Exercise Set: Functional Dependencies-I

Database Managament Course

September 1, 2024

Problem 1. Consider a relation about persons with schema *Name*, *AadharNo*, *Address*, *City*, *State*, *Pincode*, *PhoneAreaCode* and *PhoneNo*. Write down the FDs you expect to hold. What are the keys for the relation? (*Note:* Some assumptions are needed about whether area code can be shared across two cities/two states? Can Pincode be shared across two areacodes or vice-versa?

Problem 2. Consider the schema of $R(A_1, A_2 \ldots, A_n)$. As a function of n, count how many superkeys R has, if:

1. The only key is A_1.
2. The only keys are A_1 and A_2.
3. The only keys are $\{A_1, A_2\}$ and $\{A_1, A_3\}$.

Problem 3. Consider a relation with schema $R(A, B, C, D)$ and FD's $AB \to C$, $C \to D$ and $D \to A$.

1. What are all the non-trivial FDs that follow from the given FD's? (Make RHS of each FD a singleton).
2. What are all the keys of R?
3. What are all the superkeys of R that are not keys?

Problem 4. Recall the Armstrong Axioms (Reflexivity, Augmentation and Transitivity holds) as inference rules for functional dependencies. Prove that the following rules also hold using (a) closure test/first principles and (b) the axioms where applicable.

1. *Augmenting LHS.* if $A_1, \ldots, A_n \to B$ is an FD, and C is another attribute, then, $A_1, \ldots, A_n, C \to B$ holds.
2. Prove *Augmentation rule using closure test*: if $A_1, \ldots, A_n \to B$ holds, then for any attribute C, $A_1, \ldots, A_n, C \to BC$ holds.
3. *Pseudo transitivity.* Suppose the following FDs hold: $A_1, \ldots, A_n \to B_1, \ldots, B_m$ and $C_1, \ldots, C_k \to D$. The B's are among the C's (though may be not vice-versa, so $m \leq k$). Show that

$$A_1, \ldots, A_n E_1, \ldots, E_l \to D$$

holds, where, the E_j's are all the C_i's that are not among the B's.

4. *Addition.* If $A_1, \ldots, A_n \to B_1, \ldots, B_m$ and $C_1, \ldots, C_p \to D_1, \ldots, D_q$, then

$$A_1, \ldots, A_n C_1, \ldots, C_p \to B_1, \ldots, B_m D_1, \ldots, D_q$$

holds. In the above, we should remove one copy of all shared attributes between the A's and the C's, and likewise, remove one copy of all shared attributes between the B's and the D's.

Problem 5. Show that each of the following are not valid rules about FD's by giving example relations that satisfy the given FDs (following the "if") but not the FDs that follows (after the "then").

1. If $AB \to C$ then $A \to C$.
2. If $AB \to C$ and $A \to C$, then $B \to C$.
3. If $AB \to C$ then $A \to C$ *or* $B \to C$.

Problem 6. *Prove completeness of Armstrong's Axioms.* Let R be a schema and F be a set of FDs over R. Show that Armstrong's axioms are complete, that is, if it is claimed that an FD $A_1, \ldots, A_n \to B$ is implied by F, but it does not follow from the axioms, then, the claim is false. That is, show that the axioms are complete (i.e., no FDs implied from F are missed).

Problem 7. Let X and Y be sets of attributes of a schema R and let F be the set of FDs that hold. Show that if $X \subseteq Y$, then, $X^+ \subseteq Y^+$, under F.

Problem 8. Show that $(X^+)^+ = X^+$, for all subsets $X \subseteq R$.

Problem 9. Consider the relation schema $R(A, B, C)$, where, each attribute functionally determines the other two. That is, F is

$$A \to BC, B \to CA, C \to AB \quad .$$

Find all minimal bases of F.

Problem 10. Suppose we have a schema $R(A, B, C, D, E)$ with some set of FDs, and we wish to project those FDs onto the schema $S(A, B, C)$. Derive the set of FDs that hold on S if the given set of FDs over R are:

1. $AB \to DE, C \to E, D \to C, E \to A$.
2. $A \to D, BD \to E, AC \to E, DE \to B$.