

# Safe Learning for Control

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# Growing numbers of new applications



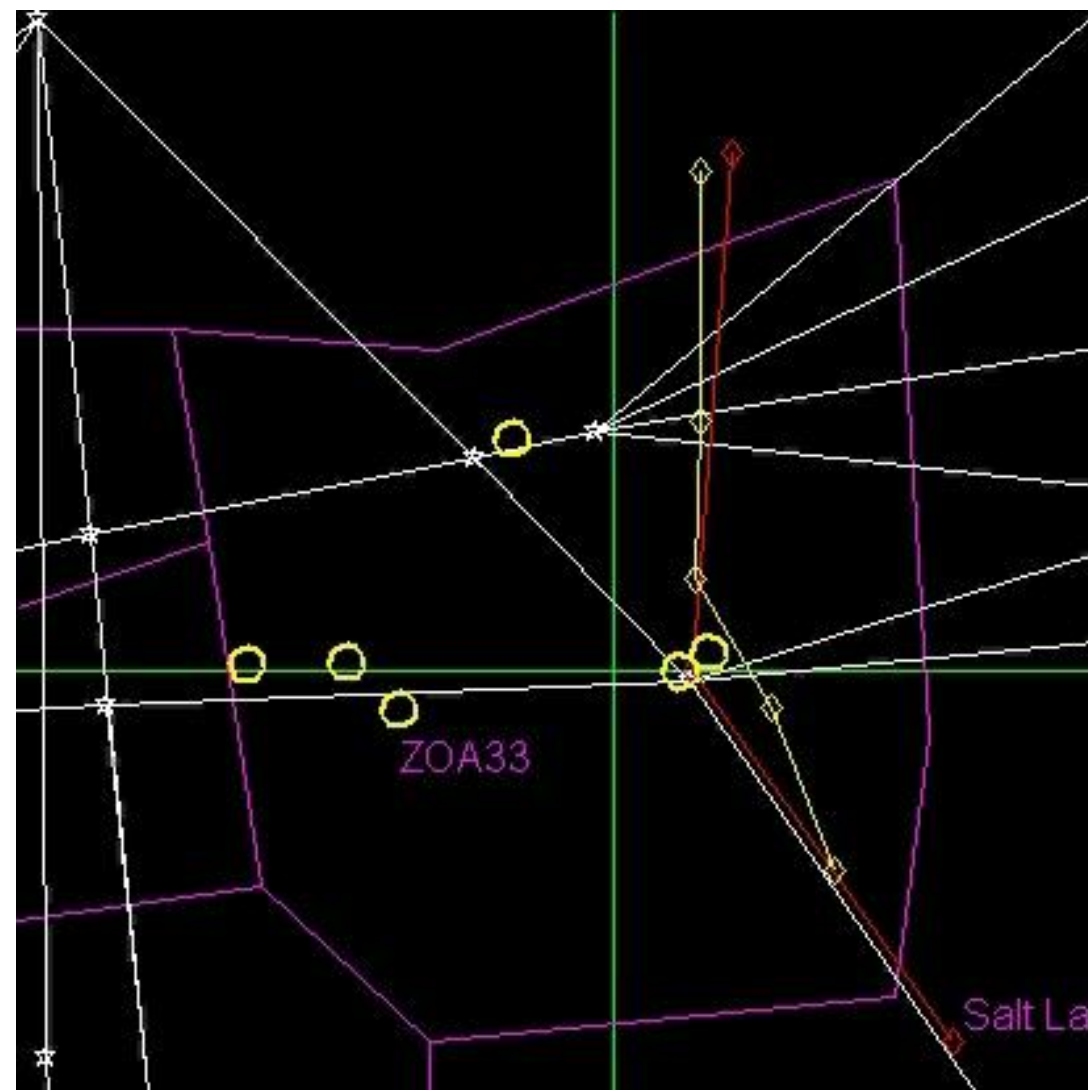
[Amazon]



[Zipline]



[Lilium]



1. Safety
2. Simplicity
3. Ability to adapt to new information

[NASA]

- Collision avoidance system
- Forced landing system
- Air taxi control

[ONR]

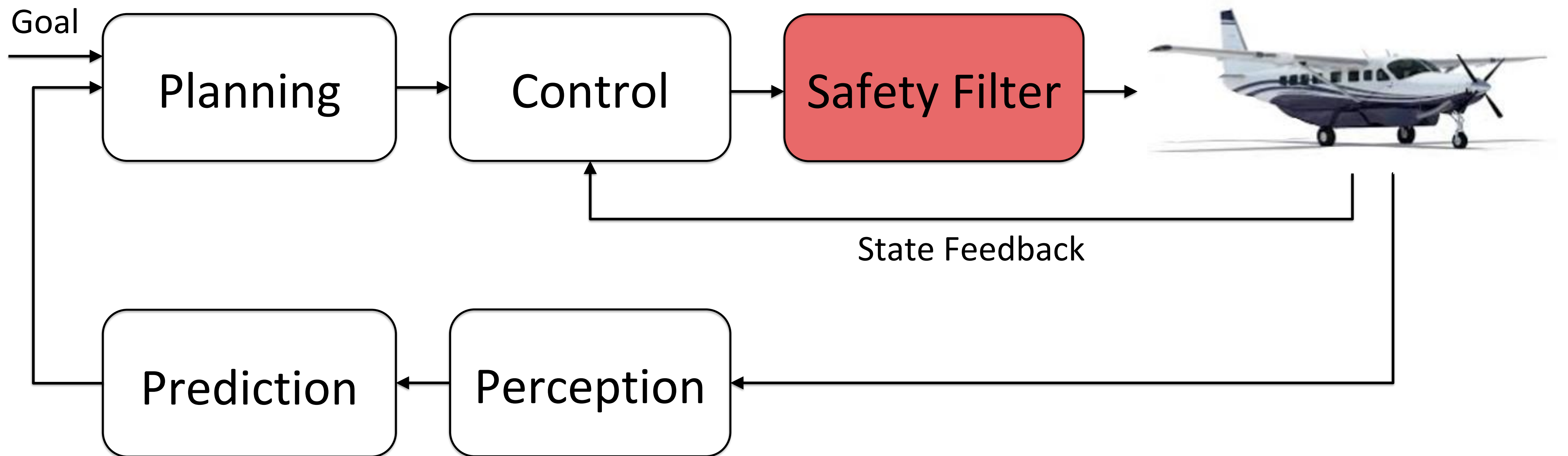
- Interdiction system



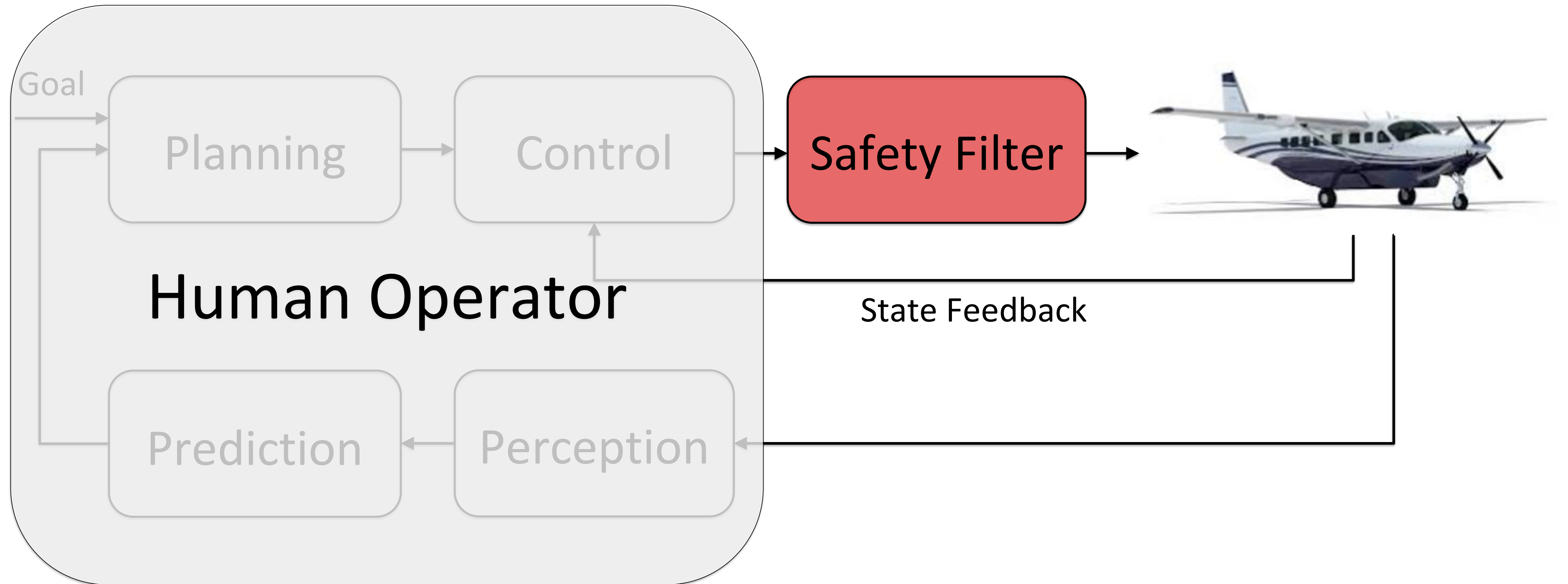
[US Coast Guard]



# Safety Filter



# Safety Filter



# Safety Analysis: Hamilton-Jacobi Reachability

$$\dot{z} = f(z, u, d)$$

State

Control

Disturbance

Compute all states for which, for all possible **disturbances**, there is a **control** which can drive the system state into a **target set** over a time horizon

Reachability as game: disturbance attempts to force system into unsafe region, control attempts to stay safe

# Safety Analysis: Hamilton-Jacobi Reachability

## 1. Cost Function

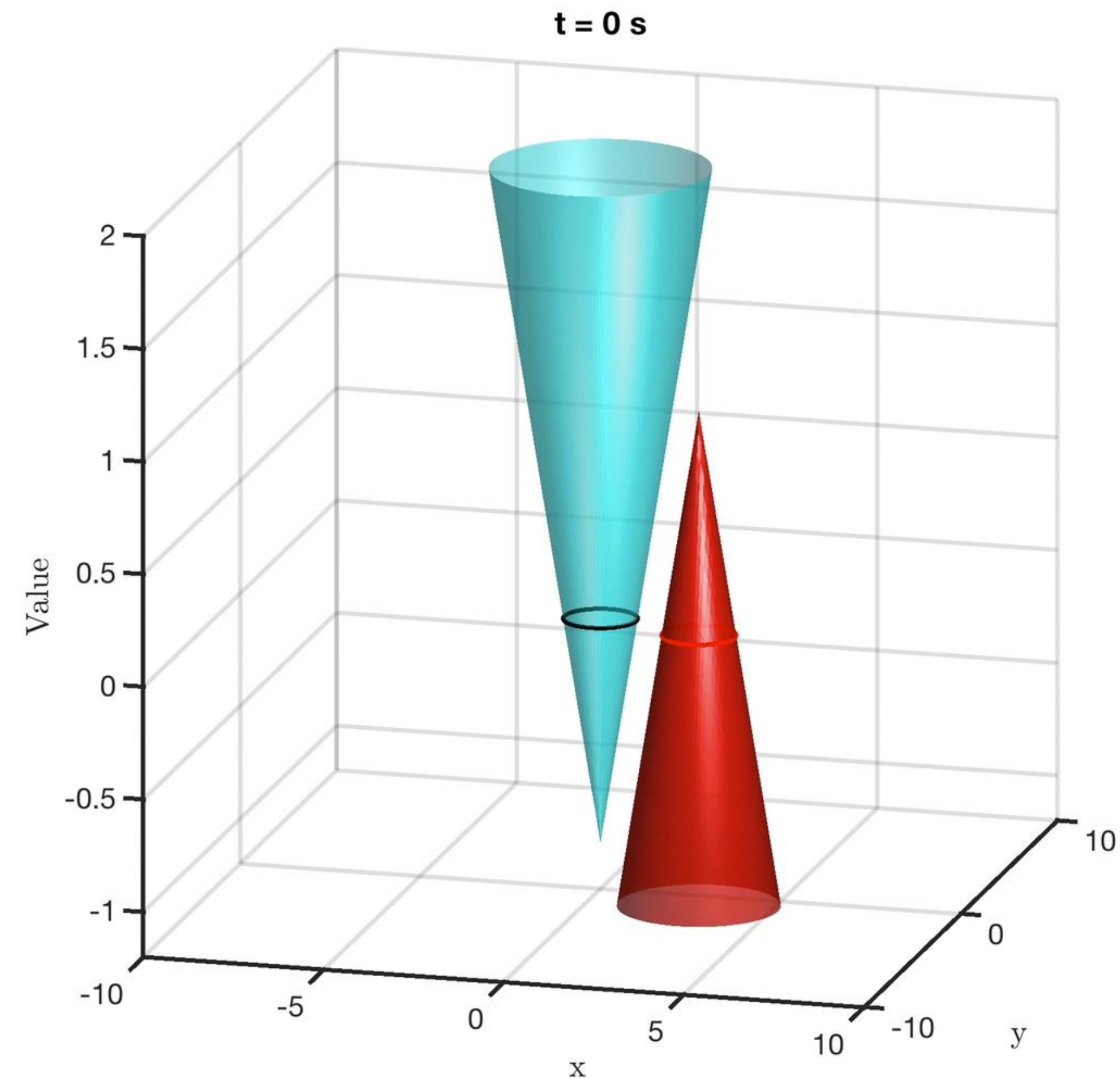
$$\begin{aligned} z \in \mathcal{L} &\leftrightarrow l(z) \leq 0 \\ z \in \mathcal{G} &\leftrightarrow g(z) \leq 0 \end{aligned}$$

## 2. Value function:

$$V(z, T) = \inf_{u(\cdot)} \sup_{d(\cdot)} \min_{t \in [0, T]} \left\{ \max_{s \in [t, T]} \left\{ l(\xi_{z,T}^{u,d}(t)), \max_{s \in [t, T]} g(\xi_{z,T}^{u,d}(s)) \right\} \right\}$$

## 3. Update equation (Hamilton-Jacobi-Isaacs PDE)

$$\max \left\{ \min \left\{ \frac{\partial V}{\partial t} + H(z, \nabla V), l(z) - V(z, t), g(z) - V(z, t) \right\} \right\} = 0$$





# Safety Analysis: Hamilton-Jacobi Reachability

## 1. Cost Function

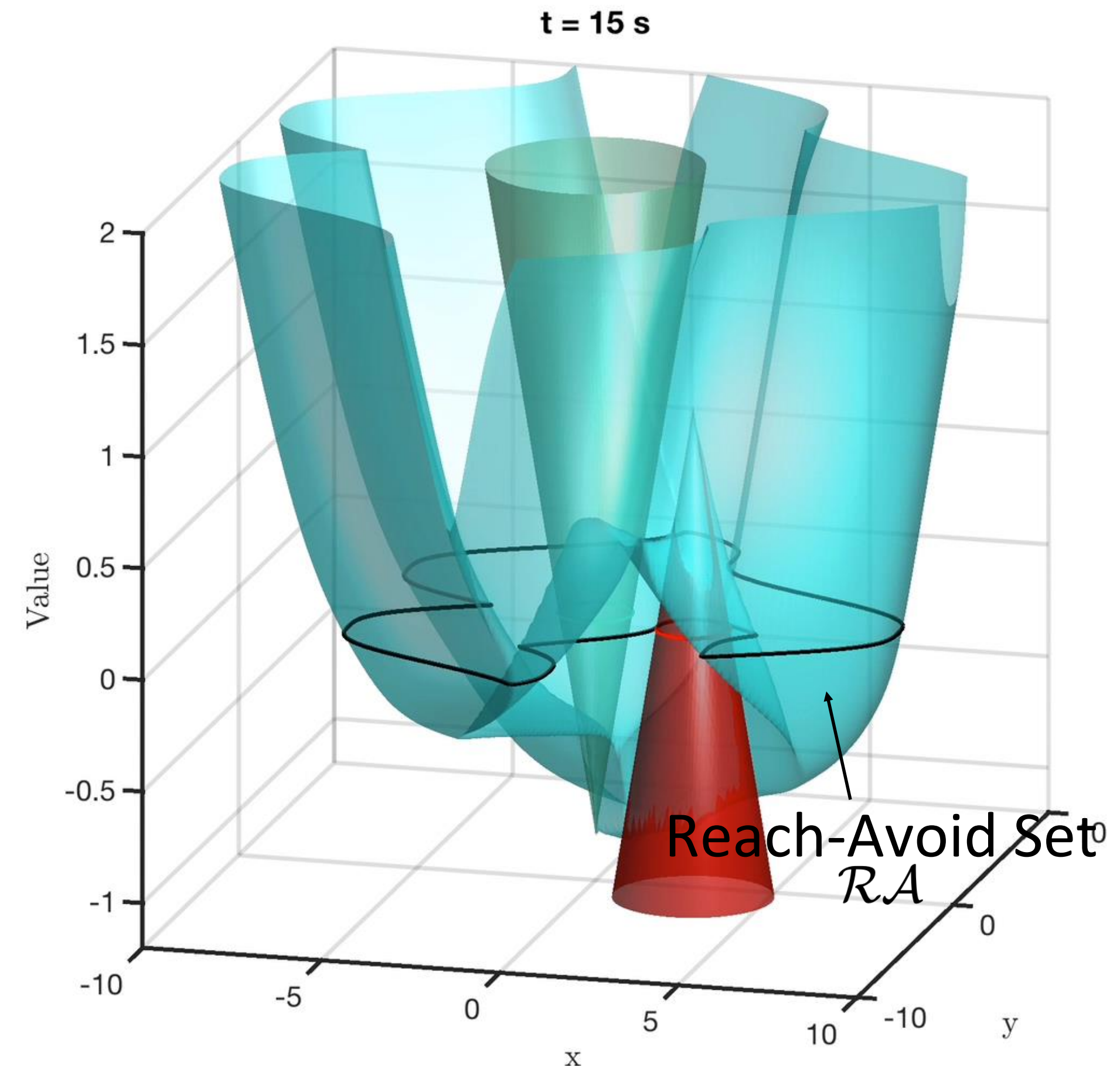
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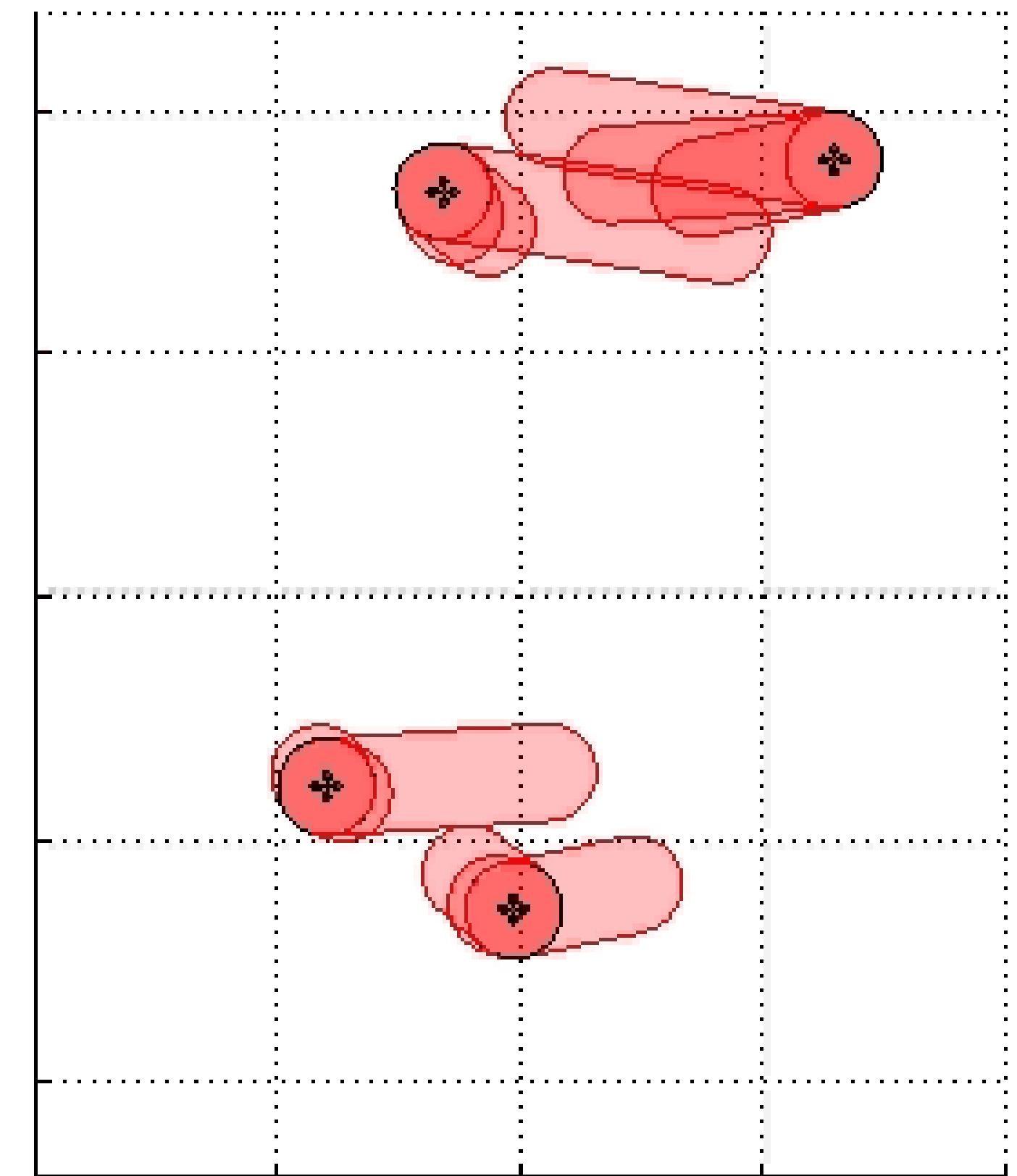
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# Pilots attempting to collide vehicles

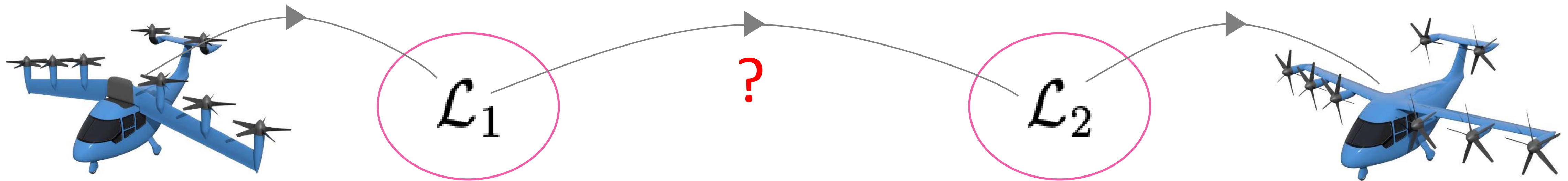




# Reachable sets for Safety Assurances



[Joby Aviation]

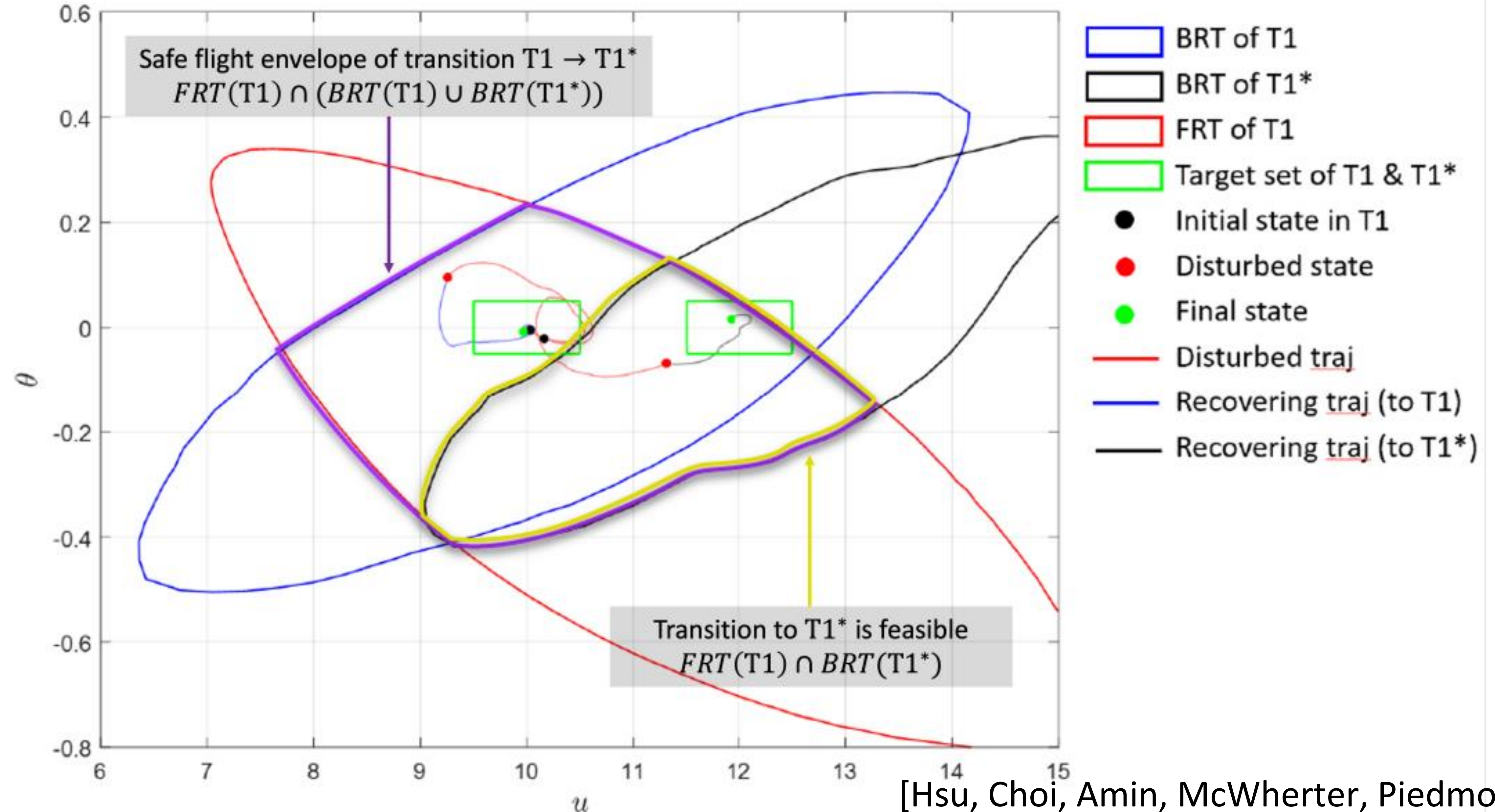


How to ensure safe transition between the trim states  
during the flight mode transition?

[Hsu, Choi, Amin, McWherter, Piedmonte]

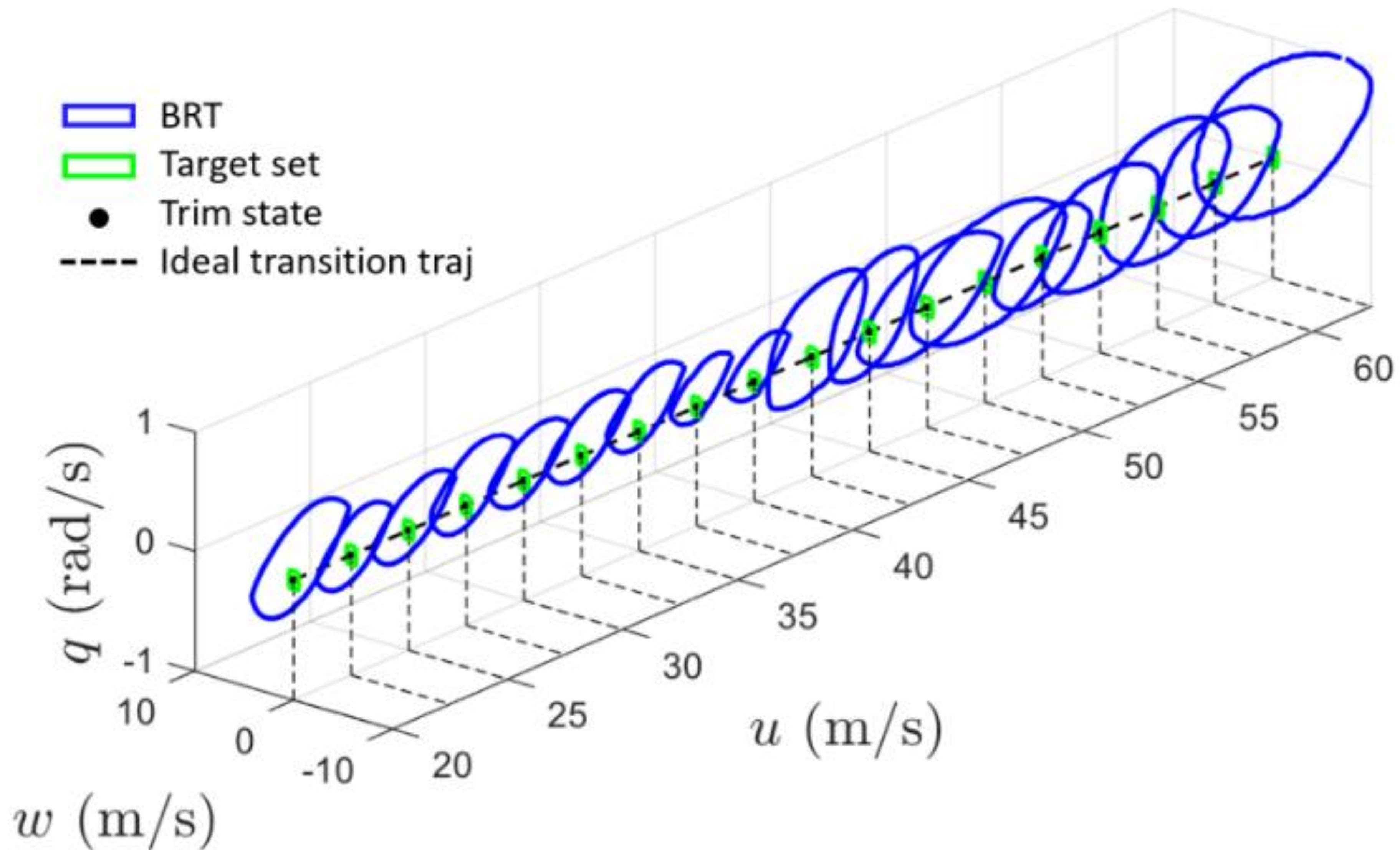


# Reachable sets for Safety Assurances

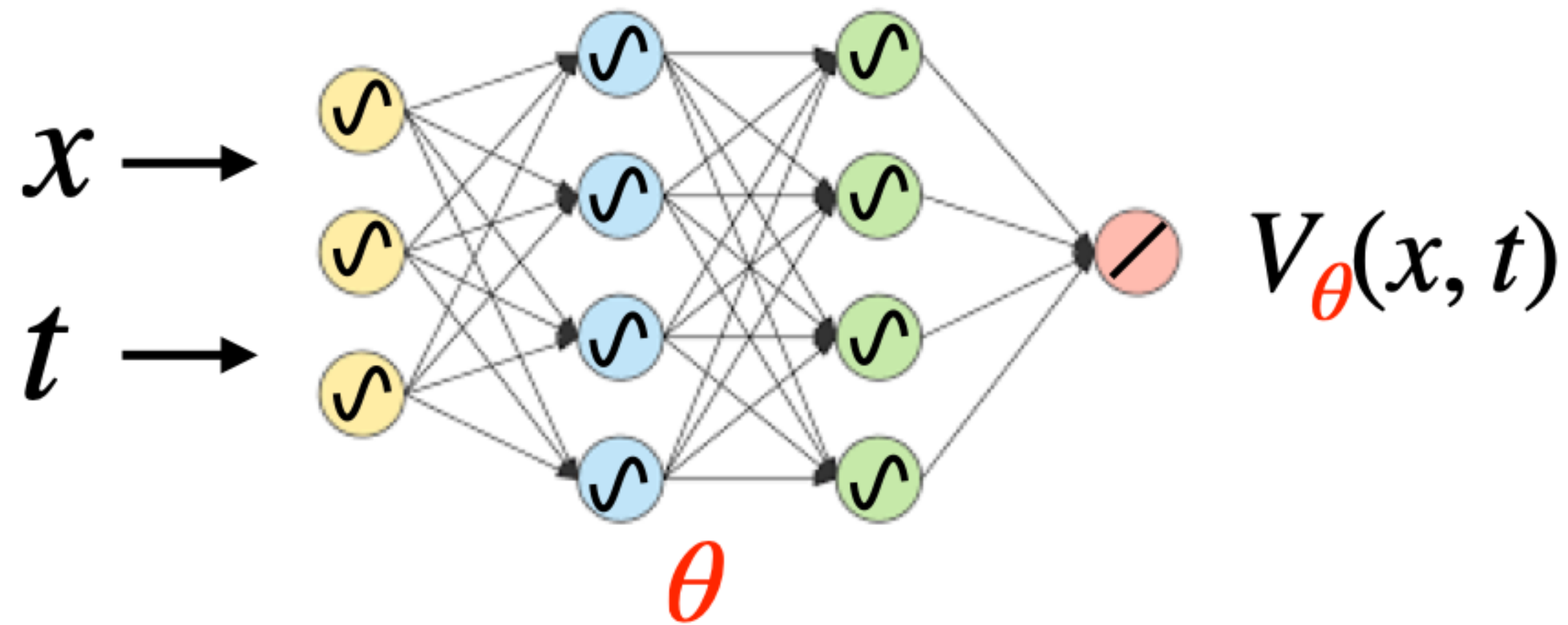




# Reachable sets for Safety Assurances



# DeepReach



Randomly Sample  
State and Time

$$\{(x_i, t_i)\}$$

Compute the Loss Function

$$h(\theta) = \sum_i \left\| \frac{\partial V_{\theta}(x_i, t_i)}{\partial t} + H(x_i, \nabla V_{\theta}(x_i, t_i)) \right\| + \lambda \|V_{\theta}(x_i, T) - l(x_i)\|$$

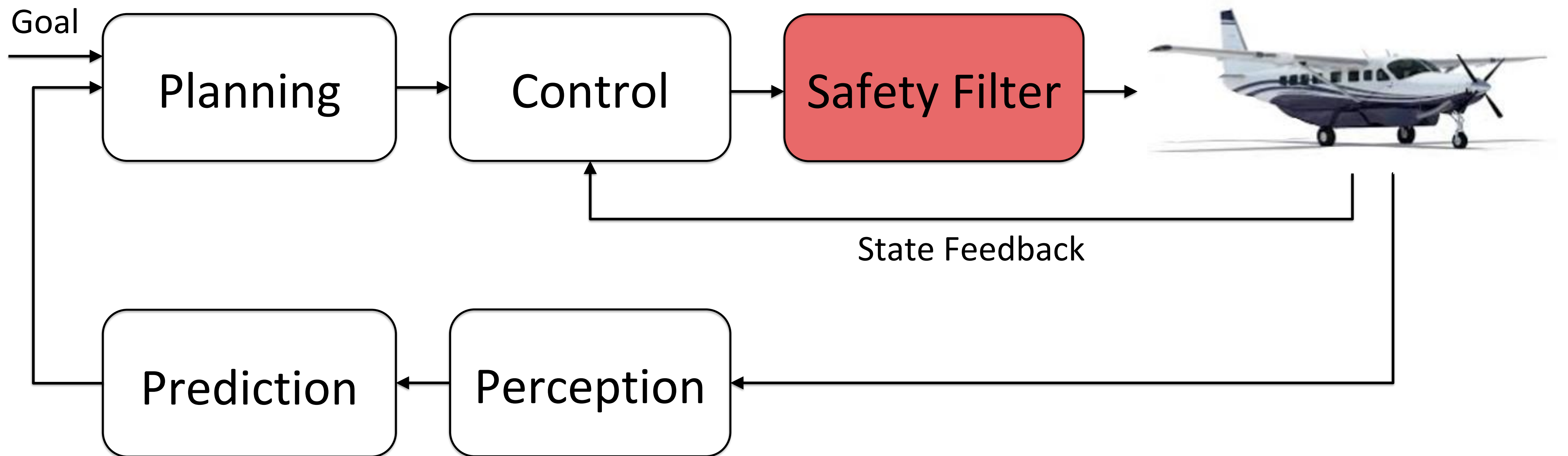
Fit the Value Function

$$\theta \leftarrow \theta - \alpha \nabla h(\theta)$$

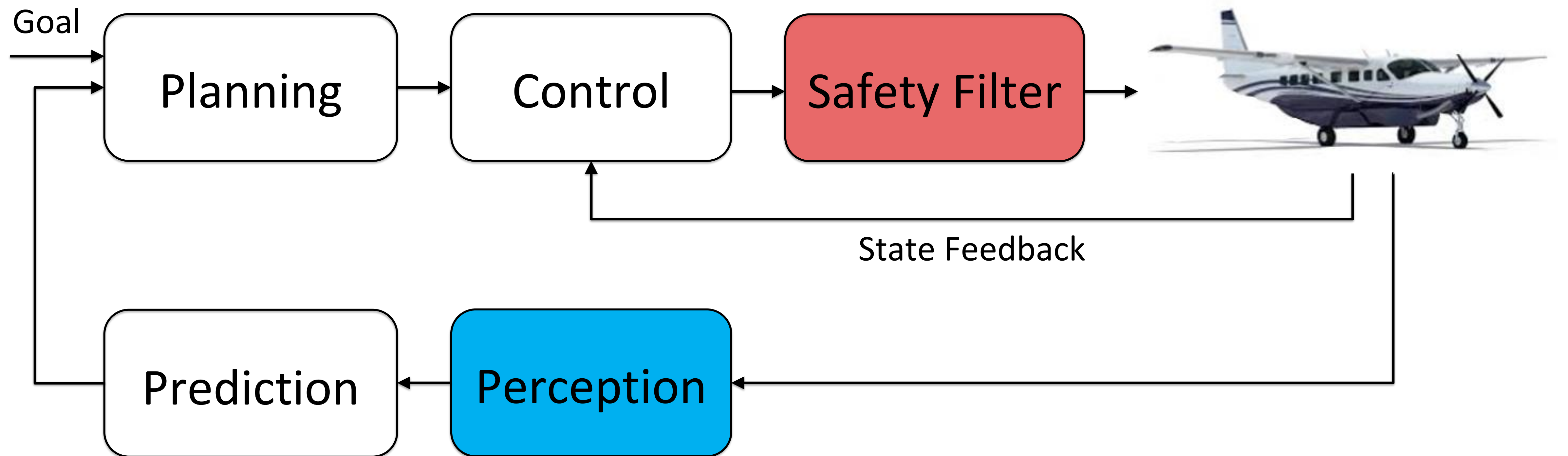
Repeat



# Safety Filter



# Safety Filter





# Capturing (and Tracking) Perception Uncertainty

**Without Incorporating Uncertainty**



**Incorporating Uncertainty**

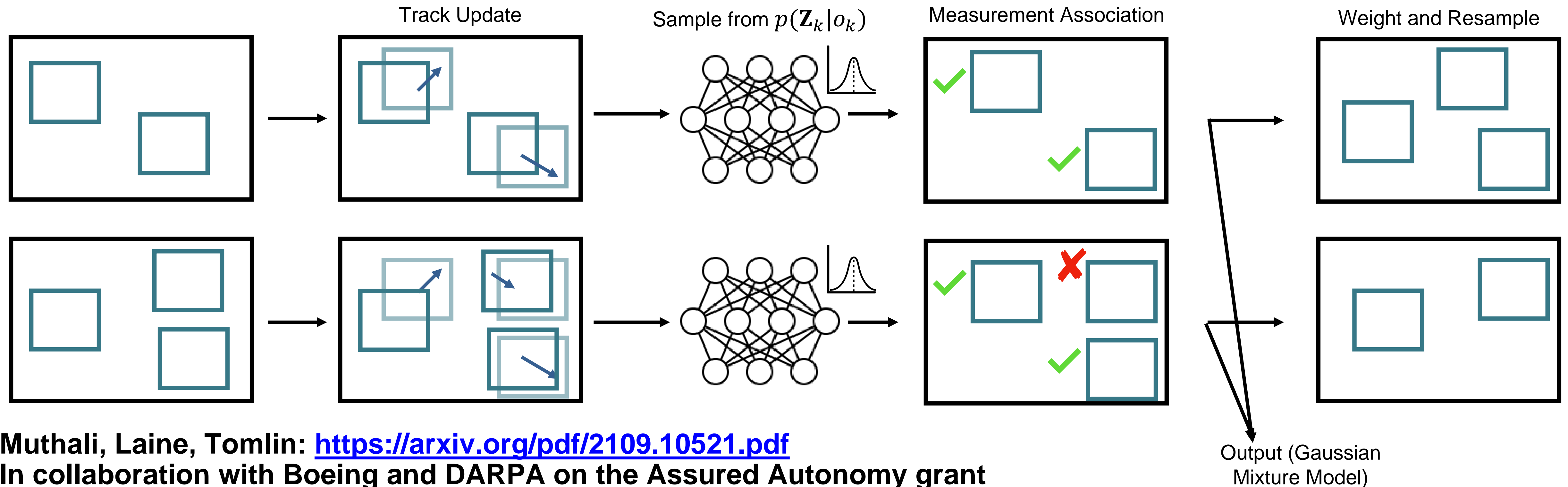


# Our Approach

## With Neural Network Uncertainty

How do we incorporate randomness in  $\mathbf{Z}_{1:k}$ ? It is possible to extend this to estimate  $p(\mathbf{X}_k | o_{1:k})$ !

$\mathbf{X}_k$  - state of tracks at stage k  
 $\mathbf{Z}_{1:k}$  - set of measurements from stages 1 to k  
 $o_{1:k}$  - set of observations from stages 1 to k



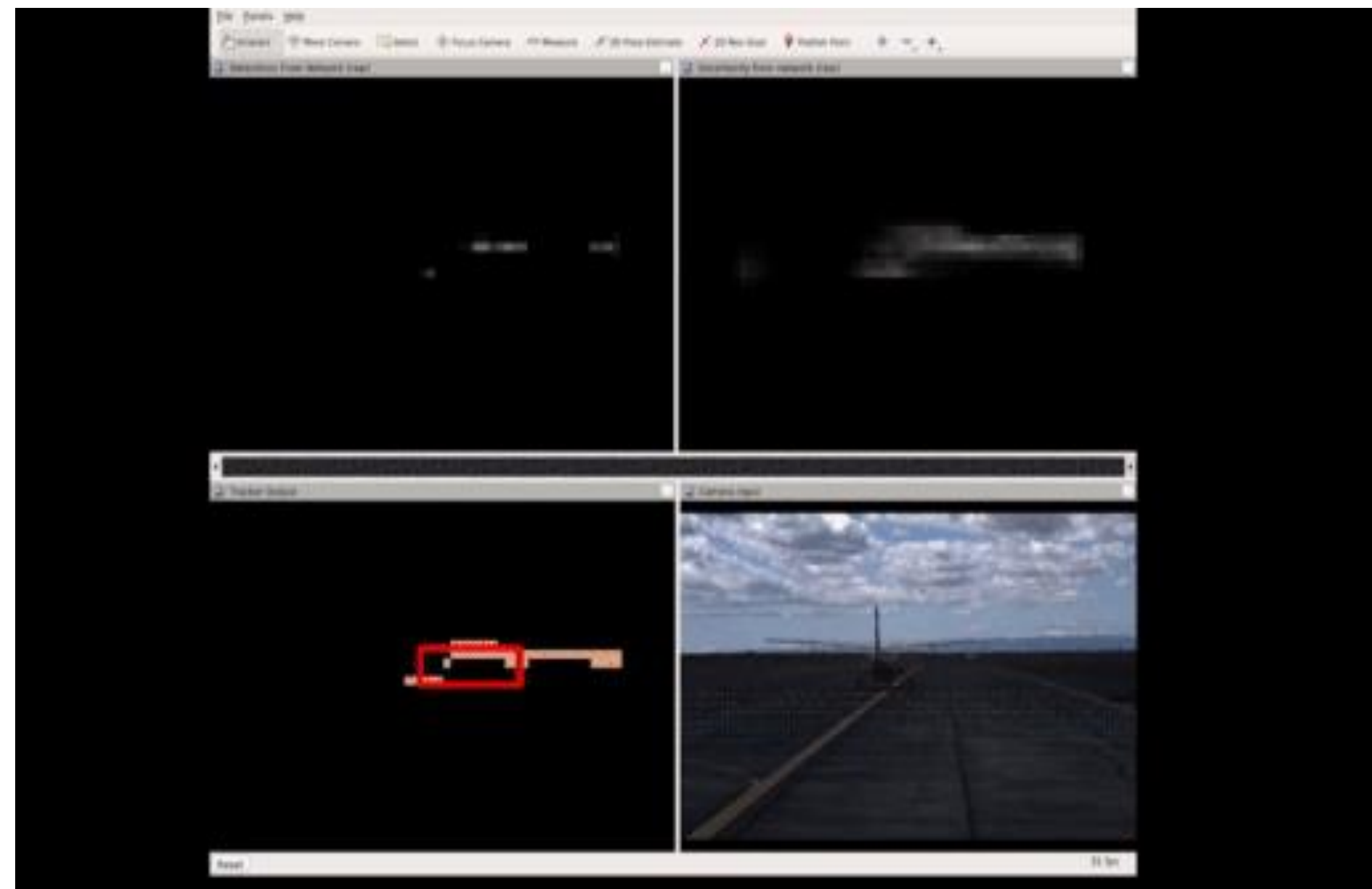
Muthali, Laine, Tomlin: <https://arxiv.org/pdf/2109.10521.pdf>

In collaboration with Boeing and DARPA on the Assured Autonomy grant



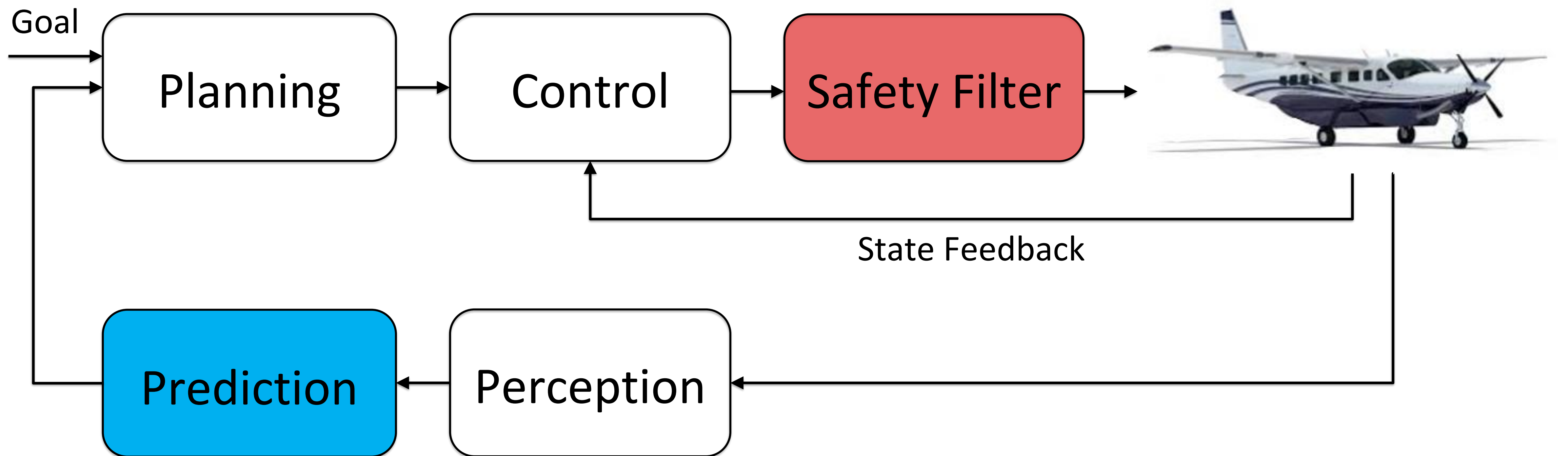
# Deployment on an Autonomous Aircraft

A sampling-free, segmentation based method to speed up the inference stage of the tracking pipeline: measurement association model leverages sparsity of scenes



Muthali, Laine, Tomlin: <https://arxiv.org/pdf/2109.10521.pdf>  
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# Safety Filter



# Predicting the behavior of other agents



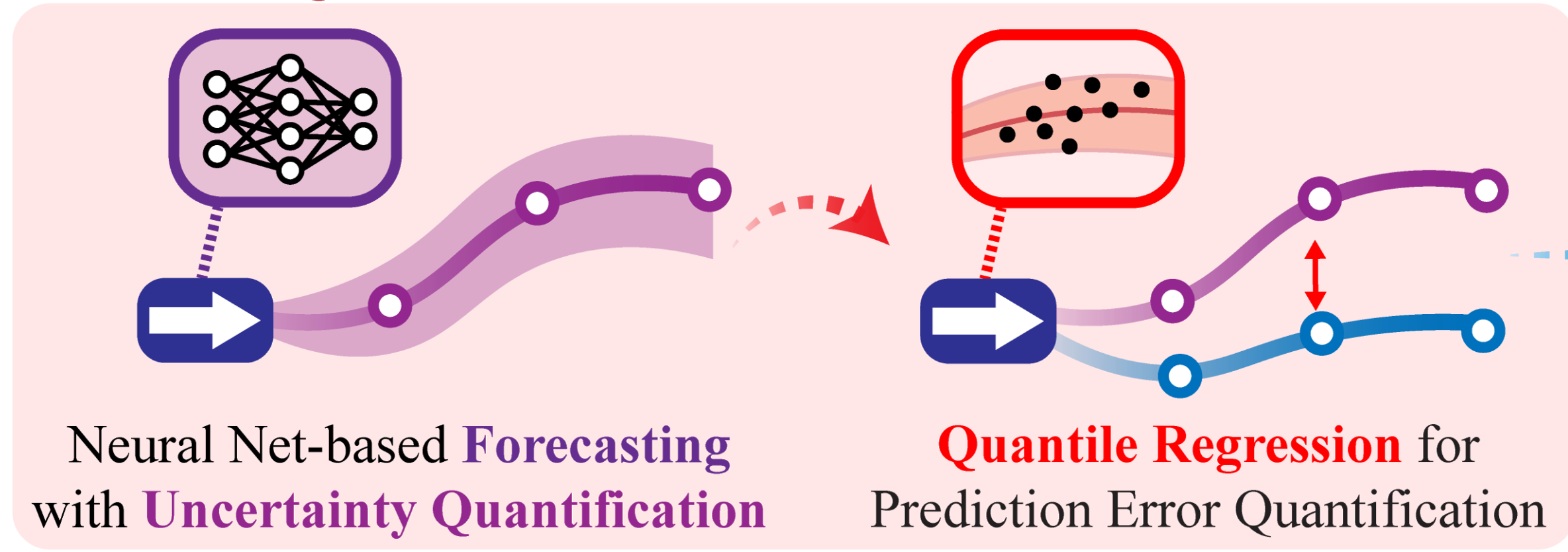
*(Photo: Transportation Safety Board of Canada)*

Based on the observed behavior of ground vehicles on or around the runway, predict with calibrated certainty if the runway will be clear for the aircraft to land

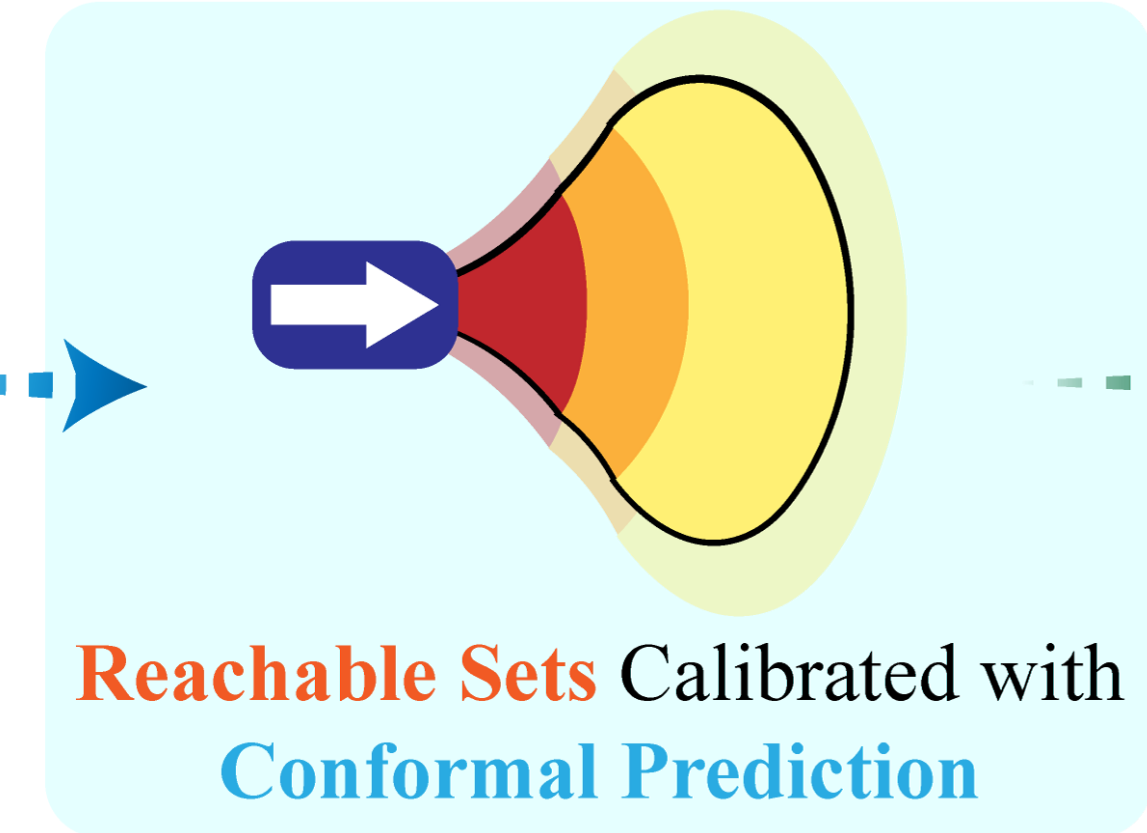


# Approach Outline

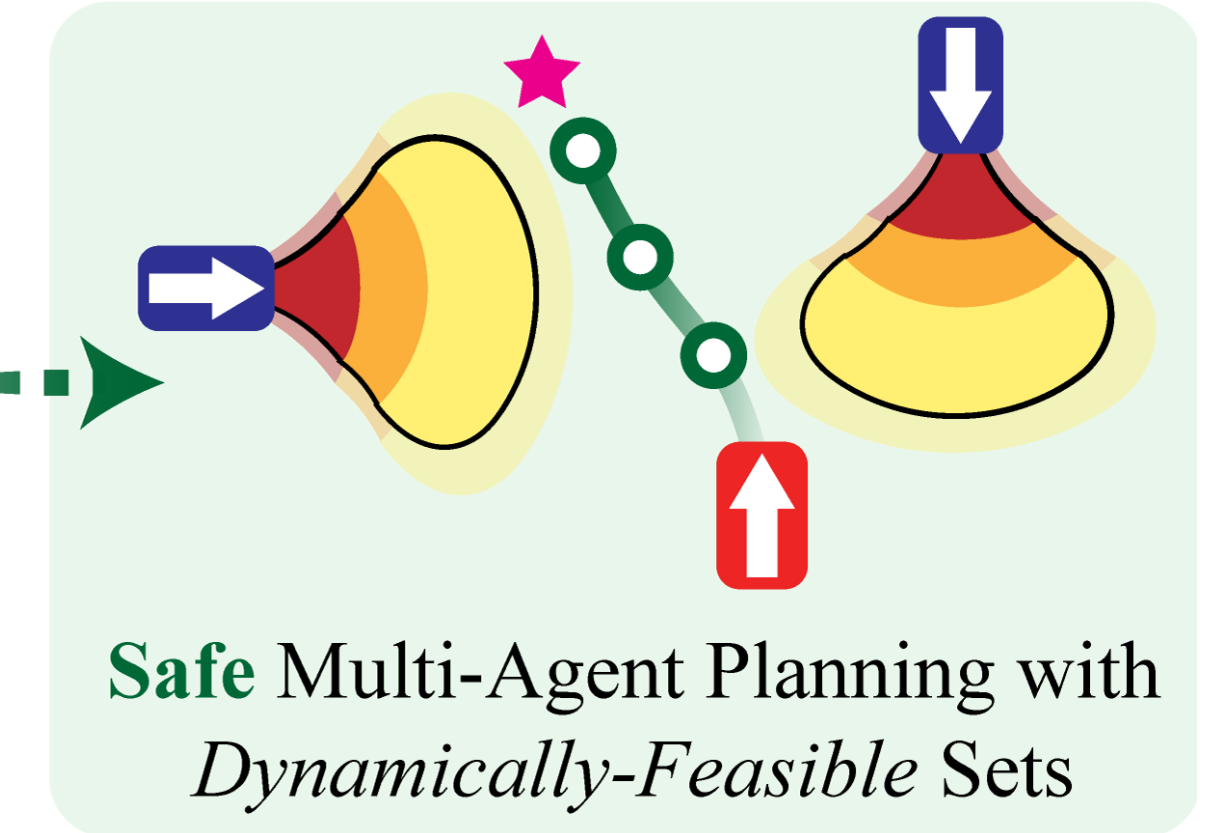
## Forecasting & Prediction



## Calibration

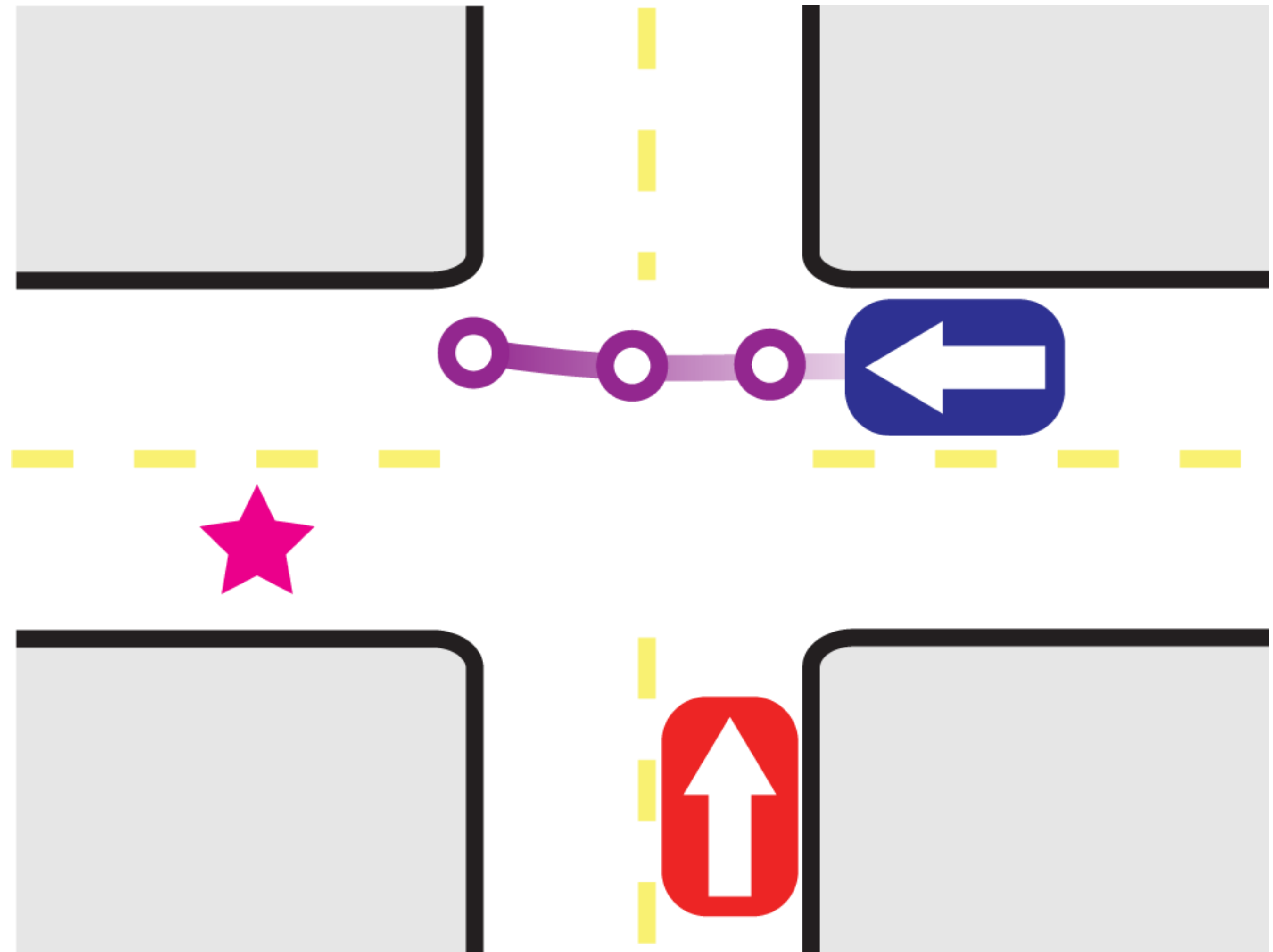


## Planning



# Forecasting and Uncertainty

- Forecast agents' actions using a trajectory prediction model
  - Trajectron++
- Obtain a measure of uncertainty
  - Conditional on each prediction





# Quantile Regression

- Use uncertainty to predict model error
- Obtain an *approximate*  $1 - \alpha$  prediction interval on model error
- Quantile regression uses a linear model
  - Easier to “understand” uncertainty by looking at regression coefficients
  - Update regression coefficients online using gradient descent

$$\mathbf{u}_{t:t+h} \in \left[ \hat{\mathbf{u}}_{t:t+h} + \hat{\mathbf{e}}_{\frac{\alpha}{2}}, \hat{\mathbf{u}}_{t:t+h} + \hat{\mathbf{e}}_{1-\frac{\alpha}{2}} \right]$$

# Conformal Prediction

- Make the *approximate* confidence intervals *exact*
  - Stretch intervals by  $\varphi(\theta)$  which is “learned” online
- Note: “stretching” is not conditional on the neural network’s prediction uncertainty
  - Quantile regression is still important!

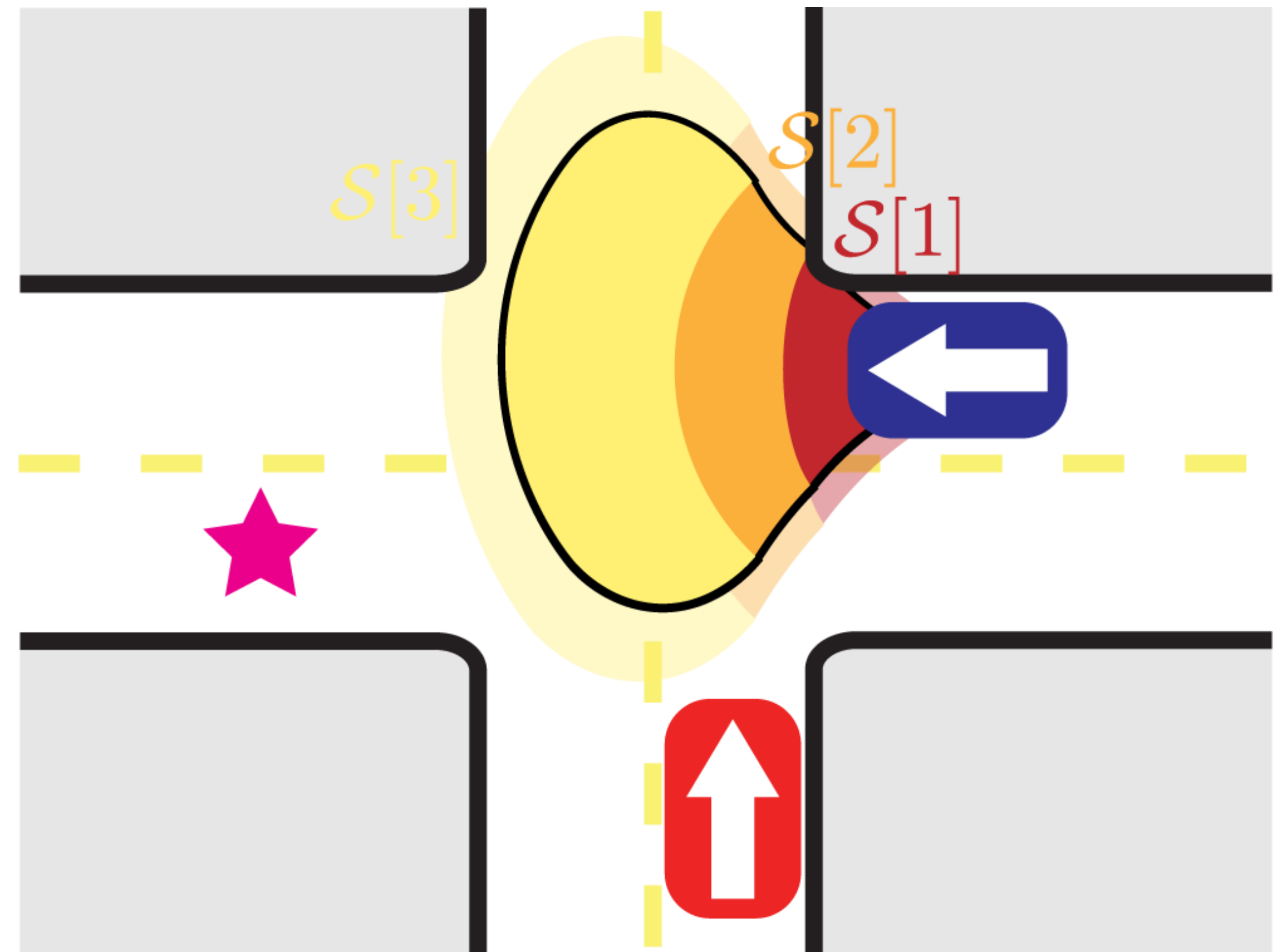
$$\mathbf{u}_{t:t+h} \in \left[ \hat{\mathbf{u}}_{t:t+h} + \hat{\mathbf{e}}_{\frac{\alpha}{2}} - \varphi(\boldsymbol{\theta}), \hat{\mathbf{u}}_{t:t+h} + \hat{\mathbf{e}}_{1-\frac{\alpha}{2}} + \varphi(\boldsymbol{\theta}) \right]$$

- Coverage rate guarantee:  $1 - \alpha + \mathcal{O}(1/t)$



# Hamilton-Jacobi Reachability

- Transform intervals in control space to sets in state space
- Incorporate dynamical constraints
- Easier to leverage for downstream planning tasks
- HJ sets are time indexed
  - $\Delta t, 2\Delta t, \dots, h$  second forward reachable tubes
  - Indexed by prediction timestep



# Multi-Agent Setting

- Require that *total* miscoverage rate does not exceed a given  $\gamma$

$$\mathbb{P}_t \left( \bigcup_{i=1}^N \left\{ \mathbf{x}_t^{(i)} \notin \mathcal{S}[t]^{(i)} \right\} \right) \leq \gamma$$

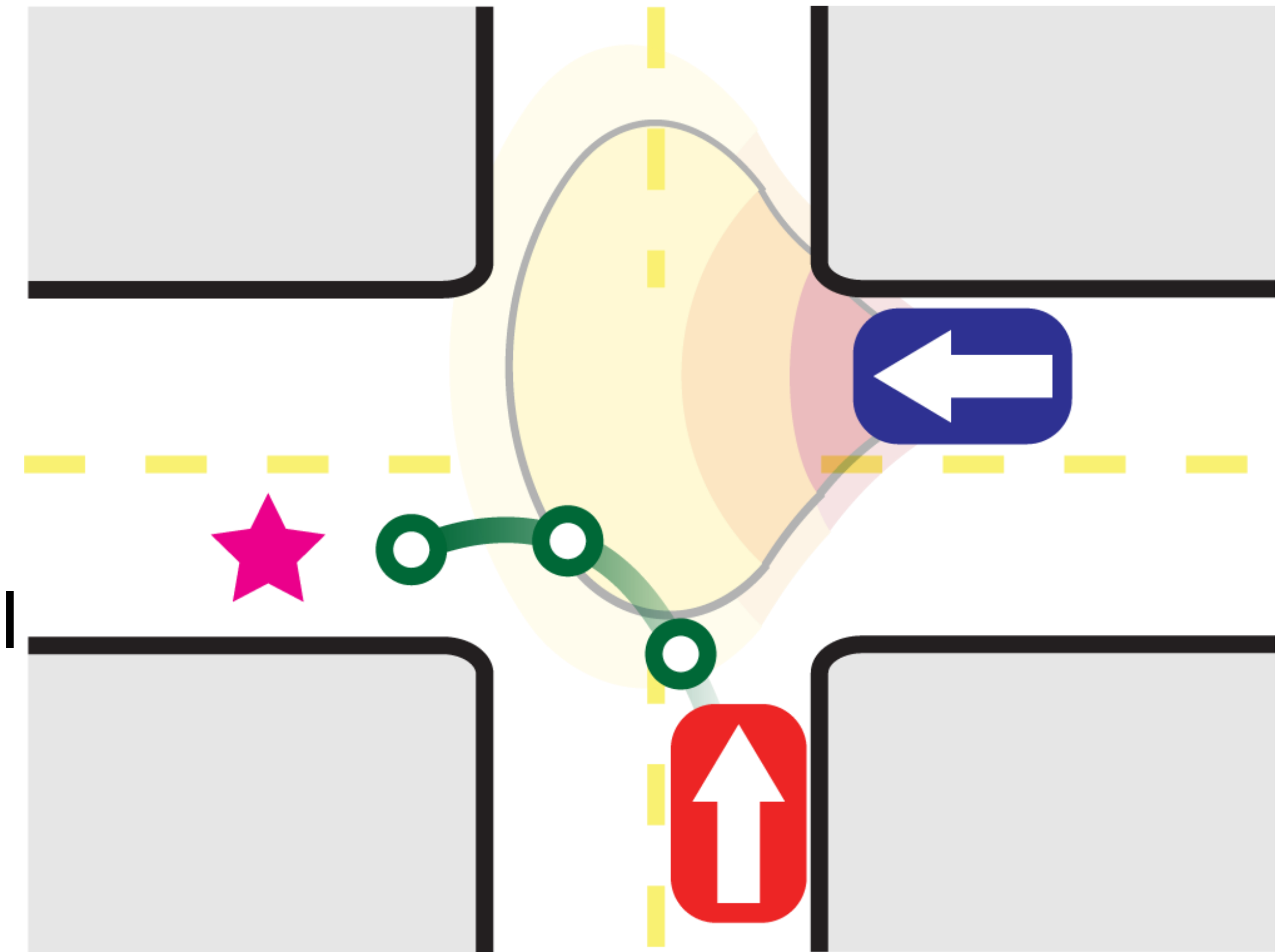
- Thus, set the desired miscoverage rate for each agent to

$$\alpha = 1 - (1 - \gamma)^{\frac{1}{N}}$$



# Ego Agent Planning

- Treat time-indexed sets as dynamic obstacles
- Compute forward reach-avoid tube using dynamic obstacles
- Derive optimal control to a desired goal inside reach-avoid tube
- Hamiltonian-maximizing control



# Case Study: Runway Clear





# Case Study: Runway Clear

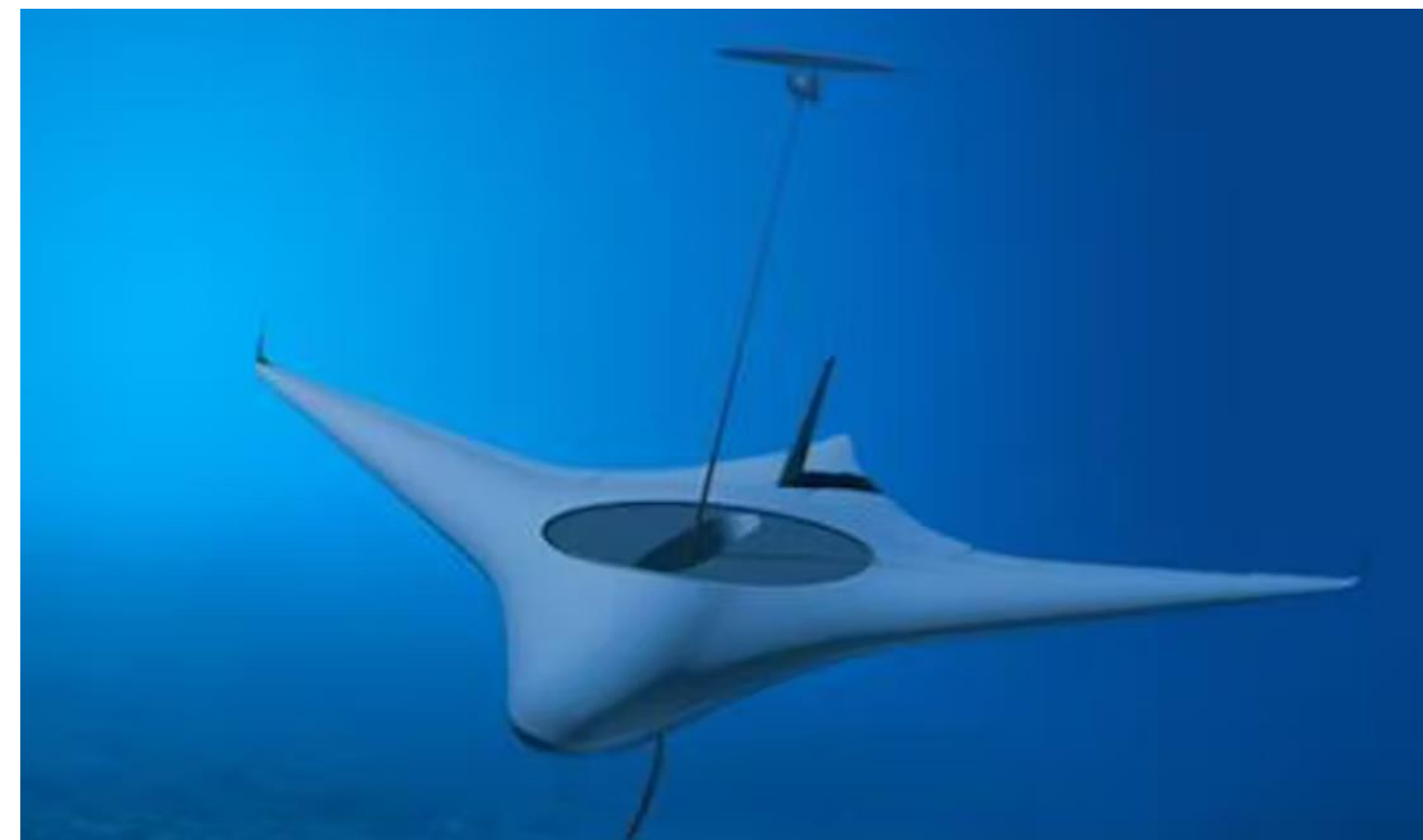




# Today: Automating platforms



[Amazon]



[Manta Ray UUV]



[Boeing]



[Zipline]

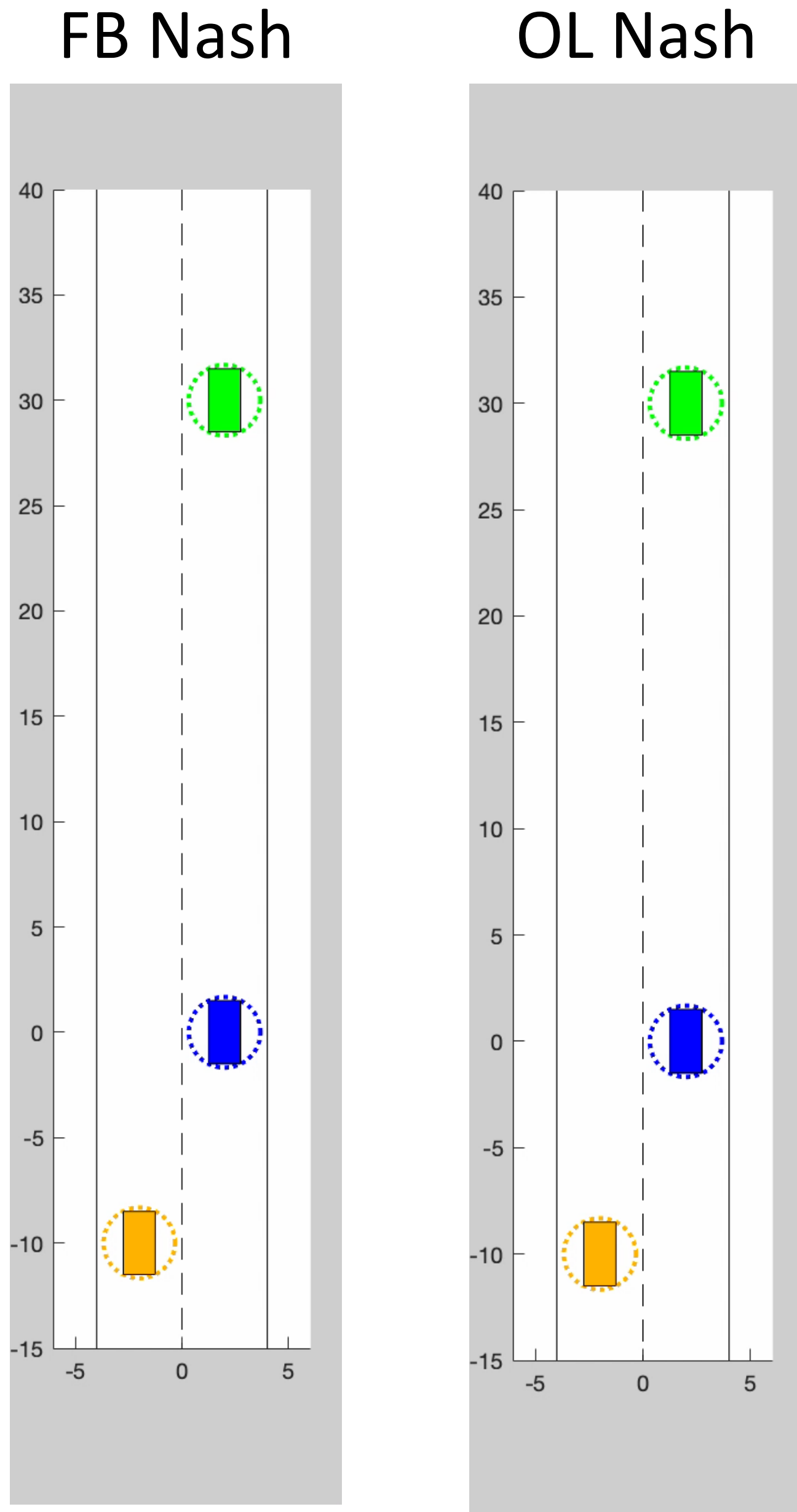
Need to make guarantees about the behavior of these platforms even as they learn and evolve



[Polaris MRZR]



# What's needed: Automating Systems



- Predictions need to be closed loop!
- Enable predictable, safe, and high-confidence interactions between agents in multiple agent systems
- Even if specification is unknown
- Distribution of future data will not necessarily follow distribution of past
- Teams of humans and robots
- Focus on systems, not components

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