Implementing Morlet Kernel in Kernel Principal Component Analysis to de-noise images

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Abstract

We introduce the implementation of a newly proposed kernel in Kernel Principal Component Analysis to filter noisy images. The Morlet Wavelet Function finds extensive application in image processing to filter out noise from distorted or corrupt images. Kernel PCA with standard Gaussian RBF kernel does the same for images likely corrupted with Gaussian noise by reducing the dimensionality of the image hence removing the redundant noises and then reconstructing it. Our project aims to validate some mathematical properties of a kernel constructed using the wavelet function and hence implement it as a valid kernel in Kernel PCA to de-noise images. We will also provide a comparative review of our proposed methodology with Linear PCA and Kernel PCA using Gaussian RBF kernel on the same set of noisy images.

1 Introduction

[1] formally introduced the Morlet wavelet kernel based on the Morlet wavelet function $\psi(x) = \cos(5x)\exp(-\frac{x^2}{2})$ as follows:

$$k(x,y) = \prod_{k=1}^{d} \cos\left(\frac{5(x_k - y_k)}{a}\right) \exp\left(-\frac{(x_k - y_k)^2}{2a^2}\right) \quad where \quad x, y \in \mathbb{R}^d$$

The paper produced a proof to validate their claim that this is indeed a positive definite kernel using a number of integral transformations. However we have taken a different approach that involves the direct use of Mercer Conditions to prove the same. Once the mathematical foundation for the project is established we will deploy the kernel in kernel principal component analysis to de-noise standard image data corrupted with both Gaussian noise and noise generated from our corresponding wavelet distribution given by:

$$\chi(x) = \cos(5x)\exp(-\frac{x^2}{2}), \quad -2.5 \leq x \leq 2.5$$

[2] proposed a simple way of mutating data with this type of noise which involves drawing a random sample $x^* \sim Uniform[-2.5, 2.5]$ and substituting it in $\chi(x)$ to get a new value $\chi(x^*)$ which will be added as noise to our image data.

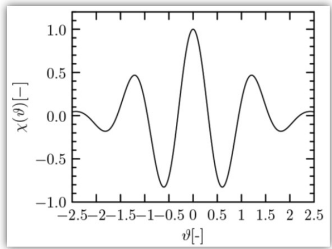

Figure 1: Morlet Wavelet Density

2 Proposed methodology

Once the images are corrupted, they are to be converted to data matrices. The corresponding data matrices then undergo a reduction in dimensionality through Kernel Principal Component Analysis with our suggested kernel. This process removes most of the redundant noises and on reconstruction we are supposed to obtain a nearly noise-free image. Other standard Linear PCA and Kernel PCA models like ones utilizing the Gaussian RBF kernel are also to be tested on the same data and the results are to be compared and interpreted.

3 Work plan and time line

Data Collection - 1 week
Literature Review - 2 weeks
Implementation and Comparison- 2 weeks
Interpretation - 1 week

4 Work plan division in our group

Data Collection - Uddalak Mukherjee
Literature Review - Both

Validating Mathematical Foundations - Uddalak Mukherjee
Implementation in Python - Biswajit Rana
Comparison with other models - Biswajit Rana
Interpretation of Results - Both

References

[1] S.Xie, A.T.Lawniczak, S.Krishnan, P.Lio Wavelet Kernel Principal Component Analysis in Noisy Multiscale data Classification ISRN Computational Mathematics 259 (2012) ID - 197352.

[2] T.You, Y.HU, P.Li, Y.Tang An improved imperialist competitive algorithm for global optimization Turkish Journal of Electrical Engineering & Computer Science (2019) doi:10.3906/elk-1811-59