Ramakrishna Mission Vivekananda Educational and Research Institute

PO Belur Math, Howrah, West Bengal 711 202

School of Mathematical Sciences

Department of Computer Science

MSc BDA : Batch 2019-21, Semester II, Mid-Semester Exam

DA310: Multivariate Statisticss

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Student Name (in block letters):	Date: 02 March 2020
Student Roll No:	Max Marks: 50
Signature:	Time: 2hrs

Answers must be properly justified to deserve full credits.

1. (10 points)

You are given the random vector $\mathbf{X}' = [X_1, X_2, \ldots, X_5]$ with mean vector $\mu'_{\mathbf{X}} = [2, , 4, -1, 3, 0]$ and variance-covariance matrix

$$\mathbf{\Sigma_X} = \begin{bmatrix} 4 & -1 & .5 & -.5 & 1 \\ -1 & 3 & 1 & -1 & 0 \\ .5 & 1 & 5 & 1 & -1 \\ -.5 & -1 & 1 & 4 & 0 \\ 1 & 0 & -1 & 0 & 2 \end{bmatrix}.$$

Partition $\mathbf{X}$ as

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \cdots\cdots \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} \mathbf{X}^{(1)} \\ \cdots\cdots \\ \mathbf{X}^{(2)} \end{bmatrix}$$

Let

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

Find $\text{Cov}(\mathbf{AX}^{(1)}, \mathbf{BX}^{(2)})$. Is it a variance-covariance matrix?

2. (10 points)

Let $\mathbf{X} = [X_1 \ X_2 \ X_3]'$ be distributed as $N_3(\mu, \Sigma)$, where $\mu' = [1, -1, 2]$ and

$$\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix},$$

(a) (2 points) Find ρ_{23}

(b) (4 points) Find the correlation between X_2 and $\frac{1}{2}X_1 + \frac{1}{2}X_3$.

(c) (4 points) What is the conditional distribution of X_2, given that $X_3 = 2$.

3. (10 points)

   The sample mean vector and the sample covariance matrix, as given below, are calculated from pairs of 42 observations.

$$\bar{x} = \begin{bmatrix} 0.564 \\ 0.603 \end{bmatrix}, \text{ and } S = \begin{bmatrix} 0.0144 & 0.0117 \\ 0.0117 & 0.0146 \end{bmatrix}.$$

   Compare the 95% T^2 and 95% Bonferroni simultaneous confidence intervals.

4. (10 points)

   Calculate T^2, for testing $H_0 : \mu = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$, using the data

$$X = \begin{bmatrix} 2 & 12 \\ 8 & 9 \\ 6 & 9 \\ 8 & 10 \end{bmatrix}.$$

   Specify the distribution of T^2 and test H_0 at the $\alpha = 0.05$ level. What conclusion do you reach?

5. (10 points)

   Find the directions of the principal components and the proportion of the total population variance explained by each when the correlation matrix is

$$\rho = \begin{bmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{bmatrix}.$$

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You may need following values:

$$t_{41}(0.05) = 1.683, t_{41}(0.025) = 2.020, t_{41}(0.0125) = 2.327,$$
$$F_{2,40}(0.025) = 4.051, F_{40,2}(0.025) = 39.473, F_{2,40}(0.05) = 3.232, F_{40,2}(0.05) = 19.471$$
$$F_{2,2}(0.05) = 19, F_{2,2}(0.025) = 39$$

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This exam has total 5 questions, for a total of 50 points and 0 bonus points.