Ramakrishna Mission Vivekananda Educational and Research Institute
PO Belur Math, Howrah, West Bengal 711 202
School of Mathematical Sciences
Department of Computer Science

80

MSc Big Data Analytics : Batch 2021-23, Semester II, Midterm
CS 342: Computer Vision
Instructor: Prof. Bhabatosh Chanda

Student Name (in block letters): SUBHANKAR NAG
Student Roll No: B2130019
Signature: Subhankar Nag.

Date: 12 April 2022
Max Marks: 100
Time: 3hrs

Answer any ten (10) questions

1. **Short questions (answer all, 2 marks each):**
   (a) What are the percentage coverage of red, green and blue filters in Bayer filter of digital colour camera?
   (b) State two differences between rod and cone cells in human retina.
   (c) List the intrinsic parameters of a camera. Considering homogeneous coordinate system, write down the intrinsic parameter matrix to map world point to image pixel.
   (d) What is meant by camera calibration and how is it done usually?
   (e) What is meant by depth of field of a camera? For what type of camera the depth of field is infinite?

2. **Short questions (answer all, 2 marks each):**
   (a) When would epipolar lines in image plane of Camera-2 be vertical and parallel due to image points in Camera-1?
   (b) Suppose there are two cameras. Which point is called the epipole of the image plane in camera-2? Whose image is it?
   (c) What is underlying strategy for edge preserving smoothing?
   (d) How is Gaussian kernel modified to achieve anisotropic diffusion for edge preserving smoothing?
   (e) Write down the kernel of bilateral noise filter. Why is it called position variant kernel?

3. **Short questions (answer all, 2 marks each):**
   (a) Why is Gamma correction required?
   (b) Suppose minimum and maximum gray levels in an image are 'a' and 'b', whereas that of available gray range are 'L' and 'M' respectively. Write the linear stretching transformation from input graylevel 'x' to output gray level 'y'.
   (c) How can smoothing technique be used to sharpen an image?
   (d) How is the concept of hysteresis thresholding used in Canny's edge detector?
   (e) What is the main objective of rectification technique?

4. Prove that $\left(p_h^{I_2}\right)^T F p_h^{I_1} = 0$, where $p_h^{I_i}$ denotes the homogeneous coordinate of an image point in i-th camera. Superscript T denotes the transposition and F is fundamental matrix of size 3×3. [10]

5. (a) Derive the Harris matrix for corner detection.
   (b) Hence, find the formula of the single response for corner detection from this matrix. [8+2]

6. Two points at b unit apart along w-direction in a 3D scene form images at the same point (x_1, y_1) on the image plane of a camera. Then the camera is shifted by d unit only along x (or u)-axis without making any rotation. New images of those two 3D points are formed at (x_2, y_1) and (x_3, y_1). Determine the focal length of the camera. [10]

7. (a) Prove the following the property of perspective projection: A set of parallel lines in 3D, n perpendicular to the optical axis, maps to a set of concurrent lines in 2D.
(b) Hence, show why is the condition 'not perpendicular to the optical axis' important?
(c) Also show what happens to the projected lines in 2D, if the set of parallel lines in 3D are perpendicular to the optical axis? [6+2+2]

8. (a) Suppose an imaging system can be modelled as a pin-hole camera with a focal length of 20 mm and the optical axis of the camera is same as the w-axis. A square metal sheet of size 80 cm $\times$ 40 cm is placed in front of the camera in such a way that w-axis hits its centre at a distance of 40 cm from the pin-hole and the sheet is perpendicular to both $w-u$ plane and $w-v$ plane. Calculate the size of the image of metal sheet.
(b) Image of this metal sheet just covers the image of another metal sheet of size 1.6 m x 0.8 m. What is the distance of the second sheet from the first one? [7+3]

9. (a) Suppose there is a camera with focal length 2 cm and image plane of size 1.6 cm (horizontally) by 1.5 cm (vertically). Given that photo sensor density along horizontal direction is 2500 cm^{-1} and in vertical direction it is 2000 cm^{-1}, calculate the pixel coordinate corresponding to a scene point at (6 m, 4 m, 40 m). Assume the origin of the system is at centre of lens of the camera and the optical axis meets the image plane at its centre. The world coordinate axes coincide with that of the camera.
(b) Now if the camera is shifted vertically downward by 1 m and then rotated by 90^o anti-clock-wise, what will be new coordinate of the image point? [6+4]

10. (a) Suppose one of two normalized cameras is at (0, 0, 0) and the other at (2, -32, -52). Second camera is rotated by 45^o anti-clockwise about z-axis. Show that (-4, 2, 1) in the second camera is the corresponding point of the point (-3/72, 4/72, 1) in the first camera.
(b) Suppose two real cameras C_1 and C_2 are related by a fundamental matrix

$$\begin{bmatrix} 0.01 & 0.02 & -3.0 \\ -0.02 & 0.01 & -0.5 \\ 0,03 & -0.04 & 2.0 \end{bmatrix}$$

Find the equation of epipolar line on the image plane of C_2 due to an image point (300,250,1) in C_1. [7+3]

11. Two cameras with same intrinsic parameters are placed in such a way that their image plane axes (i.e., x-axis and y-axis) are parallel and are aligned with u-axis and v-axis of the 3D scene. Suppose, in 3D world, the optical centers of these two cameras are at (0, 0, 0) and (25, 10, 0). Due to an object point in 3D scene the coordinate of image points in two cameras are (400, 350) and (405, b). If the focal length of the cameras is 2cm, what is the value of 'b'? [10]

12. (a) Describe the histogram equalization technique for contrast enhancement.
(b) Suppose graylevel histogram of an image having 8 levels is given by

value	0	1	2	3	4	5	6	7
freq.	3000	2000	1500	200	300	500	1000	1500

Calculate the graylevel histogram of the histogram equalized image. [3+7]

Ramakrishna Mission Vivekananda Educational and Research Institute
PO Belur Math, Dist Howrah, West Bengal 711 202, India

Additional Sheet No. 0

No. A19-007403

Signature of the Invigilator : Sarmo 12/4/22

Name - Subhankar Nag (B2130019)

1) a) 25% ~~oo~~red, 25% blue and 50% green filters are there in Bayer filter of digital Colour Camera. 2

b)

Rod cell	Cone cell
i) It is senseltive to low intensity.	i) It is senseltive to the Colour.
ii) It is Smaller in Size.	ii) It is respectively larger.

2

c) Intrisic Parameters of Camera - $f, \varphi_x, \varphi_y, d_x, d_y$.

Intrinsic Paramers matrix. - $\begin{bmatrix} \varphi_x & 0 & d_x \\ 0 & \varphi_y & d_y \\ 0 & 0 & 1/f \end{bmatrix}$ 2

d) Estimating or finding intrinsic and extrinsic Parameter of camera is known as Camera Calibration. 1

It is done ~~using~~ ~~Read~~ as follows -

Take 2 or more Pictures

Find Corresponding matches

Compute Essential / Fundamental matrix.

From this matrix we can compute the Paramets (Camera's)

e) Depending on the aperture the camera can focus to a object in a distant and blur other object, are not in that same distant. This is depth of

Aperture → −∞ ~~then~~ ~~field~~

then depth of field → ∞

0

2) a) If we shift the camera 2 vertically with respect to camera 1, that is camera 2 is just above or below the camera 1, then epipolar lines in image plane of camera 2 be vertical and parallel due image points in camera 1.

2

b) ~~Epipole in Camera 2 is the image of Camera~~

Epipole of the image plane of camera 2 is a projection of the optiche centre of camera 1 on optiche axis.

1

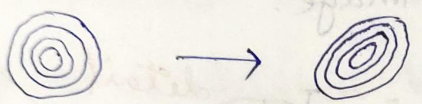
c) Strategy for edge preserving smoothing is –

i) Intra region smoothing is preferable rather than interregion smoothing.

ii) We should smooth along the direction of edge not perpendicular to it.

2

We need to smooth along the edge to smooth the edge noise but not Perpendicular to the edge otherwise the edge's intensity will be reduced. Gaussian kernel is transformed such a way that more weigth is given in direction of the edge and less weight is given Perpendicular to it.



formula? (Expression)?

?

e) Bilateral filter —

Kernel?

why?

At each Pixel, based on the neighbour hood the kernel is computed and applied, that's why it is Position invariant kernel.

3) a) Camera sensor ~~accept~~ response to light in a non-linear fashion. So we need to apply the inverse operation to get the original image. Applying Gama Correction to a image we can get a brigther image. (Darker area might be brigther). linear

2

b) $y = \frac{(M-L)}{(b-a)}(x-a) + L$

2

c) ~~smooth using Gaussian kernel~~

If we Subtract the smoothed image from original image, then we get the edges. and now if we add this edges to the original image we will get sharpend image.

$I_{original} - I_{smooth} = I_{edge}$ detail

$I_{original} + I_{edge} = I_{sharpen}$

2

d) Hysteresis Thresholding -

If the Point is higher than the threshold-high
It is an edge Point

If the Point is lower than the threshold-low
It is not an edge Point.

If the value of Point is in between 2 thresholds
If the Points neighbours are of the Part of an edge then It is also an edge.

else
It is not an edge Point.

2

e)

Ramakrishna Mission Vivekananda Educational and Research Institute

PO Belur Math, Dist Howrah, West Bengal 711 202, India

Additional Sheet No. 1

No. A19- 007404

Signature of the Invigilator : Sama 12/4/22

Name - SUBHANKAR NAG (B2130019)

e) To find the corresponding matches we need to search the image 2 to find that Point and it is search on the epipolar line. We need to transform the image 2 such a way that the epipolar line should present in horizonatally with respect to image 1 so that the computation will be ~~much~~ slightly lighter. ~~we just~~ After rectification we just scan the horizontal line to find the matching.

2

4) Let the world coordinates $P_h^W = \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$

Projected coordinates $P_h^I = \lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

The world coordinate projected on the image plane

$$P_h^I = K[R|t]P_h^W$$

[K and [R|t] are Instrinsic and Extrinsic Parameters of Camera]

In a normalized camera

$f = 1, \quad \phi_x = \phi_y = 1$

$d_x = d_y = 1 \quad$ so,

$K = I$ (identity).

$$\hat{P}_n^I = K^{-1} P_n^I = [R|t] P_n^W$$

[Homogenious to normal

$$\hat{P}_n^I = RP^W + t$$

We assume the camera 1 is aligned with world

$$\hat{P}_n^{I_1} = IP^W + 0 = P^W$$

Camera 2 is rotated(R) and ~~transformed~~ Shifted (t)

$$\hat{P}_n^{I_2} = RP^W + t = R\hat{P}_n^{I_1} + t$$

Now taking Cross Product with t

$$t \times \hat{P}_n^{I_2} = t \times R\hat{P}_n^{I_1} + t \times t$$

$$= t \times R\hat{P}_n^{I_1}$$

Now taking dot-product with $\hat{P}_n^{I_2}$

$$\hat{P}_n^{I_2} \cdot (t \times \hat{P}_n^{I_2}) = \hat{P}_n^{I_2} \cdot (t \times R\hat{P}_n^{I_1})$$

$$\Rightarrow \quad 0 = (\hat{P}_n^{I_2})^T (t \times R\hat{P}_n^{I_1})$$

$$\Rightarrow (\hat{P}_n^{I_2})^T E (\hat{P}_n^{I_1}) = 0$$

Now Putting $\hat{P}_n^I = K^{-1} P_n^I$

$$(K_2^{-1} P_n^{I_2})^T E (K_1^{-1} P_n^{I_1}) = 0$$

$$(P_n^{I_2})^T (K_2^{-1})^T E (K_1^{-1})(P_n^{I_1}) = 0$$

$$(P_n^{I_2})^T F (P_n^{I_1}) = 0$$

(Proved)

10

Two Patches, one is shifted from another

$f(x,y)$ and $f(x+\Delta x,\ y+\Delta y)$

$$f(x+\Delta x,\ y+\Delta y) = f(x,y) + \frac{\delta f}{\delta x}\Delta x + \frac{\delta f}{\delta y} + \cdots$$

Sum of squared of two Patches -

$$S = \sum_{(x,y)\in W}\left(f(x,y) - f(x,y) - \begin{bmatrix}\frac{\delta f}{\delta x} & \frac{\delta f}{\delta y}\end{bmatrix}\begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix}\right)^2$$

$$= \sum\left(- \begin{bmatrix}\frac{\delta f}{\delta x} & \frac{\delta f}{\delta y}\end{bmatrix}\begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix}\right)^2$$

$$\cancel{= \sum \begin{bmatrix}\Delta x & \Delta y\end{bmatrix}\left(\begin{bmatrix}\frac{\delta f}{\delta x} & \frac{\delta f}{\delta y}\end{bmatrix}\begin{bmatrix}\frac{\delta f}{\delta x} \\ \frac{\delta f}{\delta y}\end{bmatrix}\right)\begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix}}$$

$$\cancel{= \begin{bmatrix}\Delta x & \Delta y\end{bmatrix}\left(\sum \frac{\delta f}{\delta x}\right.}$$

$$= \sum_{(x,y)\in W} \begin{bmatrix}\Delta x & \Delta y\end{bmatrix}\left(\begin{bmatrix}\frac{\delta f}{\delta x} \\ \frac{\delta f}{\delta y}\end{bmatrix}\begin{bmatrix}\frac{\delta f}{\delta x} & \frac{\delta f}{\delta y}\end{bmatrix}\right)\begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix}$$

$$= \begin{bmatrix}\Delta x & \Delta y\end{bmatrix}\begin{pmatrix}\sum\left(\frac{\delta f}{\delta x}\right)^2 & \sum \frac{\delta f}{\delta x}\frac{\delta f}{\delta y} \\ \sum \frac{\delta f}{\delta y}\frac{\delta f}{\delta x} & \sum\left(\frac{\delta f}{\delta y}\right)^2\end{pmatrix}\begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix}$$

$$= X^T A X$$

A is the Harris matrix.

8

b) We can apply PCA on matrix A and get eigen values λ_1 and λ_2. We can formulate it by single response –

$$R = \det(A) - k(\text{trace})^2$$

$$= \lambda_1 \lambda_2 - K(\lambda_1 + \lambda_2)^2$$

where k is emperical error.

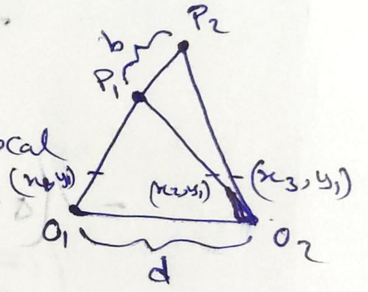
2

If $R < 0$ then edge

If R is small then flat region

If R is high then corner.

6) let w_1 and w_2 are the z axis coordinate of P_1 and P_2. f is the focal length.



$$W_1 = \frac{df}{x_2 - x_1}$$

$$W_2 = \frac{df}{x_3 - x_1}$$

we know $w_2 - w_1 = b$

$$\therefore b = \frac{df}{x_3 - x_1} - \frac{df}{x_2 - x_1}$$

$$b = df\left(\frac{x_2 - x_1 - x_3 + x_1}{(x_3 - x_1)(x_2 - x_1)}\right)$$

$$b = df\left(\frac{x_2 - x_3}{(x_3 - x_1)(x_2 - x_1)}\right)$$

<u>Focal lenght</u> $f = \frac{(x_3 - x_1)(x_2 - x_1)}{d(x_2 - x_3)}$

10

Ramakrishna Mission Vivekananda Educational and Research Institute
PO Belur Math, Dist Howrah, West Bengal 711 202, India
Additional Sheet No. 2
No. A19-007306
Signature of the Invigilator :

Name - SUBHANKAR NAG (B2130019)

9 a) $f = 2\,cm$, image plane $= 1.6 \times 1.5\ cm^2$

$\Phi_x = 2500\ cm^{-1}$, $\Phi_y = 2000\ cm^{-1}$

World coordinates (6m, 4m, 40m).

~~image Coordinates~~

Pixel Coordinates $x = -f\,\Phi_x \frac{u}{w}$

$$= \frac{2 \times 2500 \times 6}{40} = -750$$

$$y = -f\,\Phi_y \frac{v}{w}$$

$$= \frac{2 \times 2000 \times 4}{40} = -400$$

Now if we shift the center to the left top corner

a) $(x', y') = \left(-750 + \frac{1.6}{2} \times 2500,\ -400 + \frac{1.5 \times 2000}{2}\right)$

$= (1250,\ 1100)$ (Pixel Coordinate)

6

b) Camera is vertically downward by 1m and rotated anticlock wise 90°, we get new coordinate -

$$\begin{bmatrix} \cos 90^\circ & \sin 90^\circ & 0 & 0 \\ -\sin 90^\circ & \cos 90^\circ & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 40 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 40 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -6-1 \\ 40 \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \\ 40 \end{bmatrix}$$

World Coordinate $4, -\cancel{7}, 40$

$\therefore$ Pixel Coordinate -

$$\left(-f\,\Phi_x \frac{u}{w} + \frac{\Phi_x d_x}{2}, \; -f\,\Phi_y \frac{v}{w} + \frac{\Phi_y d_y}{2}\right)$$

$$= \left(-\frac{2 \times 2500 \times 4}{40} + \frac{2500 \times 1.6}{2}, \; \frac{-2 \times 2000 \times -7}{40} + \ldots\right)$$

$$= (-500 + 2000, \; 700 + 1500)$$

$$= (1500, \; 2200) \text{ (new Pixel Coordinate)}$$

2

12) a) In ~~Histogram~~ Histogram equalization we transform the histogram into a uniform distribution. In this we ~~brigh~~ focus on the frequency of an level. By intensify the higher level we are focusing on the major data that usually lie on the higher level.

In Continous domain $\int_{-\infty}^{+\infty} P(x)\, dx$

In ~~Stati dist~~ discrete domain $L_{max} \sum_{j=1}^{n_i} \frac{n p_j}{N}$

Histogram equalization automatically adjust the frequency of the greylevel where greylevel is high

1

	Freq	$\sum \frac{n_i}{N} L_{max}$	~ Result
0	3000	$\frac{3000}{10\,000} \cdot 7$	2
1	2000	$\frac{5000}{10000} \times 7$	~~3~~ 4
2	1500	$\frac{6500 \times 7}{10000}$	5
3	200	$\frac{6700 \times 7}{10000}$	5
4	300	$\frac{7000 \times 7}{10000}$	5
5	500	$\frac{7500 \times 7}{10000}$	5.
6	1000	$\frac{8500 \times 7}{10000}$	6
7	1500.	$\frac{10000 \times 7}{10000}$	7

histogram of histogram equalized image

Value	freq
0	0
1	0
2	3000
3	0
4	2000
5	2500
6	1000
7	1500

3

7) a) line in 3D that Pass through any point (x_i, y_i, z_i)

$$\frac{x - x_i}{a} = \frac{y - y_i}{b} = \frac{z - z_i}{c} = d$$

$x = x_i + ad$

$y = y_i + bd$ $\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} + d \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$z = z_i + cd$

After Perspective Projection 2D coordinate -

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} f \dfrac{x_i + da}{z_i + dc} \\ f \dfrac{y_i + db}{z_i + dc} \end{pmatrix}$$

At $d \to \infty$,

$$\lim_{d \to \infty} f \frac{x_i + da}{z_i + dc} = \lim_{d \to \infty} f \frac{\frac{x_i}{d} + a}{\frac{z_i}{d} + c} = f \frac{a}{c}$$

$$\lim_{d \to \infty} f \frac{y_i + db}{z_i + dc} = \lim_{d \to \infty} f \frac{\frac{y_i}{d} + b}{\frac{z_i}{d} + c} = f \frac{b}{c}$$

line in 3D that passes through any $(x_i, y_i z_i)$, at $d \to \infty$ does not depend on (x_i, y_i, z_i).

So A set of Parralel lines in 3D, not Perpendicular to optical axis maps to a set of Concurrent lines in 3D.

6

Ramakrishna Mission Vivekananda Educational and Research Institute
PO Belur Math, Dist Howrah, West Bengal 711 202, India
Additional Sheet No. 3
No. A19- 007301
Signature of the Invigilator :

Name :- SUBHANKAR NAG (B2130019)

10) b) epipolar line on Image plane of C_2 is l_2.

$$l_2 \Rightarrow (P^{I_1})^T \cdot F^T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 300 & 250 & 1 \end{bmatrix} \begin{bmatrix} 0.01 & -0.02 & 0.03 \\ 0.02 & 0.01 & -0.04 \\ -3.0 & -0.5 & 2.0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 3+5-3 & -6+2.5-0.5 & 9-10+2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 5 & -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$5x - 4y + 1 = 0$$

3

a) $$C_1 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & \cos(45^\circ) & \sin(45^\circ) & -32 \\ 0 & -\sin(45^\circ) & \sin(45^\circ) & -52 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -32 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -52 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -4+2 \\ \sqrt{2} - \frac{1}{\sqrt{2}} - 32 \\ \sqrt{2} - \frac{1}{\sqrt{2}} - 52 \end{bmatrix}$$

-2

$$= \begin{bmatrix} -4+2 \end{bmatrix}$$

=

)a) line .

$$C_1 = \begin{bmatrix} \cos(-45^\circ) & \sin(-45^\circ) & 0 & 2 \\ -\sin(-45^\circ) & \cos(-45^\circ) & 0 & -32 \\ 0 & 0 & 1 & -52 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 2 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -32 \\ 0 & 0 & 1 & -52 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\sqrt{2} - \sqrt{2} + 2 \\ 2\sqrt{2} + \sqrt{2} - 32 \\ 1 - 52 \end{bmatrix}$$

$$\left(\hat{P}_h^{I_2}\right)^T E \left(\hat{P}_h^{I_1}\right) = 0$$

$$\begin{bmatrix} 400 & 350 & 1 \end{bmatrix} t \times R \begin{bmatrix} 405 \\ b \end{bmatrix} = 0$$

?

No rotation sho R is identity, and only shift in x and y axis 25, 10 respectively.

$$\begin{bmatrix} 400 & 350 & 1 \end{bmatrix} t \begin{bmatrix} 405 \\ b \end{bmatrix} = 0$$

X 0