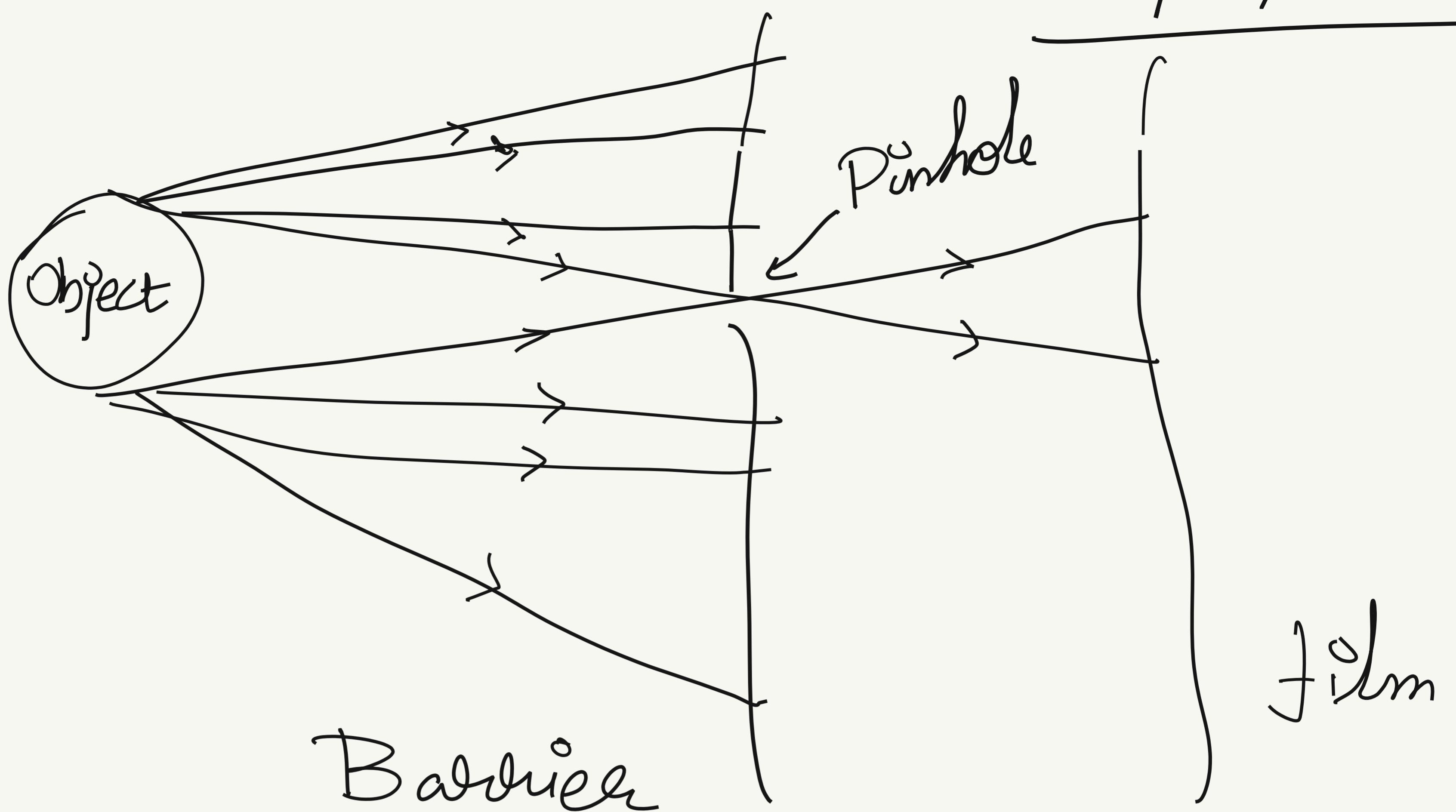
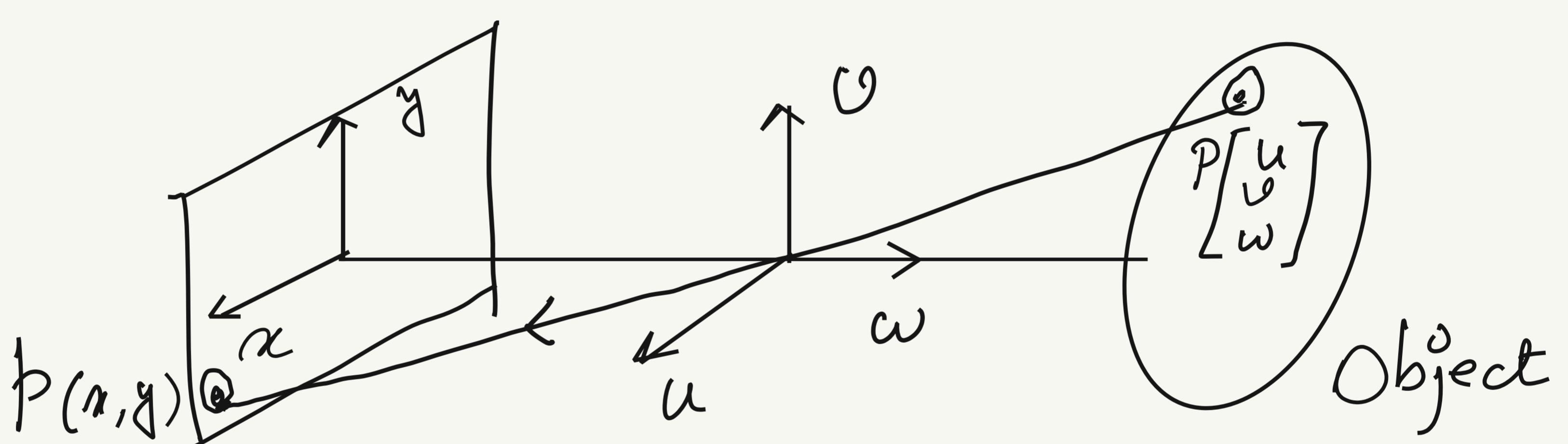


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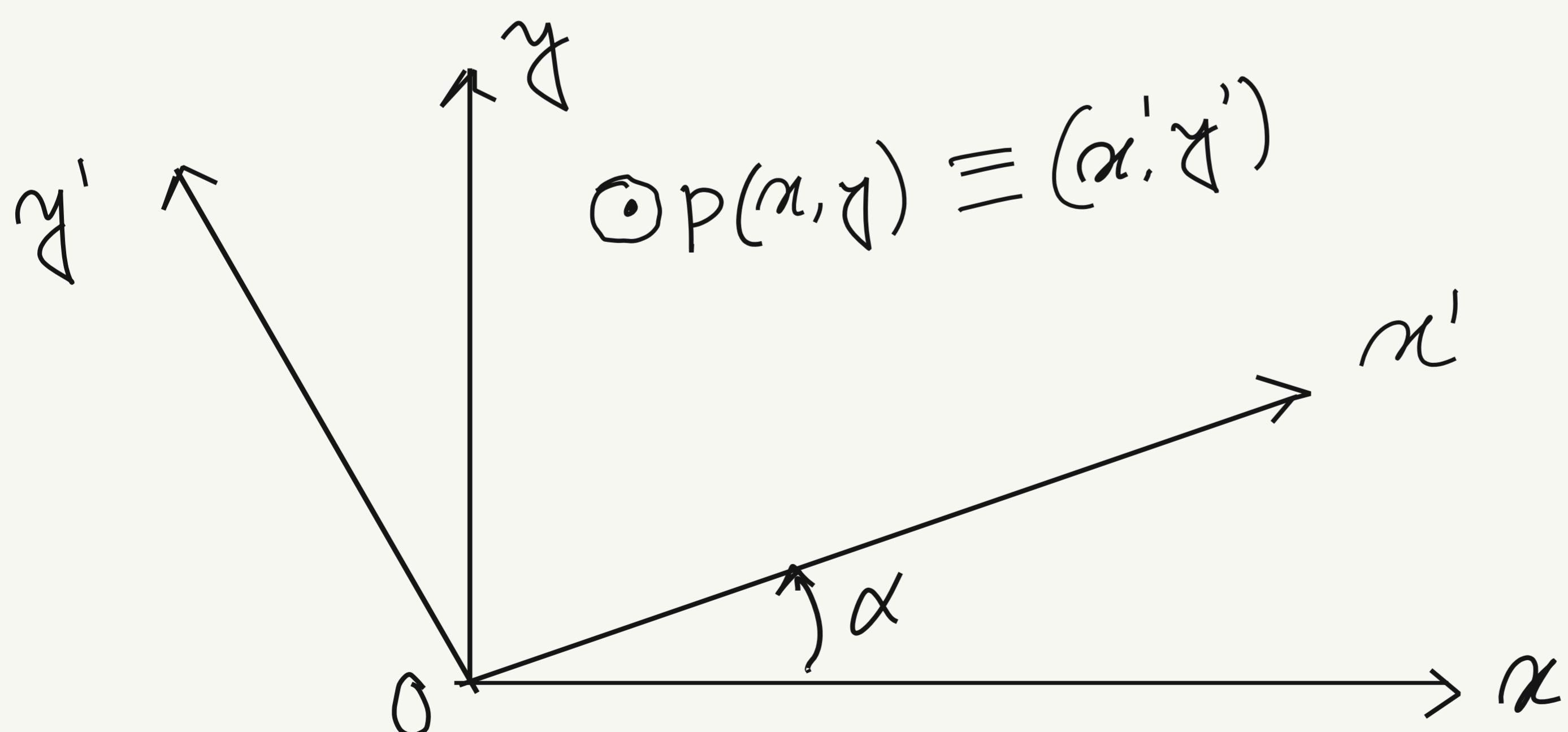


Pinhole  $\rightarrow$  Optical Center



From 3D to 2D.

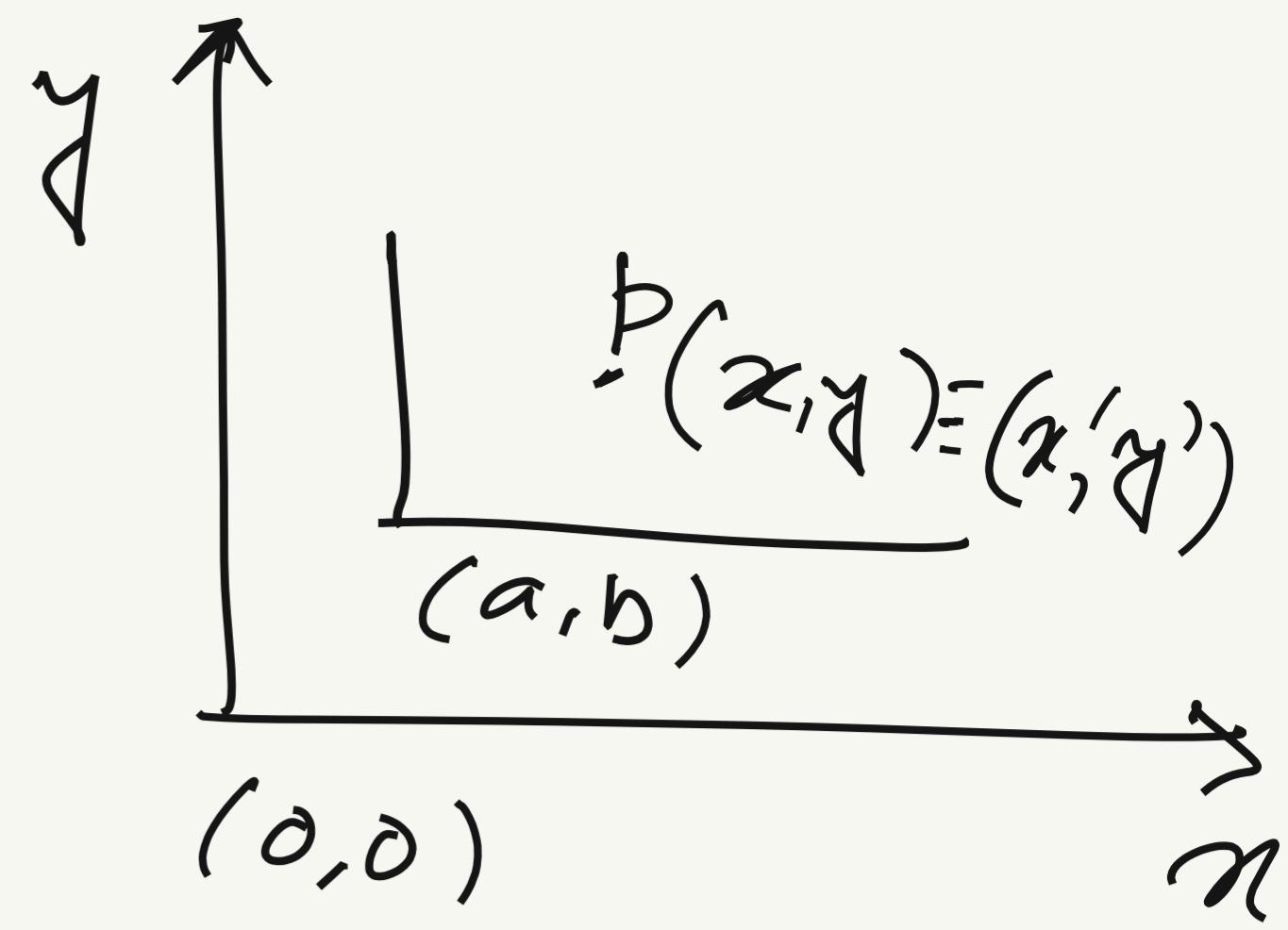
$$x = -f \cdot \frac{u}{w}, \quad y = -f \cdot \frac{v}{w}$$



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

For Scale changing,

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} S_x & 0 \\ 0 & S_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



For origin changing,

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -a \\ -b \end{pmatrix} \quad (\text{Not-linear})$$

Cartesian Co-ordinate

Homogeneous Co-ordinate

$$(x, y, z) \longrightarrow (kx, ky, kz, k)$$

$$\left(\frac{a}{d}, \frac{b}{d}, \frac{c}{d}\right) \longleftarrow (a, b, c, d)$$

In this case,

(Translation)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \quad (\text{origin changing})$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Homogeneous  
Co-ordinate  
when  $k=1$   
and we take 2D.

$$\begin{pmatrix} u \\ v \\ w/f \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \\ 1 \end{pmatrix}$$

Image  
Co-ordinate  
in Homo.  
Co-ordinate  
System

$$P_1(u_1, v_1, w_1) \quad \lambda P_1 + (1-\lambda) P_2 \quad P_2(u_2, v_2, w_2)$$

: 3D

$$\downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow$$

$$P_1\left(\frac{u_1}{w_1}, \frac{v_1}{w_1}\right) \quad P \quad P_2\left(\frac{u_2}{w_2}, \frac{v_2}{w_2}\right)$$

Transformation

: 2D

Note that

$$P = \left( f \cdot \frac{\lambda u_1 + (1-\lambda) u_2}{\lambda w_1 + (1-\lambda) w_2}, f \cdot \frac{\lambda v_1 + (1-\lambda) v_2}{\lambda w_1 + (1-\lambda) w_2} \right)$$

$$\begin{aligned} x &= f \cdot \frac{\lambda u_1 + (1-\lambda) u_2}{\lambda w_1 + (1-\lambda) w_2} \\ &= f \cdot \frac{\lambda u_1}{\lambda w_1 + (1-\lambda) w_2} + f \cdot \frac{(1-\lambda) u_2}{\lambda w_1 + (1-\lambda) w_2} \\ &= \frac{\lambda w_1}{\lambda w_1 + (1-\lambda) w_2} \times f \cdot \frac{u_1}{w_1} + \frac{(1-\lambda) w_2}{\lambda w_1 + (1-\lambda) w_2} \times f \cdot \frac{u_2}{w_2} \\ &= \frac{\lambda w_1}{\lambda w_1 + (1-\lambda) w_2} \times f \cdot \frac{u_1}{w_1} + \left[ 1 - \frac{\lambda w_1}{\lambda w_1 + (1-\lambda) w_2} \right] \times f \cdot \frac{u_2}{w_2} \end{aligned}$$

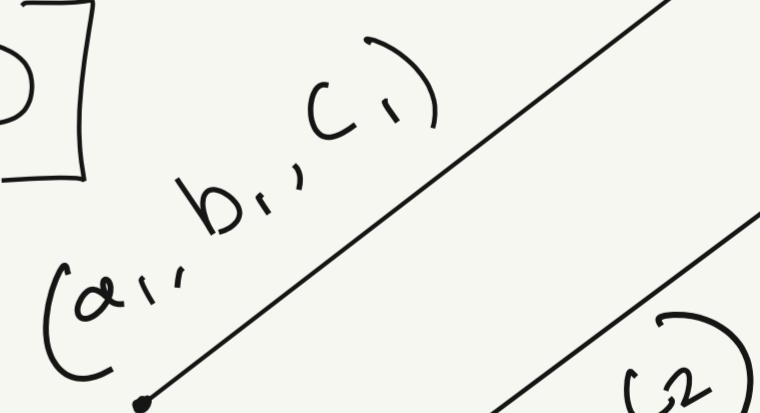
$$\text{if } x = \alpha \cdot f \frac{u_1}{w_1} + (1-\alpha) \cdot f \frac{u_2}{w_2}, \quad \alpha = \frac{\lambda w_1}{\lambda w_1 + (1-\lambda) w_2}$$

Similarly,

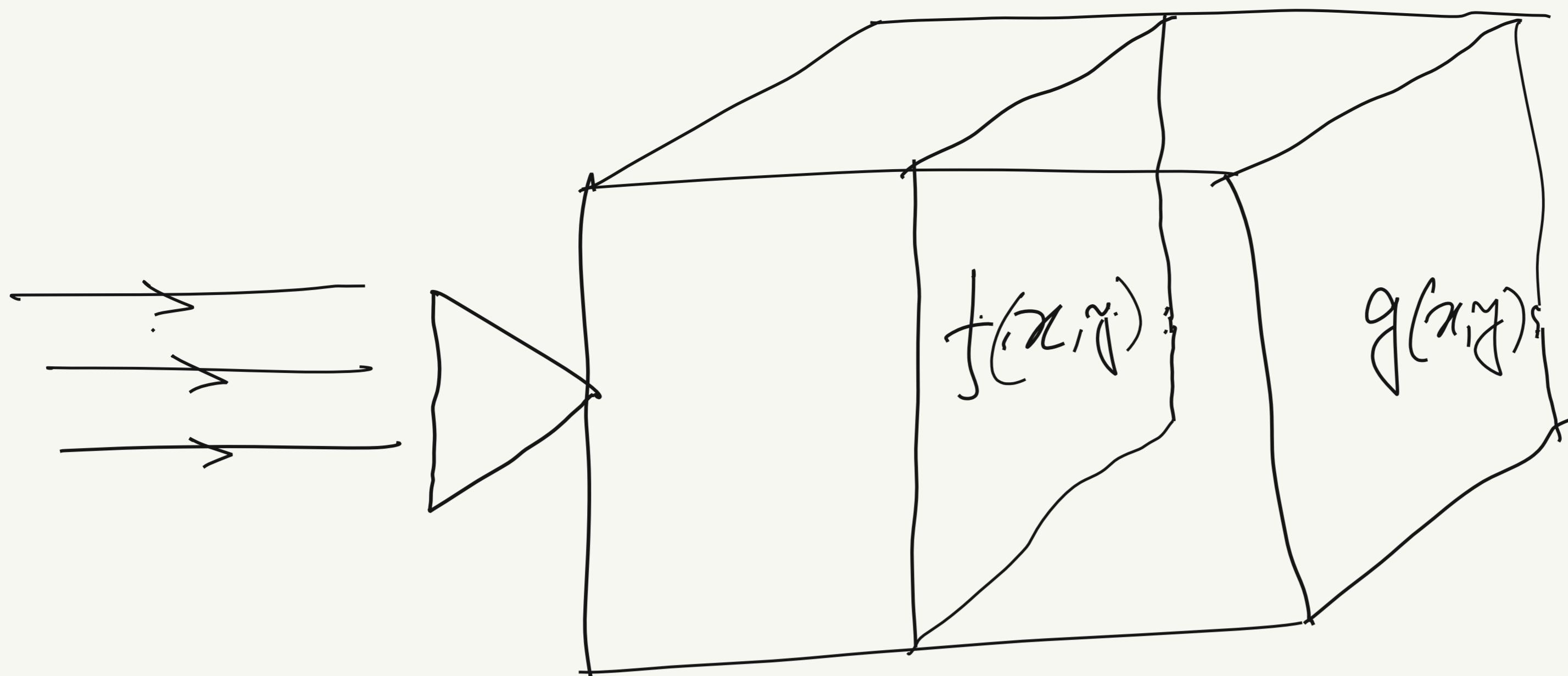
$$x = \alpha x_1 + (1-\alpha) x_2$$

$$y = \alpha y_1 + (1-\alpha) y_2$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} a_i \\ b_i \\ c_i \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix} \quad [\text{Eq of the St line in 3D}]$$



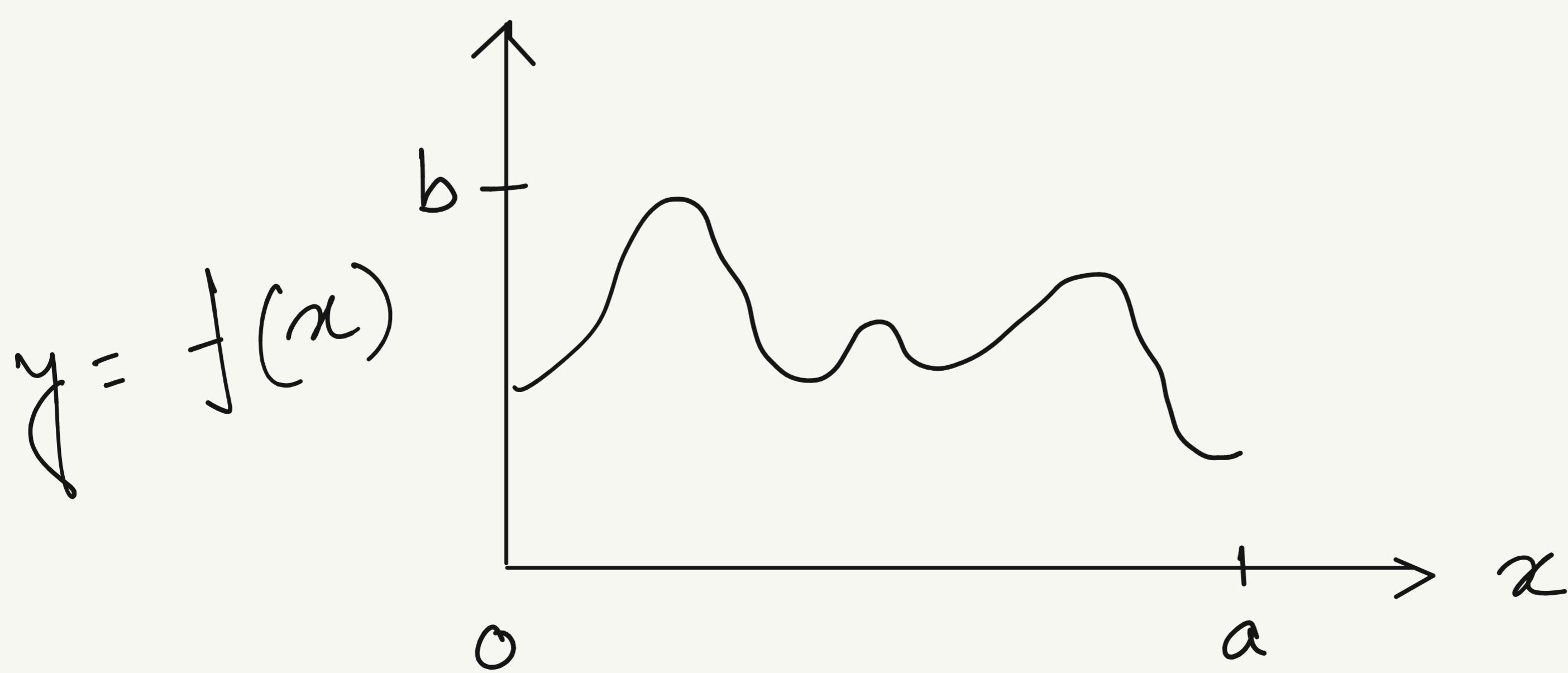
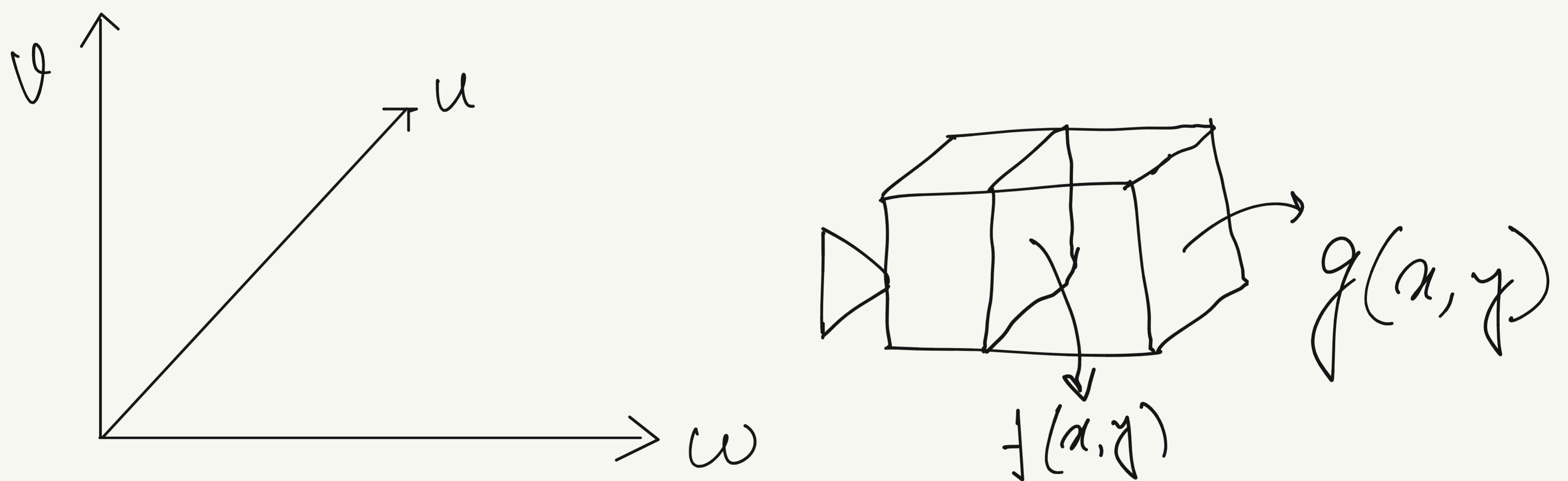
$$\begin{aligned} x &= \lim_{\lambda \rightarrow 0} f \cdot \frac{a_i + \lambda l}{c_i + \lambda n} = f \cdot \frac{l}{n} \\ y &= \lim_{\lambda \rightarrow 0} f \cdot \frac{b_i + \lambda m}{c_i + \lambda n} = f \cdot \frac{m}{n} \end{aligned}$$



Ideal Colour or Intensity Distribution  $f(x,y)$   
 Recorded Image  $g(x,y)$

The plane of  $f(x,y)$  is totally hypothetical ie that may not be present in real life.

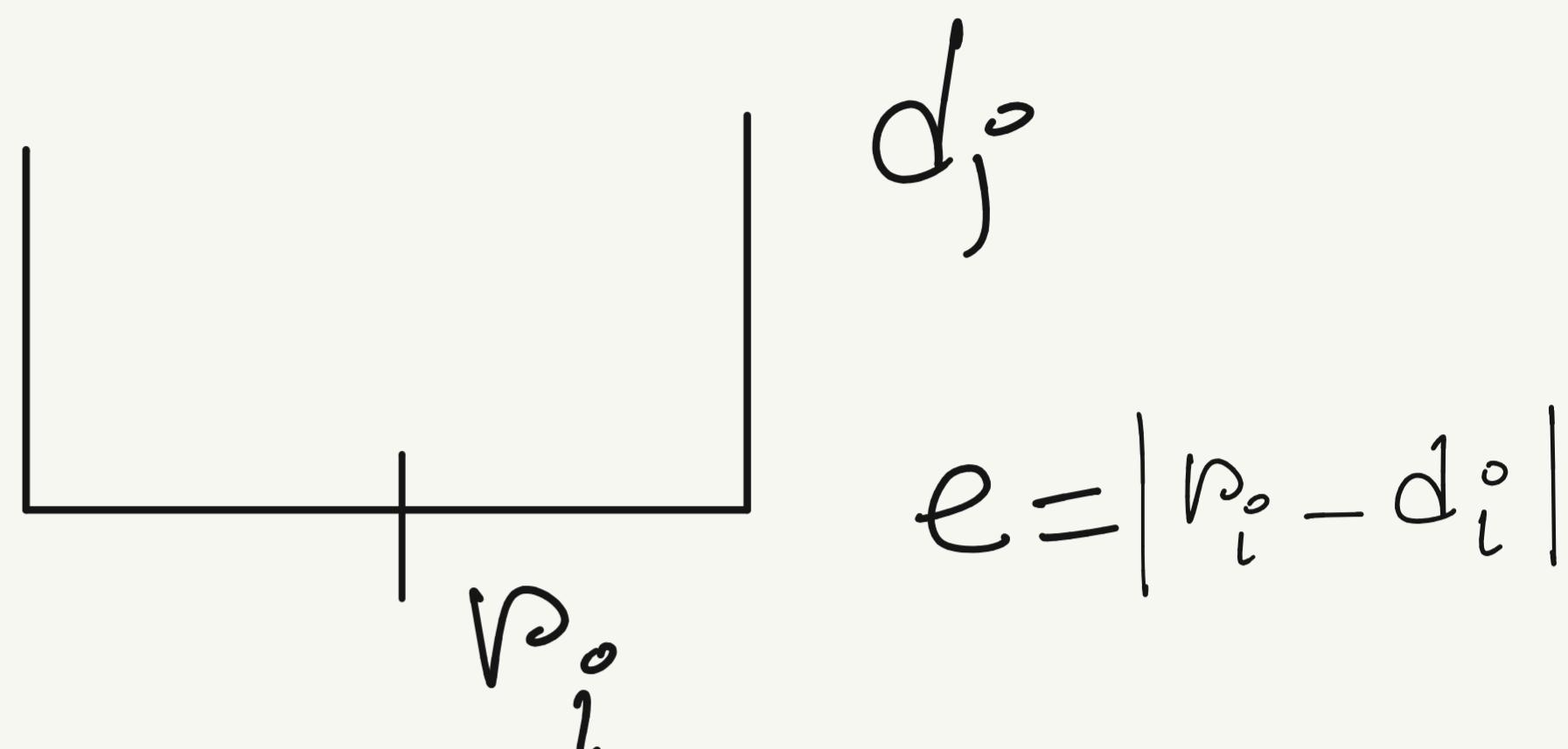
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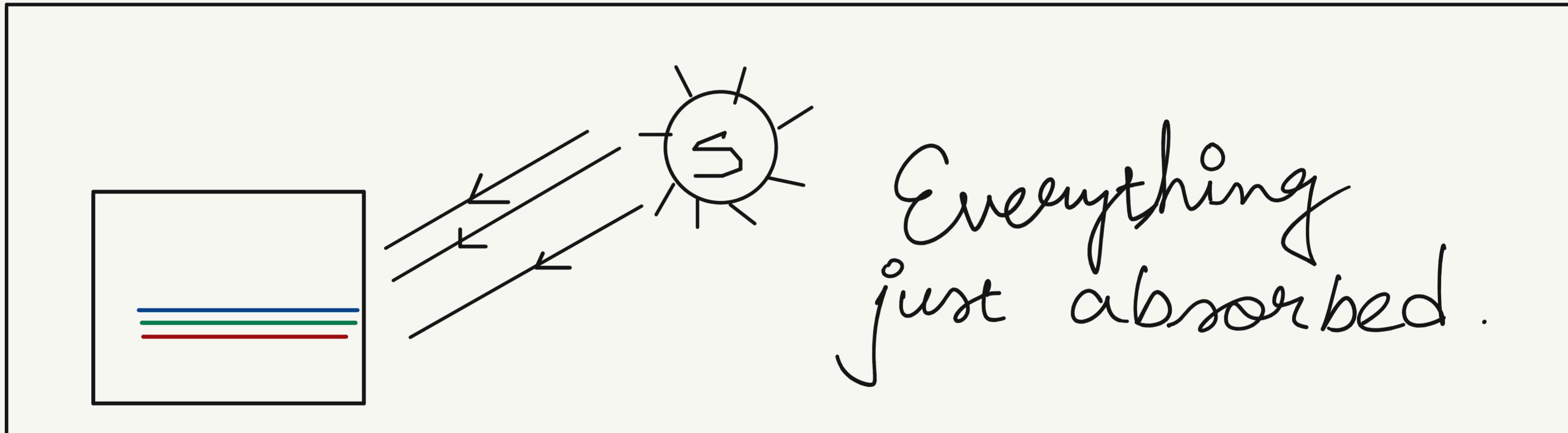
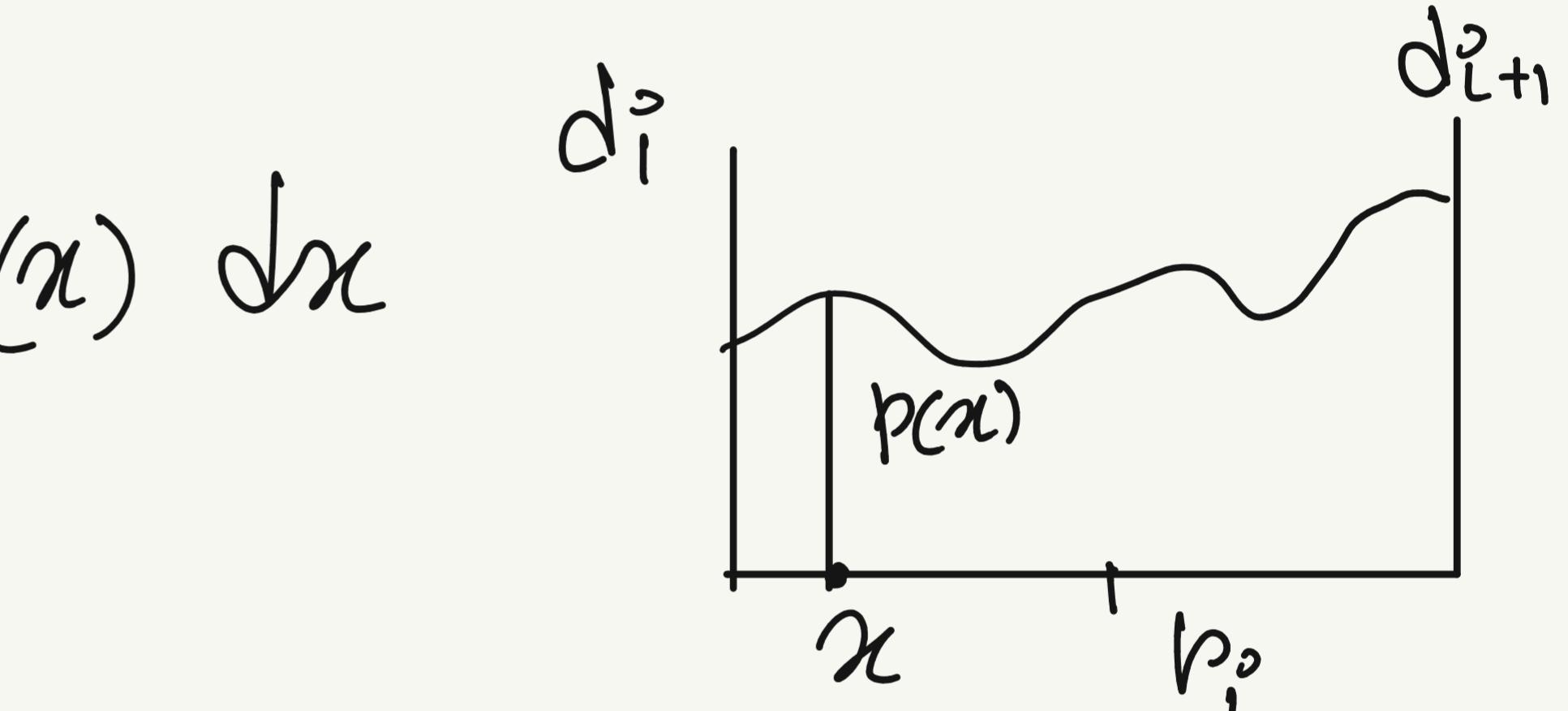
We divide the whole picture in many small blocks, called pixel.

There is any possible value between lowest (black) to highest (white).

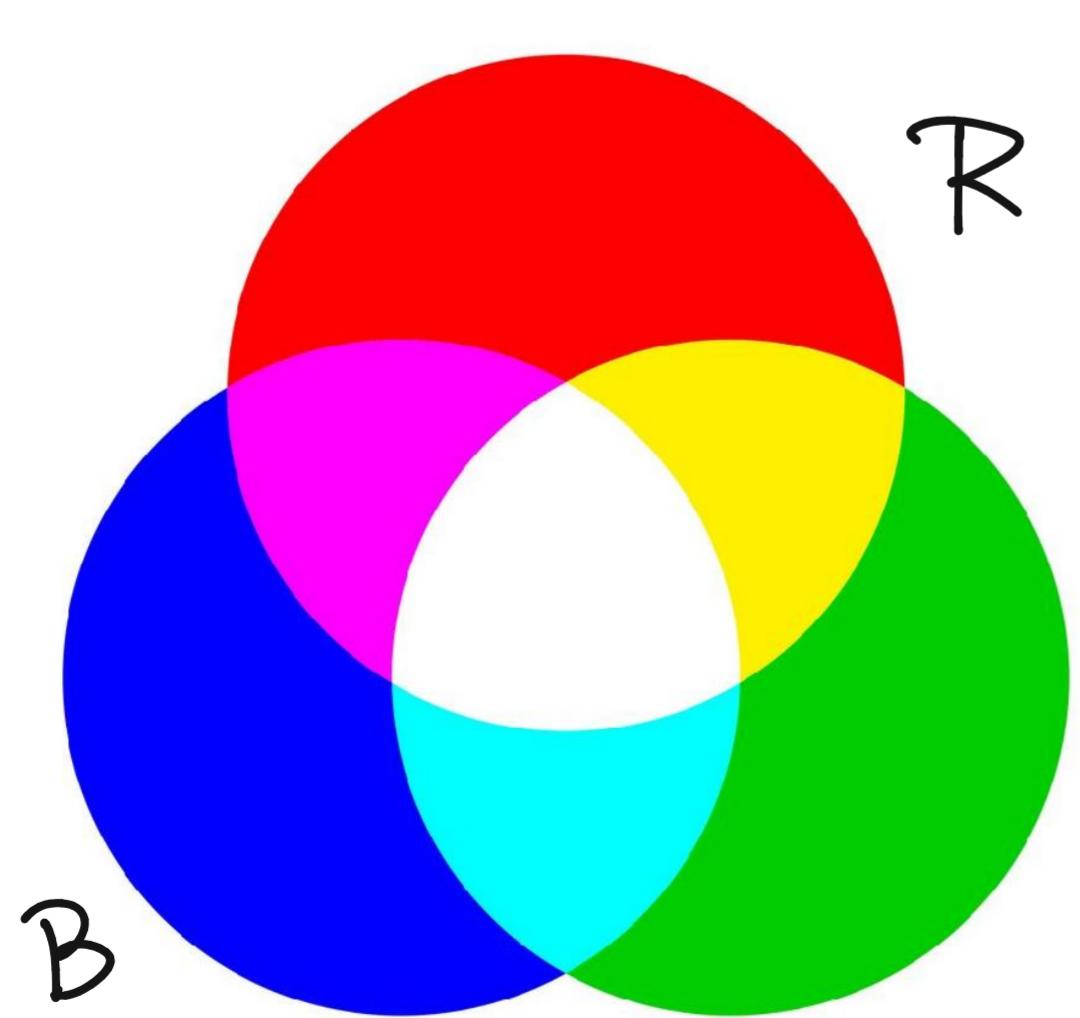
After getting  $d_i^o$   
the pixel value



$$\text{Error} = \sum_{i=0}^{d_i^o} \int_{d_i^o}^{d_{i+1}^o} (p_i^o - x)^2 p(x) dx$$

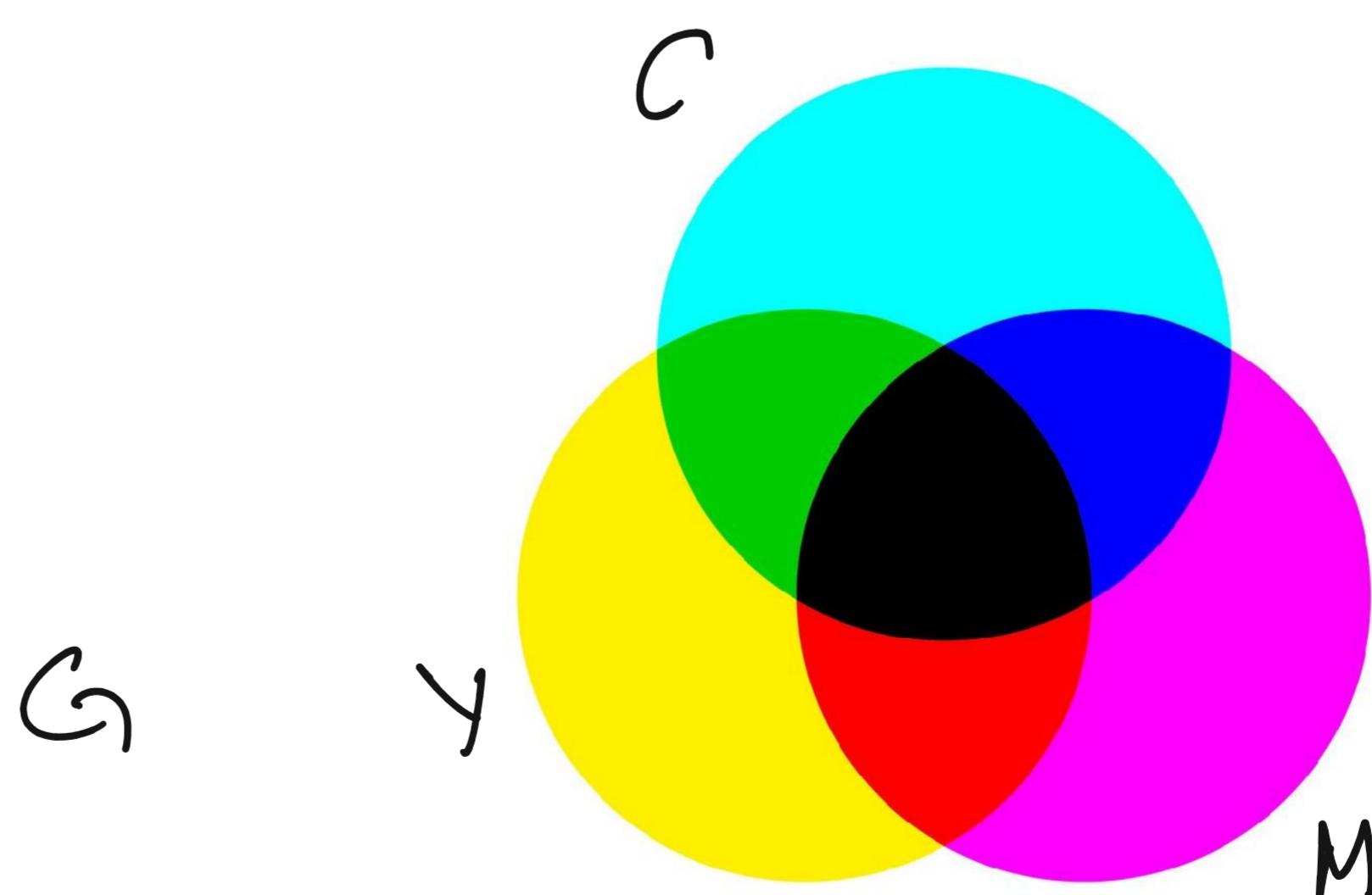


**RGB**

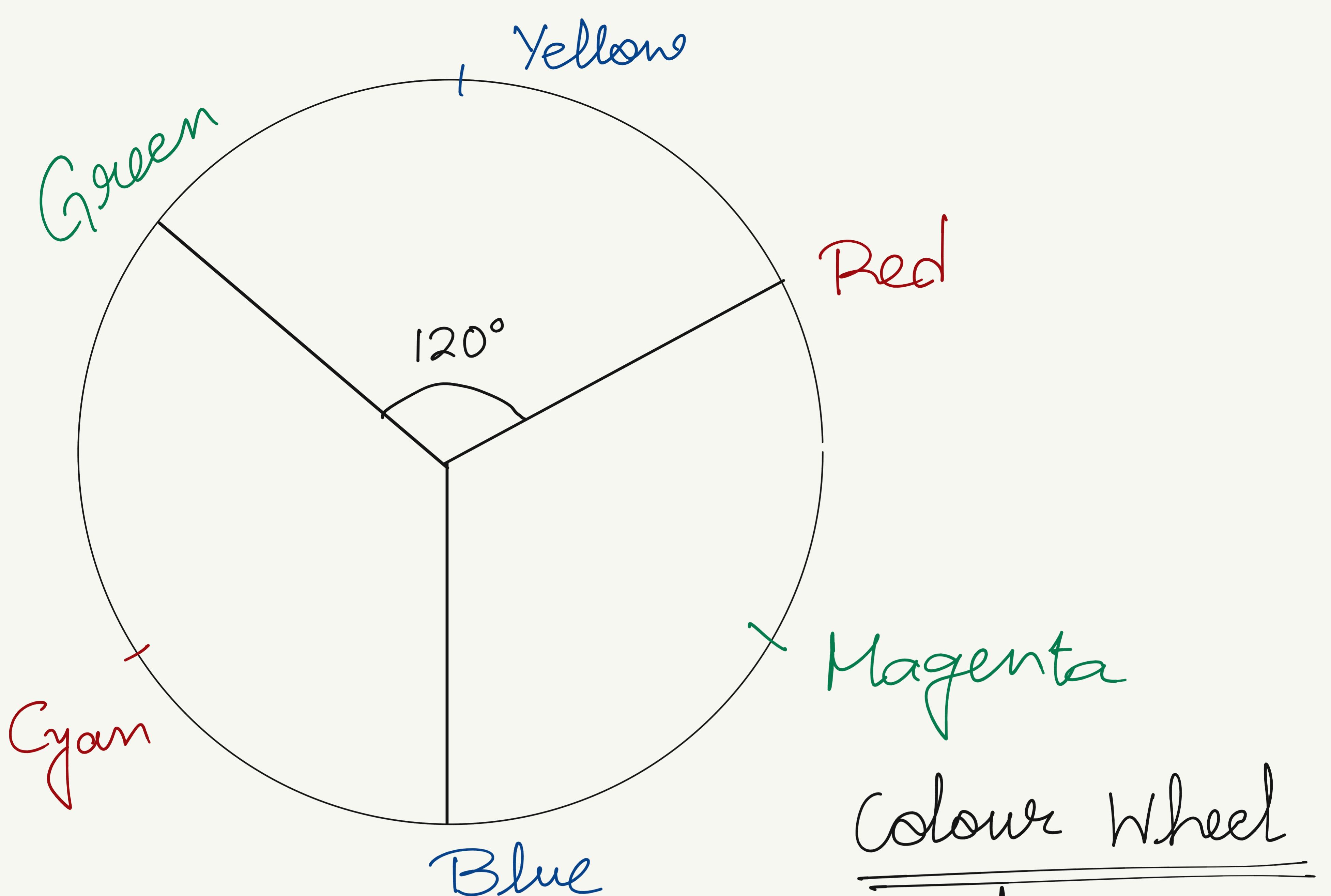


Normal  
Colours

**CMYK**

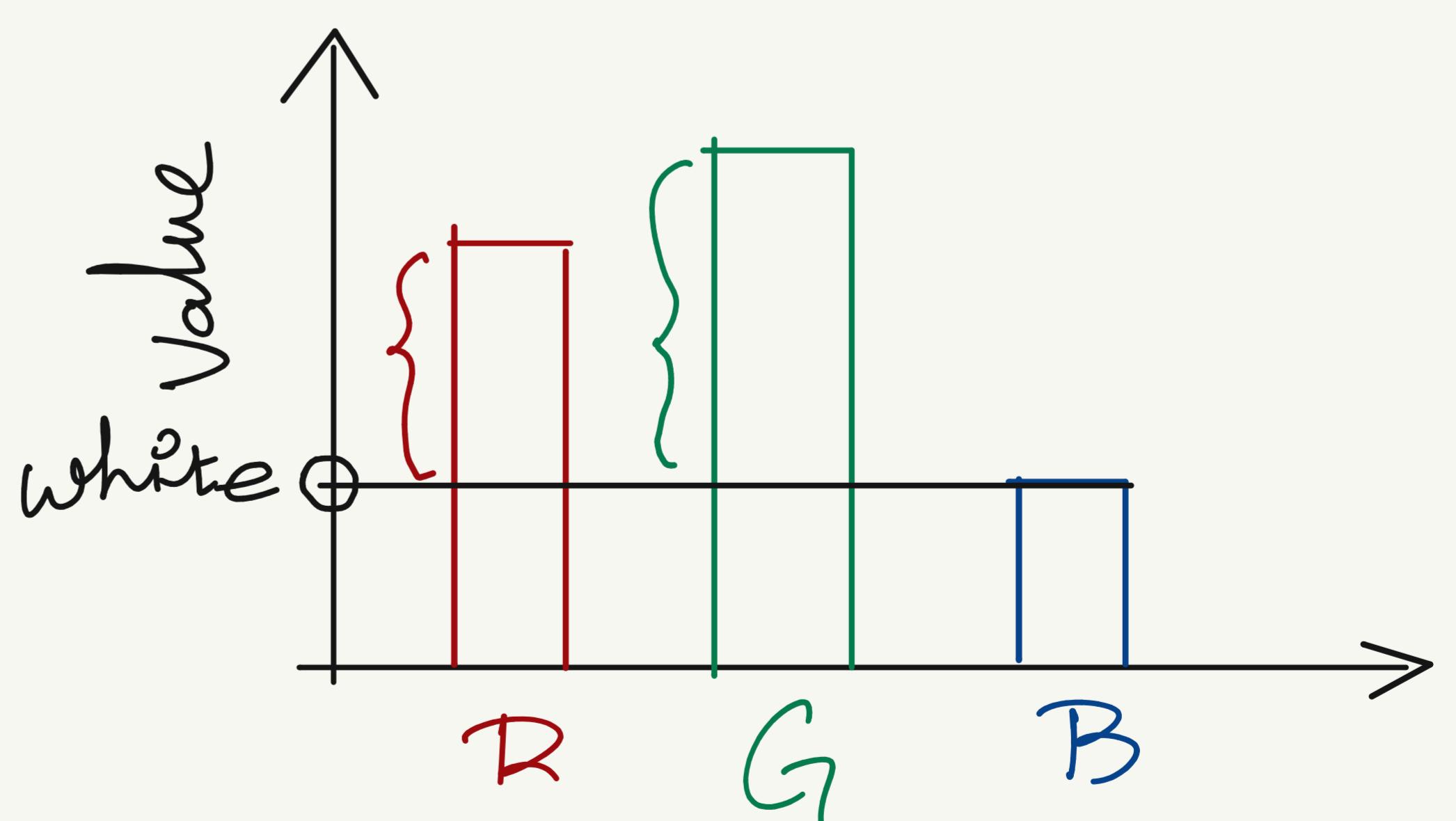


Complementary  
Colours



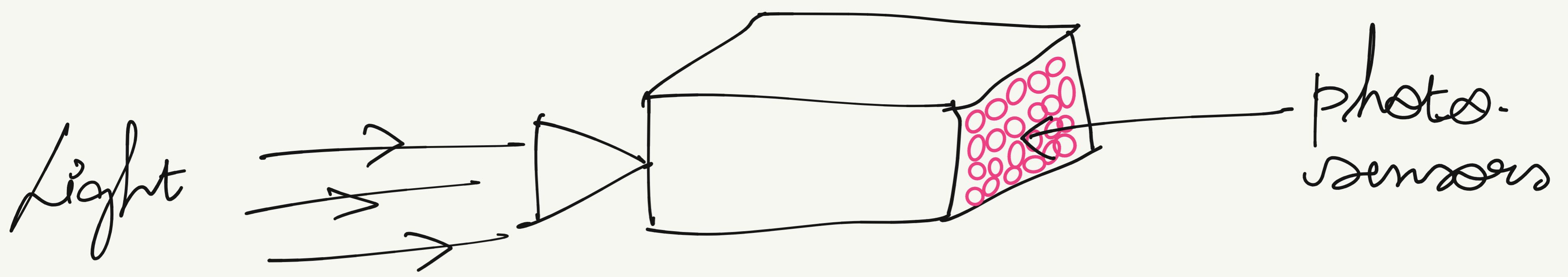
On the periphery all the colours are pure. At near the center (near white) colours are just mixtures of other colours.

$$g(x, y) = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



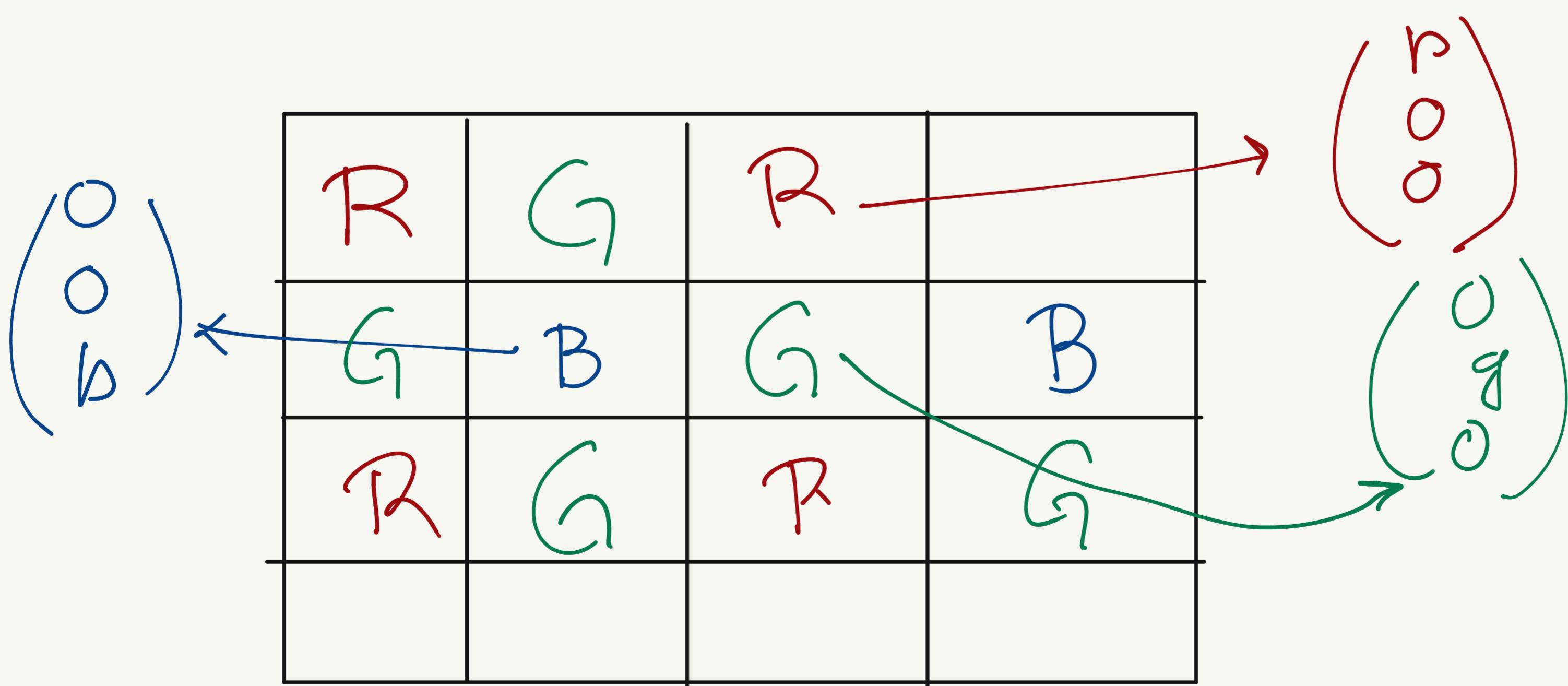
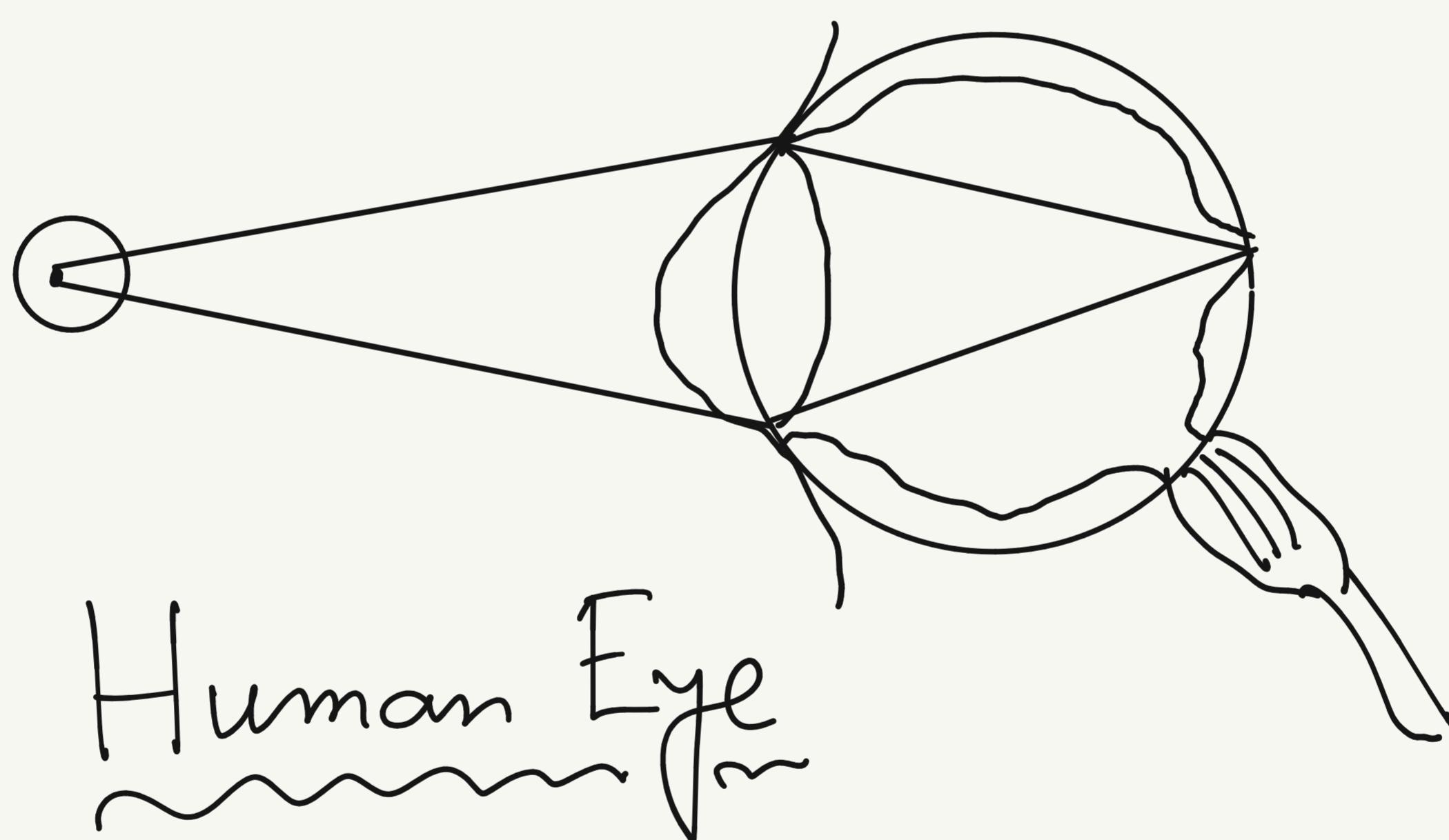
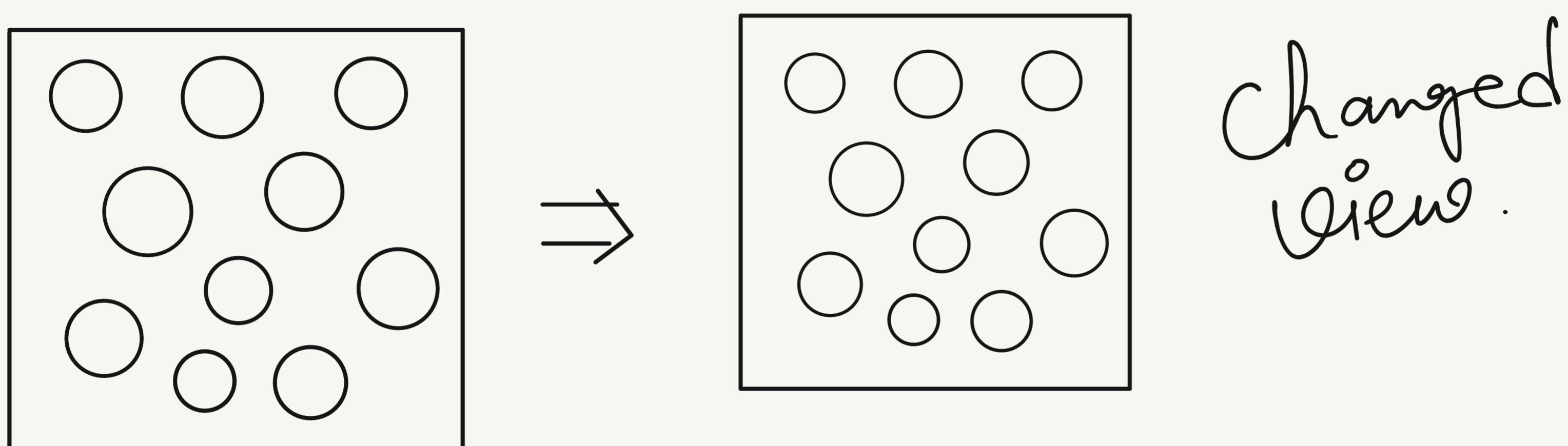
White = Red + Green + Blue

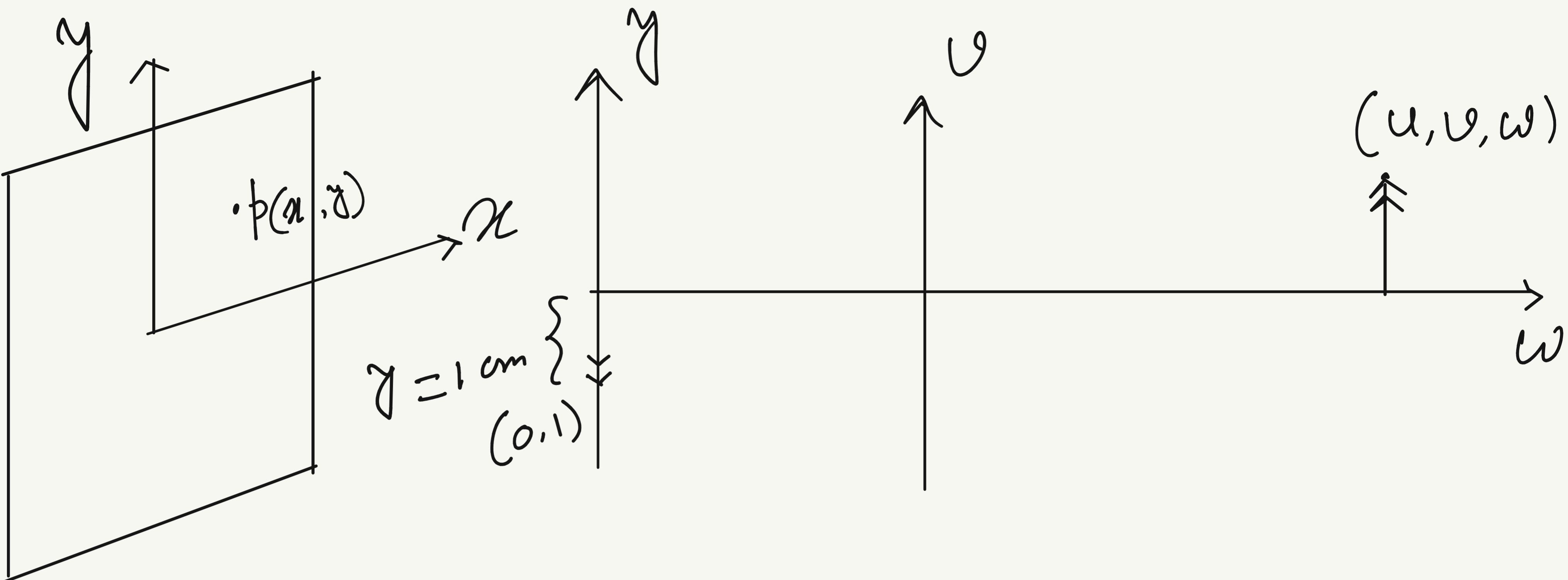
Additive  
Model



- there are many photo-sensors in the image plane.

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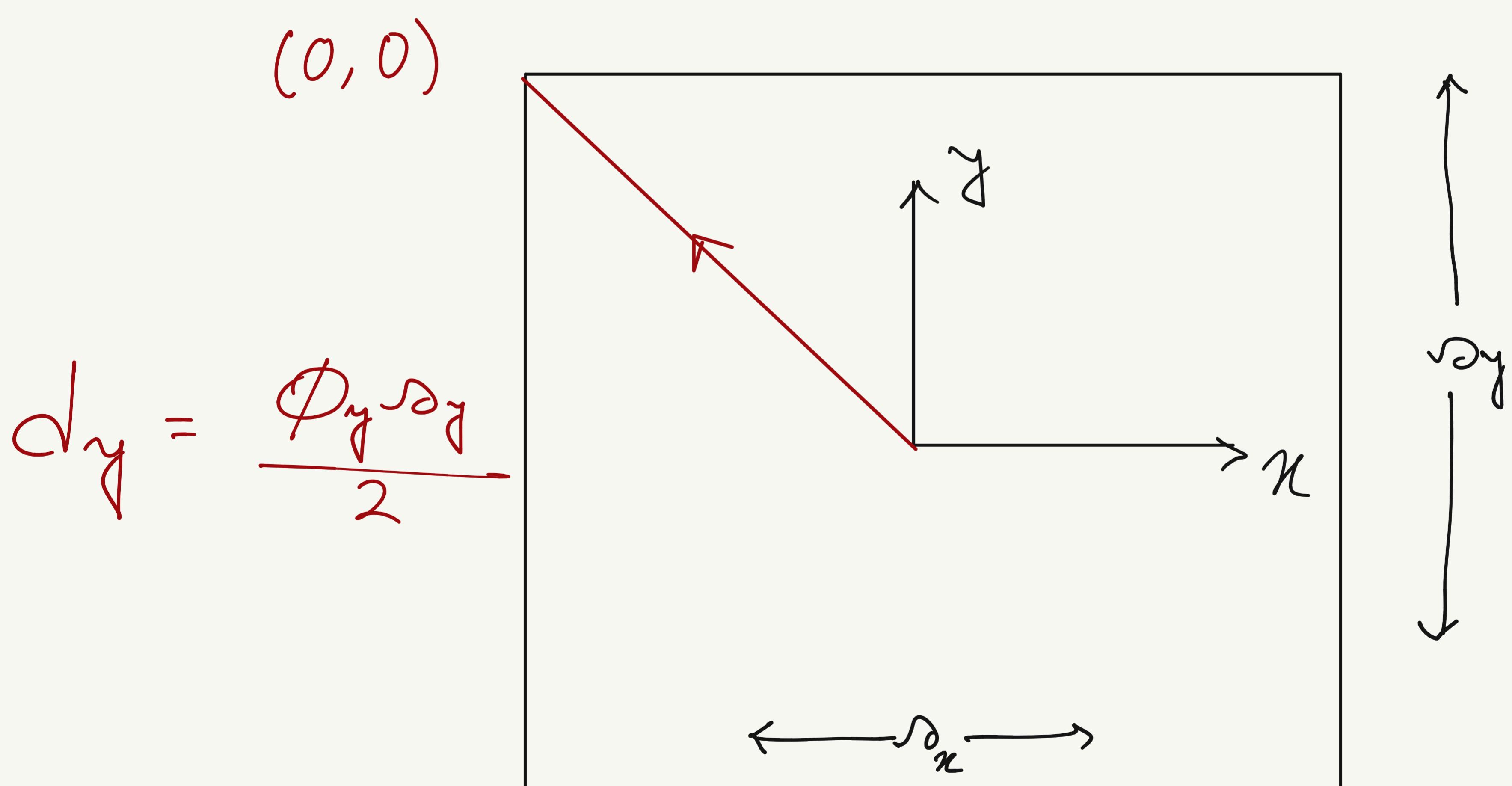




$$y \equiv \phi_y \gamma \quad [\phi_y \rightarrow \text{No. of photo sensors per unit length } (\gamma)]$$

$$x \equiv f \cdot \frac{u}{\omega} \phi_x \quad (\text{no. of pixels})$$

$$\text{Similarly, } y \equiv f \cdot \frac{v}{\omega} \cdot \phi_y$$



$$d_x = \frac{\phi_x \delta_x}{2}$$

$$x \equiv f \cdot \frac{u}{\omega} \cdot \phi_x + d_x = x' + d_x$$

$$y \equiv f \cdot \frac{v}{\omega} \phi_y + d_y = y' + d_y$$

$$\begin{bmatrix} x \\ y \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ \lambda' \end{bmatrix}$$

Also,

$$\begin{bmatrix} x \\ y \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi_x & 0 & 0 \\ 0 & \phi_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ \lambda' \end{bmatrix} = \begin{bmatrix} \phi_x & 0 & \frac{d_x}{f} \\ 0 & \phi_y & \frac{d_y}{f} \\ 0 & 0 & \frac{1}{f} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

ie  $x' = \phi_x u + \frac{\omega d_x}{f}$

$y' = \phi_y v + \frac{\omega d_y}{f}$

$\lambda = \frac{\omega}{f}$

} Homogeneous System

Now,

$$x = \frac{x'}{\lambda} = \phi_x u \cdot \frac{f}{\omega} + d_x \quad } \text{Non-homogeneous System}$$

$$y = \frac{y'}{\lambda} = \phi_y v \cdot \frac{f}{\omega} + d_y \quad }$$

$$x = f \cdot \frac{u}{\omega} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \omega = \frac{f \cdot B}{x - x'}$$

$$x' = f \cdot \frac{u - B}{\omega} \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$\therefore x' = \frac{f(u - B)}{f \cdot B / (x - x')}$$

$$u = \frac{\omega x}{f}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\begin{bmatrix} u'' \\ v'' \\ w'' \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_u \\ 0 & 1 & 0 & t_v \\ 0 & 0 & 1 & t_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u' \\ v' \\ w' \\ r \end{bmatrix}$$

ie.

$$\begin{bmatrix} u'' \\ v'' \\ w'' \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_u \\ 0 & 1 & 0 & t_v \\ 0 & 0 & 1 & t_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} & 0 \\ p_{21} & p_{22} & p_{23} & 0 \\ p_{31} & p_{32} & p_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$$

For 1<sup>st</sup> Camera :

06/03/2023

$$P \begin{pmatrix} u \\ v \\ w \end{pmatrix} \longrightarrow p \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x = f \cdot \frac{u}{w}, \quad y = f \cdot \frac{v}{w}$$

$$\begin{pmatrix} u^c \\ v^c \\ w^c \end{pmatrix} = TR \begin{pmatrix} u^w \\ v^w \\ w^w \end{pmatrix} \longrightarrow p \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x' = f \cdot \frac{u^c}{w^c}, \quad y' = f \cdot \frac{v^c}{w^c}$$

$$\begin{pmatrix} x' \\ y' \\ z \\ w^c \end{pmatrix} = K \begin{pmatrix} R_{3 \times 3} \\ t_{3 \times 1} \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} x \\ y \end{pmatrix} \text{ as}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = K (R^+ +) \begin{pmatrix} u \\ v \\ w \\ 1 \end{pmatrix}$$

$$A = [K] [R \ t]$$

$$= [KR \ Kt]$$

$$= \left[ \begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{array} \right]$$

$$K = \left[ \begin{array}{ccc} \phi_x & 0 & \frac{dx}{f} \\ 0 & \phi_y & \frac{dy}{f} \\ 0 & 0 & \frac{1}{f} \end{array} \right] \quad \text{Upper triangular}$$


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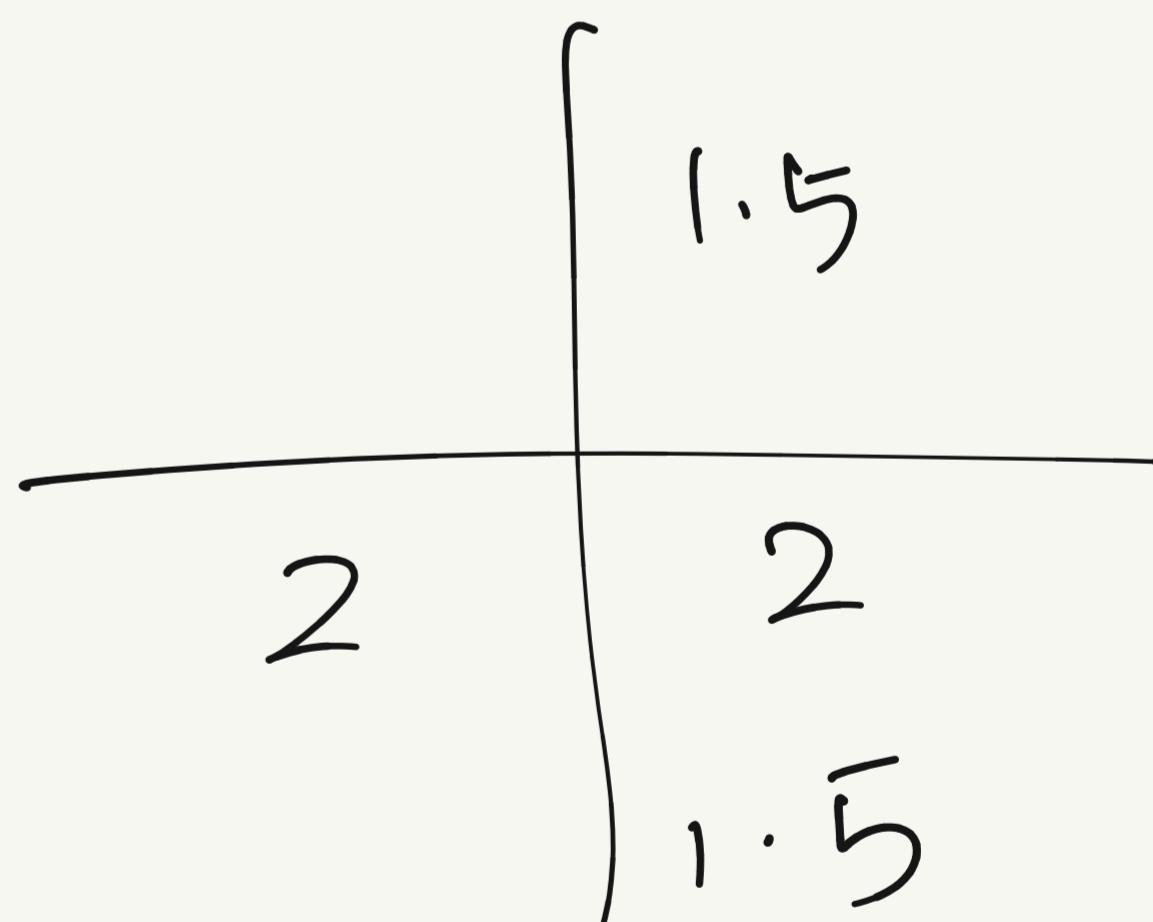
$$x = (1.5) \cdot \frac{2}{10} \quad , \quad y = (1.5) \cdot \frac{5}{10}$$

$$= 0.3 \quad , \quad = 0.75 .$$

$$0.3 \times 1200 = 360 +$$

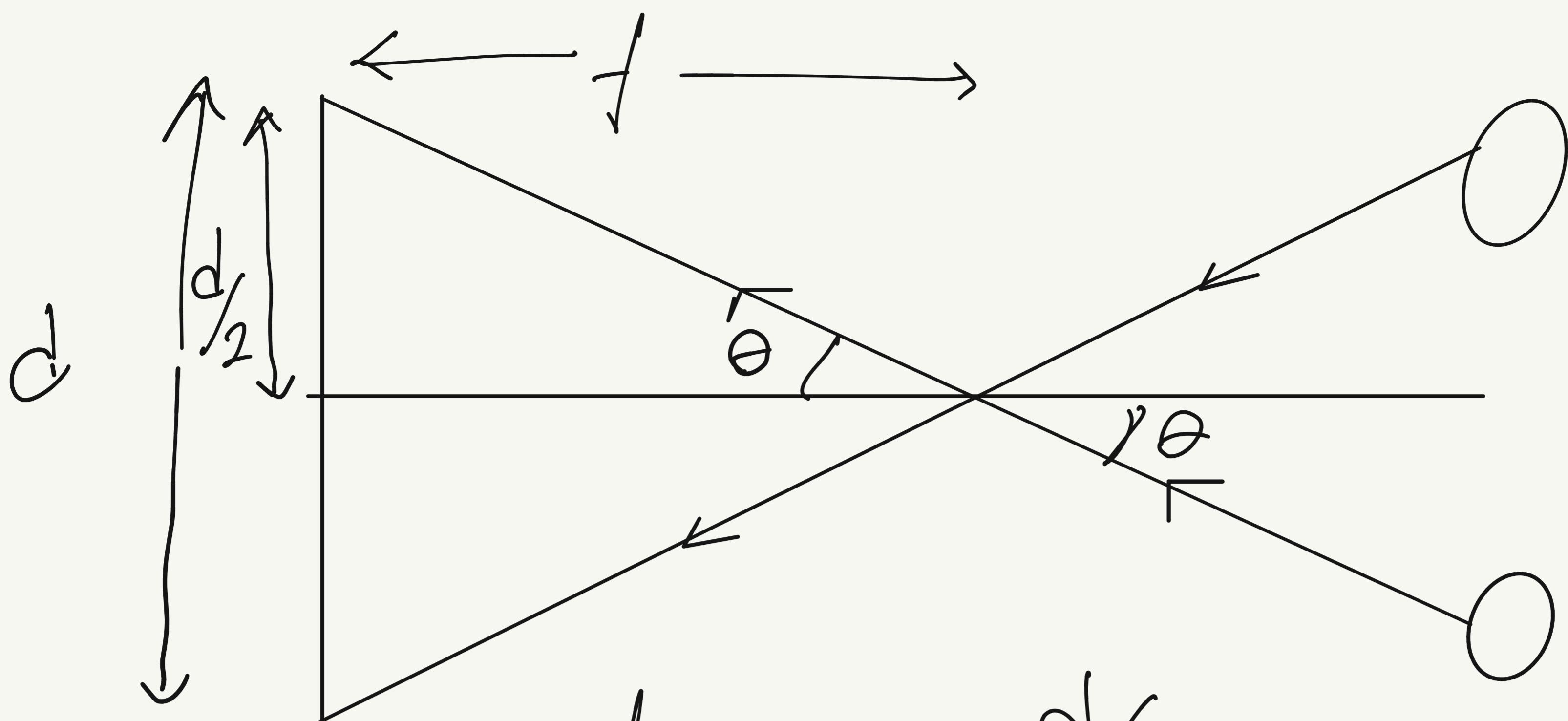
$$0.75 \times 1200 = 900 +$$

$$\frac{4 \times 1200}{2} = 2400$$

$$\frac{3 \times 1200}{2} = 1800$$


$\therefore \text{Co-ordinate} = (2760, 2700)$

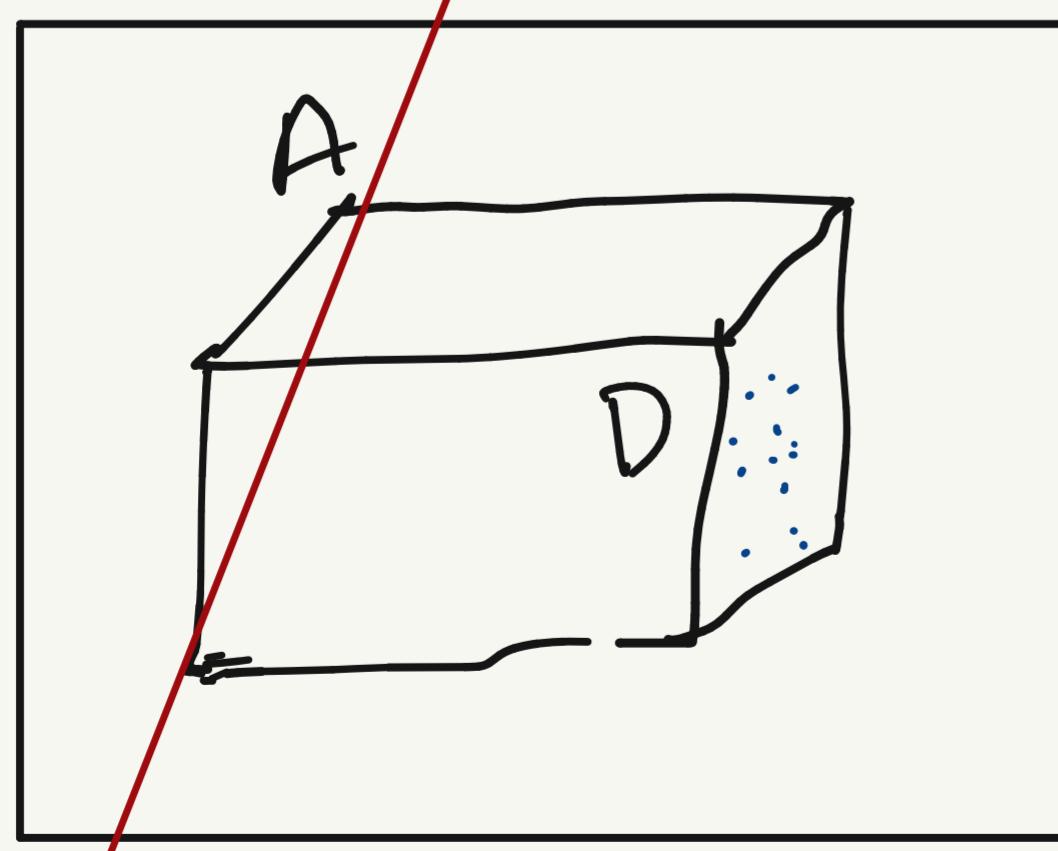
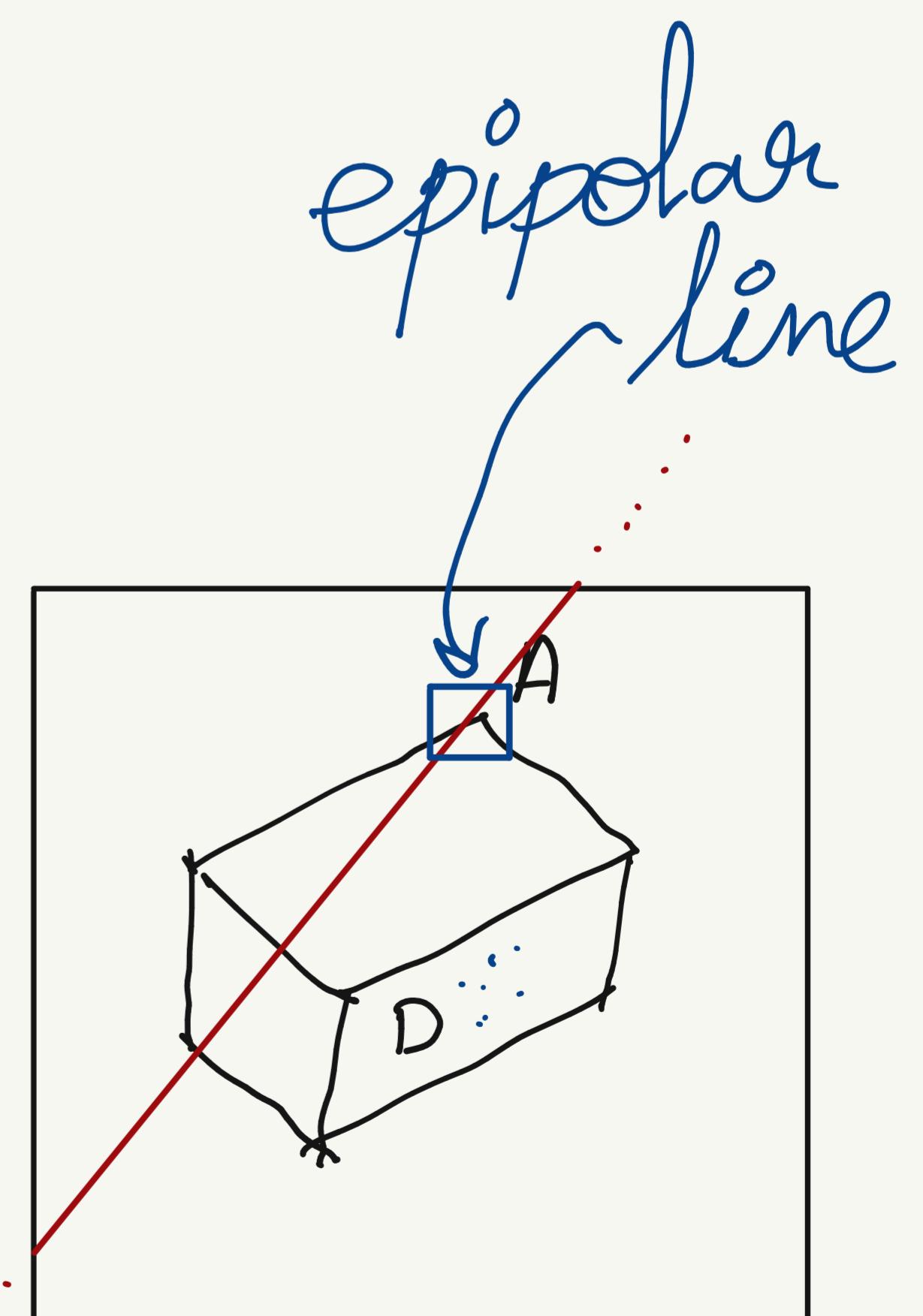
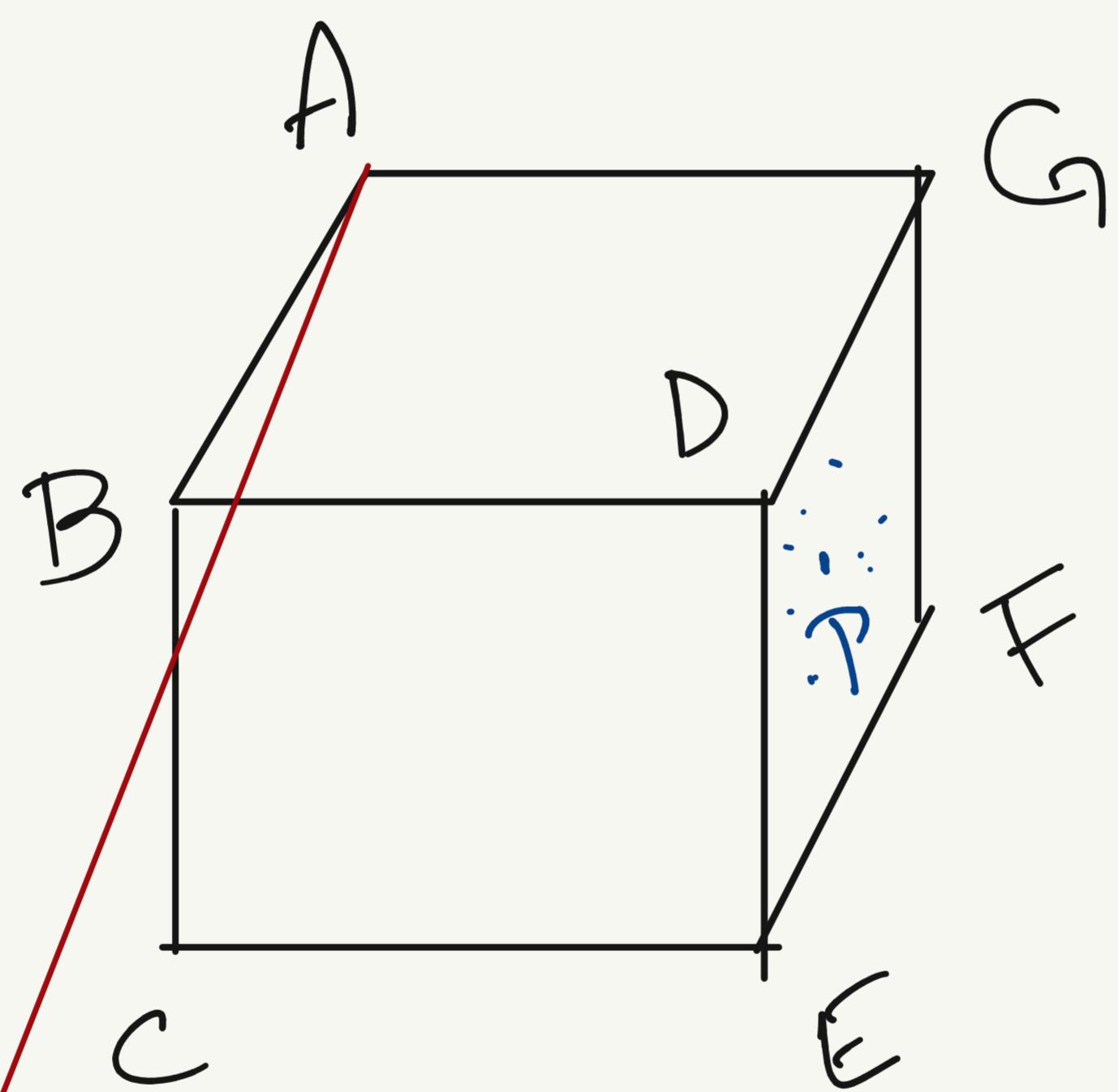
X



$$\tan \theta = \frac{d/2}{f}$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{d/2}{f} \right)$$

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$C_1$

$C_2$

To get the points we have to describe the neighbourhood of the point.

For a fixed point there must be an epipolar line, and the image of the point must lie on the epipolar line.

$$\begin{matrix} \text{Real} \\ \begin{pmatrix} \lambda x \\ \lambda y \\ \lambda \end{pmatrix} \end{matrix} = \begin{bmatrix} \phi_m & 0 & d\phi_m/dx \\ 0 & \phi_y & d\phi_y/dy \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} u^c \\ v^c \\ w^c \\ 1 \end{pmatrix} = [R \mid t] \begin{pmatrix} u^w \\ v^w \\ w^w \\ 1 \end{pmatrix}$$

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad P_h^T = \begin{pmatrix} \lambda x \\ \lambda y \\ \lambda \end{pmatrix}$$

Then,

$$\begin{pmatrix} u^c \\ v^c \\ w^c \\ 1 \end{pmatrix} = K^{-1} P_h^T$$

i.e.

$$Y = A V$$

$m \times 1$ 
 $m \times m$ 
 $m \times 1$

$$\hat{P}_h^I = RP^{\omega} + t$$

For Camera 1:  $\hat{P}_m^{I_1} = P^{\omega}$

For Camera 2:  $\hat{P}_m^{I_2} = RP^{\omega} + t$

i.e.  $\hat{P}_m^{I_2} = R \hat{P}_m^{I_1} + t$

Now,  $t \times \hat{P}_m^{I_2} = t \times R \hat{P}_m^{I_1} + t \times t$

$$= t \times R \hat{P}_m^{I_1}$$

$$\Rightarrow \hat{P}_m^{I_2} \cdot (t \times \hat{P}_m^{I_2}) = \hat{P}_m^{I_2} \cdot (t \times R \hat{P}_m^{I_1})$$

$$\Rightarrow \hat{P}_m^{I_2} \cdot \left[ \begin{matrix} t \\ R \hat{P}_m^{I_1} \end{matrix} \right] = 0$$

$$\vec{a} = a \hat{i} + b \hat{j} + c \hat{k}$$

$$\vec{t} = t_u \hat{i} + t_v \hat{j} + t_w \hat{k}$$

$$\vec{t} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t_u & t_v & t_w \\ a & b & c \end{vmatrix} = (c t_v - b t_w) \hat{i} + (a t_w - c t_u) \hat{j} + (b t_u - a t_v) \hat{k}$$

$$\vec{T}^I \rightarrow \begin{bmatrix} 0 & -t_w & t_v \\ t_w & 0 & -t_u \\ -t_v & t_u & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Thus,

$$\hat{P}_m^{I_2} \cdot \left[ \begin{array}{c|c|c} t & R & \hat{P}_m^{I_1} \\ \hline 3 \times 1 & 3 \times 3 & 3 \times 1 \end{array} \right] = 0$$

$$\Rightarrow \hat{P}_m^{I_2} (T' R \hat{P}_m^{I_1}) = 0$$

$$\Rightarrow \hat{P}_m^{I_2} (E \hat{P}_m^{I_1}) = 0$$

$$\Rightarrow (\hat{P}_m^{I_2})^T E \hat{P}_m^{I_1}$$

where,  $E = T' R$  = Essential matrix

Camera 2 :

$$\left( \begin{array}{c} x \\ y \\ z \end{array} \right)^T E \hat{P}_m^{I_2} = 0$$

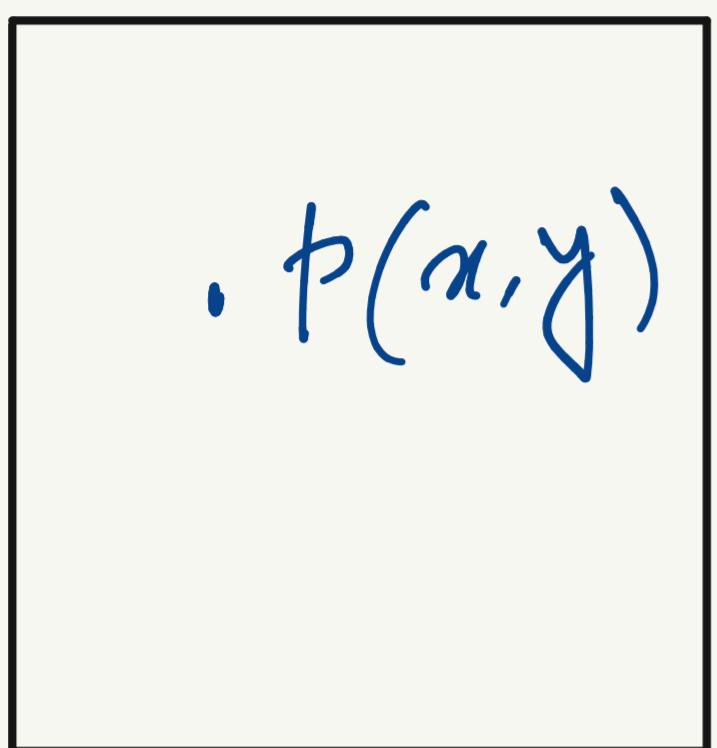
$\Rightarrow$  Equation of a st. line  
i.e. epipolar line on Camera 1.

$$(K_2^{-1} \hat{P}_m^{I_2})^T E (K_1^{-1} \hat{P}_m^{I_1}) = 0$$

$$\Rightarrow (\hat{P}_m^{I_2})^T ((K_2^{-1})^T E (K_1^{-1})) \hat{P}_m^{I_1} = 0$$

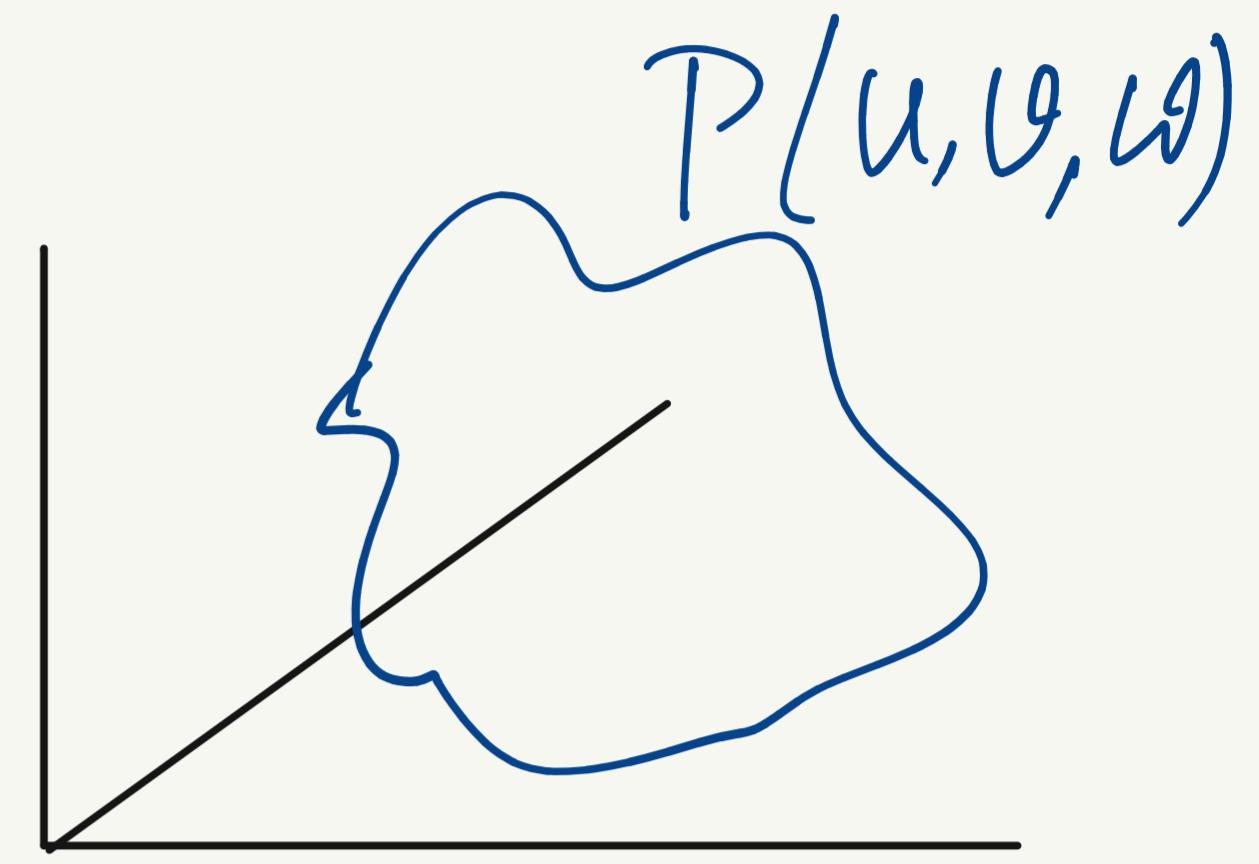
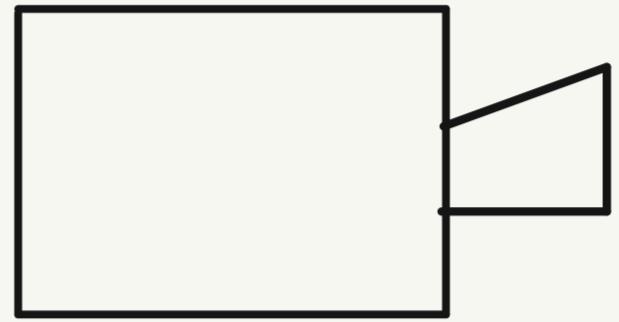
$$\Rightarrow \left( \begin{matrix} p_m^{I_2} \\ p_m^{I_1} \end{matrix} \right)^T F \left( \begin{matrix} p_m^{I_1} \\ p_m^{I_2} \end{matrix} \right) = 0$$

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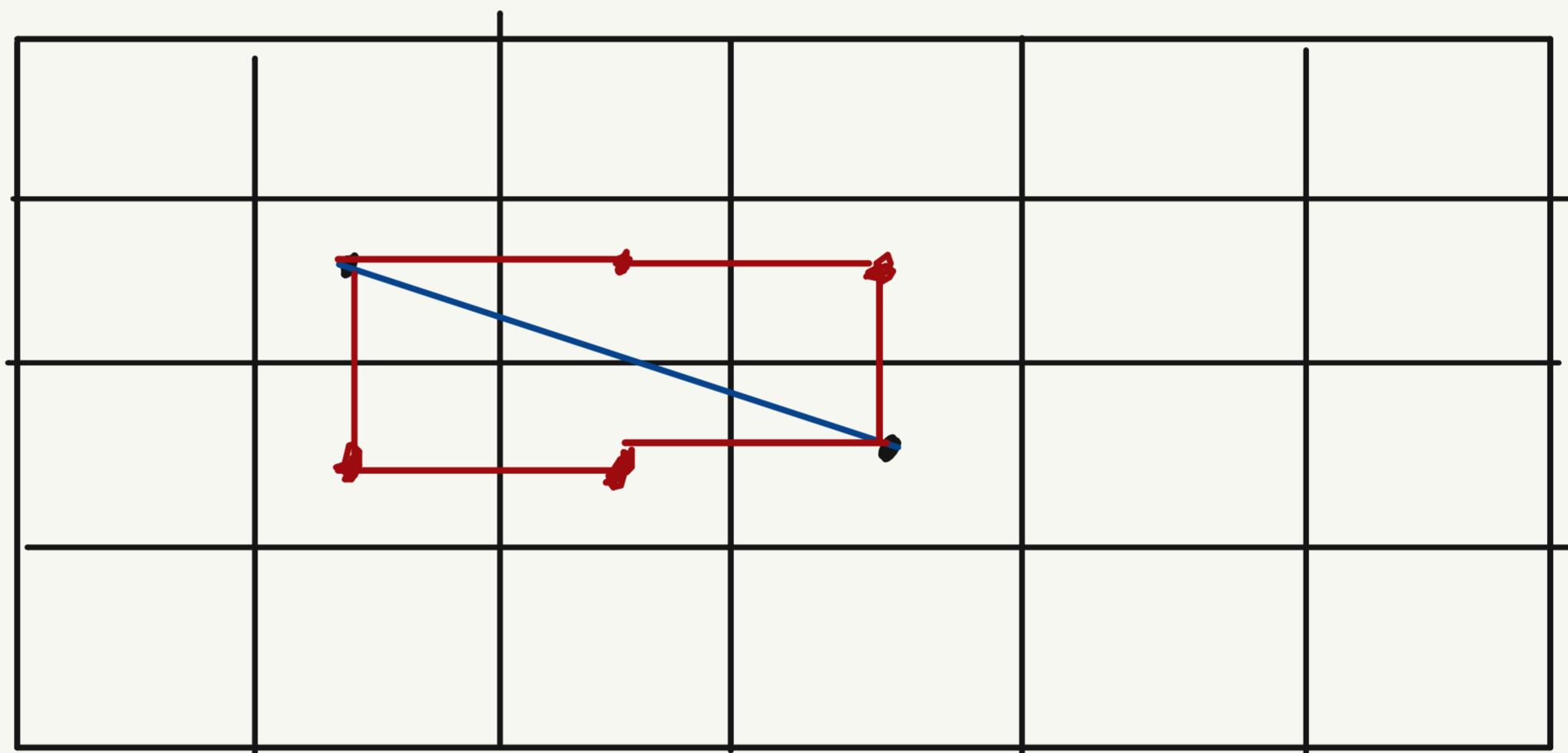


$$x = f \frac{u}{w}$$

$$y = f \frac{v}{w}$$



Object /  
Foreground:



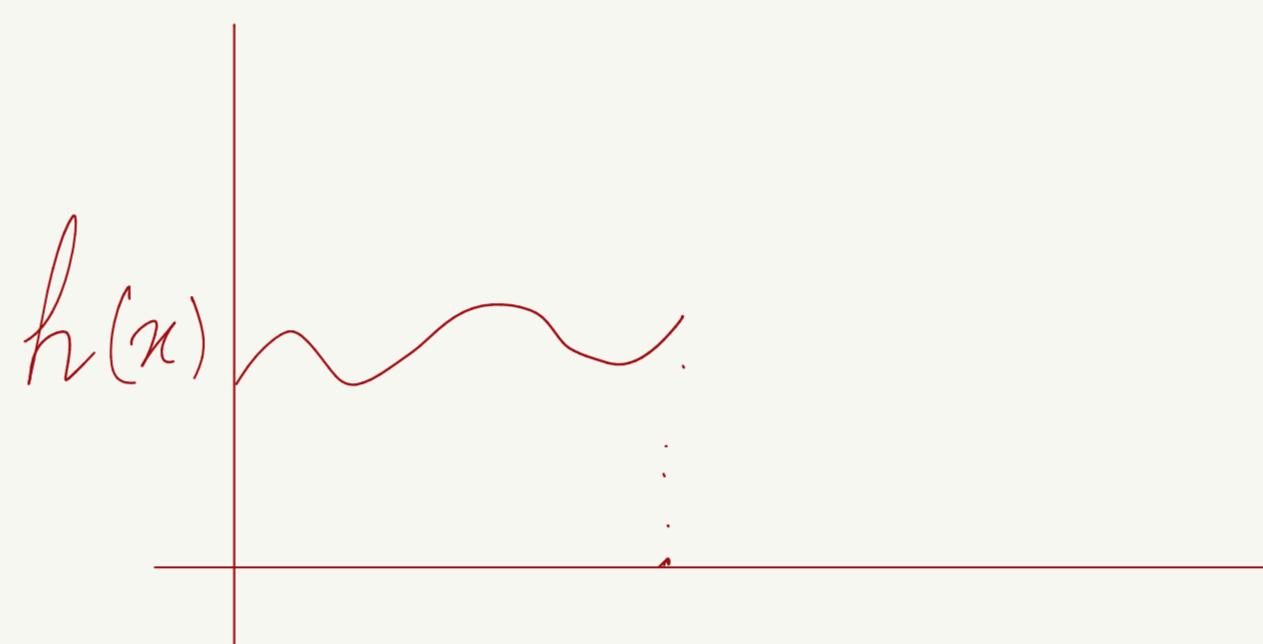
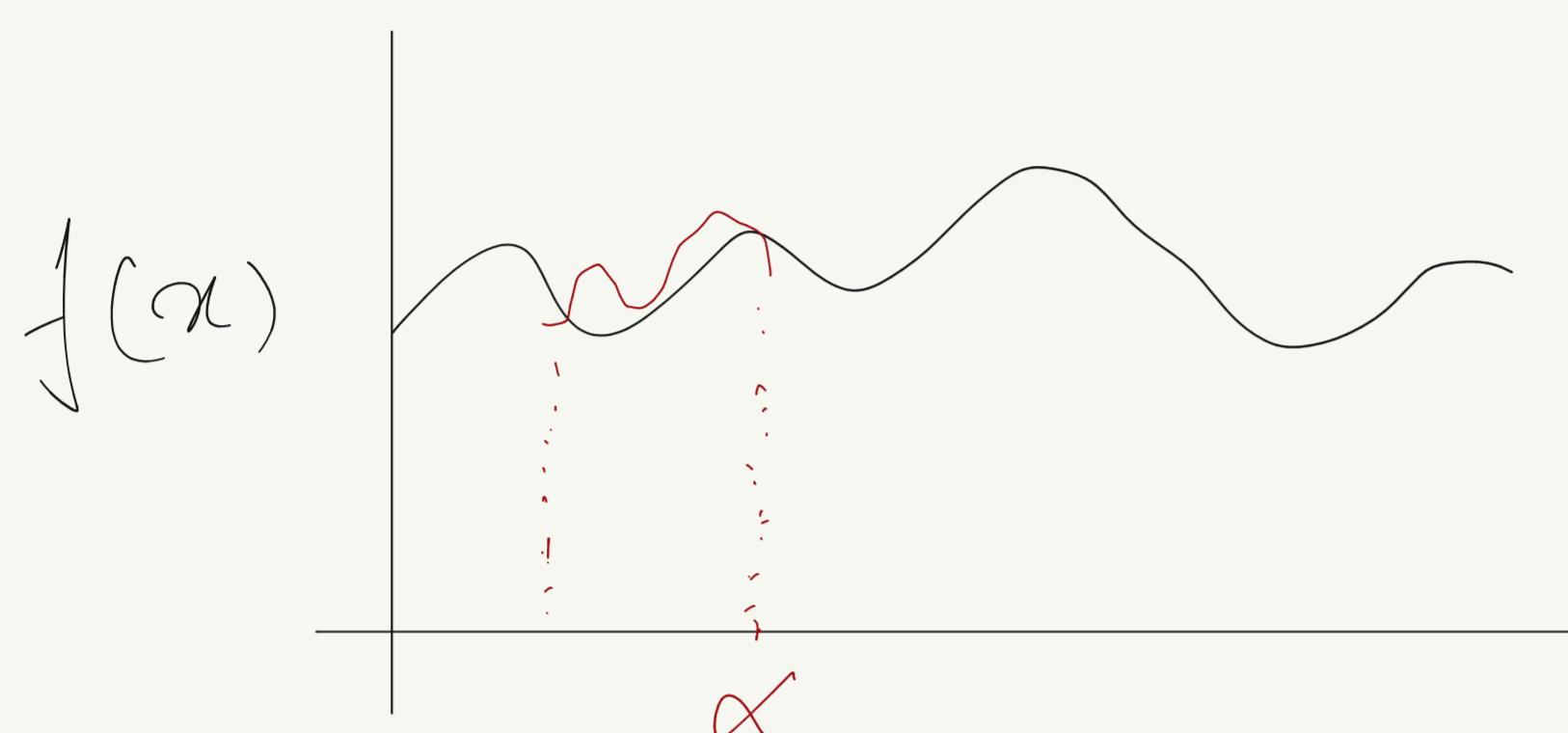
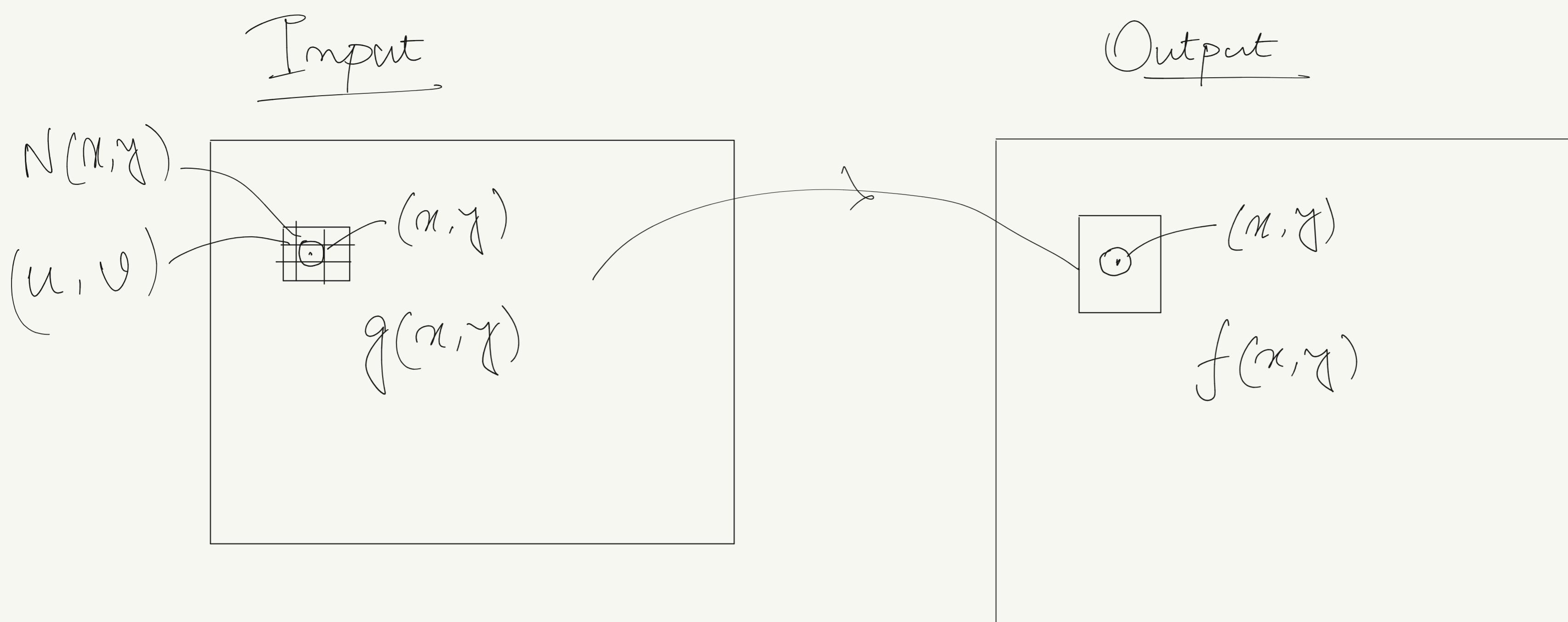
$$dist_E(P_0, Q_0) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Euclidean Distance doesn't work  
Since it is a discrete map

$$dist_H(P_0, Q_0) = |x_1 - x_2| + |y_1 - y_2|$$

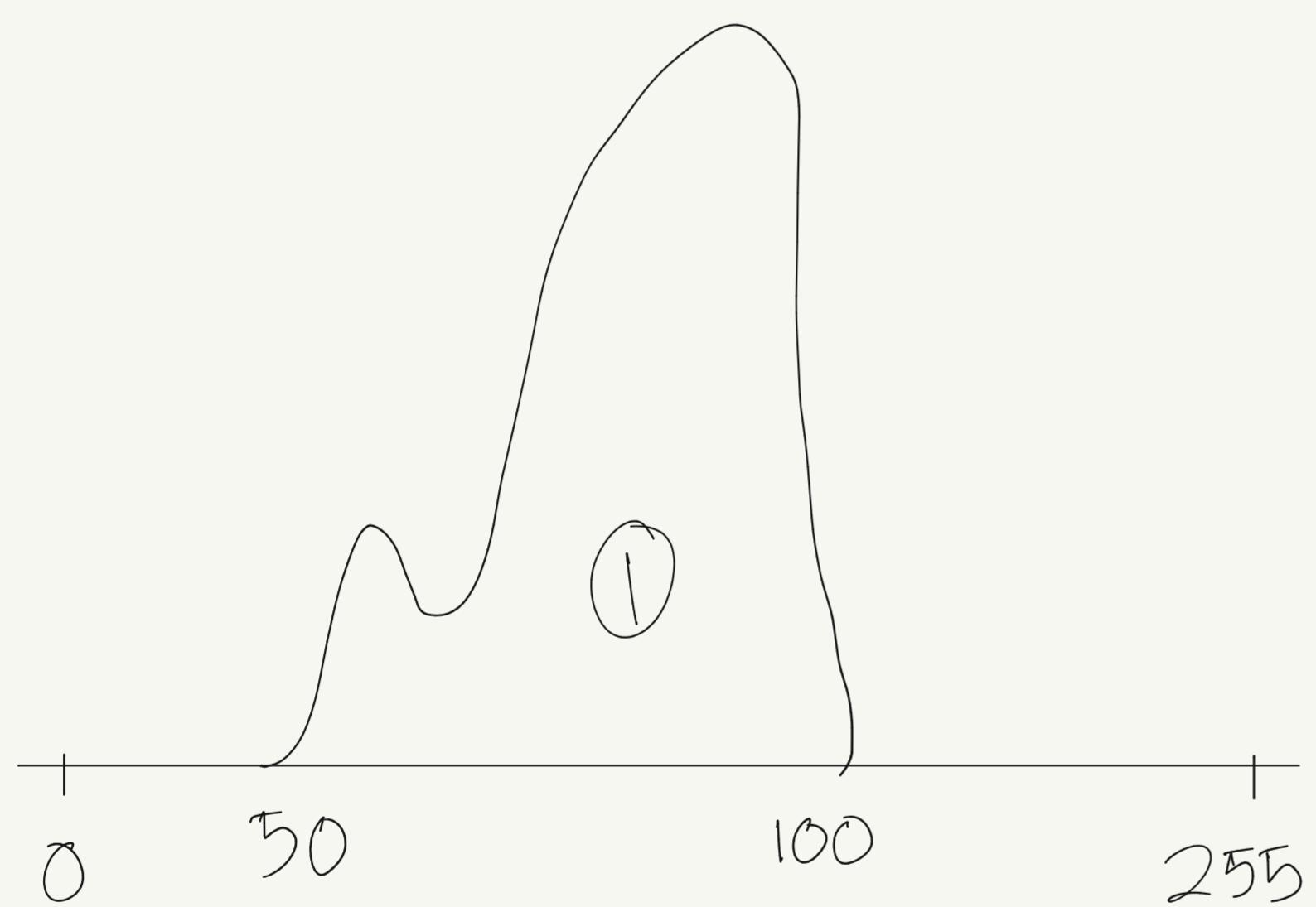
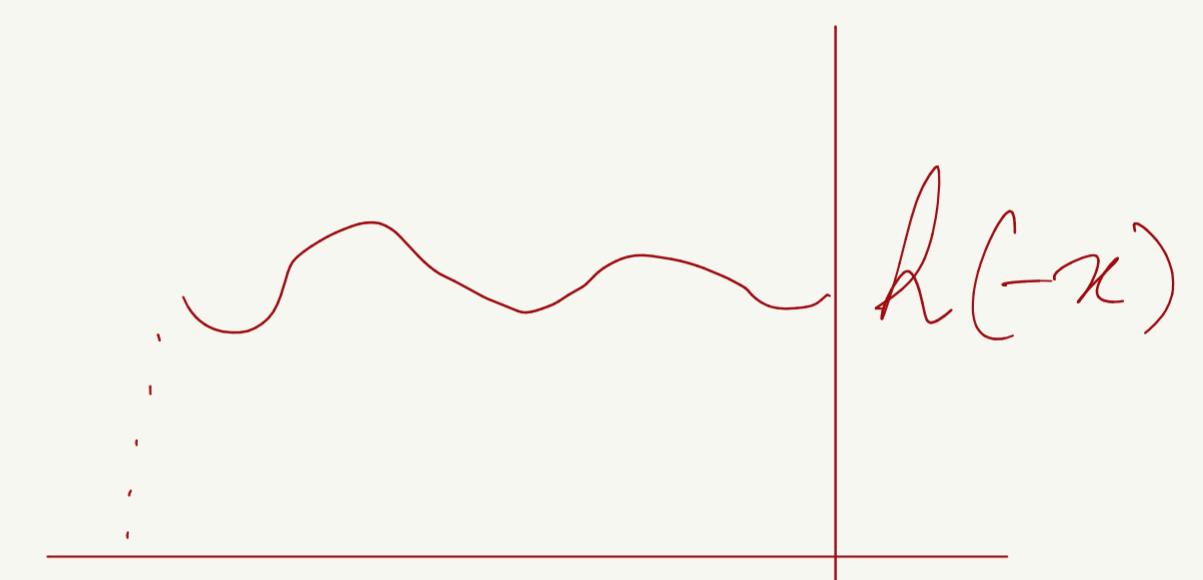
$$dist_g(P_0, Q_0) = \max \{|x_1 - x_2|, |y_1 - y_2|\}$$

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Convolution,  $f(x) * h(x)$

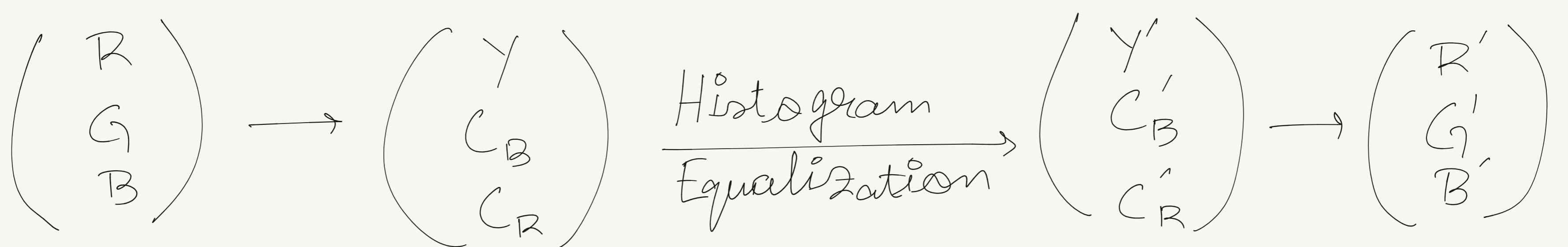
$$= \int f(x) h(x-x) dx$$



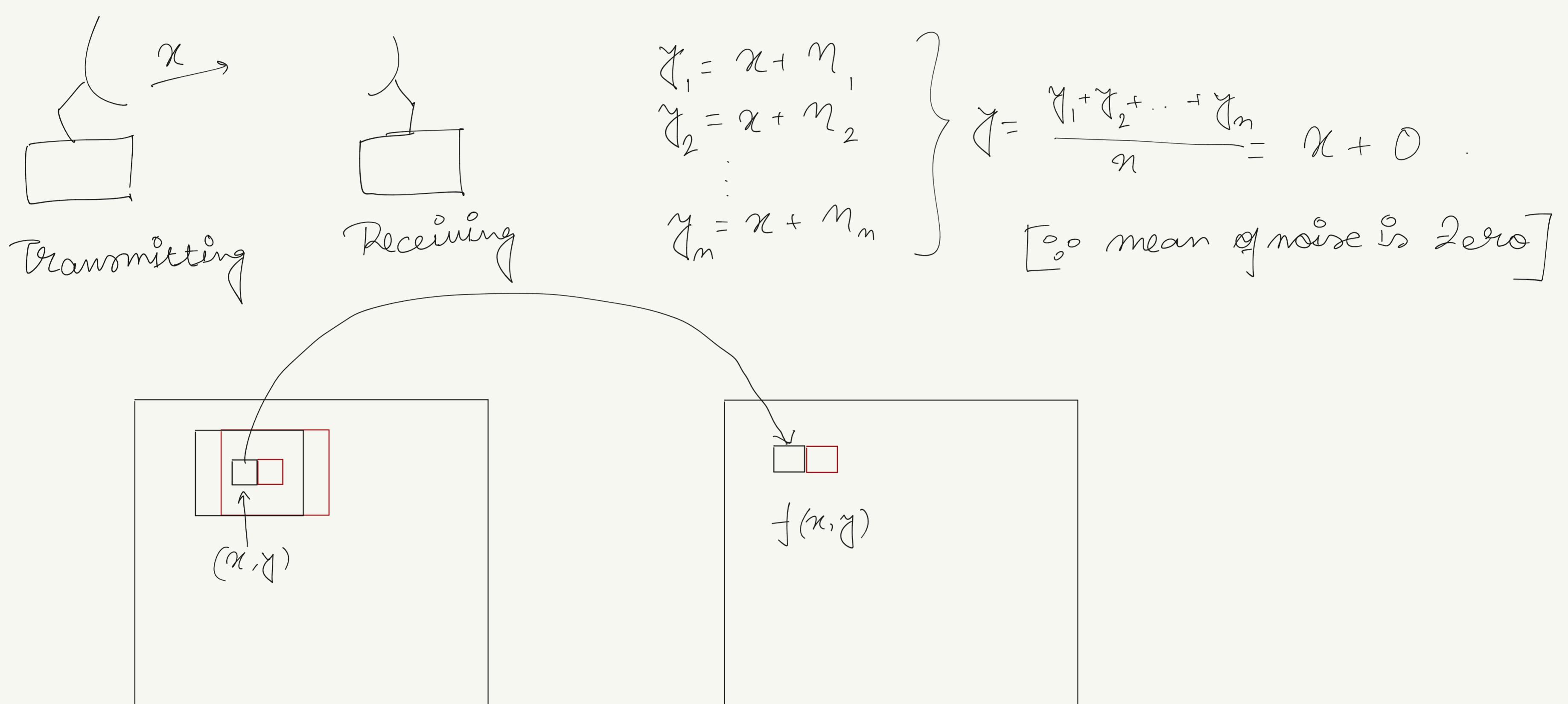
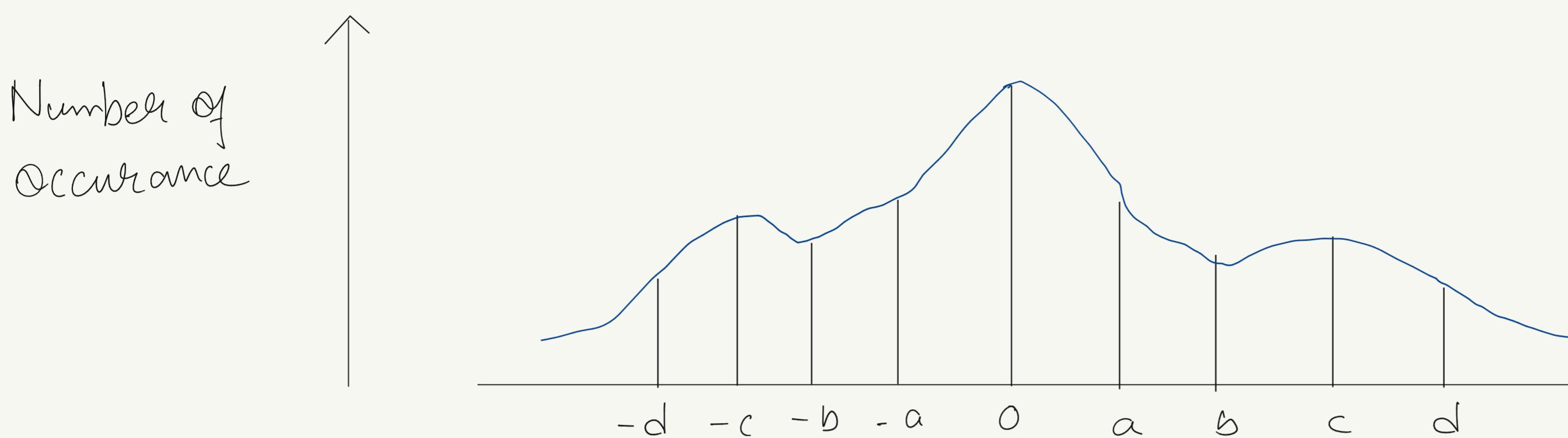
$$s = a(r - b) + c = 2.5(r - 50) + 0$$

we are changing  $(50-100)$  range to  $(0-250)$  range

If the range  $(0-255)$  exceed then the computer store the data as  $(x \bmod 256)$



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$$ax + by + cz + d = 0 \quad (\text{Eq. of a plane in 3D})$$

$$\Rightarrow \frac{a}{c}x + \frac{b}{c}y + z + \frac{d}{c} = 0$$

$$\Rightarrow z = -\frac{a}{c}x - \frac{b}{c}y - \frac{d}{c}$$

$$\text{i.e. } f(x, y) = Ax + By + C = A(x - x_0) + B(y - y_0) + C$$

But in reality,

$$g(x, y) = A(x - x_0) + B(y - y_0) + C + \gamma(x, y) \leftarrow \text{Noise}$$

$$\text{i.e. } g(x, y) = f(x, y) + \gamma(x, y)$$

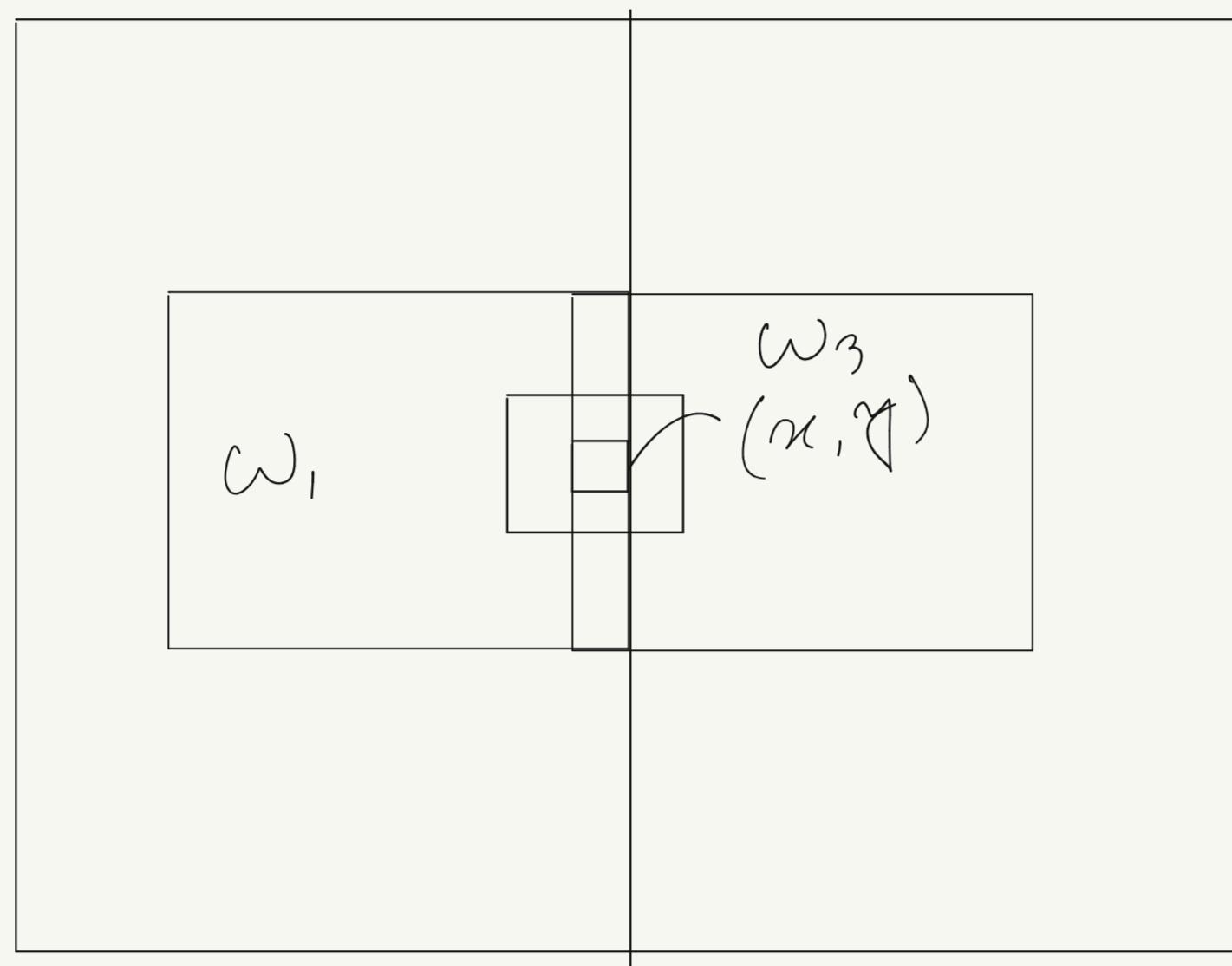
Now, if we get the values of A, B, C then  
we can estimate the  $f(x, y)$ .

thus,

$$[g(x, y) - A(x - x_0) - B(y - y_0) - C]^2 = [\eta(x, y)]^2$$

$$\text{Let, } \ell(A, B, C) = \sum_{(x, y) \in W} [g(x, y) - A(x - x_0) - B(y - y_0) - C]^2 - \sum_{(x, y) \in W} [\eta(x, y)]^2$$

Now, we are minimizing the error term  $\ell(A, B, C)$



$\text{Var}(W, \text{region}) \sim 0$   
 $\text{Var}(w_3, \text{region}) \text{ is some significant value} > 0$

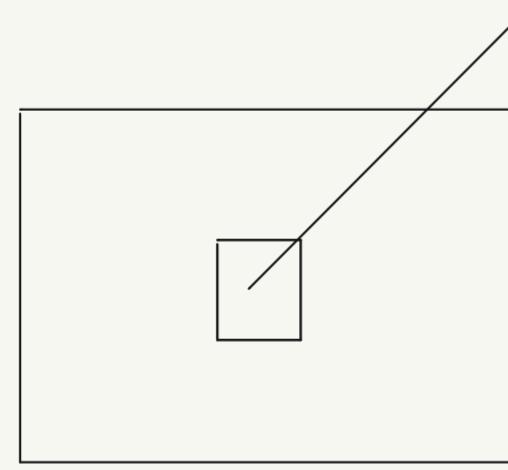
Fourier Series :

$$f(x) = \sum a_i \phi_i(x)$$

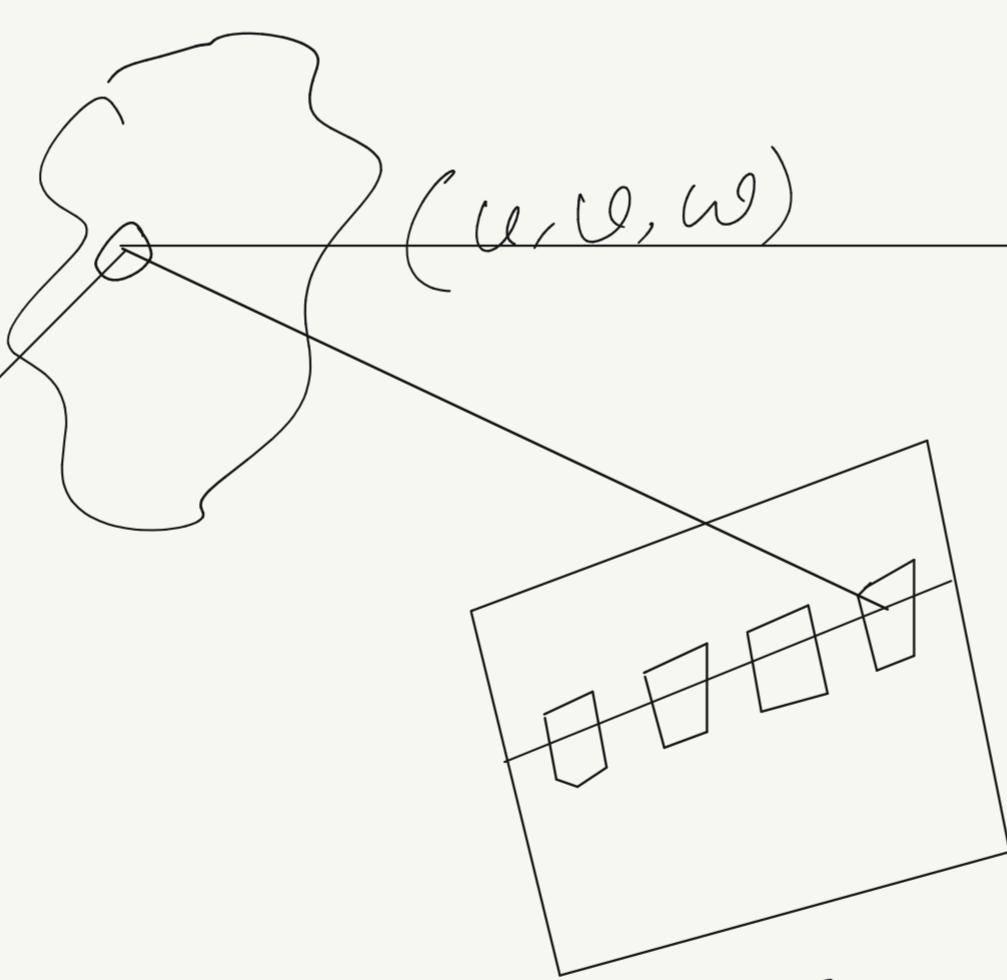
as  $i$  goes higher the frequency of  $\phi_i(x)$  goes higher.

Assignment :

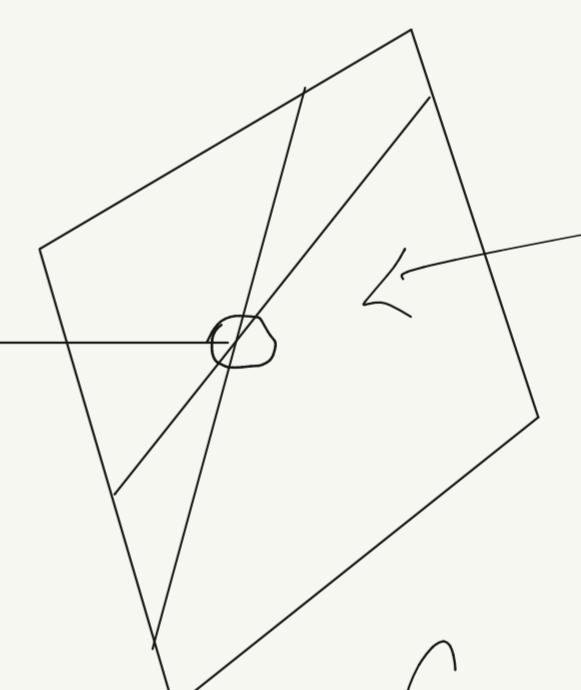
Multiview  
Geometry



$C_1$



$C_2$



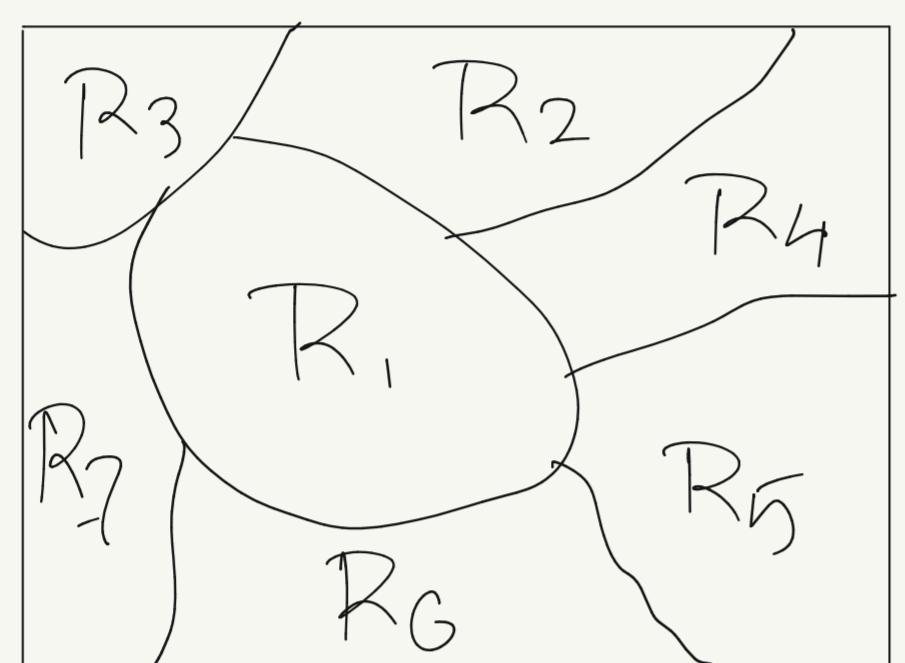
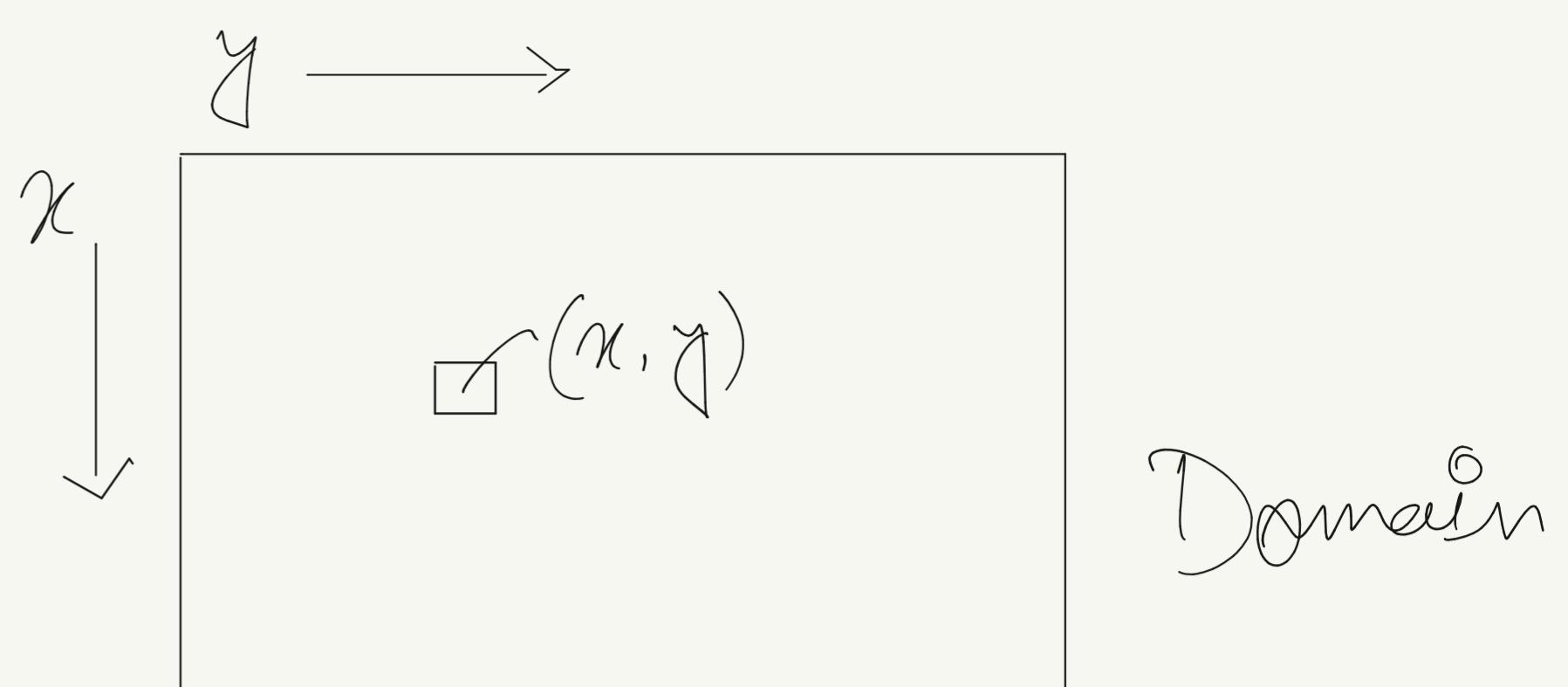
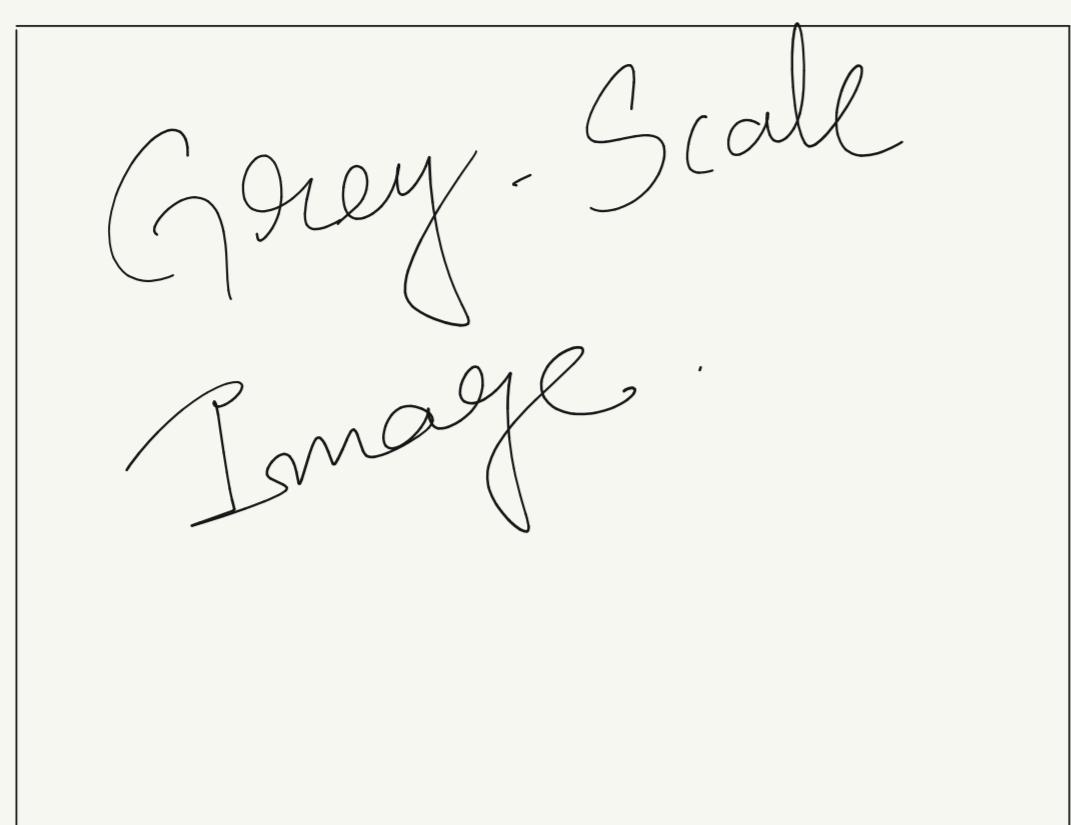
$C_3$

epipolar line

3D Zephher / Mesh Room ← packages.

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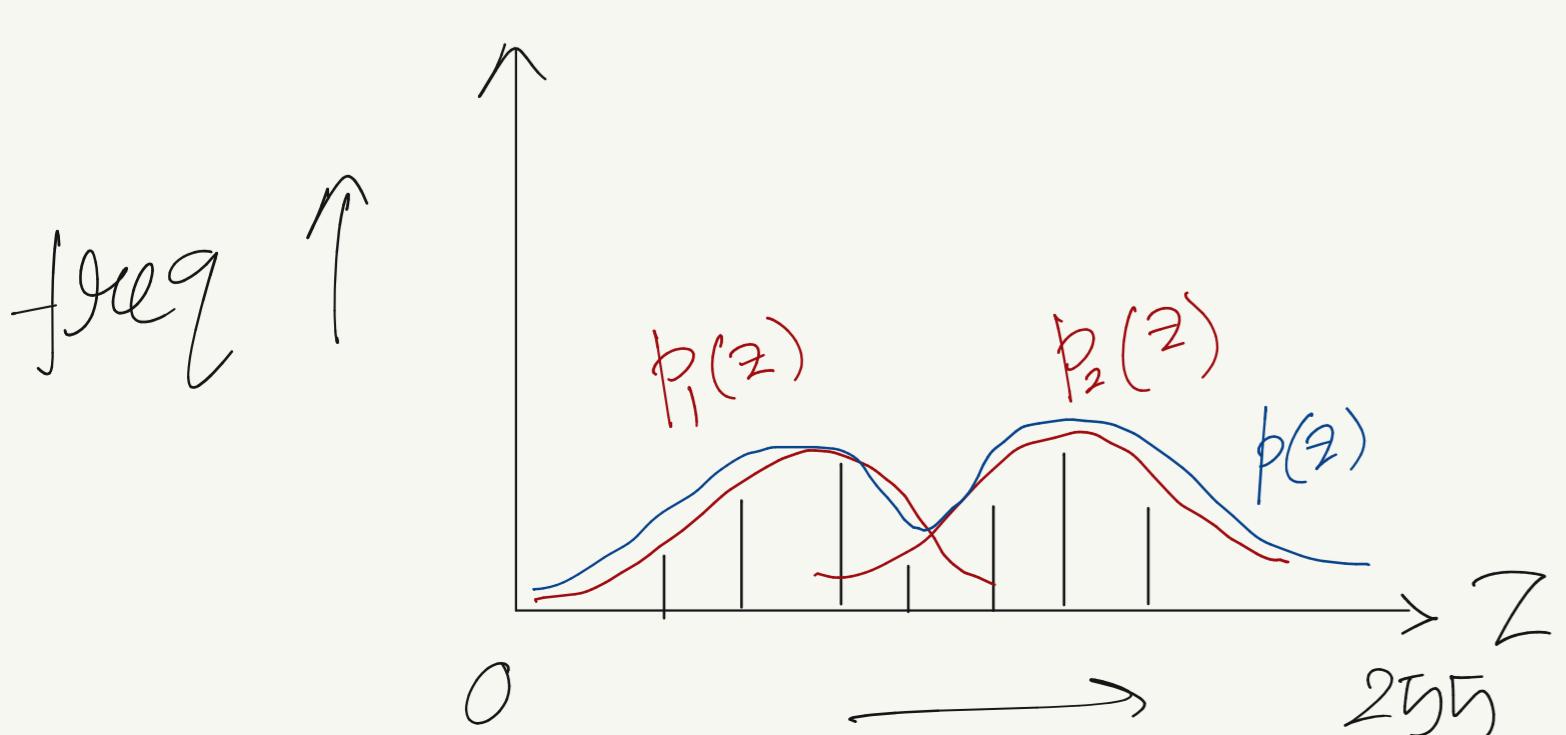
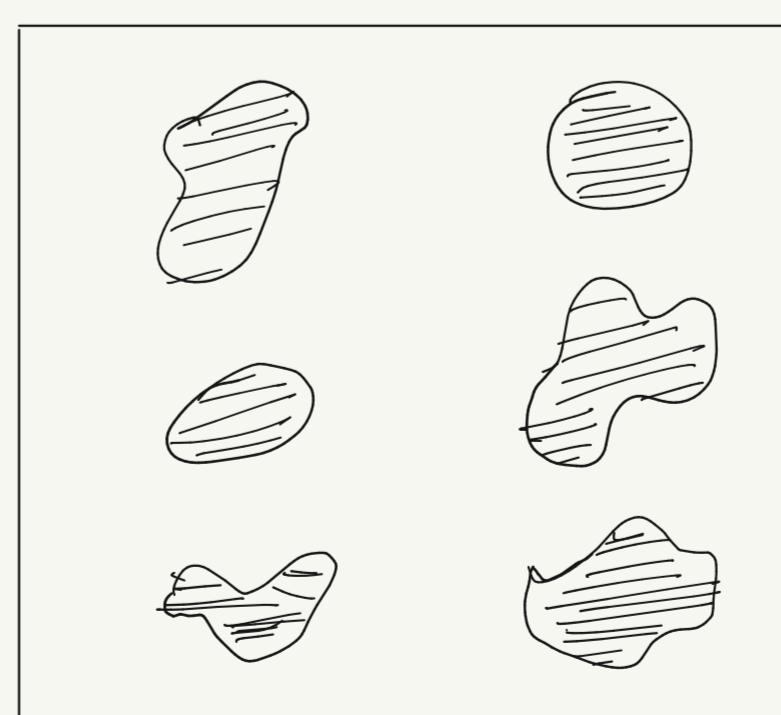
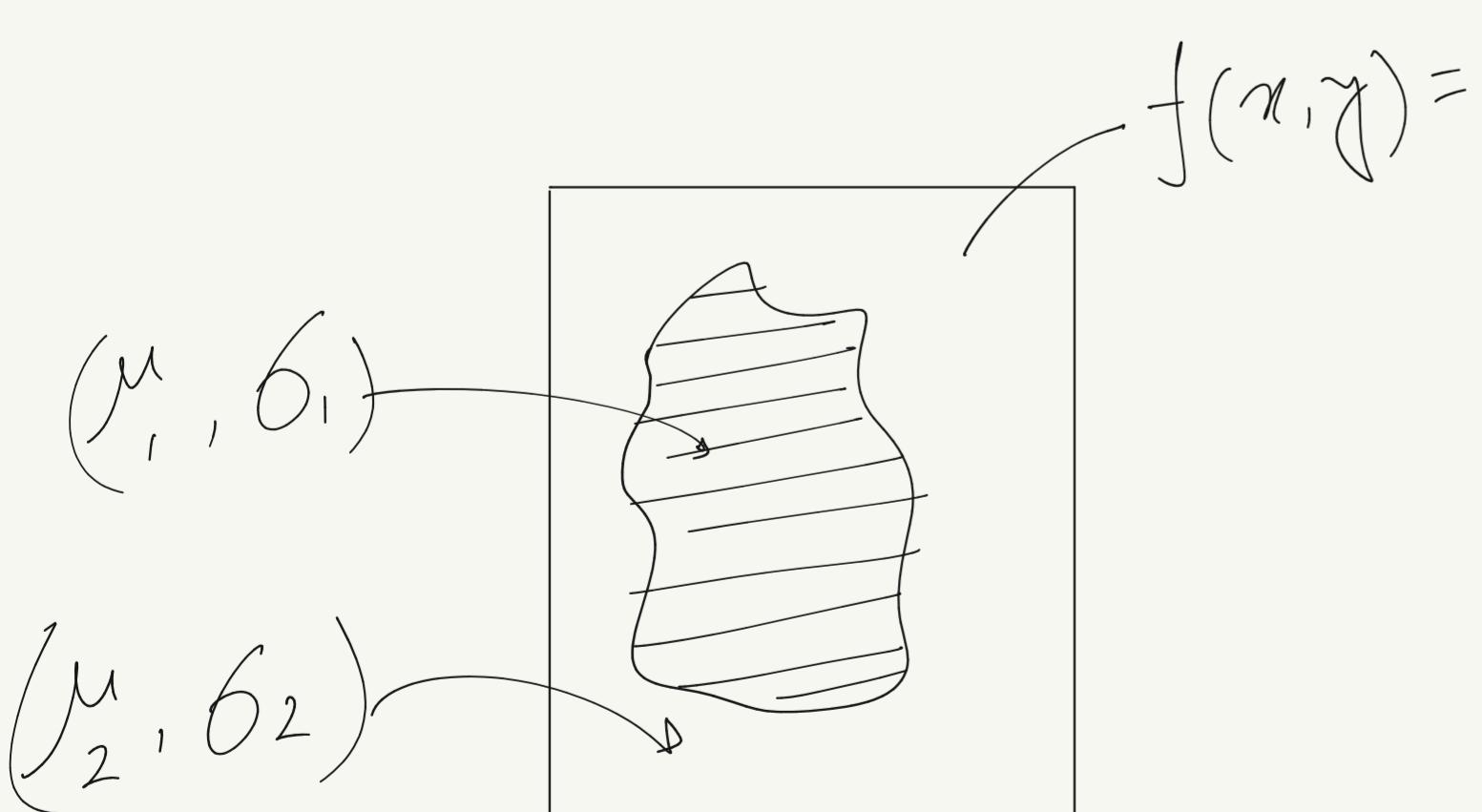
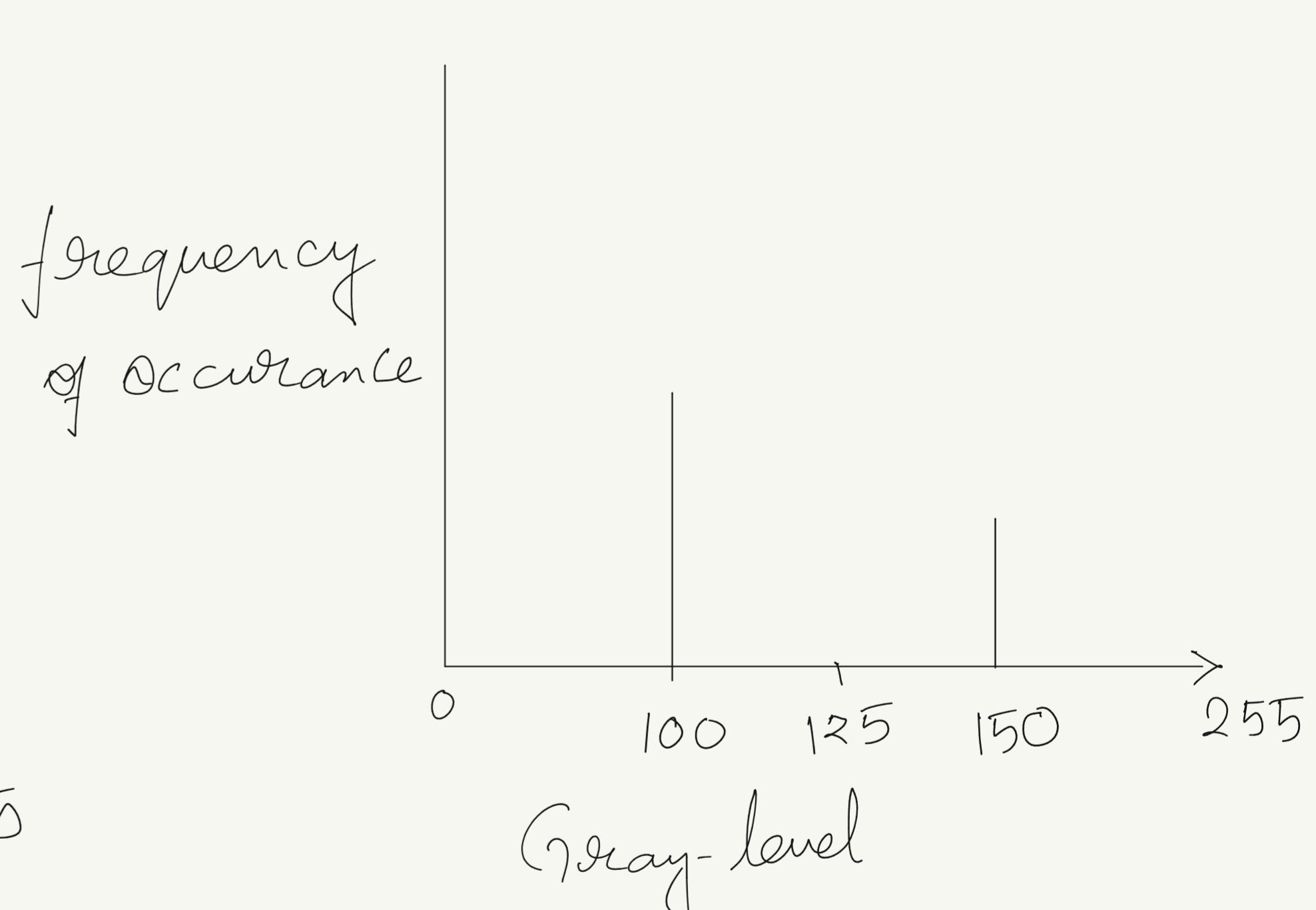
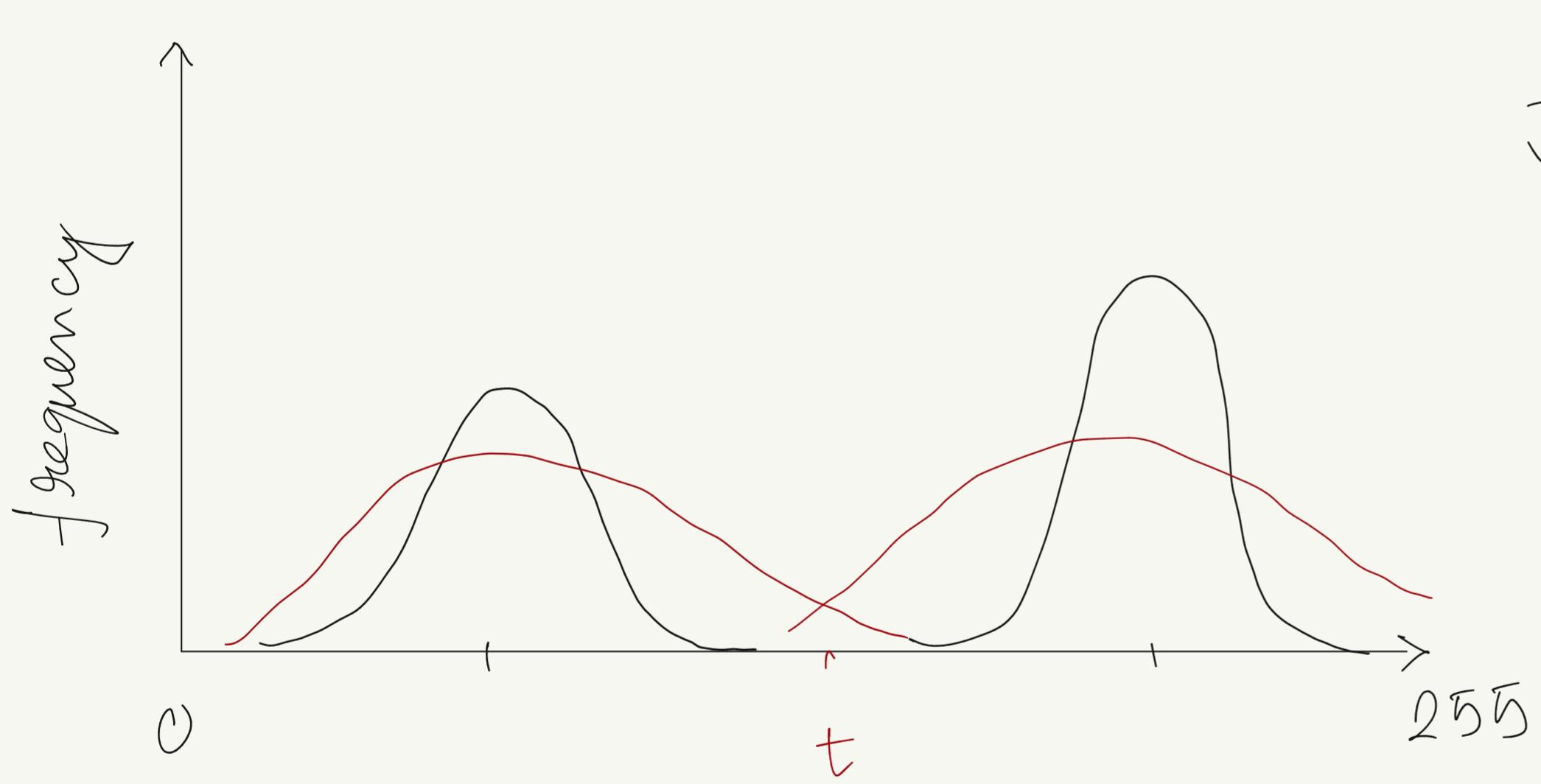
Objective :



Domain:  $R_i^o : i = \{1, 2, \dots, 7\}$

In general,  $\bigcup_{i=1}^n R_i^o = D$ ,  $R_i^o \cap R_j^o = \emptyset \forall i \neq j$ .

$\text{Prop}(R_i^o) = \text{True}$ ,  $\text{Prop}(R_i^o \cup R_j^o) = \text{False}$  if  $R_i^o$  and  $R_j^o$  are adjacent.



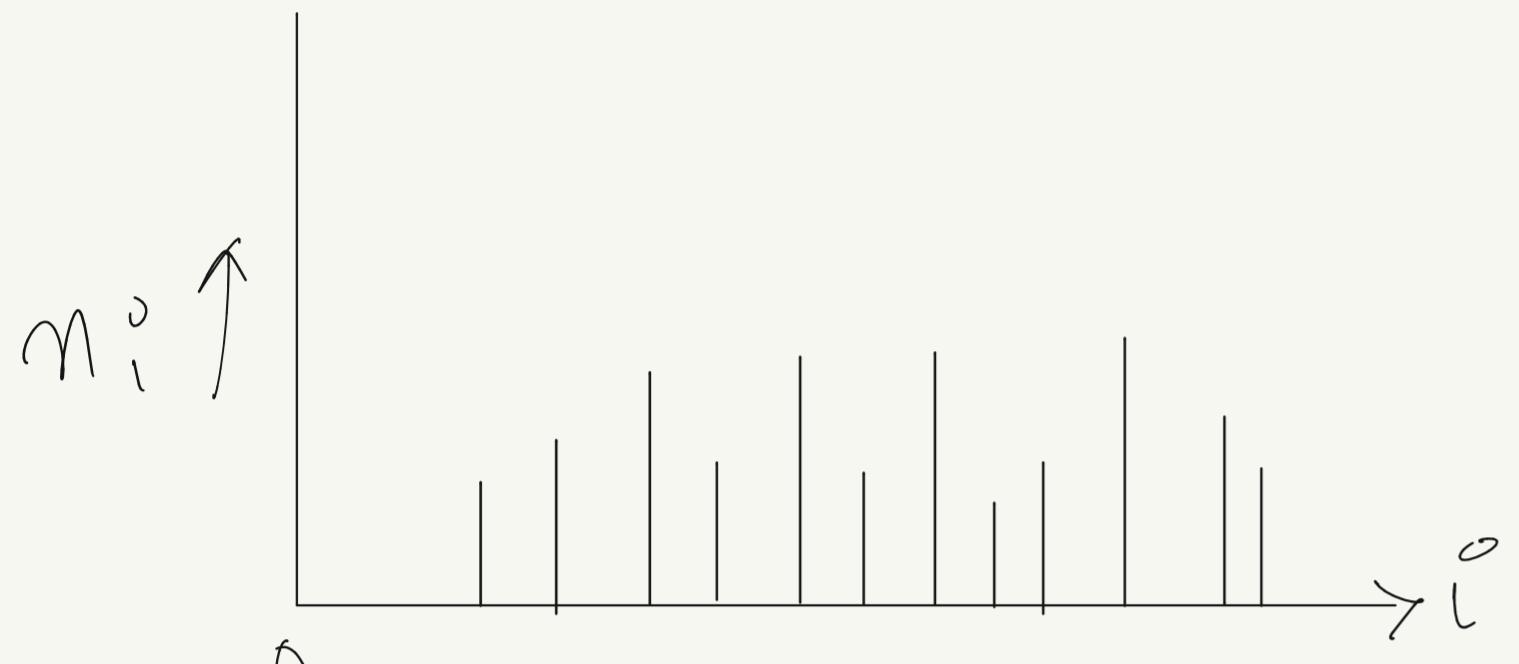
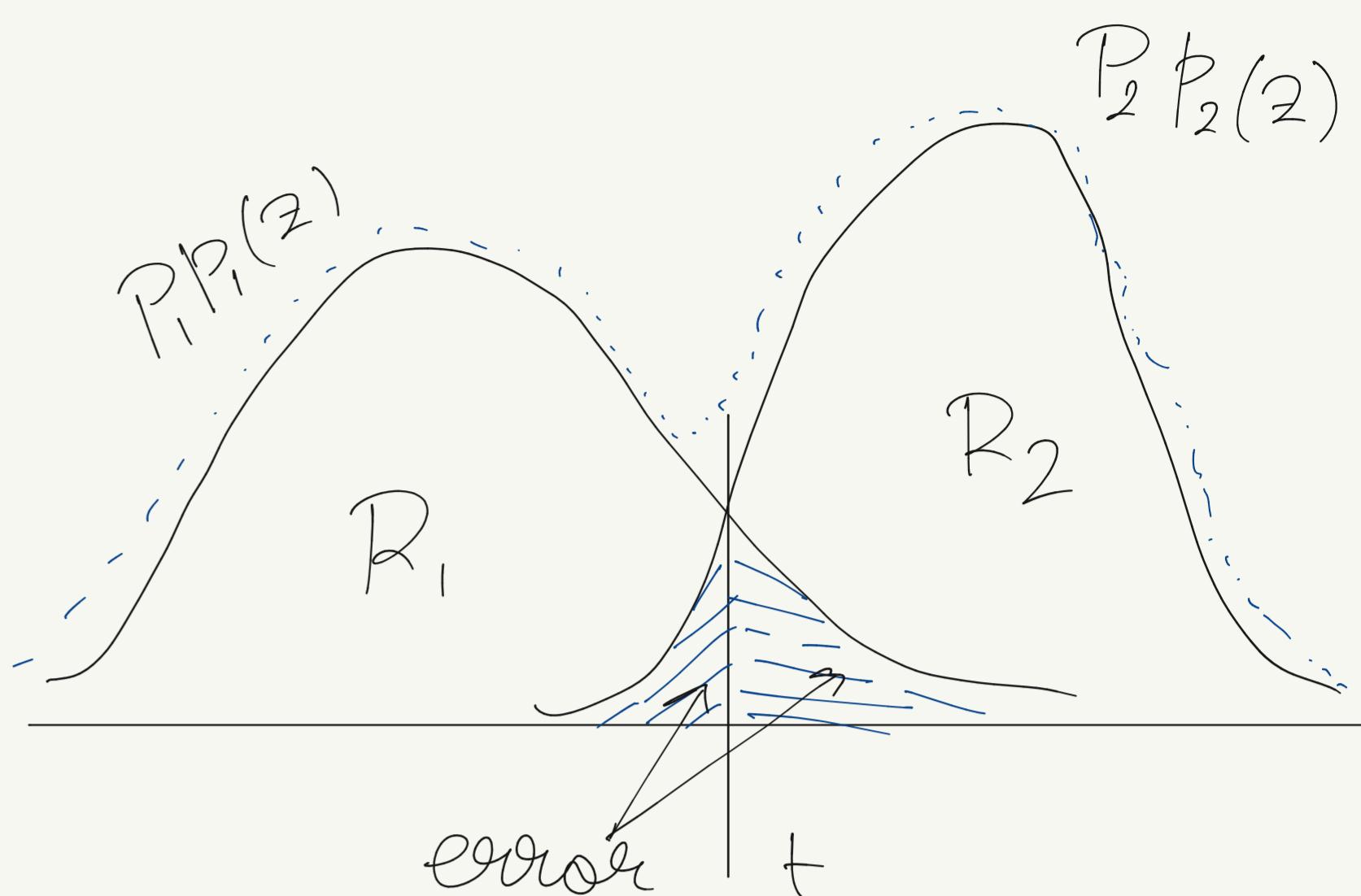
mean =  $\mu_1$ , std =  $\sigma_1$

$$p_1(z) = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{1}{2}\left(\frac{z-\mu_1}{\sigma_1}\right)^2}$$

And,  $P_2(z) = \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{1}{2} \left(\frac{z-\mu_2}{\sigma_2}\right)^2}$  (Assume,  $\mu_1 < \mu_2$ )

$$p(z) = P_1 p_1(z) + P_2 p_2(z)$$

$$e(t) = \int_{-\infty}^t P_2 p_2(z) dz + \int_t^\infty P_1 p_1(z) dz$$



Then,  $\sum_{i=0}^{255} n_i = N$

thus,  $\sum_{i=0}^{t-1} n_i = N_1$  and  $\sum_{i=t}^{255} n_i = N_2$

Let, prior probability  $= \frac{n_i}{N} = p_i$ , then,

$$\sum_{i=0}^{t-1} p_i = P_1, \quad \sum_{i=t}^{255} p_i = P_2 \quad \text{and}$$

$$\sum_{i=0}^{t-1} i p_i = \mu_1, \quad \sum_{i=t}^{255} i p_i = \mu_2, \quad \sum_{i=0}^{255} i p_i = \mu = P_1 \mu_1 + P_2 \mu_2$$

$$\sum_{i=0}^{t-1} p_i (i - \mu_1)^2 = \sigma_1^2, \quad \sum_{i=t}^{255} p_i (i - \mu_2)^2 = \sigma_2^2, \quad \sigma^2 = P_1 \sigma_1^2 + P_2 \sigma_2^2$$

$$P_1 + P_2 = 1 \Rightarrow P_2(t) = 1 - P_1(t)$$

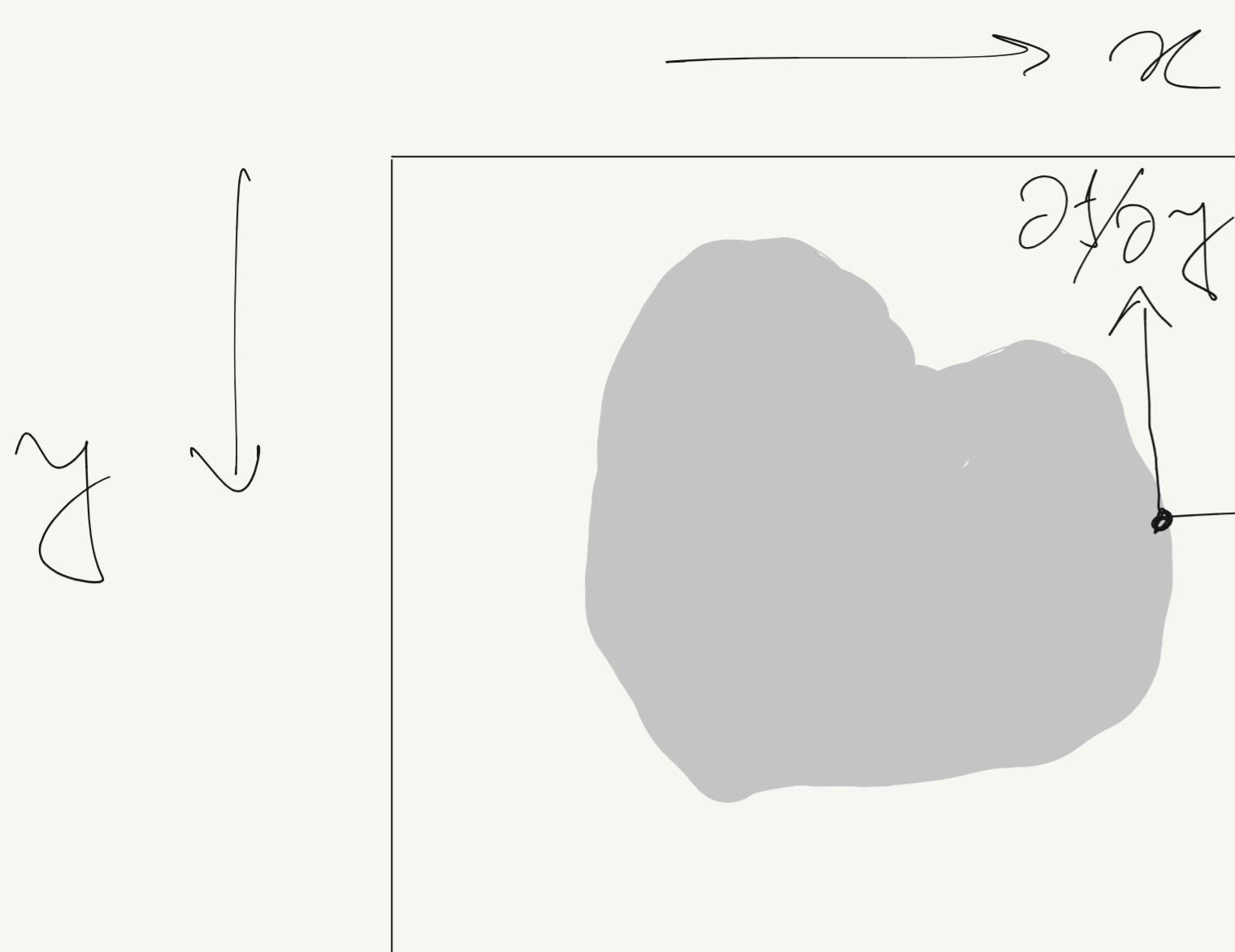
$$\sigma_b^2 = P_1(t) (\mu_1(t) - \mu)^2 + P_2(t) (\mu_2(t) - \mu)^2$$

$$P_1(t) = \sum_{i=0}^{t-1} p_i, \quad \mu_1(t) = \sum_{i=0}^{t-1} i p_i$$

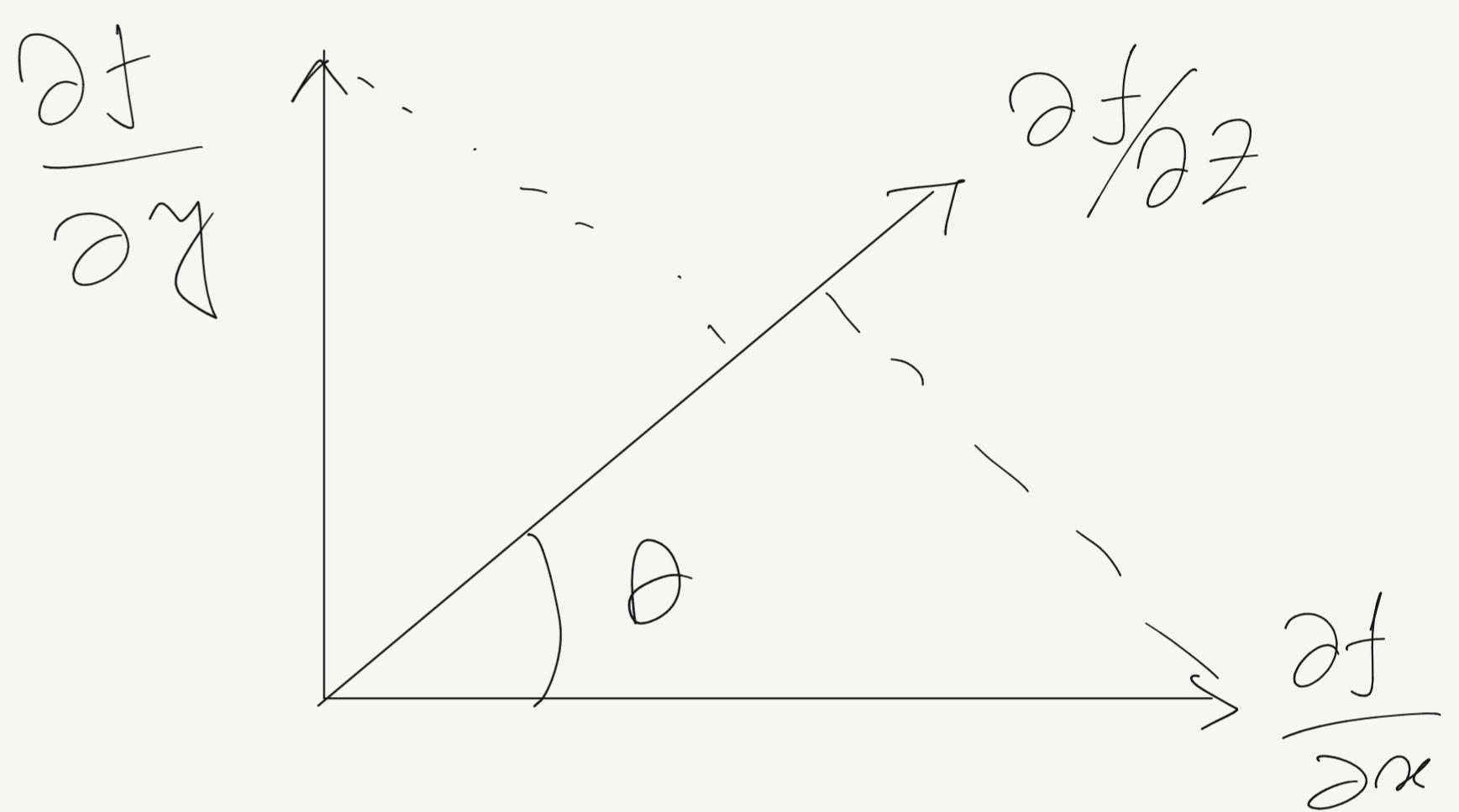
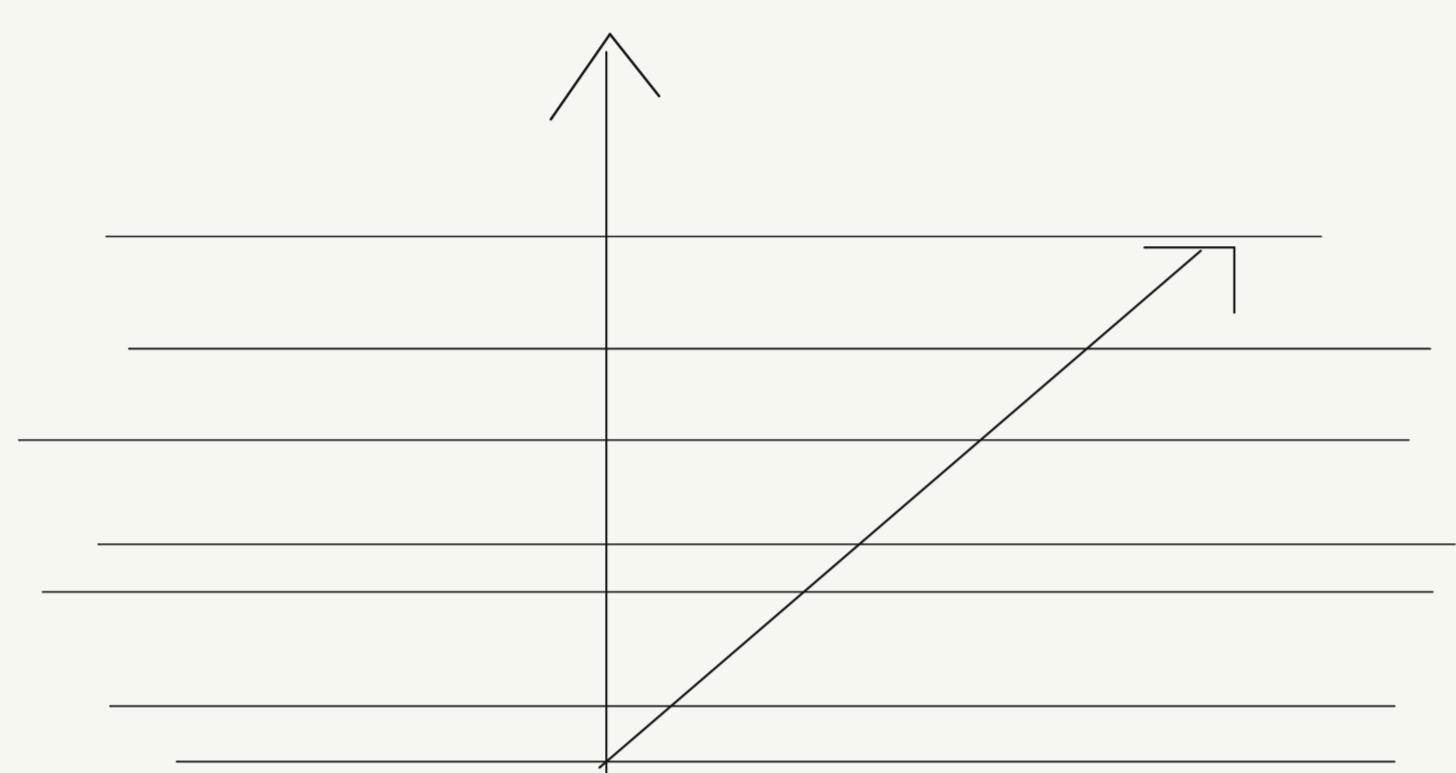
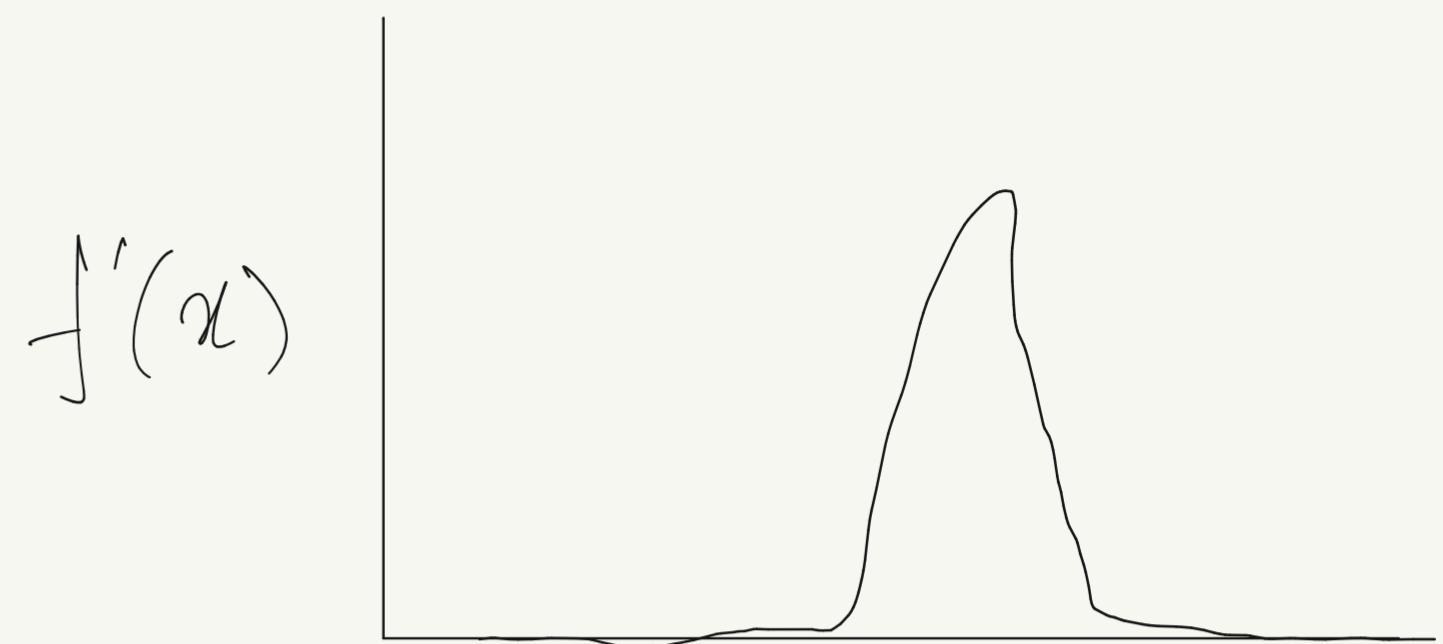
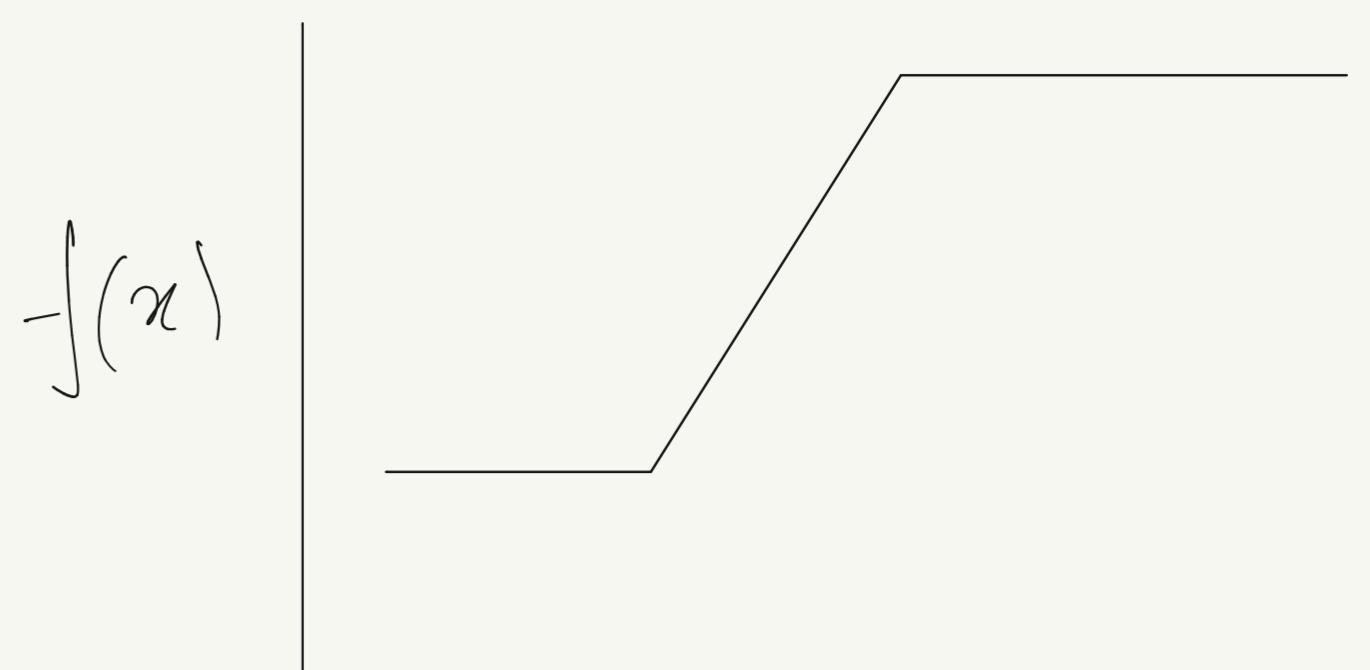
$$P_1(t+1) = P_1(t) + p_{t+1} \quad \text{and} \quad \mu_1(t+1) = \mu_t + (t+1) p_{t+1}$$

# Edge Detection

23/03/2023



$$\frac{\partial f}{\partial x}$$



$$\therefore \frac{\partial f}{\partial z} = \frac{\partial f}{\partial x} \cos \theta + \frac{\partial f}{\partial y} \sin \theta$$

$$\therefore \frac{\partial}{\partial \theta} \left( \frac{\partial f}{\partial z} \right) = - \frac{\partial f}{\partial x} \sin \theta + \frac{\partial f}{\partial y} \cos \theta = 0$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} \right)$$

$$\begin{aligned} \therefore \frac{\partial^2}{\partial \theta^2} \left( \frac{\partial f}{\partial z} \right) &= - \frac{\partial f}{\partial x} \cos \theta - \frac{\partial f}{\partial y} \sin \theta \\ &= - \left( \frac{\partial f}{\partial x} \cdot \frac{\frac{\partial f}{\partial x}}{\sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2}} \right) \\ &\quad - \left( \frac{\partial f}{\partial y} \cdot \frac{\frac{\partial f}{\partial y}}{\sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2}} \right) \end{aligned}$$

$$= - \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

$$\therefore \frac{\partial f}{\partial z} = \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

27/03/2023

## Histogram equalization :

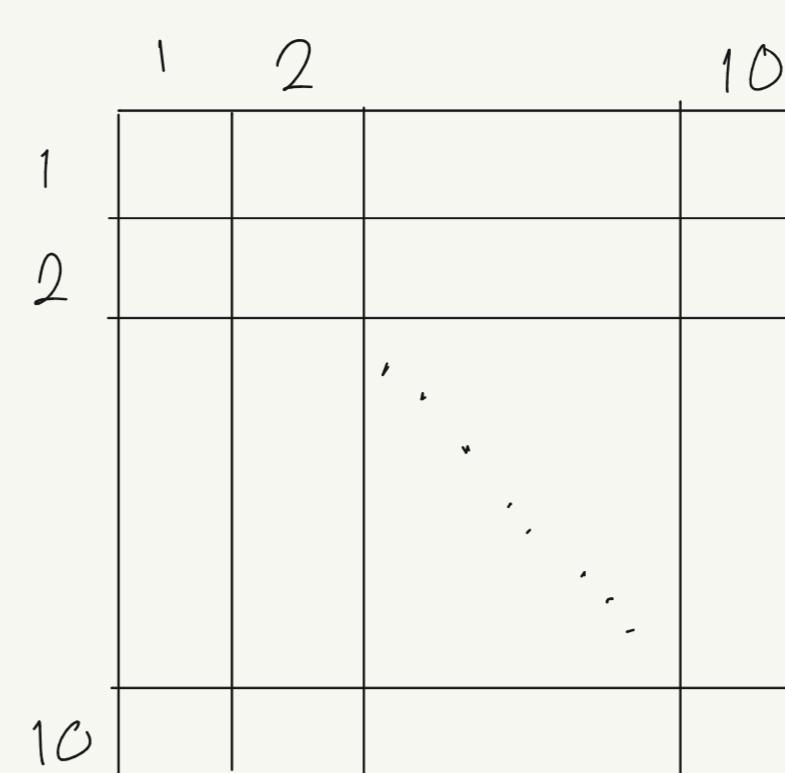
$$T: P_i \rightarrow S_i$$

$$\downarrow$$

$$\{0, 1, 2, \dots, 255\}$$

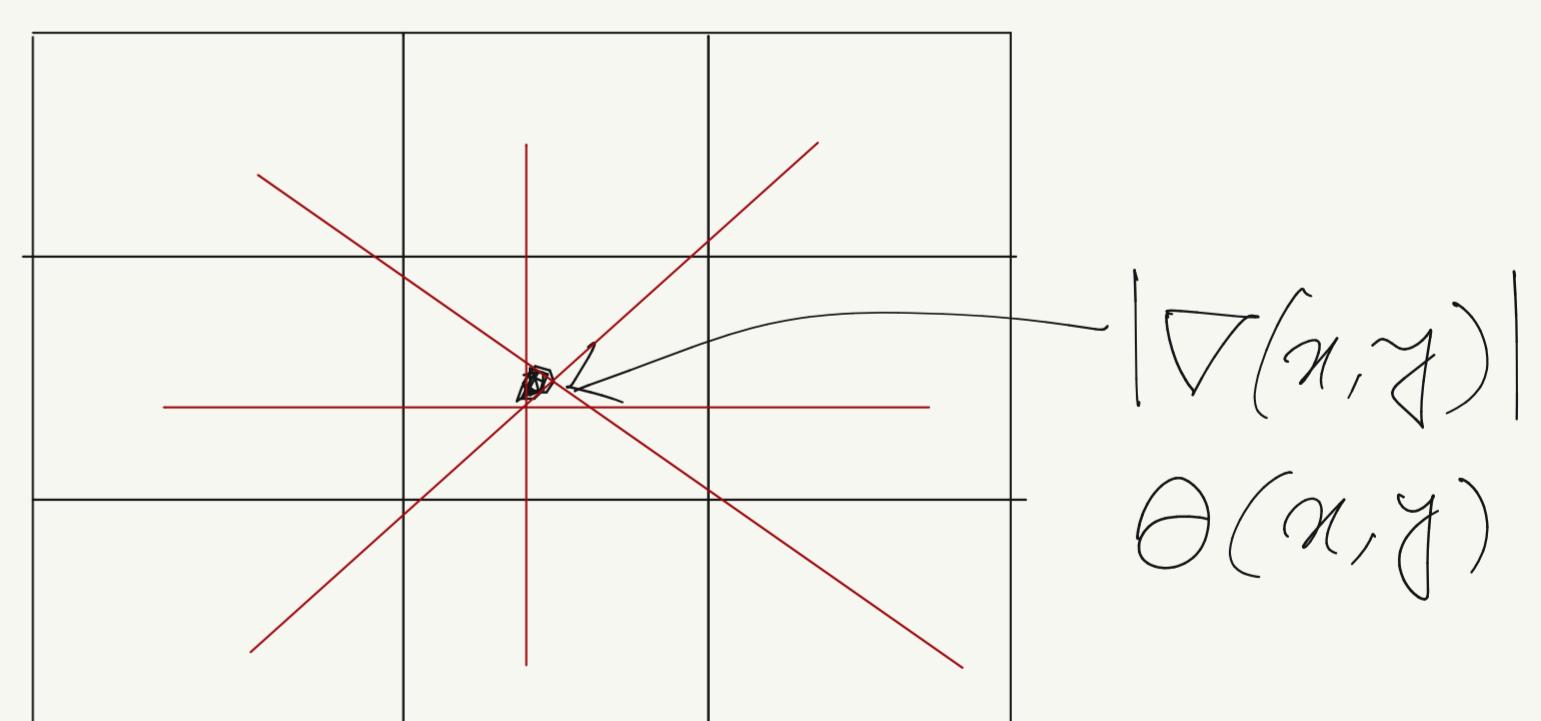
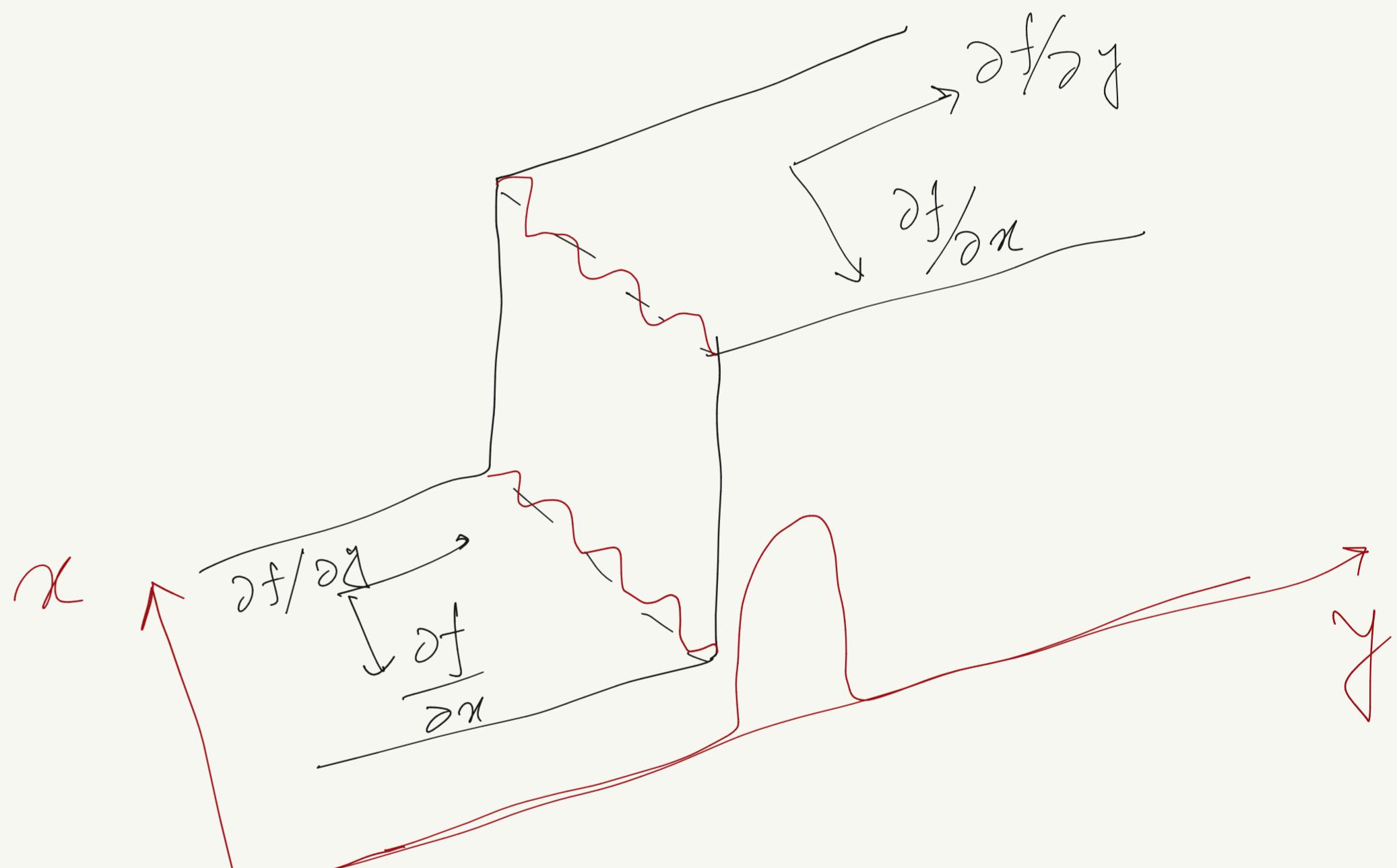
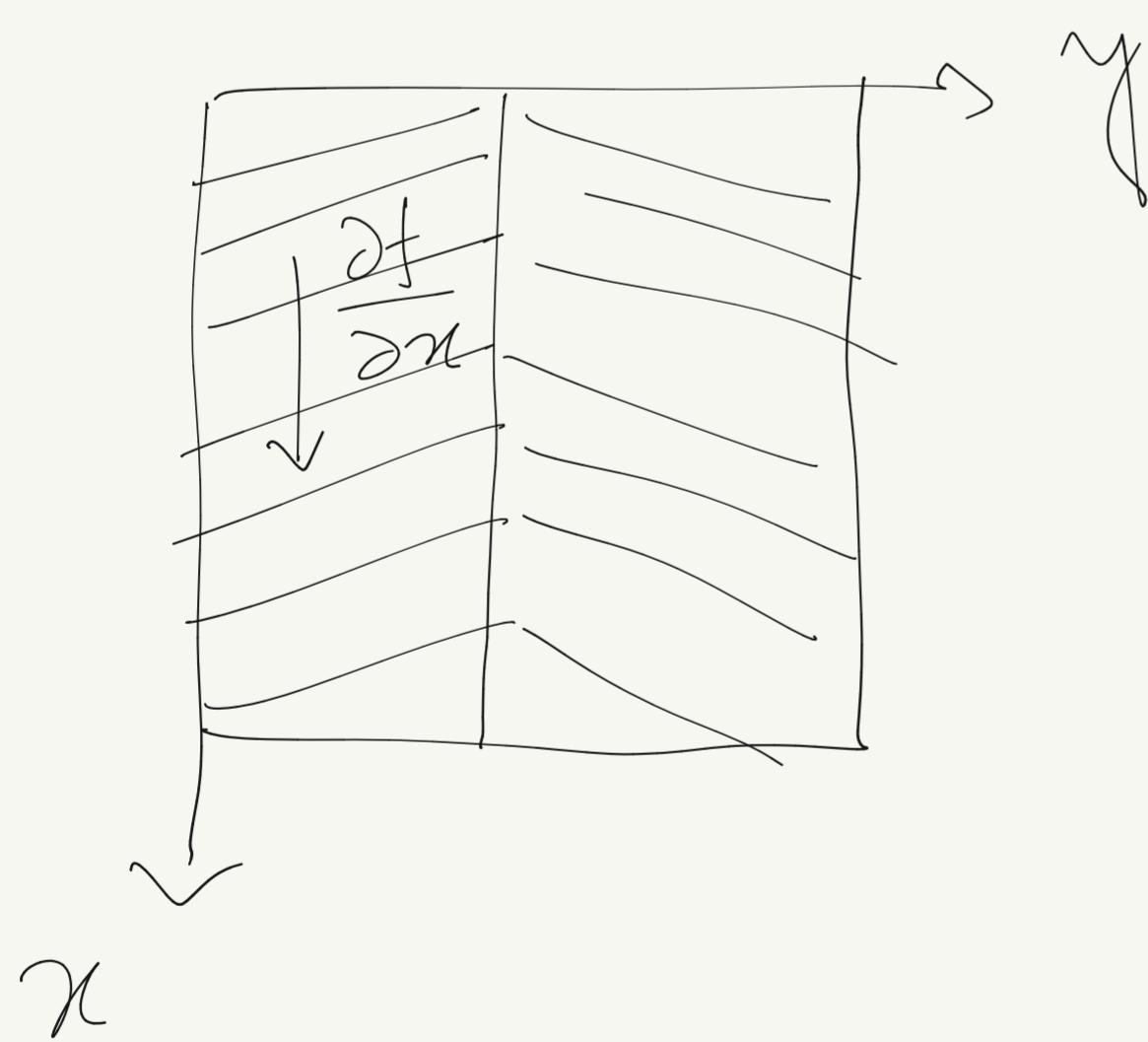
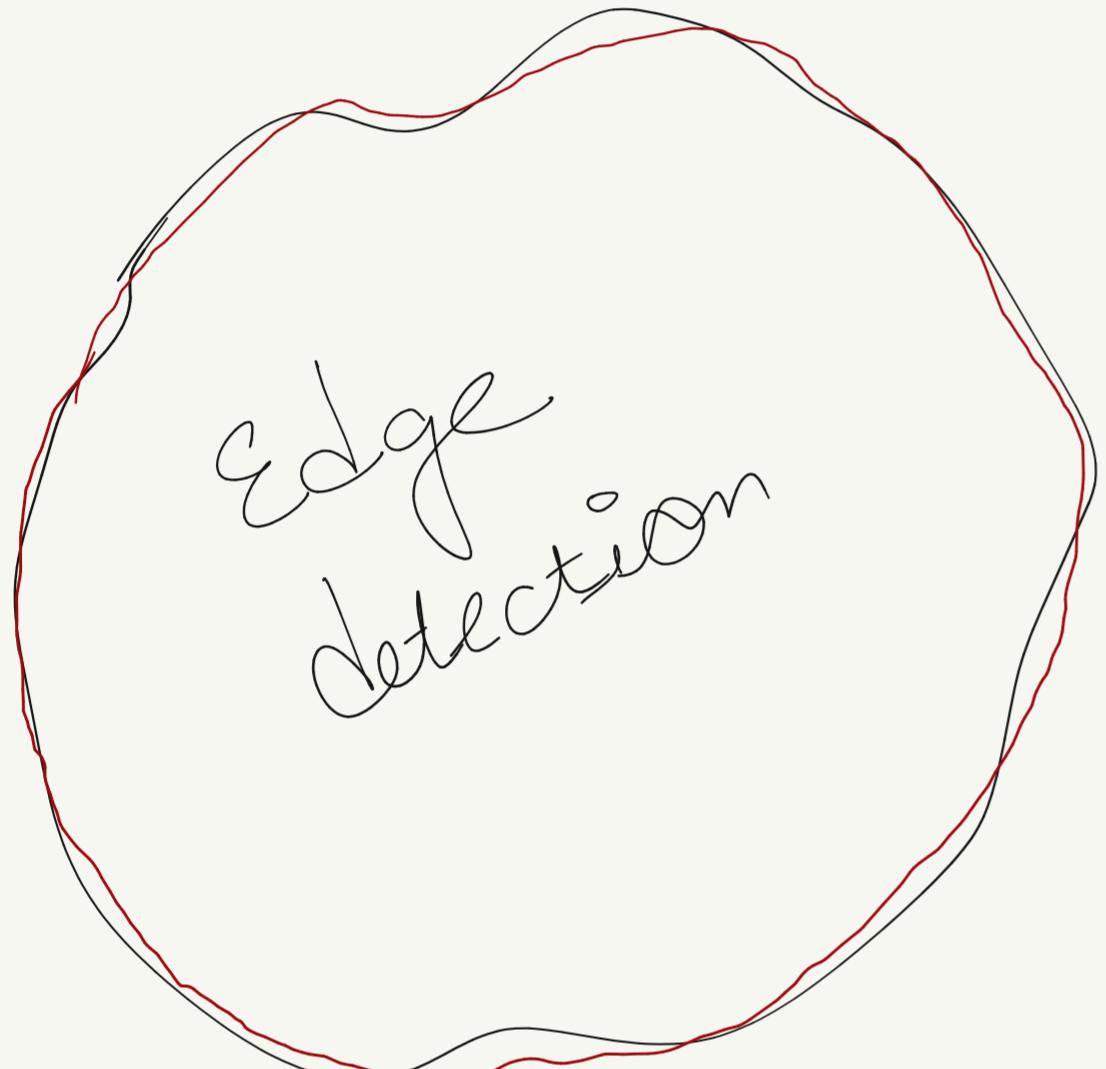
$$S_i = (L-1) C_i = (L-1) \sum_{k=0}^{i-1} p_k$$

$S_i$	0	1	2	3	4	5	6	7	Total
$x_i$	4	13	7	0	35	0	25	16	100



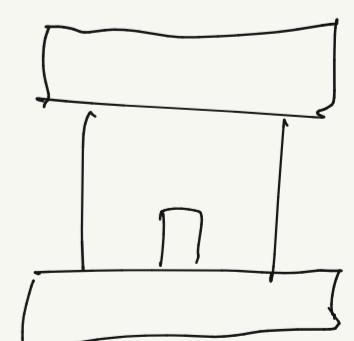
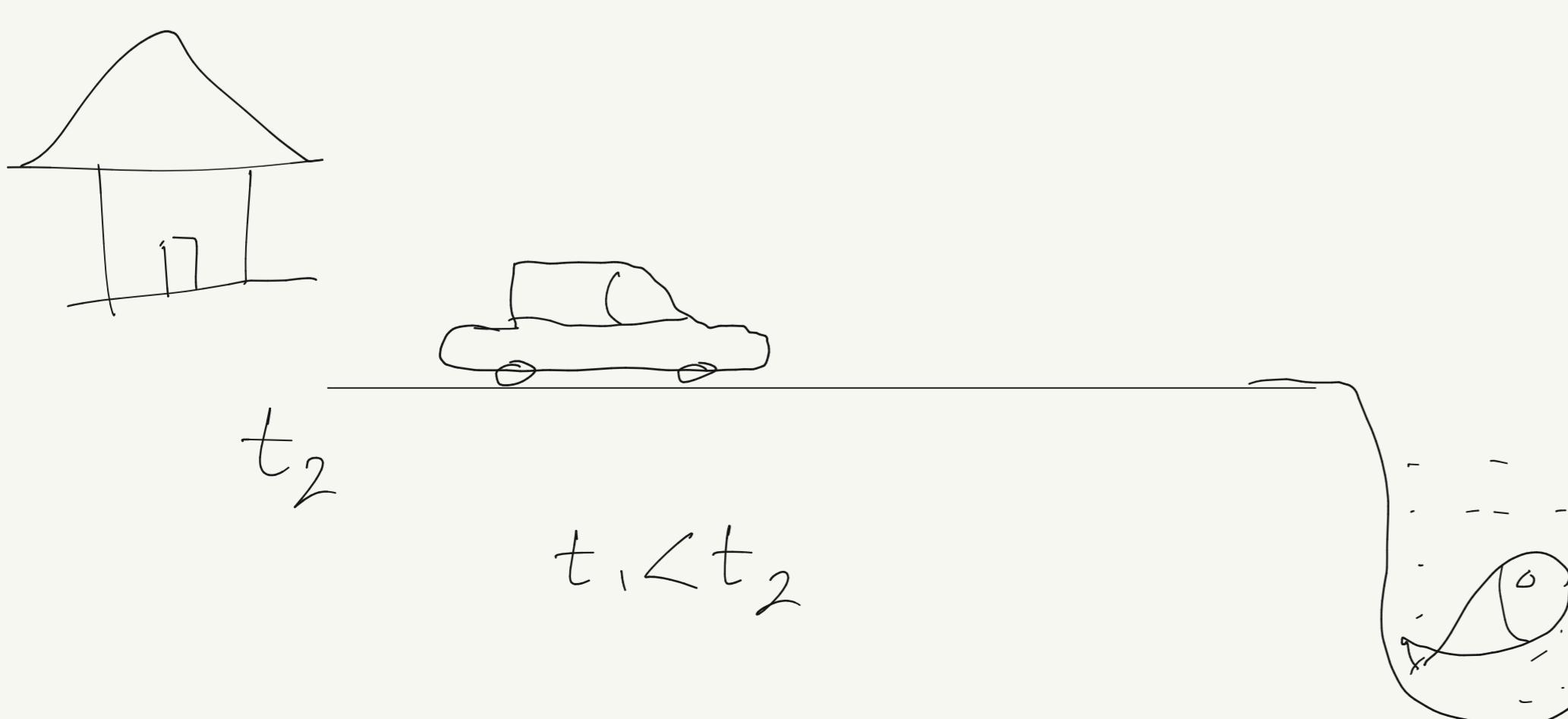
10x10

$P_i$	0	1	2	3	4	5	6	7	Total
$M_i$	0	4	13	7	35	25	16	0	100
$p_p$	0	0.04	0.13	0.07	0.35	0.25	0.16	0	1
$C_i$	0	0.04	0.17	0.24	0.59	0.84	1	1	X
$f_C$	0	0.28	1.19	1.68	4.13	5.88	7	7	X
$S_d$	0	0	1	2	4	6	7	7	X



$$0^\circ \leq \theta < 360^\circ$$

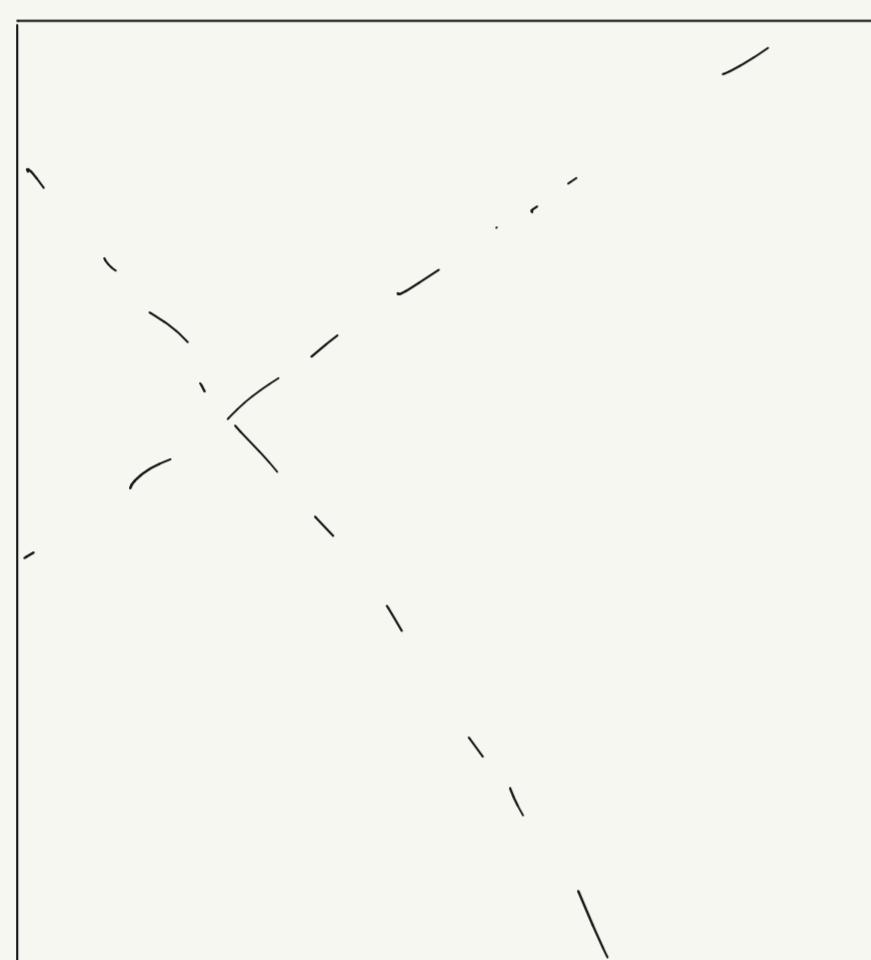
$$-22.5^\circ \leq \theta \leq 22.5^\circ$$



f	e	d	e	f
e	c	b	c	e
d	b	a	b	d
e	c	b	c	e
f	e	d	e	f

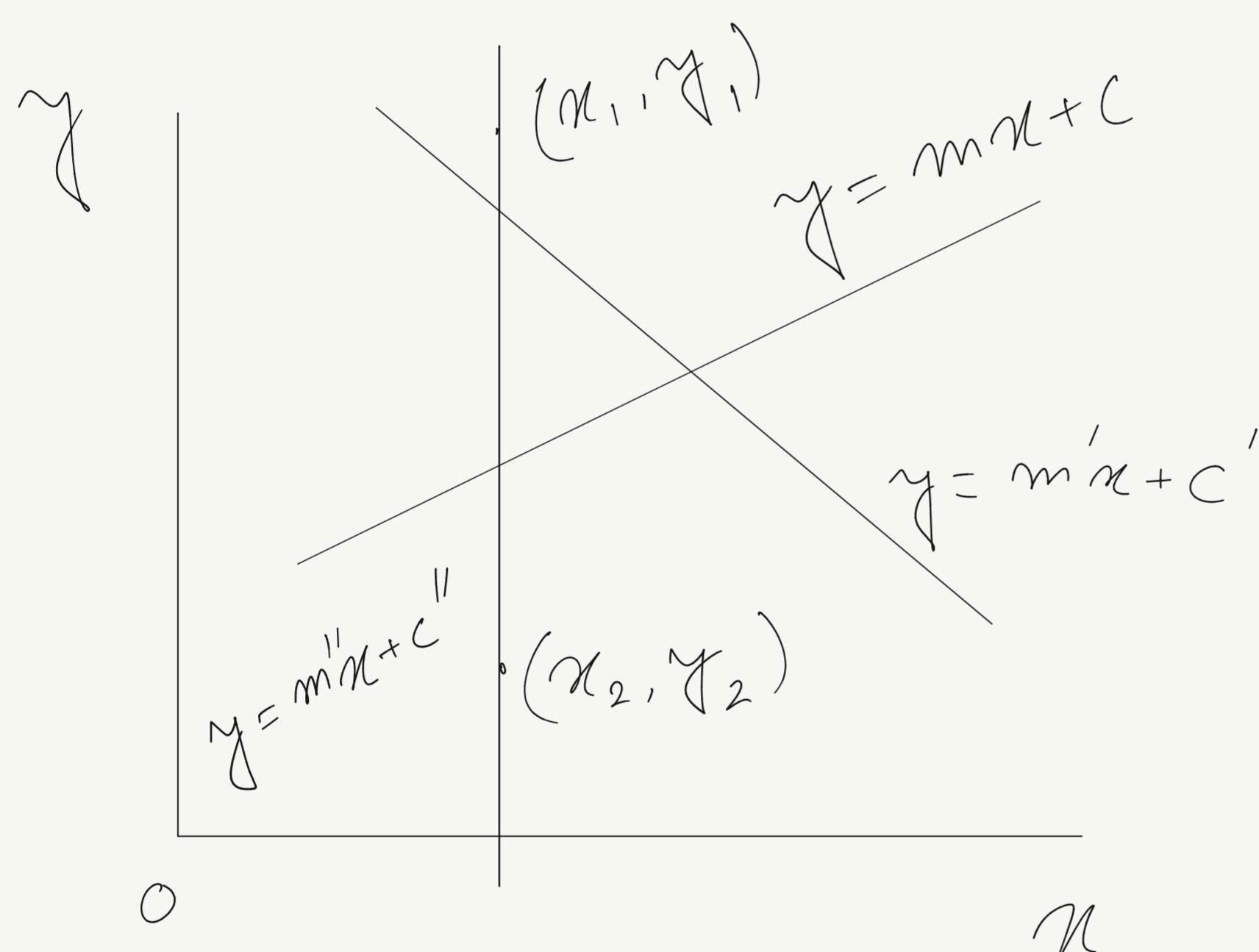
$$a > b > c > d > e > f$$

$$\frac{1}{\sum w_{ij}}$$

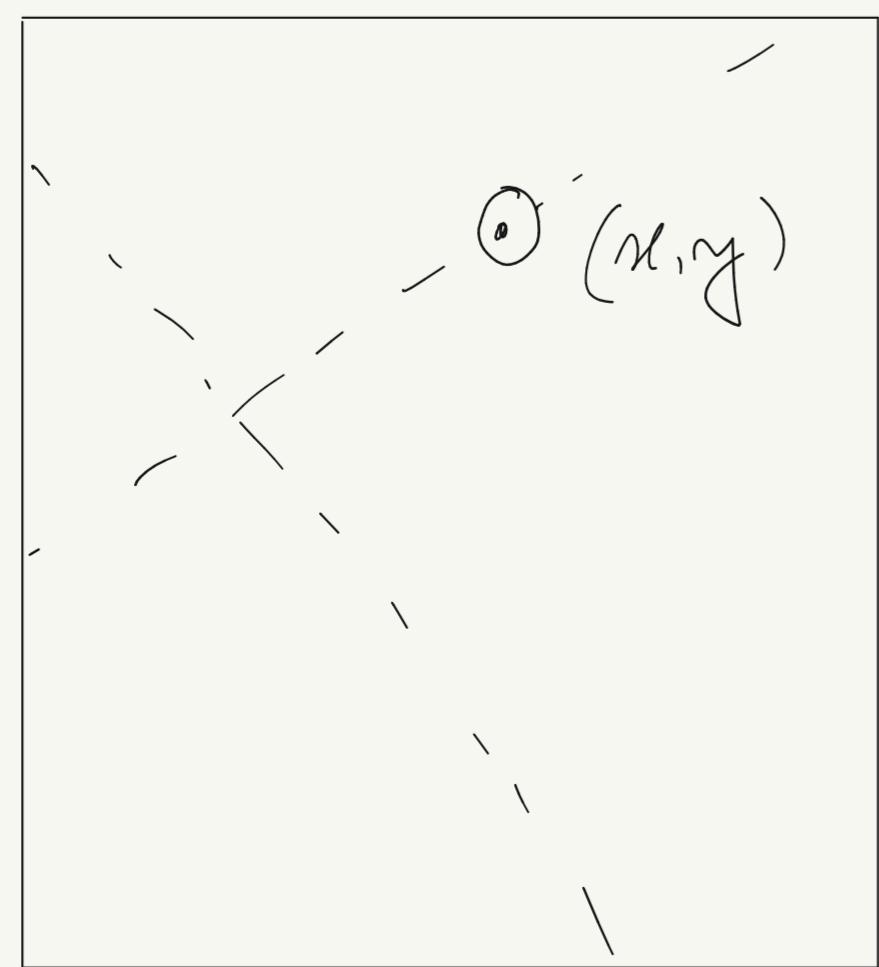
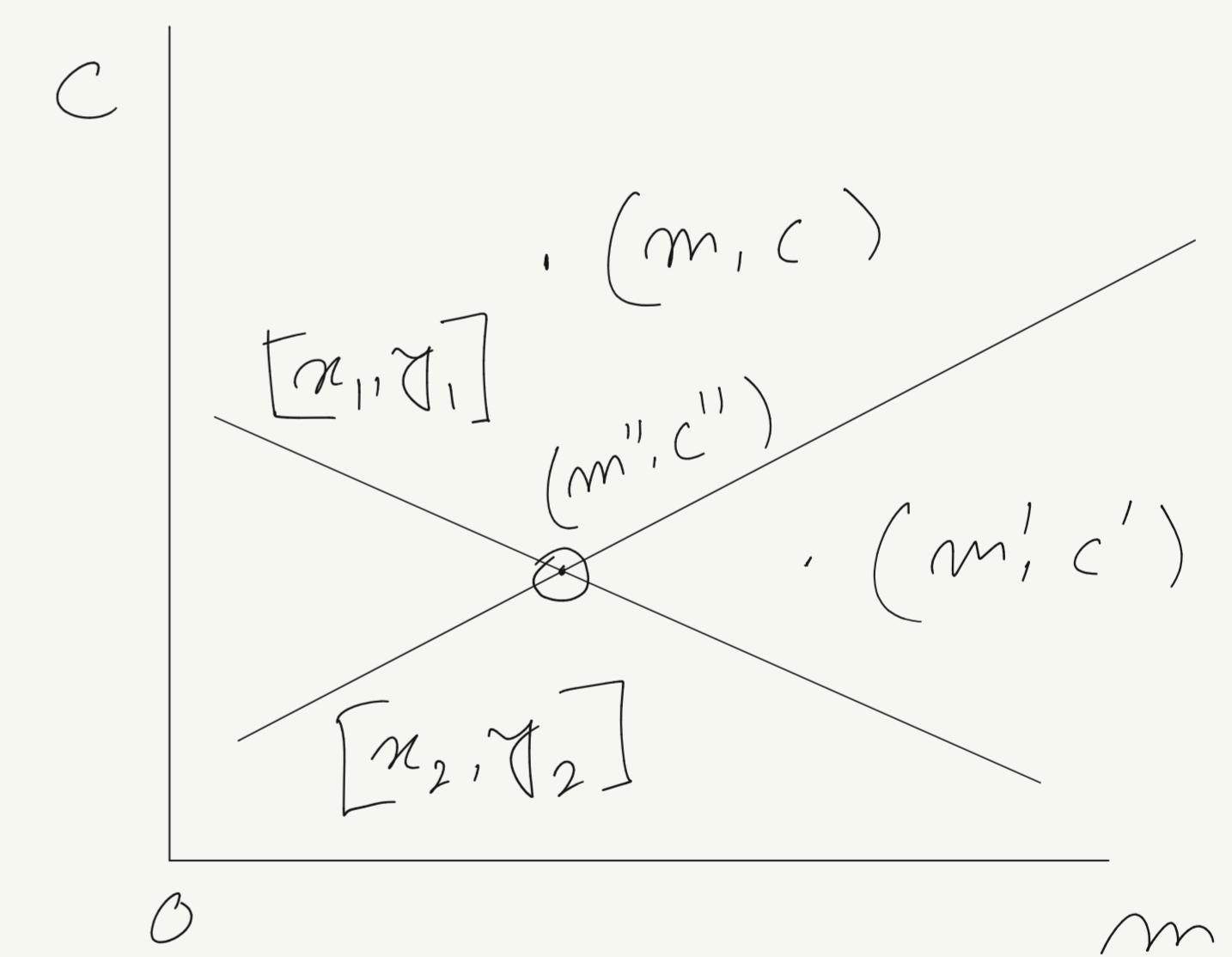


1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

Smoothing



$$y = m(x - x_1) + y_1$$

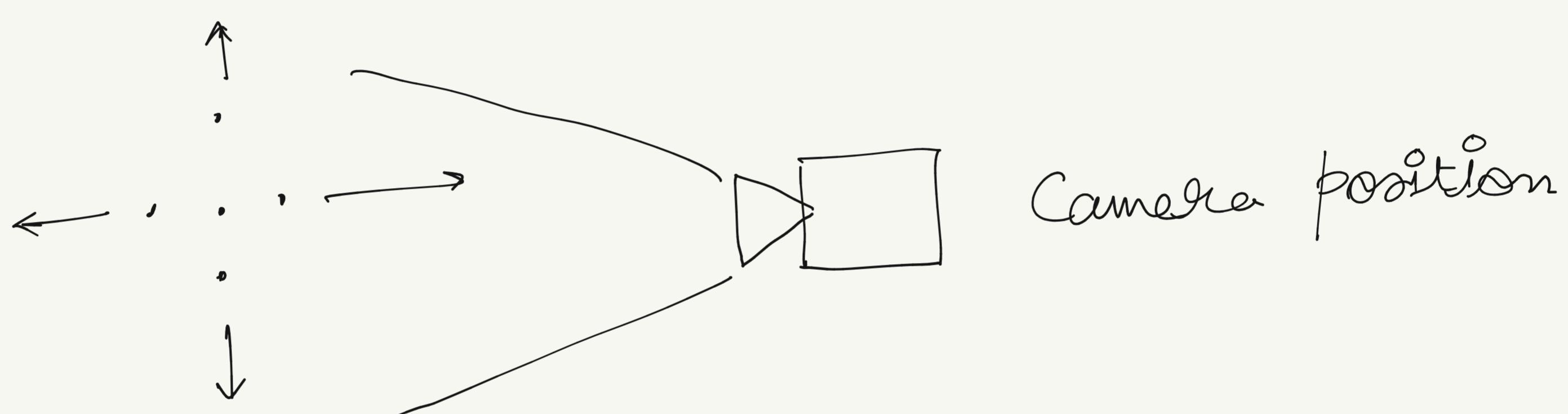
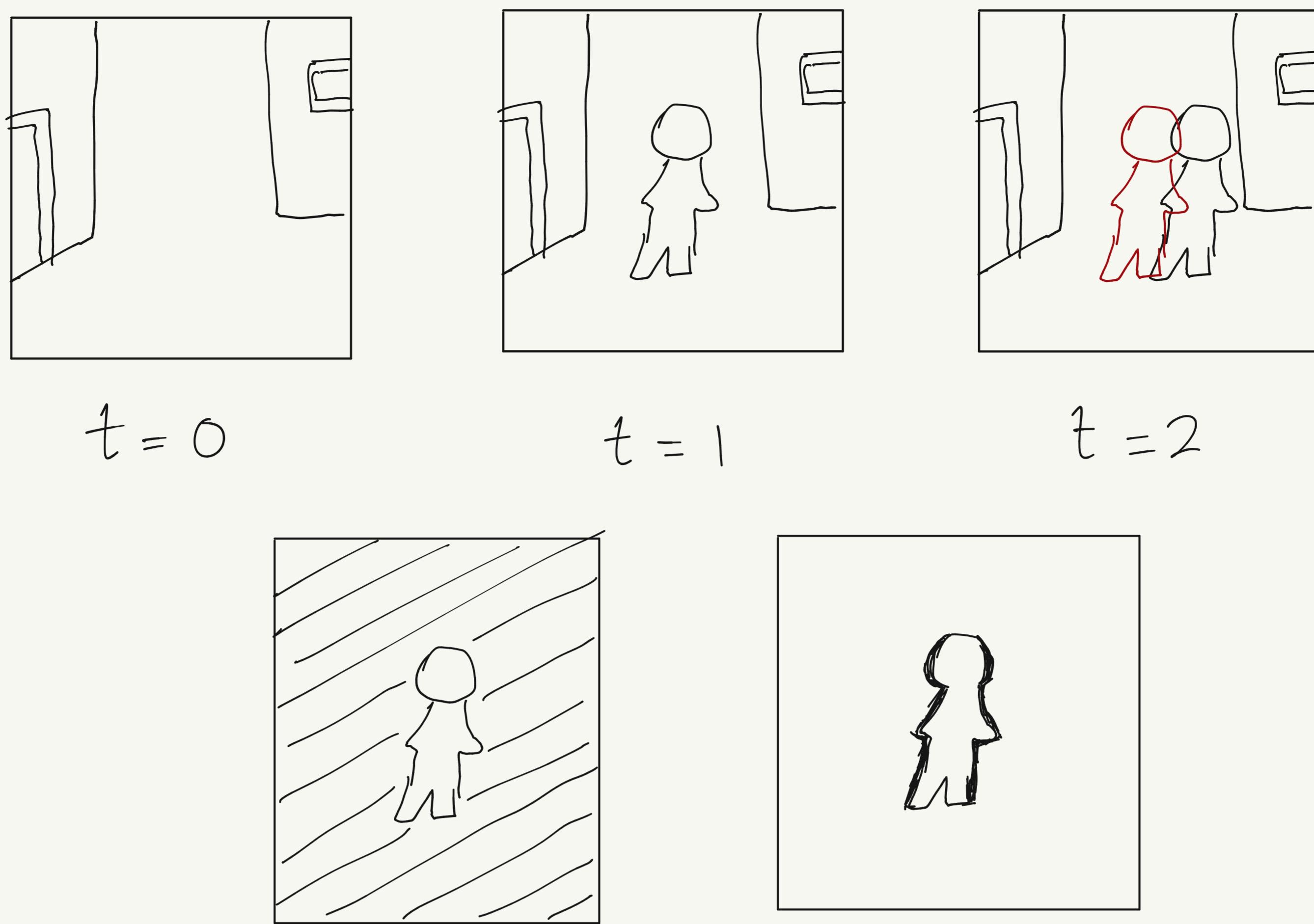


$$y = mx + c$$

	0	1	2	3	4	$\dots$	$K$
0		1		1			
1			1	1			
2			1	1	1		
3	1	1	3				
4	1	1	1				
$\vdots$				1			
L			1	1			

Accumulator

03/04/2023



Camera position

05/04/2023

$P(x)$   $x(t)$   $x(t+dt)$

$\begin{matrix} \bullet \\ P \\ \bullet \end{matrix}$   $\dots$   $\dots$   $\dots$

$x(t)$   $x(t+dt)$

$X = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$  if  $X(t) = \begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix}$

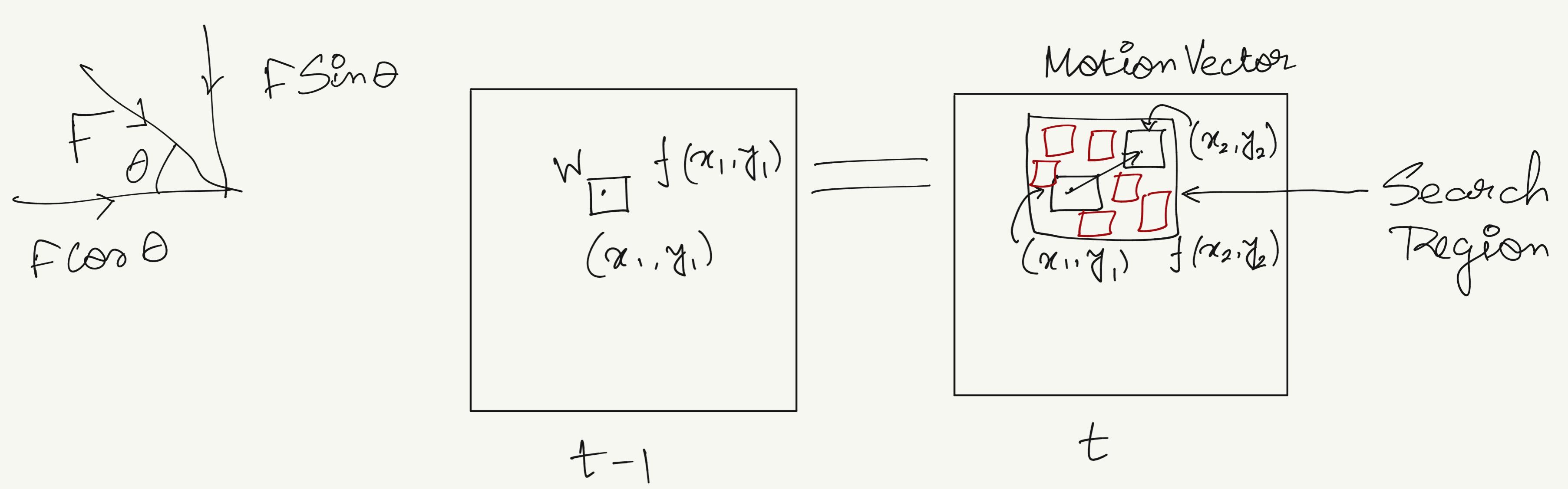
$$x(t) = f \cdot \frac{u(t)}{w(t)}, \quad y(t) = f \cdot \frac{v(t)}{w(t)}$$

$$V_x = \frac{dx(t)}{dt} = \frac{\frac{du}{dt}w - \frac{dw}{dt}u}{w^2} \cdot f = \frac{f V_u - V_w \cdot x}{w} \quad \left[ f \cdot \frac{u}{w} = x \right]$$

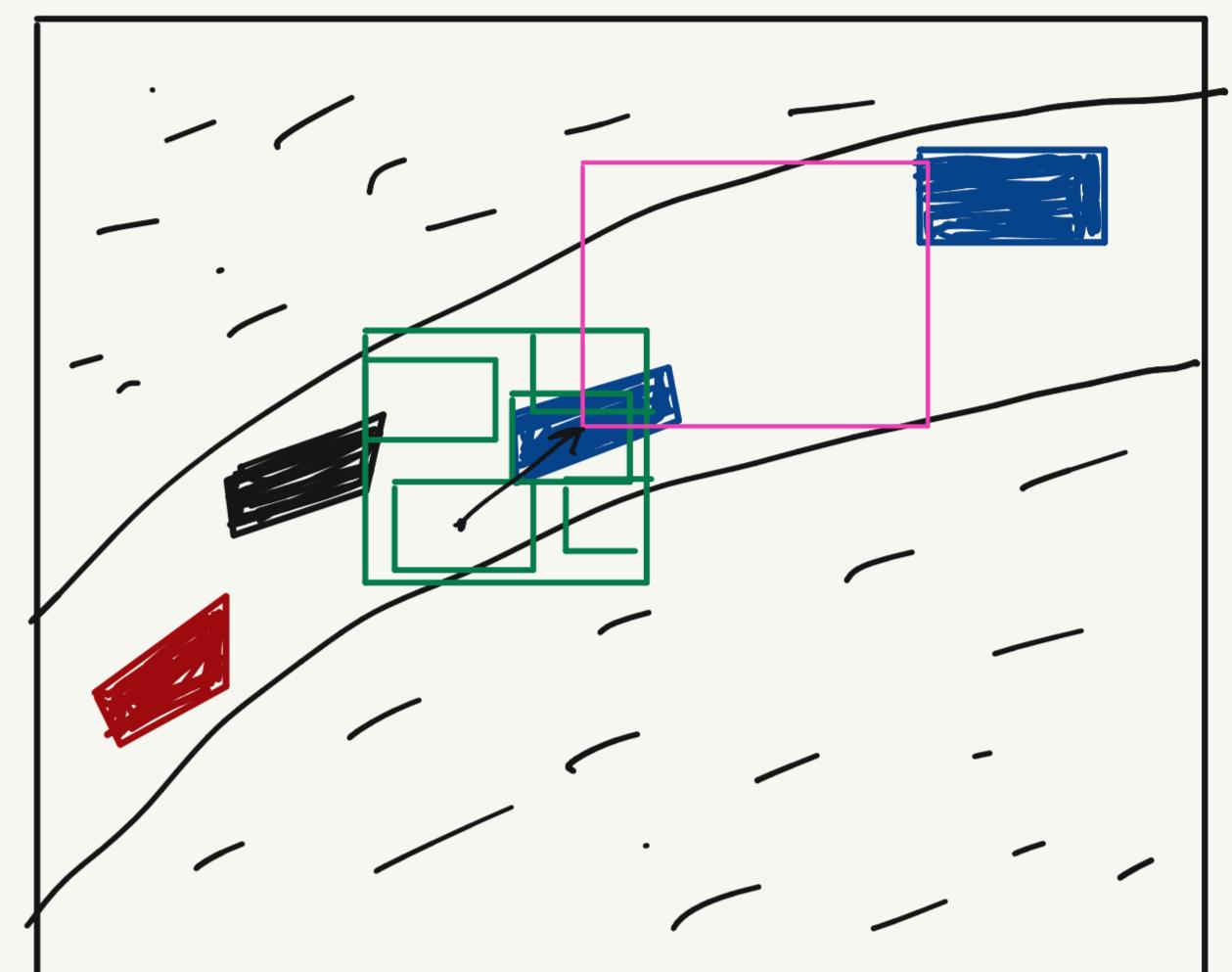
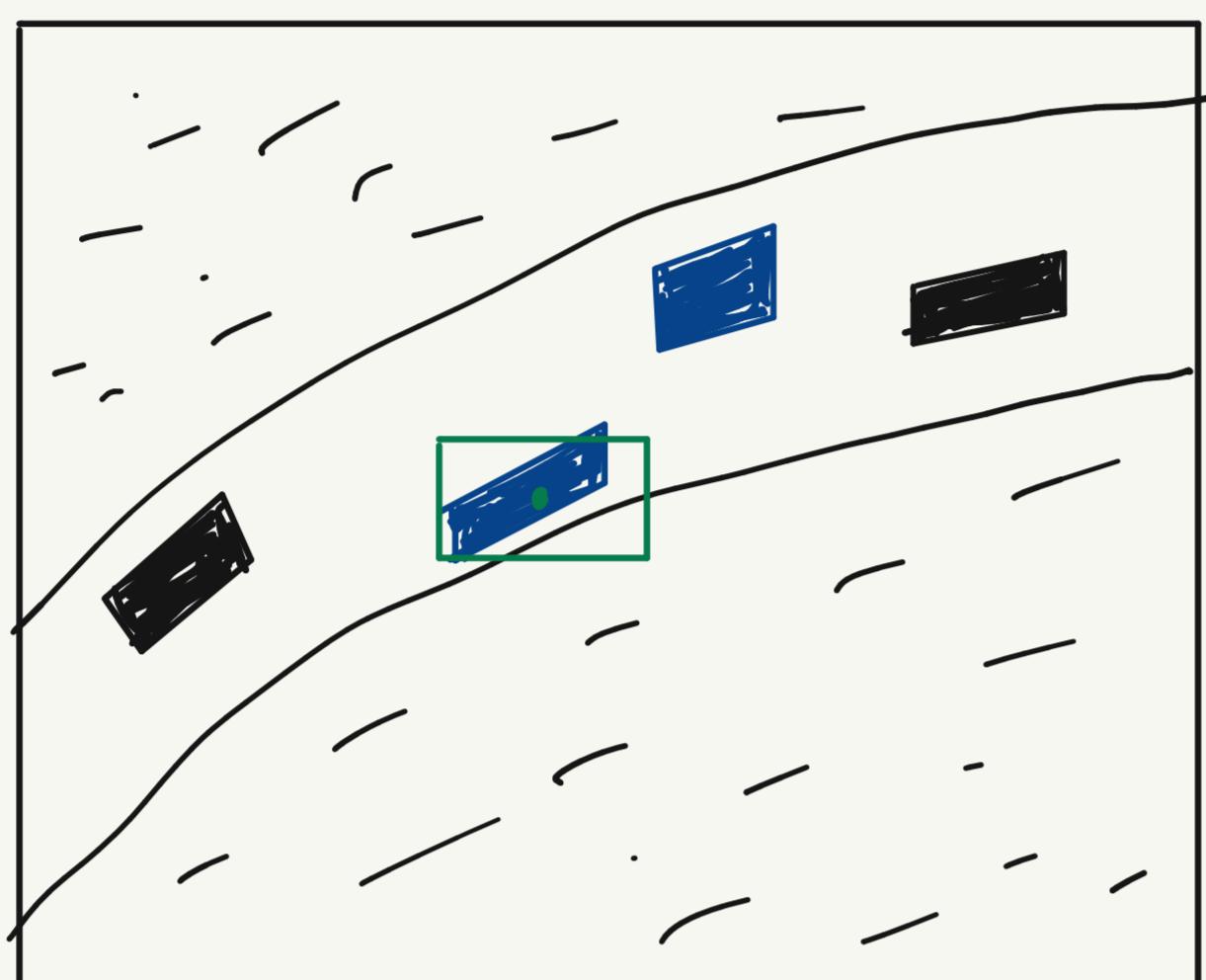
$$V_y = \frac{dy(t)}{dt} = \frac{\frac{dv}{dt}w - \frac{dw}{dt}v}{w^2} \cdot f = \frac{f V_v - V_w \cdot y}{w} \quad \left[ f \cdot \frac{v}{w} = y \right]$$

$$\therefore \begin{pmatrix} V_x \\ V_y \end{pmatrix} = \frac{1}{w} \left[ \begin{pmatrix} f V_u \\ f V_v \end{pmatrix} - V_w \begin{pmatrix} x \\ y \end{pmatrix} \right]$$

$$\therefore V_x = \frac{1}{w} \left[ \underset{2 \times 1}{\begin{matrix} V_u \\ V_v \end{matrix}} - \underset{2 \times 1}{\begin{matrix} V_w X \\ V_w Y \end{matrix}} \right] = \begin{pmatrix} V_x \\ V_y \end{pmatrix} \equiv \text{Optical Flow}$$



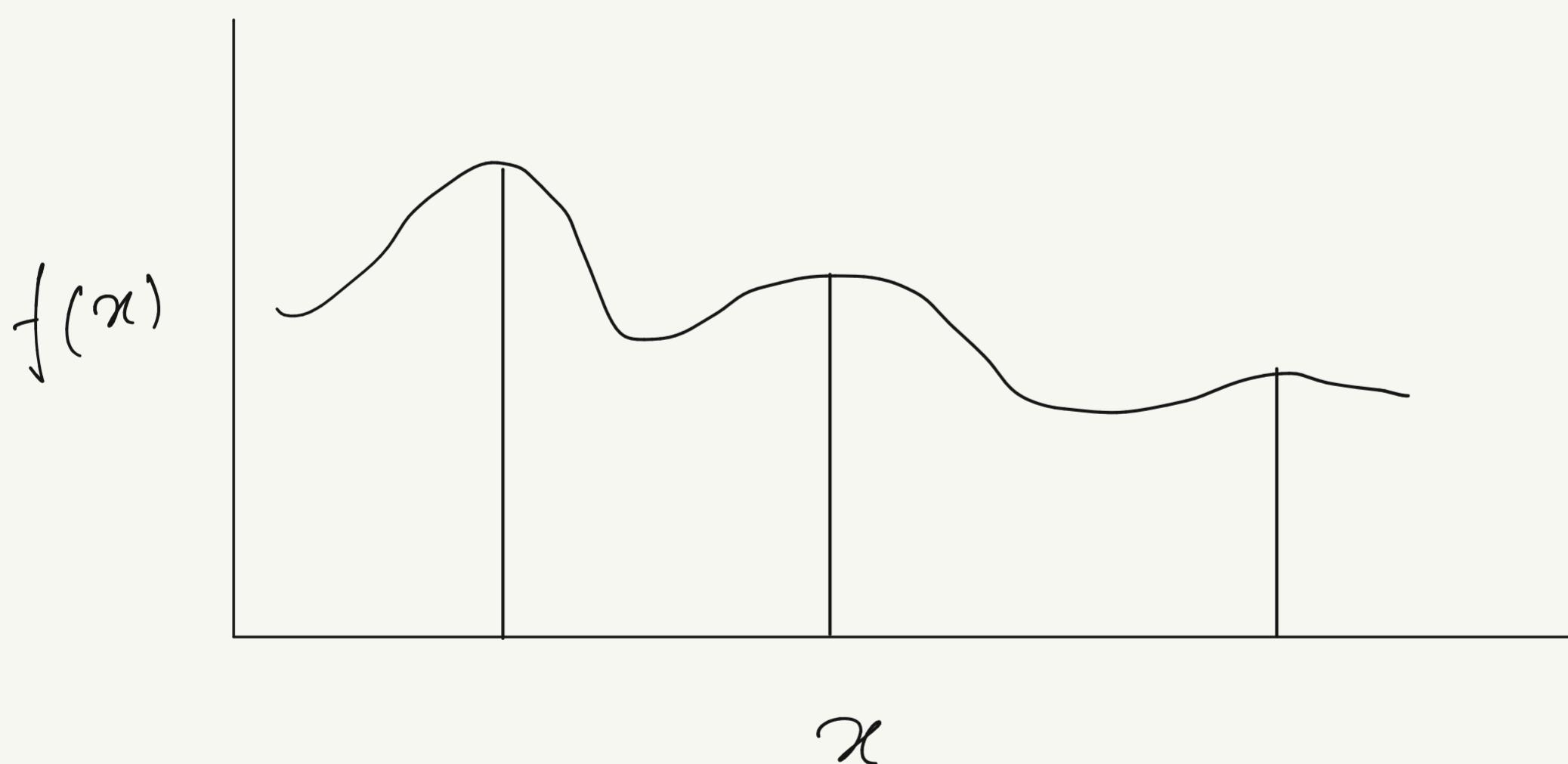
$$\sum_{(x,y) \in W} |f(x,y) - f(x+u, y+v)|^2$$



$$I(x, y, t-1) = I(x+u, y+v, t) \quad \forall (x, y)$$

$$u(x, y) = \frac{dx}{dt} \Delta t, \quad v(x, y) = \frac{dy}{dt} \Delta t$$

$$\begin{matrix} (x_1, y_1) \\ \text{---} \\ (x, y, t-1) \end{matrix} \longrightarrow \begin{matrix} (x_2, y_2) \\ \text{---} \\ (x+u, y+v, t) \end{matrix}$$



$$f(x) = x^2 \sin x + 3x$$

$$I(x, y, t) = I(x+\Delta x, y+\Delta y, t+\Delta t)$$

$$= I(x, y, t) + \frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t + H.O.T. \dots$$

$$\simeq I(x, y, t) + \frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t$$

$$\Rightarrow \frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t = 0$$

$$\Rightarrow \frac{\partial I}{\partial x} \cdot \frac{\Delta x}{\Delta t} + \frac{\partial I}{\partial y} \cdot \frac{\Delta y}{\Delta t} + \frac{\partial I}{\partial t} = 0$$

$$ax + by = c$$

$$\Rightarrow \frac{\partial I}{\partial x} \cdot v_x + \frac{\partial I}{\partial y} \cdot v_y + \frac{\partial I}{\partial t} = 0$$

$$\Rightarrow (a \ b) \begin{pmatrix} x \\ y \end{pmatrix} + \frac{\partial I}{\partial t} = 0$$

$$\Rightarrow \vec{\nabla} I \cdot \vec{v} = - I_t$$

$$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = - I_t$$

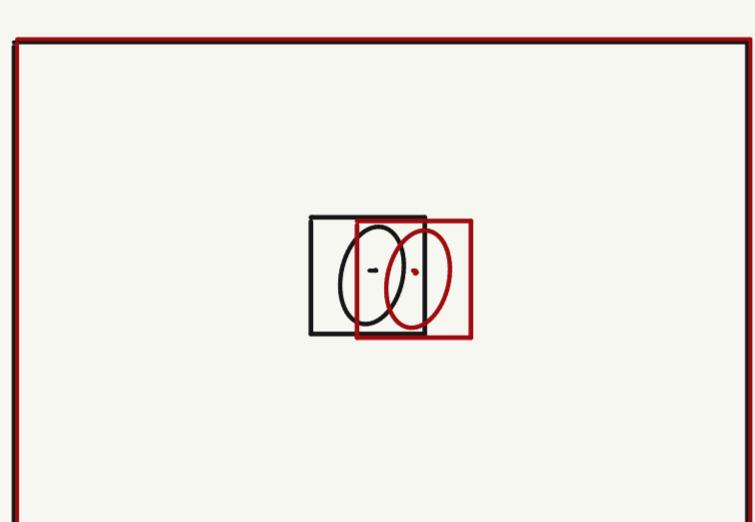
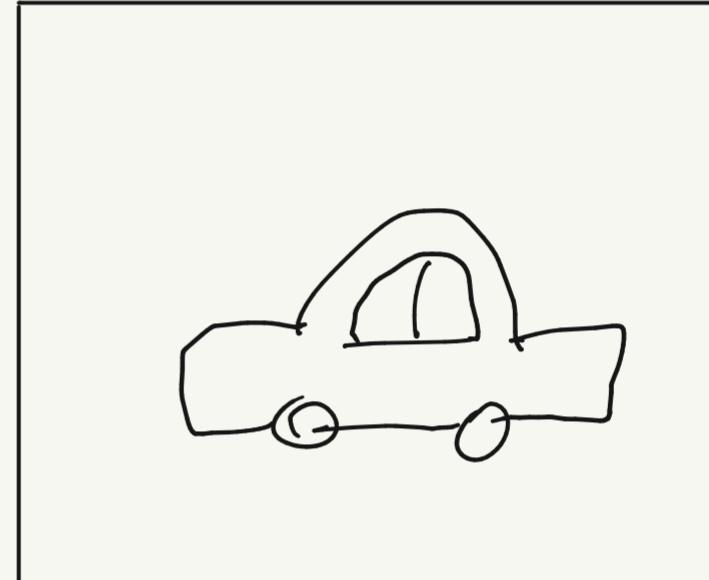
$$\Rightarrow \left( \frac{\partial I}{\partial x} \quad \frac{\partial I}{\partial y} \right) \begin{pmatrix} v_x \\ v_y \end{pmatrix} = - I_t$$

where,  $\vec{\nabla} I = \left( \frac{\partial I}{\partial x} \quad \frac{\partial I}{\partial y} \right)^T$ ,  $\vec{v} = (v_x \ v_y)^T$

17/04/2023

$$I_x u + I_y v + I_t = 0$$

$$\begin{pmatrix} u \\ v \end{pmatrix} \quad \vec{\nabla} I = (I_x, I_y)$$



$t=1$

$t=0$

Tracking the moving object.

Assignment 2 → Tracking an object first manually then automatically. (Software: Any)

# Types of Detector :

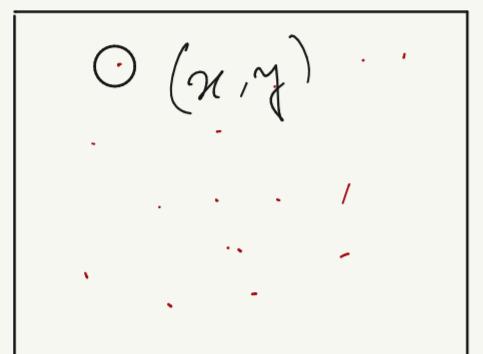
20/04/2023

- 1) Harris Corner Detector
- 2) Histogram of Oriented Gradient (HOG)
- 3) Scale Invariant Feature Transformation

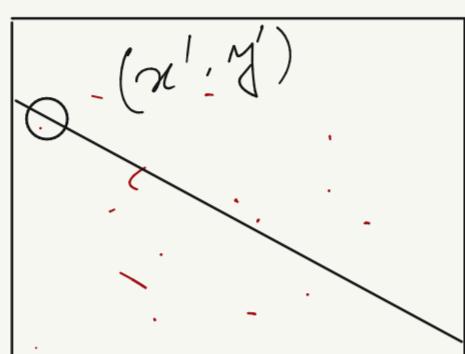
24/04/2023

① P

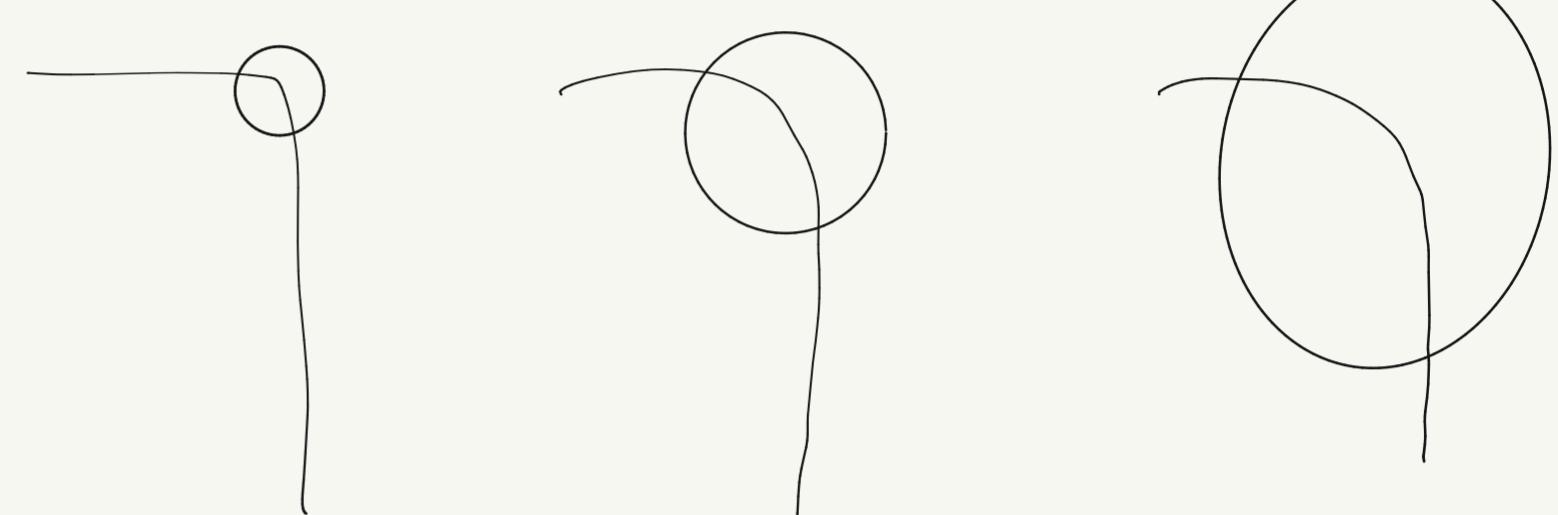
$$\omega = f \cdot \frac{B}{x - x'}$$



C<sub>1</sub>

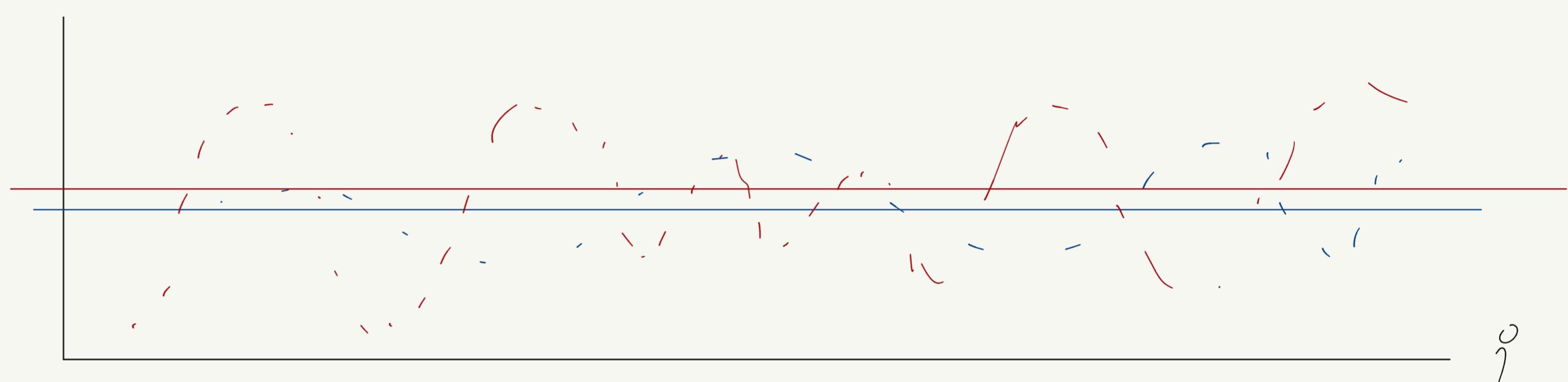


C<sub>2</sub>



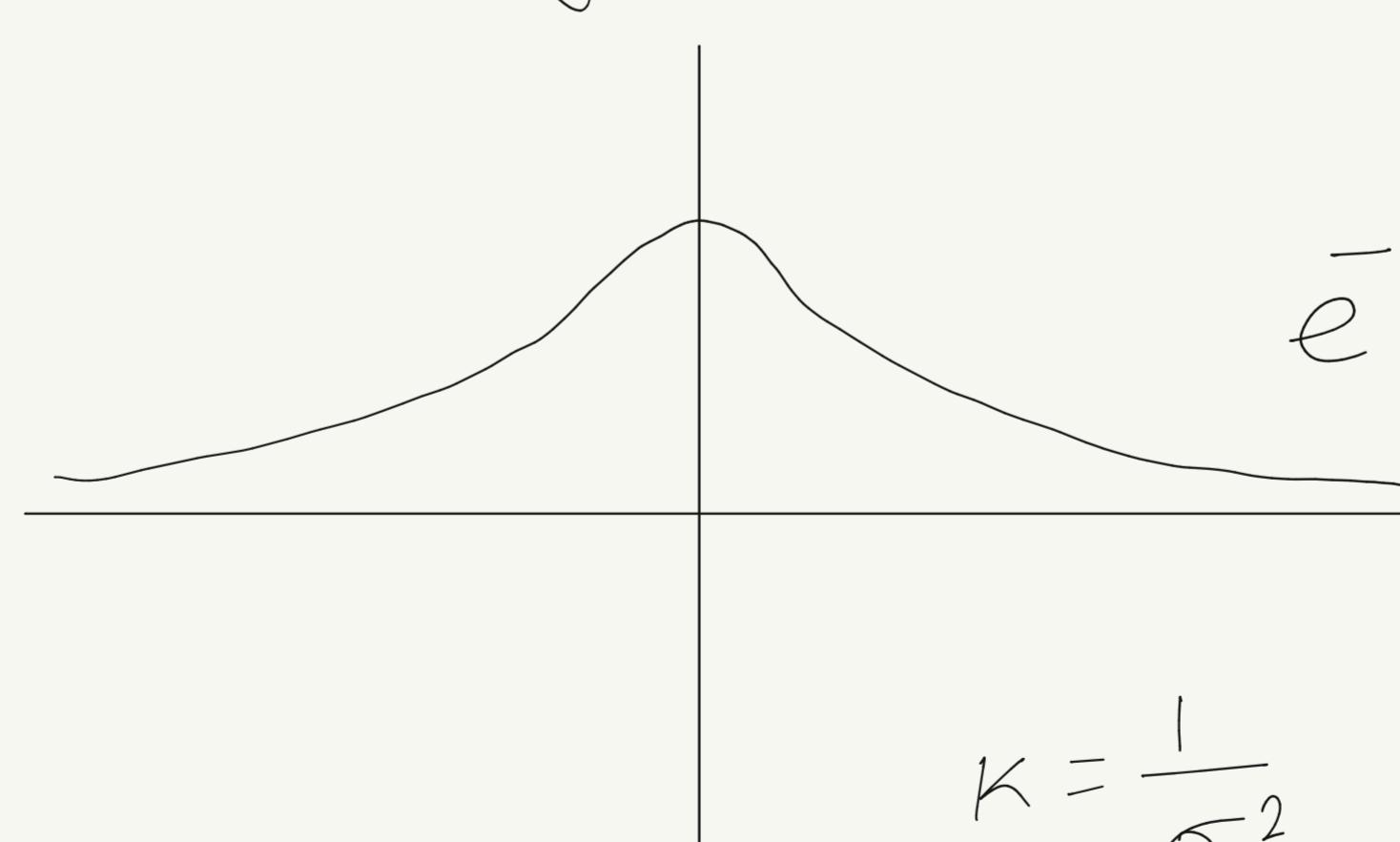
Edge and Corner of a Object

$$P(x, y) = \frac{\text{Cov}(x, y)}{\sigma_x \cdot \sigma_y} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$



$e^{-Kx^2}$  is a gaussian function.

$$\begin{aligned} \frac{d}{dx} \left( e^{-x^2/\sigma^2} \right) \\ = e^{-x^2/\sigma^2} \cdot \frac{1}{\sigma^2} (-2x) \end{aligned}$$

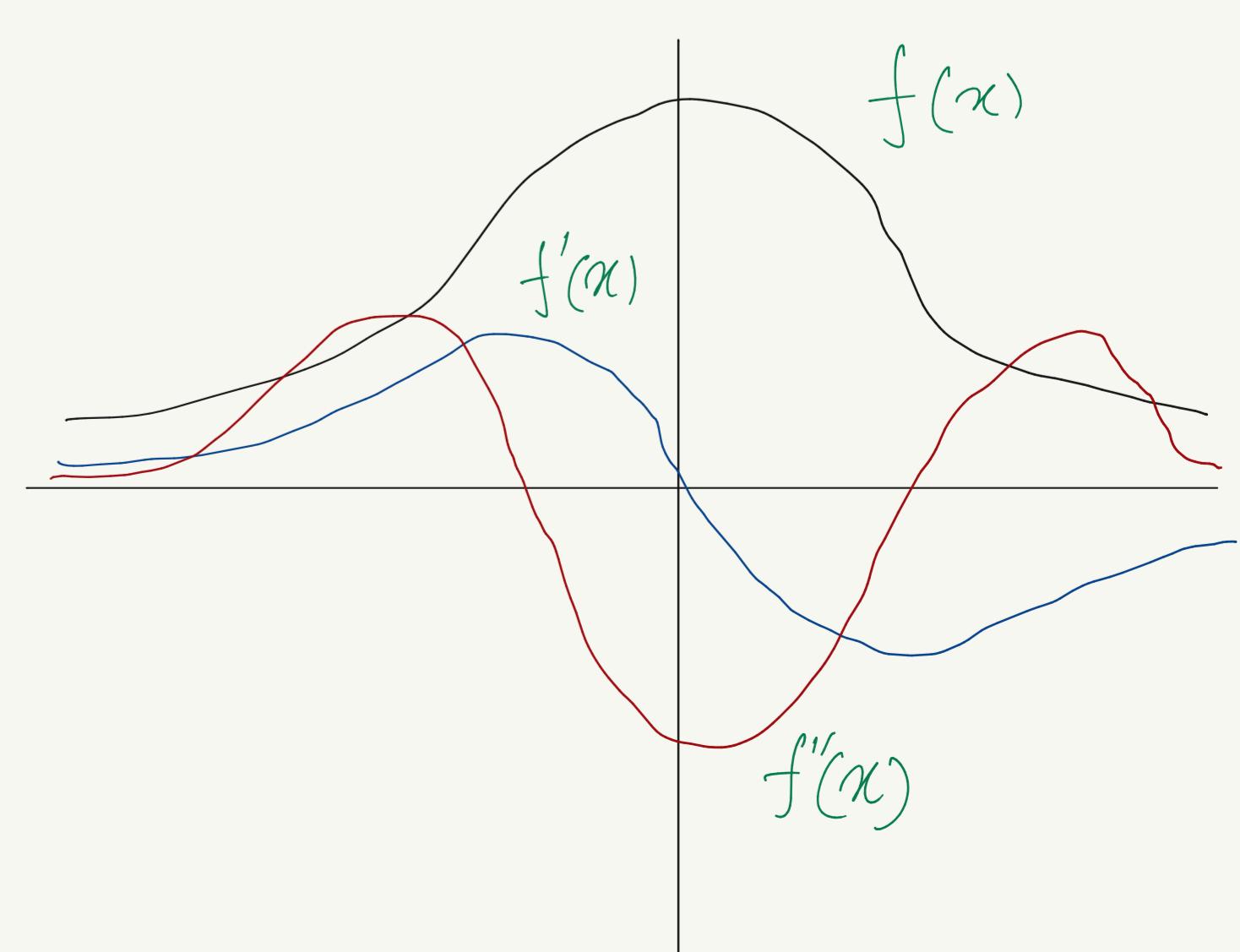
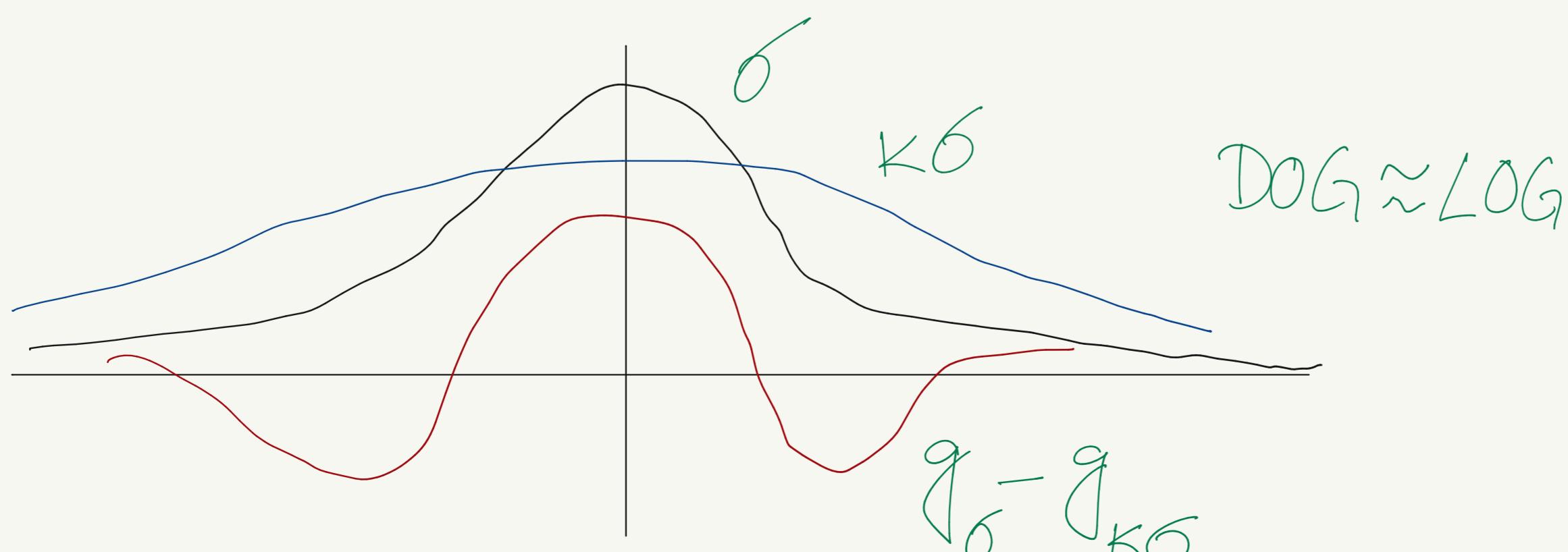


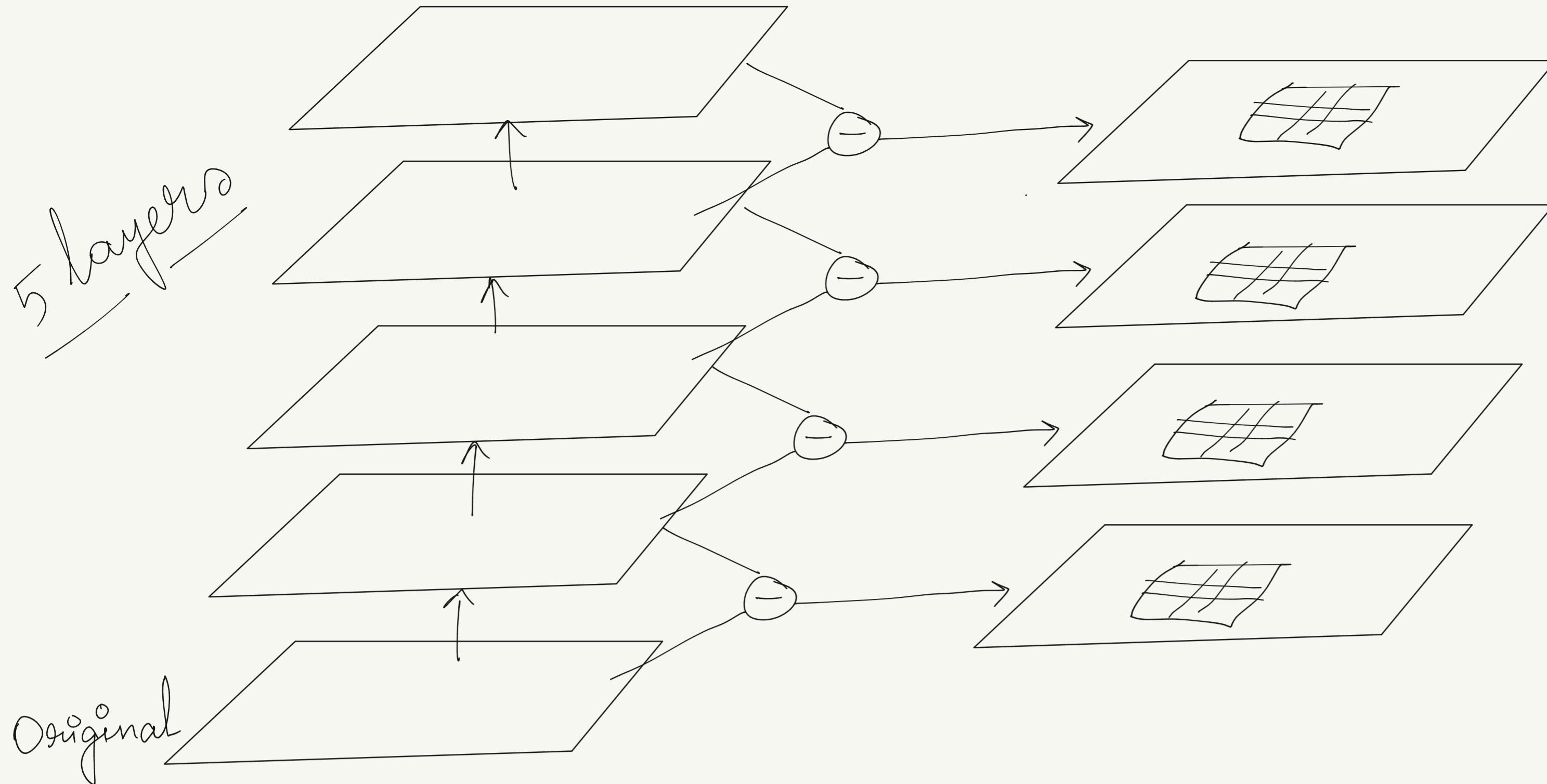
$$e^{-\frac{x^2}{\sigma^2}}$$

where,  $\sigma$  is the scale parameter

$$K = \frac{1}{\sigma^2}$$

thus, as  $\sigma$  increases  
the curve will be more flat





In Harris Corner Detector,

$$R = \text{Det}(H) - k(\text{Trace}(H)^2) = \alpha\beta - k(\alpha + \beta)^2$$

$[\alpha \gg 0, \beta \approx 0 \text{ or } \beta \gg 0, \alpha \approx 0] \rightarrow \text{Edge}$

$[\alpha \gg 0, \beta \gg 0] \rightarrow \text{Corner}$

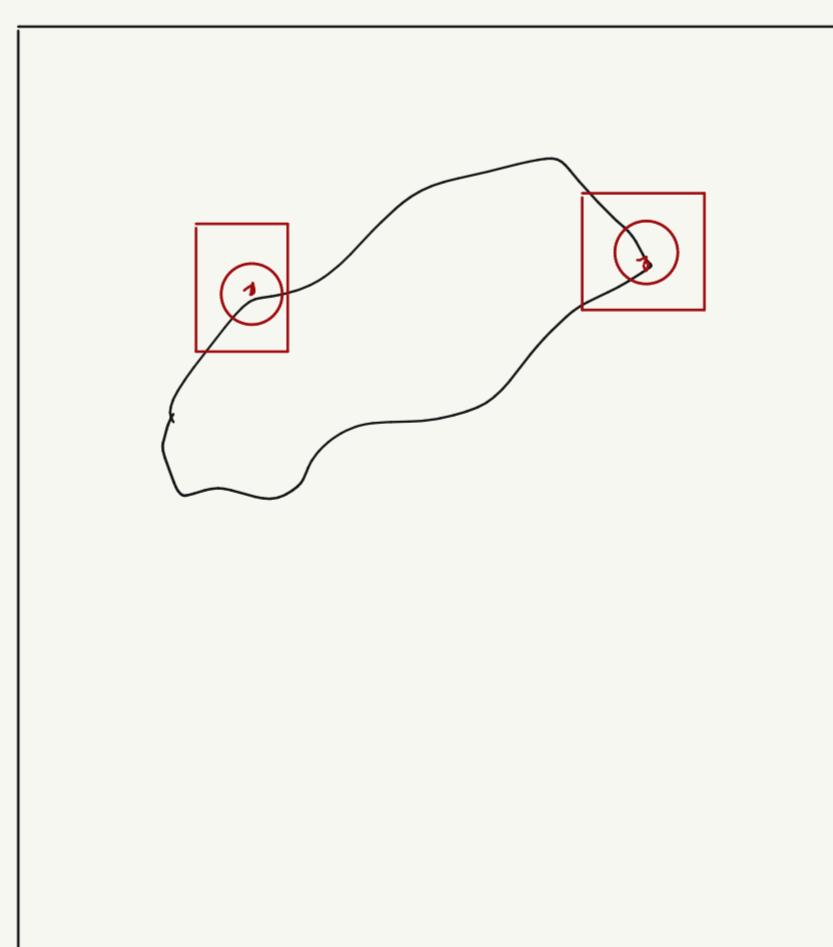
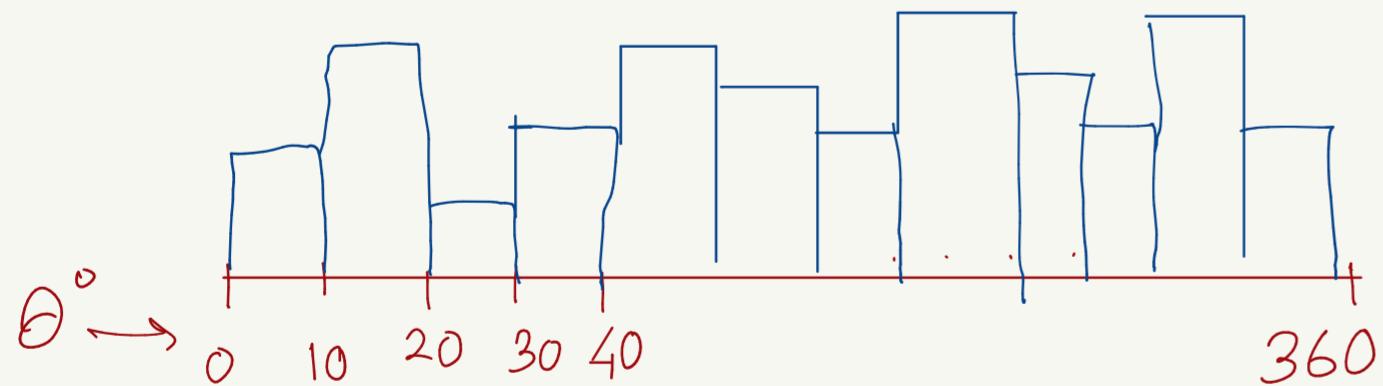
$$\frac{\text{Trace}(H)^2}{\text{Det}(H)} = \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(\beta n + \beta)^2}{\beta^2 n} = \frac{(n+1)^2}{n}$$

Let,  $\alpha = \beta n$ ,  $n > 1.0$

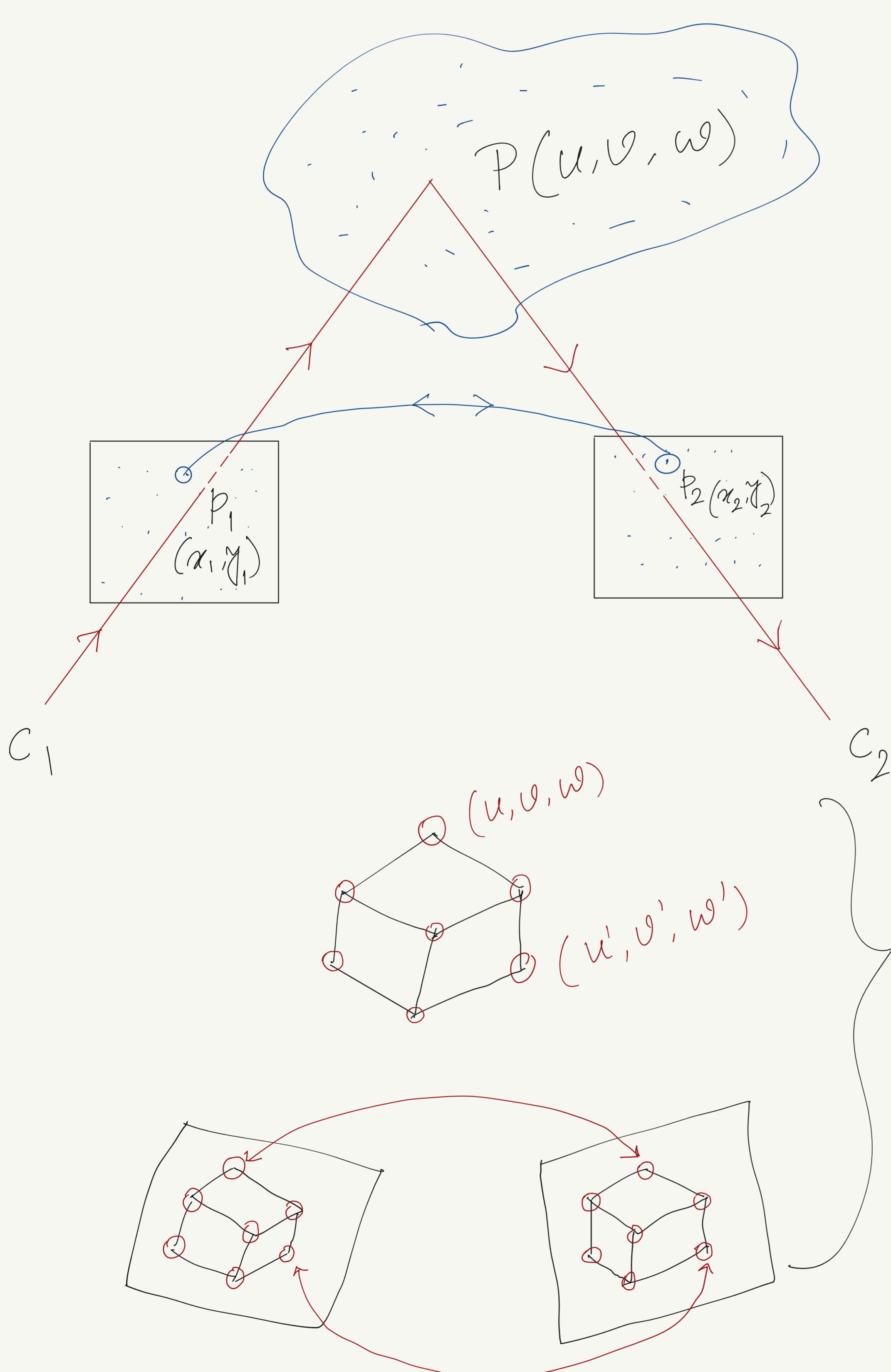
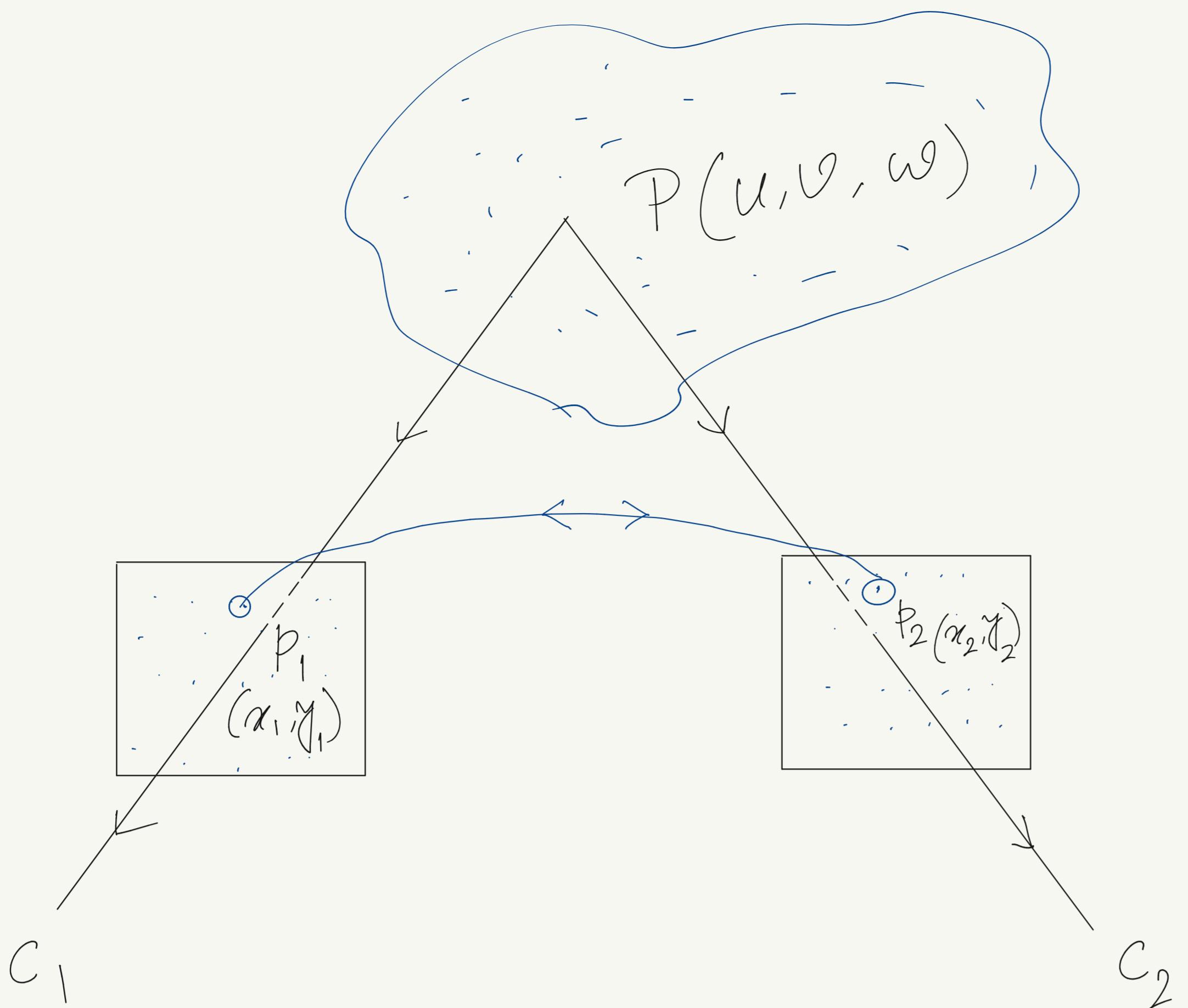
for  $n=1$ , we get the minimum value.

$$\begin{aligned} f(n) &= \frac{(n+1)^2}{n} \\ \Rightarrow f'(n) &= \frac{2(n+1)}{n} - \frac{(n+1)^2}{n^2} = 0 \\ \Rightarrow f''(n) &> 0 \\ \Rightarrow \frac{n+1}{n} &= 2 \\ \Rightarrow n+1 &= 2n \Rightarrow n=1 \end{aligned}$$

Orientation Assignment:

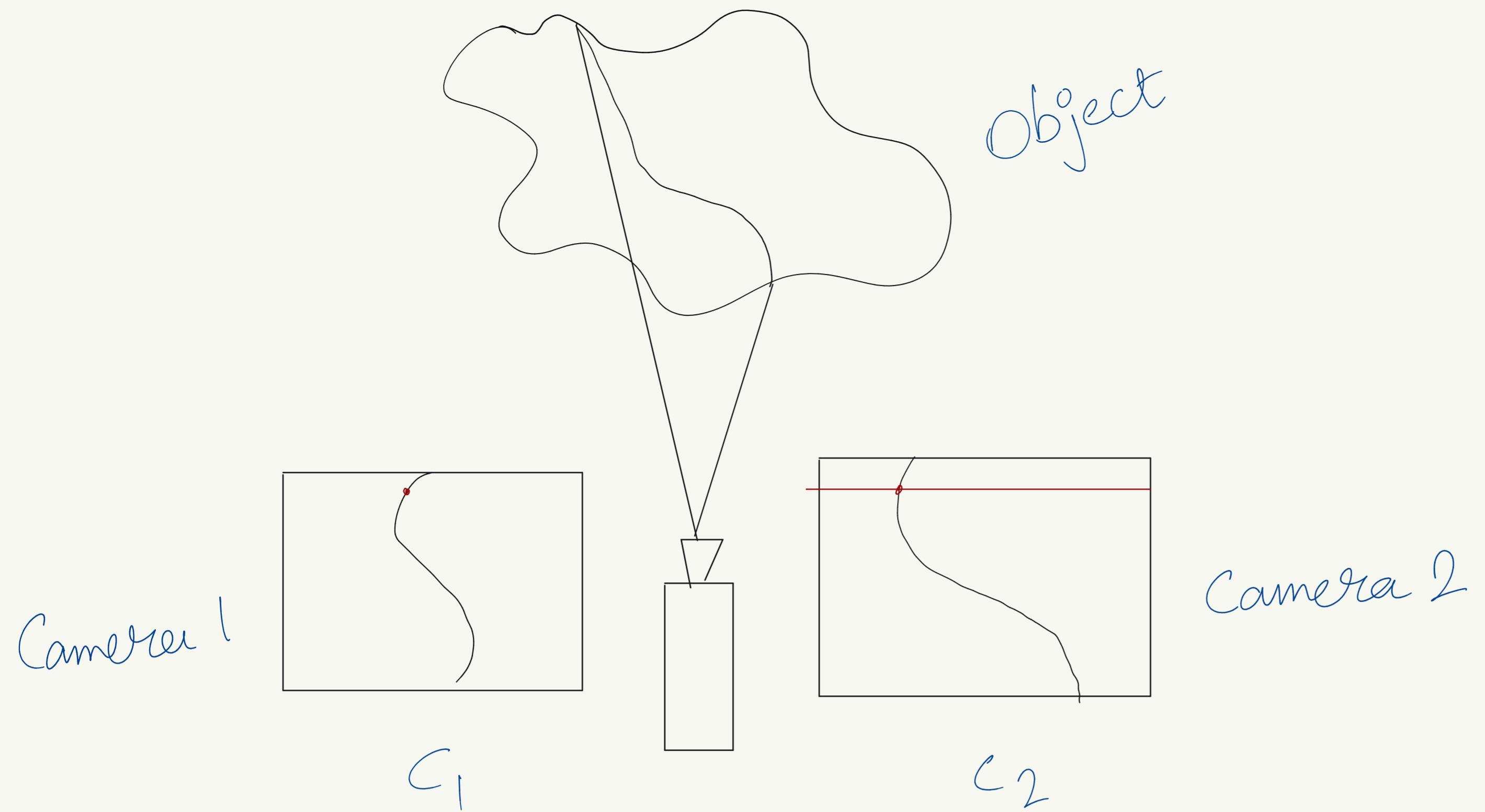


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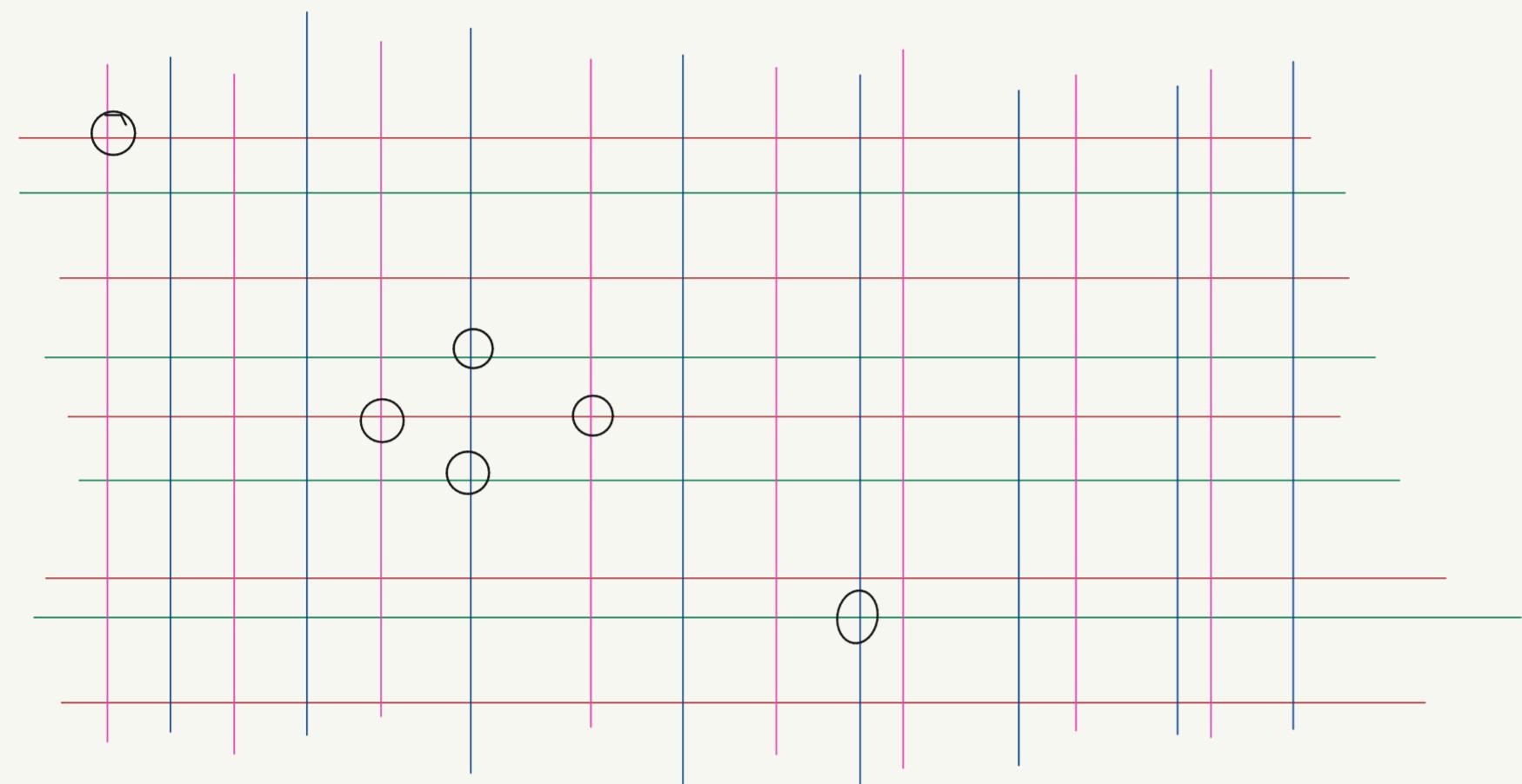


Similarly, we through  
a laser beam from  $P_1$ ,  
that hits  $P$  and then  
reflected through  $P_2$

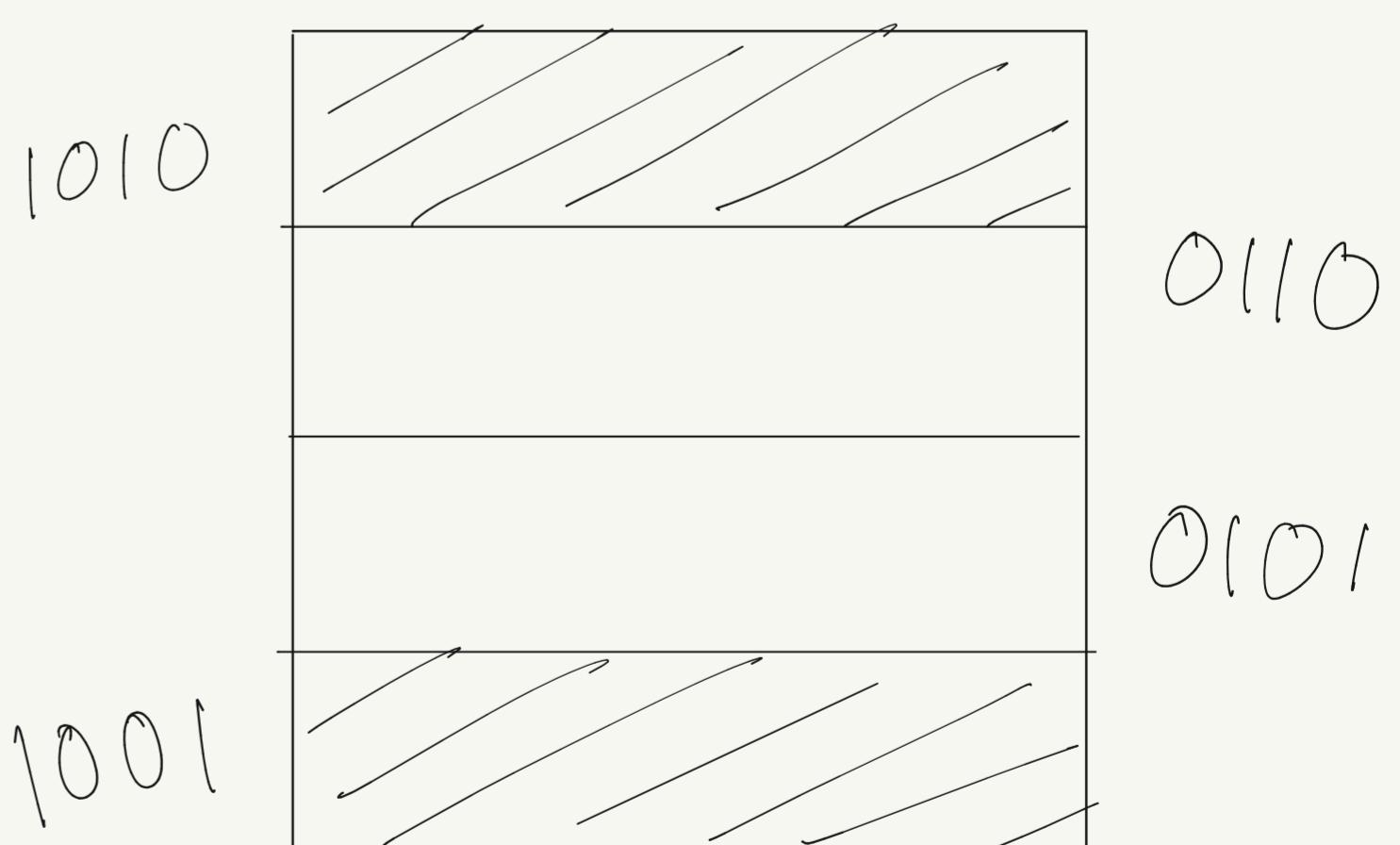
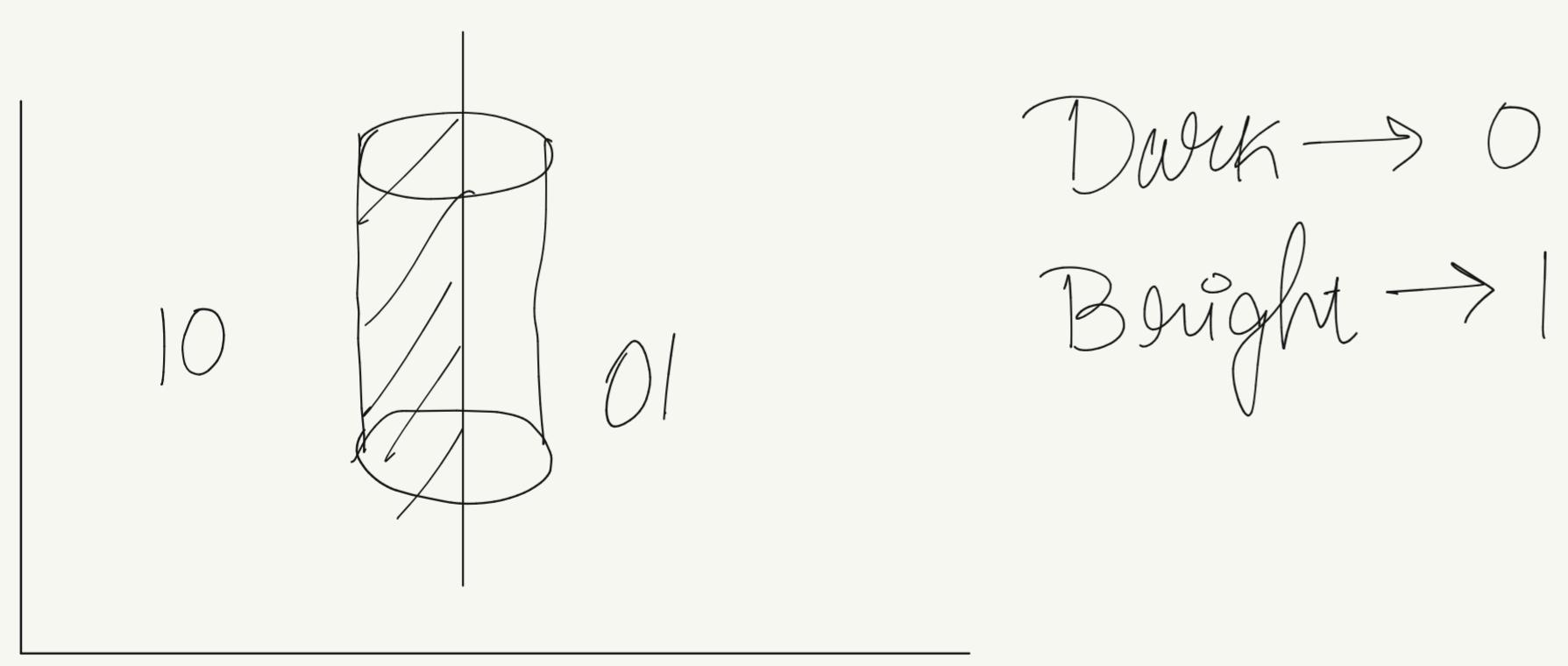
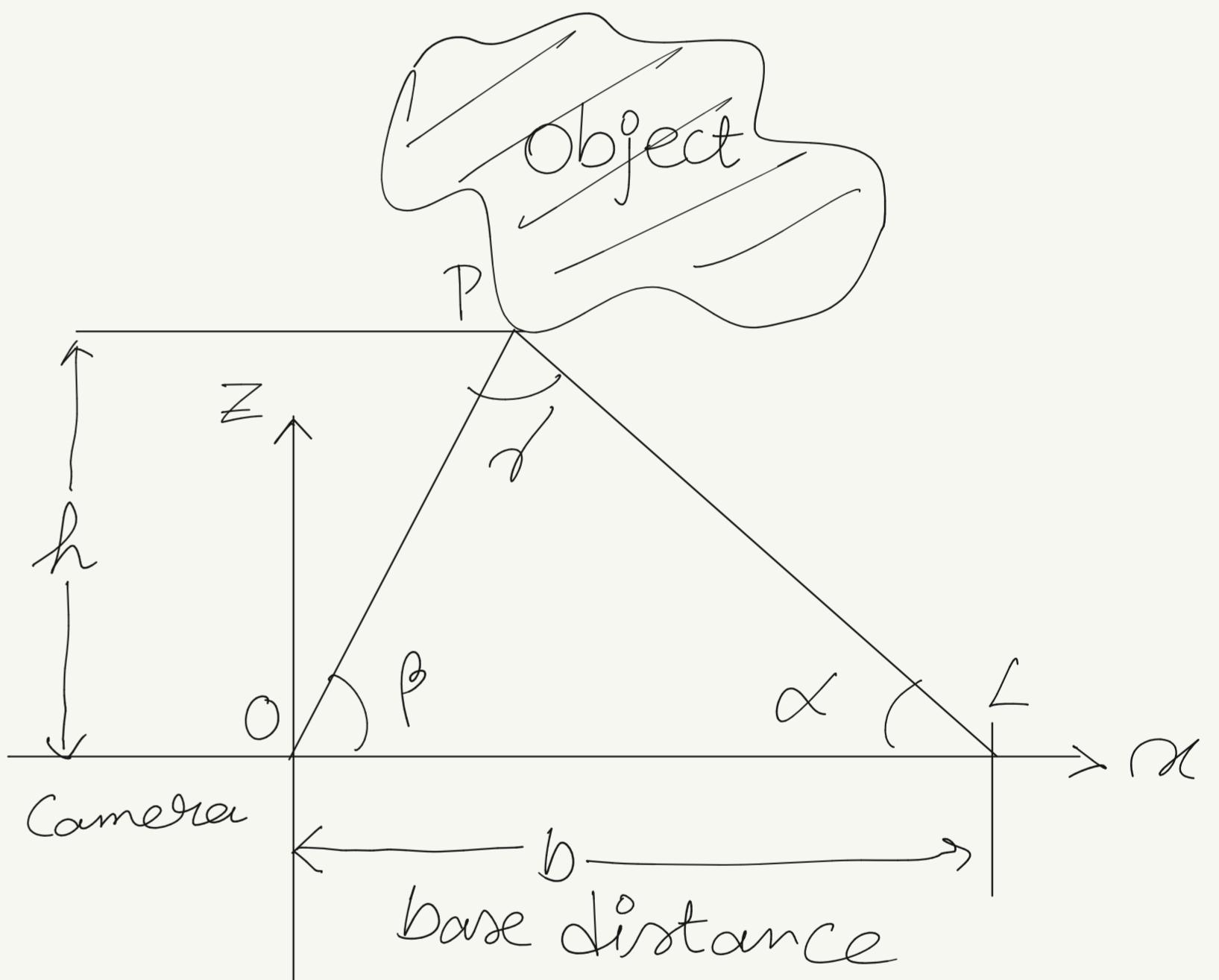
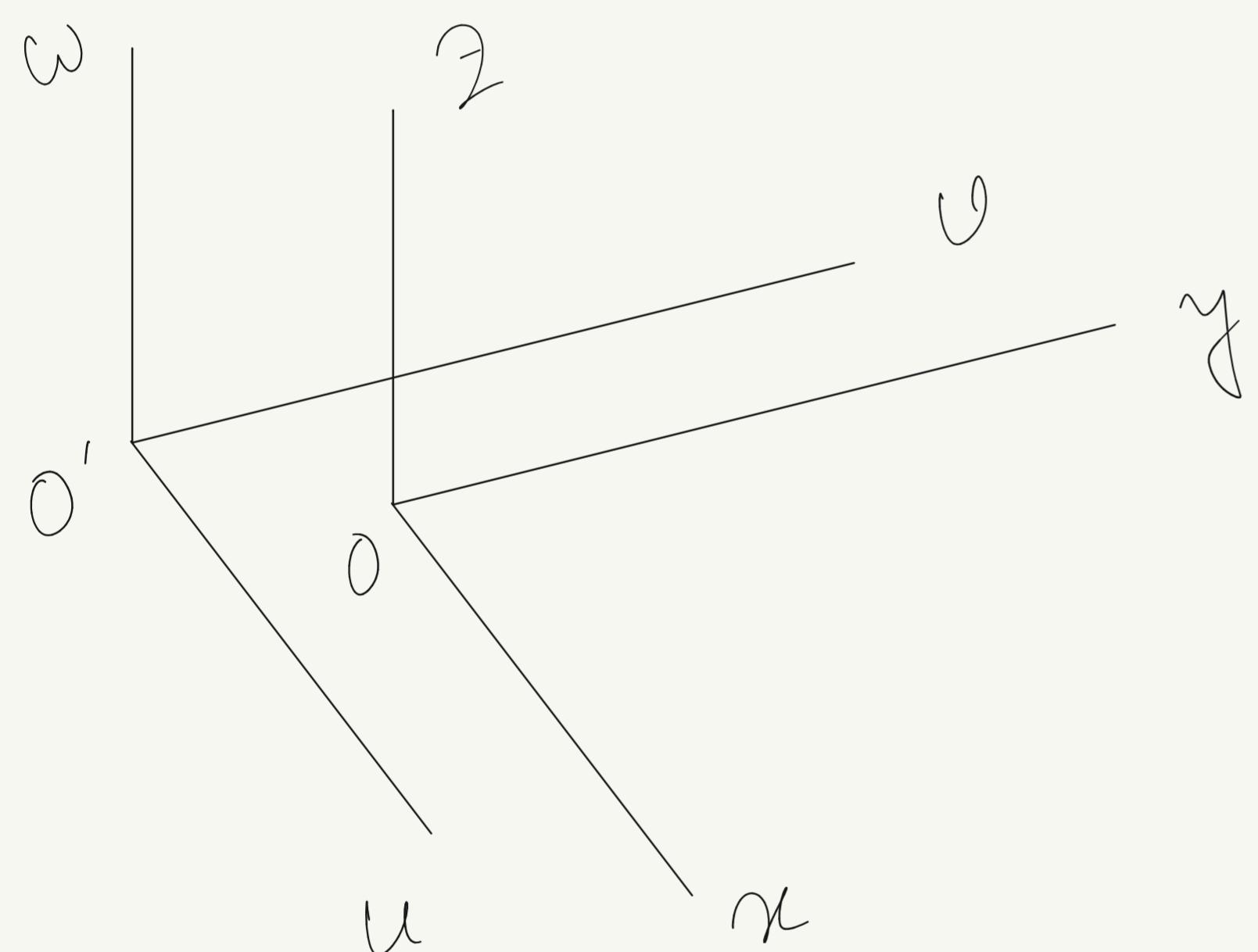
A grid of handwritten numbers and symbols, likely a math worksheet. The grid consists of 10 columns and 10 rows. The first two columns contain the number '0'. The third column contains the symbol '1'. The fourth column contains the symbol '2'. The fifth column contains the symbol '3'. The sixth column contains the symbol '4'. The seventh column contains the symbol '5'. The eighth column contains the symbol '6'. The ninth column contains the symbol '7'. The tenth column contains the symbol '8'. There are several small, illegible marks scattered throughout the grid.



04/05/2023



the locations are red-pink cross and green-blue cross



1st and 2nd light pattern  
3rd light pattern  
4th light pattern

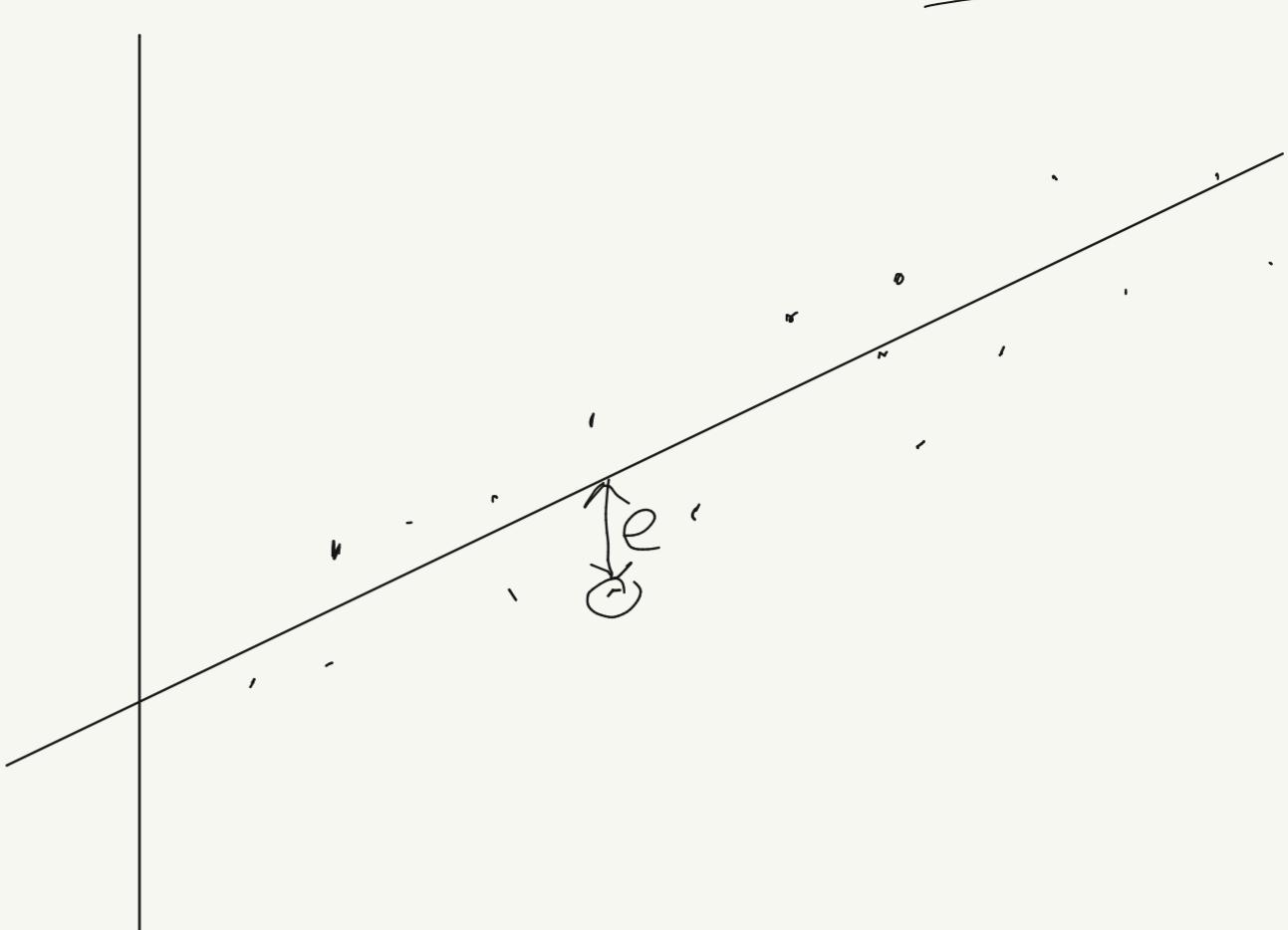
$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$

08/05/2023

## Simple Linear Regression

$$\hat{y}_i = mx_i + c - \text{error}$$

$$\therefore e = \sum_{i=1}^n (\hat{y}_i - y_i - mx_i - c)^2$$



Predicted class

		P(+)	N(-)
Actual Class	P(+)	Correctly Positive	Falsely Negative
	N(-)	Falsely Positive	Correctly Negative

$$\text{Accuracy} = \frac{CP + CN}{CP + CN + FP + FN} \times 100\%$$

$$\text{Failure} = \frac{FP + FN}{CP + CN + FP + FN} \times 100\%$$

$$FP \text{ percentage} = \frac{FP}{CP + CN} \times 100\%$$

$$FN \text{ percentage} = \frac{FN}{CP + CN} \times 100\%$$

Eg. CP = 170, FN = 0, FP = 30, CN = 0

then,

$$\text{Accuracy} = \frac{170}{200} \times 100\% = 85\%$$

$$\text{Failure} = \frac{30}{200} \times 100\% = 15\%$$

$$FP \text{ percentage} = \frac{30}{200} \times 100\% = 15\%$$

$$FN \text{ percentage} = \frac{0}{170} \times 100\% = 0\%$$

Predicted

		A	B	C	D
Actual	A	CA	Not A		
	B	False			
C	A				
D					

11/05/2023

$$\begin{array}{c} \text{dependent} \\ \downarrow \\ \omega = f(a) \\ \text{independant} \end{array}$$

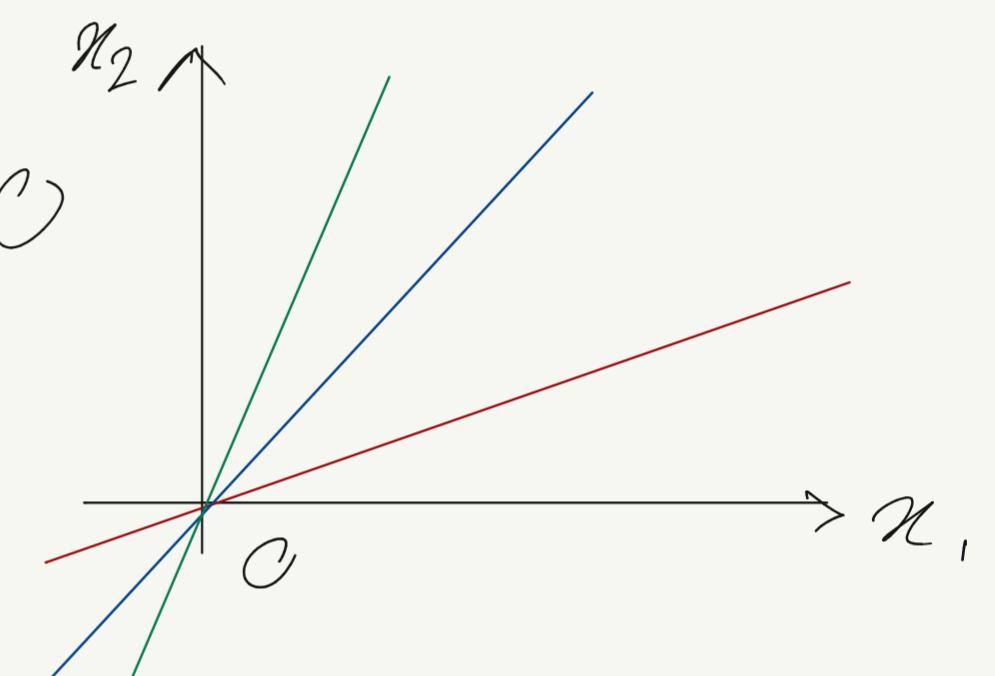
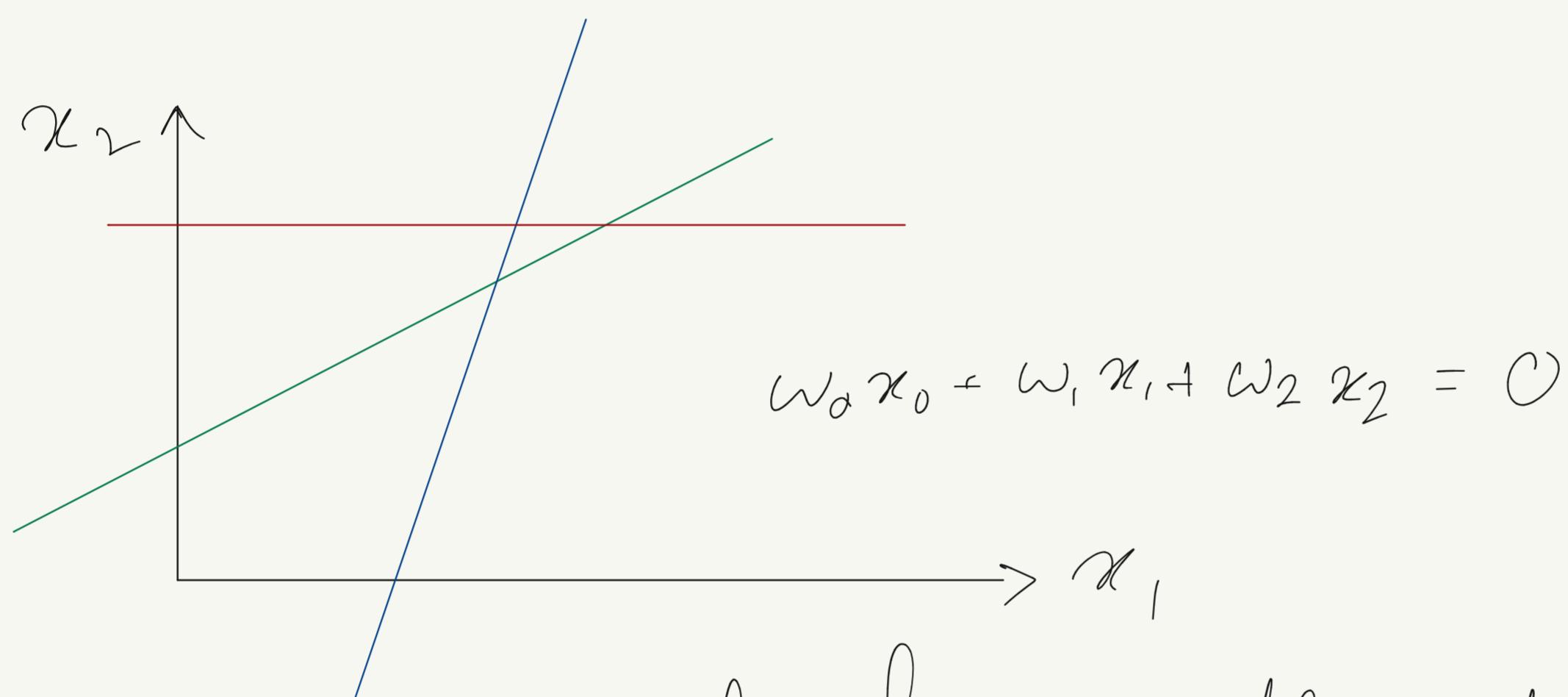
$$\text{Similarly, } p_n = f(x_1, x_2, x_3) \quad \begin{array}{c} \text{dependent} \\ \uparrow \\ \text{independant} \end{array}$$

$$\phi = f(x, y, z) = g(a, b, c) = ax + by + cz$$

$$x_1 \xrightarrow{\omega_1} \circlearrowright \rightarrow y \\ x_2 \xrightarrow{\omega_2} \delta(\sum \omega_i x_i) = \delta(z)$$

$$\text{Similarly, } y = mx + c \quad \text{and} \quad e = \sum_i \|y_i - mx_i - c\|^2$$

$$z = \sum w_i x_i$$



Let us consider the linear transformation.

$$T(z) = \omega z$$

but,  $T(z) = \omega z + k$  is not a linear transformation.

$$x_1 \xrightarrow{\omega_1} \circlearrowright \rightarrow y \\ x_2 \xrightarrow{\omega_2} \circlearrowright \rightarrow$$

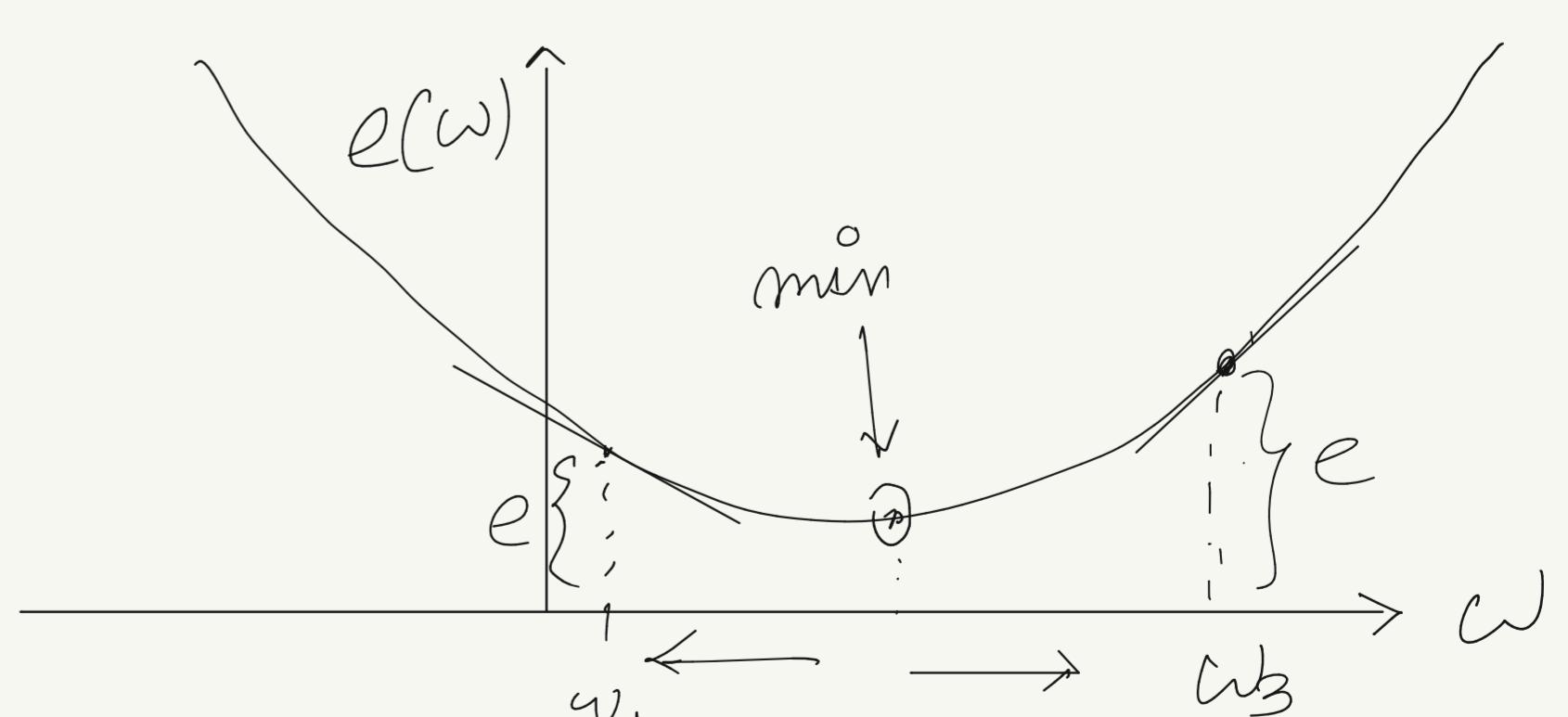
$$y = \delta(\omega_1 x_1 + \omega_2 x_2) \\ y = (\omega, x)$$



$$e(\omega_1, \omega_2) = \|y - \hat{y}\|^2$$

$$T_j, e(\omega) = \|y - \hat{y}\|^2$$

$$\omega_{\text{new}} = \omega_{\text{old}} - \left( \frac{de}{d\omega} \right) \Delta \omega$$

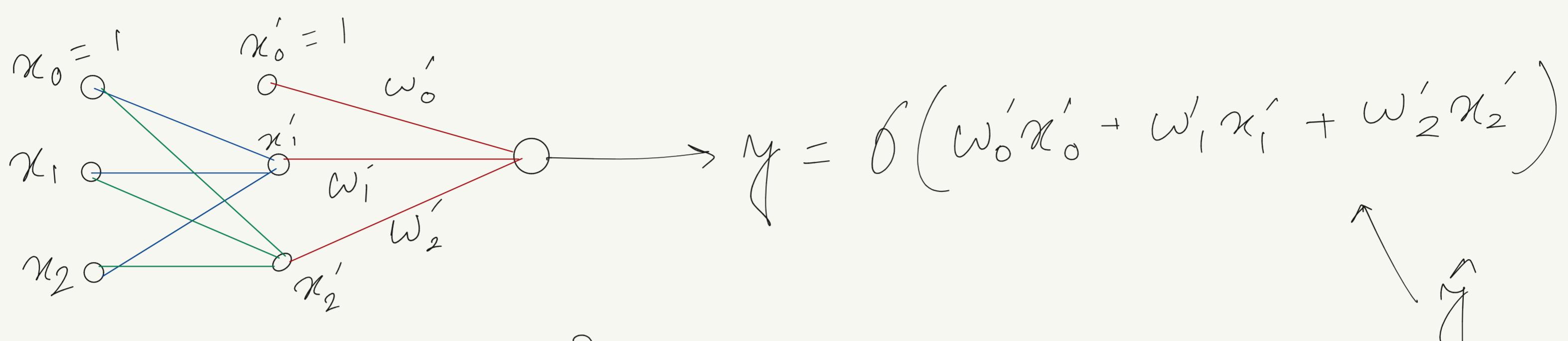
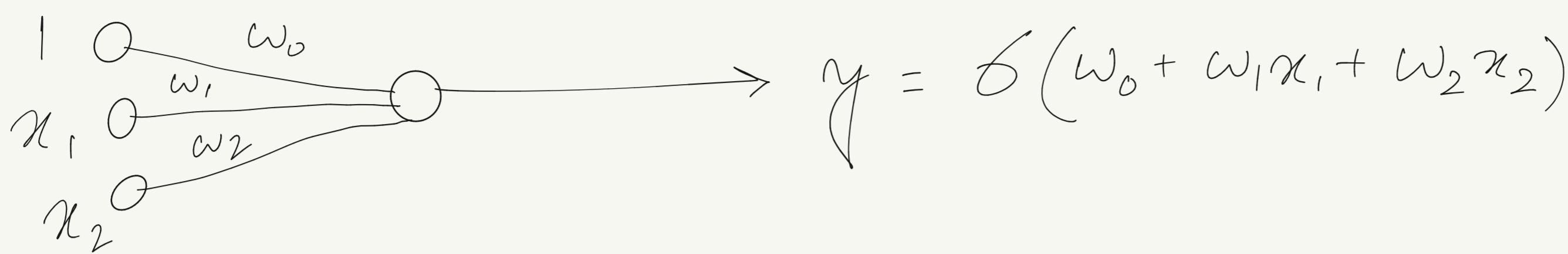


$$\Rightarrow \boxed{\omega_{\text{new}} = \omega_{\text{old}} - \eta \frac{de}{d\omega}}$$

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For the multiple variables,

$$w_{i,\text{new}} = w_{i,\text{old}} - \eta \frac{\partial E}{\partial w_i}$$



$$e(\omega') = \|y - \hat{y}\|^2$$

$$y = e^{a \sin(x^3)} \Rightarrow \frac{dy}{dx} = e^{a \sin(x^3)} \cdot a \cos(x^3) \cdot 3x^2$$

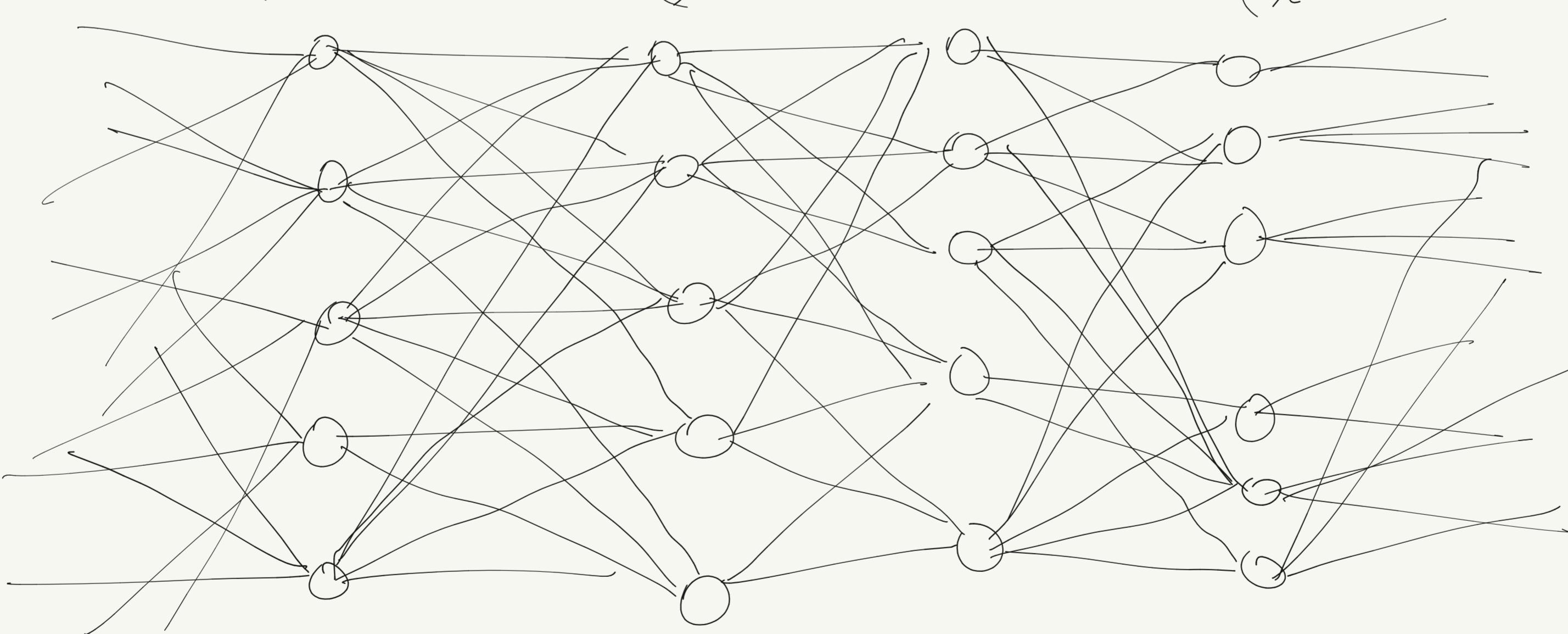
$$\Rightarrow \frac{dE}{dx} = 3a x^2 \cos(x^3) e^{a \sin(x^3)}$$

thus,

$$\hat{y} = \sigma(w_0'x_0' + w_1'x_1' + w_2'x_2') \quad (\text{for 2 layers})$$

$$= \sigma(w_0' + w_1' \sigma(w_{10} + w_{11}x_1 + w_{12}x_2) + w_2' \sigma(w_{20} + w_{21}x_1 + w_{22}x_2))$$

Layers  $\rightarrow$   $(l-2)$   $(l-1)$   $l$   $(l+1)$



Starting from 0<sup>th</sup> layer

$w$  (to layer)

$w$  (to node) (from node)

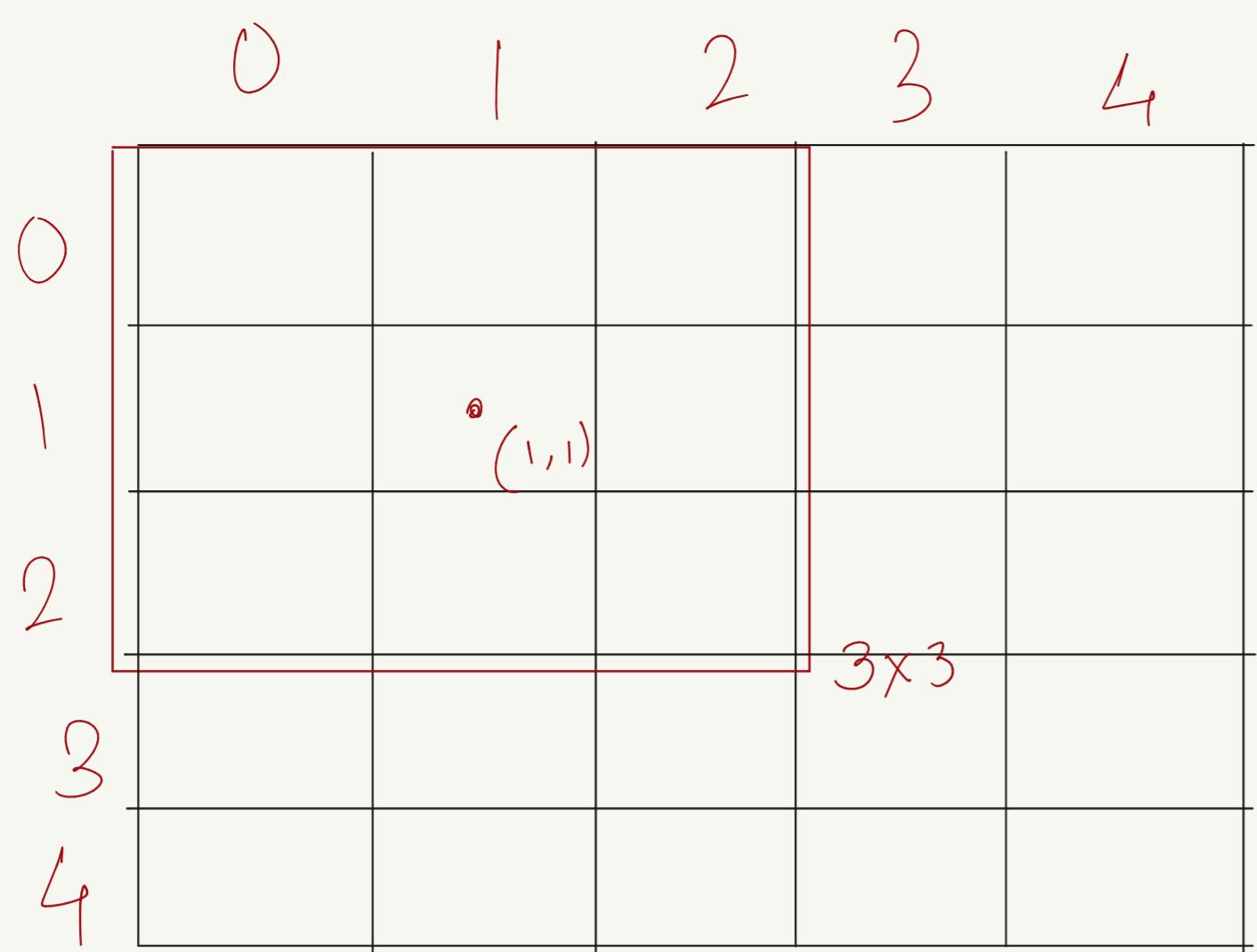
$$x_i = y_i$$

$$\hat{y}_i = \sigma(w^{i-1} x^{i-1})$$

Eq.

$$w_{33}^2(\text{new}) = w_{33}^2(\text{old}) - \eta \left( \frac{\partial E}{\partial w_{33}^2} \right)$$

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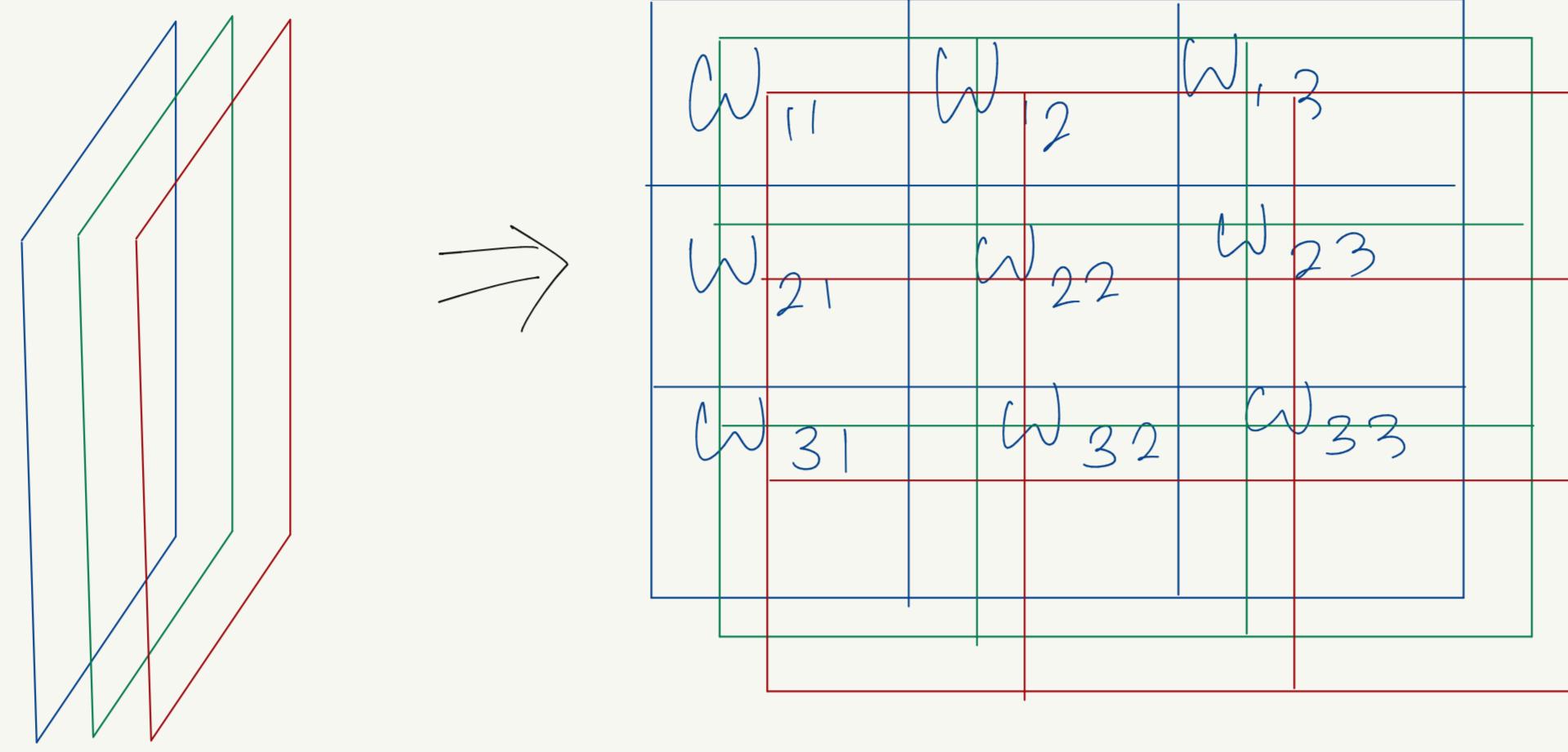


Total kernels we get,

$$(4 - 2 + 1) \times (4 - 2 + 1)$$

i.e. in general,

$$(M+1 - 3+1) \times (N+1 - 3+1)$$



1 feature map has 1 kernel  
6 feature maps have 6 kernels }

dimension  
 $5 \times 5 \times 6$

