Ramakrishna Mission Vivekananda University

Belur Math, Howrah, West Bengal

School of Mathematical Sciences, Department of Data Science

M.Sc. in Big Data Analytic 2017, Mid Semester Exam — Date: 23 March 2018

Course : **DA310: Multivariate Statistics** — Time: $1\frac{3}{4}$ hrs

Instructor : Dr. Sudipta Das — *Max marks: 30*

Student signature and Id:

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1. A morning newspaper lists the following used-car prices for a foreign compact with age x_1 measured in years and selling price x_2 measured in thousands of dollars:

x_1	1	2	3	3	4	5	6	8	9	11
x_2	18.95	19.00	17.95	15.54	14.00	12.95	8.94	7.49	6.00	3.99

   (a) Construct a scatter plot of the data.

   (b) Infer the sign of sample covariance form the scatter plot.

   (c) Find arrays $\bar{\mathbf{x}}$, $\mathbf{S_n}$ and $\mathbf{R}$.

   [2+1+4=7]

2. Define the distance from the point $P = (x_1, x_2)$ to the origin $O = (0, 0)$ as

$$d(O, P) = \max(|x_1|, |x_2|).$$

   Plot the locus of points whose squared distance from the origin is 1. [3]

3. Prove that every eigenvalue of $k \times k$ positive definite matrix $\mathbf{A}$ is positive. [3]

4. Find the maximum value of $\mathbf{x}'\mathbf{A}\mathbf{x}$ for $\mathbf{x}'\mathbf{x} = 1$, where

$$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}.$$

   [2]

5. Let $\mathbf{X} \sim N_3(\mu, \Sigma)$, where $\mu' = [1, -1, 2]$ and

$$\mathbf{\Sigma} = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}.$$

   Which random variables are independent? [2]

6. Let X_1 be $N(0, 1)$, and let

$$X_2 = \begin{cases} -X_1 & \text{if } -1 \le X_1 \le 1 \\ X_1 & \text{otherwise.} \end{cases}$$

   Show that $X_2 \sim N(0, 1)$ and (X_1, X_2) is not bivariate normal. [6]

7. You are given the random vector $\mathbf{X}' = [X_1, X_2, \ldots, X_5]$ with mean vector $\mu'_{\mathbf{X}} = [2, , 4, -1, 3, 0]$ and variance-covariance matrix

$$\boldsymbol{\Sigma}_{\mathbf{X}} = \begin{bmatrix} 4 & -1 & .5 & -.5 & 0 \\ -1 & 3 & 1 & -1 & 0 \\ .5 & 1 & 6 & 1 & -1 \\ -.5 & -1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}.$$

Partition $\mathbf{X}$ as

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \cdots\cdots \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} \mathbf{X}^{(1)} \\ \cdots\cdots \\ \mathbf{X}^{(2)} \end{bmatrix}$$

Let

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

Find $\text{Cov}(\mathbf{AX}^{(1)}, \mathbf{BX}^{(2)})$. [7]

This exam has total 7 questions, for a total of 30 points and 0 bonus points.