RKMVERI Belur
Practice Assignment
Course - Econometrics

This is a practice assignment for your endsemester preparation. No need to submit it.

1. Let $\{X_1, X_2, \cdots, X_n\}$ be a sample of size n from a normal distribution with mean μ and variance σ^2. Consider the following point estimators of μ

   (a) $\hat{\sigma}_1^2 = \dfrac{1}{n-3}\sum_{i=1}^{n}(X_i - \bar{X})^2$

   (b) $\hat{\sigma}_2^2 = (X_1 - \bar{X})^2$

   Answer the following questions with justification:

   (a) Which are the unbiased estimators?

   (b) Which are the consistent estimators?

   (c) Which is the most efficient?

   (d) Is the normality assumption needed to answer (a) to (c)?

2. Suppose $\hat{\mu}$ is an estimator of μ derived from a sample of size n. You are given that $E(\hat{\mu}) = \mu + \dfrac{2}{n}$ and $Var(\hat{\mu}) = \dfrac{4\mu}{n} + \dfrac{\mu^2}{n^2}$

   (a) Examine whether $\hat{\mu}$ is an unbiased estimator of μ.

   (b) Whether $\hat{\mu}$ is a consistent estimator of μ.

   (c) Whether $\hat{\mu}$ is asymptotically unbiased of μ.

   (d) Whether $\hat{\mu}$ is asymptotically efficient of μ.

   (e) What is the asymptotic variance of $\hat{\mu}$?

3. Comment briefly on the meaning of the following terms:

   (a) Estimated regression coefficient β

   (b) Standard error

   (c) R^2

   (d) r

   (e) Residual sum of squares

   (f) Total sum of squares

   (g) Sum of squares due to regression or Regression sum of squares

   (h) Unbiasedness

   (i) Consistency

   (j) Asymptotic unbiasedness

   (k) p-value

   (l) Type-I-error

   (m) Level of significance

   (n) Power of a test

   (o) Least squares method

   (p) Weighted least squares method

   (q) Normal equations

(r) Population and samples

(s) Parameters

(t) Random samples

(u) Estimated regression coefficient $\hat{\beta} = 2$

4. The following data present experience (x) and salary structure (y) of the faculties of Economics at the Calcutta University (in thousand of rupees) in the year 2022-23. Calculate the regression of Y on X: $Y_i = \alpha + \beta X_i + u_i$. Calculate R^2 and r. Calculate the RSS. Calculate the 95% confidence interval for β assuming that the errors are normally distributed. Predict the value of Y when $X = 3$.

X	Y
20	300
15	250
5	150
12	200
30	350

5. Show that simple linear regression line of Y on X coincides with the regression line of X on Y if and only if $r^2 = 1$, where r is the sample correlation coefficient between X and Y.

6. Let $Y_i = \alpha + \beta X_i + u_i$. Show that if $\bar{X} = 0$, then $\text{Cov}(\hat{\alpha}, \hat{\beta}) = 0$ where $\hat{\alpha}$ and $\hat{\beta}$ are the least squares estimators.

7. Let $\hat{u}_i$ be the residuals in the least squares fit of $Y_i = \alpha + \beta X_i + u_i,\ i = 1, 2, \cdots, n$. Derive the followings:

$$\sum_{i=1}^{n} \hat{u}_i = 0$$

and

$$\sum_{i=1}^{n} x_i \hat{u}_i = 0.$$

$\cdots\cdots * * * \cdots\cdots$