Unit 2

SYSTEM SIMULATION

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Monte Carlo Simulation

- It is a technique of experimental sampling with random number or method of trial which can be used to solve man problems otherwise difficult or sometimes impossible.
- This method cannot be used to attempt high accuracy and hence is suitable for those problems that doesn’t require high degree of accuracy.
- Monte Carlo method are stochastic technique and make use of random number and probability to solve the problem.

- It can be used to solve both stochastic and deterministic problems. When we solve the deterministic problem using monte Carlo method, we first convert the deterministic model into stochastic model eg:- Calculating the value of pi(π).
- Some examples of problems solved using Monte Carlo Simulation are:-
  - To find the area of irregular surface.
  - Numerical Integration
  - Gambling Game
  - Random Walk Problem.

Finding the area of Irregular Surface

a) Let us consider an irregular shape as shown in figure below:

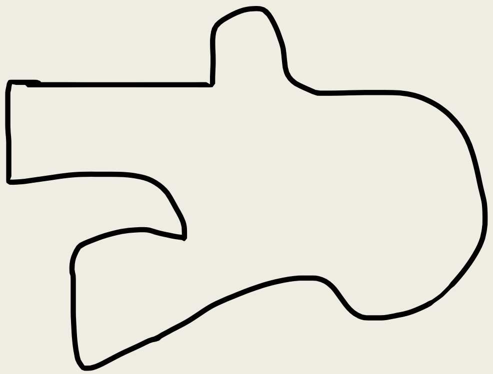
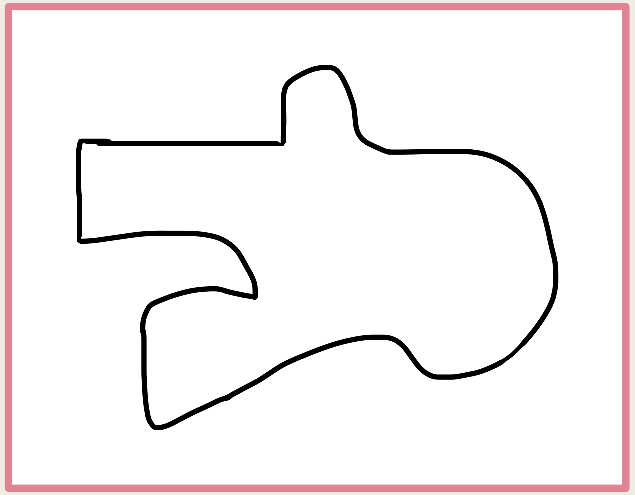
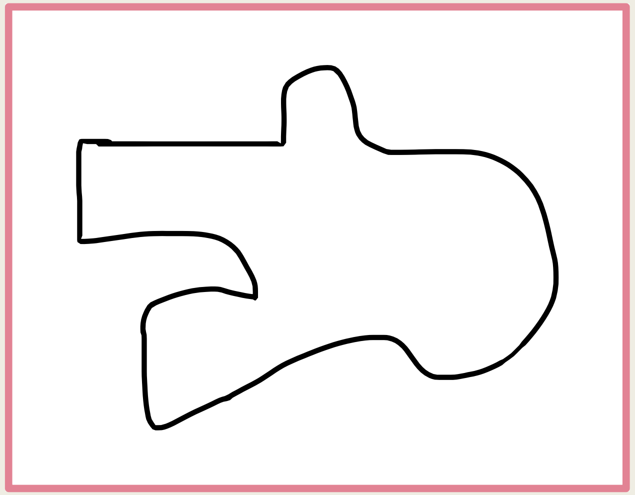


Fig: Irregular Shape

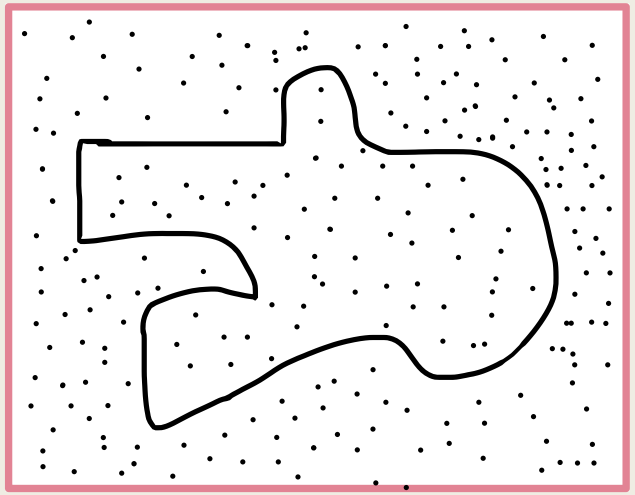
b) Enclose the shape with rectangle as shown



b) Enclose the shape with rectangle as shown



c) Inside rectangle mark random ‘N’ number of dots as shown in figure below:



- Let there be ‘M’ dots inside the irregular surface & ‘N’ be the total number of dots inside the rectangle.

d) Count the number of dots inside the irregular figure, let these be ‘M’.

If ‘F’ be the area of irregular figure.

Then geometrically,

$$\frac{F}{A} = \frac{M}{N}$$

$$\therefore \; \boldsymbol{F = \frac{M}{N} . A}$$

where, F = Area of Irregular Figure

A = Area of rectangle

M = Total no. of dots inside irregular figure.

N = Total no. of dots inside the rectangle.

Larger the number of dots, more accurate will be the result.

Suppose N = 50, M=25, A =20cm^2. Find F

$$\boldsymbol{F} = \frac{\boldsymbol{M}}{\boldsymbol{N}}.\,A$$

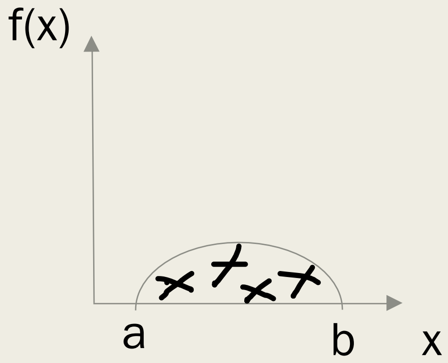
$$= \frac{\mathbf{25}}{\mathbf{50}}.\,20$$

$$\therefore F \quad = 10\text{cm}^2$$

Numerical Integration

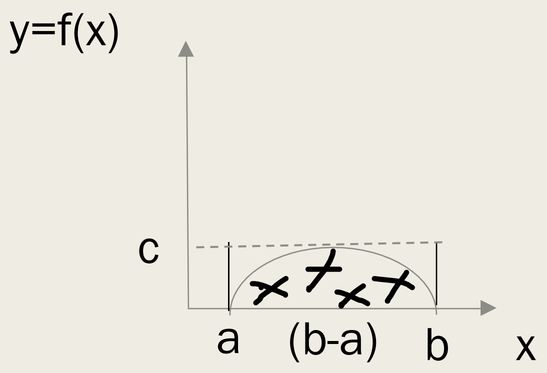
- The integral of single variable over a given range gives the area under the graph representing the function
- Let us consider a function f(x) is positive and has lower and upper boundary ‘a’ and ‘b’ respectively.

i.e. $\int_a^b f(x)dx$



- Now draw the rectangle as in figure below.. The side of rectangle are (b-a) and c.

  Here Area of rectangle, A = c * (b-a)



- Then mark random number of dots inside the rectangle and count dots. Let ’M’ be the number of dots inside the curve and ‘N’ be the dots inside the rectangle.

- Then, $\frac{Area\ of\ Irregular\ Surface}{Area\ of\ rectangle} = \frac{M}{N}$

  i.e. $\frac{Area\ of\ curve(F)}{Area\ of\ rectangle} = \frac{M}{N}$

  i.e. $\int_a^b f(x)dx = \frac{M}{N}$ x (b-a)c

- The accuracy improves as the number of ‘N’ increases.

- For an random point x,y. The value of x is selected at random between ‘a’ and ‘b’ and value of y is selected at random between 0 and c.

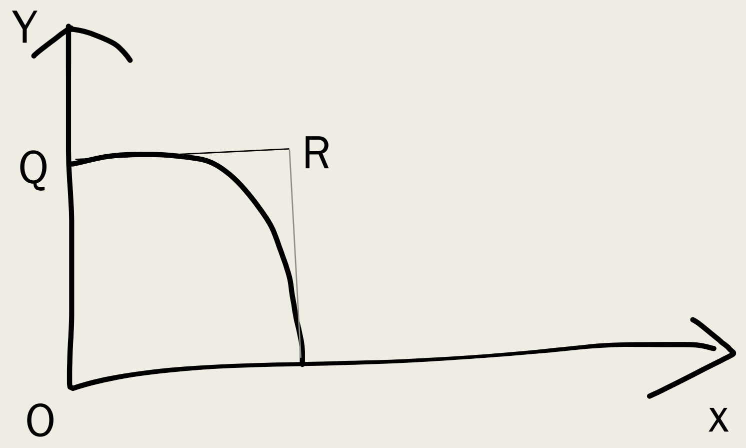
- If $y_0 \leq f(x_0)$, the point x_0, y_o is accepted in the count of M, otherwise it is rejected.

Solve $\int_2^4 x^2\,dx$ using Monte Carlo Method.
Also estimate error percentage

Solve $\int_2^5 x^3 \, dx$ using Monte Carlo Method. Also estimate error percentage

Deterministic problem through random number

- ❖ The application of Monte Carlo Method for the evaluation of pi(π) requires converting the deterministic model into stochastic model.
- ❖ Consider a quadrant of unit circle as shown in figure below



- At Point satisfy the equation

$x^2 + y^2 \leq 1$, x, $y \geq 0$ lie inside the circle quadrant.

- The equation can be written as

$$y^2 \leq 1 - x^2$$

$$\therefore y \leq \sqrt{1 - x^2}$$

- Now if (r_1, r_2) is a pair of random number in range r(0,1), we call this pair acceptable if

$$r_2 \leq \sqrt{(1 - r_1{}^2)}$$

Find the value of Pi(π) $using$ $Monte\ carlo\ Method$

Random Walk Problem

- Let 'N' be the steps of equal length be taken along the time. Let 'P' be the probability of taking steps to right and 'q' be the probability(taking steps to left)

  n_1 = no. of steps to right

  n_2 = no. of steps to left.

- The quantity(p,q,n_1,n_2) can be related b

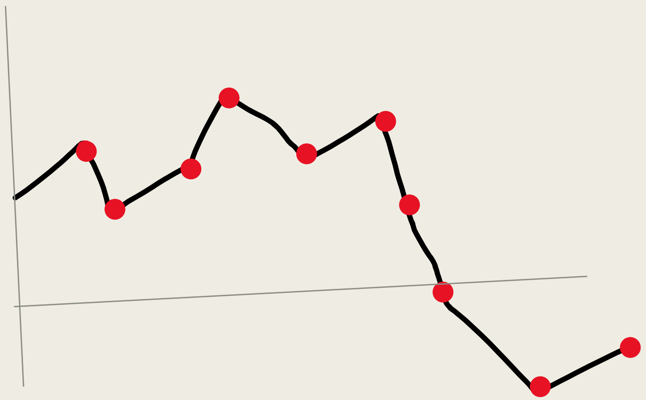
  $p + q = 1$

  $n_1 + n_2 = 1$

  ex:- flip a coin and take steps

  1. flip coin, if Head move right

     if tail move left



Drunkard Problem

- It's a type of random walk problem, the algorithm states
  1. Pick a random points in grid and make it empty.
  2. Choose a random direction North, East, South , West.
  3. Move in that direction and mark it empty.
  4. Repeat step 2 and 3 until you have emptied as man grid as desired.
  5. Steps drunkard took in direction North, East, South, East, East.

Distributed Lag Model

- When a model is large or complex, we perform numeric computation technique using computer.
- However there is a simple technique in simulation that can be applied without difficulty even for large model.

Properties of Distributed Lag Model

- Model changes only at fixed interval of time.
- Model is based on the current value of variable and values that occurred in previous intervals. Eg: Census data.
- This model consist the rule of linear algebraic expression.
- They represent continuous systems, but one in which data is available at fixed point in time.
- If current value of variable can be represented in terms of values then such variable is called **lagged variable.**
- **$X = A + BX_{-1}$**

  Where, X = current value of X

  and X_{-1} = previous value of X

❖ So, if an initial value of X is given a new value of X can be computer.

Consider the following mathematical model of National Economy

$I = 2 + 0.1 X_{-1}$

$Y = 45.45 + 2.27(I + G)$

$T = 0.2 Y$

$C = 20 + 0.7(Y-T)$

Where, C be the consumption, I be investment, T be taxes and Y be national income. For the above equations, the values of G are supplied to 5 years as below.

Year	G
1991	15
1992	20
1993	25
1994	30
1995	35

If initial value of Y_{-1} is 75, calculate the consumption for 1st and 2nd Year.

Cobweb Model

- Consider the static mathematical model for market, demand and supply model. We have,

$$D = a - bp$$

$$S = c + dp$$

$$D = S$$

- ❖ Generally supply should be dependent on demand from previous marketing period.
- ❖ But the demand however will respond to current price.
- ❖ So, writing the market model in distributed lag model we get,

$D = a - bp$

$S = c + dp_{-1}$

$D = S$

- ❖ So given an initial value of price P_0 . The value of ‘S’ can be calculated. This determines the value of D.
- ❖ Model of this type are called cobweb model because when model is solved graphically, the combinations of spiral, supply and demand curve taken from of cobweb shape.

Draw Cobweb Model for the following market . Also draw market fluctuation of market price for the following values. Is the market stable?

- Given $P_0 = 1.0$

  $a = 12.4$

  $b = 1.2$

  $c = 1.0$

  $d = 0.9$

❖ The market model is given as

$$D = a - bp$$

$$S = c + dp_{-1}$$

$$D = S$$

Queuing System

- Most systems of interest in a simulation stud contain a process in which there is a demand for service.
- The system may service entities(customer, packets) at a rate which is lesser than the rate at which the entities arrive.
- The entities are then said to join in waiting line. The line where the entities wait is called **queue.**
- **The combination of all entities in system being served and being waiting for service will be called as queuing system.**
- **Queuing results due to congestion in the system.**

Problem in Queuing System

- A queuing problem is essentially a problem of being balancing the cost of waiting time against the cost of idle time for services facilities in the system.
- This balance requires an analysis of the queuing system like idle time, average waiting time, queue length etc.
- The problem arises due to stochastic nature of time between the arrival of customers as well as time it takes to serve the each customer.
- This problem can be solved with the help of simulation.

Elements/Characteristics of Queuing System

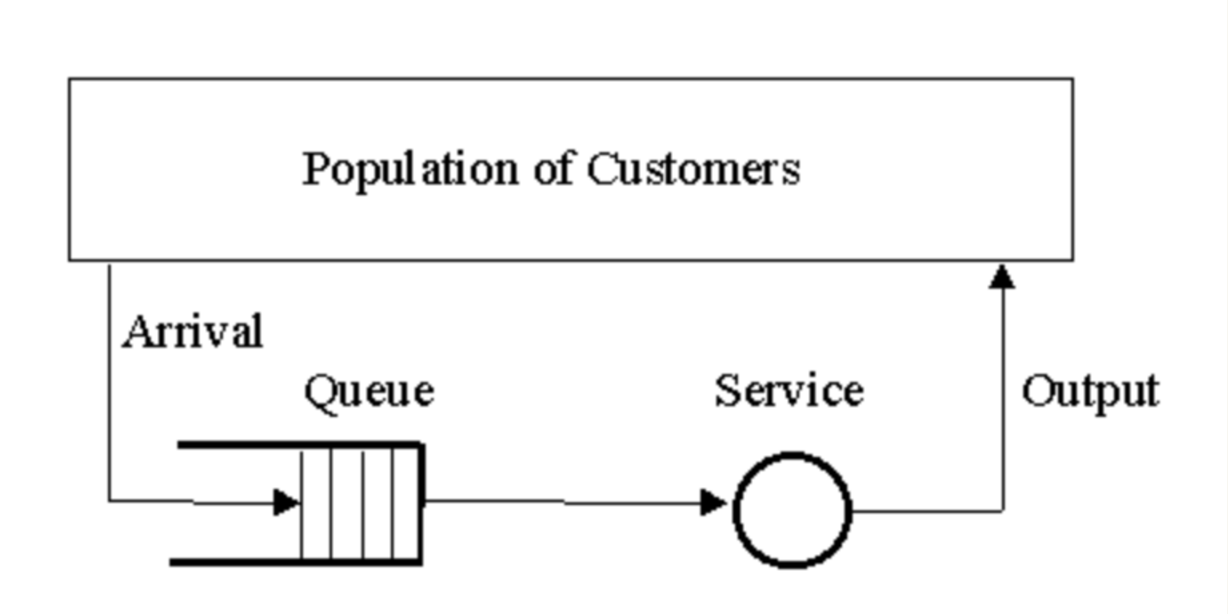


Fig: Single Server Queuing Model

- In the above figure, Customer can be student waiting for registration number, packets waiting for route in router etc.
- If the customer after arriving can enter the system center, there is no problem. Otherwise the have to wait for service and form a queue.
- They remain in queue until they are provided the service.
- But sometimes, the queue become too long ,at that time the will leave the queue without being served.
- In order to model a queuing system, we need to have in-depth about what constitutes a queuing system.
- There are three basic elements common to all queuing system:
  - Arrival Pattern
  - Service Process
  - Queuing Discipline

Arrival Pattern

- In a queuing system, before entities can be processed or subjected to waiting, the must enter the system.
- Depending upon the environment, entities can arrive smoothly in unpredictable form.
- The also can arrive one at a time or in batch.
- They arrive independently.
- So, arrival pattern simply defines the way how entitles enter into a system.
- Eg: A special arrival process which is useful for modelling process is Poisson's arrival process.

Service Process

- Once the entity enters the system, they must be served..
- An entity who arrives and finds the server idle enters the service immediately and service time s_1, s_2, s_3 ....s_n of the successive entities are independent.
- An entity who arrives and finds the server bus joins the end of the queue..
- Then the entities are processed in FCFS order or according to some kind of priority rule.

Queuing Discipline

- It represents the way, queue is organized or is the order in which service is provided such as:
  - LIFO
  - FIFO
  - SIRO
  - Priority
  - SPTF

LIFO(Last in First Out)

- According to this rule, the service is offered to the customer who has arrived recently.

FIFO()

- According to this rule, the service is offered on the basis of arrival time of the custo
- A customer who comes first gets the service first.

SIRO(Serve in Random Order)

- According to this rule, the service is offered on the basis of arrival time of the customer.
- A customer who comes first gets the service first.

Priority

- A special number called priority is assigned to each customer waiting in queue.
- Then, according to this number, customer is chosen for service..

SPTF(Shortest Priority First)

- Here, customer with shortest service time will be chosen for the service.

Calculate the following Parameter for SSQM where inter arrival time and service time are given. Assume the first customer arrives at time t=0.

IAT	-	8	6	1	8	3	8	7	2	3
ST	4	1	4	3	2	4	5	4	5	3

Calculate the following Parameter for SSQM where inter arrival time and service time are given. Assume the first customer arrives at time t==0.

IAT	-	4	1	4	2	4
ST	2	3	1	4	2	3

Types of System Simulation

- We distinguish between continuous and discrete system as being in which smooth or sudden changes occur.
- The distinction between continuous and discrete model is not made during the classification of model,
- Distinction is not important for mathematical or analytical model, but really important for simulation.
- The general computational technique which used with two kind of model differ significantly.

Numeric computational technique for discrete Model

Numerical Computation Technique for Continuous Model

S.N.	Attributes	Analytical Method	Simulation Based Method
1	Computation Time	Usually Faster than Simulation	Time consuming(depends on the number of iterations)
2	Final Results	Exact/Precise	Approximate
3	Complexity Consideration	Simplification is needed for large-scale systems.	It would be easier to evaluate complex systems through simulation
4	Further Results Extraction	It is possible to drive other factors like MTTF(Mean time to failure)/MTBF(Mean time between failure) from results directly.	Only Statistical Calculation

Experimental nature of Simulation

- Simulation technique make no specific attempt to isolate the relationship between any particular variables instead it observe the way in which all variable of the model changes with time.
- The relationship between the variable must be derivable from the observation.
- Many simulation runs have to be made to understand the relationship between the variable must be derived from the observation.
- Many simulation runs have to be made to understand the relationship involved in the system.
- So the use of simulation in between ‘o’, ‘k’ and ‘m’ would had to be discovered in vehicle suspension system by observing the value of result in the motion being non oscillatory.

Steps in Simulation

Step 1: Define the Problem

- An initial set up is to describe problem to be solved in concise manner. The description must be enough to answer the question asked and what need to be taken in order to answer the question.

Step 2: Define the Problem

- Based on the problem definition, a model must be defined.

Step 3: Simulate

- After defining a model we have to decide either to use simulation or analytical method to solve the problem.
- If the problem can be solved analytically, then solve problem using analytically method and stop.
- If the problem can not be solved analytically redefine the model and solve numerically i.e. perform simulation.

Step 4: Plan the study

- When it is decided to simulate we must plan the study by deciding the major parameters to be varied, the number of cases to be conducted and the order in which the runs are to be made.

Step 5: Write the program

- Simulation is done by digital computer, hence program must be written.

Step 6: Validate Model

- The model must be valid before the beginning of major set of runs.

Step 7: Run the Model

- The stage will then move into the stage of executing a series of runs according to study pal.
- It is essential to repeat the run with different set of random number so that more than one sample result is available.

Step 8 : Verify result

- After making multiple runs, we get various samples and those samples must be verified.
- If the approximation is found, stop the process otherwise redefine the model and repeat the entire process.

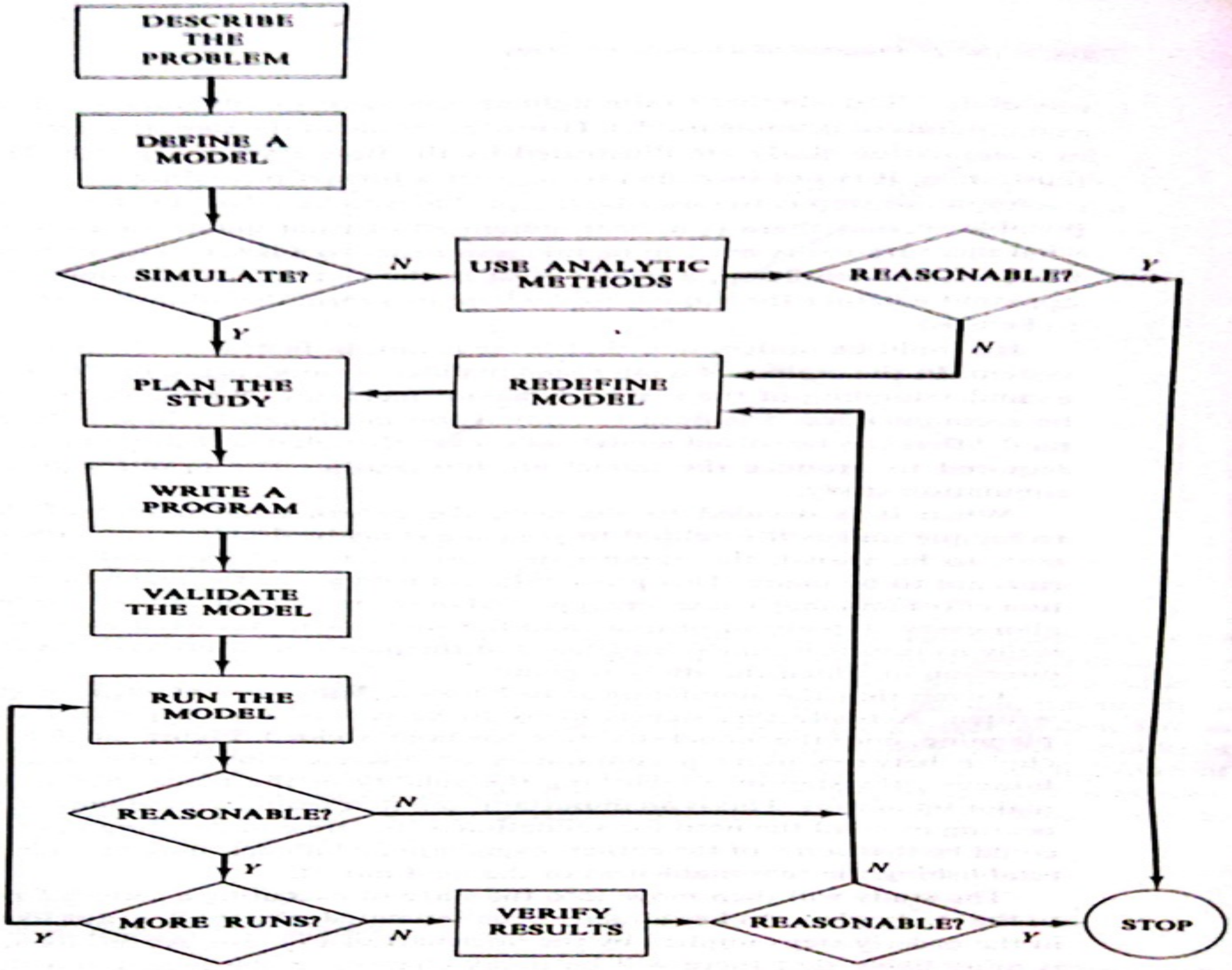


Figure 3-6. The process of simulating.