Title: Extremum and concavity of a single variable Function

Aim : To Find the maximum and minimum by using Second derivative test, visualize the curve with maximum point and the minimum point, visualization of first derivative test for local maxima and minima and visualization of concavity.

Questions:

1. Find local maxima and minima for $y = \frac{x^4}{4} - 2x^2 + 4$ and visualize the concavity.
2. An open-top box is to be made by cutting small congruent squares from the comers of a 12-in.-by-12-in. sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?
3. Find local maxima and minima for $y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3}$ and visualize the concavity
4. Find two positive numbers whose sum is 300 and whose product is maximum.

MATLAB Code:

```
% Extremum and concavity of a single variable function
clc
clear all
format compact
syms x real
f = input('Enter the function f(x):')
fx = diff(f,x)
c = solve(fx) % to get the critical points
cmin = min(double(c))
cmax = max(double(c))
figure(1)
ezplot(f,[cmin-2,cmax+2])
hold on
fxx=diff(fx,x)
for i=1:size(c)
T1=subs(fxx,x,c(i))
T3=subs(f,x,c(i))
if (double(T1)==0)
sprintf('The test fails at x=%d',double(c(i)))
else
if (double(T1)<0)
sprintf('The maximum point x is %d',double(c(i)))
sprintf('The value of the function is %d',double(T3))
else
sprintf('The minimum point x is %d',double(c(i)))
sprintf('The value of the function is %d',double(T3))
```

```
end
end
plot(double(c(i)), double(T3), 'r*', 'markersize', 10)
end
% plotting inflection points for testing concavity
de=polynomialDegree(fxx);
if(de==0)
sprintf('the given polynomial is second degree or less')
else
d = solve(fxx) % finding inflection points
for i = 1:1:size(d)
T2 = subs(f, x ,d(i) );
R1=sign(subs(fxx,x,d(i)+0.0001));
L1=sign(subs(fxx,x,d(i)-0.0001));
check=abs(L1-R1)
if (check==2)
sprintf('The point x=%d is a point of inflection',double (d(i)))
else
sprintf('The point x=%d is not a point of inflection',double (d(i)))
end
plot(double(d(i)), double(T2), 'g*', 'markersize', 15);
end
end
% Identifying maxima and minima through first derivative test
figure(2)
ezplot(fx,[cmin-2,cmax+2])
title('Plotting first derivative of f and critical points')
hold on
for i = 1:1:size(c)
T4 = subs(fx, x ,c(i) );
plot(double(c(i)), double(T4), 'r*', 'markersize', 15);
end
figure(3)
ezplot(fxx,[cmin-2,cmax+2])
hold on
if(de==0)
sprintf('the given polynomial is second degree or less, second derivative plot is not possible')
else
for i = 1:1:size(d)
T4 = subs(fxx, x ,d(i) );
plot(double(d(i)), double(T4), 'r*', 'markersize', 15);
end
title('Plotting second derivative of f and inflection points ')
end
```

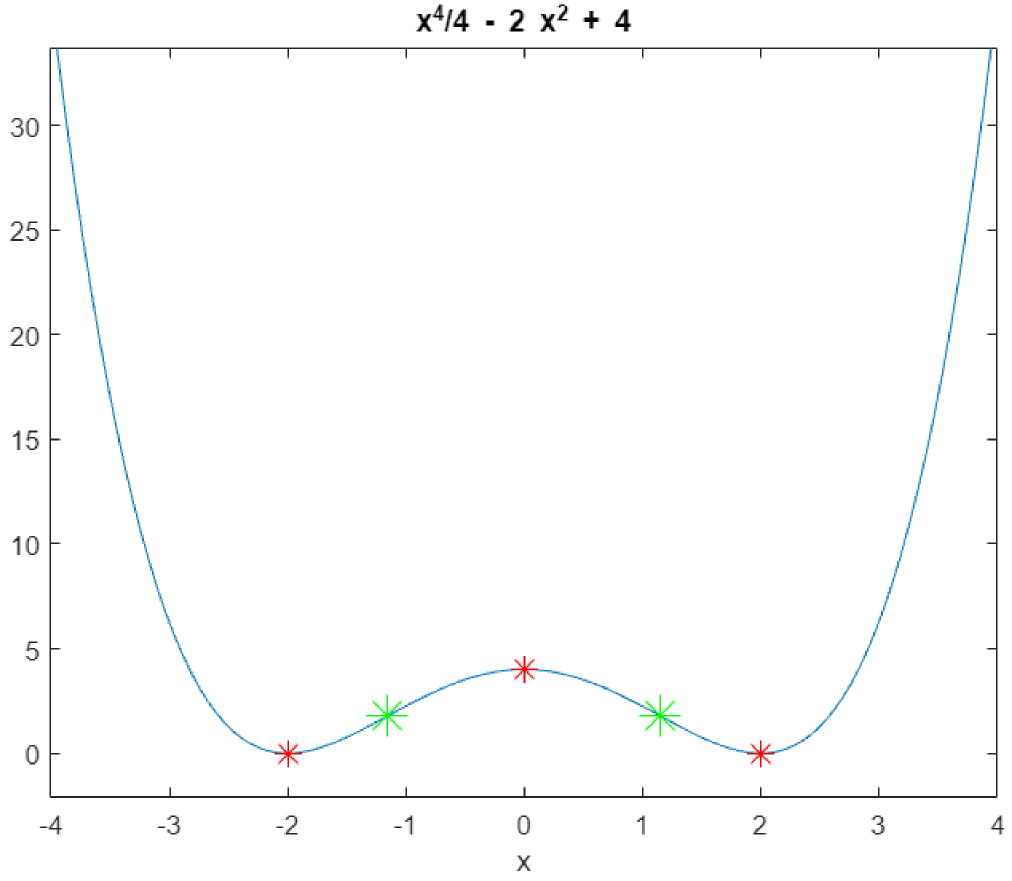
Output 1 :

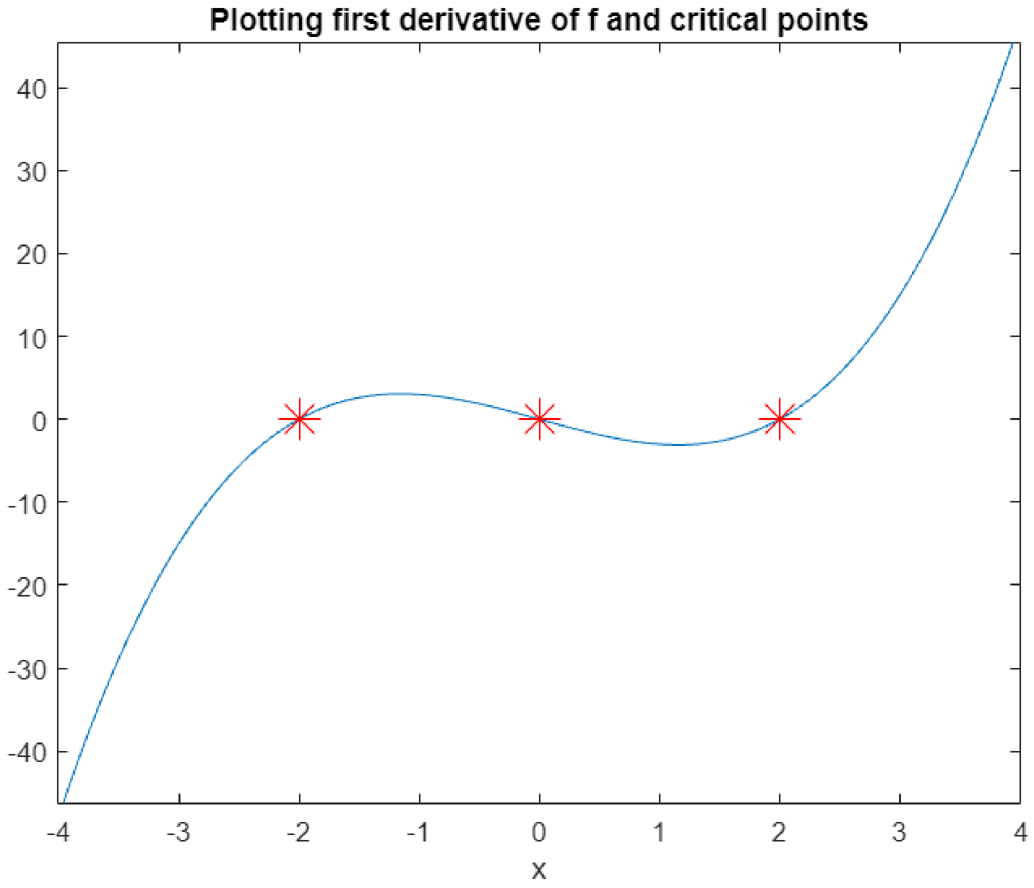
Numerical Output

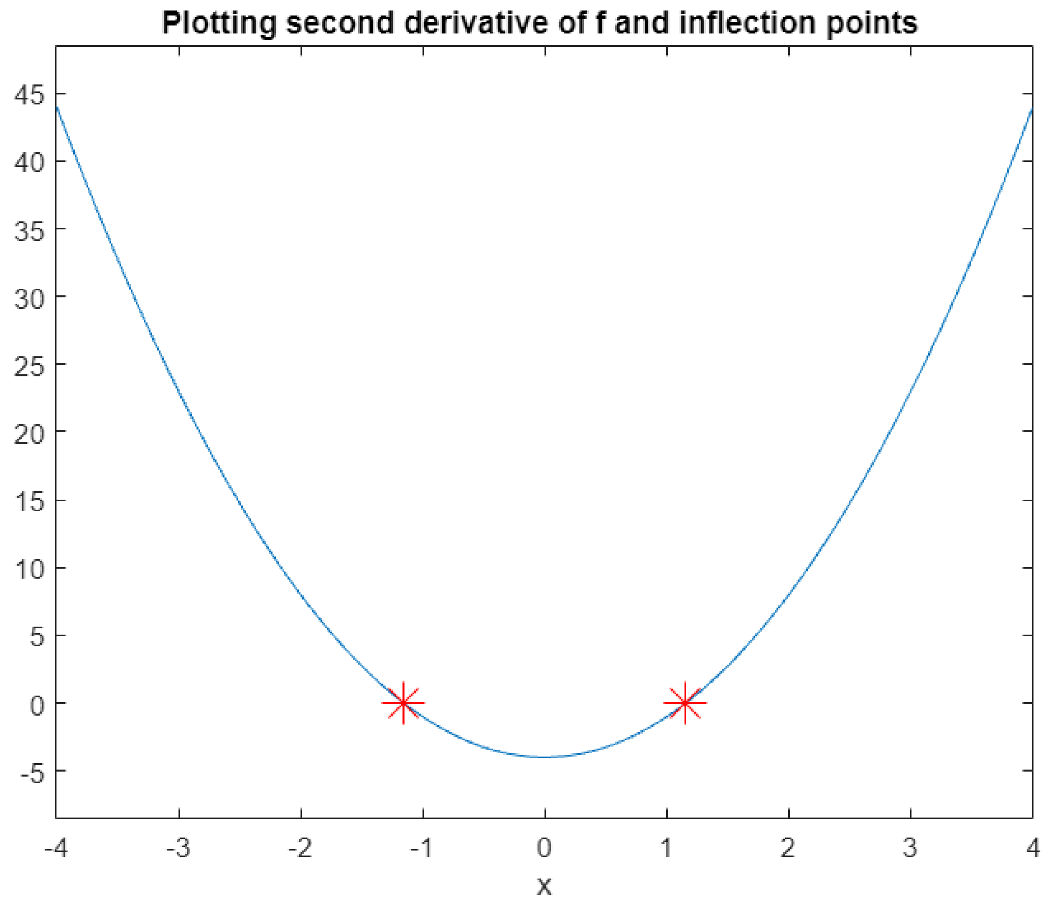
```
Enter the function f(x):
(x^4/4)-(2*x^2)+4

f =
x^4/4 - 2*x^2 + 4
fx =
x^3 - 4*x
c =
-2
 0
 2
cmin =
    -2
cmax =
     2
fxx =
3*x^2 - 4
T1 =
8
T3 =
0
ans =
    'The minimum point x is -2'
ans =
    'The value of the function is 0'
T1 =
-4
T3 =
4
ans =
    'The maximum point x is 0'
ans =
    'The value of the function is 4'
T1 =
8
T3 =
0
ans =
    'The minimum point x is 2'
ans =
    'The value of the function is 0'
d =
-(2*3^(1/2))/3
 (2*3^(1/2))/3
```

check =
2
ans =
    'The point x=-1.154701e+00 is a point of inflection'
check =
2
ans =
    'The point x=1.154701e+00 is a point of inflection'

Figure Output




Plotting first derivative of f and critical points
40
30
20
10
0
-10
-20
-30
-40
-4
-3
-2
-1
0
1
2
3
4
x

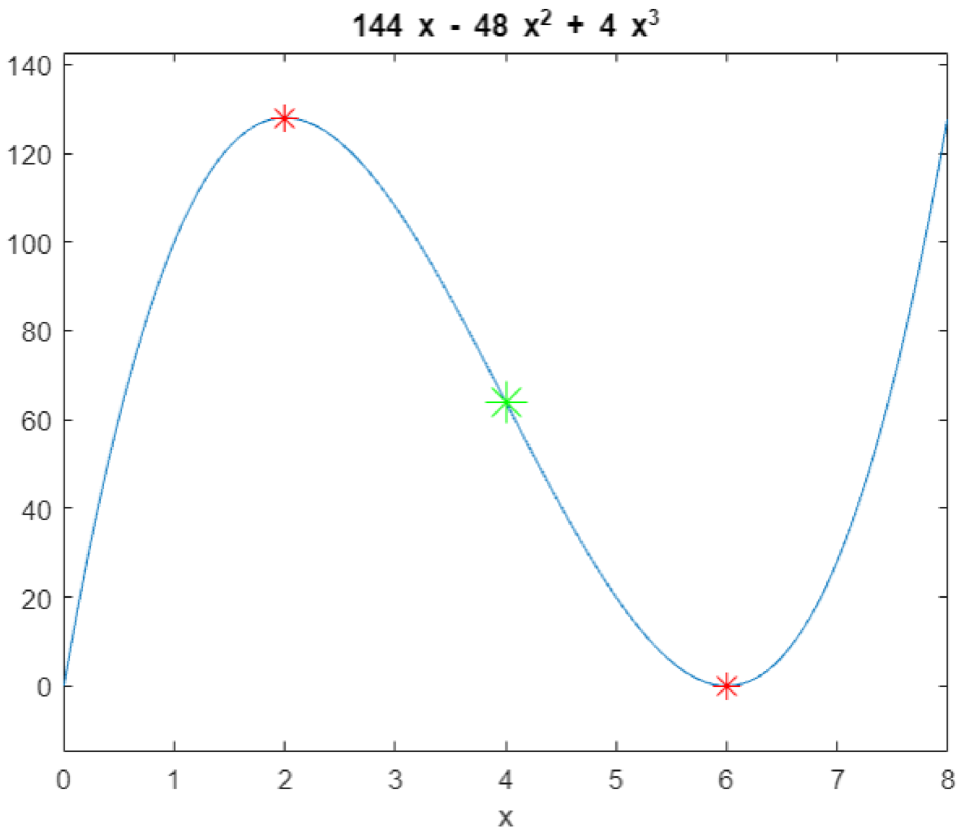

Plotting second derivative of f and inflection points
45
40
35
30
25
20
15
10
5
0
-5
-4
-3
-2
-1
0
1
2
3
4
x

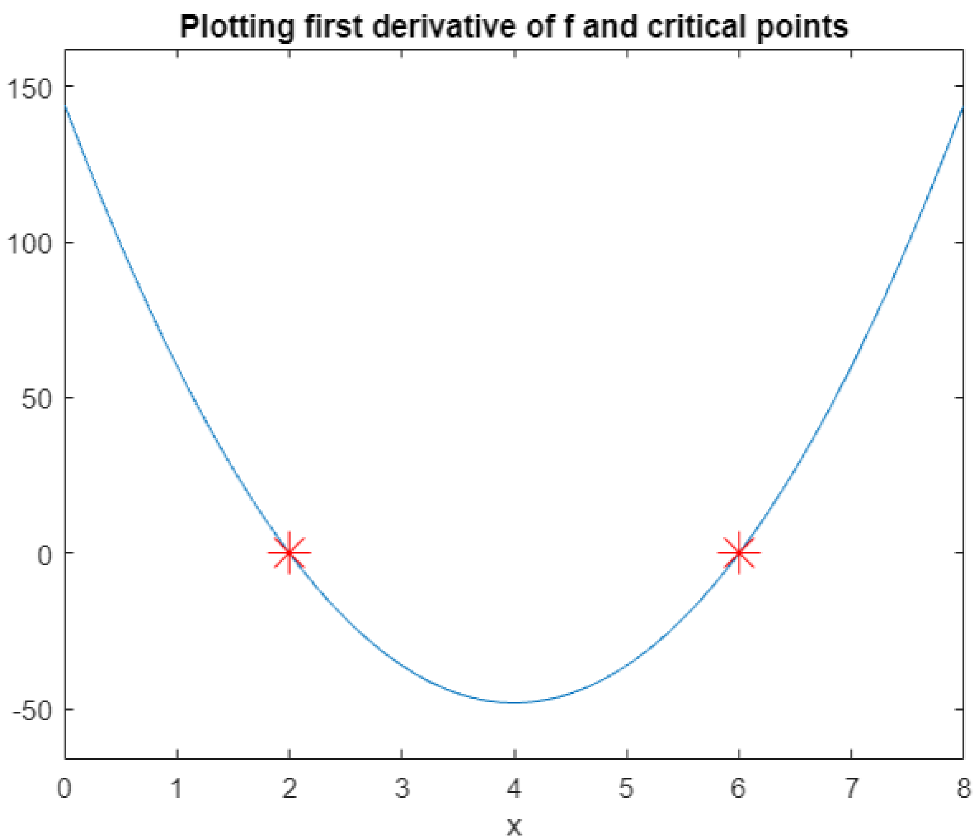
Output 2 :

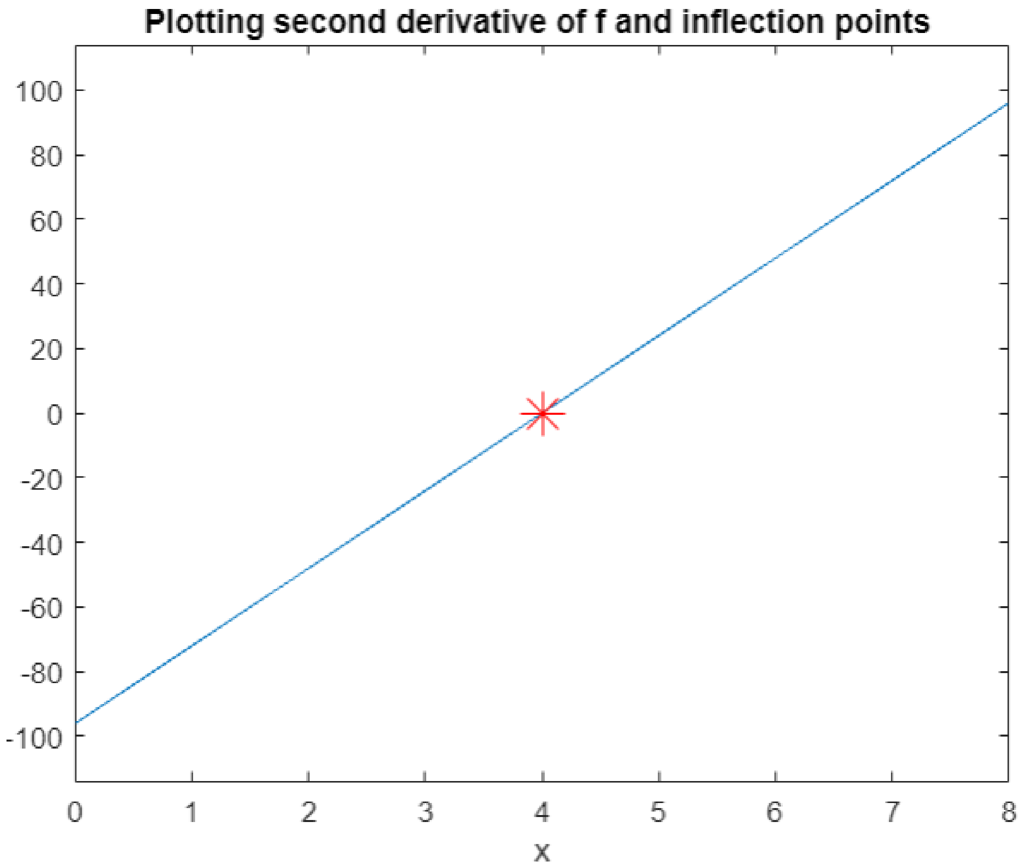
Numerical Output

```
Enter the function f(x):
(4*x^3)-(48*x^2)+144*x
f =
4*x^3 - 48*x^2 + 144*x
fx =
12*x^2 - 96*x + 144
c =
2
6
cmin =
     2
cmax =
     6
fxx =
24*x - 96
T1 =
-48
T3 =
128
ans =
    'The maximum point x is 2'
ans =
    'The value of the function is 128'
T1 =
48
T3 =
0
ans =
    'The minimum point x is 6'
ans =
    'The value of the function is 0'
d =
4
check =
2
ans =
    'The point x=4 is a point of inflection'
```

Figure Output







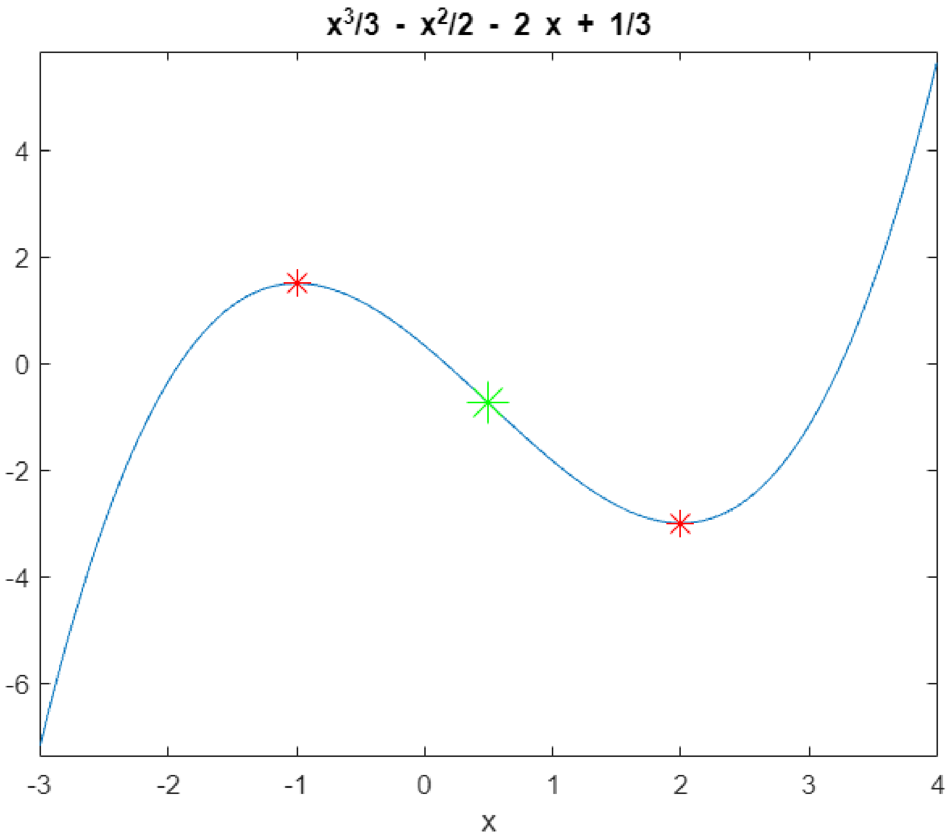
Output 3 :

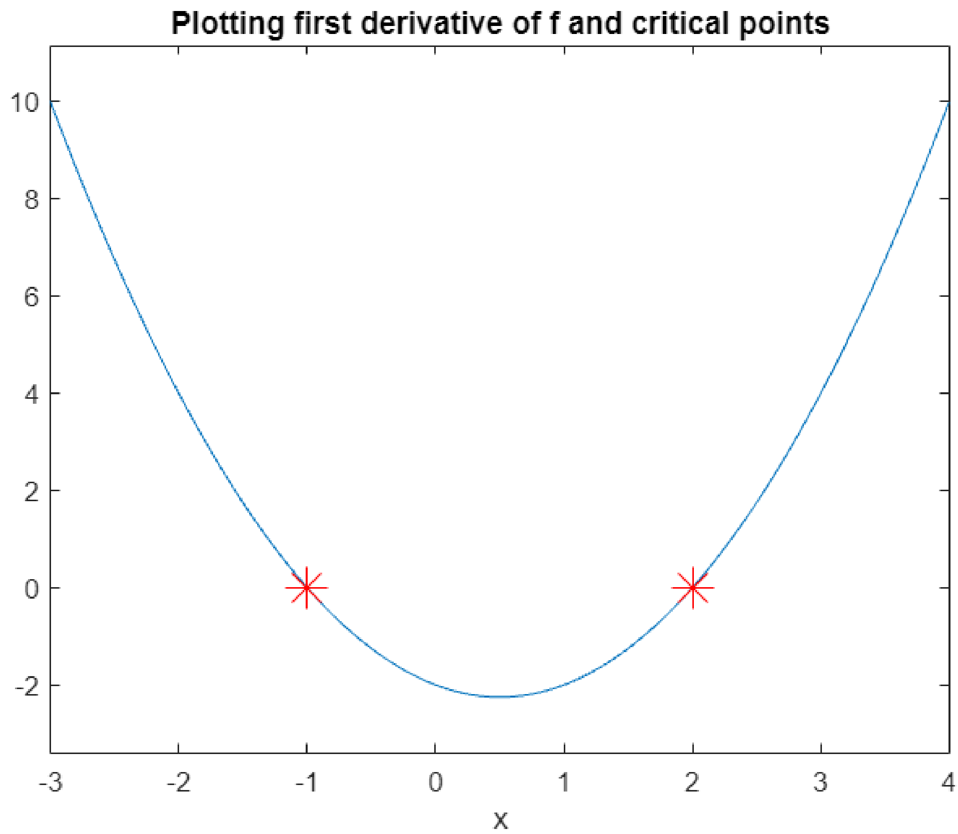
Numerical Output

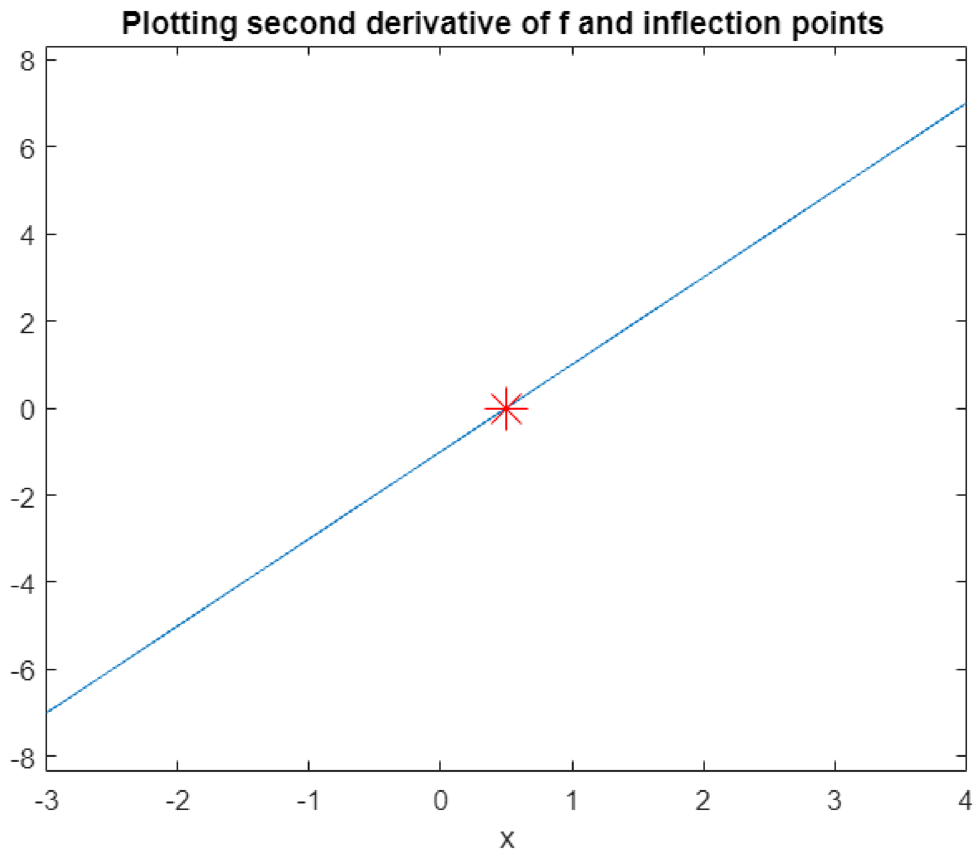
Enter the function f(x):
(x^3/3)-(x^2/2)-2*x+(1/3)
f =
x^3/3 - x^2/2 - 2*x + 1/3
fx =
x^2 - x - 2
c =
-1
 2
cmin =
   -1
cmax =
    2
fxx =
2*x - 1
T1 =
-3
T3 =
3/2
ans =
   'The maximum point x is -1'
ans =
   'The value of the function is 1.500000e+00'

```
T1 =
3
T3 =
-3
ans =
    'The minimum point x is 2'
ans =
    'The value of the function is -3'
d =
1/2
check =
2
ans =
    'The point x=5.000000e-01 is a point of inflection'
```

Figure Output




Plotting first derivative of f and critical points
10
8
6
4
2
0
-2
-3
-2
-1
0
1
2
3
4
x

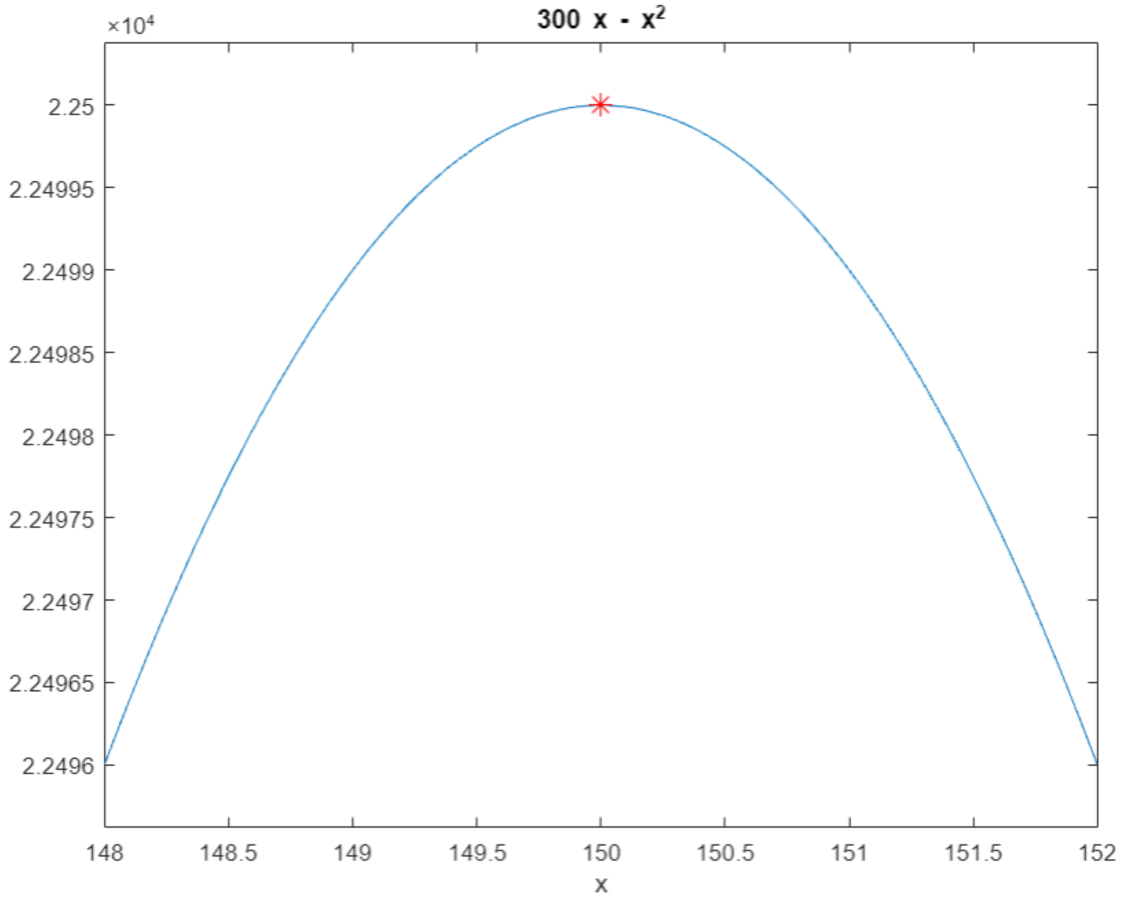

Plotting second derivative of f and inflection points
8
6
4
2
0
-2
-4
-6
-8
-3
-2
-1
0
1
2
3
4
x

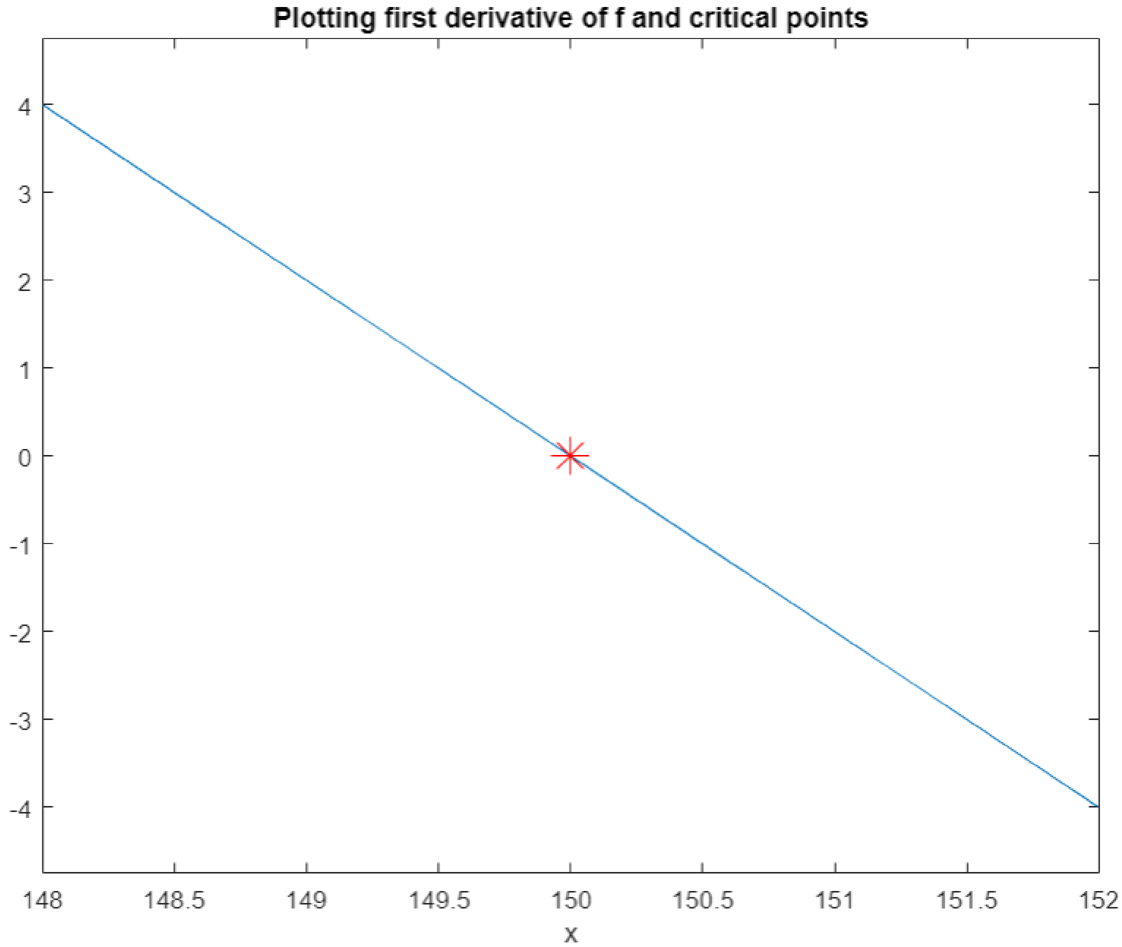
Output 4:

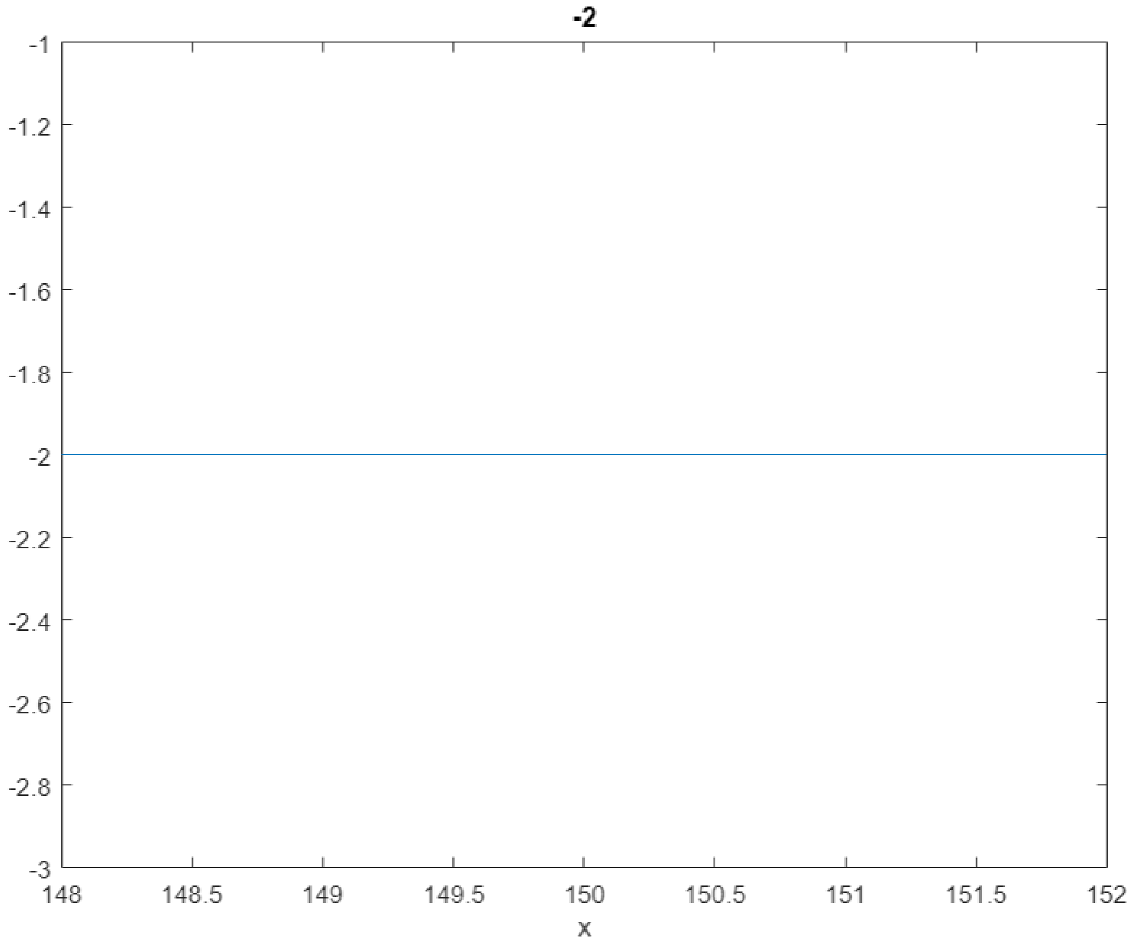
Numerical Output

```
Enter the function f(x):
300*x-x^2
f =
- x^2 + 300*x
fx =
300 - 2*x
c =
150
cmin =
   150
cmax =
   150
fxx =
-2
T1 =
-2
T3 =
22500
ans =
    'The maximum point x is 150'
ans =
    'The value of the function is 22500'
ans =
    'the given polynomial is second degree or less'
ans =
    'the given polynomial is second degree or less, second derivative plot is not possible'
```

Figure Output







-2
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-2.6
-2.8
-3
148
148.5
149
149.5
150
150.5
151
151.5
152
x