Finance

Midterm / Semester I 2021-22

Time - 2 hours / Maximum Score - 50

NOTE : A standard calculator is allowed as per rule. A normal table will be supplied. *There are 55 marks allotted in the paper. You can answer as much as you can. Maximum you can score is 50.* SHOW ALL YOUR WORK TO GET THE FULL CREDIT. RESULTS USED MUST BE CLEARLY STATED.

1. Consider the portfolio optimization problem with N risky assets. Let w_p be any frontier portfolio with corresponding exoected return μ_p. Assume that there is no risk-free asset and $\mu_p \neq \mu_{mvp}$. Let w_q be any other feasible portfolio.

   (a) ($3\times3 = 9$ marks) Define a frontier portfolio, the corresponding zero covariance portfolio and a feasible portfolio in the above context, after clearly describing the problems mathematically and with appropriate diagram.

   (b) (8 marks) Let r_q be return vector that can be represented as $r_q = \alpha_1 r_{zc(p)} + \alpha_0 r_p + \varepsilon_{qp}$, where r_p is the return corresponding to μ_p with $E(\varepsilon_{qp}|r_p, r_{zc(p)}) = 0$. Obtain α_0 and α_1, that mininizes the $var(\varepsilon_{qp})$.

   (c) (5 marks) Would α_1 be equal to $1 - \alpha_0$? Say, yes or no, and justify your answer.

   (a) Consider a market with 3 risky assets with mean vector $\mu = (0.06, 0.04, 0.02)$ and covariance matrix, $V = \begin{pmatrix} 0.83 & 0.49 & 0.57 \\ 0.49 & 1.46 & 0.76 \\ 0.57 & 0.76 & 0.63 \end{pmatrix}$.

   (a) Calculate

   (i) (5 marks) the minimum variance portfolio vector w_{mvp};

   (ii) (5 marks) the minimum variance portfolio vector when the short sale is not allowed;

   (iii) (5 marks) the zero tangent portfolio vector $w_{tangent(p)}$ corresponding to portfolio vector w_p which is the frontier portfolio corresponding to the expected return 0.05;

   (iv) (6 marks) the standard deviation corresponding to all the portfolio return in (i), (ii) and (iii).

(b) (6+6=12 marks) Suppose, the market also has a riskfree asset whose return is 5%. If you have to advise some investor who wish to form a portfolio whose expected return is 8%, how would you advise the investorlike to invest so that your variance is minimum (i) with shortsale, (ii) without shortsale. Calculate the standard deviation of the corresponding portfolio return.