Title: Evaluating Triple integrals

Aim: To Evaluate the triple integrals within the given plane and curves.

Question:

Example Problems

1. Evaluate the iterated integral $\int_0^1 \int_0^z \int_0^{x+z} 6xz\, dy\, dx\, dz$
2. Evaluate the triple integral $\iiint_E 6xy\, dV$, where E lies under the plane z =1+x+y and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, y=0 and x=1.
3. Evaluate the triple integral $\iiint_E y\, dV$, where E is bounded by the planes x =0, y = 0, z=0, and 2x+2y+z = 4.

Converting Rectangular to cylindrical or spherical coordinates:

4. A solid E lies within the cylinder $x^2 + y^2 = 1$, below the plane z =4, and above the paraboloid $z = 1 - x^2 - y^2$. The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.
5. Use Matlab to draw the solid enclosed by the paraboloids $z = x^2 + y^2$ and $z = 5 - x^2 - y^2$
6. Draw a sphere of radius 5 with centre at (0,0,0)
7. Draw a hemisphere of radius 3 with centre at (0,0,0)
8. Evaluate $\iiint_E e^{\sqrt{x^2+y^2+z^2}}\, dV$, where E is enclosed by the sphere $x^2 + y^2 + z^2 = 9$ in the first octant.
9. Evaluate $\iiint_E z\, dV$, where E is enclosed by the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

Exercise problems:

10. Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy – plane, and below the cone $z = \sqrt{x^2 + y^2}$.
11. Sketch the solid whose volume is given by the integral and evaluate the integral $\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_1^2 \rho^2 \sin(\varphi)\, d\rho d\varphi d\theta$
12. Evaluate $\iiint_E \sqrt{x^2 + y^2}\, dV$, where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

EXAMPLE PROBLEMS

Question 1:

MATLAB CODE

```
clc
clear all
syms x y z
sol = int(int(int(6*x*z,y,0,x+z),x,0,z),z,0,1)
```

Output

```
sol =
1
```

Question 2:

MATLAB CODE

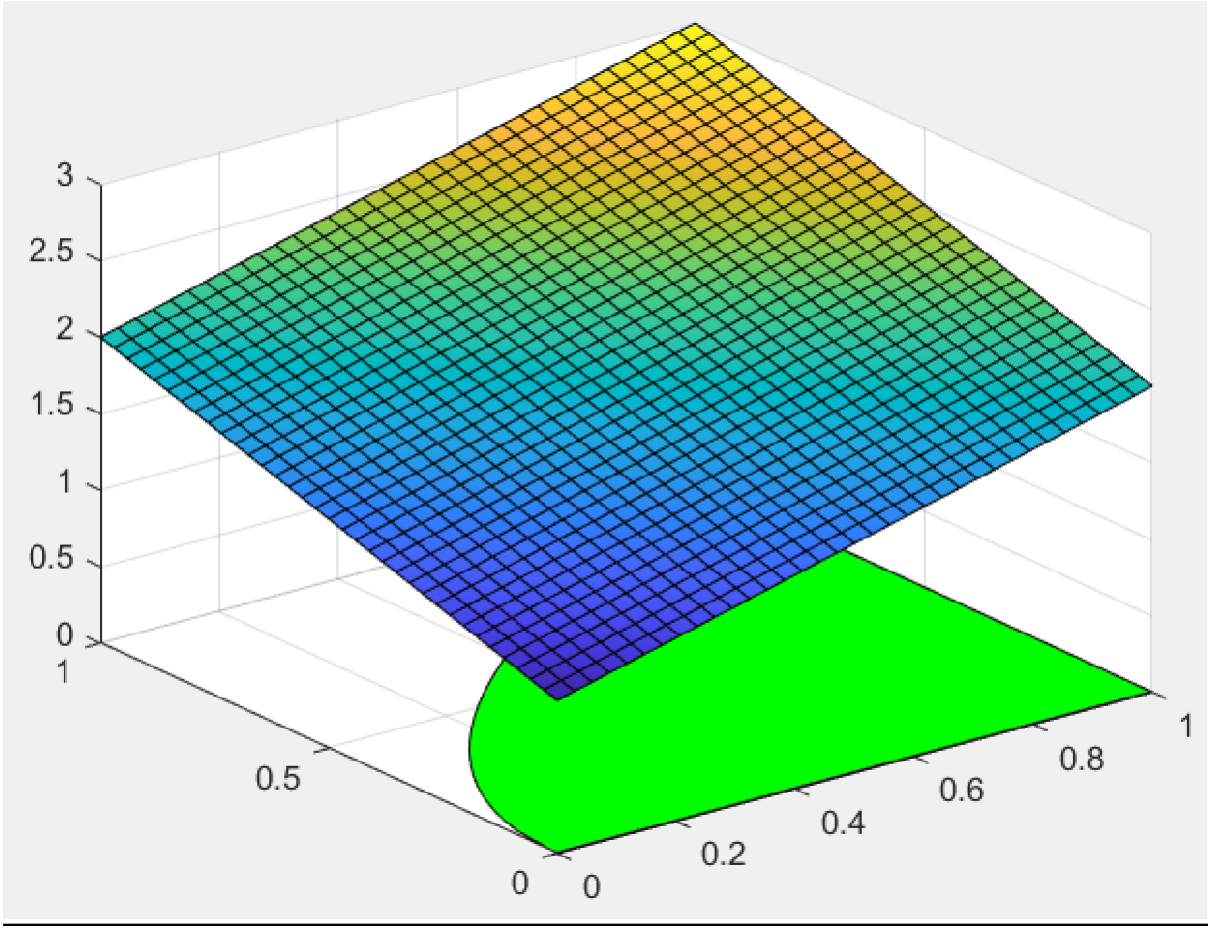
```
clc
clear all
syms x y z
sol = int(int(int(6*x*y,z,0,1+x+y),y,0,sqrt(x)),x,0,1)
z=fsurf(1+x+y,[0 1 0 1])
hold on
y1=sqrt(x)
y2= 0
x1  =  linspace(0,1);
yy1  =subs(y1,x,x1);
yy2  =  subs(y2,x,x1);
x1 = [x1,fliplr(x1)];
yy = [yy1,fliplr(yy2)];
fill(x1,yy,'g')
```

Output

```
sol =
65/28
z =
  FunctionSurface with properties:

     Function: x + y + 1
       XRange: [0 1]
       YRange: [0 1]
    EdgeColor: [0 0 0]
    LineStyle: '-'
    FaceColor: 'interp'

  Show all properties
y1 =
x^(1/2)
y2 =
     0
```



Question 3:

MATLAB CODE

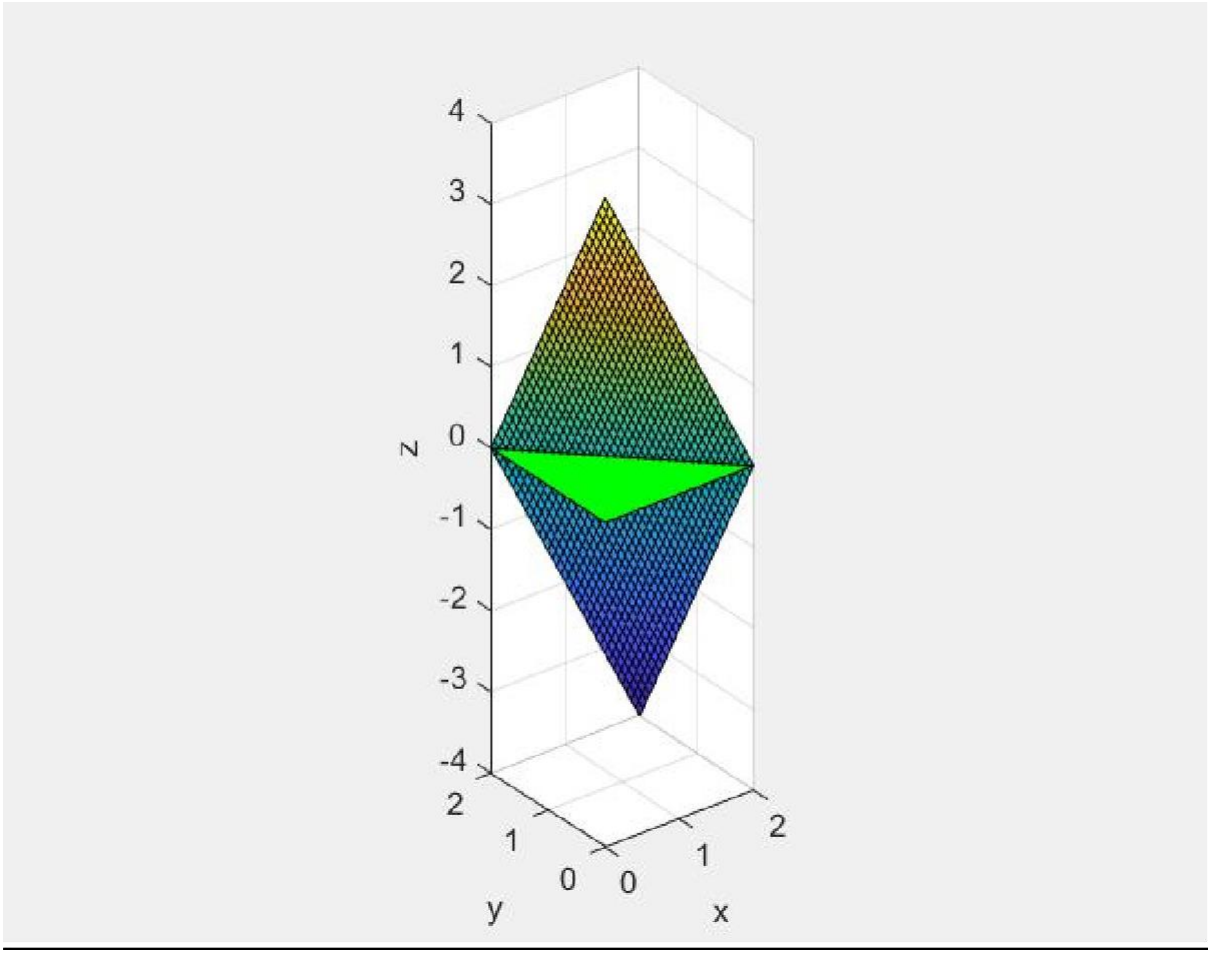
```
clc
clear all
syms x y z
sol = int(int(int(y,y,0,(4-z-2*x)/2),z,0,4-2*x),x,0,2)
z=fsurf(4-2*x-2*y,[0 2 0 2])
hold on
y1=2-x;
y2=0;
x1  =  linspace(0,2);
yy1  =subs(y1,x,x1);
yy2  =  subs(y2,x,x1);
x1 = [x1,fliplr(x1)];
yy = [yy1,fliplr(yy2)];
fill(x1,yy,'g')
hold off
axis equal
xlabel('x')
ylabel('y')
zlabel('z')
```

Output

```
sol =
4/3
z =
  FunctionSurface with properties:

     Function: 4 - 2*y - 2*x
       XRange: [0 2]
       YRange: [0 2]
    EdgeColor: [0 0 0]
    LineStyle: '-'
    FaceColor: 'interp'

  Show all properties
```



Question 4:

MATLAB CODE

```
clc
clear all
syms r z theta K
Ma= int(int(int((K*r)*r, z, 1-r^2,4), r ,0, 1),theta,0,2*pi) % integration
x = r*cos(theta),
y = r*sin(theta),
s = sym(4)
fsurf(x,y,1-r^2, [0 1 0 2*pi], 'g', 'EdgeColor', 'none'); % plotting paraboloid
hold on
fsurf(1*cos(theta), 1*sin(theta), r, 'y', [0 4 0 2*pi], 'EdgeColor', 'none')
%plotting % cylinder of radius 1 with height z = 4
fsurf(x,y,s, [0 1 0 2*pi], 'k', 'EdgeColor', 'none'); % plotting circular plane
z=4.
hold  on
axis equal;
xlabel('x');
ylabel('y');
zlabel('z');
alpha 0.5
```

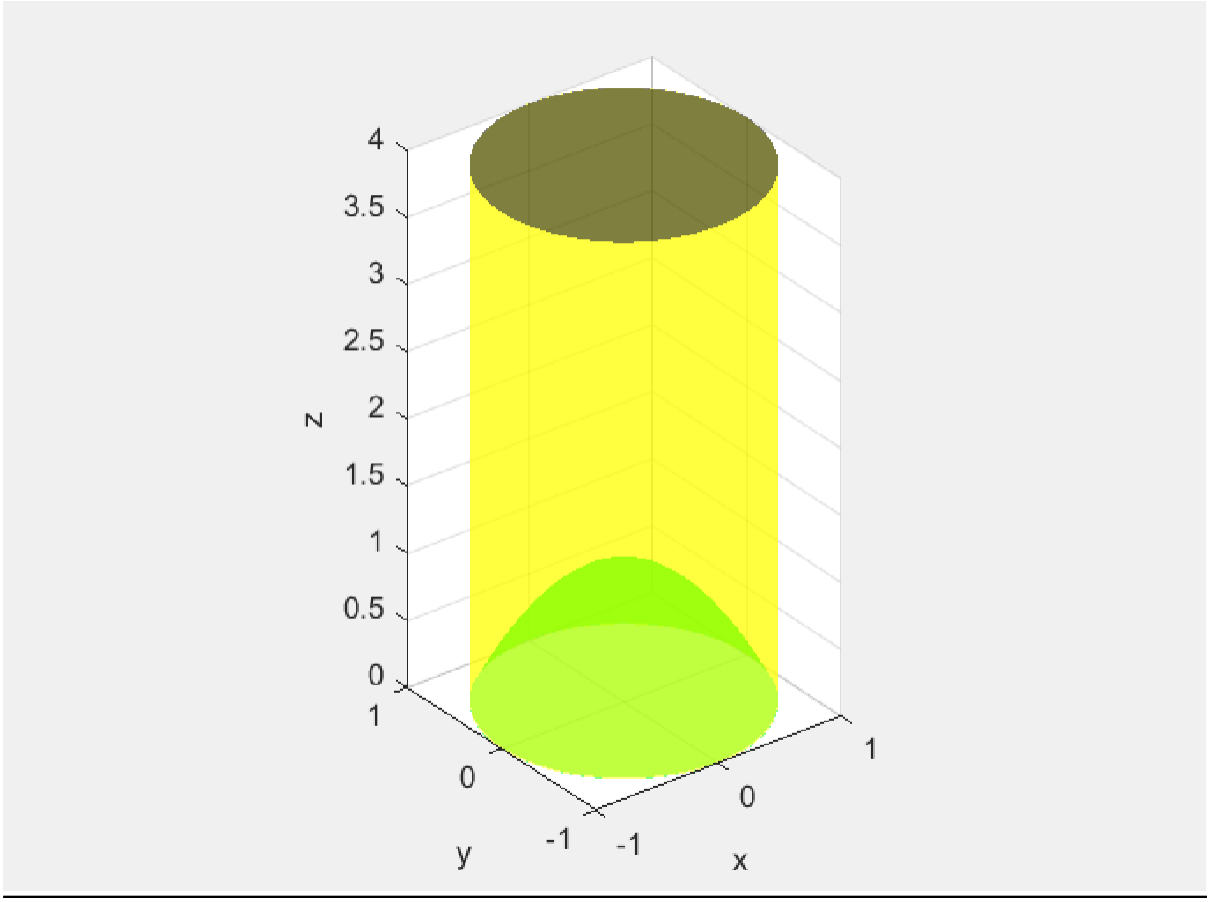
Output

Ma =
(12*pi*K)/5

x =
r*cos(theta)

y =
r*sin(theta)

s =
4

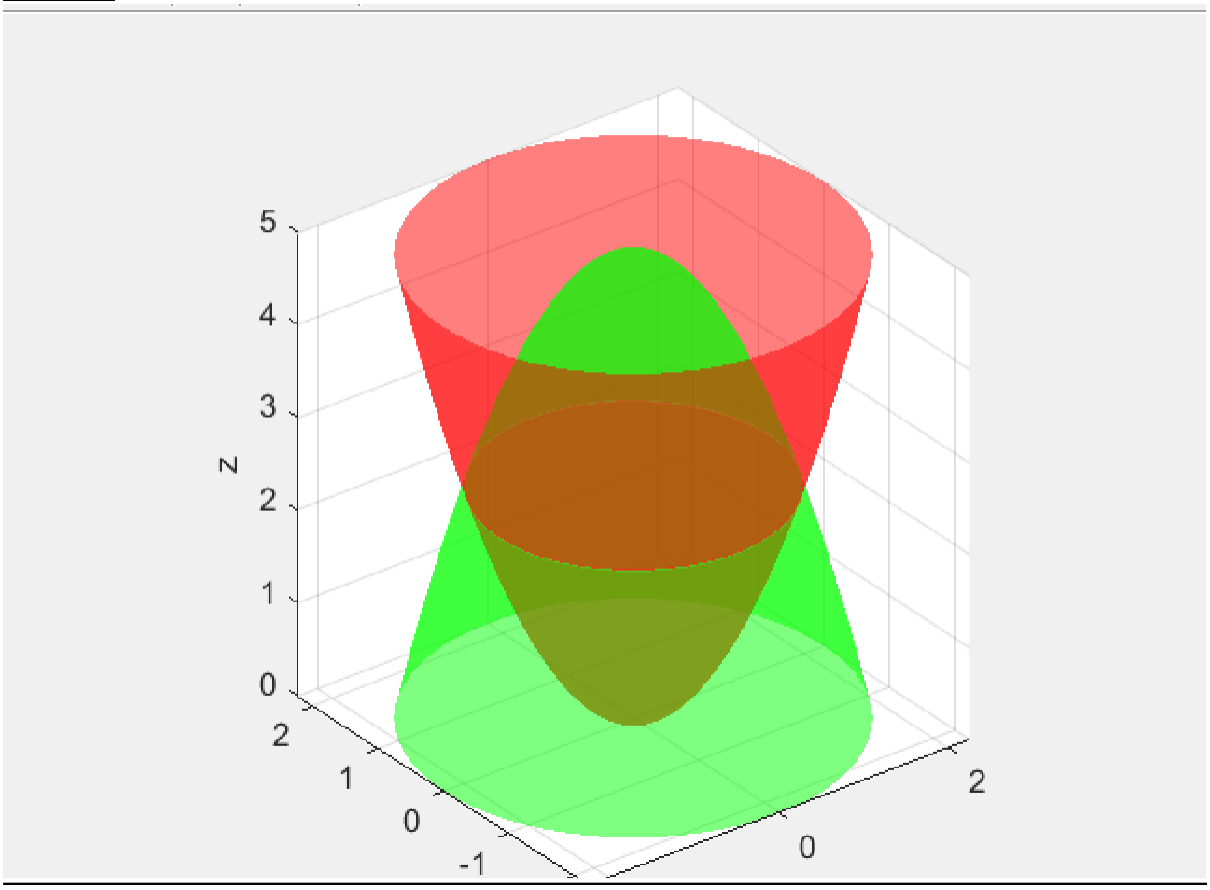

4
3.5
3
2.5
2
1.5
1
0.5
0
z
1
0
-1
y
-1
0
1
x

Question 5:

MATLAB CODE

```
clc
clear all
syms r z theta
x = r*cos(theta); y = r*sin(theta);
fsurf(x,y,5-r^2,[0 sqrt(5) 0 2*pi], 'g', 'EdgeColor', 'none');
hold on
fsurf(x,y,r^2, [0 sqrt(5) 0 2*pi], 'r', 'EdgeColor', 'none');
axis equal; xlabel('x'); ylabel('y'); zlabel('z');
alpha 0.5
```

Output

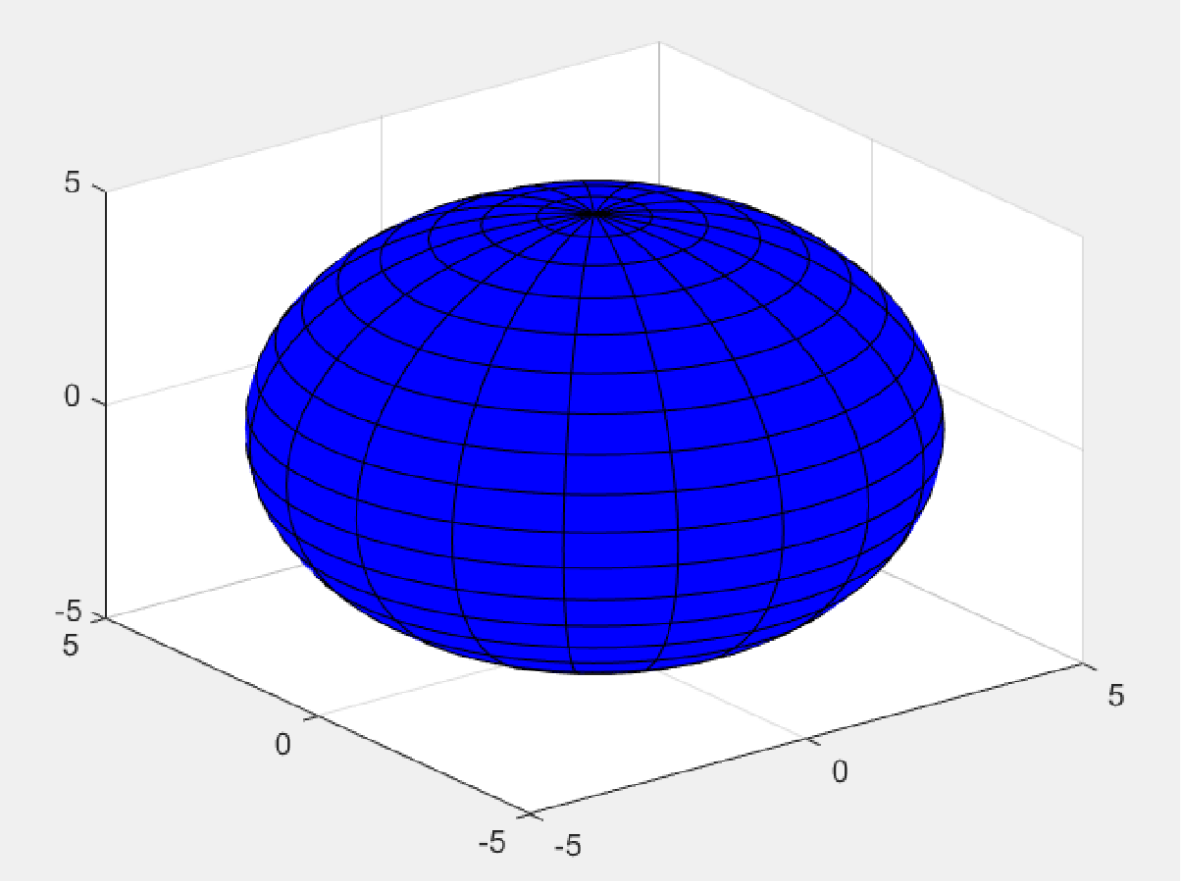


Question 6:

MATLAB CODE

```
clc
clear all
syms r z phi rho theta
rho=5;
x= rho*sin(phi)*cos(theta);
y = rho*sin(phi)*sin(theta);
z= rho*cos(phi) ;
fsurf(x,y,z, [0 pi 0 2*pi], 'b', 'MeshDensity', 20);
```

Output

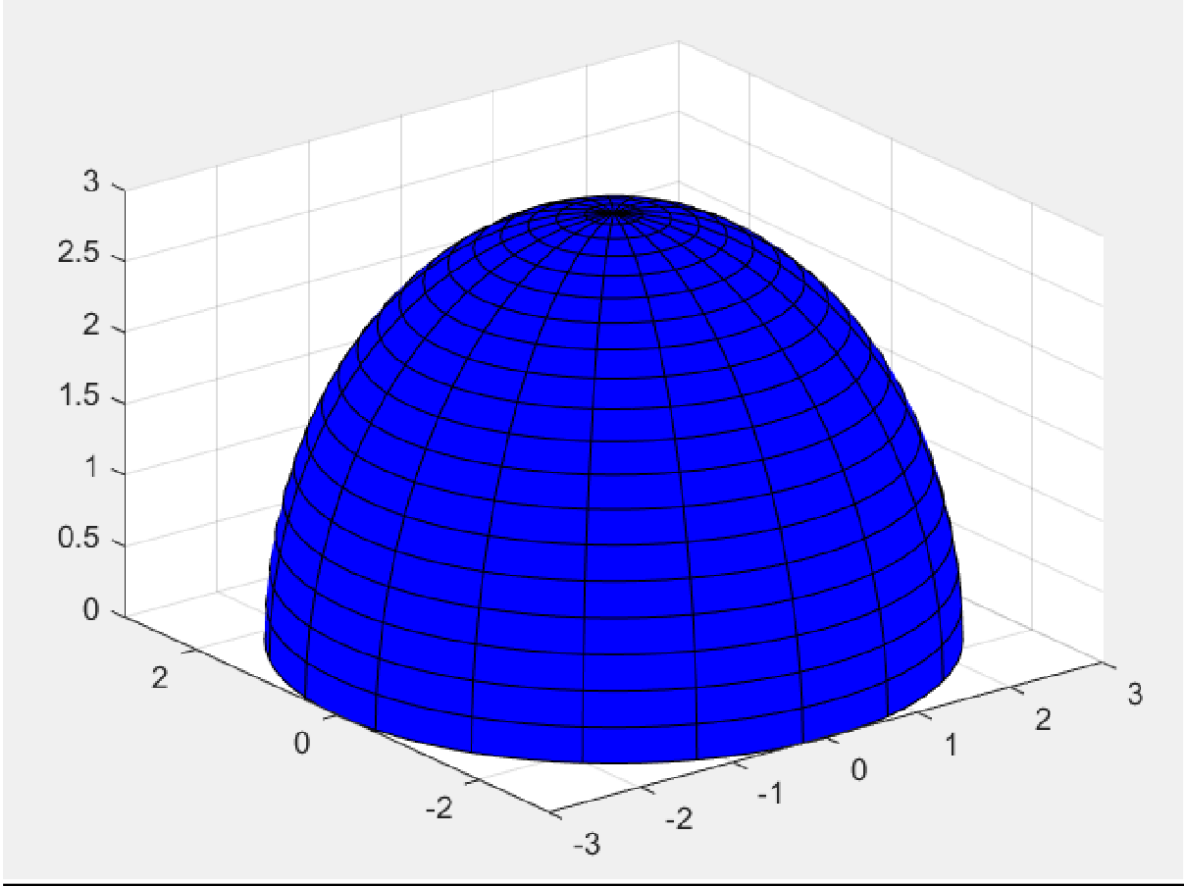


Question 7:

MATLAB CODE

```
clc
clear all
syms r z phi rho theta
rho=3;
x= rho*sin(phi)*cos(theta);
y = rho*sin(phi)*sin(theta);
z= rho*cos(phi) ;
fsurf(x,y,z, [0 pi/2 0 2*pi], 'b', 'MeshDensity', 20);
```

Output



Question 8:

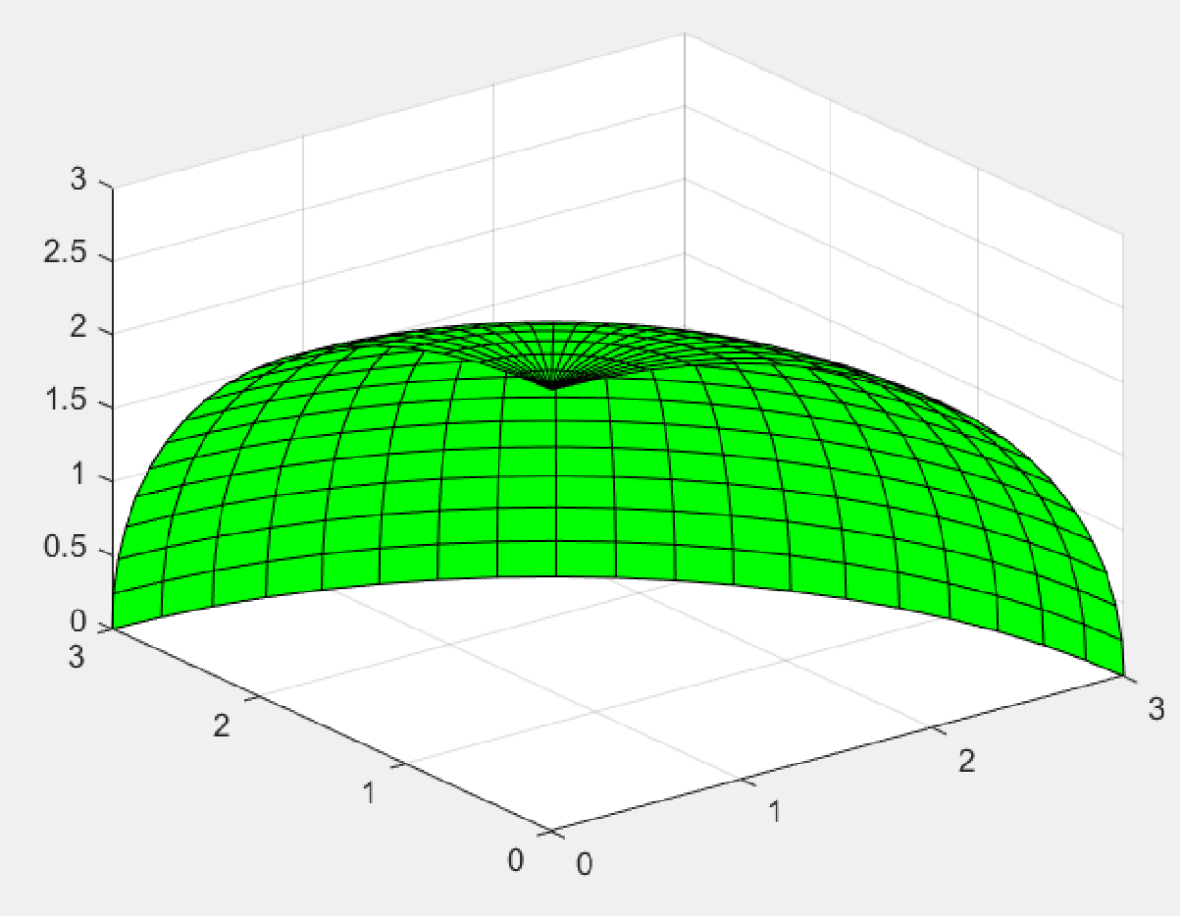
MATLAB CODE

```
clc
clear all
syms r phi rho theta
Sol=int(int(int((exp(rho))*(rho)^2*sin(phi), rho,0,3), phi ,0, pi/2),theta,0,pi/2)
rho=3;
x = rho*sin(phi)*cos(theta);
y = rho*sin(phi)*sin(theta);
z = rho*cos(phi) ;
fsurf(x,y,z, [0 pi/2 0 pi/2], 'g', 'MeshDensity', 20);
```

Output

Sol =

(pi*(5*exp(3) - 2))/2



Question 9:

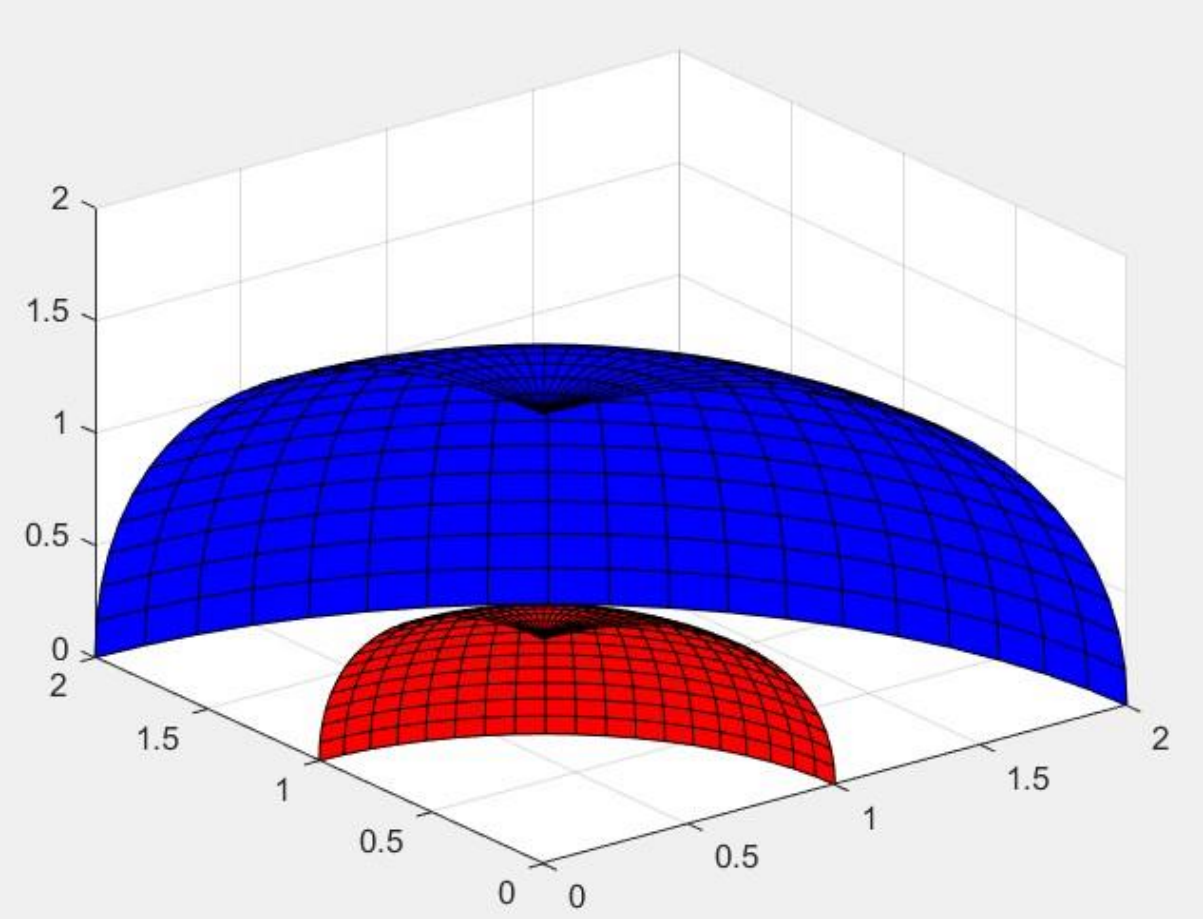
MATLAB CODE

```
clc
clear all
syms r phi rho theta
Sol=int(int(int((rho*cos(phi))*(rho)^2*sin(phi), rho,1,2), phi ,0,
pi/2),theta,0,pi/2)
rho=1;
x = rho*sin(phi)*cos(theta);
y = rho*sin(phi)*sin(theta);
z = rho*cos(phi) ;
fsurf(x,y,z, [0 pi/2 0 pi/2], 'r', 'MeshDensity', 20);
hold on
rho=2;
x = rho*sin(phi)*cos(theta);
y = rho*sin(phi)*sin(theta);
z = rho*cos(phi) ;
fsurf(x,y,z, [0 pi/2 0 pi/2], 'b', 'MeshDensity', 20);
hold off
```

Output

Sol =

(15*pi)/16



PRACTICE PROBLES:

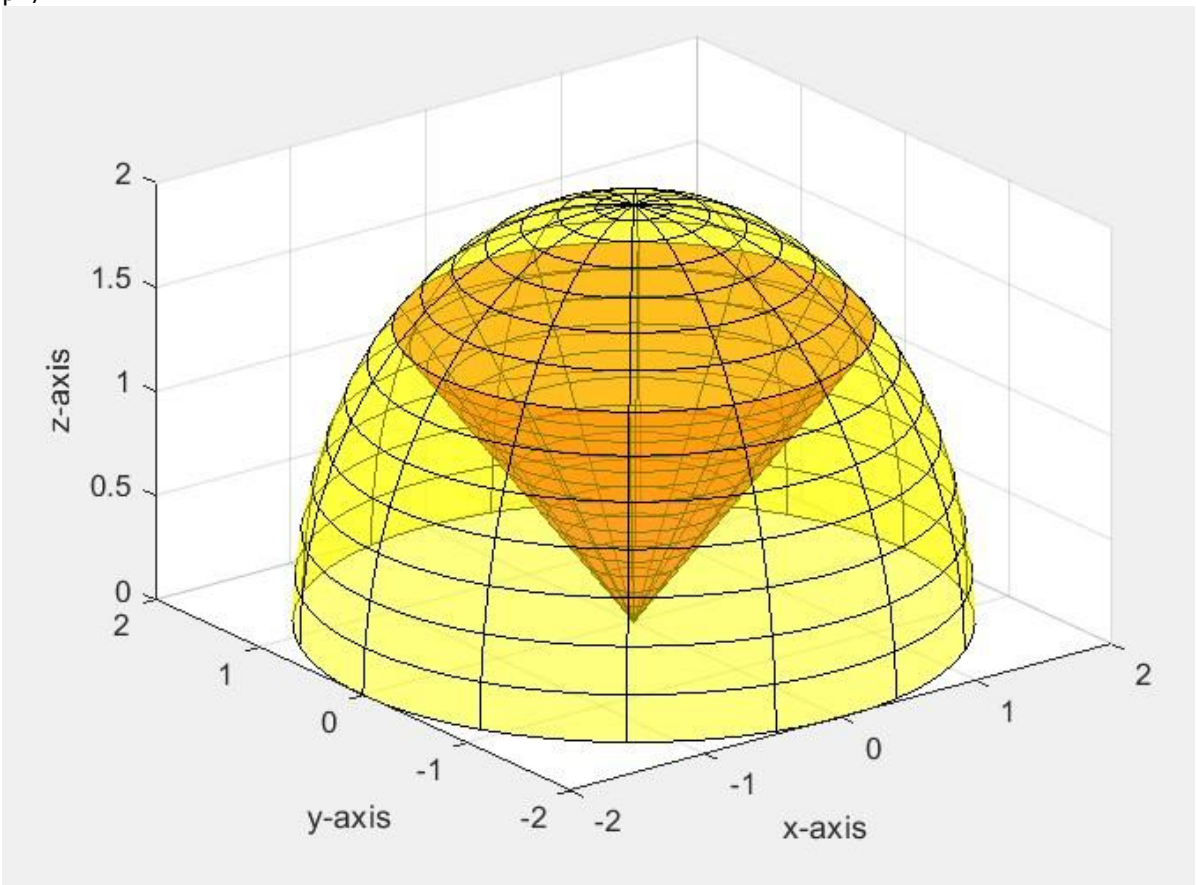
Question 10:

MATLAB CODE

```
clc
clear all
format compact
syms phi rho theta t beta
vol= int(int(int((rho)^2*sin(phi), rho, 0,cos(phi)), phi, pi/4,pi/2), theta,
0,2*pi)
fsurf(t*cos(beta),t*sin(beta),t,[0 2*pi 0 sqrt(2)], 'r', 'MeshDensity',15)
hold on
rho = 2;
x = rho*sin(phi)*cos(theta);
y = rho*sin(phi)*sin(theta);
z = rho*cos(phi);
xlabel('x-axis')
ylabel('y-axis')
zlabel('z-axis')
fsurf(x,y,z, [0 pi/2 0 2*pi], 'y', 'MeshDensity', 15);
alpha 0.5
```

Output

```
vol =
pi/24
```



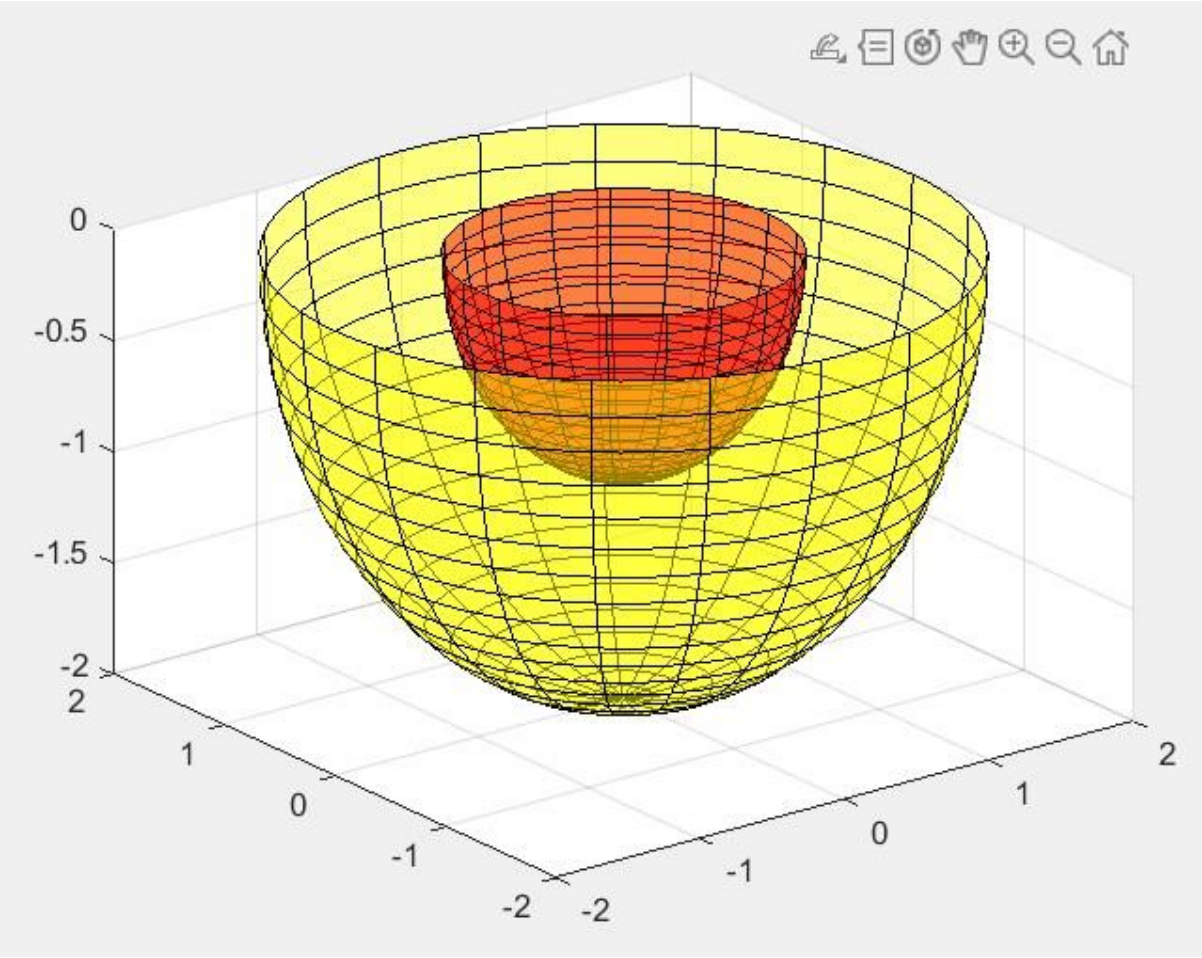
Question 11:

MATLAB CODE

```
clc
clear all
format compact
syms rho phi theta
vol=int(int(int(rho^2*sin(phi),rho,1,2),phi,pi/2,pi),theta,0,2*pi)
rho=1;
x = rho*sin(phi)*cos(theta);
y = rho*sin(phi)*sin(theta);
z = rho*cos(phi) ;
fsurf(x,y,z, [pi/2 pi 0 2*pi], 'r', 'MeshDensity', 20);
hold on
rho=2;
x = rho*sin(phi)*cos(theta);
y = rho*sin(phi)*sin(theta);
z = rho*cos(phi) ;
fsurf(x,y,z, [pi/2 pi 0 2*pi], 'y', 'MeshDensity', 20);
hold off
alpha 0.5
```

Output

```
vol =
(14*pi)/3
```



Question 12:

MATLAB CODE

```
clc
clear all
format compact
syms r theta x y z
vol=int(int(int(r^2, z, -5,4), r, 0,4), theta, 0,2*pi)
fsurf(r*cos(theta),r*sin(theta),r,[0 4 0 2*pi], 'r', 'MeshDensity',20)
hold on
r = 4;
x = r*cos(theta);
y = r*sin(theta);
fsurf(x,y,z, [0 2*pi 0 4], 'g ', 'MeshDensity', 20)
hold off
alpha 0.5
```

Output

```
vol =
384*pi
```