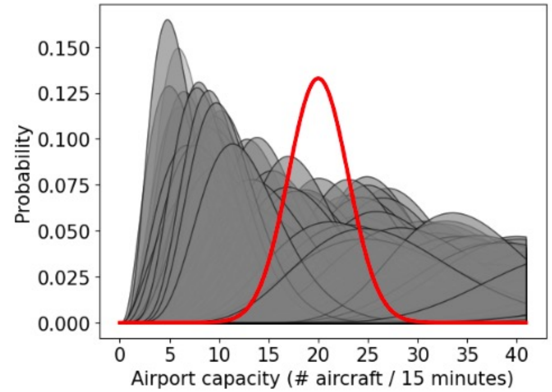
- Empirical distribution is Gaussian
- Sampled Gaussian subset of full ambiguity set $\mathcal{P}_\epsilon(\hat{P})$
- Accept/reject criteria with $\epsilon = 0.005$

(but it need not be Gaussian …)

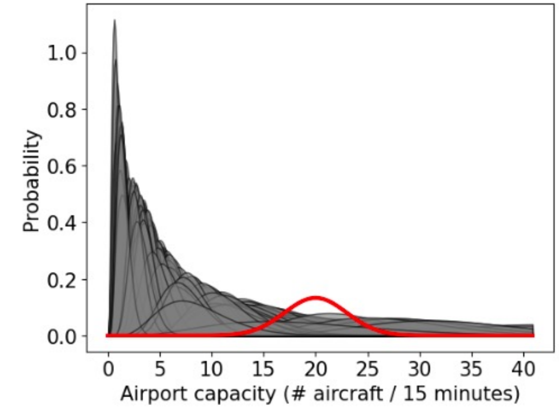




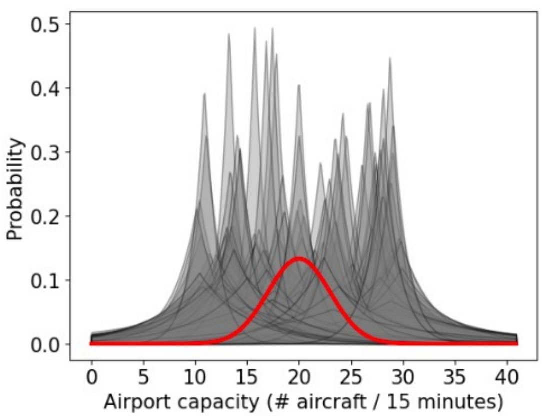
(a) Gaussian



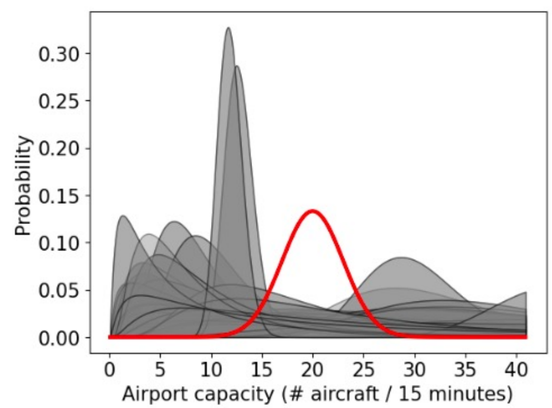
(b) Gamma



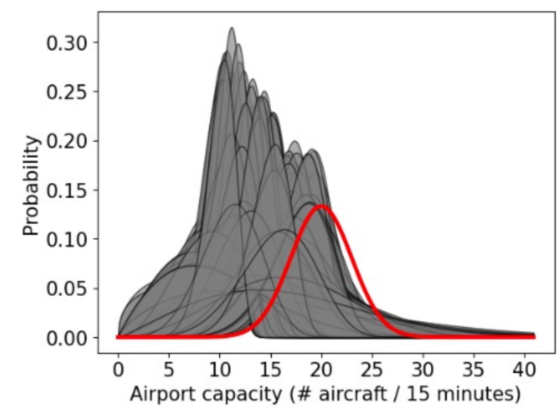
(c) Erlang



(d) Laplace



(e) Lognorm



(f) Weibull

$\epsilon = 0.005$

Distributionally Robust Multi-Airport Ground Holding Problem (dr-MAGHP)

- Objective function:

$$\min_{u,v} \left\{ \sum_{f\in F} (C_g g_f + C_a a_f) + \boxed{\underbrace{\max_{p\in\mathcal{P}^{(g)}_{\epsilon_1}(\widehat{P}^{(g)})}}_{\text{👿}} \overbrace{\mathbb{E}_p\left[Q\left(u,\xi^{(g)}\right)\right]}^{\text{ground delay}}} + \boxed{\underbrace{\max_{p\in\mathcal{P}^{(a)}_{\epsilon_2}(\widehat{P}^{(a)})}}_{\text{👿}} \overbrace{\mathbb{E}_p\left[Q\left(v,\xi^{(a)}\right)\right]}^{\text{airborne hold}}} \right\}$$

  - … with capacity, connectivity, coverage, integrality constraints
  - Departure capacity distribution (+ ambiguity set)
  - Arrival capacity distribution (+ ambiguity set)
- Two-stage problem, optimize by minimizing expected wait times …

  … across worst-case distribution

dr-MAGHP Deterministic Equivalent Formulation (idea)

- Use Lagrangian dual to transform inner maximization problems to minimization problems

$$\boxed{\max_{p \in \mathcal{P}_{\epsilon_1}^{(g)}\left(\widehat{P}^{(g)}\right)} \mathbb{E}_p\left[Q\left(u, \xi^{(g)}\right)\right]}$$

$$\boxed{\max_{p \in \mathcal{P}_{\epsilon_2}^{(a)}\left(\widehat{P}^{(a)}\right)} \mathbb{E}_p\left[Q\left(v, \xi^{(a)}\right)\right]}$$

- Reformulation to semi-infinite linear program (Esfahani & Kuhn, 2017)

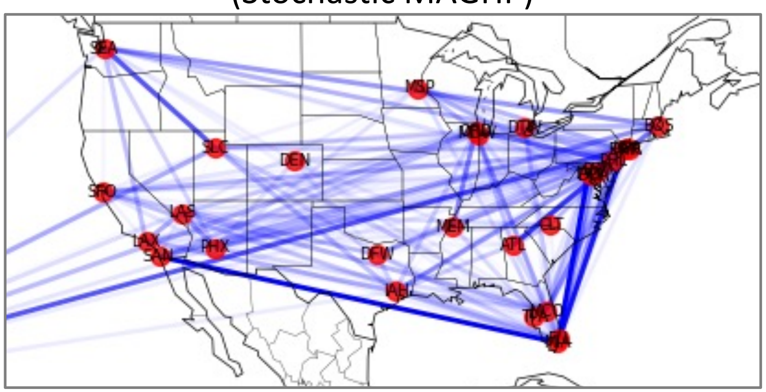
- Rigorously discretize (finite reducibility, weak discretization, solvability) (López & Still, 2007)

Evaluate + Compare

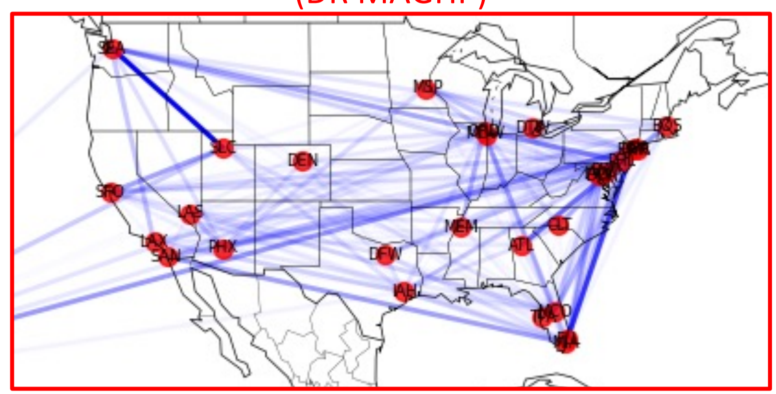
- Comparison against a deterministic policy and stochastic policy

- ϵ choice??

Ground Holding Policy
(Stochastic MAGHP)



Ground Holding Policy
(DR MAGHP)



Ground Holding Problems (GHPs)

- Decision variable:

Robustify decisions against distributional uncertainty …

dr-MAGHP formulation

$$\boxed{x_{it}} = \begin{cases} 1, & \text{if aircraft } i \text{ is assigned to land in time period } t, \\ 0, & \text{otherwise} \end{cases}$$

- Formulation:

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… uncertain of your predicted capacity distributions

$t = 1, \dots, T,\ T+1$

increasing uncertainty

Wu & **L** (2023)

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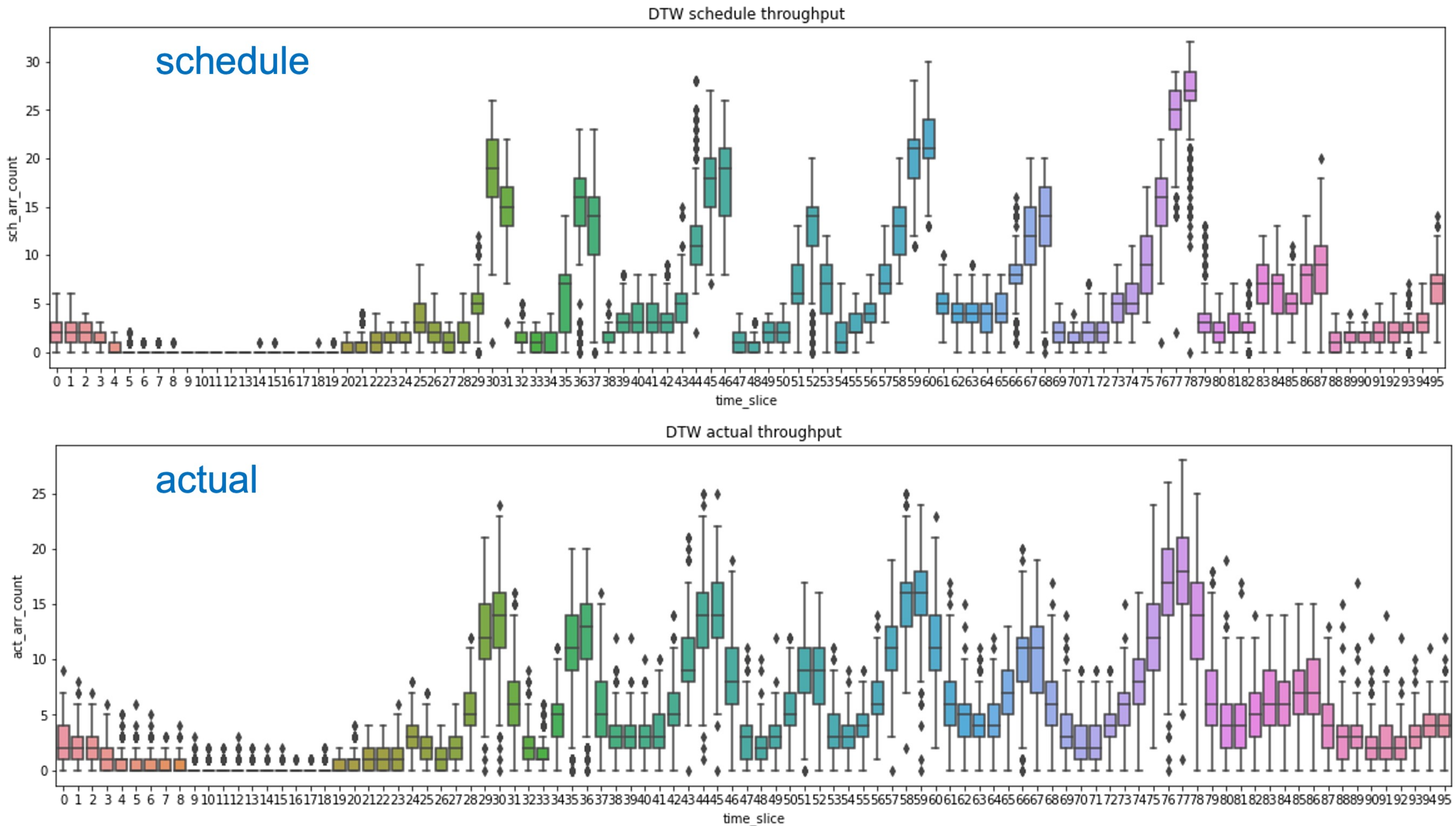
Airport Capacity Prediction

- Capacity prediction input critical for GDP planning (and for dr-MAGHP)
- Factors that influence airport capacity:
  - Separation requirement, fleet mix
  - Runway configuration
  - Visibility, cloud ceiling
  - Wind speed/direction
  - Arrival/departure priorities
  - …

Airport Capacity Prediction



Wu, Zhou, Zhu, Li, **L** (in prep)

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Airport Capacity Prediction

Model

- Target variable is discrete, finite (i.e., integral capacities)
- Must address class imbalance
- Affected by weather, runway, time-of-day, etc.
- Specific to each airport

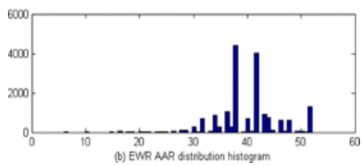
Data

- US DOT Bureau of Transportation Statistics
- FAA Aviation System Performance Metrics
- FAA Airport Capacity Benchmarks
- Weather data (High-Resolution Rapid Refresh, or HRRR)

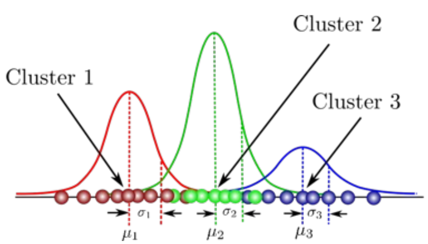
Airport Capacity Prediction

Step 1

Historical Capacity



GMM clustering



Step 2

Historical Weather feature + Capacity cluster label

Classification model

Step 3

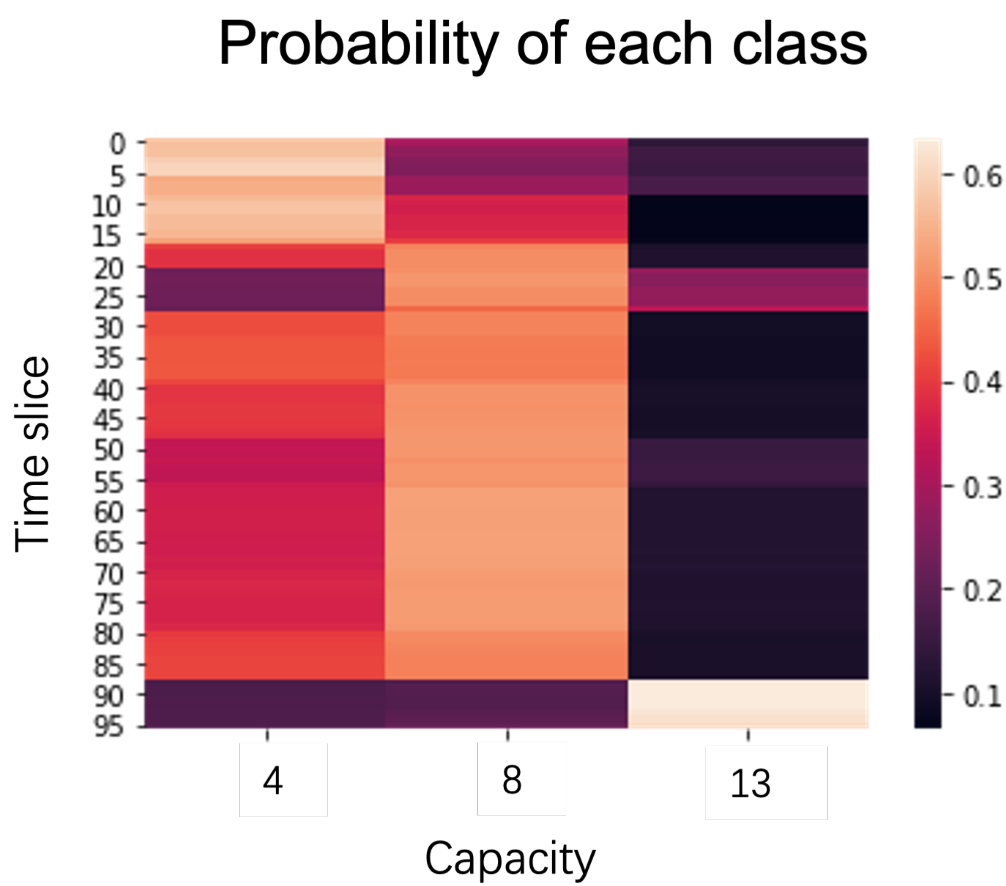
forecast Weather feature + Classification model

$N(\mu_1, \sigma_1)$

$N(\mu_2, \sigma_2)$

$N(\mu_3, \sigma_3)$

Airport Capacity Prediction



Wu, Zhou, Zhu, Li, **L** (in prep)

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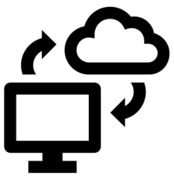
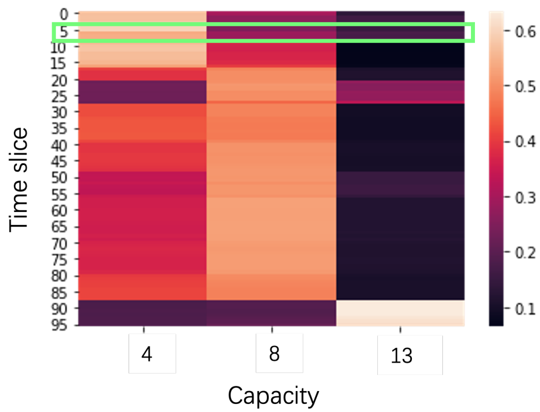
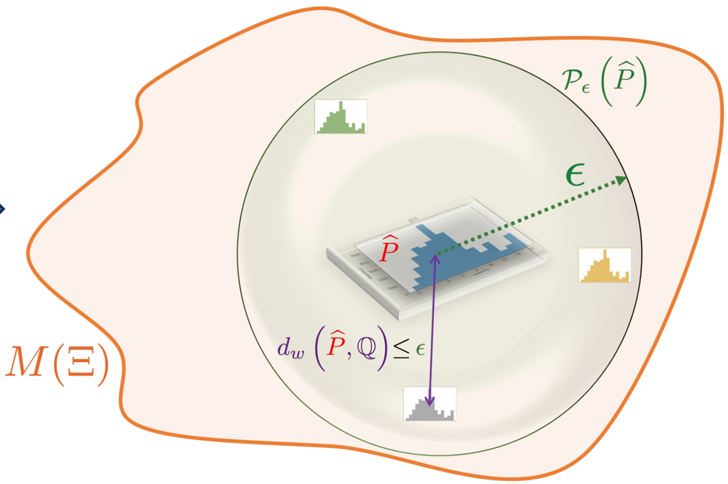
Ongoing: Integration

ML Model







$$\min_{u,v} \left\{ \sum_{f\in F} \left(C_g g_f + C_a a_f\right) + \max_{p\in\mathcal{P}^{(g)}_{\epsilon_1}\left(\widehat{P}^{(g)}\right)} \mathbb{E}_p\left[Q\left(u,\xi^{(g)}\right)\right] + \max_{p\in\mathcal{P}^{(a)}_{\epsilon_2}\left(\widehat{P}^{(a)}\right)} \mathbb{E}_p\left[Q\left(v,\xi^{(a)}\right)\right]\right\}$$

Testing & Evaluation
Open Questions …

Open Questions

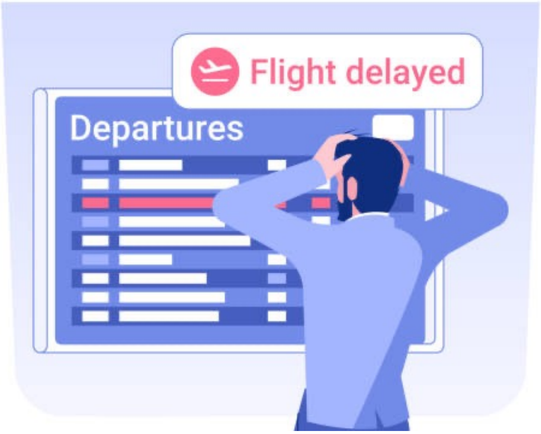
- Data-driven optimization of ambiguity set size
  - What ϵ (or ϵ_t) to use?
- Geometric characterizations of the ambiguity set for discrete, non-negative distributions
- Adaptive discretization schemes

Concluding Remarks

- For the future, Information-Centric National Airspace System, AI/ML will play key roles in strategic traffic management and ensuring efficiency

- Given such predictions have heavy uncertainty, how can we be robust to such uncertainty?

- Predictive (ML) + Prescriptive (DRO)



Thank You!

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