# A Guide to Solar Power Forecasting using ARMA models

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**Abstract**

We describe a simple and succinct methodology to develop hourly auto-regressive moving average (ARMA) models to forecast power output from a photovoltaic generator. We illustrate how to use statistical tests to validate the model and construct hourly samples. These samples can be used as scenarios for energy planning or in stochastic optimization models.

*Keywords:* ARMA, solar power, photovoltaic, forecasting, scenario generation

## 1. Introduction

Increasing penetration of renewable energy sources, such as wind and solar, in the electricity grid requires an accurate representation of the uncertainty to guarantee feasibility of the system operations, and for efficiently planning new transmission lines and generation capacities. To address these challenges, advanced decision-making models that lie at the interface of statistics and operation research fields have been widely explored. Representation of the uncertainty in renewable energy is typically done by either using *samples* or a *set representation* from the underlying stochastic process. The former generally requires forecasting tools for generating synthetic samples or scenarios that are used for feeding decision-making optimization models [**?** ]. The latter requires a simple representation of the stochastic process in order to embed it into more sophisticated decision-making tools [**?** ]. In both cases, but especially so in the latter, complex forecasting models result in models that are hard to integrate.

In this article, we present a forecasting model that is easy-to-embed into more sophisticated decision-making models, which at the same time also serves as a tool for generating samples of renewable energy forecasts. In particular, we focus on forecasting hourly photovoltaic (PV) solar power generation, but the methodology is not limited to this technology. Solar power differs from wind power due to its diurnal nature, and can have much greater ramps than wind [1].

Forecasting methods for solar power are broadly divided into two categories: (i) physics-based models—these models predict solar power from numerical weather predictions and solar irradiation data, and (ii) statistical models—these models forecast solar power directly from historical data. Comparisons of these two methods are available; see, e.g., [2, 3]. There are other approaches available as well which combine these two methods [**?** ]. In this article, we center on statistical methods alone, and specifically the use of auto-regressive moving average (ARMA) models to develop our forecasts. Despite their limitations [**?** ], ARMA models are widely used to forecast wind power [4, 5], as well as solar power [6**?** ], because of their ease of implementation and parameter selection. Yet, accurate and fast methods to generate solar power scenarios are often unavailable or significantly complex, and normal approximations are frequently used; see, e.g., [7]. Here, we describe a summary of the methodology to forecast solar power using ARMA models. The software codes and generated scenarios are available on request. The presented models can be applied either to a local PV generating plant or at the regional level.

The main contribution of this article is to provide a step-by-step approach and easy-to-implement ARMA model to forecast PV solar power generation. The proposed model is able to capture the important statistical features of the parameters, while maintaining simplicity. The model allows modelers to embed it into more complex decision-making structures, statisticians to have an all-in-one place ARMA model design for PV power generation, and policymakers and electrical engineers to have a scenario generation tool.

## 2. Methods

We take hourly year-long historical solar power output from a site described in [8]. This zone has an altitude of 595m, a nominal power of 1560 MW, a panel tilt of 36°, and a 38° clockwise panel orientation from the north. Further installation specifics are available in zone 1 from Table 1 of [8], while technical specifications are available in [9]. We use approximately nine months of data for training. The data does not have any solar power for the ten hours [20:00-5:00], and hence we restrict the forecasts in these hours to be zero as well. Equivalently, a criteria based on the solar zenith angle can be used; i.e., 0° at sunrise and 90° when the sun is directly overhead. For each of the remaining 14 hours of the day, we build an ARMA$(p, q)$ model. One limitation of this approach is that it does not capture the correlations between different hours; see, e.g., [**?** ]. For each hour, we verify the stationarity of the time series and test a number of ARMA$(p, q)$ models to find the best one. We use statistical tests on the residuals to validate the models. Finally, we use Monte Carlo sampling from the best ARMA model, for each hour, to create hourly scenarios. Below we provide more details.

### *2.1. The ARMA model*

We recall here to the general formulation of ARMA models for modeling time series. Given a time series $x_t$, we can model the level of its current observation depending on the level of its $p$ lagged observations, $x_{t-1}, x_{t-2}, \ldots, x_{t-p}$, plus an additional white noise error term $\epsilon_t$. This model is known as an *autoregressive* model of order $p$, AR$(p)$:

$$x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \epsilon_t \tag{1}$$

Next, assume that the level of the current observation is affected not only by the current white noise error term, but also by the previous $q$ white noise errors. This model is known as a *moving average* model of order $q$, MA$(q)$:

$$x_t = \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} \tag{2}$$

An ARMA$(p, q)$ model combines both equations (1) and (2). Observe that the output variable depends linearly on the current and various past values (which is an advantage compared to other high fidelity forecasting models). An important question is how many representative lagged observations should be considered in order to have good fidelity while keeping the model as simple as possible. We discuss this in the proceeding sections.

$$x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} \tag{3}$$

### *2.2. Stationarity*

An ARMA model may be a suitable forecasting tool if a time-series is stationary. We test the hourly data for stationarity using the Augmented Dickey-Fuller (ADF) test [10]. The ADF test has a null hypothesis that the series includes a unit root (or, is non-stationary). We reject the null hypothesis at a level 0.05 if the test-statistic exceeds its 0.95 level quantile. For all the 14 hours of the day, the null hypothesis is rejected suggesting the series may be stationary, and hence an ARMA model may be suitable. If the series were not stationary, an ARIMA model may be suitable; see, e.g., [11].

### *2.3. Selecting parameters of the ARMA model*

Next, we estimate the parameters of the ARMA model, $p$, the order of the autoregressive part and, $q$, the order of the moving average part. For each hour, we construct 16 models with both $p$ and $q$ between one and four, and compute the log-likelihood objective function value. Next, for each hour, we calculate the Bayesian information criteria (BIC) for the 16 models using $p + q + 1$ parameters. The BIC penalizes for models with more parameters, and the smallest value of the BIC gives the best model, for each hour. Table 1 provides our estimated $p$ and $q$ values for the 14 hours of the day. We note that none of the hours have an order value exceeding two.

Table 1: Estimated $p$ and $q$ values for ARMA($p, q$) models for 14 hours of the day

| Hour | 6:00 | 7:00 | 8:00 | 9:00 | 10:00 | 11:00 | 12:00 | 13:00 | 14:00 | 15:00 | 16:00 | 17:00 | 18:00 | 19:00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| q | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 2 | 1 |

### *2.4. Prediction*

Figure 1 plots a day-ahead prediction using the above constructed ARMA models; i.e., one hour ahead predictions from the 14 ARMA models. A number of metrics are available to evaluate the prediction; see, e.g., [12]. We use a few of them here. The mean absolute error between the actual and the predicted series is 39.6 MW, or 3.3% of the maximum actual value. The root mean square error between the actual and the predicted series is 61.0 MW, or 5.1% of the maximum actual value.

We further verify autocorrelation in the series, for each hour, using the Ljung-Box test [13] on the residuals for lags of 5, 10, and 15. The Ljung-Box test has a null hypothesis that the residuals are uncorrelated up to a given lag. We reject the null hypothesis at a level 0.05 if its test-statistic exceeds its 0.95 level quantile. For all the 14 hours of the day, the null hypothesis is not rejected suggesting a zero autocorrelation in the series, or the model choice may be appropriate.

With increasing penetration of solar power in the electricity grid, a number of stochastic optimization models for bidding, storage, and generation have been developed; see, e.g., [14, 15].

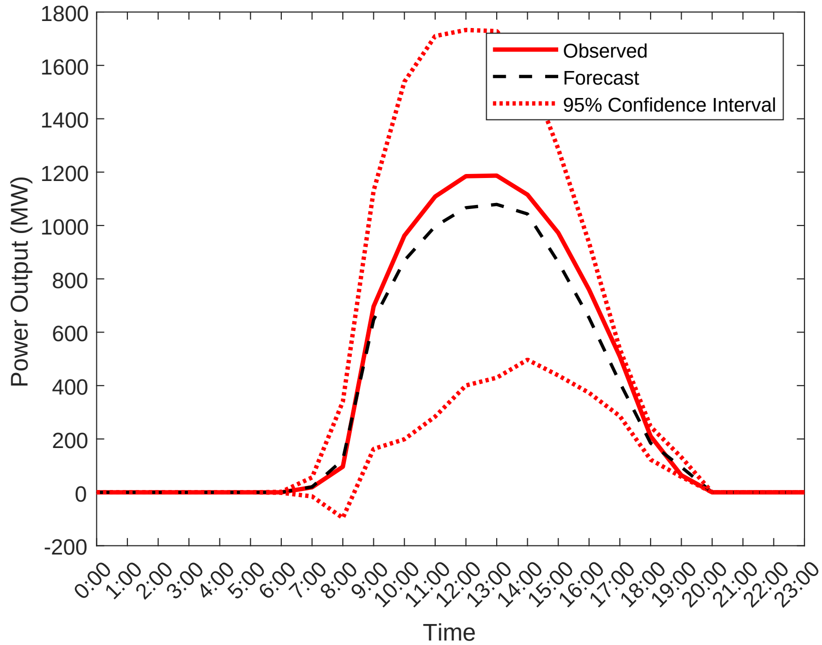


Figure 1: Day-ahead actual and predicted values using ARMA models from Table 1

Stochastic optimization models rely on the availability of a large number of scenarios. We can use Monte Carlo sampling to generate hourly solar power scenarios. The output from an ARMA model is real valued, and hence can be negative. In our analysis, we truncate the negative powered outputs to 0. For the 14 hours of the day, this sampling resulted in 1.6% of the outputs with estimated power output below -5MW. Figure 2 plots 2000 day-ahead scenarios as well as the median and 10 percentile values.

## 3. Conclusions

In this article, we present a simple step-by-step scheme for fitting an ARMA model to historical solar power data. The proposed model provides an easy-to-implement linear tool to forecast future hourly scenarios, or for embedding into decision-making models. We introduce statistical tests to check the applicability of various models, identify model parameters, and to finally forecast scenarios. If significant statistical evidence is not present in support of a test, the chosen model is not expected to perform well. This can lead to erroneous conclusions; see, e.g., [16, 17]. The proposed

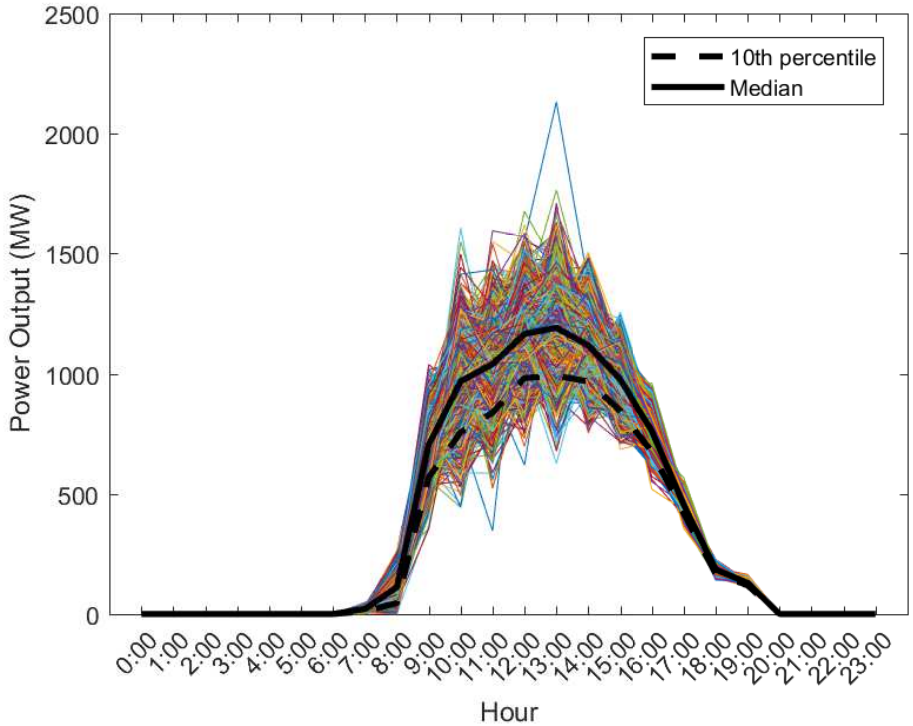


Figure 2: 2000 hourly scenarios for solar power generated using the ARMA models from Table 1. The dashed black line is the median hourly value, and the solid black line is the 10 percentile solar power value.

ARMA model performs better than a Smart Persistence model (see Appendix). In summary, the methodology in this article can be directly applied to historical data, both for a single PV source and a PV site, to create future scenarios for use in stochastic or robust optimization models for power system operation and planning.

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## Appendix

We compare the developed model against two other forecasting methods. First, we fit a single ARMA model, as opposed to hour-by-hour, on the entire time series. Second, we use the Smart-Persistence model of [**?** ], which assumes the solar power at hour $t$ is the mean of the previous $h$ hours. We use $h = 2$ in here. Table 2 compares the three models; the proposed model performs better in both of the chosen metrics.

Table 2: Comparison of three forecasting models. MAE denotes the mean absolute error and RMSE denotes the root mean squared error.

| | Model | | |
|---|---|---|---|
| | Hourly ARMA | Single ARMA | Smart-Pers |
| MAE (MW) | 39.6 | 48.2 | 45.7 |
| RMSE (MW) | 61 | 112.51 | 102.6 |