



# Relational Database Design

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# Multivalued Dependencies

- Suppose we record names of children, and phone numbers for instructors:
  - *inst\_child*(ID, child\_name)
  - *inst\_phone*(ID, phone\_number)
- If we were to combine these schemas to get
  - *inst\_info*(ID, child\_name, phone\_number)
  - Example data:
    - (99999, David, 512-555-4321)
    - (99999, William, 512-555-1234)
    - (99999, David, 512-555-1234)
    - (99999, William, 512-555-4321)
- This relation is in BCNF
  - Why?

# Multivalued Dependencies (MVDs)

- Let  $R$  be a relation schema and let  $\alpha \subseteq R$  and  $\beta \subseteq R$ . The **multivalued dependency**

$$\alpha \twoheadrightarrow \beta$$

holds on  $R$  if in any legal relation  $r(R)$ , for all pairs for tuples  $t_1$  and  $t_2$  in  $r$  such that  $t_1[\alpha] = t_2[\alpha]$ , there exist tuples  $t_3$  and  $t_4$  in  $r$  such that:

$$\begin{aligned} t_1[\alpha] &= t_2[\alpha] = t_3[\alpha] = t_4[\alpha] \\ t_3[\beta] &= t_1[\beta] \\ t_3[R - \beta - \alpha] &= t_2[R - \beta - \alpha] \\ t_4[\beta] &= t_2[\beta] \\ t_4[R - \beta - \alpha] &= t_1[R - \beta - \alpha] \end{aligned}$$

# MVD (Cont.)

- Tabular representation of  $\alpha \twoheadrightarrow \beta$

	$\alpha$	$\beta$	$R - \alpha - \beta$
$t_1$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$a_{j+1} \dots a_n$
$t_2$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$b_{j+1} \dots b_n$
$t_3$	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$b_{j+1} \dots b_n$
$t_4$	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$a_{j+1} \dots a_n$

# Example

- Let  $R$  be a relation schema with a set of attributes that are partitioned into 3 nonempty subsets.

$Y, Z, W$

- We say that  $Y \twoheadrightarrow Z$  ( $Y$  **multidetermines**  $Z$ ) if and only if for all possible relations  $r(R)$

$\langle y_1, z_1, w_1 \rangle \in r$  and  $\langle y_1, z_2, w_2 \rangle \in r$

then

$\langle y_1, z_1, w_2 \rangle \in r$  and  $\langle y_1, z_2, w_1 \rangle \in r$

- Note that since the behavior of  $Z$  and  $W$  are identical it follows that

$Y \twoheadrightarrow Z$  if  $Y \twoheadrightarrow W$

# Example (Cont.)

- In our example:

$ID \twoheadrightarrow child\_name$

$ID \twoheadrightarrow phone\_number$

- The above formal definition is supposed to formalize the notion that given a particular value of  $Y$  ( $ID$ ) it has associated with it a set of values of  $Z$  ( $child\_name$ ) and a set of values of  $W$  ( $phone\_number$ ), and these two sets are in some sense independent of each other.
- Note:
  - If  $Y \rightarrow Z$  then  $Y \twoheadrightarrow Z$
  - Indeed we have (in above notation)  $Z_1 = Z_2$   
The claim follows.

# Armstrong Axioms in MVD

2). An MVD  $X \twoheadrightarrow Y$  in  $R$  is called a trivial MVD if any one of Condition follows

- $Y$  is a subset of  $X$
- $X \cup Y = R$ .

Ex  $R(A, B, C)$

$A \twoheadrightarrow B$  non-trivial

$B \twoheadrightarrow AC$  trivial  $X \cup Y = R$

$AB \twoheadrightarrow A$  trivial.

augmentation

3)  $X \twoheadrightarrow Y \quad V \in W$

$WX \twoheadrightarrow VY$

transitivity

4)  $X \twoheadrightarrow Y, Y \twoheadrightarrow Z$

$\models X \twoheadrightarrow (Z - Y)$

Replication

5)  $X \rightarrow Y \Rightarrow X \twoheadrightarrow Y$  but reverse not true.

name	area code	phone	beach	rent
a	b	c	d	e
x	101	55511	B	AB
x	101	55511	PW	P
X	102	55549	B	AB
X	102	55549	W	P

name  $\twoheadrightarrow$  area code, phone  
 name  $\twoheadrightarrow$  beach, rent but  
 not name  $\twoheadrightarrow$  area code nor  
 name  $\twoheadrightarrow$  phone

# Use of Multivalued Dependencies

- We use multivalued dependencies in two ways:
  1. To test relations to **determine** whether they are legal under a given set of functional and multivalued dependencies
  2. To specify **constraints** on the set of legal relations. We shall thus concern ourselves *only* with relations that satisfy a given set of functional and multivalued dependencies.
- If a relation  $r$  fails to satisfy a given multivalued dependency, we can construct a relations  $r'$  that does satisfy the multivalued dependency by adding tuples to  $r$ .



# Theory of MVDs

- From the definition of multivalued dependency, we can derive the following rule:

- If  $\alpha \rightarrow \beta$ , then  $\alpha \twoheadrightarrow \beta$

That is, every functional dependency is also a multivalued dependency

- The **closure**  $D^+$  of  $D$  is the set of all functional and multivalued dependencies logically implied by  $D$ .
  - We can compute  $D^+$  from  $D$ , using the formal definitions of functional dependencies and multivalued dependencies.
  - We can manage with such reasoning for very simple multivalued dependencies, which seem to be most common in practice

# Fourth Normal Form

- A relation schema  $R$  is in **4NF** with respect to a set  $D$  of functional and multivalued dependencies if for all multivalued dependencies in  $D^+$  of the form  $\alpha \twoheadrightarrow \beta$ , where  $\alpha \subseteq R$  and  $\beta \subseteq R$ , at least one of the following hold:
  - $\alpha \twoheadrightarrow \beta$  is trivial (i.e.,  $\beta \subseteq \alpha$  or  $\alpha \cup \beta = R$ )
  - $\alpha$  is a superkey for schema  $R$
- If a relation is in 4NF it is in BCNF

# Restriction of Multivalued Dependencies

- The restriction of  $D$  to  $R_i$  is the set  $D_i$  consisting of
  - All functional dependencies in  $D^+$  that include only attributes of  $R_i$
  - All multivalued dependencies of the form

$$\alpha \twoheadrightarrow\twoheadrightarrow (\beta \cap R_i)$$

where  $\alpha \subseteq R_i$  and  $\alpha \twoheadrightarrow\twoheadrightarrow \beta$  is in  $D^+$

# 4NF Decomposition Algorithm

*result* := {*R*};

*done* := false;

compute  $D^+$ ;

Let  $D_i$  denote the restriction of  $D^+$  to  $R_i$

**while** (**not** *done*)

**if** (there is a schema  $R_i$  in *result* that is not in 4NF) **then**

**begin**

            let  $\alpha \twoheadrightarrow \beta$  be a nontrivial multivalued dependency  
            that holds

            on  $R_i$  such that  $\alpha \rightarrow R_i$  is not in  $D_i$ , and  $\alpha \cap \beta = \phi$ ;

*result* := (*result* -  $R_i$ )  $\cup$  ( $R_i - \beta$ )  $\cup$  ( $\alpha, \beta$ );

**end**

**else** *done* := true;

Note: each  $R_i$  is in 4NF, and decomposition is lossless-join

# Example

Ex: test if the relation  $R(A, B, C, D, E)$  and  $D = \{A \twoheadrightarrow D, A \rightarrow B, A \rightarrow C\}$ .  
 Sol: it is not in 4NF because.  
 •  $A \twoheadrightarrow D$  is not a trivial MVD.  
 •  $A$  is not a superkey. Key AD

• Decompose into  $R_1(A, D)$ ,  $f_1 = \{A \twoheadrightarrow D\}$   
 and  $R_2(A, B, C)$   $f_2 = \{A \rightarrow B, A \rightarrow C\}$   
 • In  $R_1$ :  $A \twoheadrightarrow D$  is trivial MVD, thus in 4NF.  
 • In  $R_2$ :  $A$  is <sup>the</sup> Key, thus in 4NF.  $\Rightarrow A \rightarrow BC$   
 $\boxed{A \twoheadrightarrow BC}$   
 trivial

# Example

■  $R = (A, B, C, G, H, I)$

$F = \{ A \twoheadrightarrow B$

$B \twoheadrightarrow HI$

$CG \twoheadrightarrow H \}$

■  $R$  is not in 4NF since  $A \twoheadrightarrow B$  and  $A$  is not a superkey for  $R$

■ Decomposition

a)  $R_1 = (A, B)$

( $R_1$  is in 4NF)

b)  $R_2 = (A, C, G, H, I)$   
 $R_4$ )

( $R_2$  is not in 4NF, decompose into  $R_3$  and

c)  $R_3 = (C, G, H)$

( $R_3$  is in 4NF)

d)  $R_4 = (A, C, G, I)$   
 $R_6$ )

( $R_4$  is not in 4NF, decompose into  $R_5$  and

•  $A \twoheadrightarrow B$  and  $B \twoheadrightarrow HI \Rightarrow A \twoheadrightarrow HI$ , (MVD transitivity), and

• and hence  $A \twoheadrightarrow I$  (MVD restriction to  $R_4$ )

e)  $R_5 = (A, I)$

( $R_5$  is in 4NF)

f)  $R_6 = (A, C, G)$

( $R_6$  is in 4NF)

# Further Normal Forms

- **Joint dependencies** generalize multivalued dependencies
  - lead to **project-join normal form (PJNF)** (also called **fifth normal form**)
- A class of even more general constraints, leads to a normal form called **domain-key normal form(also called sixth Normal form)**.
- Problem with these generalized constraints: are hard to reason with, and no set of sound and complete set of inference rules exists.
- Hence rarely used

# Overall Database Design Process

- We have assumed schema  $R$  is given
  - $R$  could have been generated when converting E-R diagram to a set of tables.
  - $R$  could have been a single relation containing *all* attributes that are of interest (called **universal relation**).
  - Normalization breaks  $R$  into smaller relations.
  - $R$  could have been the result of some ad hoc design of relations, which we then test/convert to normal form.



# ER Model and Normalization

- When an E-R diagram is carefully designed, identifying all entities correctly, the tables generated from the E-R diagram should not need further normalization.
- However, in a real (imperfect) design, there can be functional dependencies from non-key attributes of an entity to other attributes of the entity
  - Example: an *employee* entity with attributes *department\_name* and *building*, and a functional dependency *department\_name* → *building*
  - Good design would have made department an entity
- Functional dependencies from non-key attributes of a relationship set possible, but rare --- most relationships are binary

# Denormalization for Performance

- Occasionally database designers choose a schema that has redundant information
- They use the redundancy to improve performance for specific applications.
- The penalty paid for not using a normalized schema is the extra work (in terms of coding time and execution time) to keep redundant data consistent.
- The process of taking a normalized schema and making it non-normalized is called **denormalization**
- Designers use it to tune performance of systems to support time-critical operations.
- A better alternative is to use the normalized schema, and additionally store the join of them as a **materialized view**.

# Denormalization for Performance

- May want to use non-normalized schema for performance
- For example, displaying *prereqs* along with *course\_id*, and *title* requires join of *course* with *prereq*
- Alternative 1: Use denormalized relation containing attributes of *course* as well as *prereq* with all above attributes
  - faster lookup
  - extra space and extra execution time for updates
  - extra coding work for programmer and possibility of error in extra code
- Alternative 2: use a materialized view defined as  $course \bowtie prereq$ 
  - Benefits and drawbacks same as above, except no extra coding work for programmer and avoids possible errors

# Other Design Issues

- Some aspects of database design are not caught by normalization
- Examples of bad database design, to be avoided:

Instead of *earnings* (*company\_id*, *year*, *amount* ), use

- *earnings\_2004*, *earnings\_2005*, *earnings\_2006*, etc., all on the schema (*company\_id*, *earnings*).
  - ▶ Above are in BCNF, but make querying across years difficult and needs new table each year
- *company\_year* (*company\_id*, *earnings\_2004*, *earnings\_2005*, *earnings\_2006*)
  - ▶ Also in BCNF, but also makes querying across years difficult and requires new attribute each year.
  - ▶ Is an example of a **crosstab**, where values for one attribute become column names
  - ▶ Used in spreadsheets, and in data analysis tools

**END OF CHAPTER**

# PROOF OF CORRECTNESS OF 3NF ALGORITHM

# Correctness of 3NF Decomposition Algorithm

- 3NF decomposition algorithm is dependency preserving (since there is a relation for every FD in  $F_c$ )
- Decomposition is lossless
  - A candidate key ( $C$ ) is in one of the relations  $R_i$  in decomposition
  - Closure of candidate key under  $F_c$  must contain all attributes in  $R$ .
  - Follow the steps of attribute closure algorithm to show there is only one tuple in the join result for each tuple in  $R_i$

# Correctness of 3NF Decomposition Algorithm (Cont'd.)

Claim: if a relation  $R_i$  is in the decomposition generated by the

above algorithm, then  $R_i$  satisfies 3NF.

- Let  $R_i$  be generated from the dependency  $\alpha \rightarrow \beta$
- Let  $\gamma \rightarrow B$  be any non-trivial functional dependency on  $R_i$ . (We need only consider FDs whose right-hand side is a single attribute.)
- Now,  $B$  can be in either  $\beta$  or  $\alpha$  but not in both. Consider each case separately.



# Correctness of 3NF Decomposition (Cont'd.)

## ■ Case 1: If $B$ in $\beta$ :

- If  $\gamma$  is a superkey, the 2nd condition of 3NF is satisfied
- Otherwise  $\alpha$  must contain some attribute not in  $\gamma$
- Since  $\gamma \rightarrow B$  is in  $F^+$  it must be derivable from  $F_c$ , by using attribute closure on  $\gamma$ .
- Attribute closure not have used  $\alpha \rightarrow \beta$ . If it had been used,  $\alpha$  must be contained in the attribute closure of  $\gamma$ , which is not possible, since we assumed  $\gamma$  is not a superkey.
- Now, using  $\alpha \rightarrow (\beta - \{B\})$  and  $\gamma \rightarrow B$ , we can derive  $\alpha \rightarrow B$  (since  $\gamma \subseteq \alpha \beta$ , and  $B \notin \gamma$  since  $\gamma \rightarrow B$  is non-trivial)
- Then,  $B$  is extraneous in the right-hand side of  $\alpha \rightarrow \beta$ ; which is not possible since  $\alpha \rightarrow \beta$  is in  $F_c$ .
- Thus, if  $B$  is in  $\beta$  then  $\gamma$  must be a superkey, and the second condition of 3NF must be satisfied.

# Correctness of 3NF Decomposition (Cont'd.)

- Case 2:  $B$  is in  $\alpha$ .
  - Since  $\alpha$  is a candidate key, the third alternative in the definition of 3NF is trivially satisfied.
  - In fact, we cannot show that  $\gamma$  is a superkey.
  - This shows exactly why the third alternative is present in the definition of 3NF.

Q.E.D.

*Thank You*