CDS, IISc

Assignment-2

DS 211, Fall 2022: Numerical Optimization

Due: Nov 15, 2022
23:59 Hours

Submission instructions:

- The assignment is to be submitted in ONE single notebook.
- Submit the .ipynb file through this Teams Assignment.
- The submitted notebook should be a fresh file.
- If your IISc email ID is $< username > @iisc.ac.in$, then name the file $< username > \_Assgn\_2$ E.g. akankshamr_Assgn_2 for my email ID akankshamr@iisc.ac.in
- Make sure you follow the question-wise instructions for each problem.
- Before submission, execute the 'Restart and Run all' command from the Runtime/Kernel tab. Verify that there are no errors and that you are getting the output you expect.

Problem 1

Conjugate Gradient

(a) 1.5 marks Code the Conjugate Gradient Method and apply it to solve a simple Hx=b system, with H's dimension as 10×10.

(b) 1.5 marks Construct matrices with various eigenvalue distributions (clustered and non-clustered) and apply the above CG method to them. Comment on the behaviour of the CG method in terms of convergence.

Problem 2

Linear Programming

(a) 1.0 marks Your start-up will face the cash requirements shown in Table 1 in the next eight quarters (positive entries represent cash needs while negative entries represent cash surpluses).
The company has three borrowing possibilities.

Table 1: Cash Flow (in Crores of INR)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
100	500	100	-600	-500	200	600	-900

- a 2-year loan available at the beginning of Q1, with a 1% interest per quarter.

- The other two borrowing opportunities are available at the beginning of every quarter: a 6-month loan with a 1.8% interest per quarter, and a quarterly loan with a 2.5% interest for the quarter.

Any surplus can be invested at a 0.5% interest per quarter.
Formulate a LP that maximizes the wealth of the company at the beginning of Q9.

(b) 1.0 marks Write a program that implements the revised simplex algorithm.

(c) 1.0 marks Solve the above LP using your program and report the final results.

(d) 0.5 marks Use PuLP and scipy.optimize.linprog to verify your answer. Change the optimization algorithm used by the solver - try interior point, revised-simplex and dual-simplex. Comment on findings.

Problem 3

Quadratic Programming

(a) 0.5 marks Consider the problem

$$min\ x_1^2 + 2x_2^2 - 2x_1 - 6x_2 - 2x_1x_2,$$

$$s.t.\ \ x_1 + x_2 \leq 2,\ \ -x_1 + 2x_2 \leq 2,\ \ x_1,\ x_2 \geq 0.$$

Solve geometrically (Plot the geometry and solution points to visualize)

(b) 1.0 marks Implement the active set method for quadratic programs (Algo 16.3 in Nocedal and Wright (2006)).

(c) 1.0 marks Choose three initial starting points: one in the interior of the feasible region, one at the vertex, and one at a non-vertex point on the boundary of the feasible region. Comment on the convergence properties.

(d) 1.0 marks Use CVXOPT and solve the above QP. What algorithm does CVXOPT use to solve QP? Understand and comment on your findings.