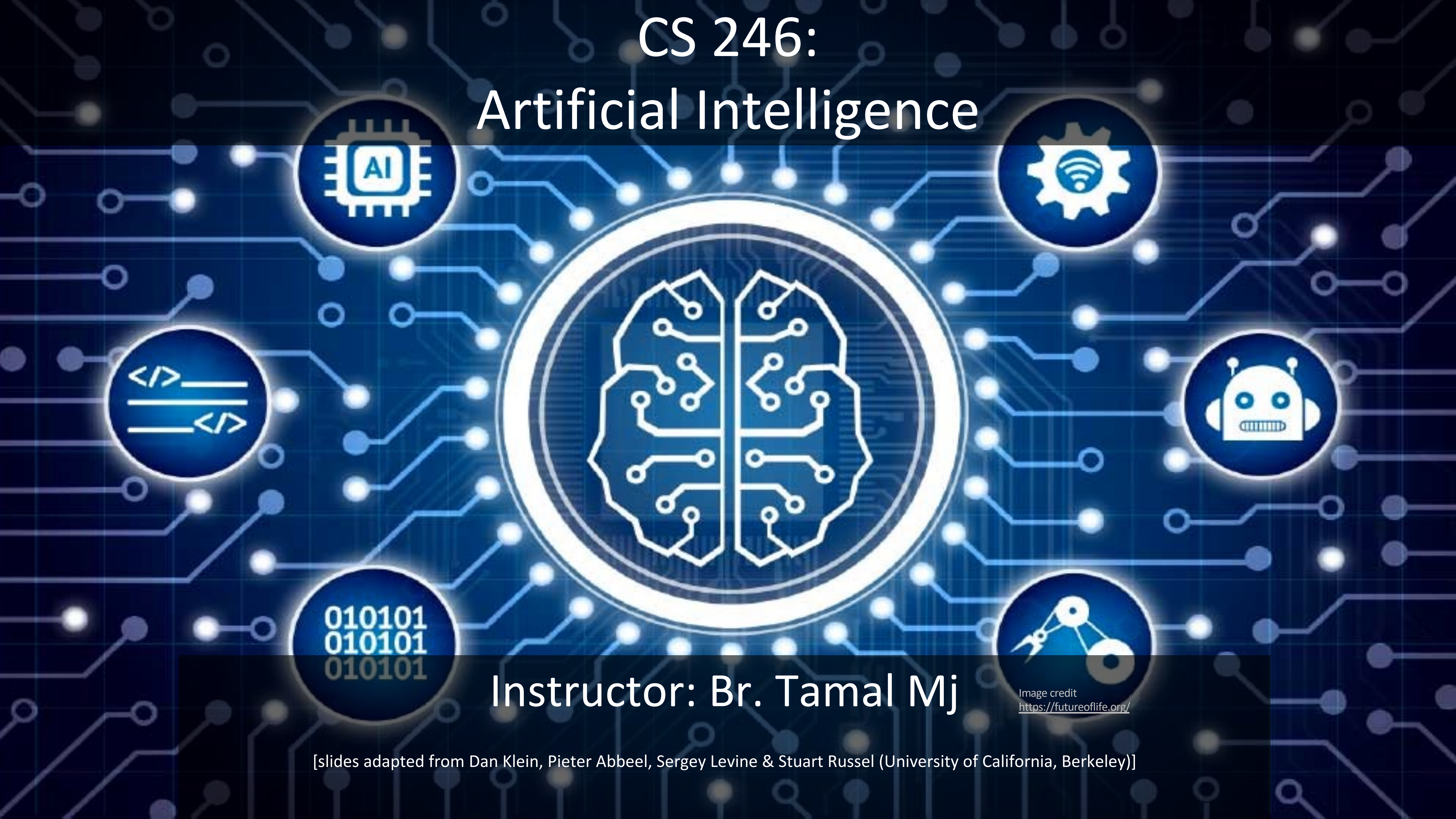
CS 246:
Artificial Intelligence



Instructor: Br. Tamal Mj

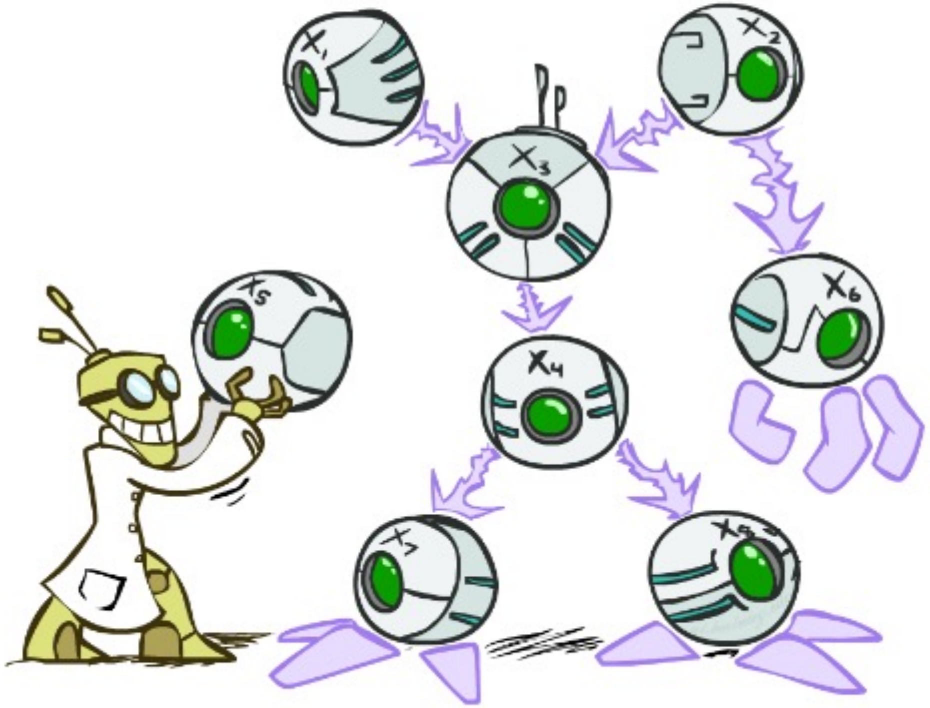
Image credit
https://futureoflife.org/

[slides adapted from Dan Klein, Pieter Abbeel, Sergey Levine & Stuart Russel (University of California, Berkeley)]

Om Saha Naav[au]-Avatu
Saha Nau Bhunaktu
Saha Viiryam Karavaavahai
Tejasvi Naav[au]-Adhiitam-
Astu Maa Vidvissaavahai
Om Shaantih Shaantih
Shaantih

Om, May we all be protected
May we all be nourished
May we work together with great energy
May our intelect be sharpened (may our study be effective)
Let there be no Animosity amongst us
Om, peace (in me), peace (in nature), peace (in divine forces)

Bayes' Nets



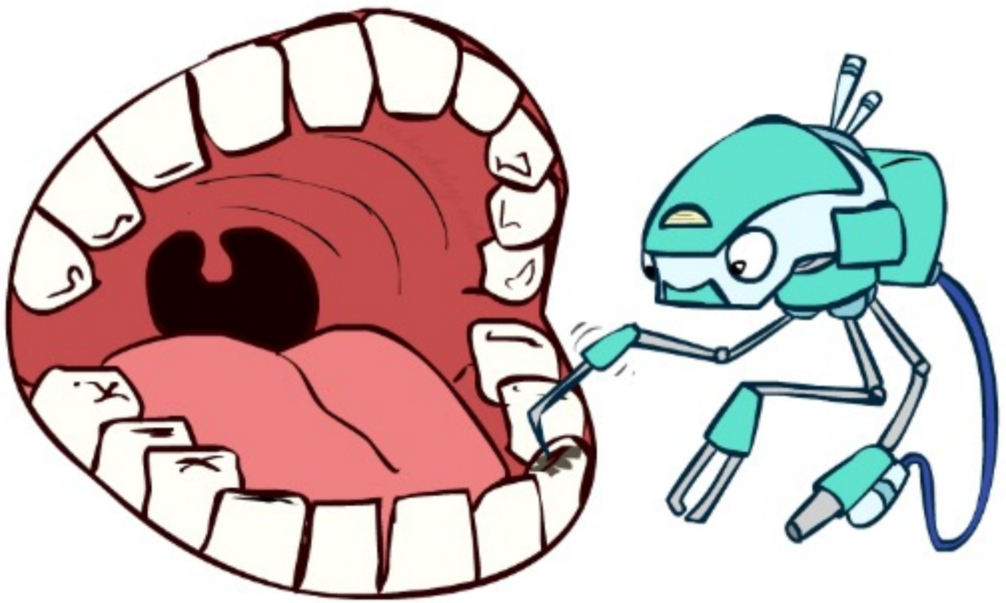
Conditional Independence

- P(Toothache, Cavity, Catch)

- If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:
  - P(+catch | +toothache, +cavity) = P(+catch | +cavity)

- The same independence holds if I don't have a cavity:
  - P(+catch | +toothache, -cavity) = P(+catch| -cavity)

- Catch is *conditionally independent* of Toothache given Cavity:
  - P(Catch | Toothache, Cavity) = P(Catch | Cavity)



- Equivalent statements:
  - P(Toothache | Catch , Cavity) = P(Toothache | Cavity)
  - P(Toothache, Catch | Cavity) = P(Toothache | Cavity) P(Catch | Cavity)
  - One can be derived from the other easily

Conditional Independence

- Unconditional (absolute) independence very rare (why?)

- *Conditional independence* is our most basic and robust form of knowledge about uncertain environments.

- X is conditionally independent of Y given Z $X \perp\!\!\!\perp Y \mid Z$

  if and only if:

$$\forall x, y, z : P(x, y|z) = P(x|z)P(y|z)$$

  or, equivalently, if and only if

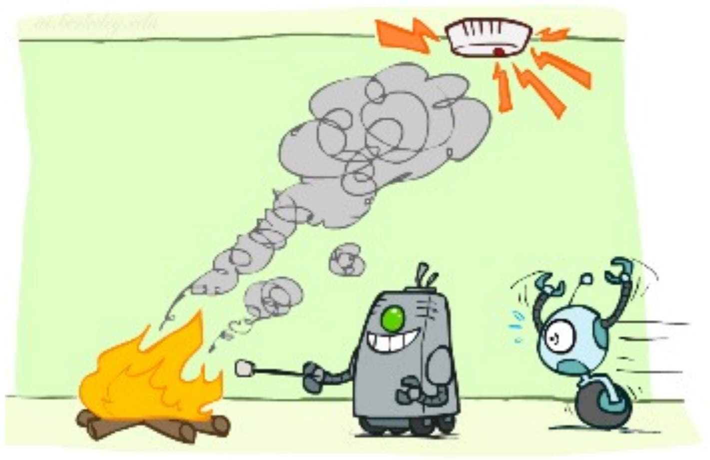
$$\forall x, y, z : P(x|z, y) = P(x|z)$$

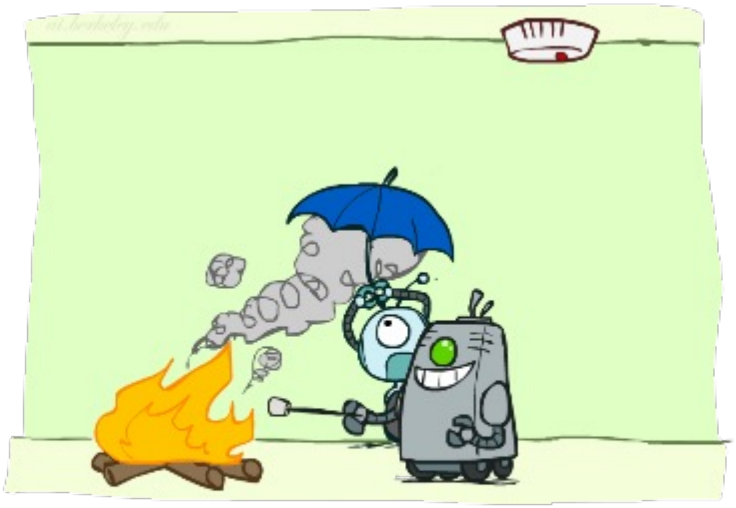
Conditional Independence

- What about this domain:
  - Traffic
  - Umbrella
  - Raining

Conditional Independence

- What about this domain:
  - Fire
  - Smoke
  - Alarm





Conditional Independence and the Chain Rule

- Chain rule: $P(X_1, X_2, \ldots, X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2)\ldots$

- Trivial decomposition:

$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) =$$
$$P(\text{Rain})P(\text{Traffic} \mid \text{Rain})P(\text{Umbrella} \mid \text{Rain}, \text{Traffic})$$

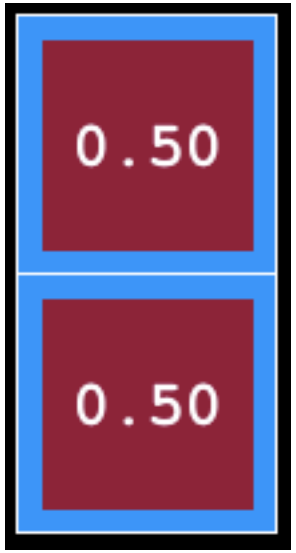
- With assumption of conditional independence:

$$P(\text{Traffic}, \text{Rain}, \text{Umbrella}) =$$
$$P(\text{Rain})P(\text{Traffic} \mid \text{Rain})P(\text{Umbrella} \mid \text{Rain})$$

- Bayes'nets / graphical models help us express conditional independence assumptions

Ghostbusters Chain Rule

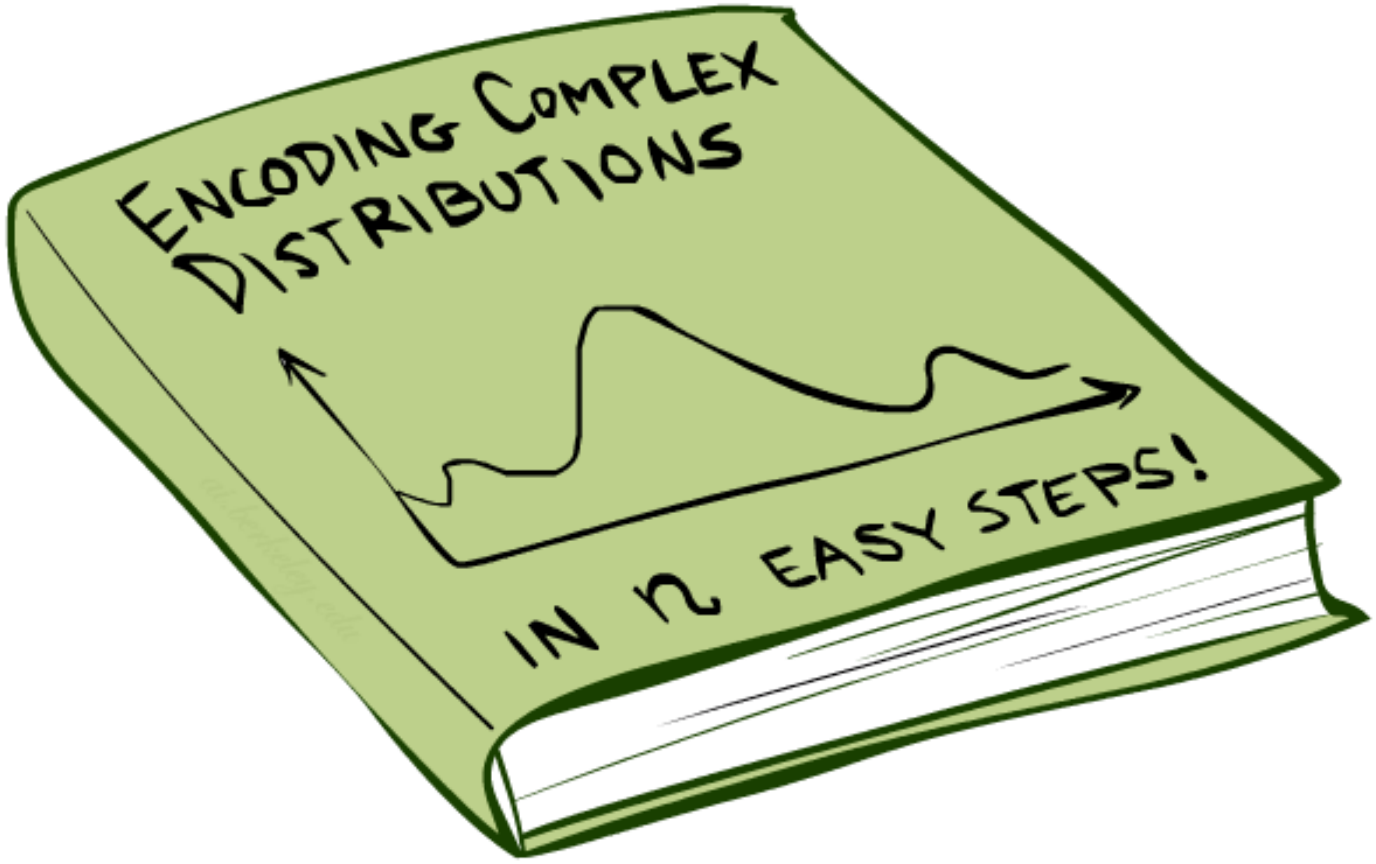
- Each sensor depends only on where the ghost is
- That means, the two sensors are conditionally independent, given the?
  - ghost position
- T: Top square is red
  B: Bottom square is red
  G: Ghost is in the top
- Given:
  P(+g) = 0.5
  P(-g) = 0.5
  P(+t | +g) = 0.8
  P(+t | -g) = 0.4
  P(+b | +g) = 0.4
  P(+b | -g) = 0.8



P(T,B,G) = P(G) P(T|G) P(B|G)

T	B	G	P(T,B,G)
+t	+b	+g	0.16
+t	+b	-g	0.16
+t	-b	+g	0.24
+t	-b	-g	0.04
-t	+b	+g	0.04
-t	+b	-g	0.24
-t	-b	+g	0.06
-t	-b	-g	0.06

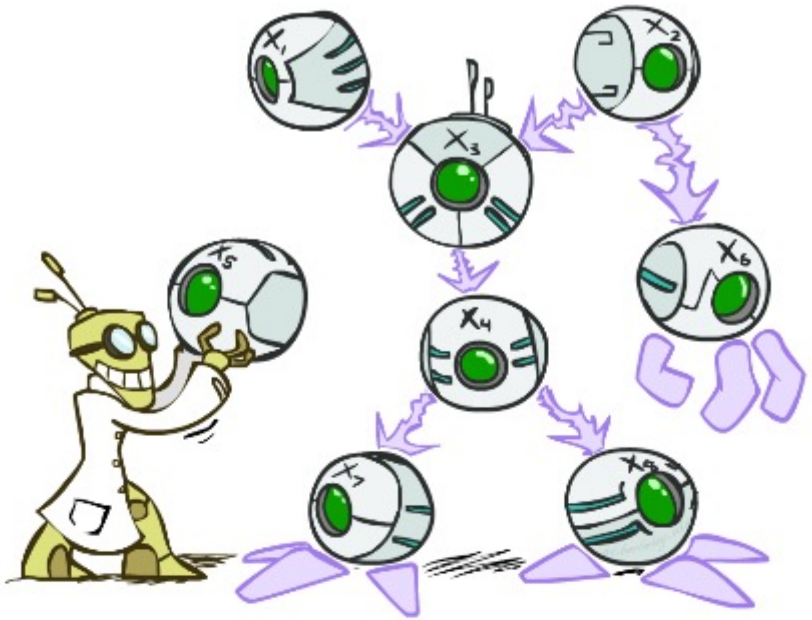
Bayes' Nets: Big Picture



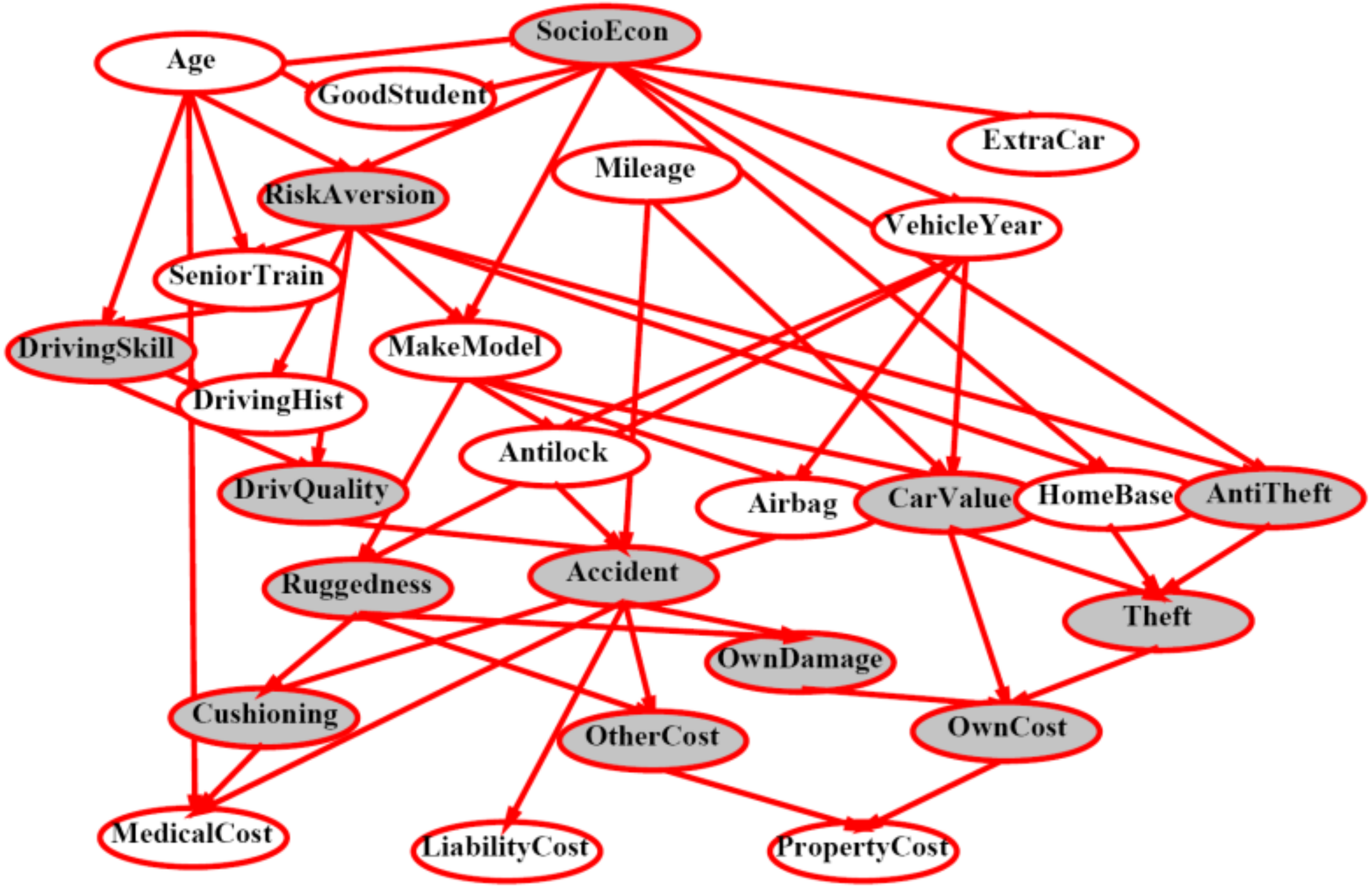
Bayes' Nets: Big Picture

- Two problems with using full joint distribution tables as our probabilistic models:
  - Unless there are only a few variables, the joint is WAY too big to represent explicitly
  - Hard to learn (estimate) anything empirically about more than a few variables at a time

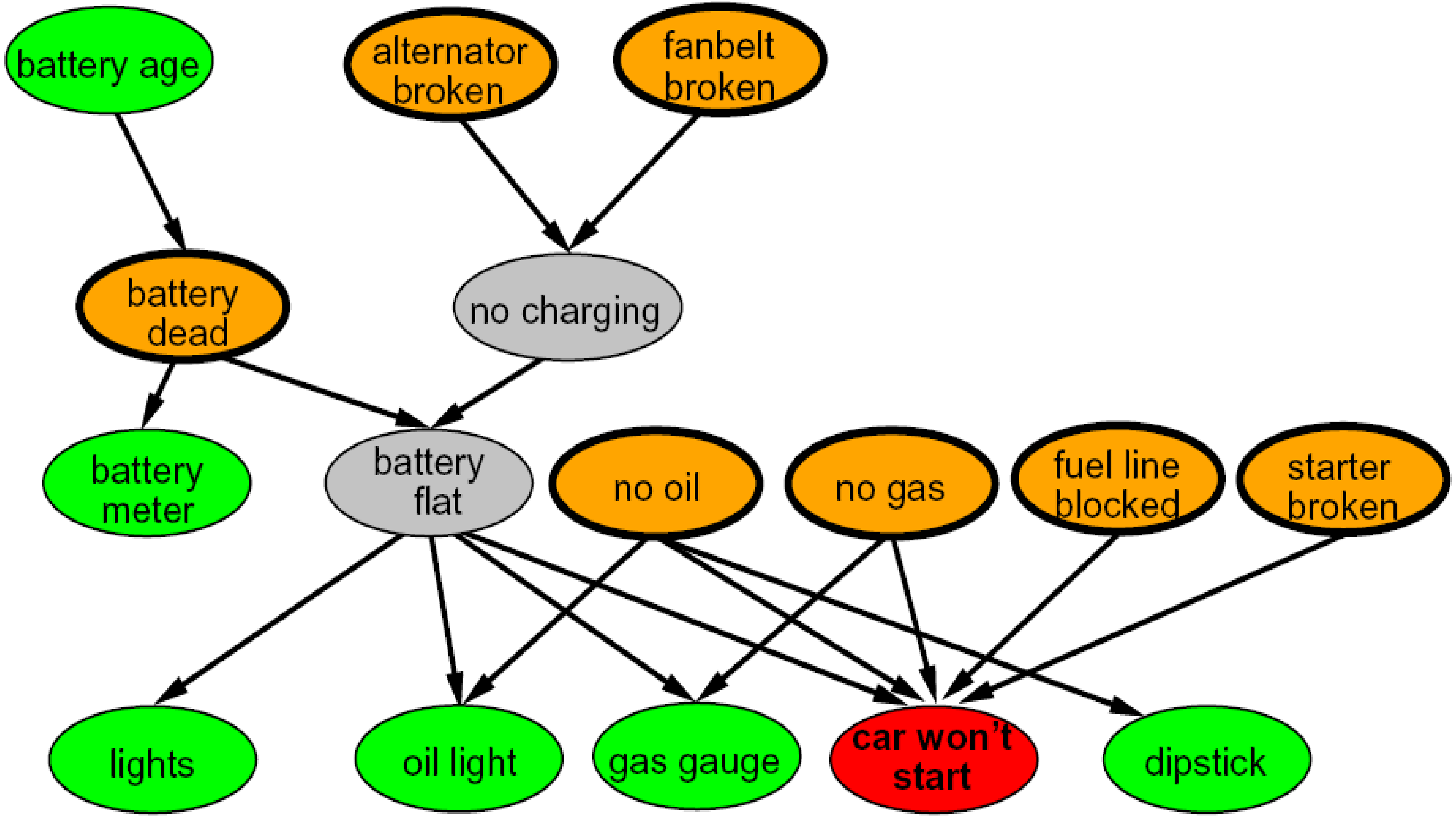
- Bayes' nets: a technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)
  - More properly called graphical models
  - We describe how variables locally interact
  - Local interactions chain together to give global, indirect interactions
  - For about 15 min, we'll be vague about how these interactions are specified



Example Bayes' Net: Insurance



Example Bayes' Net: Car

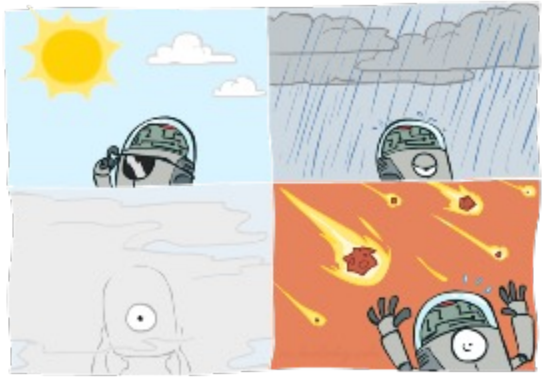


Why to study Bayes Net

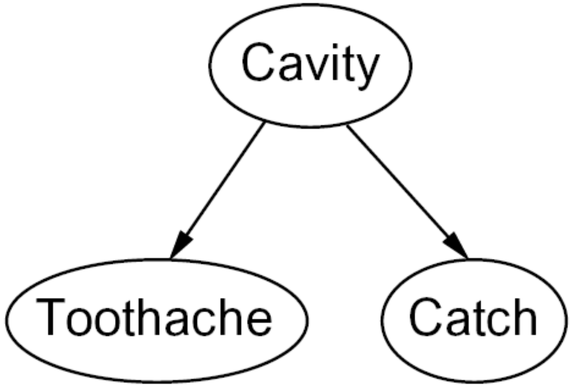
- **Understand Causality**:
  - Move beyond correlations to causal insights, crucial in fields like medicine and finance.
- **Handle Uncertainty**:
  - Make structured decisions in uncertain environments using probabilities.
- **Simplify Complexity**:
  - Break down complex systems into manageable parts, valuable in AI and robotics.
- **Infer Missing Data**:
  - Make predictions even with incomplete information
- **Competitive Advantage**:
  - Specialized skill for data science and AI, making you stand out in the job market.

Graphical Model Notation

- Nodes: variables (with domains)
  - Can be assigned (observed) or unassigned (unobserved)



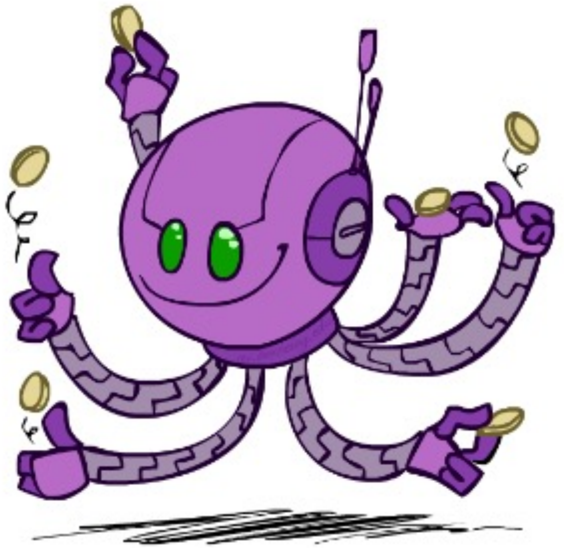
- Arcs: interactions
  - Similar to CSP constraints
  - Indicate "direct influence" between variables
  - Formally: encode conditional independence (more later)



- For now: imagine that arrows mean direct causation (in general, they don't!)

Example: Coin Flips

- N independent coin flips



X_1 $\quad$ X_2 $\quad \cdots \quad$ X_n

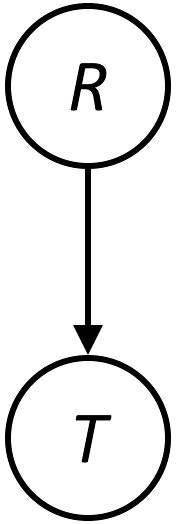
- No interactions between variables: absolute independence

Example: Traffic

- Variables:
  - R: It rains
  - T: There is traffic

- Model 1: independence

- Model 2: rain causes traffic



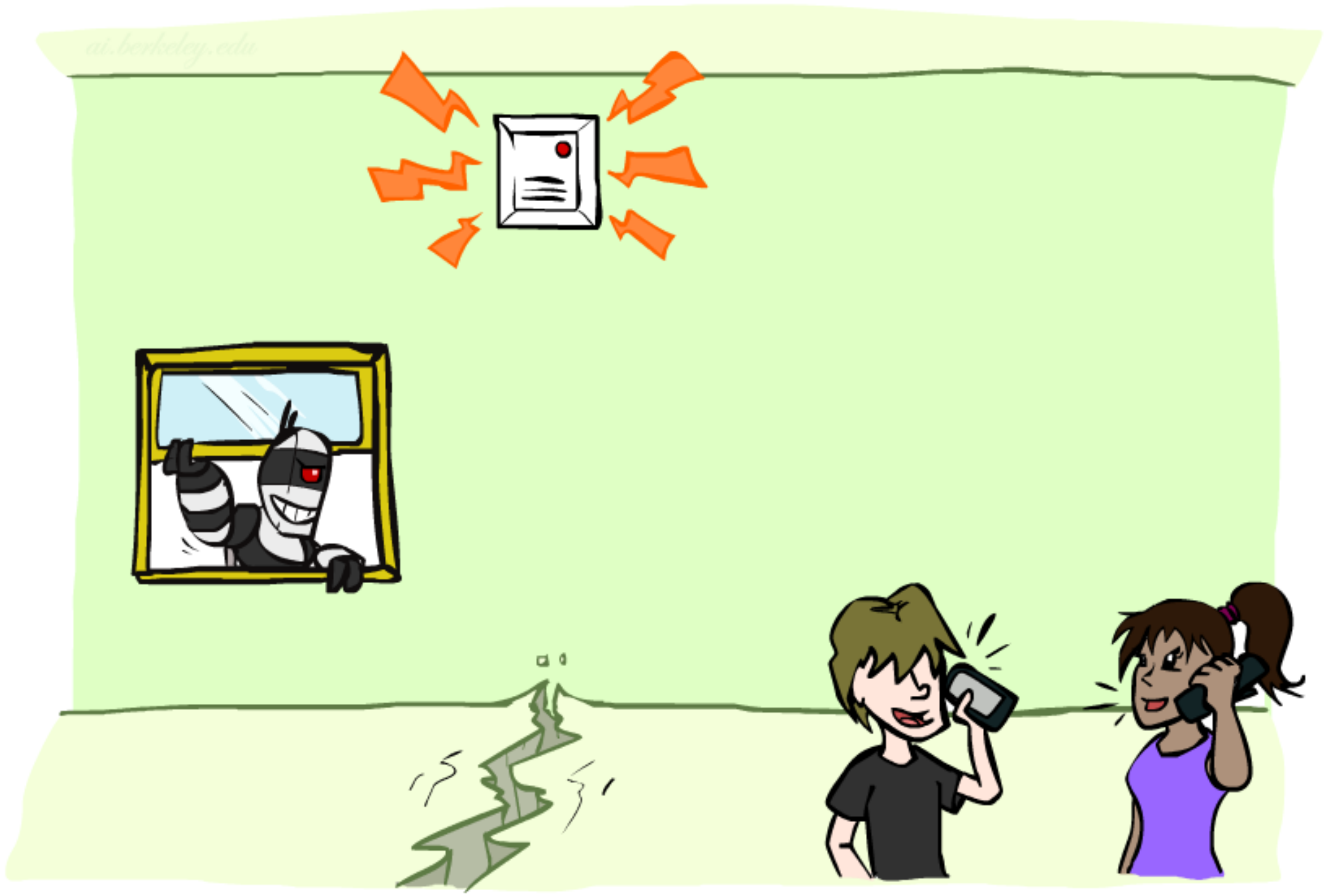
- Why is an agent using model 2 better?

Example: Traffic II

- Let's build a causal graphical model!
- Variables
  - T: Traffic
  - R: It rains
  - L: Low pressure
  - D: Roof drips
  - B: Ballgame
  - C: Cavity

Example: Alarm Network

- Variables
  - B: Burglary
  - A: Alarm goes off
  - M: Mary calls
  - J: John calls
  - E: Earthquake!



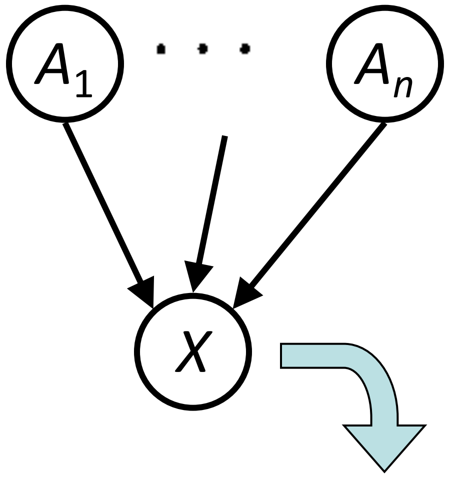
Bayes' Net Semantics

Bayes' Net Semantics

- A set of nodes, one per variable X
- A directed, acyclic graph (DAG)
- A conditional distribution for each node
  - A collection of distributions over X, one for each combination of parents' values

$$P(X \mid a_1 \ldots a_n)$$

  - CPT: conditional probability table
  - Description of a noisy "causal" process



$$P(X \mid a_1 \ldots a_n)$$

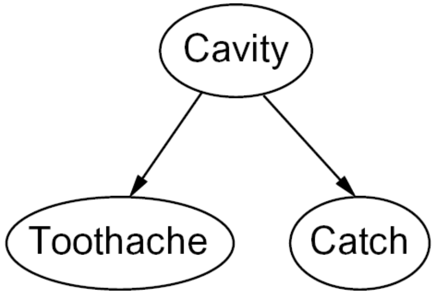
A Bayes net = Topology (graph) + Local Conditional Probabilities

Probabilities in BNs

- Bayes' nets implicitly encode joint distributions
  - As a product of local conditional distributions
  - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$

  - Example:



$$P(+\text{cavity}, +\text{catch}, -\text{toothache})$$

Probabilities in BNs

- Why are we guaranteed that setting

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$

results in a proper joint distribution?

- Chain rule (valid for all distributions): $P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid x_1 \ldots x_{i-1})$

- Assume conditional independences: $P(x_i \mid x_1, \ldots, x_{i-1}) = P(x_i \mid \text{parents}(X_i))$

  → Consequence: $P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$

- Not every BN can represent every joint distribution
  - The topology enforces certain conditional independencies

Example: Coin Flips

X_1 X_2 . . . X_n

$P(X_1)$

h	0.5
t	0.5

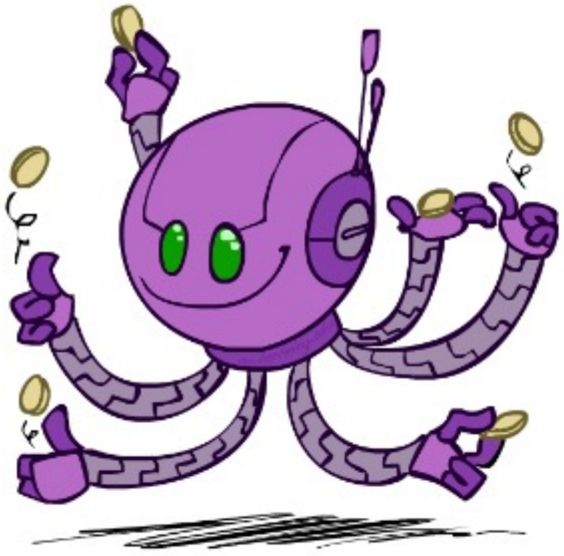
$P(X_2)$

h	0.5
t	0.5

. . .

$P(X_n)$

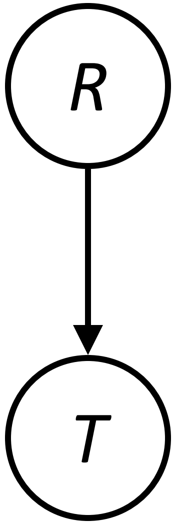
h	0.5
t	0.5



$P(h, h, t, h) =$

Only distributions whose variables are absolutely independent can be represented by a Bayes' net with no arcs.

Example: Traffic



$P(R)$

+r	1/4
-r	3/4

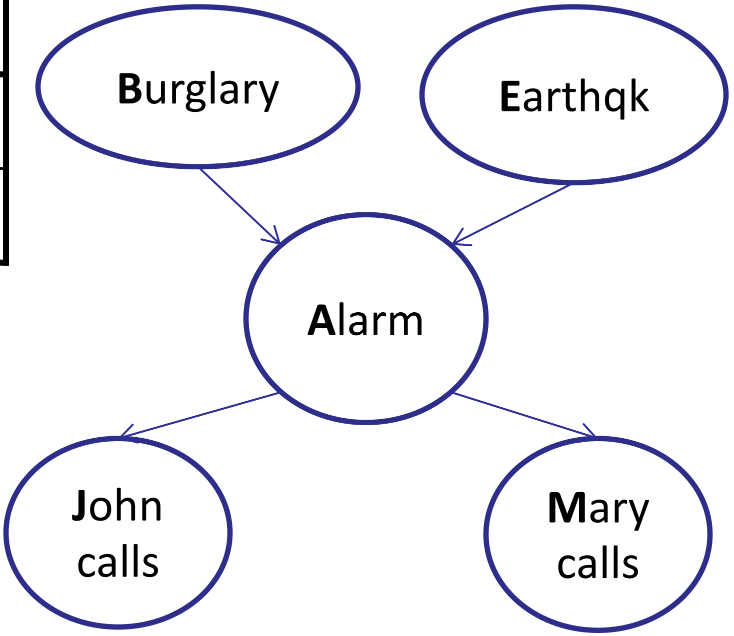
$P(T|R)$

+r	+t	3/4
	-t	1/4
-r	+t	1/2
	-t	1/2

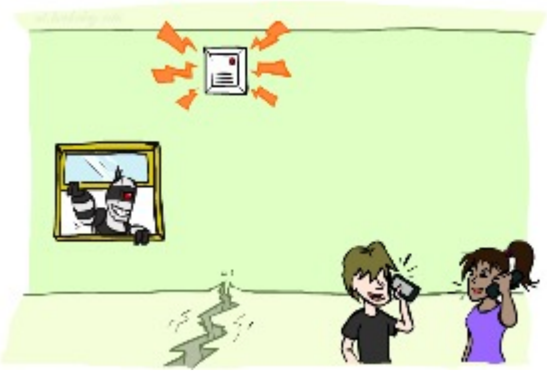
$$P(+r, -t) =$$

Example: Alarm Network

B	P(B)
+b	0.001
-b	0.999



E	P(E)
+e	0.002
-e	0.998



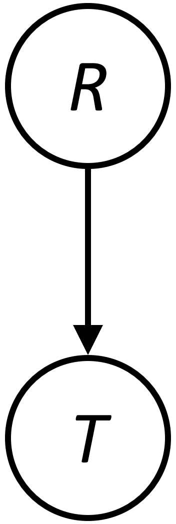
B	E	A	P(A|B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999

A	J	P(J|A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

A	M	P(M|A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

Example: Traffic

- Causal direction



$P(R)$

+r	1/4
-r	3/4

$P(T|R)$

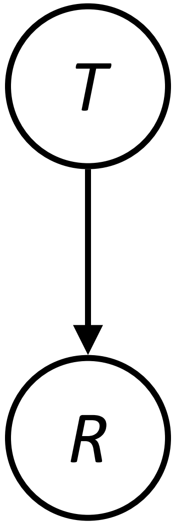
+r	+t	3/4
	-t	1/4
-r	+t	1/2
	-t	1/2

$P(T, R)$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16

Example: Reverse Traffic

- Reverse causality?



$P(T)$

+t	9/16
-t	7/16

$P(R|T)$

+t	+r	1/3
	-r	2/3

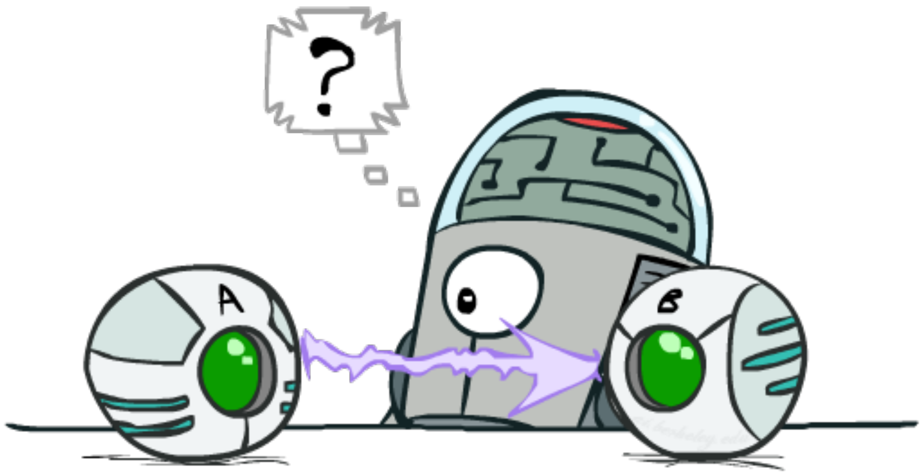
-t	+r	1/7
	-r	6/7

$P(T, R)$

+r	+t	3/16
+r	-t	1/16
-r	+t	6/16
-r	-t	6/16

Causality?

- When Bayes' nets reflect the true causal patterns:
  - Often simpler (nodes have fewer parents)
  - Often easier to think about
  - Often easier to elicit from experts



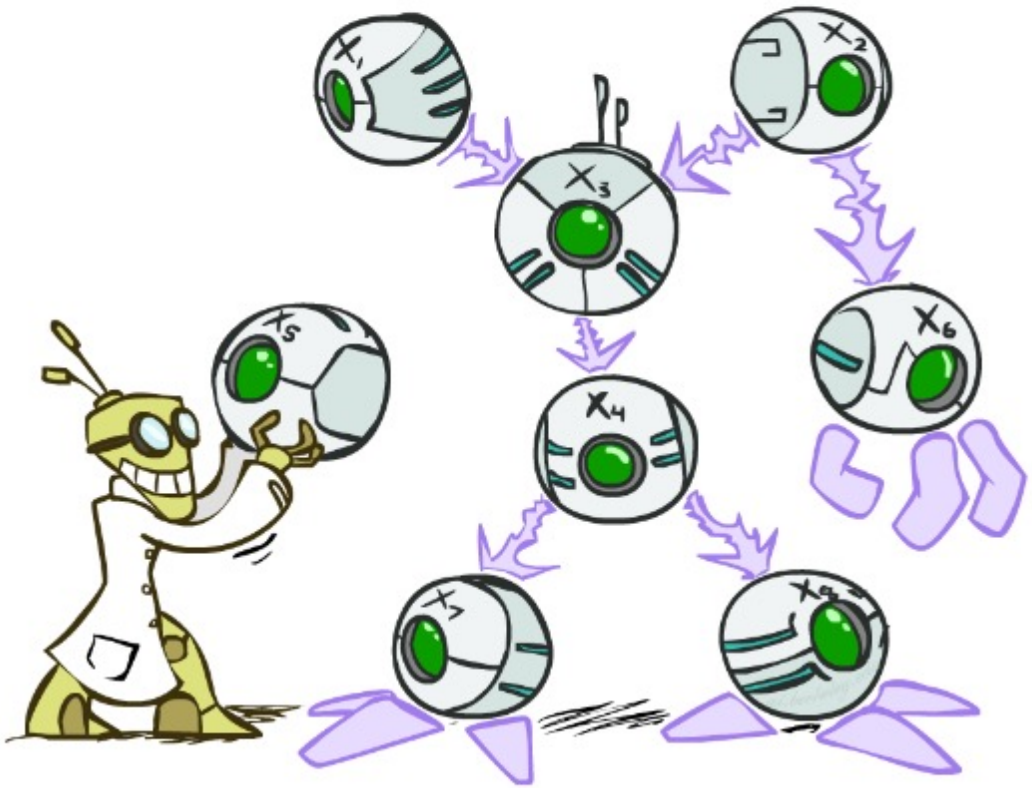
- BNs need not actually be causal
  - Sometimes no causal net exists over the domain (especially if variables are missing)
  - E.g. consider the variables *Traffic* and *Drips*
  - End up with arrows that reflect correlation, not causation

- What do the arrows really mean?
  - Topology may happen to encode causal structure
  - Topology really encodes conditional independence

$$P(x_i \mid x_1, \ldots, x_{i-1}) = P(x_i \mid \text{parents}(X_i))$$

Bayes’ Nets

- So far: how a Bayes’ net encodes a joint distribution
- Next: how to answer queries about that distribution
  - Today:
    - First assembled BNs using an intuitive notion of conditional independence as causality
    - Then saw that key property is conditional independence
  - Main goal: answer queries about conditional independence and influence
- After that: how to answer numerical queries (inference)



Bayes' Nets: Independence

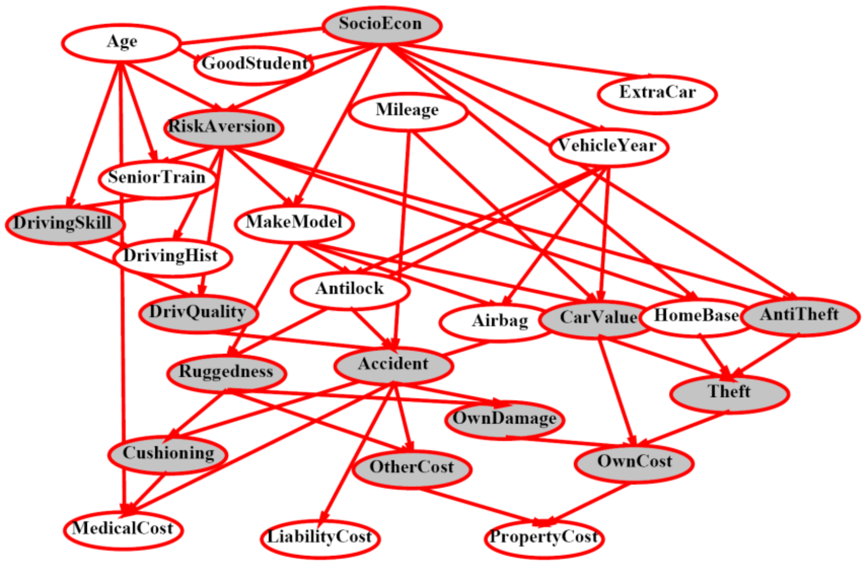
Probability Recap

- Conditional probability $$P(x|y) = \frac{P(x,y)}{P(y)}$$
- Product rule $$P(x,y) = P(x|y)P(y)$$
- Chain rule $$\begin{aligned} P(X_1, X_2, \ldots X_n) &= P(X_1)P(X_2|X_1)P(X_3|X_1,X_2)\ldots \\ &= \prod_{i=1}^{n} P(X_i|X_1,\ldots,X_{i-1}) \end{aligned}$$
- X, Y independent if and only if: $\forall x,y : P(x,y) = P(x)P(y)$
- X and Y are conditionally independent given Z if and only if:

$$\forall x,y,z : P(x,y|z) = P(x|z)P(y|z) \qquad X \perp\!\!\!\perp Y|Z$$

Bayes' Nets

- A Bayes' net is an efficient encoding of a probabilistic model of a domain



- Questions we can ask:
  - Inference: given a fixed BN, what is P(X | e)?
  - Representation: given a BN graph, what kinds of distributions can it encode?
  - Modeling: what BN is most appropriate for a given domain?

Bayes' Net Semantics

- A directed, acyclic graph, one node per random variable
- A conditional probability table (CPT) for each node
  - A collection of distributions over X, one for each combination of parents' values

$$P(X|a_1 \ldots a_n)$$

- Bayes' nets implicitly encode joint distributions
  - As a product of local conditional distributions
  - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

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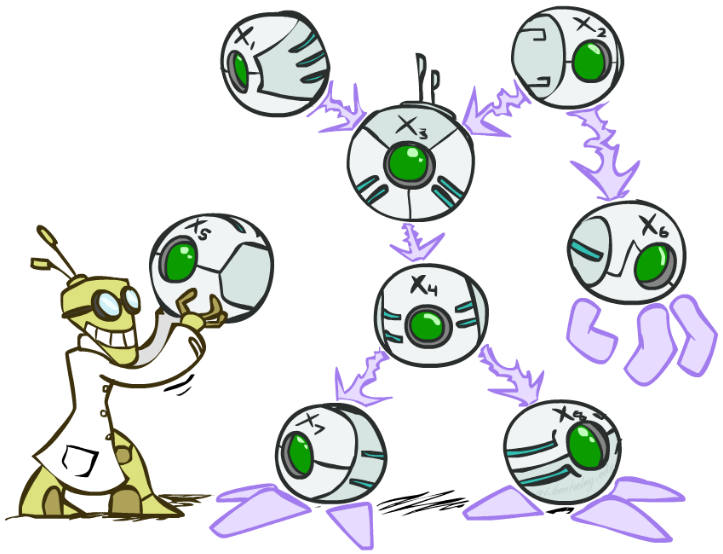
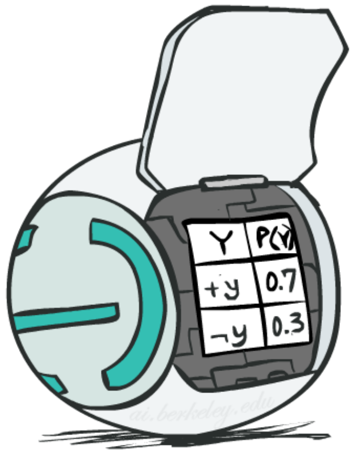
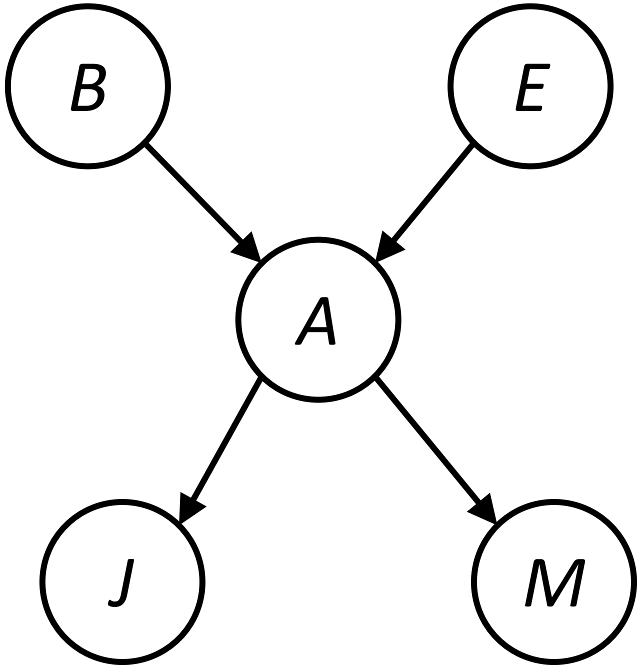




Example: Alarm Network

B	P(B)
+b	0.001
-b	0.999



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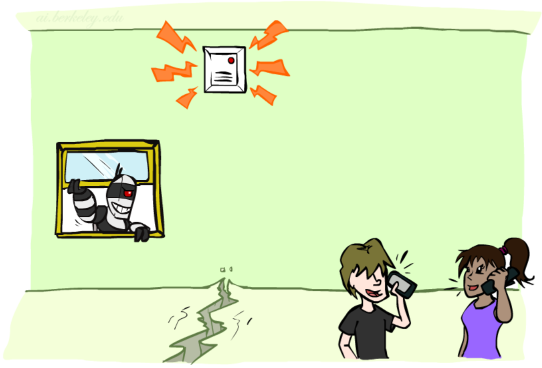

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+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

A	M	P(M|A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

B	E	A	P(A|B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999

$$P(+b, -e, +a, -j, +m) =$$

Example: Alarm Network

B	P(B)
+b	0.001
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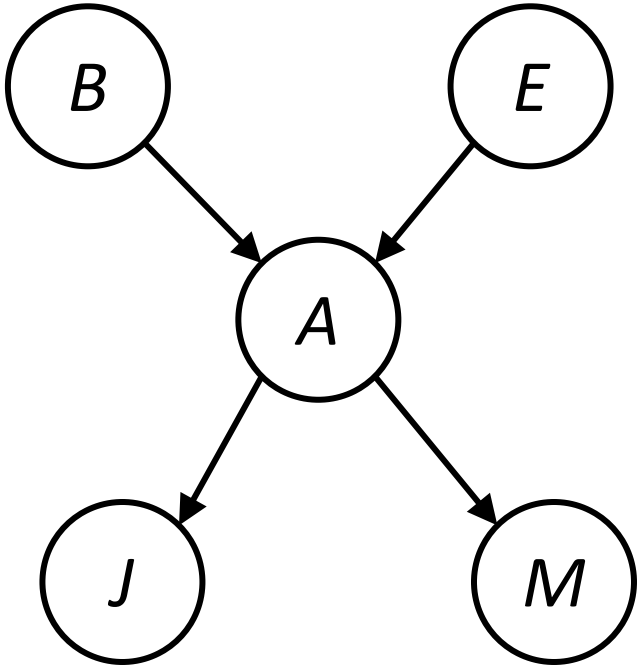

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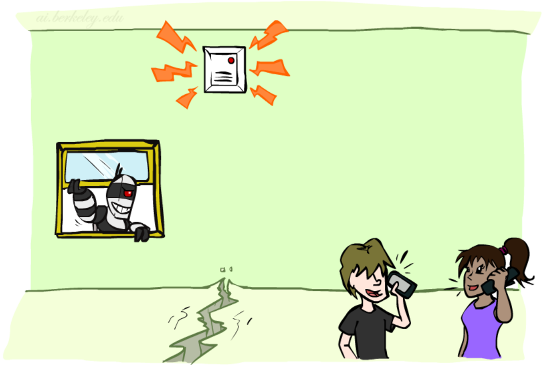

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-a	-j	0.95

A	M	P(M|A)
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-a	+m	0.01
-a	-m	0.99

B	E	A	P(A|B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999

$$P(+b, -e, +a, -j, +m) =$$

$$P(+b)P(-e)P(+a|+b, -e)P(-j|+a)P(+m|+a) =$$

$$0.001 \times 0.998 \times 0.94 \times 0.1 \times 0.7$$

Size of a Bayes' Net

- How big is a joint distribution over N Boolean variables?

  2^N

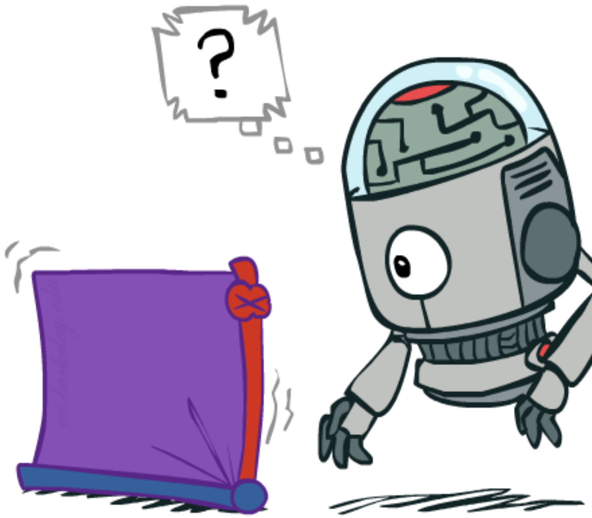
- How big is an N-node net if nodes have up to k parents?

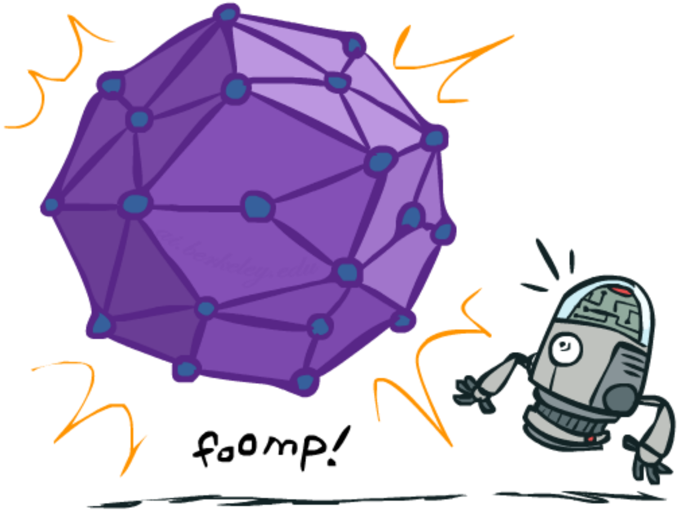
  $O(N * 2^{k+1})$

- Both give you the power to calculate

  $$P(X_1, X_2, \ldots X_n)$$

- BNs: Huge space savings!
- Also easier to elicit local CPTs
- Also faster to answer queries (coming)





Bayes' Nets

- ✓ Representation
- Conditional Independences
- Probabilistic Inference
- Learning Bayes' Nets from Data

Conditional Independence

- X and Y are **independent** if

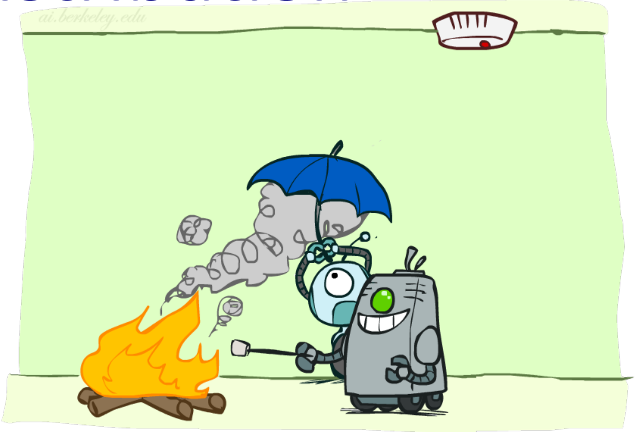
$$\forall x, y \;\; P(x, y) = P(x)P(y) \dashrightarrow X \perp\!\!\!\perp Y$$

- X and Y are **conditionally independent** given Z

$$\forall x, y, z \;\; P(x, y|z) = P(x|z)P(y|z) \dashrightarrow X \perp\!\!\!\perp Y|Z$$

- (Conditional) independence is a property of a distribution

- Example: $Alarm \perp\!\!\!\perp Fire|Smoke$

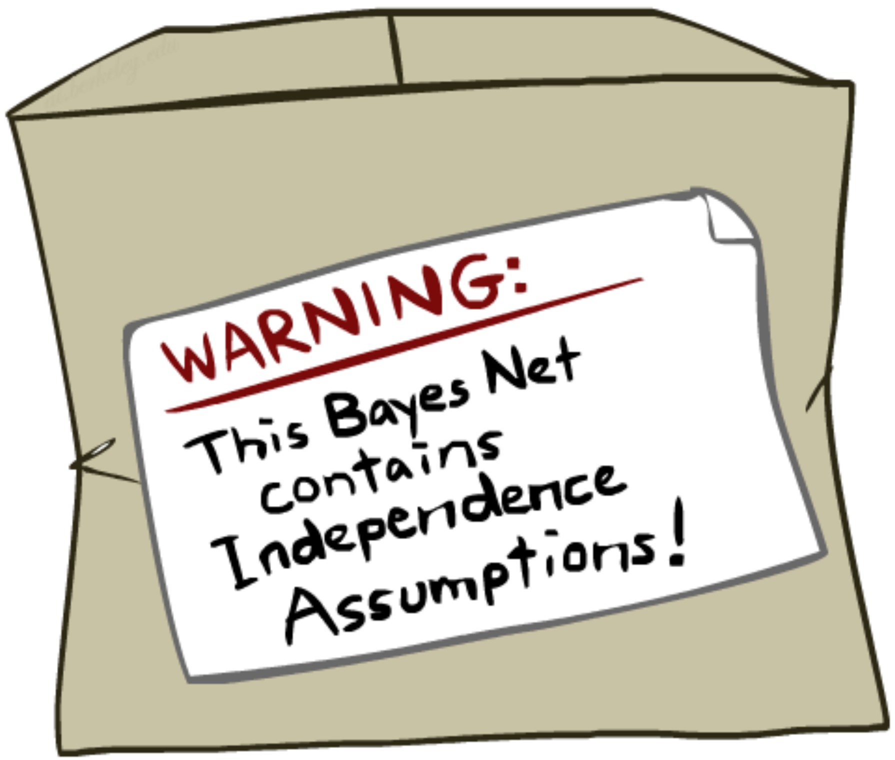


Bayes Nets: Assumptions

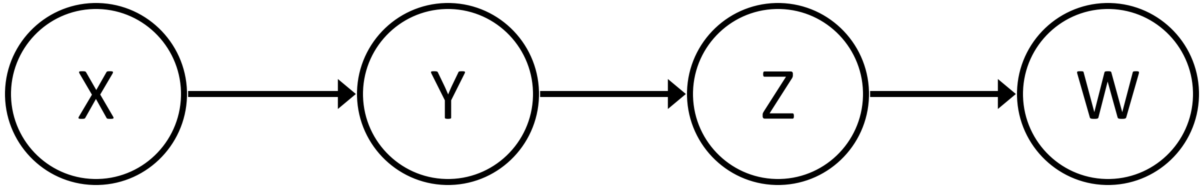
- Assumptions we are required to make to define the Bayes net when given the graph:

$$P(x_i|x_1 \cdots x_{i-1}) = P(x_i|parents(X_i))$$

- Beyond above "chain rule → Bayes net" conditional independence assumptions
  - Often additional conditional independences
  - They can be read off the graph
- Important for modeling: understand assumptions made when choosing a Bayes net graph



Example



- Conditional independence assumptions directly from simplifications in chain rule:

$$P(x, y, z, w) = P(x) \cdot P(y \mid x) \cdot P(z \mid x, y) \cdot P(w \mid x, y, z)$$

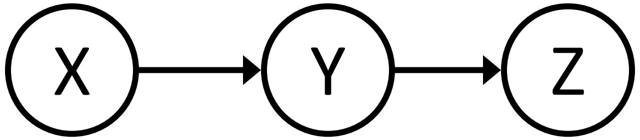
$$P(x, y, z, w) = P(x) \cdot P(y \mid x) \cdot P(z \mid y) \cdot P(w \mid z)$$

- Additional implied conditional independence assumptions?

$X \perp\!\!\!\perp Z \mid Y$ $\qquad$ $Y \perp\!\!\!\perp W \mid Z$

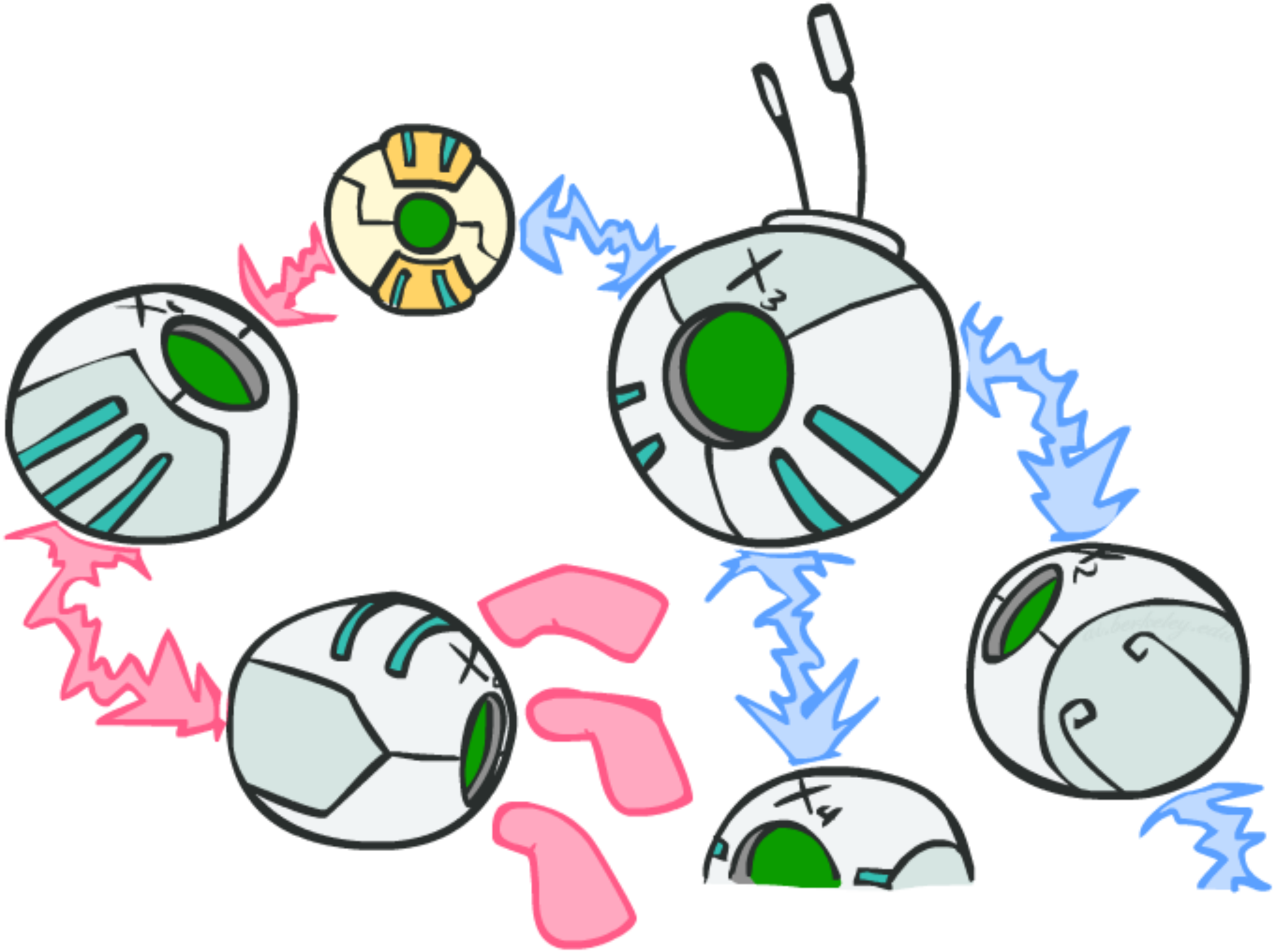
Independence in a BN

- Important question about a BN:
  - Are two nodes independent given certain evidence?
  - If yes, can prove using algebra (tedious in general)
  - If no, can prove with a counter example
  - Example:



  - Question: are X and Z necessarily independent?
    - Answer: no. Example: low pressure causes rain, which causes traffic.
    - X can influence Z, Z can influence X (via Y)
    - Addendum: they *could* be independent: how?

D-separation: Outline



D-separation: Outline

- Study independence properties for triples
- Analyze complex cases in terms of member triples
- D-separation: a condition / algorithm for answering such queries

Causal Chains

- This configuration is a "causal chain"

X: Low pressure　　Y: Rain　　Z: Traffic

$$P(x, y, z) = P(x)P(y|x)P(z|y)$$

- Guaranteed X independent of Z ? *No!*
  - One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.
  - Example:
    - Low pressure causes rain causes traffic, high pressure causes no rain causes no traffic
    - In numbers:

      P(+y | +x) = 1, P(-y | - x) = 1,
      P(+z | +y) = 1, P(-z | -y) = 1

Causal Chains

- This configuration is a "causal chain"

X: Low pressure Y: Rain Z: Traffic

$$P(x,y,z) = P(x)P(y|x)P(z|y)$$

- Guaranteed X independent of Z given Y?

$$P(z|x,y) = \frac{P(x,y,z)}{P(x,y)}$$

$$= \frac{P(x)P(y|x)P(z|y)}{P(x)P(y|x)}$$

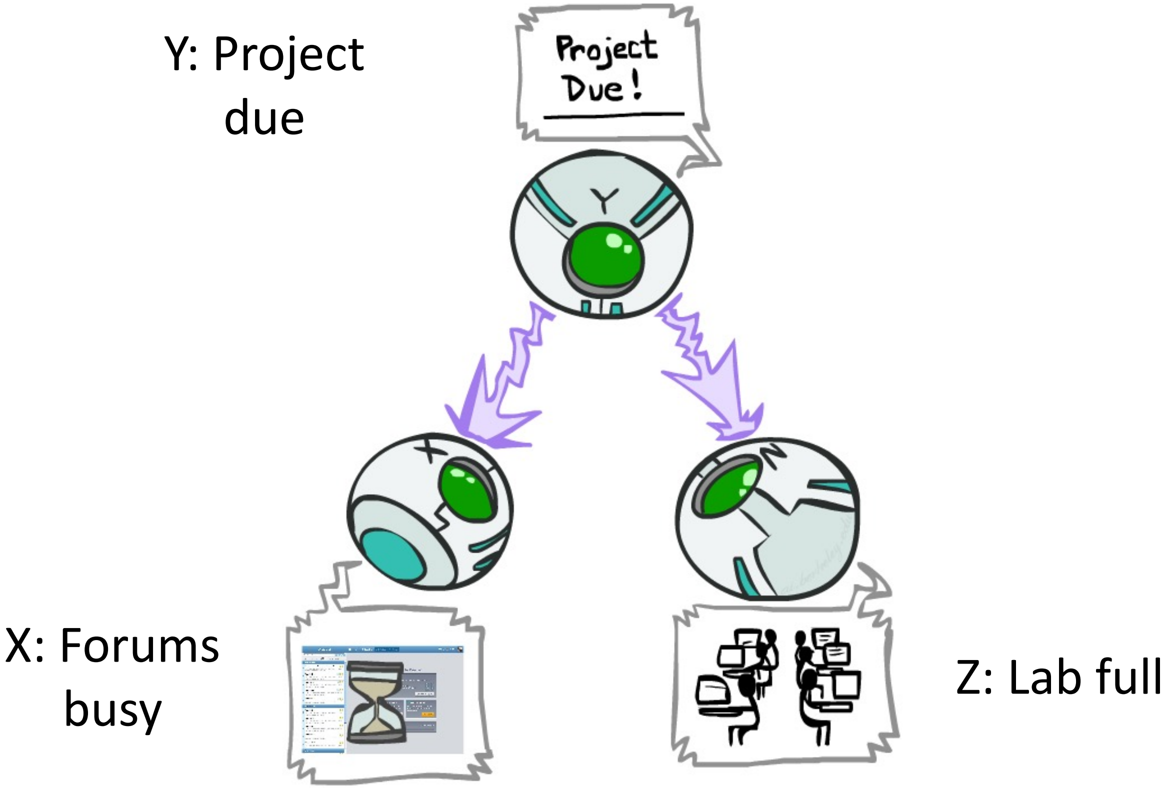
$$= P(z|y)$$

Yes!

- Evidence along the chain "blocks" the influence

Common Cause

- This configuration is a "common cause"



$$P(x, y, z) = P(y)P(x|y)P(z|y)$$

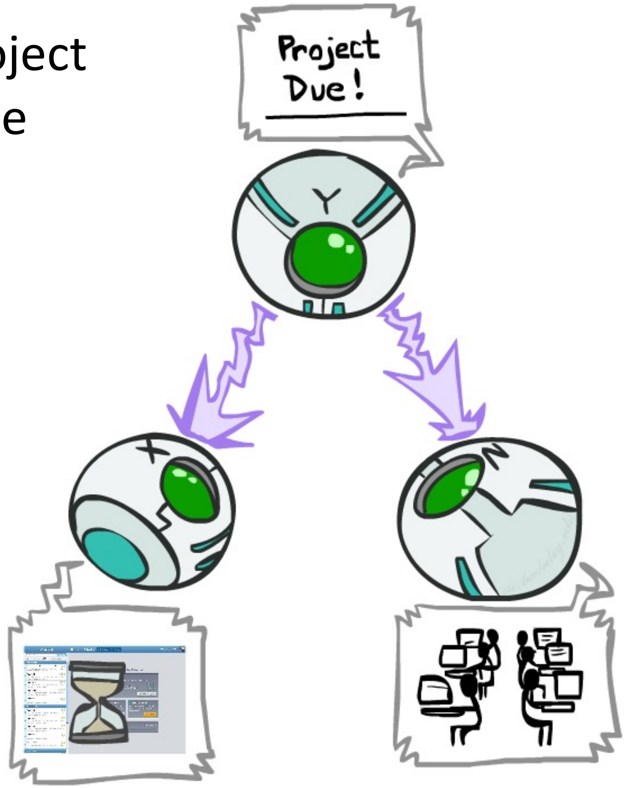
- Guaranteed X independent of Z ? *No!*
  - One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.
  - Example:
    - Project due causes both forums busy and lab full
    - In numbers:

P(+x | +y) = 1, P(-x | -y) = 1,
P(+z | +y) = 1, P(-z | -y) = 1

Common Cause

- This configuration is a "common cause"

Y: Project due



X: Forums busy

Z: Lab full

$$P(x, y, z) = P(y)P(x|y)P(z|y)$$

- Guaranteed X and Z independent given Y?

$$P(z|x, y) = \frac{P(x, y, z)}{P(x, y)}$$

$$= \frac{P(y)P(x|y)P(z|y)}{P(y)P(x|y)}$$

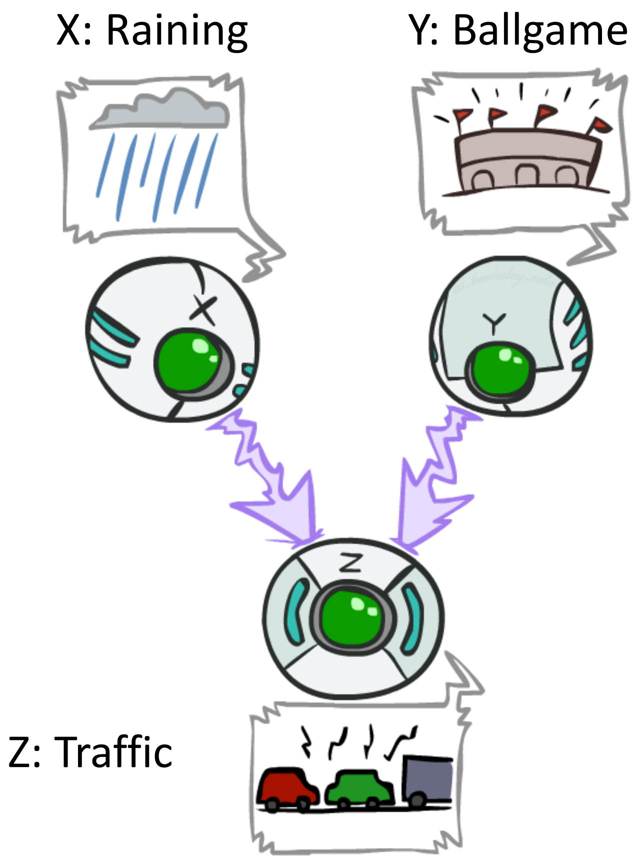
$$= P(z|y)$$

Yes!

- Observing the cause blocks influence between effects.

Common Effect

- Last configuration: two causes of one effect (v-structures)

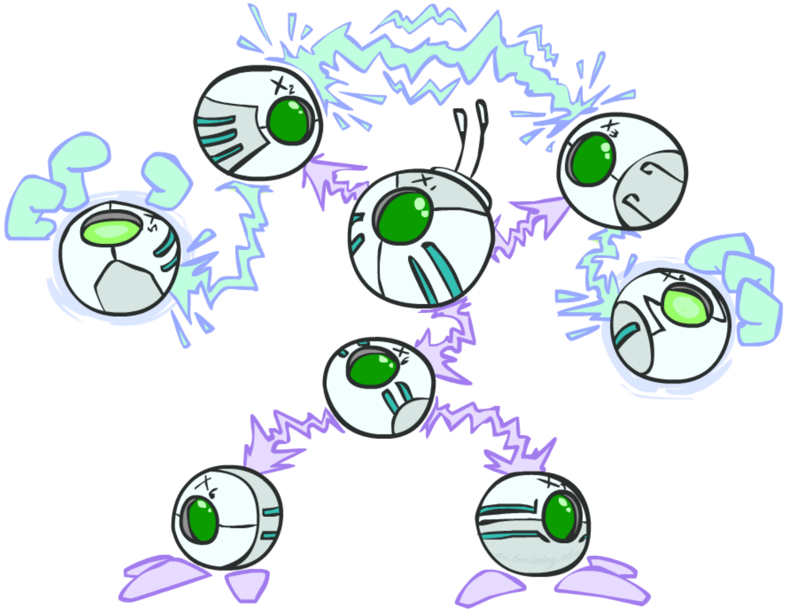


- Are X and Y independent?
  - *Yes*: the ballgame and the rain cause traffic, but they are not correlated
  - Still need to prove they must be (try it!)
- Are X and Y independent given Z?
  - *No*: seeing traffic puts the rain and the ballgame in competition as explanation.
- This is backwards from the other cases
  - Observing an effect activates influence between possible causes.

The General Case

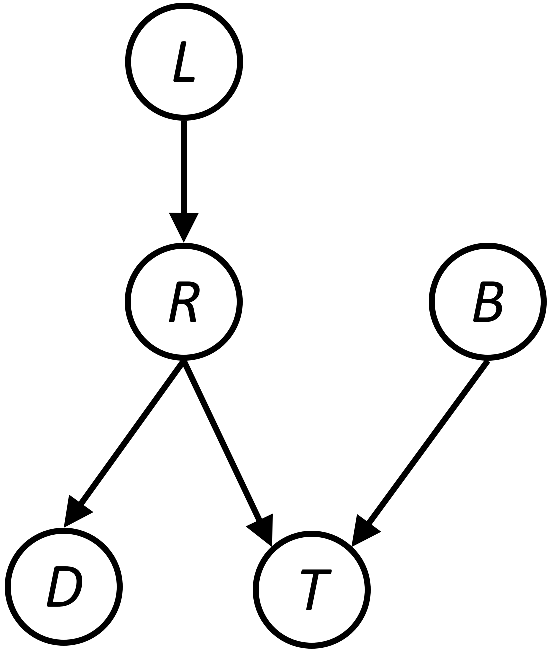
The General Case

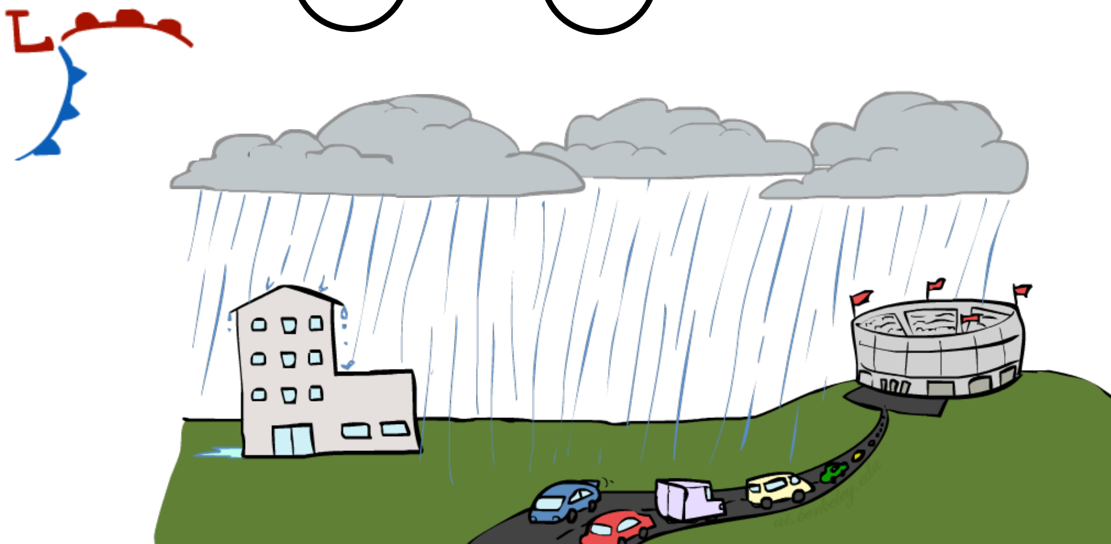
- General question: in a given BN, are two variables independent (given evidence)?
- Solution: analyze the graph
- Any complex example can be broken into repetitions of the three canonical cases



Reachability

- Recipe: shade evidence nodes, look for paths in the resulting graph
- Attempt 1: if two nodes are connected by an undirected path blocked by a shaded node, they are conditionally independent
- Almost works, but not quite
  - Where does it break?
  - Answer: the v-structure at T doesn't count as a link in a path unless "active"

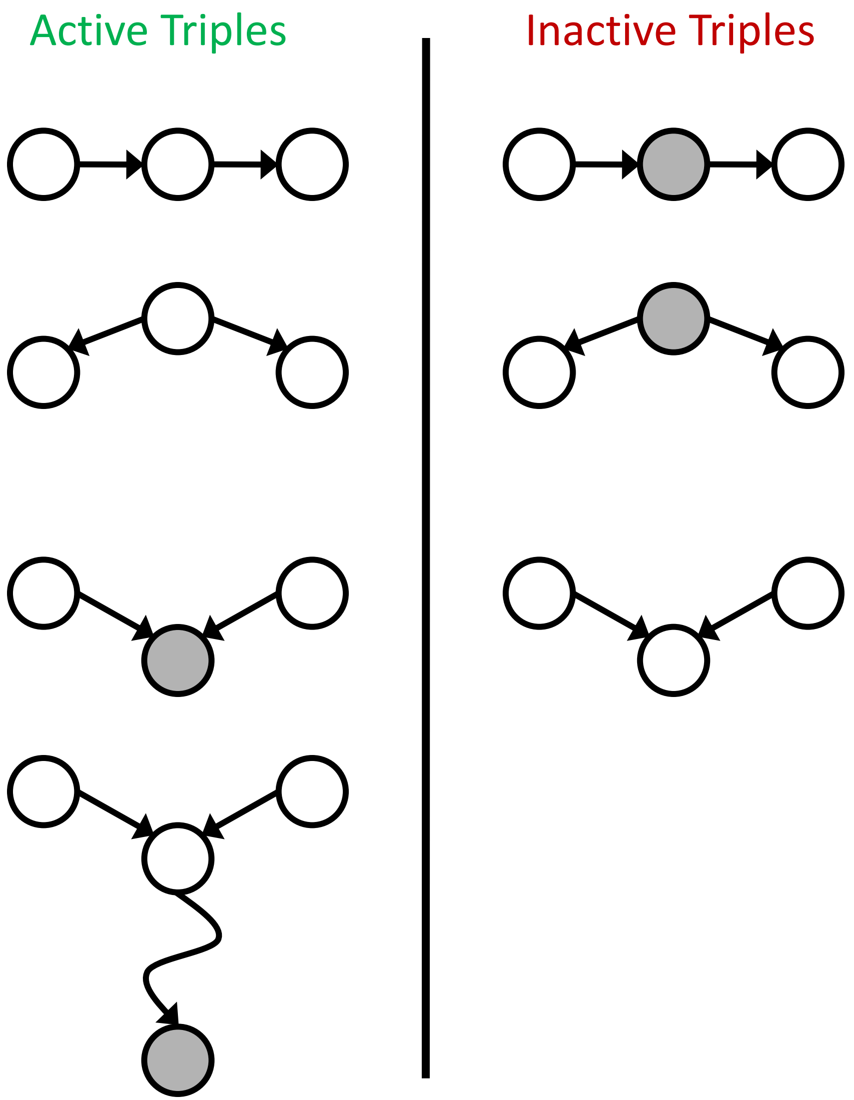




Active / Inactive Paths

- Question: Are X and Y conditionally independent given evidence variables {Z}?
  - Yes, if X and Y "d-separated" by Z
  - Consider all (undirected) paths from X to Y
  - No active paths = independence!
- A path is active if each triple is active:
  - Causal chain A $\rightarrow$ B $\rightarrow$ C where B is unobserved (either direction)
  - Common cause A $\leftarrow$ B $\rightarrow$ C where B is unobserved
  - Common effect (aka v-structure)
    A $\rightarrow$ B $\leftarrow$ C where B *or one of its descendents* is observed
- All it takes to block a path is a single inactive segment

Any active path breaks the independence



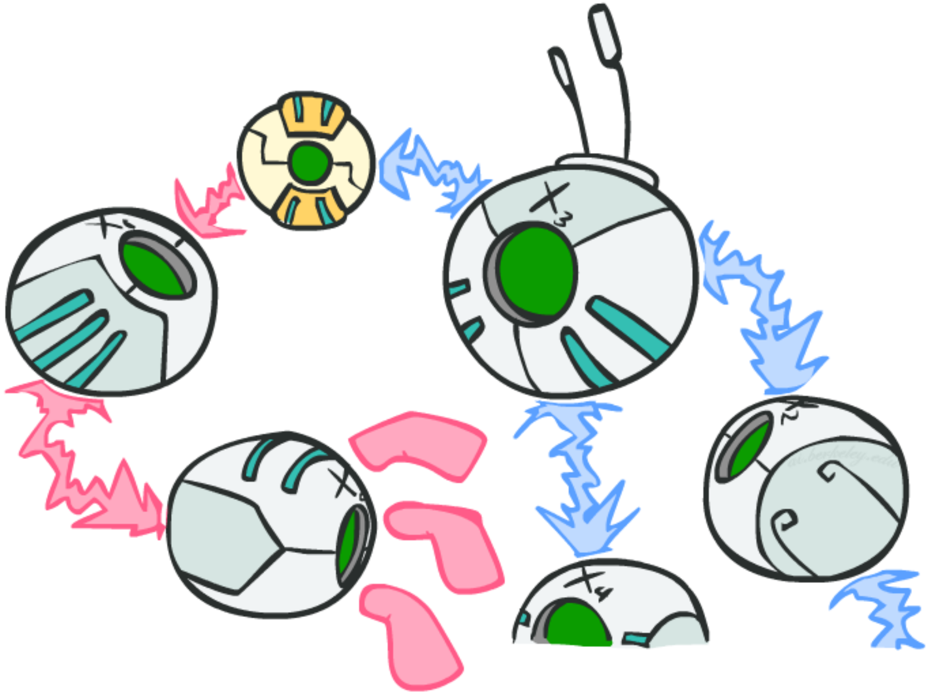
D-Separation

- Query: $X_i \perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$?
- Check all (undirected!) paths between X_i and X_j
  - If one or more active, then independence not guaranteed

    $$X_i \not\perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$$

  - Otherwise (i.e. if all paths are inactive), then independence is guaranteed

    $$X_i \perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$$

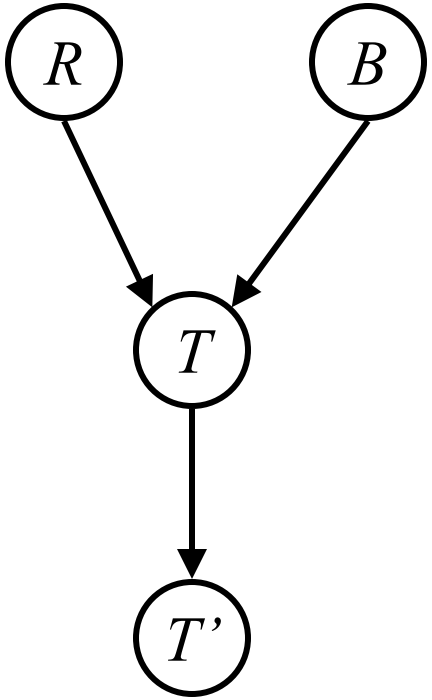


Example

$R \perp\!\!\!\perp B$ *Yes*

$R \perp\!\!\!\perp B | T$

$R \perp\!\!\!\perp B | T'$



Example

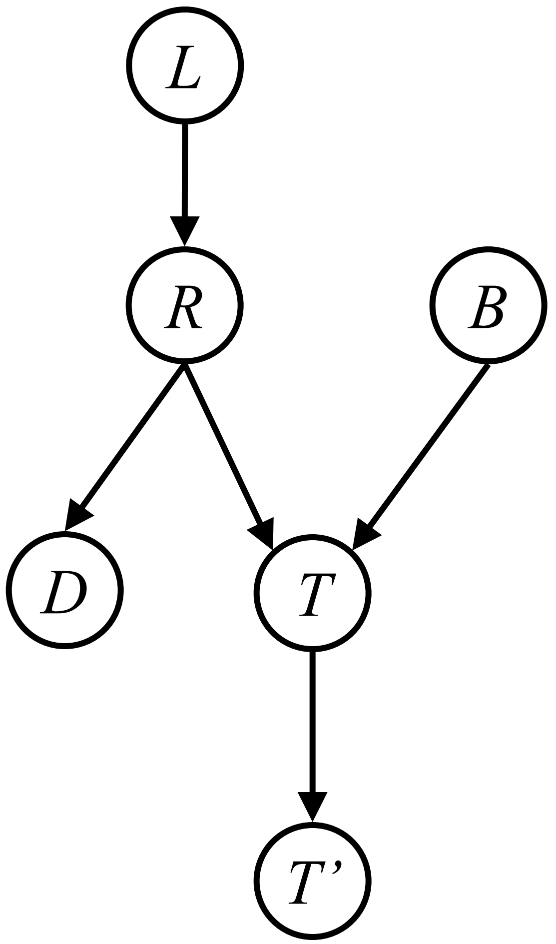
$L \perp\!\!\!\perp T' | T$ Yes

$L \perp\!\!\!\perp B$ Yes

$L \perp\!\!\!\perp B | T$

$L \perp\!\!\!\perp B | T'$

$L \perp\!\!\!\perp B | T, R$ Yes



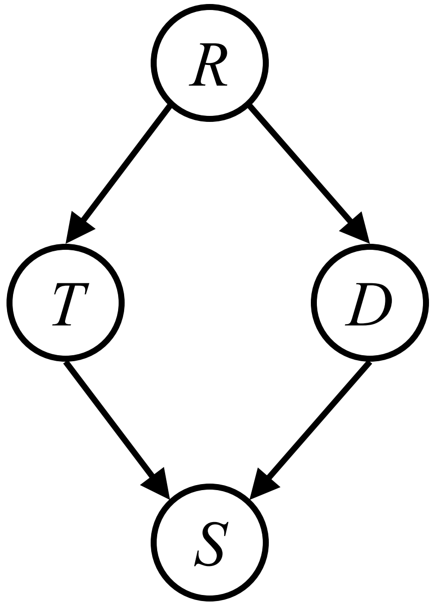
Example

- Variables:
  - R: Raining
  - T: Traffic
  - D: Roof drips
  - S: I'm sad
- Questions:

$$T \perp\!\!\!\perp D$$

$$T \perp\!\!\!\perp D | R \qquad \textit{Yes}$$

$$T \perp\!\!\!\perp D | R, S$$

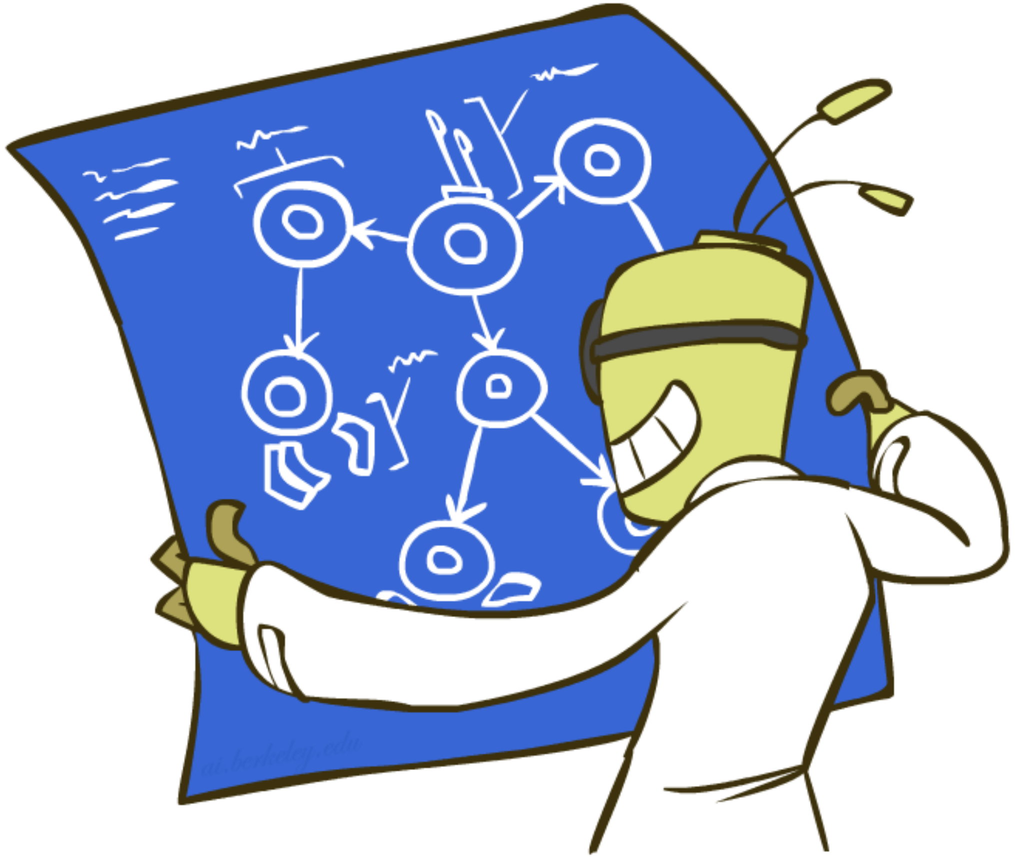


Structure Implications

- Given a Bayes net structure, can run d-separation algorithm to build a complete list of conditional independences that are necessarily true of the form

$$X_i \perp\!\!\!\perp X_j | \{X_{k_1}, ..., X_{k_n}\}$$

- This list determines the set of probability distributions that can be represented



(Easier) d-separation

- Problem:
  - Is P(A l BDF) = P(A l DF)?”
- Alogorithm:
  - we can convert it into an independence question like this:
  - “Are A and B independent, given D and F?”
  - $A \perp B \mid D, F$
- Then follow the algorithm…

Source: http://web.mit.edu/jmn/www/6.034/d-separation.pdf

Algorithm

- 1. Draw the ancestral graph.
  - Construct the “ancestral graph” of all variables mentioned in the probability expression.
  - This is a reduced version of the original net, consisting only of the variables mentioned and all of their ancestors (parents, parents’ parents, etc.)
- 2. “Moralize” the ancestral graph by “marrying” the parents.
  - For each pair of variables with a common child, draw an undirected edge (line) between them.
  - (If a variable has more than two parents, draw lines between every pair of parents.)

3. "Disorient" the graph

- by replacing the directed edges (arrows) with undirected edges (lines).

4. Delete the givens and their edges.

- If the independence question had any given variables, erase those variables from the graph and erase all of their connections, too.
- Note that “given variables” as used here refers to the question “Are A and B conditionally independent, given D and F?”, not the equation “P(A | BDF) =? P(A | DF)”, and thus does not include B.

- 5. Read the answer off the graph.
  - If the variables are disconnected in this graph,
    - they are guaranteed to be independent.
  - If the variables are connected in this graph,
    - they are not guaranteed to be independent.*
    - Note that “are connected” means “have a path between them,” so if we have a path X-Y-Z, X and Z are considered to be connected, even if there’s no edge between them.
  - If one or both of the variables are missing
    - (because they were givens, and were therefore deleted)
    - they are independent.

Notes

- Variables are not guaranteed to be independent
  - We can say “the variables are dependent, as far as the Bayes net is concerned” or
  - “the Bayes net does not require the variables to be independent,”
- But we cannot guarantee dependency using d-separation alone,
  - because the variables can still be numerically independent (e.g. if P(A|B) and P(A) happen to be equal for all values of A and B).

Examples

1. Are A and B conditionally independent, given D and F?

(Same as “P(A|BDF) =? P(A|DF)” or “P(B|ADF) =? P(B|DF)”)

2. Are A and B marginally independent? (Same as “P(A|B) =? P(A)” or “P(B|A) =? P(B)”)

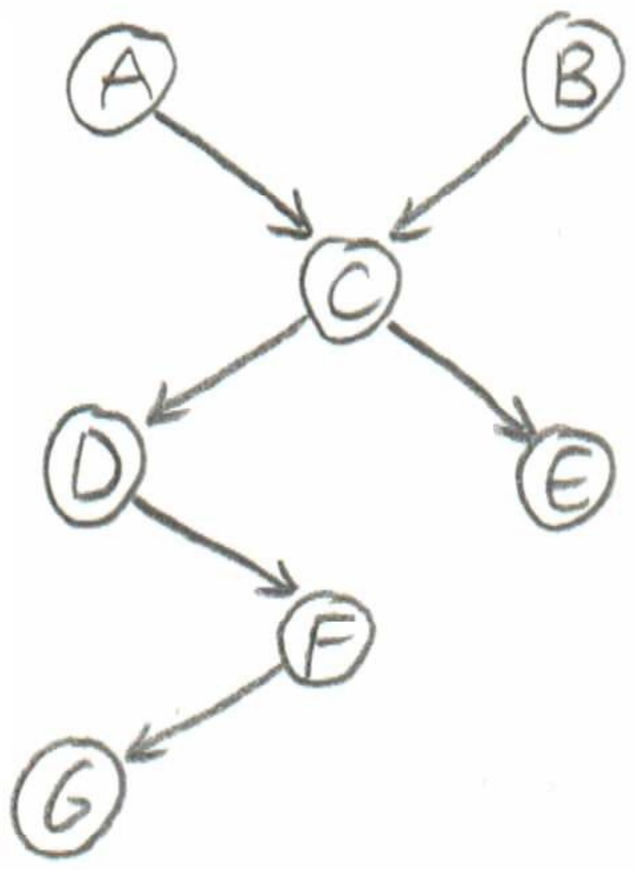
3. Are A and B conditionally independent, given C?

4. Are D and E conditionally independent, given C?

5. Are D and E marginally independent?

6. Are D and E conditionally independent, given A and B?

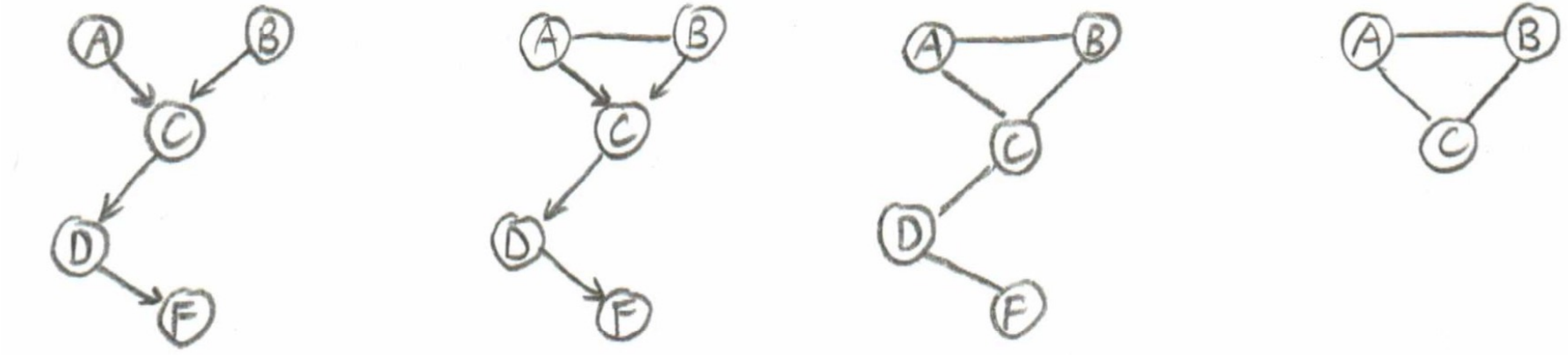
7. P(D|BCE) =? P(D|C)



Example

1. Are A and B conditionally independent, given D and F?
(Same as "P(A|BDF) =? P(A|DF)" or "P(B|ADF) =? P(B|DF)")

Draw ancestral graph | Moralize | Disorient | Delete givens

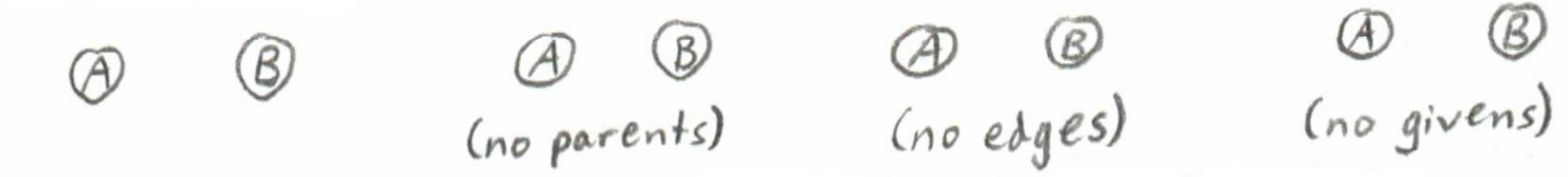


Answer: No, A and B are connected, so they are not required to be conditionally independent given D and F.

Example

2. Are A and B marginally independent? (Same as "P(A|B) =? P(A)" or "P(B|A) =? P(B)")

Draw ancestral graph | Moralize | Disorient | Delete givens



Answer: Yes, A and B are not connected, so they are marginally independent.

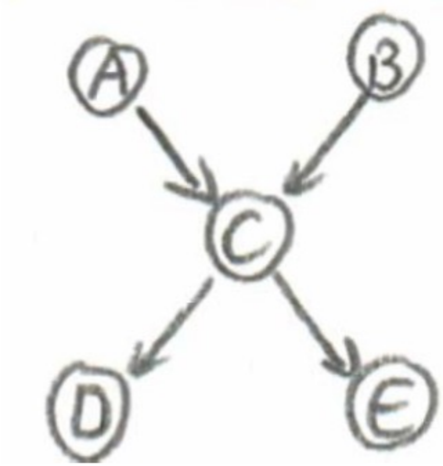
3. Are A and B conditionally independent, given C?

Draw ancestral graph Moralize Disorient Delete givens

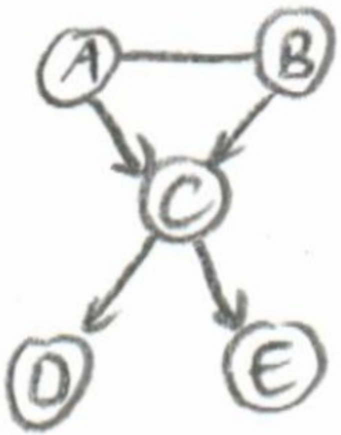
Answer: No, A and B are connected, so they are not required to be conditionally independent given C.

4. Are D and E conditionally independent, given C?

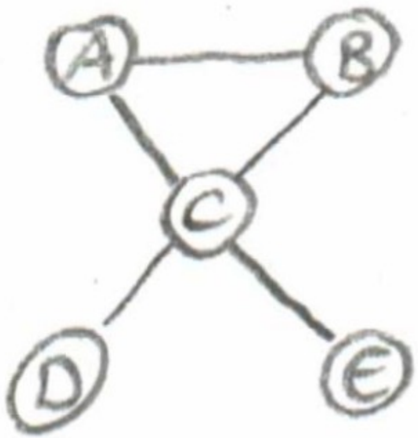
Draw ancestral graph



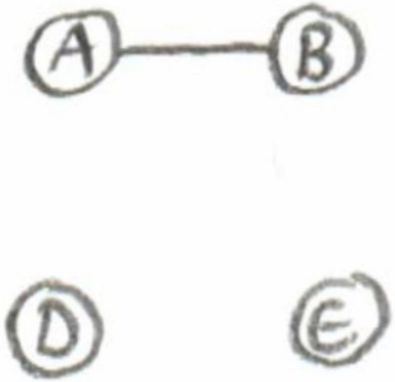
Moralize



Disorient



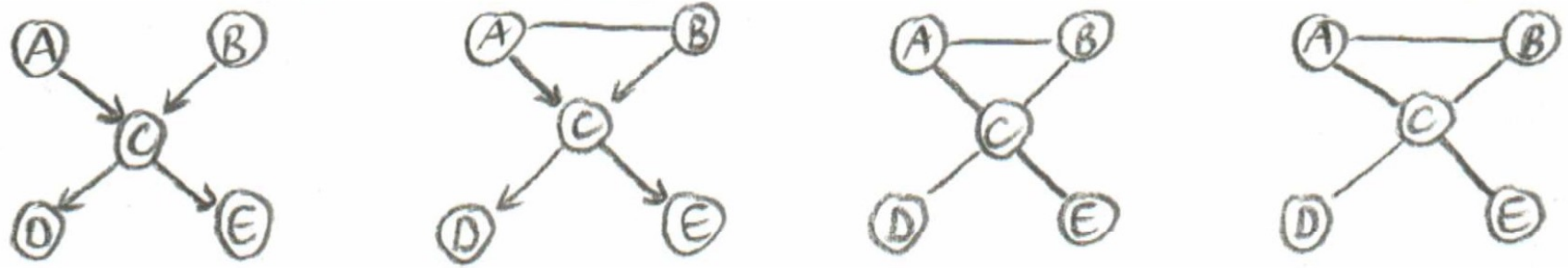
Delete givens



Answer: Yes, D and E are not connected, so they are conditionally independent given C.

5. Are D and E marginally independent?

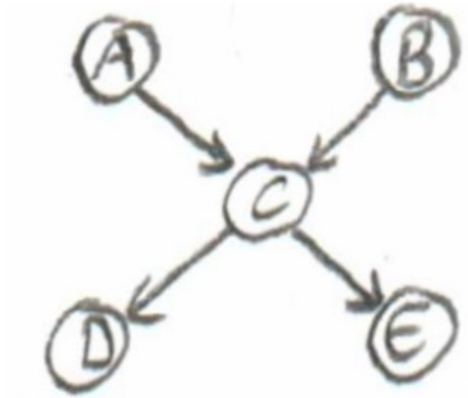
Draw ancestral graph | Moralize | Disorient | Delete givens



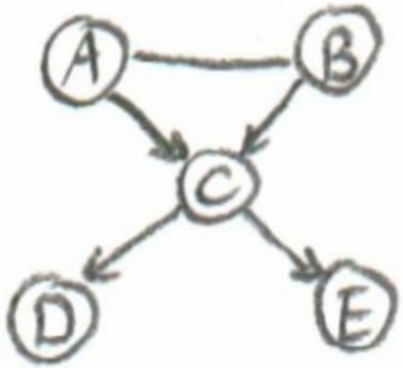
Answer: No, D and E are connected (via a path through C), so they are not required to be marginally independent.

6. Are D and E conditionally independent, given A and B?

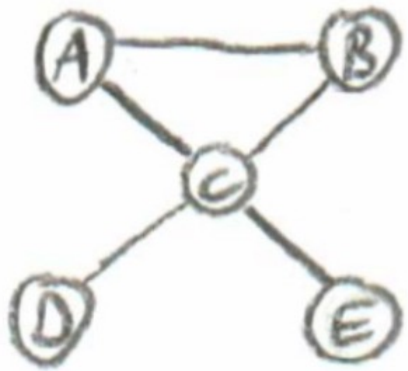
Draw ancestral graph



Moralize



Disorient



Delete givens

Answer: No, D and E are connected (via a path through C), so they are not required to be conditionally independent given A and B.

7. P(D|CEG) =? P(D|C)

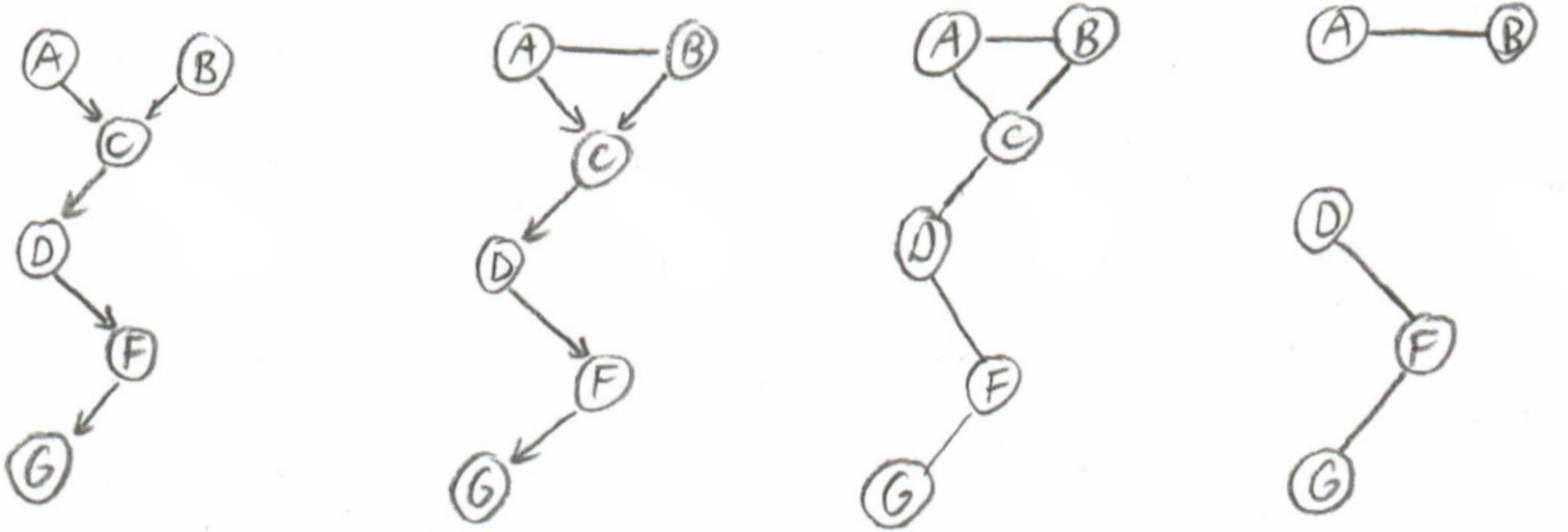
Rewrite as independence questions "Are X and Y conditionally independent, given {givens}?":

- Are D and E conditionally independent, given C? AND
- Are D and G conditionally independent, given C?

(a) Are D and E conditionally independent, given C? Yes; see example 4.

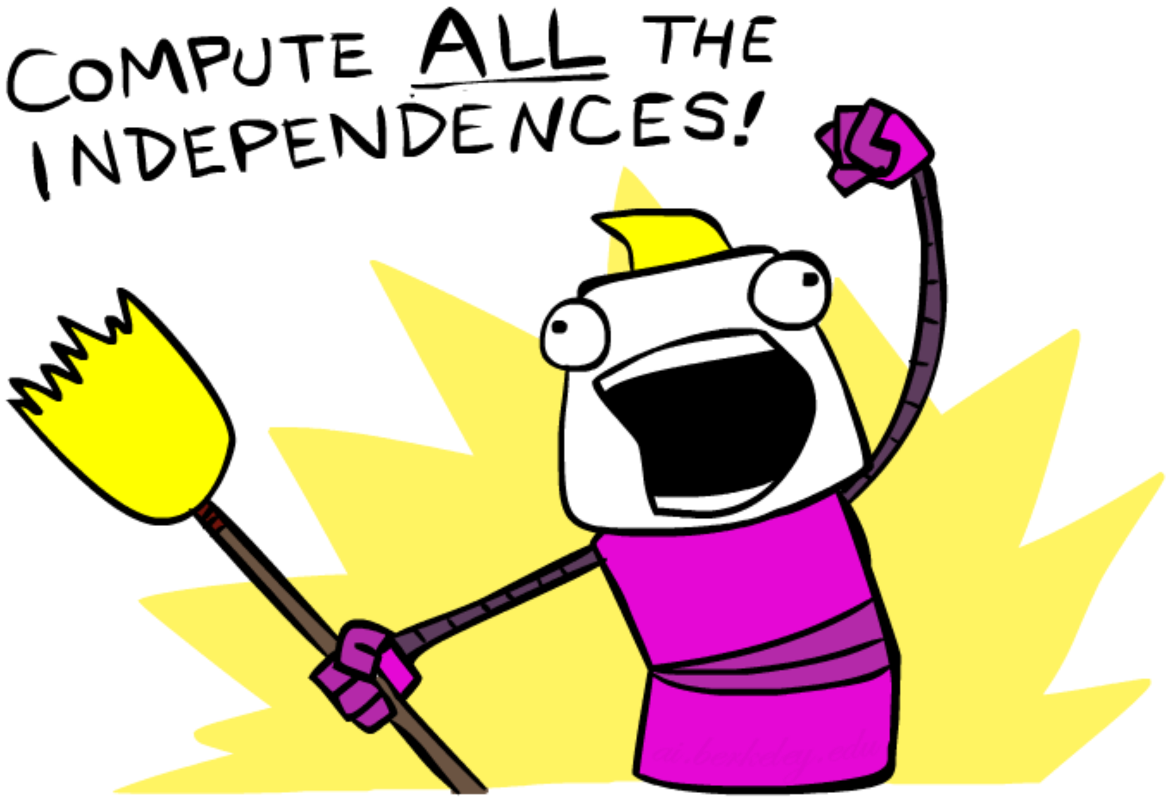
(b) Are D and G conditionally independent, given C? No, because they are connected (via F):

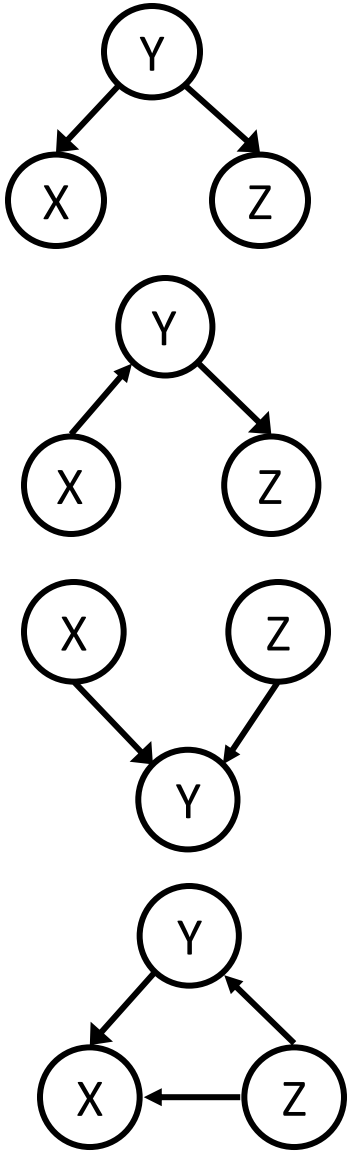
Draw ancestral graph Moralize Disorient Delete givens



Overall answer: No. D and E are conditionally independent given C, but D and G are not required to be. Therefore we cannot assume that P(D|CEG) = P(D|C).

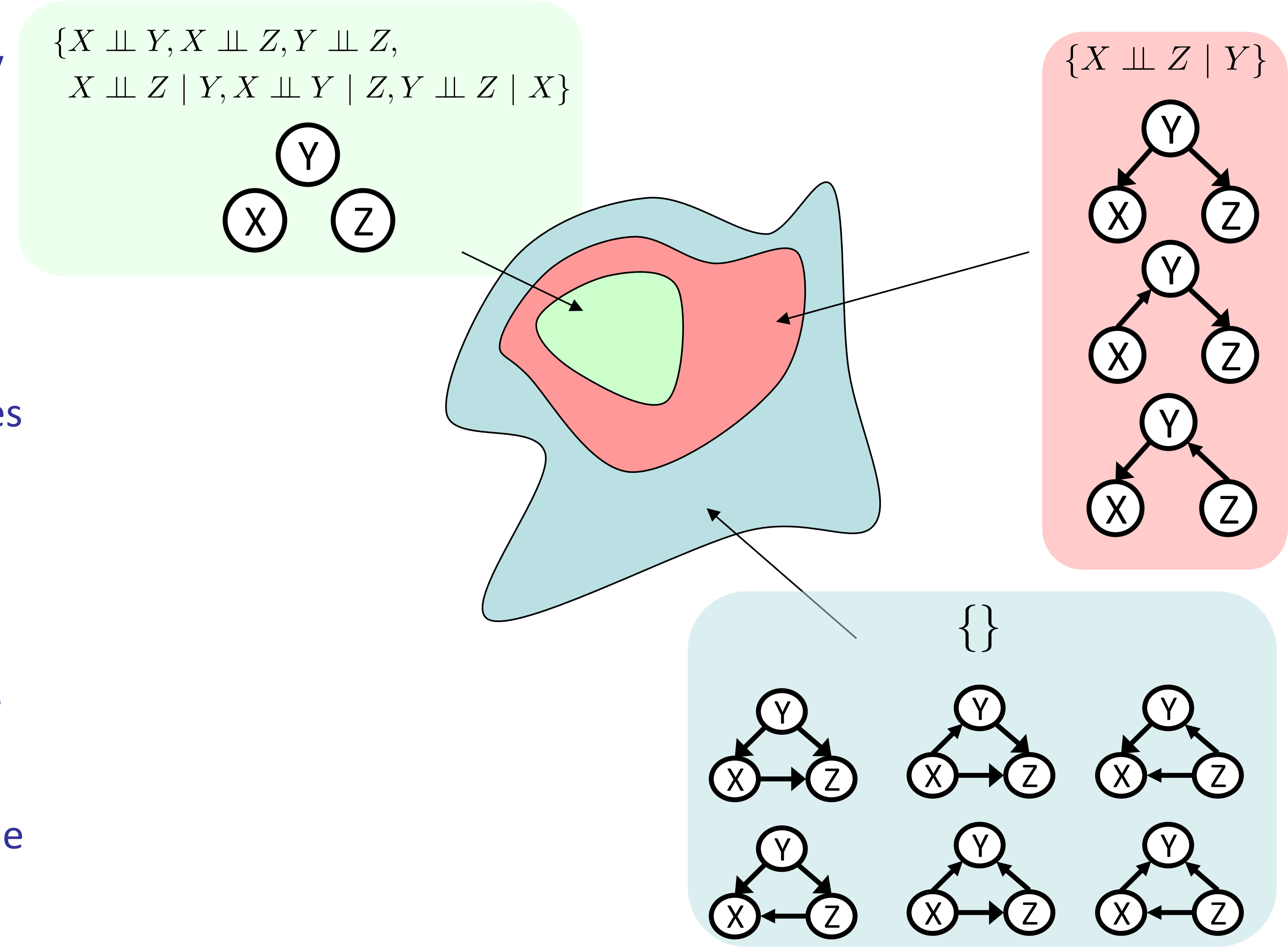
Computing All Independences





Topology Limits Distributions

- Given some graph topology G, only certain joint distributions can be encoded
- The graph structure guarantees certain (conditional) independences
- (There might be more independence)
- Adding arcs increases the set of distributions, but has several costs
- Full conditioning can encode any distribution



Bayes Nets Representation Summary

- Bayes nets compactly encode joint distributions
- Guaranteed independencies of distributions can be deduced from BN graph structure
- D-separation gives precise conditional independence guarantees from graph alone
- A Bayes' net's joint distribution may have further (conditional) independence that is not detectable until you inspect its specific distribution

Bayes' Nets

- ✓ Representation
- ✓ Conditional Independences
- Probabilistic Inference
  - Enumeration (exact, exponential complexity)
  - Variable elimination (exact, worst-case exponential complexity, often better)
  - Probabilistic inference is NP-complete
  - Sampling (approximate)
- Learning Bayes' Nets from Data