

Roulette Problem :

We are going to play 2 games of Roulette .

- First game is as follows :

odd	even	other
1	2	0
3	4	00
\vdots	\vdots	
35	36	

(Red) (Blue) (Green)

The player has 2 options ^(Red & Blue), and bets 1 rs.

and if he chooses Red and the ball falls in other colours

then he will lose 1 rs.

and if the ball falls in red he will win 1 rs.

So, ~~if~~ Let X be a r.v. takes values $+1, -1$.

$$\therefore P(X=+1) = \frac{18}{38} \quad , \quad P(X=-1) = \frac{20}{38}$$

$$\therefore E(X) = 1 \cdot \frac{18}{38} - 1 \cdot \frac{20}{38} = -\frac{1}{19}$$

So, the Expected outcome is $-\frac{1}{19}$.

- 2nd Game is as follows :

col 1	col 2	col 3	others
1	13	25	0
2	14	26	00
\vdots	\vdots	\vdots	
12	24	36	

(Red) (Blue) (Yellow) (Green)

The player has 3 options (Red, Blue, Yellow).

If the player chooses Red and ~~the ball~~ bets 1 rs. Now if

the ball falls in Red coloured number the he will win 2 rs.

and if the ~~ball~~ ball falls in other colour the he will lose 1 rs.

So, Let X be a r.v. takes values $+2, -1$.

$$\therefore P(X=+2) = \frac{12}{38} \quad , \quad P(X=-1) = \frac{26}{38}$$

$$\therefore E(X) = 2 \cdot \frac{12}{38} - 1 \cdot \frac{26}{38} = -\frac{1}{19}$$

We can see expectation from both the games are same.

So, player cannot Decide from this.

If we look at the variance of the first game.

$$\text{Var}(X) = \frac{18}{38} + \frac{20}{38} - \left(-\frac{1}{19}\right)^2 = \frac{360}{361} = 0.99$$

and for 2nd game,

$$\begin{aligned}\text{Var}(X) &= 4 \cdot \frac{12}{38} + 1 \cdot \frac{26}{38} - \left(-\frac{1}{19}\right)^2 \\ &= \frac{702}{361} = 1.944\end{aligned}$$

So, variance in game 1 is lower than variance in game 2.

1. Target: The target is to maximize the stability of returns and minimizing the risk, Aiming for more consistent outcome over the long term.
2. Which Game to Play?: Based on the target Game 1 should be chosen.
3. My choice is best for my Target:

- From the Expectation, we see it is same $-\frac{1}{19}$ which implies, on avg. the player loses $\frac{1}{19}$ th of the bet from both game.

- From the variance, we see, game 1 has less variance, and game 2 has higher variance. Which implies that

- ~~the~~ for the game 1, the outcomes are more concentrated around the expectation, which means this game is less risky and has consistent outcome.

- for the game 2, the outcome are more spreaded from the expectation, which implies that ~~the~~ this game is more risky and has less consistent outcome.

As the game 1 aligns with our Target so we choose game 1.