Relational Database Design

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Review

- Relational Model

Closure of a set of Functional Dependencies

- Given a set *F* of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.
  - For example: If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
- The set of **all** functional dependencies logically implied by *F* is the **closure** of *F*.
- We denote the *closure* of *F* by **F^+**.
- F^+ is a superset of *F*.

Closure of a set of Functional Dependencies

- Let R be the universal set of attributes and F be the set of functional dependencies on R. for α,β,γ,D being subset of R.
- We can find $F^{+,}$ the closure of F, by repeatedly applying **Armstrong's Axioms:**
  - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ **(reflexivity)**
  - if $\alpha \rightarrow \beta$, then $\gamma\ \alpha \rightarrow\ \gamma\ \beta$ **(augmentation)**
  - if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow\ \gamma$ **(transitivity)**
  - If $\alpha \rightarrow \beta$, and $\alpha \rightarrow\ \gamma$, then $\alpha \rightarrow \beta\gamma$ (**Union Rule**)
  - If $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ *and* $\alpha \rightarrow\ \gamma$ (**Decomposition Rule**)
  - if $\alpha \rightarrow \gamma$, and γ D $\rightarrow \beta$, *then* α D $\rightarrow \beta$ *(***Pseudotransivity Rule***)*
- These rules are
  - **sound** (generate only functional dependencies that actually hold), and
  - **complete** (generate all functional dependencies that hold).

Example to find F^+

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- some members of F^+
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ *and* $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$ and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$,

      and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$,

      and then transitivity

Procedure for Computing F+

- To compute the closure of a set of functional dependencies F:

$F^+ = F$
repeat
    for each functional dependency f in F^+
        apply reflexivity and augmentation rules on f
        add the resulting functional dependencies to F^+
    for each pair of functional dependencies f_1 and f_2 in F^+
        if f_1 and f_2 can be combined using transitivity
            then add the resulting functional dependency to F^+
until F^+ does not change any further

NOTE: We shall see an alternative procedure for this task later

Closure of Attribute set

- Given a set of attributes α, define the ***closure*** of α **under** *F* (denoted by α^+) as the set of attributes that are functionally determined by α under *F*

- Algorithm to compute α^+, the closure of α under *F* *(Bernstein's Algorithm)*

  - *result* := α;
    while (changes to *result*) **do**
      for each $\beta \rightarrow \gamma$ **in** *F* **do**
        begin
        if $\beta \subseteq$ *result* **then** *result* := *result* $\cup \gamma$
        end

Example of Attribute set closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- $(AG)^+$
  1. *result* = AG
  2. *result* = $ABCG$ $(A \rightarrow C$ and $A \rightarrow B)$
  3. *result* = $ABCGH$ $(CG \rightarrow H$ and $CG \subseteq AGBC)$
  4. *result* = $ABCGHI$ $(CG \rightarrow I$ and $CG \subseteq AGBCH)$
- Is AG a candidate key?
  1. Is AG a super key?
     1. Does $AG \rightarrow R$? == Is $(AG)^+ \supseteq R$
  2. Is any subset of AG a superkey?
     1. Does $A \rightarrow R$? == Is $(A)^+ \supseteq R$
     2. Does $G \rightarrow R$? == Is $(G)^+ \supseteq R$

Uses of Attribute closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if α is a superkey, we compute α^+, and check if α^+ contains all attributes of *R*.
- Testing functional dependencies
  - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^+), just check if $\beta \subseteq \alpha^+$.
  - That is, we compute α^+ by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^+, and for each $S \subseteq \gamma^+$, we output a functional dependency $\gamma \rightarrow S$.

Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B,\ B \rightarrow C,\ A \rightarrow C\}$
  - Parts of a functional dependency may be redundant
    - E.g.: on RHS: $\{A \rightarrow B,\ B \rightarrow C,\ A \rightarrow CD\}$ can be simplified to
      $\{A \rightarrow B,\ B \rightarrow C,\ A \rightarrow D\}$
    - E.g.: on LHS: $\{A \rightarrow B,\ B \rightarrow C,\ AC \rightarrow D\}$ can be simplified to
      $\{A \rightarrow B,\ B \rightarrow C,\ A \rightarrow D\}$
- Intuitively, a canonical cover of F is a "minimal" set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies

Extraneous Attributes

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
  - Attribute A is **extraneous** in α if $A \in \alpha$ and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - Attribute A is **extraneous** in β if $A \in \beta$ and the set of functional dependencies $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.
- *Note:* implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one
- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$
  - B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (I.e. the result of dropping B from $AB \rightarrow C$).
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting C

Testing if an Attribute is Extraneous

- Consider a set *F* of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in *F*.
- To test if attribute $A \in \alpha$ is extraneous in α
  1. compute $(\{\alpha\} - A)^+$ using the dependencies in *F*
  2. check that $(\{\alpha\} - A)^+$ contains β; if it does, *A* is extraneous in α
- To test if attribute $A \in \beta$ is extraneous in β
  1. compute α^+ using only the dependencies in
     $$F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\},$$
  2. check that α^+ contains *A*; if it does, *A* is extraneous in β

Canonical Cover

- A **canonical cover** for *F* is a set of dependencies F_c such that
  - *F* logically implies all dependencies in F_c, and
  - F_c logically implies all dependencies in *F*, and
  - No functional dependency in F_c contains an extraneous attribute, and
  - Each left side of functional dependency in F_c is unique.
- To compute a canonical cover for *F*:

  repeat

  Use the union rule to replace any dependencies in *F*

  $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$

  Find a functional dependency $\alpha \rightarrow \beta$ with an extraneous attribute either in α or in β

  /* Note: test for extraneous attributes done using F_c, not F*/

  If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$

  until *F* does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

Computing a Canonical Cover

- $R = (A, B, C)$
  $F = \{A \rightarrow BC$
  $B \rightarrow C$
  $A \rightarrow B$
  $AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
    - Yes: in fact, $B \rightarrow C$ is already present!
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
    - Yes: using transitivity on $A \rightarrow B$ *and* $B \rightarrow C$.
      - Can use attribute closure of A in more complex cases
- The canonical cover is: $A \rightarrow B$
  $B \rightarrow C$

Alternate way of computing Canonical Cover

- A **canonical cover** for *F* is a set of dependencies F_C such that
  - *F* logically implies all dependencies in F_C, and
  - F_C logically implies all dependencies in *F*, and
  - F_C does not contain any redundant *F.D*

To compute a canonical cover for *F*:

repeat

Use the decomposition rule to replace any dependencies in *F*

$\alpha_1 \rightarrow \beta_1 \beta_2$ with $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$

Find a functional dependency $\alpha \rightarrow \beta$ which is redundant i.e removing that F.D and finding that F.D implied by other F.D's

/* Note: test for redundant is done using $F^*=F-\{\alpha \rightarrow \beta\}^*$/

If an redundant attribute is found, delete it from $\alpha \rightarrow \beta$

until *F* does not change

Decomposition of Relational schema

- The Algorithm for relational database design may require a relation schema to be decomposed into two or more relation schema as a part of normalization process.
- Any Decomposition in relation database design must satisfy the following desirable properties:

    1. Lossless join Decomposition
    2. Dependency Preservation

Combine Schemas?

- Suppose we combine *instructor* and *department* into *inst_dept*
  - *(No connection to relationship set inst_dept)*
- Result is possible repetition of information

ID	*name*	*salary*	*dept_name*	*building*	*budget*
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
32343	El Said	60000	History	Painter	50000
45565	Katz	75000	Comp. Sci.	Taylor	100000
98345	Kim	80000	Elec. Eng.	Taylor	85000
76766	Crick	72000	Biology	Watson	90000
10101	Srinivasan	65000	Comp. Sci.	Taylor	100000
58583	Califieri	62000	History	Painter	50000
83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

What About Smaller Schemas?

- Suppose we had started with *inst_dept*. How would we know to split up (**decompose**) it into *instructor* and *department*?
- Write a rule "if there were a schema (*dept_name, building, budget*), then *dept_name* would be a candidate key"
- Denote as a **functional dependency**:

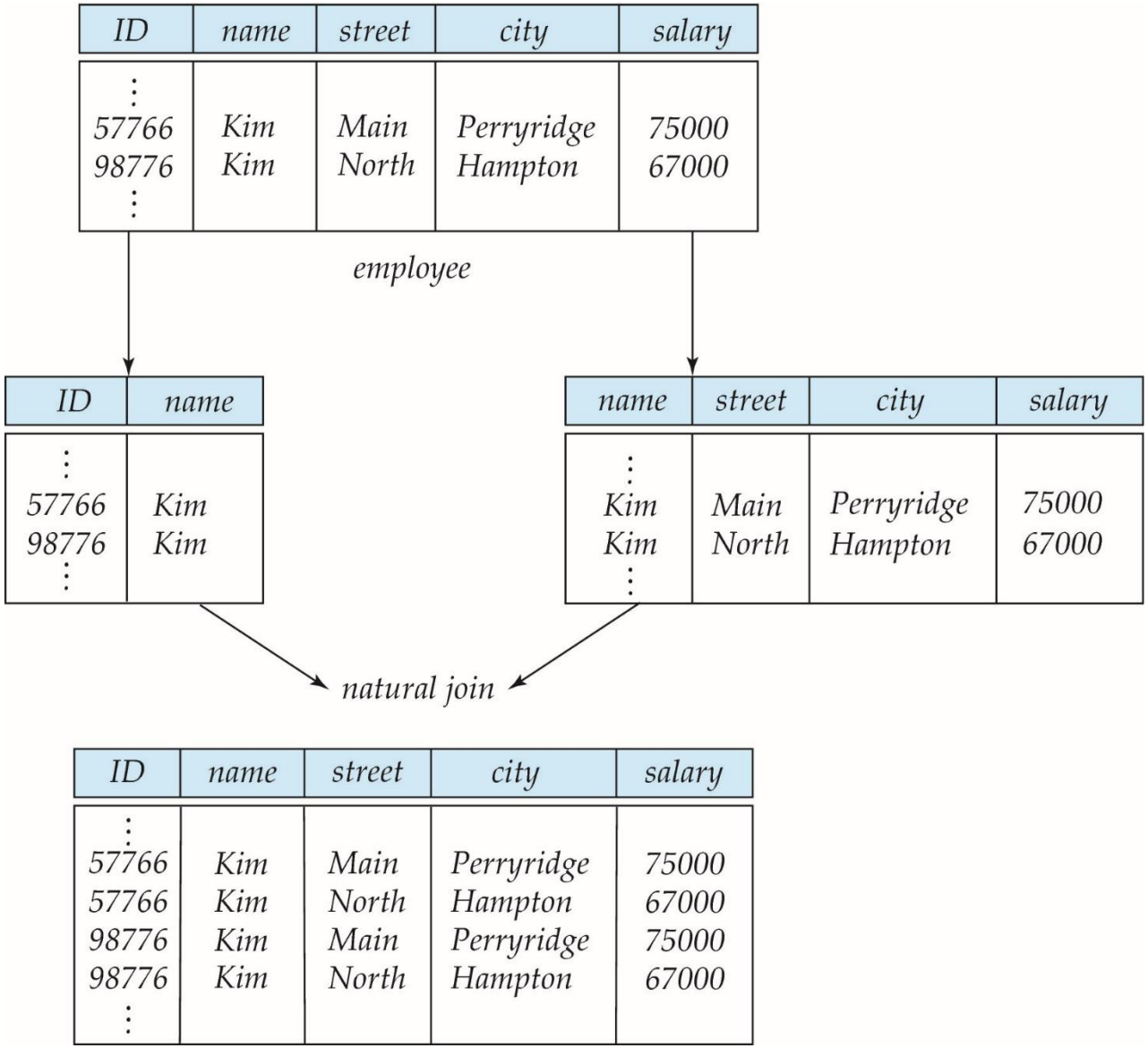
  $dept\_name \rightarrow building, budget$

- In *inst_dept*, because *dept_name* is not a candidate key, the building and budget of a department may have to be repeated.
  - This indicates the need to decompose *inst_dept*
- Not all decompositions are good. Suppose we decompose *employee(ID, name, street, city, salary)* into

  employee1 (*ID*, *name*)

  employee2 (*name*, *street, city, salary*)
- The next slide shows how we lose information -- we cannot reconstruct the original *employee* relation -- and so, this is a **lossy decomposition**.

A Lossy Decomposition



Example of Lossless-Join Decomposition

- **Lossless join decomposition**
- Decomposition of $R = (A, B, C)$

$R_1 = (A, B)$ $R_2 = (B, C)$

A	B	C
α	1	A
β	2	B

r

A	B
α	1
β	2

$\Pi_{A,B}(r)$

B	C
1	A
2	B

$\Pi_{B,C}(r)$

$\Pi_A (r) \bowtie \Pi_B (r)$

A	B	C
α	1	A
β	2	B

Lossless-join Decomposition

- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

$$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

- A decomposition of R into R_1 and R_2 is lossless join if at least one of the following dependencies is in F^+:
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$
- The above functional dependencies are a sufficient condition for lossless join decomposition; the dependencies are a necessary condition only if all constraints are functional dependencies

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C\}$
  - Can be decomposed in two different ways
- $R_1 = (A, B), \quad R_2 = (B, C)$
  - Lossless-join decomposition:

    $$R_1 \cap R_2 = \{B\} \text{ and } B \rightarrow BC$$

  - Dependency preserving
- $R_1 = (A, B), \quad R_2 = (A, C)$
  - Lossless-join decomposition:

    $$R_1 \cap R_2 = \{A\} \text{ and } A \rightarrow AB$$

  - Not dependency preserving
    (cannot check $B \rightarrow C$ without computing $R_1 \bowtie R_2$)

Dependency Preservation

- Let F_i be the set of dependencies F^+ that include only attributes in R_i.
  - A decomposition is **dependency preserving**, if

    $$(F_1 \cup F_2 \cup \ldots \cup F_n)^+ = F^+$$

  - If it is not, then checking updates for violation of functional dependencies may require computing joins, which is expensive.

Testing for Dependency Preservation

- To check if a dependency $\alpha \rightarrow \beta$ is preserved in a decomposition of R into $R_1, R_2, \ldots, R_n$ we apply the following test (with attribute closure done with respect to F)
  - *result* = α
    while (changes to *result*) do
      for each R_i in the decomposition
        $t = (result \cap R_i)^+ \cap R_i$
        $result = result \cup t$
  - If *result* contains all attributes in β, then the functional dependency $\alpha \rightarrow \beta$ is preserved.
- We apply the test on all dependencies in F to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute F^+ and $(F_1 \cup F_2 \cup \ldots \cup F_n)^+$

Thank You