ME 505 Coding Assignment 1

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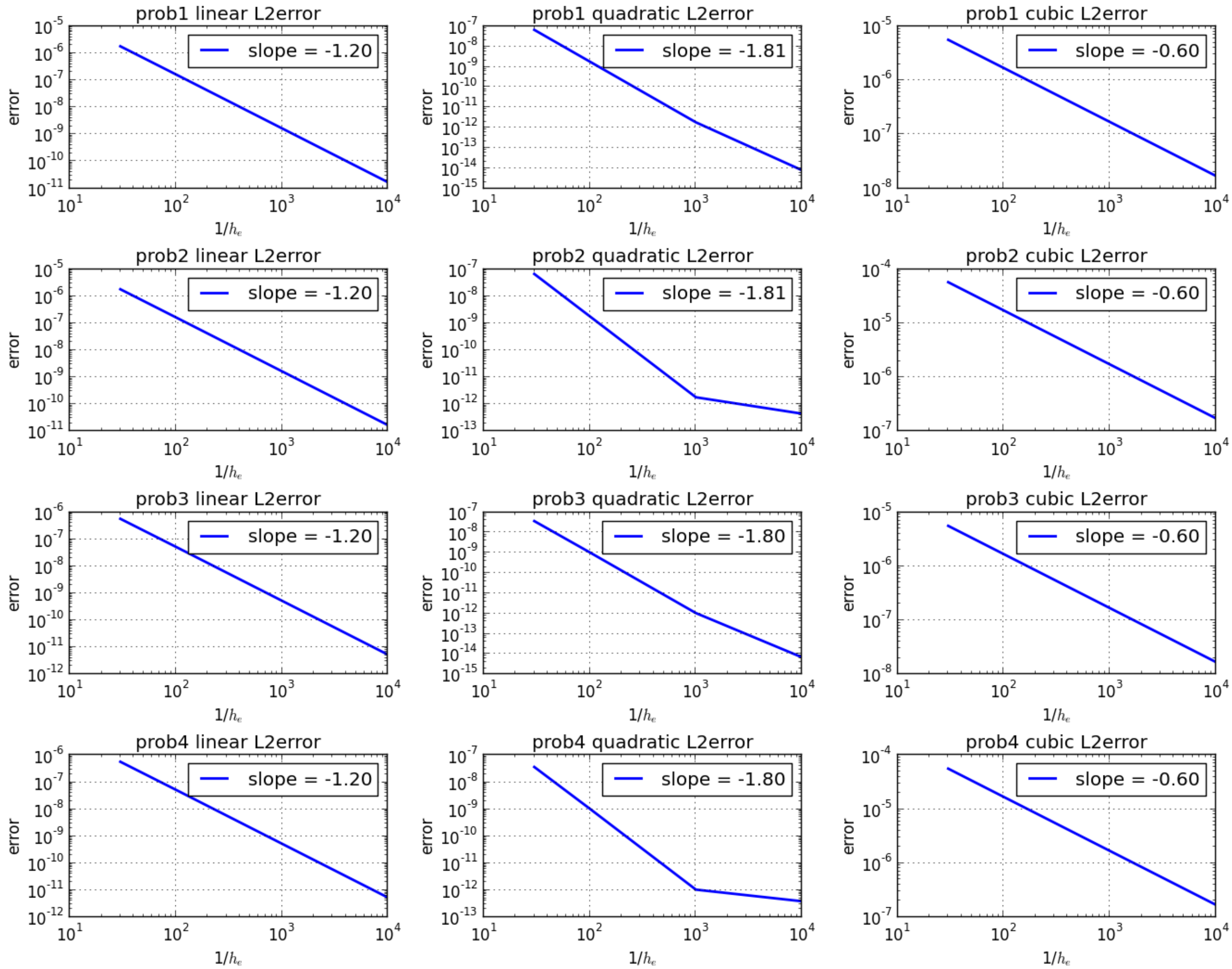
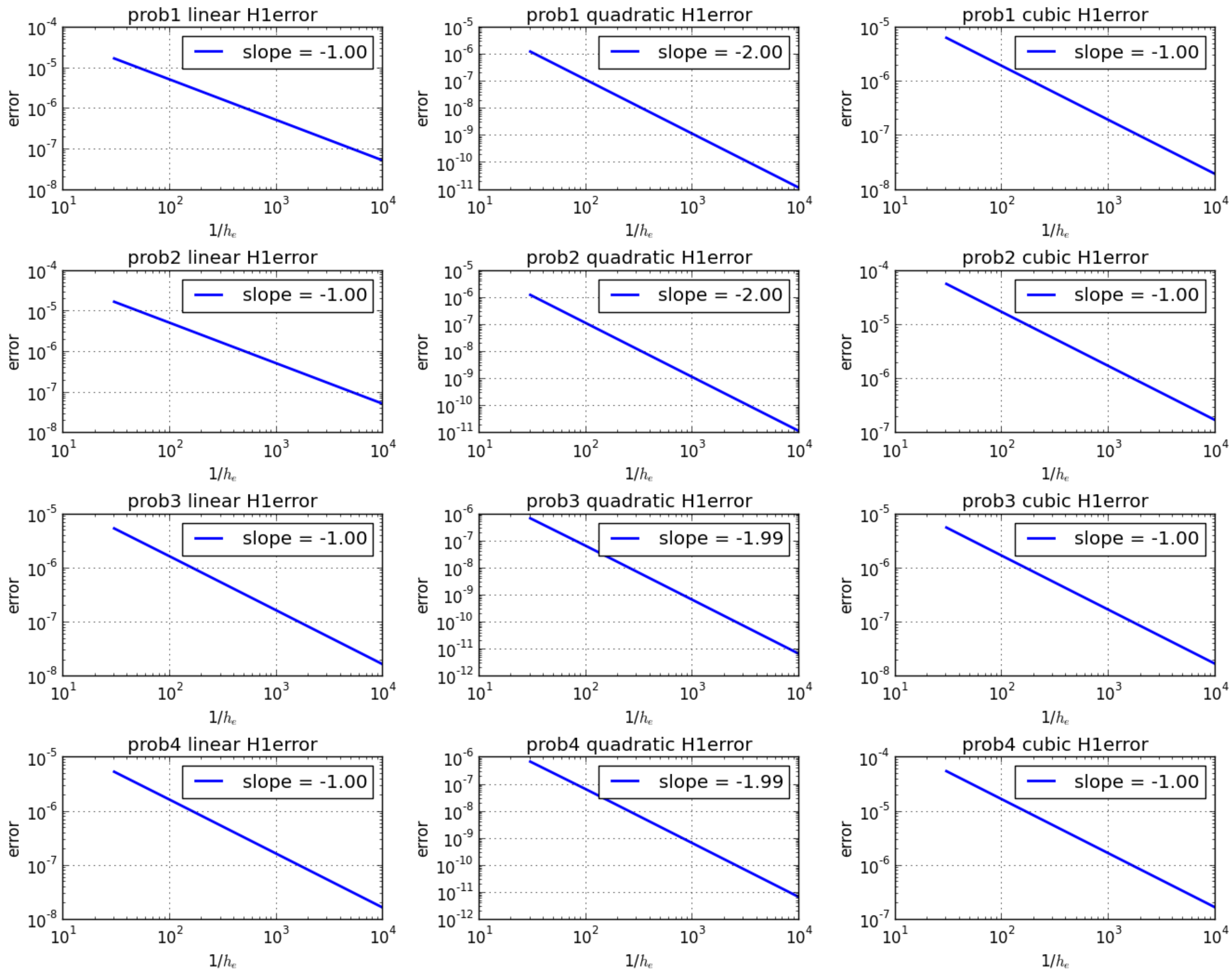
I. L_2 Error Norm Map



Comment:

The slopes are shown on the plots, and we can conclude some results from the features of total 12 plots. First, all 12 plots show great error convergence. With mesh refinement, the error reduce to some value below 1e-6, which gives great accuracy. Second, the boundary conditions do not matter a lot for the error convergence. Using different b.c.s will give the same slope of error decreasing vs. mesh refinement, indicating that the Finite Element Method is robust to solve 1D PDE problem with simple geometry. Additionally, when comparing the results using different polynomial approximation orders, the slopes are not constant. A quadratic polynomial approximation gives best error convergence rate (-1.8) on a loglog plot. It is interesting that a high order approximation (cubic polynomial) does give a more accurate solution. Probably it is due to the instability of high order approximation. Even though we have accurate nodal values, the evaluated basis functions oscillate on each element since it has more parameters to adjust and interpolate. As meshes being refined, the evaluated results become more like jittering, hence even if the error is decreasing, the decreasing rate (i.e. slope) is not as good as expected. Besides, there is a turning point on quadratic error plot where Neumann b.c. applies. This could due to the machine precision since the Nuemann b.c. reqires the derivate values which needs more number information, but a floating point precision is limited to near 1e-15 on a certain computer.

II. H_2 Error Norm Map



Comment:

The slopes are shown on the plots, it is similar to L_2 error norm. Again, all 12 plots show great error convergence with mesh refinement, and a quadratic polynomial approximation gives best error decreasing slope. Specifically, for H_1 error norm, the cubic H_1 error is not as bad as L_2 case, the slope is even the same as a linear approximation. This could probably because H_1 norm also considers the first order derivative of a solved value. Since the cubic approximation is second order smooth (while linear approximation is not continuous on nodes), its advantage and disadvantage finally give out a balanced slope. Furthermore, if talking about H_2 norm, I think the cubic approximation would be much better than linear, and probably as good as quadratic approximation.