Ramakrishna Mission Vivekananda Educational and Research Institute

PO Belur Math, Howrah, West Bengal 711 202

School of Mathematical Sciences

Department of Computer Science

MSc BDA : Batch 2022-24, Semester II, MidSem Exam

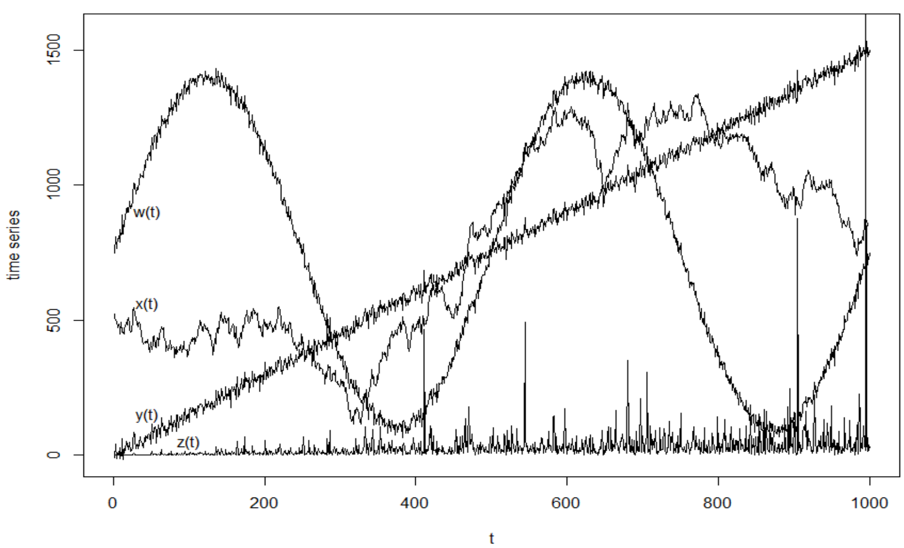
DA311: Time Series

Dr. Sudipta Das

Student Name (in block letters):	Date: 10 April 2022
Student Roll No:	Max Marks: 70
Signature:	Time: 2.25 hrs

Answers must be properly justified to deserve full credits.

1. (6 points) The following figure shows the overlaid time plots of four time series, $w(t), x(t), y(t)$ and $z(t)$ for $t = 1, \ldots, 1000$. For which of the four series would it be reasonable to look for a stationary model, after first order differencing of the given series at lag 1? Explain.



2. (16 points) Suppose that the series $\{y_t\}$ is modeled by $y_t = \mu_t + x_t$, where $\mu_t = \mu_{t-1} + \epsilon_t$ is not observable and $x_t = \omega_t - \theta\omega_{t-1}$. Assume that $\epsilon_t \sim WN(0, \sigma_\epsilon^2), \omega_t \sim WN(0, \sigma_\omega^2)$, and $Cov(\epsilon_t, \omega_s) = 0$ for all (t, s). Please answer the following questions:

   (a) (4 points) What is the mean and the variance of y_t?

   (b) (2 points) Is y_t stationary?

   (c) (10 points) If y_t is stationary, what is the autocovariance function of y_t? Otherwise,, suggest an appropriate transformation to induce stationarity in y_t. Then, calculate the mean and autocovariance function of the transformed stationary series.

3. (16 points) Consider the following ARMA(2,1) model

$$Y_t = 1.1Y_{t-1} - 0.24Y_{t-2} + Z_t - 0.2Z_{t-1},$$

where Z_t is white noise $(0, \sigma^2)$.

(a) (4 points) Show that this process is causal and invertible.

(b) (12 points) Calculate the ACF of y_t.

4. (16 points) For the following MA(1) process

$$X_t = Z_t + \theta Z_{t-1}, t = 0, \pm 1, \dots,$$

the best linear predictor of X_{n+1} based on $X_1, \dots, X_n$ is $X_{n+1}^{(n)} = \boldsymbol{\phi_n}'\mathbf{X_n}$, where $\boldsymbol{\phi_n}$ satisfies

$$\Gamma_n \boldsymbol{\phi_n} = \boldsymbol{\gamma_n}.$$

(a) (10 points) Show that for $1 \le j < n$,

$$\phi_{n,n-j} = (-\theta)^{-j}(1 + \theta^2 + \cdots + \theta^{2j})\phi_{nn}.$$

(b) (6 points) Hence, find ϕ_{nn}, that the value at lag n of the partial ACF of this MA(1) process.

5. (16 points) Consider the following MA(1) model, where $\{Z_t\} \sim WN(0, 1)$:

$$Y_t = Z_t + \theta Z_{t-1}.$$

where $\{Z_t\} \sim WN(0, 1)$.

(a) (8 points) Find the moment estimator of θ.

(b) (8 points) Find the bias of this estimator.

This exam has total 5 questions, for a total of 70 points and 0 bonus points.

Best of luck!!