0 Claims

I claim $evac(i,\epsilon) \in O(n(i+1)lg(\frac{n(i+1)}{min\{\epsilon,\frac{1}{2}\}}))$ and there is an algorithm which can compute $evac(i,\epsilon)$ in $O(lg(n)+lg(i+1)+lg(lg(\frac{1}{min\{\epsilon,\frac{1}{2}\}})))$, to proof it one of the ways is finding an upper bound like K I also claim that K could be $16n(i+1)lg(\frac{n(i+1)}{min\{\epsilon,\frac{1}{2}\}})$, and I will show that it's true in section 1 then I will proof the correctness of first two claims.

1 Finding an upper bound

In this section I showed by a recursive way that there is a constant c_1 that in any case $evac(i,\epsilon) < c_1 n(i+1)lg(\frac{n(i+1)}{min\{\epsilon,1\}})$ due to the definition of $evac(i,\epsilon)$ for any $c_2 \geq c_1$ if $k := c_2 n(i+1)lg(\frac{n(i+1)}{min\{\epsilon,1\}})$ then $e(i) \leq \epsilon$ and it's obvious that for $\epsilon_1 < \epsilon \rightarrow evac(i,\epsilon) \leq evac(i,\epsilon_1)$ so it's OK to set $\epsilon := min\{\epsilon,\frac{1}{2}\}$, and for somehow I split the cases into two general cases. At first I solve the problem for $K \in R$ allowed then I solved it for integer values of K allowed.

1.1 Lemmas and definitions

Here are some definitions and four lemmas that will help to make the proof easier:

$$\epsilon := min\{\epsilon,\frac{1}{2}\},\ K := c_1 n(i+1)lg(\frac{n(i+1)}{\epsilon}),\ k := c_2 n(i+1)lg(\frac{n(i+1)}{\epsilon})$$

1. $(x \leq y \wedge y \leq z) \rightarrow x \leq z$
2. $\forall x > 0:\ lg(x) < x$
3. $\forall x > 1:\ \frac{1}{2x} < lg(x) - lg(x-1)$
4. $\forall x \geq 15:\ 0 < \frac{x}{2} - lg(x) - 2$

1.2 Case $i = 0$:

$$\frac{(n-1)^k}{n^{k-1}} \leq \epsilon \leftrightarrow klg(n-1) - (k-1)lg(n) \leq lg(\epsilon) \leftrightarrow$$

$$0 \leq lg(\epsilon) + (k-1)lg(n) - klg(n-1) \leftrightarrow 0 \leq lg(\frac{\epsilon}{n}) + k(lg(n) - lg(n-1))$$

Using L1 and L3 it's OK to make right side smaller like this:

$$\frac{k}{2n} = \frac{c_2}{2}lg(\frac{n}{\epsilon}) \leq k(lg(n) - lg(n-1))$$

$$\xleftarrow[L1,L3]{\text{explained above}} 0 \leq lg(\frac{\epsilon}{n}) + \frac{c_2}{2}lg(\frac{n}{\epsilon}) \leftrightarrow 0 \leq (\frac{c_2}{2} - 1)lg(\frac{n}{\epsilon})$$

$$\xleftarrow[L1]{1 \leq lg(\frac{n}{\epsilon})} 0 \leq \frac{c_2}{2} - 1 \leftarrow 15 \leq c_2 \leftrightarrow \mathbf{c_1 := 15}$$

1.3 Case $0 < i$:

$$C(k,i)\frac{(n-1)^{k-i}}{n^{k-1}} \le \epsilon \leftrightarrow lg(C(k,i)) + (k-i)lg(n-1) - (k-1)lg(n) \le lg(\epsilon)$$

Using L1 it's OK to make the left side bigger like this:

$$C(k,i) \le k^i \leftrightarrow lg(C(k,i)) \le ilg(k)$$

$$\xleftarrow[L1]{\text{explained above}} ilg(k) + (k-i)lg(n-1) - (k-1)lg(n) \le lg(\epsilon)$$
$$\leftrightarrow ilg(k) \le lg(\epsilon) + ilg(n-1) - lg(n) + k(lg(n) - lg(n-1))$$

Using L1 and L3 it's OK to make right side smaller like this:

$$\frac{k}{2n} = \frac{c_2}{2}(i+1)lg(\frac{n(i+1)}{\epsilon}) \le k(lg(n) - lg(n-1))$$

$$\xleftarrow[L1,L3]{\text{explained above}} ilg(k) \le lg(\frac{\epsilon}{n}) + \frac{c_2}{2}(i+1)lg(\frac{n(i+1)}{\epsilon}) + ilg(n-1)$$
$$\leftrightarrow ilg(k) \le (\frac{c_2}{2}(i+1) - 1)lg(\frac{n}{\epsilon}) + \frac{c_2}{2}(i+1)lg(i+1) + ilg(n-1)$$

Using L1 and L2 it's OK to make left side bigger like this:

$$c_2 n(i+1)lg(\frac{n(i+1)}{\epsilon}) \le c_2 n(i+1)\frac{n(i+1)}{\epsilon} \le c_2\frac{n^2(i+1)^2}{\epsilon^2}$$

$$\xleftarrow[L1,L2]{\text{explained above}} 2ilg(\frac{n(i+1)}{\epsilon}) + ilg(c_2) \le (\frac{c_2}{2}(i+1) - 1)lg(\frac{n(i+1)}{\epsilon}) + lg(i+1) + ilg(n-1)$$

$$\leftrightarrow ilg(c_2) \le (\frac{c_2}{2}(i+1) - 2i - 1)lg(\frac{n(i+1)}{\epsilon}) + lg(i+1) + ilg(n-1)$$

$$\leftrightarrow ilg(c_2) \le (\frac{c_2}{2}i + \frac{c_2}{2} - 2i - 1)lg(\frac{n(i+1)}{\epsilon}) + lg(i+1) + ilg(n-1)$$

$$\leftrightarrow ilg(c_2) \le i(\frac{c_2}{2} - 2)lg(\frac{n(i+1)}{\epsilon}) + (\frac{c_2}{2} - 1)lg(\frac{n(i+1)}{\epsilon}) + lg(i+1) + ilg(n-1)$$

$$\xleftarrow[L1]{0 \le lg(i+1) + ilg(n-1) + (\frac{c_2}{2} - 1)lg(\frac{n(i+1)}{\epsilon})} ilg(c_2) \le i(\frac{c_2}{2} - 2)lg(\frac{n(i+1)}{\epsilon})$$

$$\xleftarrow[L1]{1 \le lg(\frac{n(i+1)}{\epsilon}) \wedge 0 < i} lg(c_2) \le (\frac{c_2}{2} - 2) \leftrightarrow 0 \le \frac{c_2}{2} - lg(c_2) - 2$$

$$\xleftarrow{L4} c_1 = 15 \le c_2 \leftarrow \mathbf{c_1 := 15}$$

2 Proof of evac's value complexity:

$$evac(i,\epsilon) \leq ceil(K) < K+1 \text{ so if } c_1 := 16 \text{ then: } evac(i,\epsilon) < K$$
$$\rightarrow evac(i,\epsilon) \in O(n(i+1)lg(\frac{n(i+1)}{min\{\epsilon, \frac{1}{2}\}}))$$

3 Proof of calculator's time complexity:

Since if $c_1 := 16$ then $evac(i,\epsilon) < K$ (it have a upper bound) the answer of $evac(i,\epsilon)$ is binary searchable, it could be done by two binary searches one for finding the value of k that maximizes $e(i)$ and second the to find the $evac(i,\epsilon)$ in a bounded sequence with decreasing $e(i)$ for k in sequence, and binary search complexity is from $O(lg(K))$ so:

$$O(lg(n(i+1)lg(\frac{n(i+1)}{min\{\epsilon, \frac{1}{2}\}})))$$
$$= O(lg(n) + lg(i+1) + lg(lg(n) + lg(i+1) + lg(\frac{1}{min\{\epsilon, \frac{1}{2}\}})))$$
$$= O(lg(n) + lg(i+1) + lg(lg(\frac{1}{min\{\epsilon, \frac{1}{2}\}})))$$