Ramakrishna Mission Vivekananda Educational and Research Institute
PO Belur Math, Howrah, West Bengal 711 202
School of Mathematical Sciences
Department of Computer Science

MSc BDA : Batch 2021-23, Semester II, MidSem Exam
DA311: Time Series
Dr. Sudipta Das

*S*tudent Name (in block letters): Date: 11 April 2022
*S*tudent Roll No: *M*ax Marks: 80
*S*ignature: Time: 2.5hrs

Answers must be properly justified to deserve full credits.

1. (8 points) Are the following processes stationary. ($\{Z_t\} \sim WN(0, \sigma^2)$)
   (a) (4 points) $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$
   (b) (4 points) $X_t = Z_t Z_{t-1}$

2. (8 points) Are the following ARMA processes causal as well as invertible. ($\{Z_t\} \sim WN(0, \sigma^2)$)
   (a) (4 points) $X_t + 1.8X_{t-1} + 0.72X_{t-2} = Z_t + 0.2Z_{t-1}$
   (b) (4 points) $X_t + 1.7X_{t-1} + 0.72X_{t-2} = Z_t$

3. (16 points) Consider the model $Y_t = Z_t Y_{t-1}$ where $Z_t \sim WN(0, \sigma^2)$
   (a) (6 points) Find the conditional variance of Y_t given Y_{t-1}.
   (b) (10 points) Show that under the assumption of second order stationarity if $\sigma^2 \neq 1$ then the variance of Y_t is either zero or infinite.

4. (16 points) For the following MA(1) process

$$X_t = Z_t + \theta Z_{t-1}, t = 0, \pm 1, \ldots,$$

   the best linear predictor of X_{n+1} based on $X_1, \ldots, X_n$ is $X_{n+1}^{(n)} = \phi_{\mathbf{n}}{}' \mathbf{X_n}$, where $\phi_{\mathbf{n}}$ satisfies

$$\Gamma_n \phi_{\mathbf{n}} = \gamma_{\mathbf{n}}.$$

   Show that for $1 \leq j < n$,

$$\phi_{n,n-j} = (-\theta)^{-j}(1 + \theta^2 + \cdots + \theta^{2j})\phi_{nn}.$$

   Hence, find ϕ_{nn}, that the value at lag n of the partial ACF of this MA(1) process.

5. (16 points) The sample autocorrelation at lag 1, from a set of 498 observations, is calculated as -0.66 and the maximum of the correlations from lag 2 to lag 20 is .08. Assuming that the sample size is large, test the claim that the sample is from an MA(1) process, at a 5% level of significance.
   (Bartlett's formula: $w_{ij} = \sum_{k=1}^{\infty} \{\rho(k+i) + \rho(k-i) - 2\rho(i)\rho(k)\}\{\rho(k+j) + \rho(k-j) - 2\rho(j)\rho(k)\}$)

6. (16 points) Given two observations x_1 and x_2 from the causal AR(1) process satisfying

$$X_t = \phi X_{t-1} + Z_t, \ \{Z_t\} \sim N(0, \sigma^2),$$

   and assuming that $|x_1| \neq |x_2|$, find the maximum likelihood estimates of ϕ and σ^2.

This exam has total 6 questions, for a total of 80 points and 0 bonus points.
Best of luck!!