Title: Line integral

Aim:. To Find the Line Integral

Question:

1. Find the work done by the force field $\vec{F}(x,y) = x^2\vec{\imath} - xy\vec{\jmath}$ in moving a particle along the quarter – circle $\vec{r}(t) = \cos(t)\,\vec{\imath} + \sin(t)\vec{\jmath}$, $0 \leq t \leq \frac{\pi}{2}$

2. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, $\vec{F}(x,y) = (x-y)\vec{\imath} + xy\vec{\jmath}$, C is the arc of the circle $x^2 + y^2 = 4$ traversed counter-clockwise from (2, 0) to(0, -2).

3. Find the work done by the force field $\vec{F}(x,y) = (y+z,\ x+z, x+y)$ on a particle that moves along the line segment (1, 0, 0) to (3, 4, 2).

4. Find the work done by the force field $\vec{F}(x,y) = x\,sin(y)\vec{\imath} + y\vec{\jmath}$ on a particle that moves along the curve $y = x^2$ from (-1, 1) to (2, 4).

5. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, $\vec{F}(x,y,z) = y\,sin(z)\vec{\imath} + z\,sin(x)\vec{\jmath} + x\,sin(y)\vec{k}$ and $\vec{r}(t) = \cos(t)\,\vec{\imath} + \sin(t)\,\vec{\jmath} + \sin(5t)\,\vec{k}$, $0 \leq t \leq \pi$.

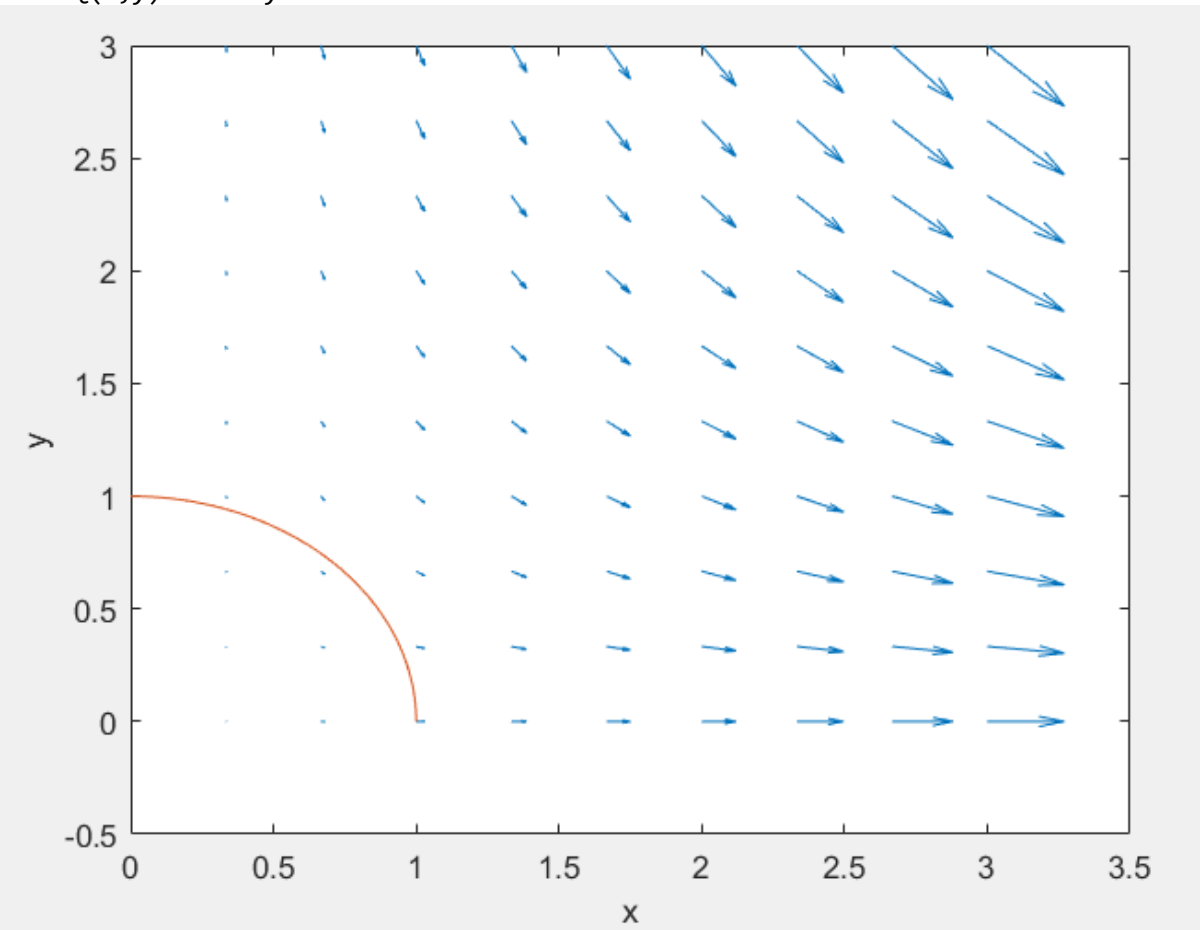
MATLAB CODE

```
clc
clear all
format compact
syms t x y
f=input('enter the f vector as i and j order in vector form: ')
rbar = input('enter the r vector as i and j order in vector form: ')
lim=input('enter the limit of integration: ')
vecfi=input('enter the vector field range ')
% knowledge of the curve is essential
drbar=diff(rbar,t);
sub = subs(f,[x,y],rbar);
f1=dot(sub,drbar)
WD=int(f1,t,lim(1),lim(2)) % work done
P = inline(vectorize(f(1)), 'x', 'y');
Q = inline(vectorize(f(2)), 'x', 'y')
x = linspace(vecfi(1),vecfi(2), 10);
y = x;
[X,Y] = meshgrid(x,y);
U = P(X,Y);
V = Q(X,Y);
quiver(X,Y,U,V)
hold on
fplot(rbar(1),rbar(2),[lim(1),lim(2)])
axis on
xlabel('x')
ylabel('y')
hold off
```

Question 1:

Output:

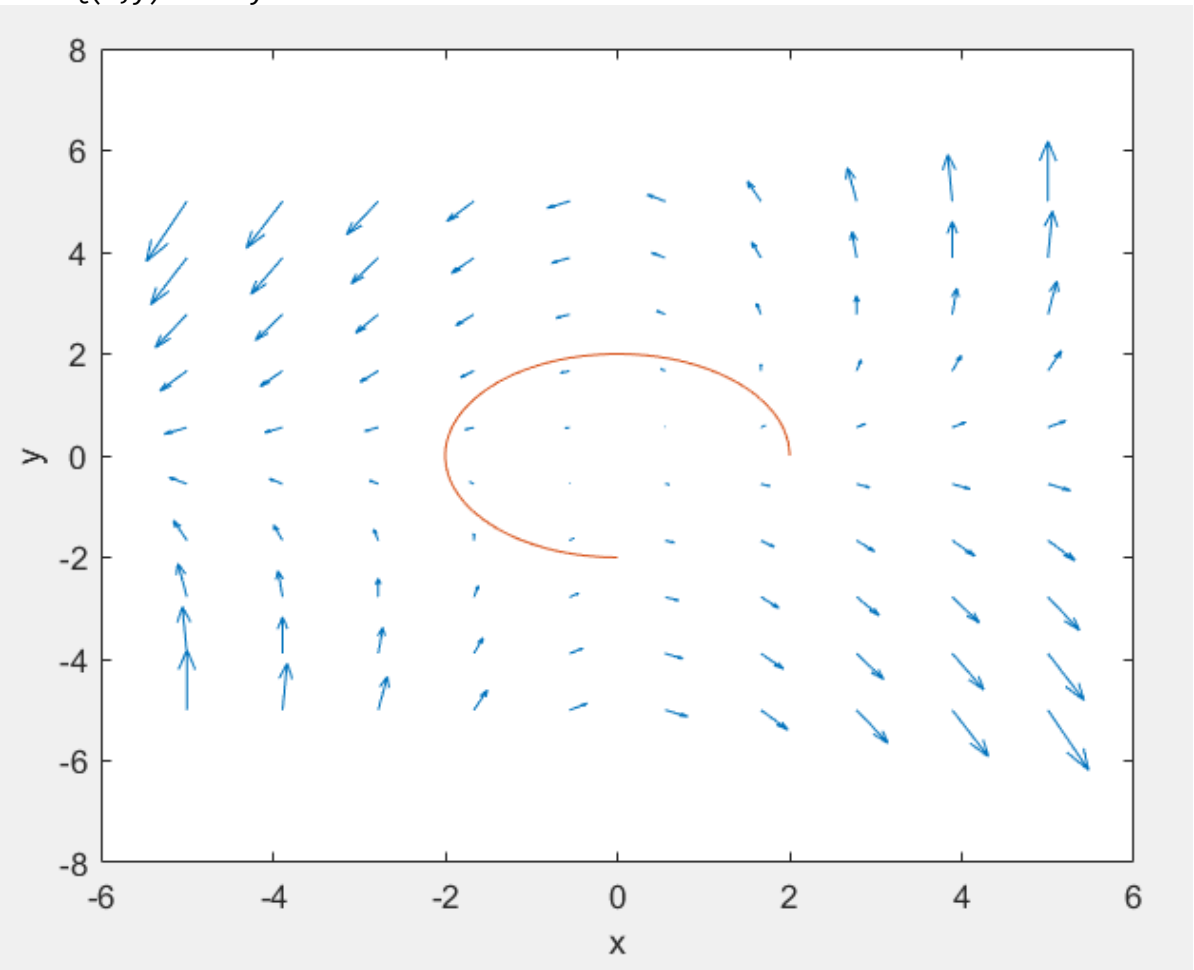
```
enter the f vector as i and j order in vector form: [x^2 -x*y]
f =
[x^2, -x*y]
enter the r vector as i and j order in vector form: [cos(t) sin(t)]
rbar =
[cos(t), sin(t)]
enter the limit of integration: [0 pi/2]
lim =
         0    1.5708
enter the vector field range [0 3]
vecfi =
     0     3
f1 =
- sin(t)*cos(conj(t))^2 - sin(conj(t))*cos(t)*cos(conj(t))
WD =
-2/3
Q =
     Inline function:
     Q(x,y) = -x.*y
```



Question 2:

Output:

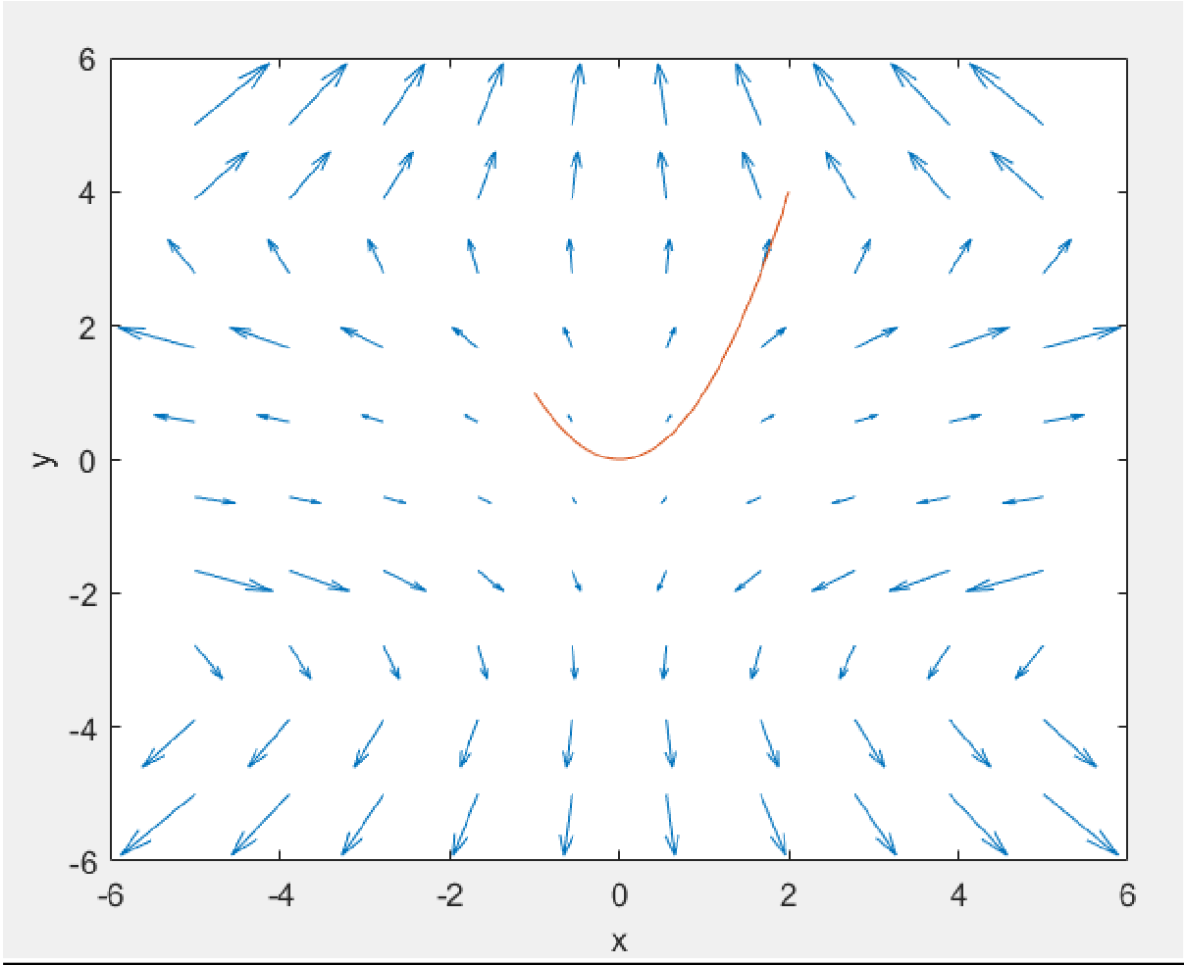
```
enter the f vector as i and j order in vector form: [x-y x*y]
f =
[x - y, x*y]
enter the r vector as i and j order in vector form: [2*cos(t) 2*sin(t)]
rbar =
[2*cos(t), 2*sin(t)]
enter the limit of integration: [0 3*(pi/2)]
lim =
         0    4.7124
enter the vector field range [-5 5]
vecfi =
    -5     5
f1 =
8*cos(conj(t))*sin(conj(t))*cos(t) - 2*sin(t)*(2*cos(conj(t)) - 2*sin(conj(t)))
WD =
3*pi + 2/3
Q =
     Inline function:
     Q(x,y) = x.*y
```



Question 4:

Output:

```
enter the f vector as i and j order in vector form: [x*sin(y) y]
f =
[x*sin(y), y]
enter the r vector as i and j order in vector form: [t t^2]
rbar =
[t, t^2]
enter the limit of integration: [-1 2]
lim =
    -1     2
enter the vector field range [-5 5]
vecfi =
    -5     5
f1 =
2*t*conj(t)^2 + sin(conj(t)^2)*conj(t)
WD =
cos(1)/2 - cos(4)/2 + 15/2
Q =
     Inline function:
     Q(x,y) = y
```



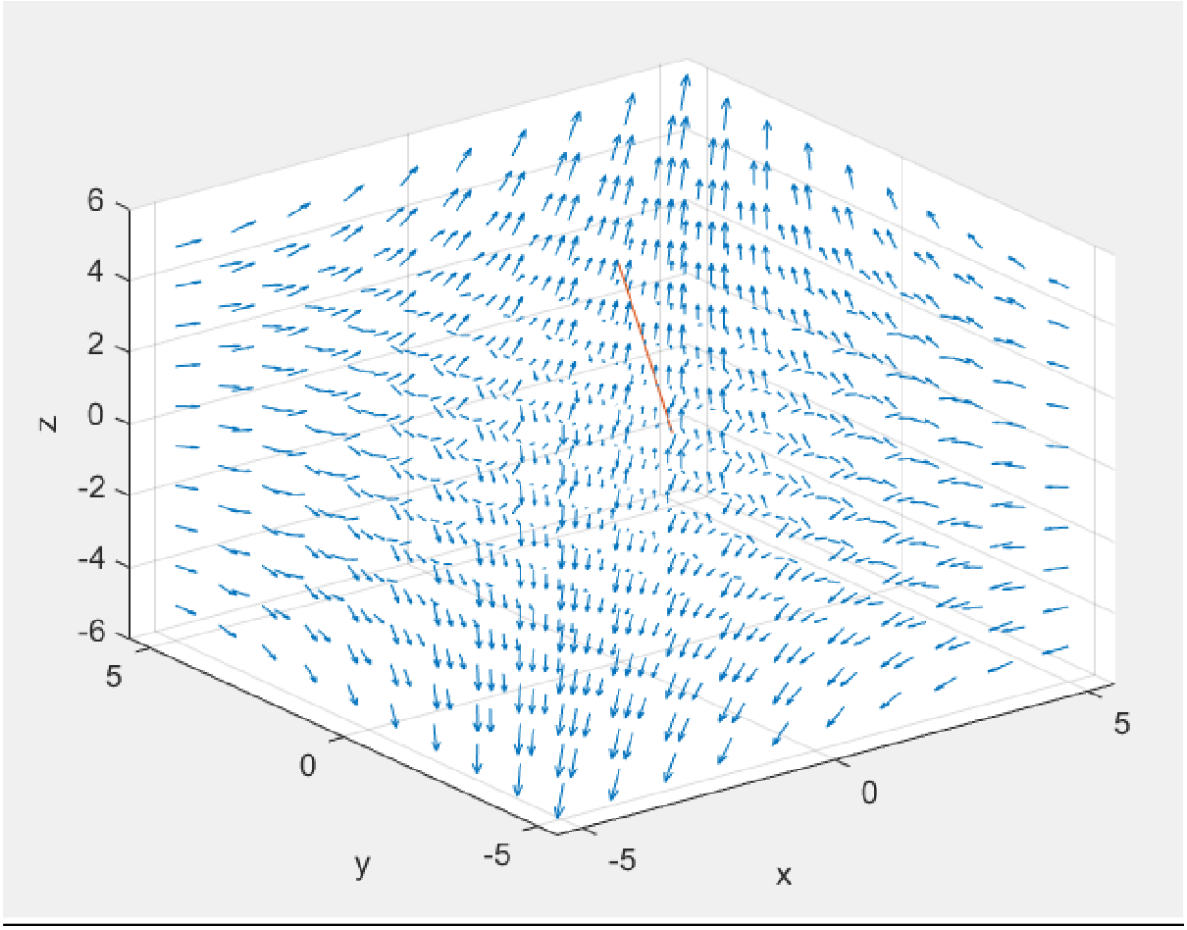
MATLAB CODE

```
% workdone
clc
clear all
format compact
syms t x y z
f=input('enter the f vector as i j and k order in vector form: ')
rbar = input('enter the r vector as i j and k order in vector form: ')
lim=input('enter the limit of integration: ')
vecfi=input('enter the vector field range ')
% knowledge of the curve is essential
drbar=diff(rbar,t);
sub = subs(f,[x,y,z],rbar);
f1=dot(sub,drbar)
WD=int(f1,t,lim(1),lim(2)) % work done
wd=vpa(WD,4)
P = inline(vectorize(f(1)), 'x', 'y', 'z');
Q = inline(vectorize(f(2)), 'x', 'y', 'z');
R = inline(vectorize(f(3)), 'x', 'y', 'z');
x = linspace(vecfi(1),vecfi(2), 10);
y = x;
z = x;
[X,Y,Z] = meshgrid(x,y,z);
U = P(X,Y,Z);
V = Q(X,Y,Z);
W = R(X,Y,Z);
quiver3(X,Y,Z,U,V,W)
hold on
fplot3(rbar(1),rbar(2),rbar(3),[lim(1),lim(2)])
axis on
xlabel('x')
ylabel('y')
zlabel('z')
hold off
```

Question 3:

Output:

```
enter the f vector as i j and k order in vector form: [y+z x+z x+y]
f =
[y + z, x + z, x + y]
enter the r vector as i j and k order in vector form: [2*t+1 4*t 2*t]
rbar =
[2*t + 1, 4*t, 2*t]
enter the limit of integration: [0 1]
lim =
     0     1
enter the vector field range [-5 5]
vecfi =
    -5     5
f1 =
40*conj(t) + 6
WD =
26
wd =
26.0
```



Question 5:

Output:

```
enter the f vector as i j and k order in vector form: [y*sin(z) z*sin(x) x*sin(y)]
f =
[y*sin(z), z*sin(x), x*sin(y)]
enter the r vector as i j and k order in vector form: [cos(t) sin(t) sin(5*t)]
rbar =
[cos(t), sin(t), sin(5*t)]
enter the limit of integration: [0 pi]
lim =
         0    3.1416
enter the vector field range [-1 1]
vecfi =
    -1     1
f1 =
5*cos(5*t)*cos(conj(t))*sin(sin(conj(t))) +
sin(5*conj(t))*sin(cos(conj(t)))*cos(t) - sin(sin(5*conj(t)))*sin(conj(t))*sin(t)
WD =
int(5*cos(5*t)*cos(conj(t))*sin(sin(conj(t))) +
sin(5*conj(t))*sin(cos(conj(t)))*cos(t) - sin(sin(5*conj(t)))*sin(conj(t))*sin(t),
t, 0, pi)
wd =
-0.1363
```