Aim: To study Lagrange's Multiplier Method for 2 variables.

Question:

1. Find the extreme values of the function $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.
2. Find the extreme values of the function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.
3. Find the extreme values of the function $f(x,y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.
4. **Maximum on a line:**
   Find the maximum value of the function $f(x,y) = 49 - x^2 - y^2$ takes on the line $x + 3y = 10$.
   (Function of three variables with one constraint)
5. **Minimum distance to the origin:**
   Find the points on the surface $z^2 = xy + 4$ closest to the origin.
6. Find the minimum value of x^2yz^3 subject to $2x + y + 3z = 3$.

MATLAB CODE FOR Q.1 TO Q4:

```
clc
clear all
format compact
syms x y lam real
f= input('Enter f(x,y) to be extremized : ')
g= input('Enter the constraint function g(x,y) : ')
F=f-lam*g
Fd=jacobian(F,[x y lam])
[ax,ay,alam]=solve(Fd,x,y,lam)
ax=double(ax); ay=double(ay);
T = subs(f,{x,y},{ax,ay}); T=double(T)
epxl=min(ax);
epxr=max(ax);
epyl=min(ay);
epyu=max(ay);
D=[epxl-1 epxr+1 epyl-5 epyu+5 ]
ezcontourf(f,D,400)
hold on
```

```
h = ezplot(g,D);
set(h,'Color',[1,0.7,0.9])
for i = 1:length(T);
    fprintf('The critical point (x,y) is(%1.3f,%1.3f).',ax(i),ay(i))
    fprintf('The value of the function is %1.3f\n',T(i))
    plot(ax(i),ay(i),'r*','markersize',25)
end
TT=sort(T)
f_min=TT(1)
f_max=TT(end)
```

<u>Output 1:</u>

Changes in Code:

```
D=[epxl-0.5 epxr+0.5 epyl-0.5 epyu+0.5 ]
ezcontourf(f,D,400)
```

<u>Numerical Output</u>

Enter f(x,y) to be extremized : x^2+2*y^2

f =

x^2 + 2*y^2

Enter the constraint function g(x,y) : x^2+y^2-1

g =

x^2 + y^2 - 1

F =

x^2 + 2*y^2 - lam*(x^2 + y^2 - 1)

Fd =

[2*x - 2*lam*x, 4*y - 2*lam*y, - x^2 - y^2 + 1]

ax =

-1

1

0

0

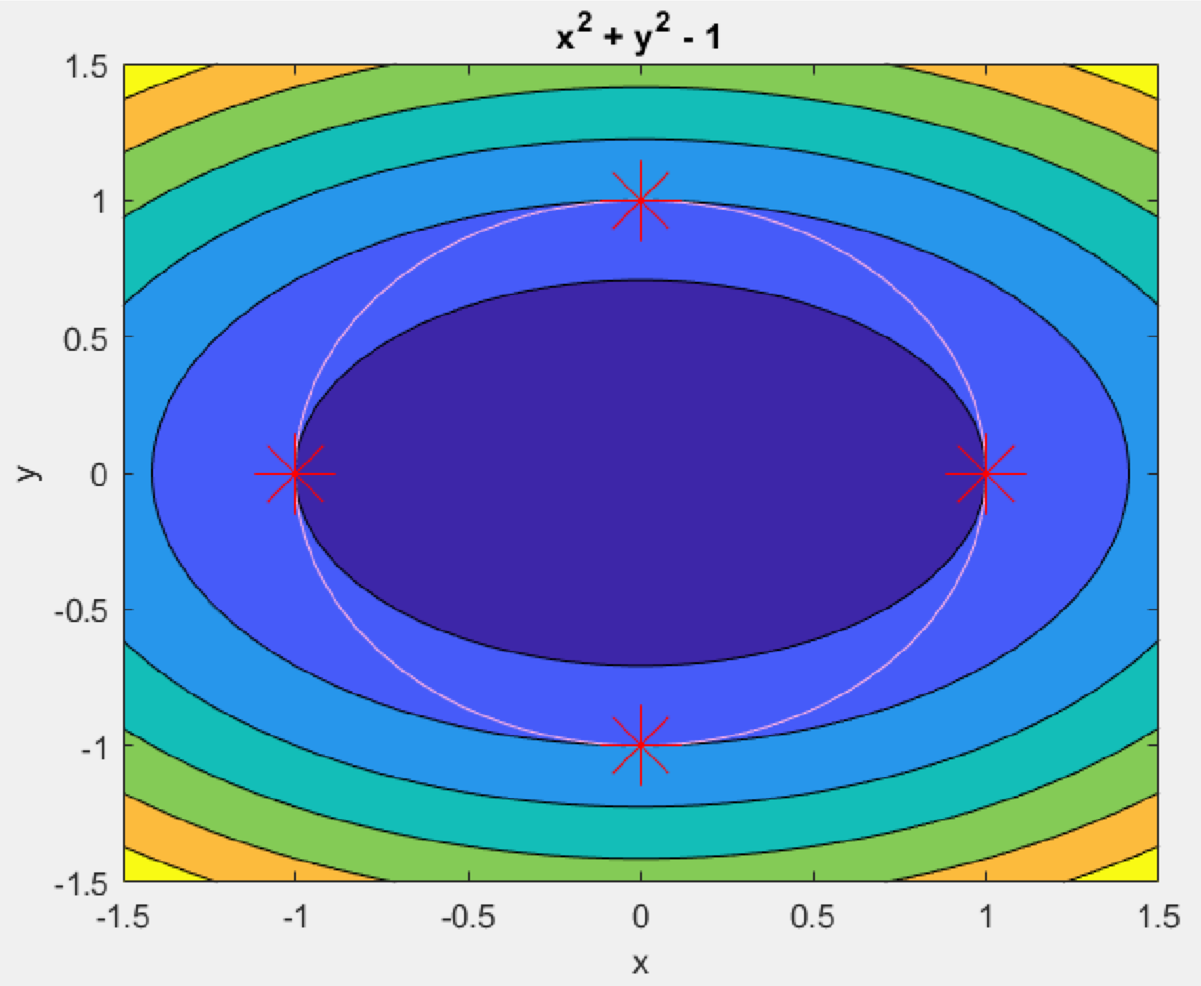
ay =

0

0

```
-1
 1
alam =
1
1
2
2
T =
     1
     1
     2
     2
D =
   -1.5000    1.5000   -1.5000    1.5000
The critical point (x,y) is (-1.000,0.000).The value of the function is 1.000
The critical point (x,y) is (1.000,0.000).The value of the function is 1.000
The critical point (x,y) is (0.000,-1.000).The value of the function is 2.000
The critical point (x,y) is (0.000,1.000).The value of the function is 2.000
TT =
     1
     1
     2
     2
f_min =
      1
f_max =
      2
```

Figure Output



Output 2:

Numerical Output

```
Enter f(x,y) to be extremized : 3*x+4*y

f =

3*x + 4*y

Enter the constraint function g(x,y) : x^2+y^2-1

g =

x^2 + y^2 - 1

F =

3*x + 4*y - lam*(x^2 + y^2 - 1)

Fd =

[3 - 2*lam*x, 4 - 2*lam*y, - x^2 - y^2 + 1]

ax =

-3/5

 3/5

ay =

-4/5

 4/5

alam =

-5/2

 5/2

T =

   -5

    5

D =

  -1.6000    1.6000   -5.8000    5.8000

The critical point (x,y) is (-0.600,-0.800).The value of the function is -5.000

The critical point (x,y) is (0.600,0.800).The value of the function is 5.000

TT =

   -5
```

5

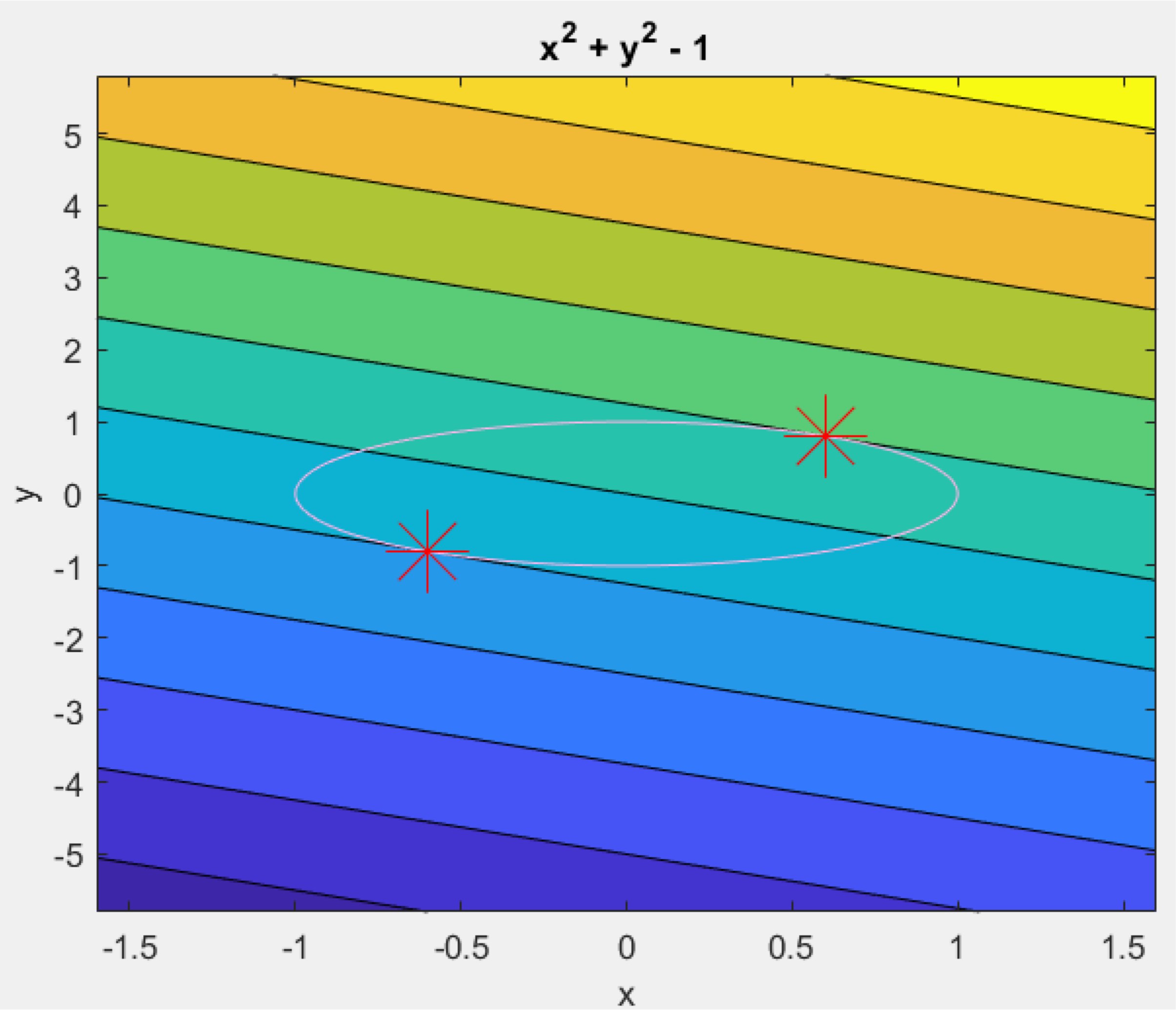
f_min =

-5

f_max =

5

Figure Output



Output 3:

Changes in Code:

```
D=[epxl-0.5 epxr+0.5 epyl-3 epyu+3 ]
ezcontourf(f,D,600)
```

Numerical Output

Enter f(x,y) to be extremized : x*y

f =

x*y

Enter the constraint function g(x,y) : x^2/8+y^2/2-1

g =

x^2/8 + y^2/2 - 1

F =

x*y - lam*(x^2/8 + y^2/2 - 1)

Fd =

[y - (lam*x)/4, x - lam*y, - x^2/8 - y^2/2 + 1]

ax =

2

-2

-2

2

ay =

-1

1

-1

1

alam =

-2

-2

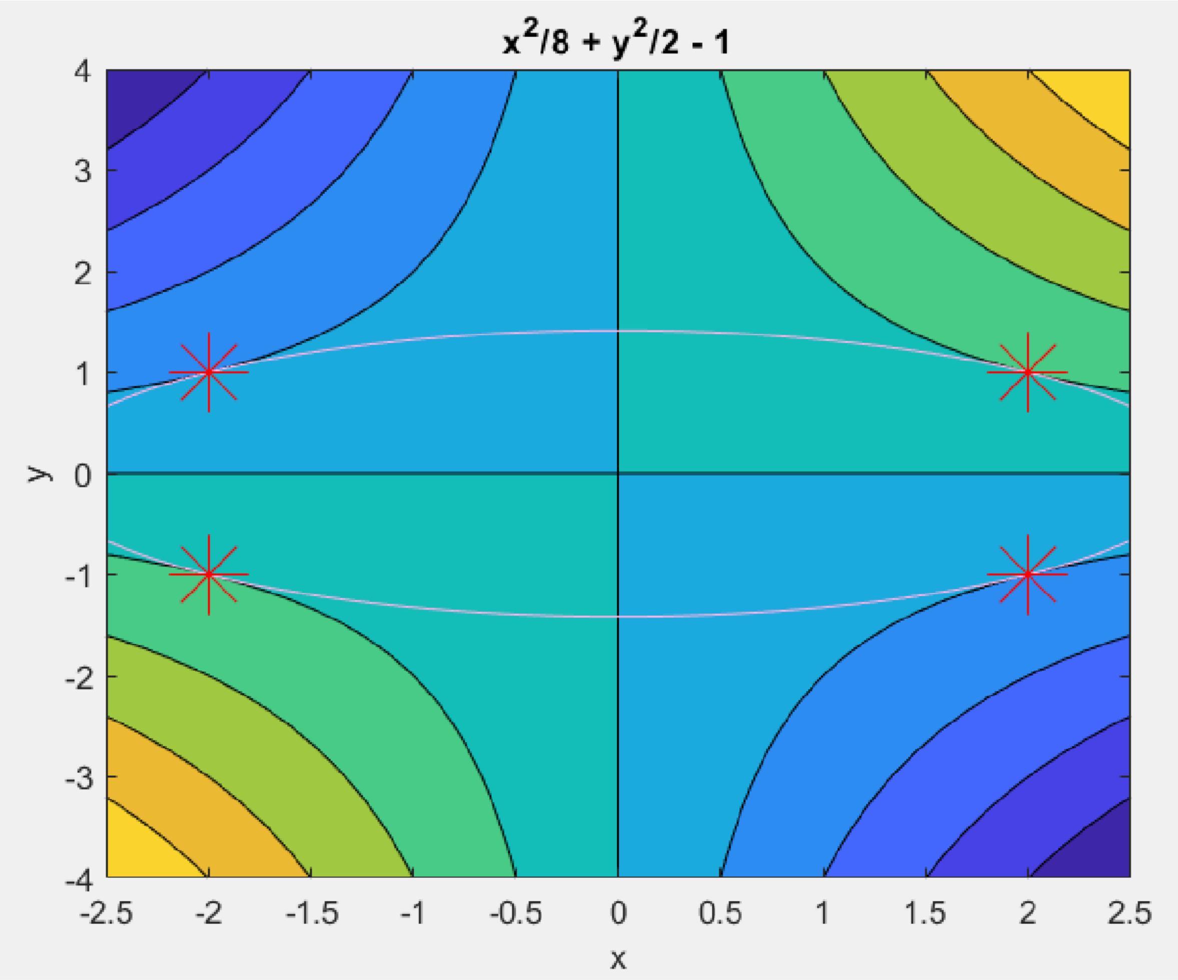
2

2

T =

```
    -2
    -2
     2
     2
D =
   -2.5000    2.5000   -4.0000    4.0000
The critical point (x,y) is (2.000,-1.000).The value of the function is -2.000
The critical point (x,y) is (-2.000,1.000).The value of the function is -2.000
The critical point (x,y) is (-2.000,-1.000).The value of the function is 2.000
The critical point (x,y) is (2.000,1.000).The value of the function is 2.000
TT =
    -2
    -2
     2
     2
f_min =
    -2
f_max =
     2
```

Figure Output



Output 4:

Changes in Code:

```
D=[epxl-1 epxr+1 epyl-3 epyu+3 ]
ezcontourf(f,D,600)
```

Numerical Output

Enter f(x,y) to be extremized : 49-x^2-y^2

f =

- x^2 - y^2 + 49

Enter the constraint function g(x,y) : x+3*y-10

g =

x + 3*y - 10

F =

49 - x^2 - y^2 - lam*(x + 3*y - 10)

Fd =

[- lam - 2*x, - 3*lam - 2*y, 10 - 3*y - x]

ax =

1

ay =

3

alam =

-2

T =

39

D =

0 2 0 6

The critical point (x,y) is (1.000,3.000).The value of the function is 39.000

TT =

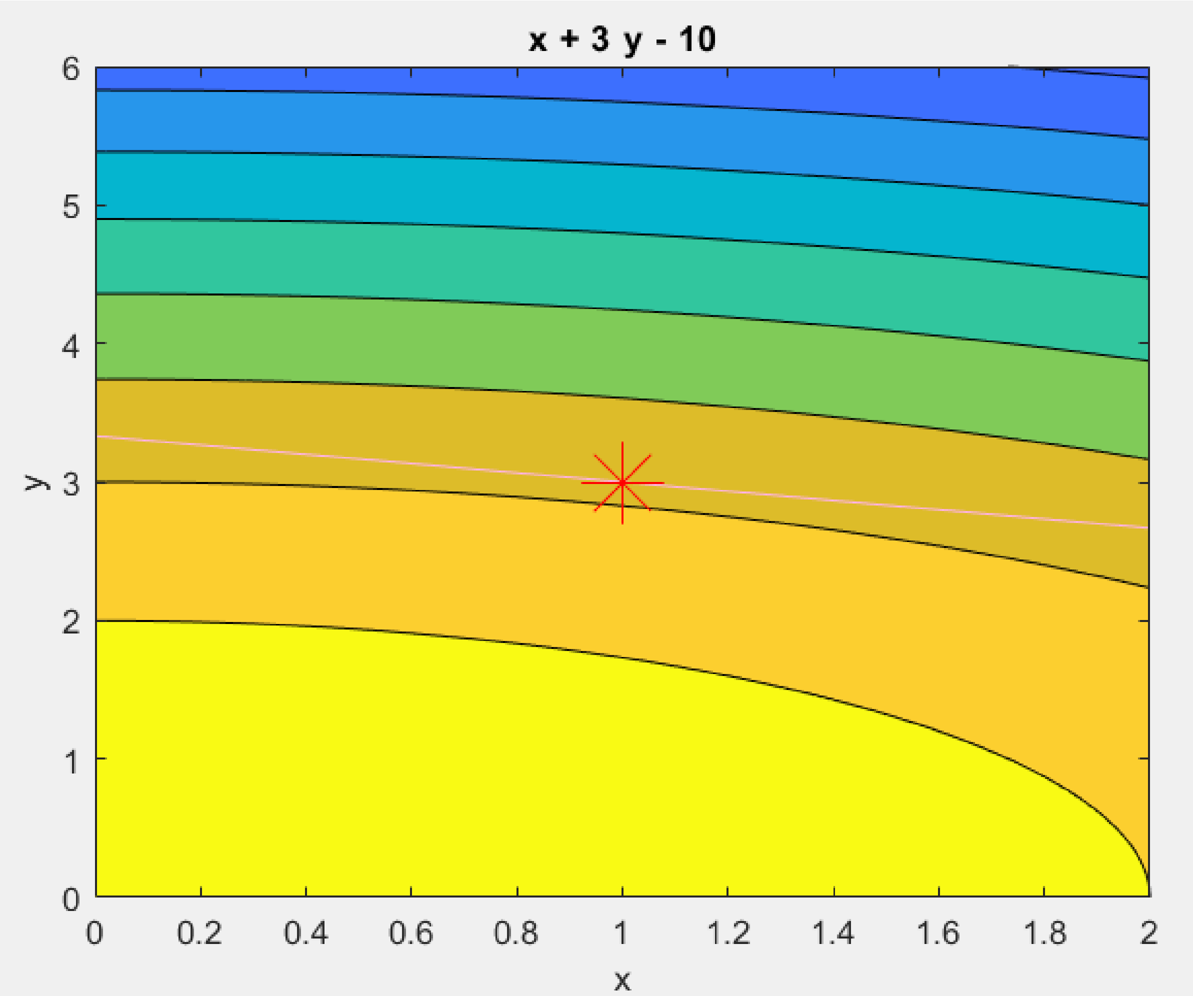
39

f_min =

39

f_max =

Figure Output



MATLAB CODE FOR Q5 and Q6:

```
% Code for Langrage's Multipliers for 3 variables with 1 constraint
clc
clear all
format compact
syms x y z l X Y Z a b c
f = input("enter function to be extremized:")
g = input("enter constraint function:")
L = f + l*g
Jk = jacobian(L,[x y z l])
[P,Q,M,N]= solve(Jk,[x y z l],"Real",true)
for N=1:size(P)
    A(N) = subs(f,[x y z],[P(N) Q(N) M(N)]);
end;
max_value = max(A);
min_value = min(A);
R = [P,Q,M]
ranger = [min(R) max(R)];
ranger = double(ranger);
for i=1:size(P)
   fprintf("the critical points (x,y,z) are(%1.3f,%1.3f,%1.3f)\n:",P(i),Q(i),M(i))
   fprintf("the value of the function at that point:(%1.3f)\n",A(i))
end;
fprintf("the maximum value of the function is: %1.3f\n",max_value)
fprintf("the minimum value of the function is: %1.3f\n",min_value)
figure
ranger = [-6 6] % add your own range for plotting graph here
F = subs(f,[x y z],[X Y Z])
Fs = fimplicit3(F,ranger)
Fs.XRange = [-6 6]; %add x range here
Fs.YRange = [-6 6]; %add y range here
Fs.ZRange = [-6 6]; %add z range here
Fs.LineStyle = "none";
Fs.EdgeColor = 'none';
Fs.FaceAlpha = 0.8;
hold on
G = subs(g,[x y z],[a b c]);
Gs = fimplicit3(G, ranger);
plot3(P, Q, M) %add your own formatting parameters.
```

Output 5:

Numerical Output

enter function to be extremized:x*y+4-z^2

f =

- z^2 + x*y + 4

enter constraint function:x*y+4-z^2

g =

- z^2 + x*y + 4

L =

x*y + l*(- z^2 + x*y + 4) - z^2 + 4

Jk =

[y + l*y, x + l*x, - 2*z - 2*l*z, - z^2 + x*y + 4]

P =

-4

0

0

Q =

1

0

0

M =

0

-2

2

N =

-1

-1

-1

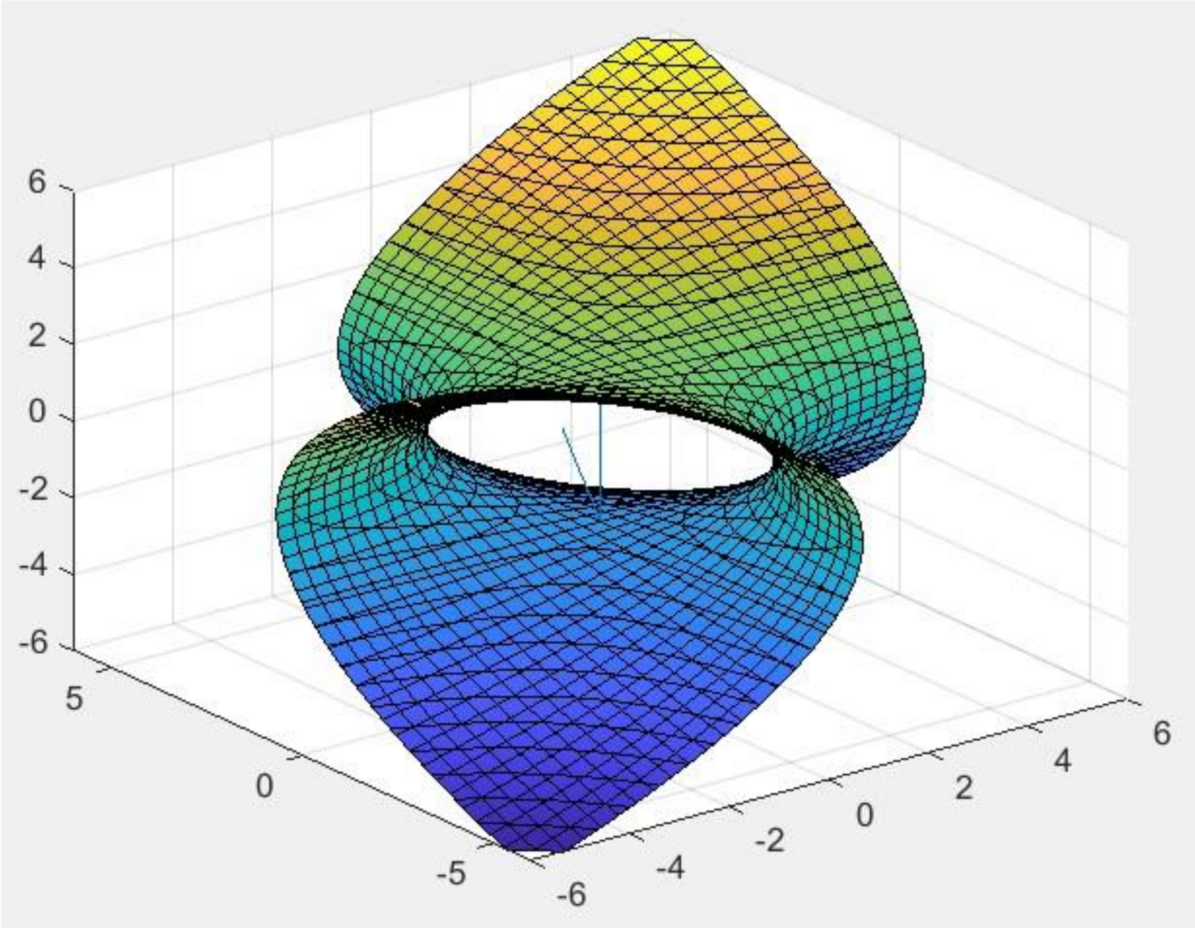
R =

[-4, 1, 0]

```
[ 0, 0, -2]
[ 0, 0,  2]
the critical points (x,y,z) are (-4.000,1.000,0.000)
:the value of the function at that point:(0.000)
the critical points (x,y,z) are (0.000,0.000,-2.000)
:the value of the function at that point:(0.000)
the critical points (x,y,z) are (0.000,0.000,2.000)
:the value of the function at that point:(0.000)
the maximum value of the function is: 0.000
the minimum value of the function is: 0.000
ranger =
   -6     6
F =
- Z^2 + X*Y + 4
Fs =
  ImplicitFunctionSurface with properties:

     Function: - Z^2 + X*Y + 4
       XRange: [-6 6]
       YRange: [-6 6]
       ZRange: [-6 6]
    EdgeColor: [0 0 0]
    LineStyle: '-'
    FaceColor: 'interp'
```

Figure Output



Output 6:

Numerical Output

```
enter function to be extremized:(x^2)*y*(z^3)

f =

x^2*y*z^3

enter constraint function:2*x+y+3*z-3

g =

2*x + y + 3*z - 3

L =

l*(2*x + y + 3*z - 3) + x^2*y*z^3

Jk =

[2*x*y*z^3 + 2*l, x^2*z^3 + l, 3*y*x^2*z^2 + 3*l, 2*x + y + 3*z - 3]

P =

1/2

3/2

 0

Q =

1/2

 0

 3

M =

1/2

 0

 0

N =

-1/32

  0

  0

R =

[1/2, 1/2, 1/2]

[3/2,  0,   0]
```

```
[ 0,   3,   0]
the critical points (x,y,z) are (0.500,0.500,0.500)
:the value of the function at that point:(0.016)
the critical points (x,y,z) are (1.500,0.000,0.000)
:the value of the function at that point:(0.000)
the critical points (x,y,z) are (0.000,3.000,0.000)
:the value of the function at that point:(0.000)
the maximum value of the function is: 0.016
the minimum value of the function is: 0.000
ranger =
   -6     6
F =
X^2*Y*Z^3
Fs =
  ImplicitFunctionSurface with properties:


     Function: X^2*Y*Z^3
       XRange: [-6 6]
       YRange: [-6 6]
       ZRange: [-6 6]
     EdgeColor: [0 0 0]
  LineStyle: '-'
  FaceColor: 'interp'
```

Figure Output