Title: Integration -Solids of Revolution

Aim: To Find the volume of Solids of Revolution

Questions:

1: Visualize and find the volume of the region in the first quadrant bounded above by the line $y = \sqrt{2}$, below by the curve $y = \sec(x)\tan(x)$, and on the left by the $y-$axis, about the line $y = \sqrt{2}$.

2: Visualize and find the volume of the solid generated by revolving the region bounded by curve $y = \sin(x)$, $0 \le x \le \pi$, about the line $y = c, 0 \le c \le 1$ by taking $c = 0, 0.2, 0.4, 0.6, 0.8, 1$. Can you identify the range/exact value of c that minimize and maximize the volume of the solid?

3: Modify the above code appropriately to visualize and find the volume of the solid of revolution by the curve $y = \tan\left(\frac{\pi}{4}y\right), 0 \le y \le 1$ about the $y-$axis.

4. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

5. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ $0 \le x \le 4$ about the line $y = 1$.

MATHLAB CODE: (For Question 1,2,4 and 5)*

```
clc
clear all
format compact
clearvars
syms x
f = input('Enter the function: ')
fL = input('Enter the interval on which the function is defined: ')
yr = input('Enter the axis of rotation y = c(enter only c value): ')
iL = input('Enter the integration limits: ')
Volume = pi*int((f-yr)^2,iL(1),iL(2));
disp(['Volume is: ', num2str(double(Volume))])
fx = inline(vectorize(f))
xvals = linspace(fL(1),fL(2),201);
xvalsr = fliplr(xvals);
xivals = linspace(iL(1),iL(2),201);
```

```
xivalsr = fliplr(xivals);
xlim = [fL(1) fL(2)+0.5]
ylim = fx(xlim)
figure('Position',[100 200 560 420])
subplot(2,1,1)
hold on;
plot(xvals,fx(xvals),'-b','LineWidth',2);
fill([xvals xvalsr], ...
[fx(xvals) ones(size(xvalsr))*yr],[0.8 0.8 0.8],'FaceAlpha',0.8)
plot([fL(1) fL(2)],[yr yr],'-r','LineWidth',2);
legend('Function Plot','Filled Region', ...
'Axis of Rotation','Location','Best');
title('Function y=f(x) and Region');
set(gca,'XLim',xlim)
xlabel('x-axis');
ylabel('y-axis');
subplot(2,1,2)
hold on;
plot(xivals,fx(xivals),'-b','LineWidth',2);
fill([xivals xivalsr],[fx(xivals) ones(size(xivalsr))*yr], ...
[0.8 0.8 0.8],'FaceAlpha',0.8)
fill([xivals xivalsr],[ones(size(xivals))*yr -fx(xivalsr)+2*yr], ...
[1 0.8 0.8],'FaceAlpha',0.8)
plot(xivals,-fx(xivals)+2*yr,'-m','LineWidth',2);
plot([iL(1) iL(2)],[yr yr],'-r','LineWidth',2);
title('Rotated Region in xy-Plane');
set(gca,'XLim',xlim)
xlabel('x-axis');
ylabel('y-axis');
[X,Y,Z] = cylinder(fx(xivals)-yr,100);
figure('Position',[700 200 560 420])
Z = iL(1) + Z.*(iL(2)-iL(1));
surf(Z,Y+yr,X,'EdgeColor','none','FaceColor','flat','FaceAlpha',0.6)
;
hold on;
plot([iL(1) iL(2)],[yr yr],'-r','LineWidth',2);
xlabel('X-axis');
ylabel('Y-axis');
zlabel('Z-axis');
view(22,11);
```

Output 1:

Numerical Output:

Enter the function: sec(x)*tan(x)

f =

tan(x)/cos(x)

Enter the interval on which the function is defined: [0,pi/4]

fL =

    0   0.7854

Enter the axis of rotation y = c(enter only c value): sqrt(2)

yr =

  1.4142

Enter the integration limits: [0,pi/4]

iL =

    0   0.7854

Volume is: 2.3014

fx =

  Inline function:

  fx(x) = tan(x)./cos(x)

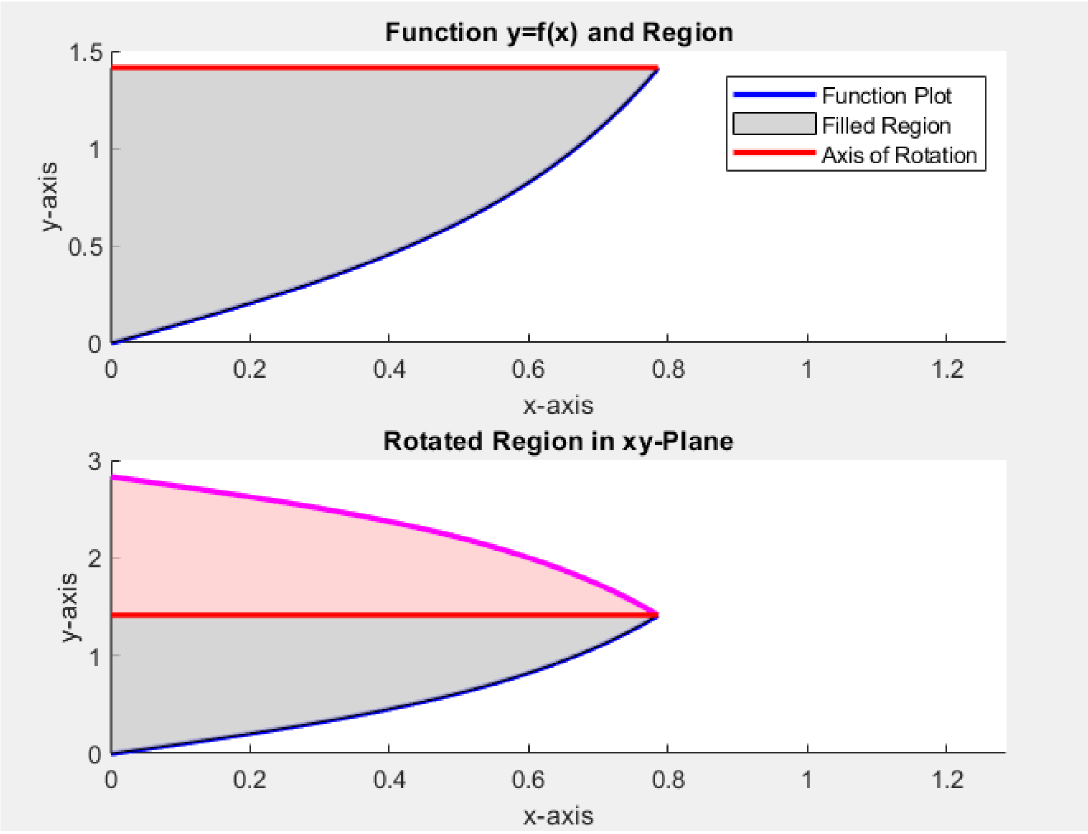
xlim =

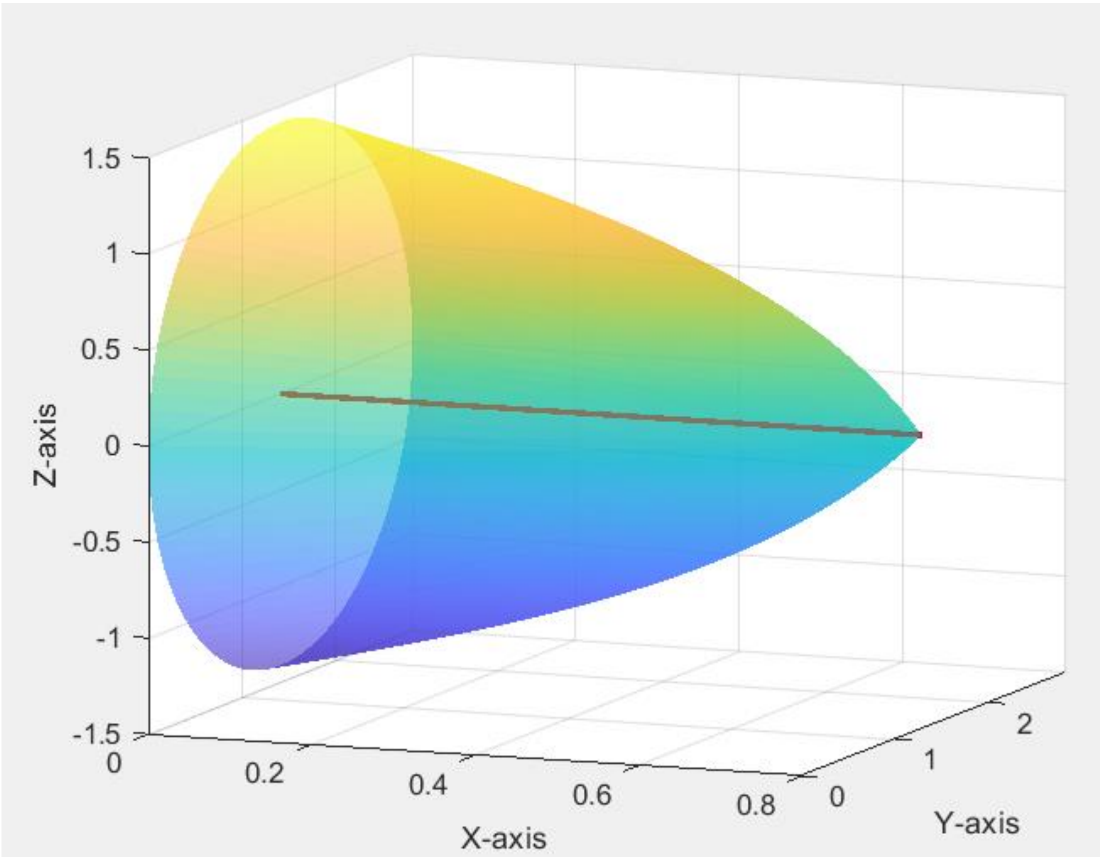
    0   1.2854

ylim =

    0   12.1057

Figure Output:





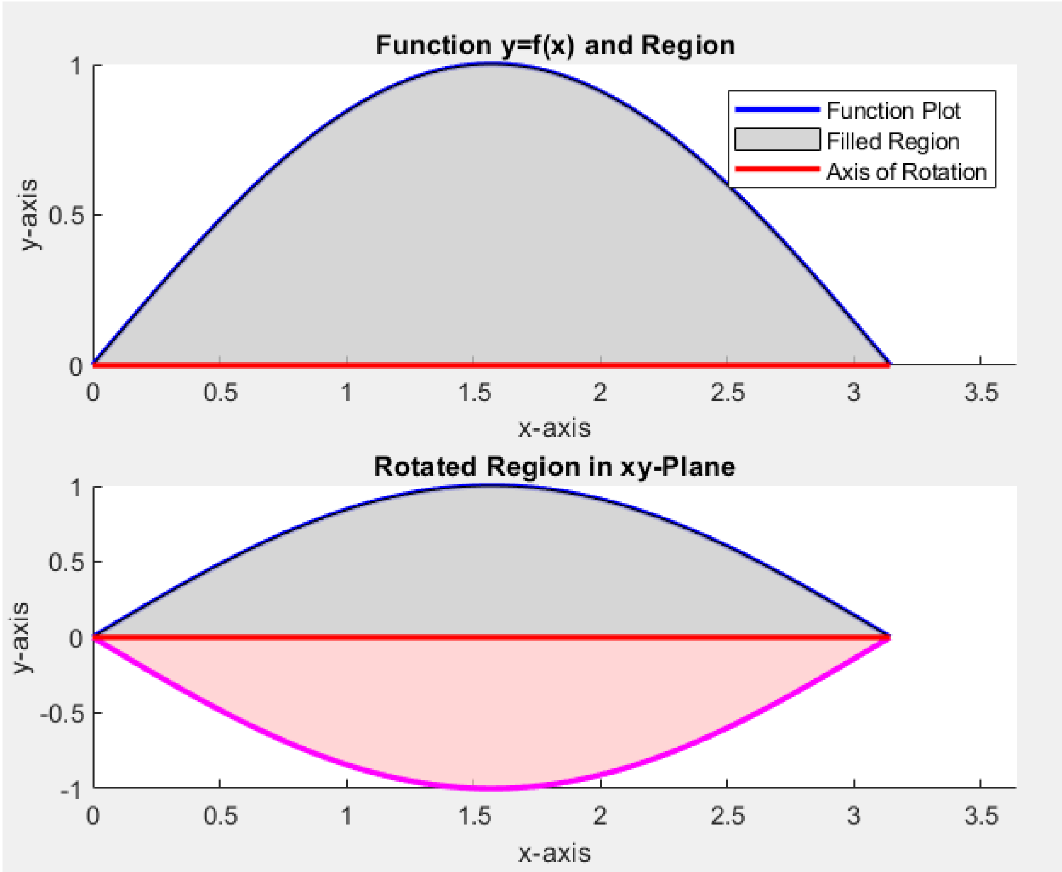
Output 2:

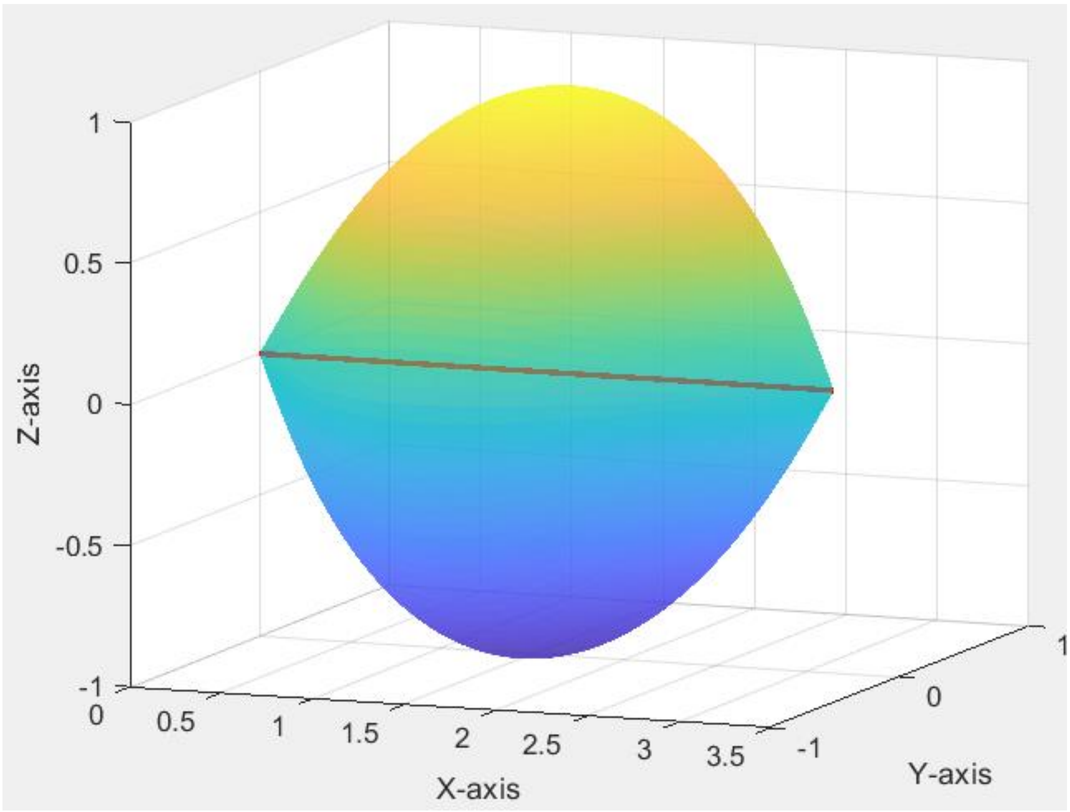
WHEN C=0

Numerical Output:

```
Enter the function: sin(x)

f =

sin(x)

Enter the interval on which the function is defined: [0,pi]

fL =

        0    3.1416

Enter the axis of rotation y = c(enter only c value): 0

yr =

     0

Enter the integration limits: [0,pi]

iL =

        0    3.1416

Volume is: 4.9348

fx =

     Inline function:

     fx(x) = sin(x)

xlim =

        0    3.6416

ylim =

        0    -0.4794
```

Figure Output:



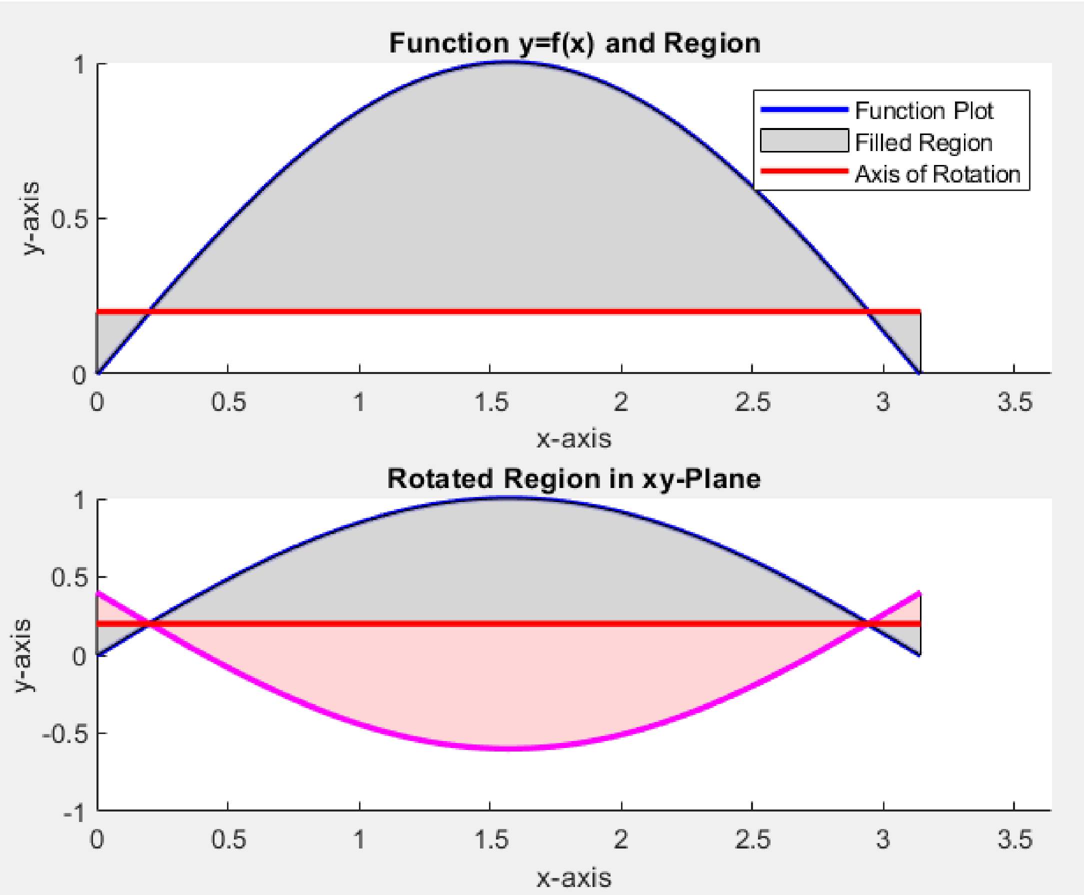


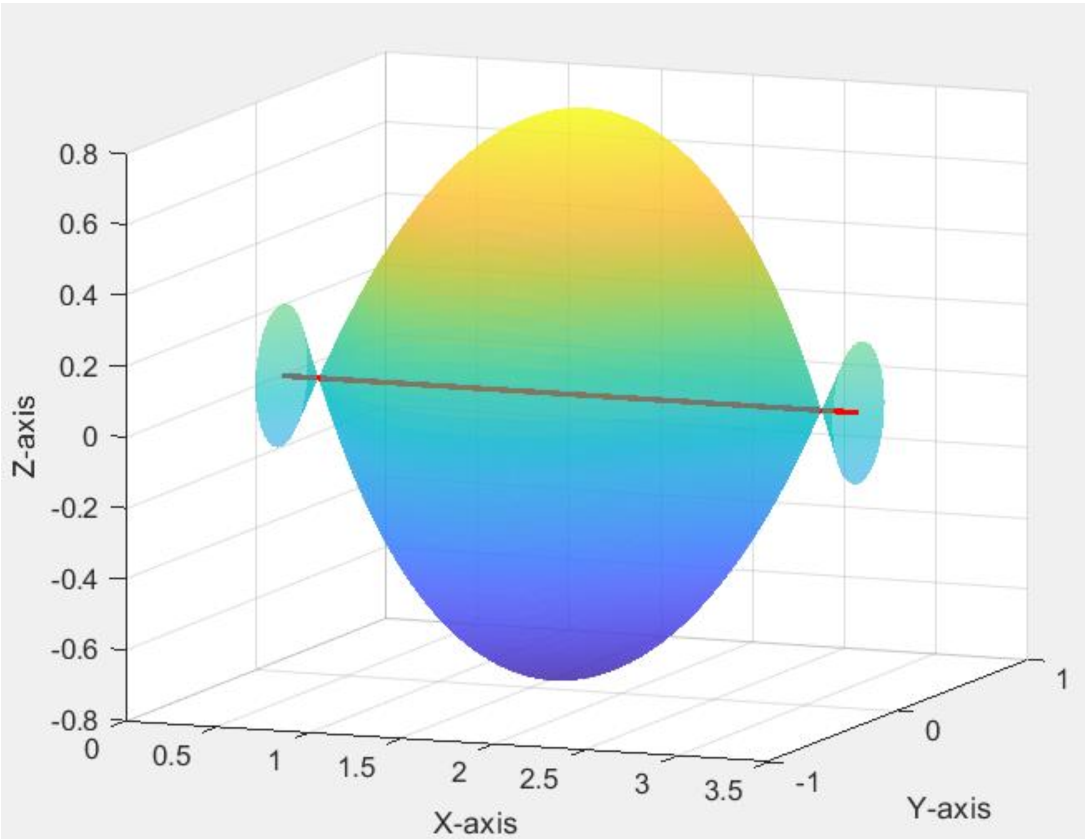
WHEN C=0.2

NUMERICAL OUTPUT:

```
Enter the function: sin(x)

f =

sin(x)

Enter the interval on which the function is defined: [0,pi]

fL =

        0    3.1416

Enter the axis of rotation y = c(enter only c value): 0.2

yr =

    0.2000

Enter the integration limits: [0,pi]

iL =

        0    3.1416

Volume is: 2.8163

fx =

     Inline function:

     fx(x) = sin(x)

xlim =

        0    3.6416

ylim =

        0   -0.4794
```

Figure Output:



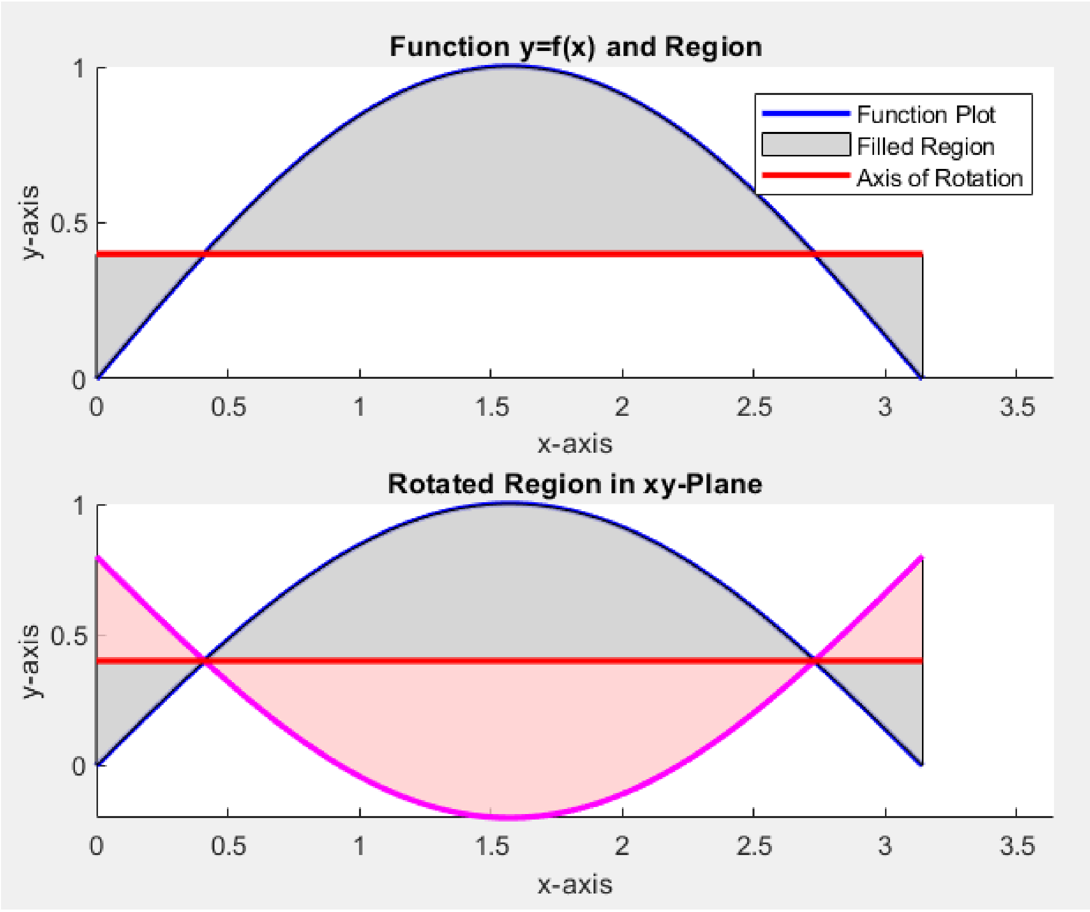


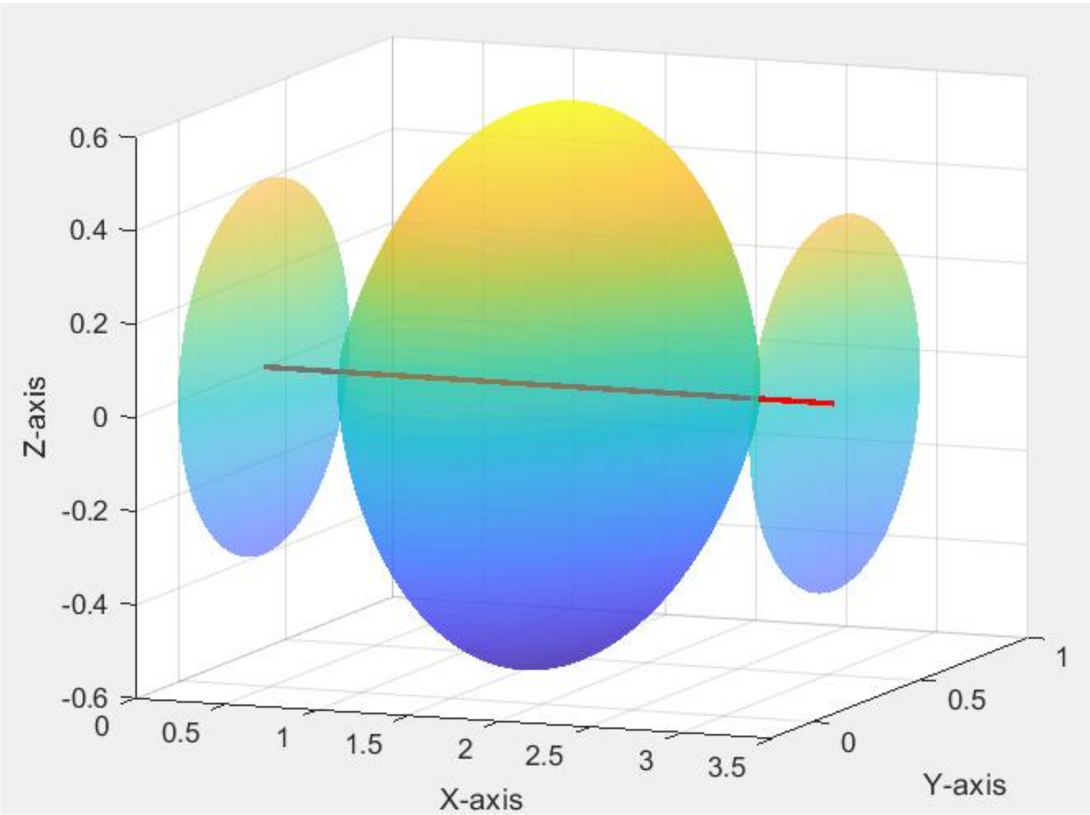
WHEN C=0.4

Numerical Output:

```
Enter the function: sin(x)

f =

sin(x)

Enter the interval on which the function is defined: [0,pi]

fL =

        0    3.1416

Enter the axis of rotation y = c(enter only c value): 0.4

yr =

    0.4000

Enter the integration limits: [0,pi]

iL =

        0    3.1416

Volume is: 1.4874

fx =

     Inline function:

     fx(x) = sin(x)

xlim =

        0    3.6416

ylim =

        0   -0.4794
```

Figure Output:



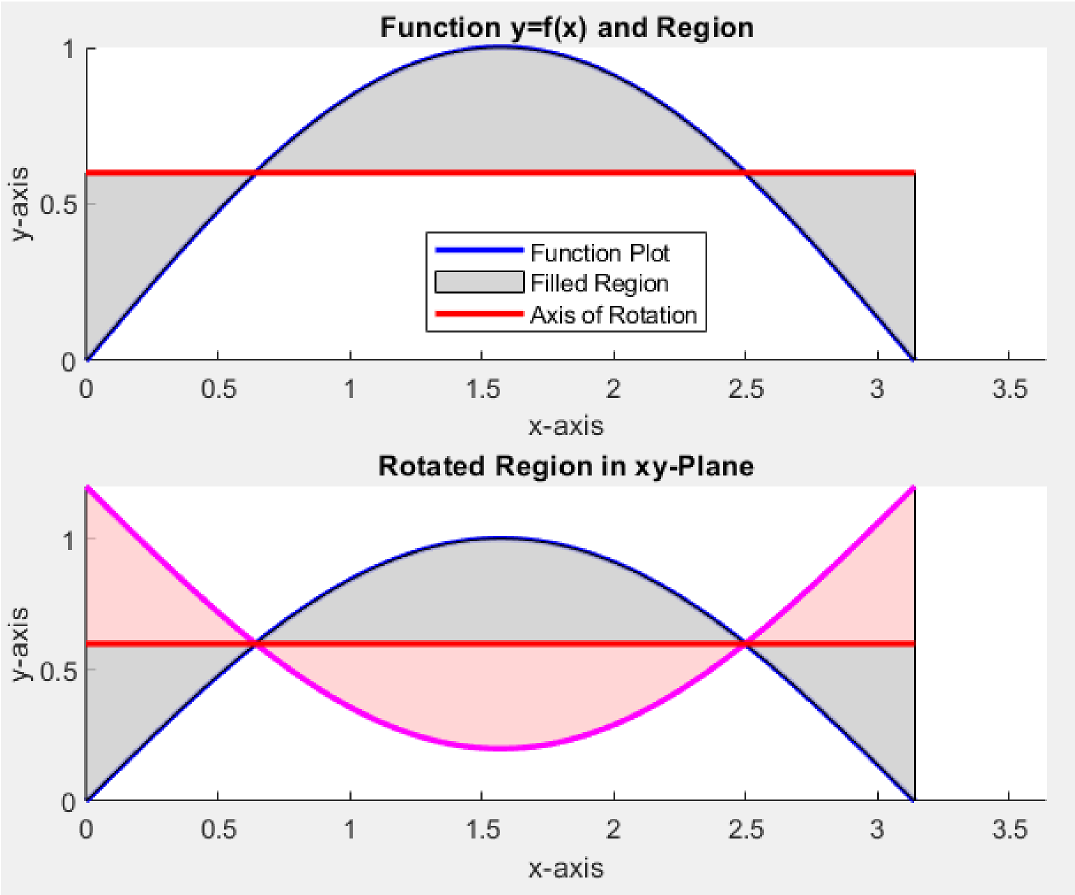


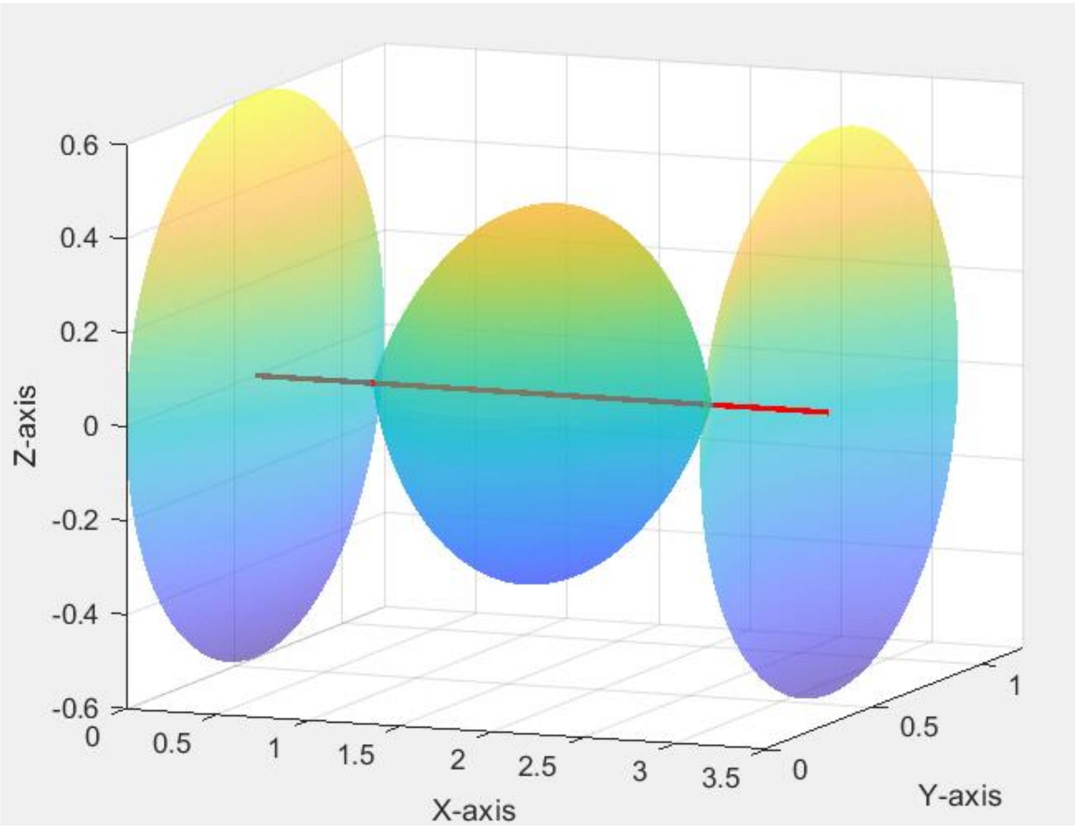
WHEN C=0.6

Numerical Output:

```
Enter the function: sin(x)

f =

sin(x)

Enter the interval on which the function is defined: [0,pi]

fL =

        0    3.1416

Enter the axis of rotation y = c(enter only c value): 0.6

yr =

   0.6000

Enter the integration limits: [0,pi]

iL =

        0    3.1416

Volume is: 0.94804

fx =

     Inline function:

     fx(x) = sin(x)

xlim =

        0    3.6416

ylim =

        0   -0.4794
```

Figure Output:



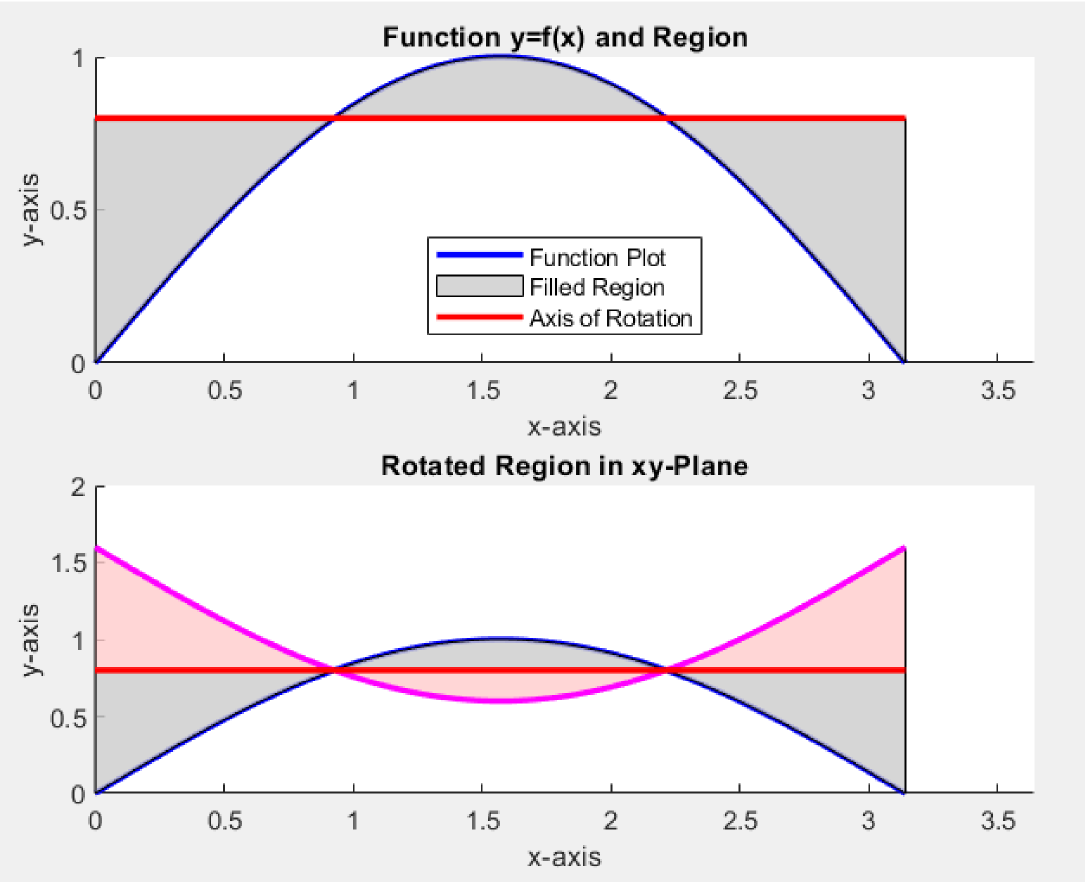


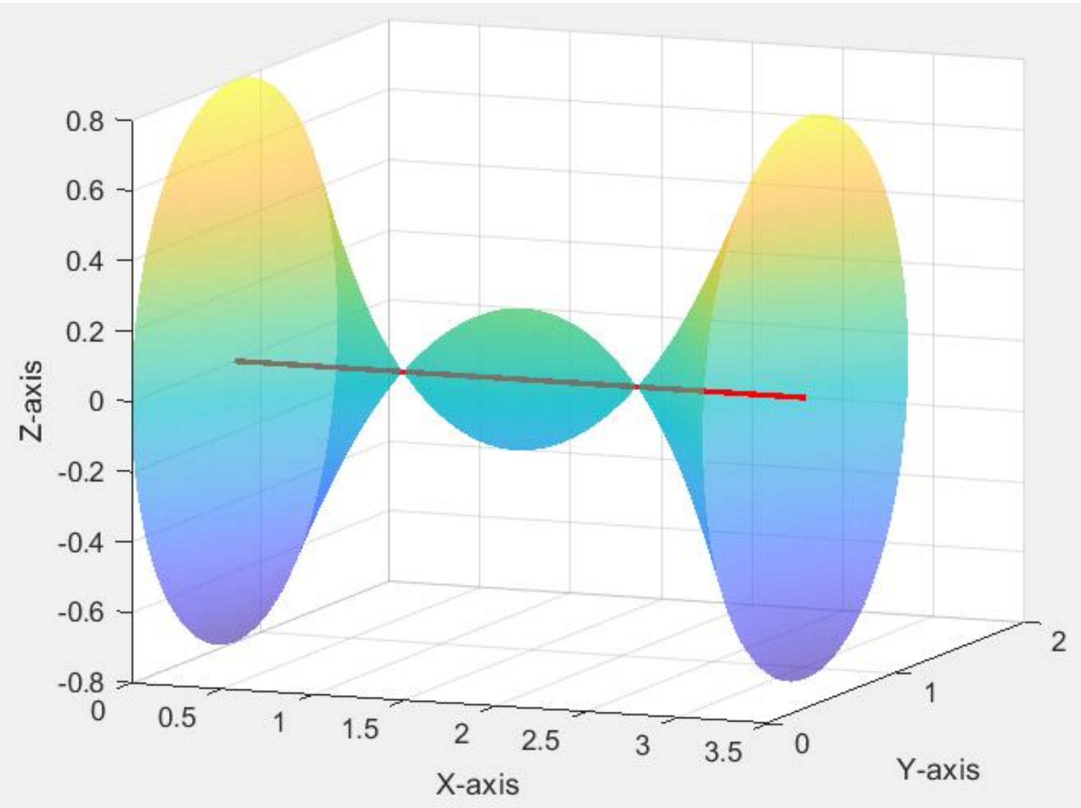
WHEN C=0.8

Numerical Output:

```
Enter the function: sin(x)

f =

sin(x)

Enter the interval on which the function is defined: [0,pi]

fL =

        0    3.1416

Enter the axis of rotation y = c(enter only c value): 0.8

yr =

    0.8000

Enter the integration limits: [0,pi]

iL =

        0    3.1416

Volume is: 1.1983

fx =

     Inline function:

     fx(x) = sin(x)

xlim =

        0    3.6416

ylim =

        0   -0.4794
```

Figure Output:





WHEN C=1

Numerical Output:

Enter the function: sin(x)

f =

sin(x)

Enter the interval on which the function is defined: [0,pi]

fL =

0   3.1416

Enter the axis of rotation y = c(enter only c value): 1

yr =

1

Enter the integration limits: [0,pi]

iL =

0   3.1416

Volume is: 2.238

fx =

Inline function:

fx(x) = sin(x)

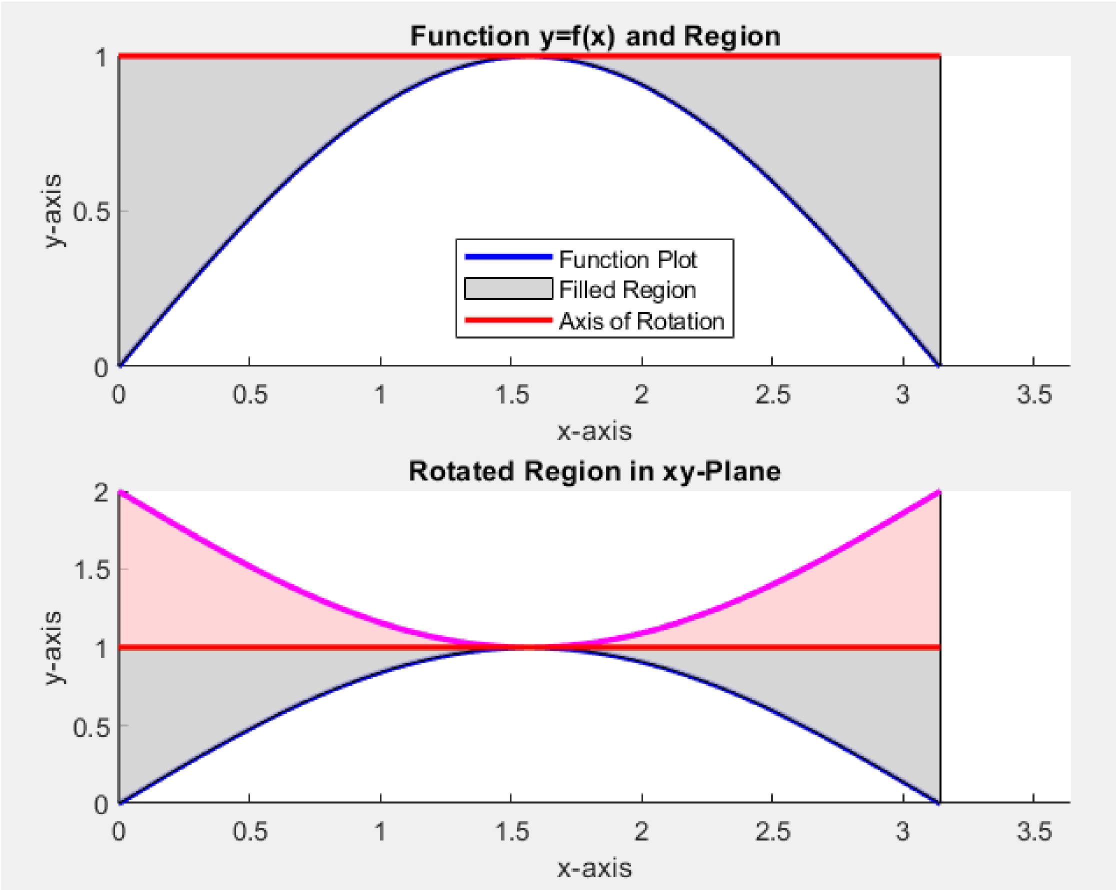
xlim =

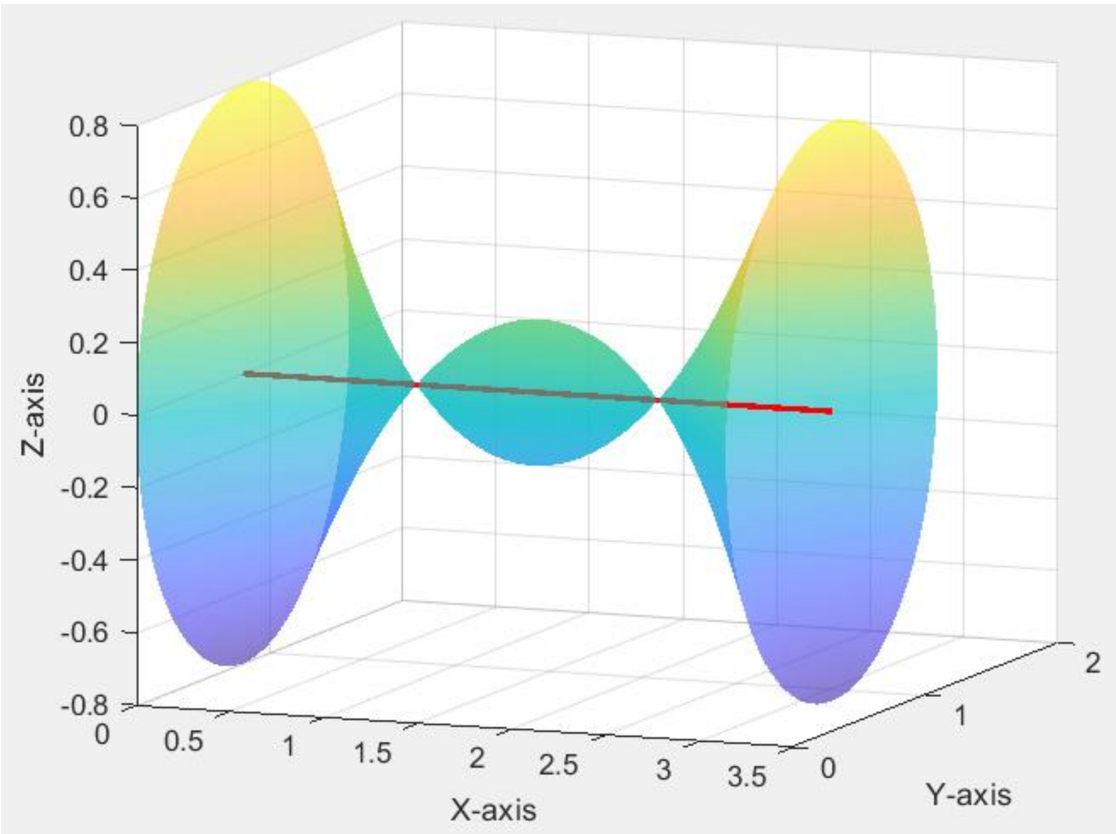
0   3.6416

ylim =

0   -0.4794

Figure Output:





MATLAB Code: (For Question 3)

```
clc
clear all
format compact
clearvars
syms y
f = input('Enter the function: ')
fL = input('Enter the interval on which the function is defined: ')
xr = input('Enter the axis of rotation x = c(enter only c value): ')
iL = input('Enter the integration limits: ')
Volume = pi*int((f-xr)^2,iL(1),iL(2));
disp(['Volume is: ', num2str(double(Volume))])
fy = inline(vectorize(f))
yvals = linspace(fL(1),fL(2),201);
yvalsr = fliplr(yvals);
yivals = linspace(iL(1),iL(2),201);
yivalsr = fliplr(yivals);
ylim = [fL(1) fL(2)+0.5]
xlim = fy(ylim)
figure('Position',[100 200 560 420])
subplot(2,1,1)
hold on;
plot(yvals,fy(yvals),'-b','LineWidth',2);
fill([yvals yvalsr], ...
[fy(yvals) ones(size(yvalsr))*xr],[0.8 0.8 0.8],'FaceAlpha',0.8)
plot([fL(1) fL(2)],[xr xr],'-r','LineWidth',2);
legend('Function Plot','Filled Region', ...
'Axis of Rotation','Location','Best');
title('Function x=f(y) and Region');
set(gca,'YLim',ylim)
ylabel('y-axis');
xlabel('x-axis');
subplot(2,1,2)
hold on;
plot(yivals,fy(yivals),'-b','LineWidth',2);
fill([yivals yivalsr],[fy(yivals) ones(size(yivalsr))*xr], ...
[0.8 0.8 0.8],'FaceAlpha',0.8)
fill([yivals yivalsr],[ones(size(yivals))*xr -fy(yivalsr)+2*xr], ...
[1 0.8 0.8],'FaceAlpha',0.8)
plot(yivals,-fy(yivals)+2*xr,'-m','LineWidth',2);
plot([iL(1) iL(2)],[xr xr],'-r','LineWidth',2);
title('Rotated Region in yx-Plane');
set(gca,'YLim',ylim)
ylabel('y-axis');
xlabel('x-axis');
[Y,X,Z] = cylinder(fy(yivals)-xr,100);
figure('Position',[700 200 560 420])
Z = iL(1) + Z.*(iL(2)-iL(1));
```

```
surf(Z,X+xr,Y,'EdgeColor','none','FaceColor','flat','FaceAlpha',0.6)
;
hold on;
plot([iL(1) iL(2)],[xr xr],'-r','LineWidth',2);
xlabel('X-axis');
ylabel('Y-axis');
zlabel('Z-axis');
view(22,11);
```

<u>Output 3:</u>

<u>Numerical Output:</u>

Enter the function: tan(pi*y/4)

f =

tan((pi*y)/4)

Enter the interval on which the function is defined: [0,1]

fL =

    0   1

Enter the axis of rotation x = c(enter only c value): 0

xr =

    0

Enter the integration limits: [0,1]

iL =

    0   1

Volume is: 0.85841

fy =

    Inline function:

    fy(y) = tan((y.*pi)./4)

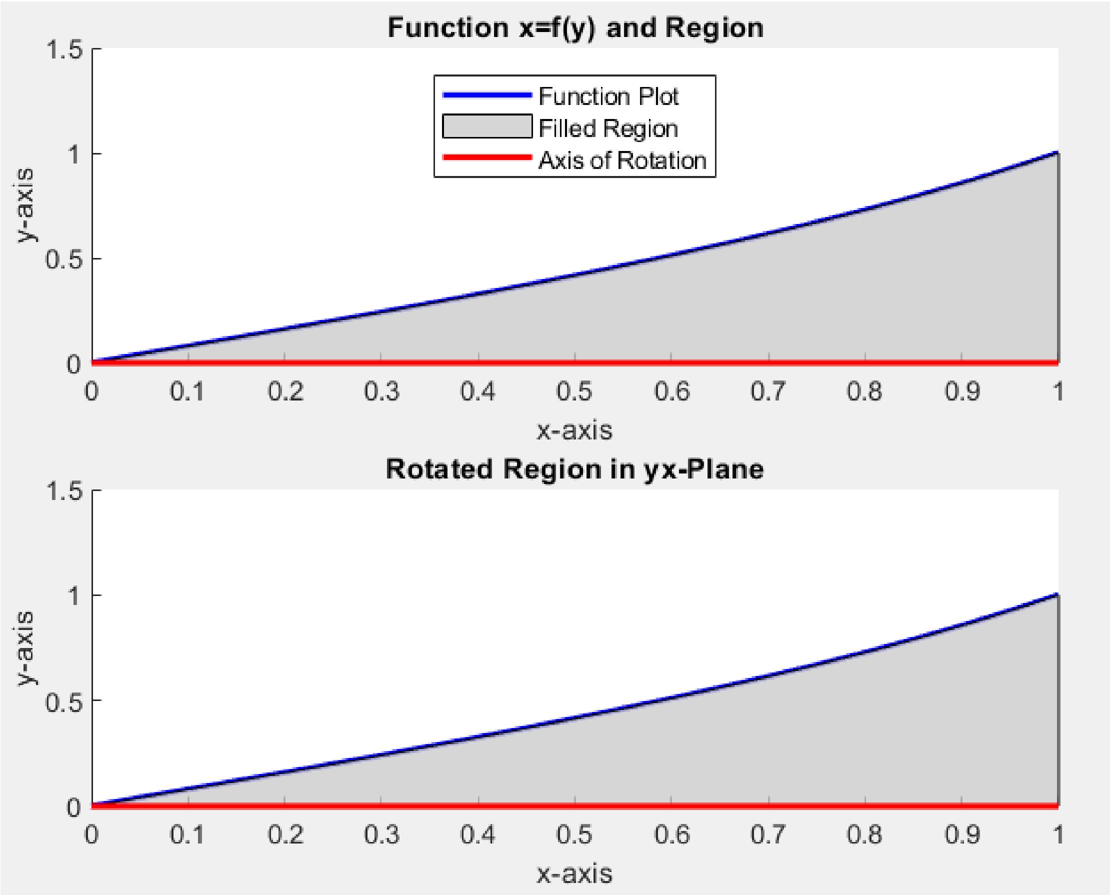
ylim =

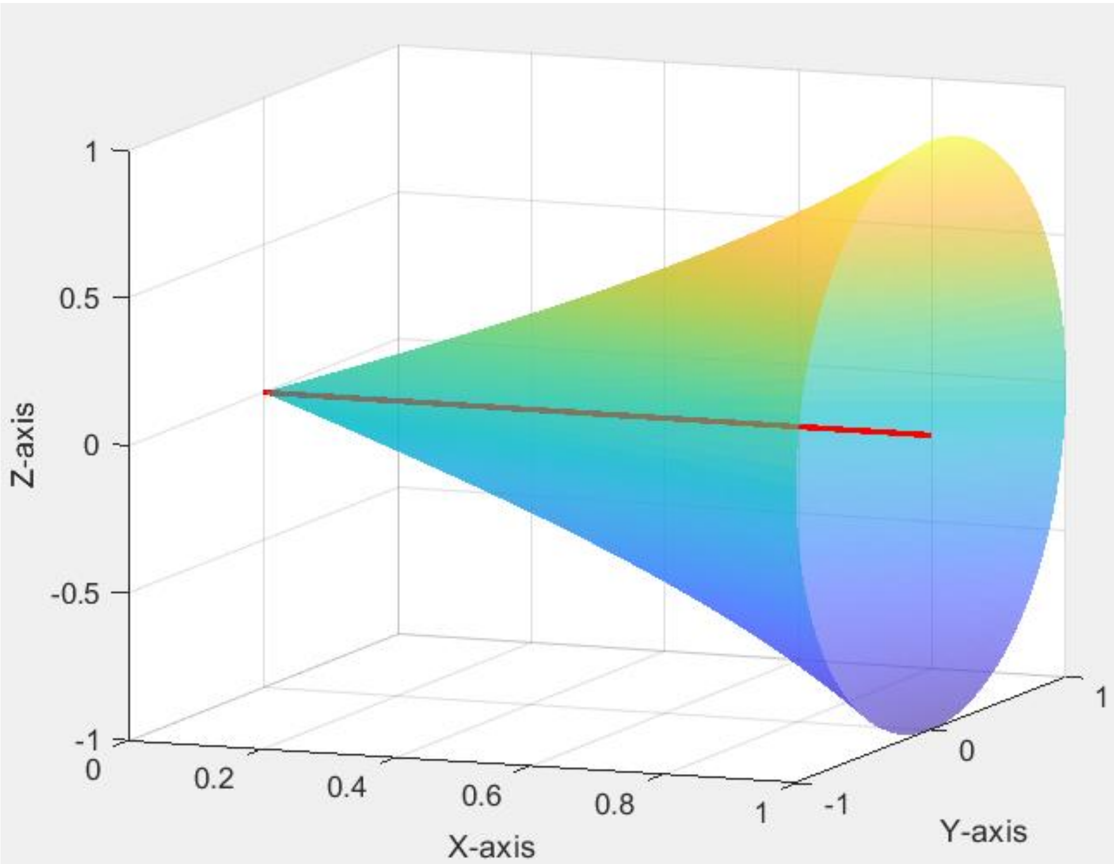
        0   1.5000

xlim =

        0   2.4142

Figure Output:



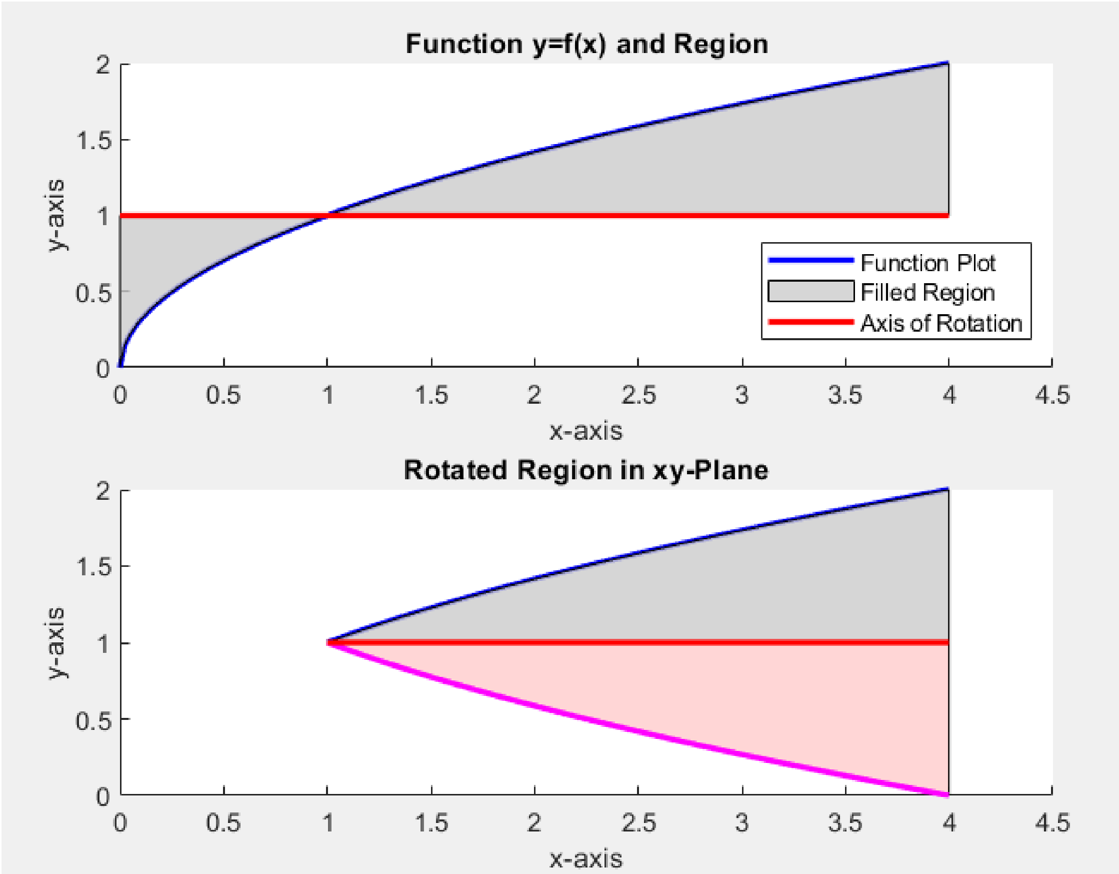


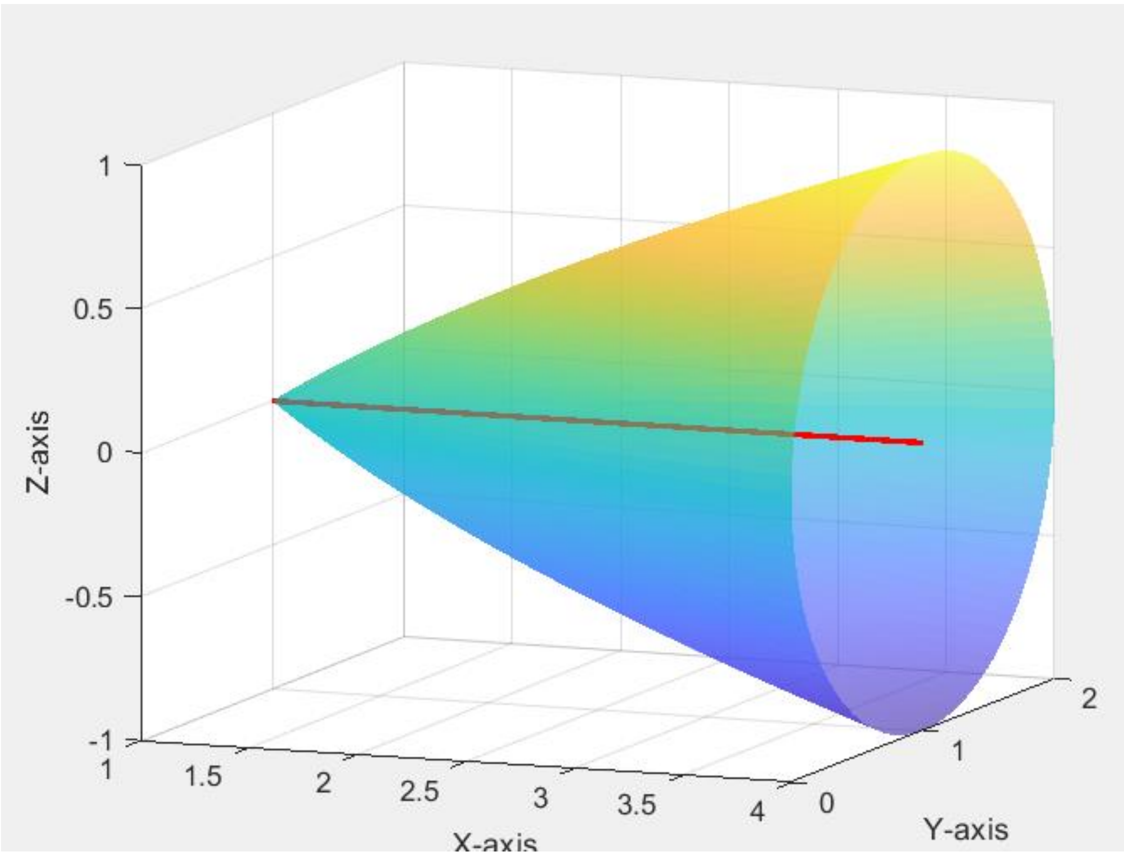
Output 4:

Numerical Output:

```
Enter the function: sqrt(x)

f =

x^(1/2)

Enter the interval on which the function is defined: [0,4]

fL =

   0    4

Enter the axis of rotation y = c(enter only c value): 1

yr =

   1

Enter the integration limits: [1,4]

iL =

   1    4

Volume is: 3.6652

fx =

   Inline function:

   fx(x) = x.^(1./2)

xlim =

       0    4.5000

ylim =

       0    2.1213
```

Figure Output:





Output 5:

Numerical Output:

```
Enter the function: sqrt(x)

f =

x^(1/2)

Enter the interval on which the function is defined: [0,4]

fL =

     0     4

Enter the axis of rotation y = c(enter only c value): 1

yr =

     1

Enter the integration limits: [0,4]

iL =

     0     4

Volume is: 4.1888

fx =

     Inline function:

     fx(x) = x.^(1./2)

xlim =

         0    4.5000

ylim =

         0    2.1213
```

Figure Output: