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1 Mathematical Modeling

1.1 Notation

Indices and Sets:

$t \in T$	set of hours
$w \in \Omega$	set of scenarios

Parameters: First Stage

η	efficiency of battery $0 < \eta < 1$
$\overline{X}$	maximum energy that can be stored in the battery [MWh]
$\underline{X}$	minimum energy that must remain in the battery [MWh]
μ_t	revenue earned for delivering a unit of energy at hour t [$/MWh]
λ_c	operational cost of charging the battery [$/MWh]
λ_d	operational cost of charging the battery [$/MWh]
λ_t^q	selling price of energy at hour t [$/MWh]
$\overline{P}$	maximum charging rate of battery [MWh]
$\overline{Q}$	maximum discharging rate of battery [MWh]

Parameters: Second Stage

s_t^ω	solar energy available at hour t under scenario ω [MWh]

Decision Variables: First Stage

y_t	energy promised to deliver at hour t [MWh]

Decision Variables: Second Stage

p_t^ω	energy charged to battery at hour t in scenario ω [MWh]
q_t^ω	energy discharged from battery at hour t in scenario ω [MWh]

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x_t^ω energy stored in battery at hour t in scenario ω [MWh]
w_t^ω binary variable; 1 if battery is charging at hour t and 0 if discharging

$$\max \sum_{t \in T} \mu_t y_t - \mathbb{E}[\lambda_c p_t^\omega + \lambda_d q_t^\omega] \tag{1a}$$

$$\text{s.t. } \mathbb{P}\left(y_t \leq s_t^\omega + q_t^\omega - p_t^\omega, \forall t \in T\right) \geq \alpha \tag{1b}$$

$$x_{t+1}^\omega = x_t^\omega + \eta p_t^\omega - \frac{1}{\eta} q_t^\omega, \qquad \forall t = 2, 3, \dots T-1, \forall \omega \in \Omega \tag{1c}$$

$$p_t^\omega \leq \overline{P} w_t^\omega, \qquad \forall t \in T, \forall \omega \in \Omega \tag{1d}$$

$$q_t^\omega \leq \overline{Q}(1 - w_t^\omega), \qquad \forall t \in T, \forall \omega \in \Omega \tag{1e}$$

$$\underline{X} \leq x_t^\omega \leq \overline{X}, \qquad \forall t \in T, \forall \omega \in \Omega \tag{1f}$$

$$w_t^\omega \in \{0, 1\} \qquad \forall t \in T, \forall \omega \in \Omega \tag{1g}$$

$$p_t^\omega, q_t^\omega \geq 0. \tag{1h}$$

The objective function in (1a) aims to maximize profit; i.e., revenue from the promise minus the cost of operating (charging or discharging) the battery. A more sophisticated model could include piecewise linear or quadratic costs; see, e.g., [1,2]. The joint chance constraint in equation (1b) requires that the joint probability of meeting the promised energy to sell, by the available solar power (via s^ω) and discharging the battery, (via q^ω), for the entire time horizon meets a threshold; excess solar energy can be used to charge the battery (via p^ω). If the solar power is excess than needed, we might need to curtail it. However, we do not penalize this curtailment and hence equation (1b) is expressed as a inequality. Here, α is a positive quantity just less than one, such as 0.95. Constraint (1c) relates the energy stored by the battery in a subsequent hour with the previous hour; here, the quantity $\eta p_t^\omega - \frac{1}{\eta} q_t^\omega$ is the energy charged or discharged by the battery at hour t. Note that both p_t^ω or q_t^ω cannot simultaneously be positive, this is ensured by constraints (1d)-(1e). Further, since $\eta < 1$, a fraction of energy is lost while both charging and discharging; i.e., we consume more than 100% while charging and supply less than a 100% while discharging. Further, constraint (1f) ensures the minimum and maximum amount of energy stored in the battery , and thus equations (1d)-(1f) restrict the amount the battery can be charged or discharged. The remaining constraints ensure the binary and non-negative restrictions on the relevant decision variables.

The probabilistic constraint in equation (1b) can be reformulated using a big-M approach as follows:

$$y_t - q_t^\omega + p_t^\omega \leq s_t^\omega + M_t^\omega z^\omega, \forall t \in T, \omega \in \Omega \tag{2a}$$

$$\textstyle\sum_{\omega \in \Omega} z^\omega \leq \lfloor N\varepsilon \rfloor \tag{2b}$$

$$z^\omega \in \{0, 1\}, \qquad \forall \omega \in \Omega. \tag{2c}$$

Here, $\lfloor \cdot \rfloor$ rounds its argument down to the nearest integer.

Proposition 1 *From equation* (1c) *and equation* (1f) *we have* $q_t^\omega - p_t^\omega \leq x_t^\omega - x_{t+1}^\omega \leq \overline{X} - \underline{X}, \forall t = 2, 3, \dots T-1, \forall \omega \in \Omega$. *Then, we can use the result from Proposition 1 of [3], and a sufficiently large value of* M_t^ω *is given by* $\overline{X} - \underline{X} + s_t^{\omega(\lfloor N\varepsilon \rfloor + 1, t)} - s_t^\omega$. *Here,* $s_t^{\omega(l,t)}$ *denotes the* l^{th} *largest solar power value at time* t.

2 Data Sources

For the computational experiments in this article, we use $\eta = 0.9$. We use $\overline{X} = 960$kWh from the lead-acid battery described in [4]. To compute the cost of charging or discharging the battery, we again use the formula given by [4]: $\lambda_c = \lambda_d = \frac{\alpha}{390\overline{X}} I$, where α is an effective weighting factor, I is the initial investment cost to purchase the battery, and $390\overline{X}$ is an approximation for the total throughput of the battery [5]. We assume $\alpha = 1$, and use a lead-acid battery cost of \$200/kWh from [6]. An approximately similiar battery cost is available from [7]. Further, we assume $\underline{X} = 0.2\overline{X}, \overline{P} = \overline{Q} = 0.5\overline{X}$, and the battery is 50% charged initially (i.e., $x_1^\omega = 0.5\overline{X}$).

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