ME505CA4 Stability Analysis

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For the stability analysis, I used a mesh with of 10 x 10 x 1, and the computing domain is [0, 1], [0, 1], and [0, 0.1] in x, y, and z-coordinates respectively. The L_2 norms between steady state solution and a transient solution is shown in Figure 1. I changed the time step size from 0.1s up to 15 s, in all three cases with $\alpha = 0$, 0.5, 1.0. Note that the x-axis is the iteration number, instead of real time now, because we are more inters ted in showing the convergence speed instead of accuracy of different schemes.

From the plots, the Backward Euler and Mid-Point scheme shows unconditionally stable with increasing time step. The L_2 error decreases faster with a larger time step. This makes sense because physically, the real time at the same iteration but a larger step size is longer, and the state should be closer to a steady state. However, for the Forward Euler scheme with $\alpha = 0$, the solution becomes unstable when $\Delta t \geq 5.77s$. The line plot of $\Delta t = 8.65s$ and $\Delta t = 12.97s$ are not obviously observed on the plot because the L_2 norm explode at the second iteration. Hence we verifies that the Forwar Euler is conditionally stable.

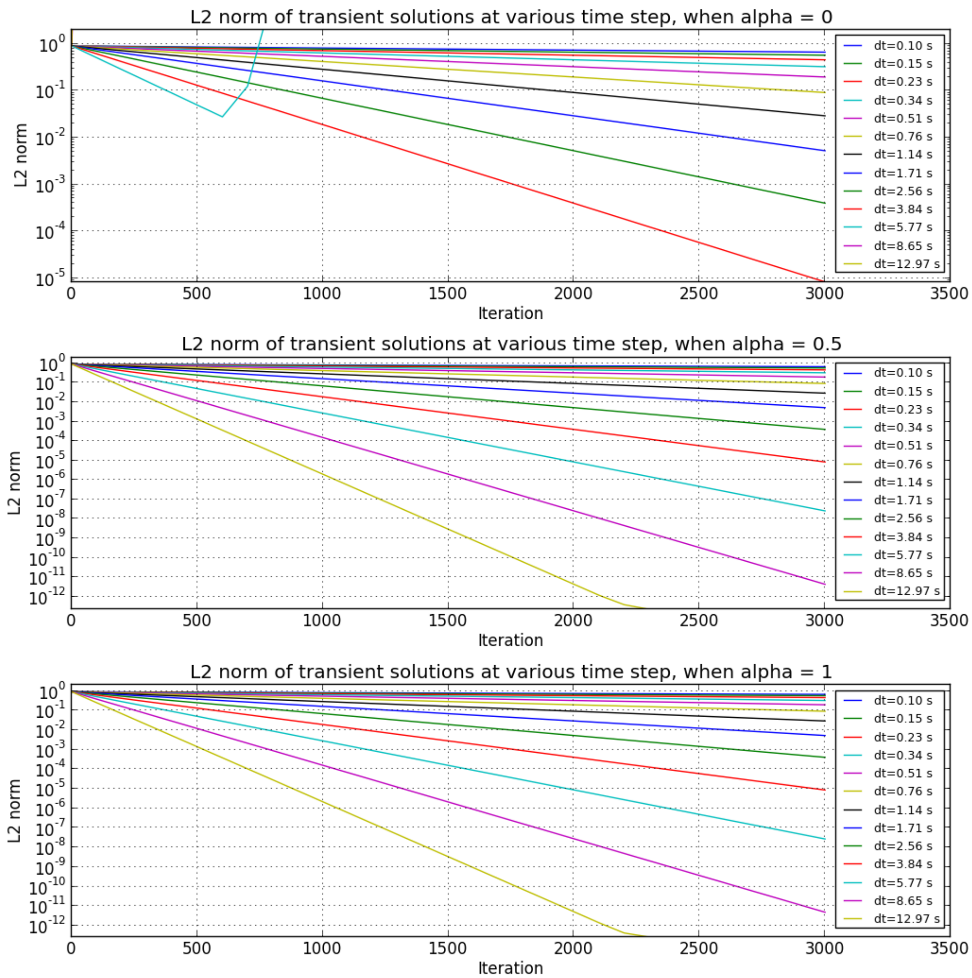

Figure 1: L2 norm plot for various alpha, from t = 0 to 3000 s