Exercise Set: Functional Dependencies and BCNF

DBMS Course

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Problem 1. You are given a few relation schemas and sets of FDs that hold on them. For each given schema and set of FDs on them, do the following:

1. Indicate all keys.
2. Indicate all BCNF violations. [*Do not forget to consider FDs that are not in the given set, but follow from them.*]
3. Decompose the relations, as necessary, into collections of relations that are in BCNF. Give brief arguments at each step.

1. $R(A, B, C, D)$ with FDs $AB \to C, C \to D$ and $D \to A$.
2. $R(A, B, C, D)$ with FDs $B \to C$ and $B \to D$.
3. $R(A, B, C, D)$ with FDs $AB \to C, BC \to D, CD \to A$ and $AD \to B$.

Problem 2. Suppose R is a schema with a set F of FDs that hold on them. Suppose we check each FD that belongs to F as to whether it satisfies the BCNF condition. If every FD in F passes this check, then is it true that R under F is in BCNF? Prove or disprove (disprove by showing a counter example). Solve the same problem for 3NF check.

Problem 3. Consider the basic step in BCNF decomposition of a relation schema R. A non-trivial FD $X \to A$ is found to violate the BCNF condition. There are two options: (i) Choose $X \cup A$ as one relation schema and $R - \{A\}$ to be another relation schema. (ii) A second option to choose X^+ as one relation schema and $(R - X^+) \cup X$ as another schema.

1. Are they both correct? Prove or disprove.
2. Design an example to illustrate (for e.g., let $R(A, B, C, D)$ be a schema with FD's $A \to B$ and $A \to C$. In (i) we can decompose R according to $A \to B$ or to $A \to C$. Do we ultimately get the same result if we expand the BCND violation to $A \to BC$.

Problem 4. Let R be the example in the above problem, but let the FDs be $A \to B$ and $B \to C$.

1. Compare decomposition using $A \to B$ first against decomposing by $A \to BC$ first.

2. Compare decomposition using $B \to C$ first against decomposing by $A \to BC$ first.

Problem 5. Suppose we have a relation schema $R(A, B, C)$ with FD $A \to B$ and we decide to decompose this schema into $S(A, B)$ and $T(B, C)$. Give an example of a relation $r(R)$ whose projection into S and T and subsequent rejoining does not yield the same relation instance. (i.e., the join is not lossless).

Problem 6. Consider the relation $R(A, B, C, D)$ with FD $B \to AD$. A proposed decomposition is $\{A, B\}, \{B, C\}$ and $\{C, D\}$. Is this decomposition lossless? Prove either way.

Problem 7. Let $R(A, B, C, D, E)$ be decomposed into relations that are each set of three attributes: $\{A, B, C\}, \{B, C, D\}, \{A, C, E\}$. (i) For each of the following FDs, use the chase test to tell whether the decomposition of R is lossless. (ii) For those that are not lossless, give an example of an instance of R that, upon projection into the decomposed relation, yields more than R when re-joined.

1. $B \to E$ and $CE \to A$.
2. $AC \to E$ and $BC \to D$.

Problem 8. For each of the sets of FDs in the above problem, is the decomposition dependency preserving?

Problem 9. For each of the relation schemas and sets of FDs of Problem 1:

1. Indicate all the 3NF violations.
2. Decompose the relations, as necessary, into collections of relations that are in 3NF.

Problem 10. Consider the relation $Courses(C, T, H, R, S, G)$ whose attributes may be though informally as course, teacher, hour, room, student and grade. Let the set of FDs for *Courses* be

$$C \to T, HR \to C, HT \to R, HS \to R, \text{ and } CS \to G \quad .$$

1. What are all the keys for *Courses*?
2. Verify that the given FD set is a minimal basis.
3. Use the 3NF synthesis algorithm to find a lossless join, dependency-preserving decomposition of R into 3NF relations? Are any of the relations not in BCNF?

Problem 11. Consider the relation $R(A, B, C, D, E)$ with FDs $AB \to C$, $C \to B$ and $A \to D$.

1. Is it a minimal basis?
2. If we partition into $\{A, B, C\}, \{B, C\}$ and $\{A, D\}$, is it lossless?
3. Give a 3NF lossless join, dependency preserving decomposition.

Problem 12. Let $R = \{A, B, C\}$ and $S = \{A, B\}$ and $T = \{B, C\}$ be the decomposition of R. Given $r(R)$, let $s(S) = \pi_S(r)$ and $t(T) = \pi_T(r)$. We are given the condition that for any value $b \in \pi_B(R)$, one of the following two conditions hold:

$$(i) \quad |\sigma_{B=b}(S)| = 1, \text{ or, } \quad (ii) \quad |\sigma_{B=b}(T)| = 1 \quad .$$

Is the decomposition lossless?