DS 298: Work Assignment - 2

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The goal is to examine the error in Algorithm-1 for matrix multiplication across different values of dimension n and sample size c. The analysis will be performed for three distinct categories of matrices. The goal is to investigate how the errors vary based on these parameters.

Class I matrices:

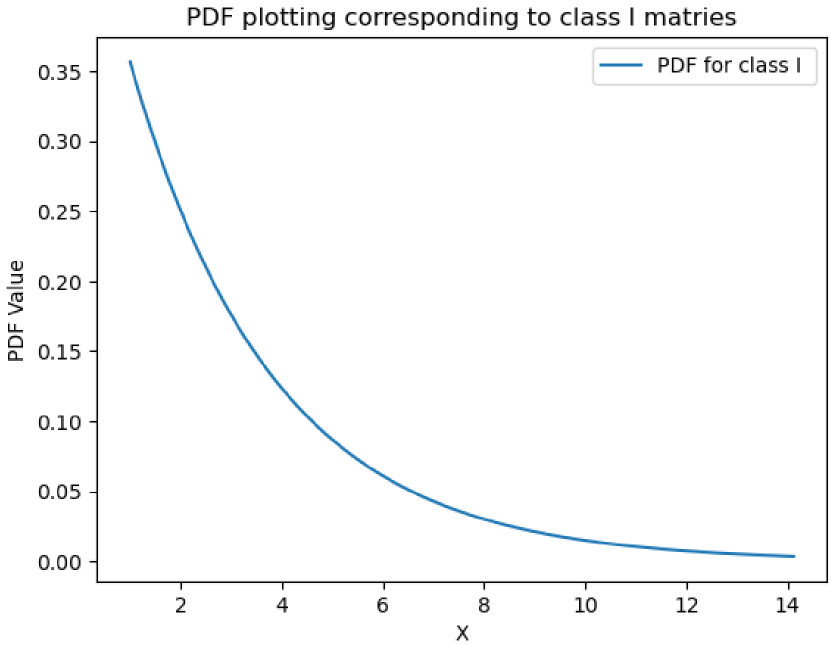
Matrices falling under Class-I are characterized by a probability distribution of the norms of columns in A and rows in B, denoted by $\|A^{(k)}\|$ and $\|B_{(k)}\|$, respectively. The distribution is defined by two independent random variables X_1 and X_2, which follow an exponential probability density function $ae^{-\frac{5x}{x_{\max}}}$.Here, the constants a, x_{min}, and x_{max} are selected such that x_{min} is 1 and $x_{max} = \sqrt{m}$.

Here, the pdf $f(x) = ae^{\frac{-5x}{\sqrt{m}}}$ so

$$\int_1^{\sqrt{m}} f(x)\, dx = 1 \implies \int_1^{\sqrt{m}} ae^{\frac{-5x}{\sqrt{m}}}\, dx = 1 \implies a = \frac{5}{\sqrt{m}(e^{-5/\sqrt{m}} - e^{-5})}.$$

Now CDF is $u = F(x) = \int_1^x ae^{\frac{-5x}{\sqrt{m}}}\, dx$ or, $x = -\frac{\sqrt{m}}{5} \ln\left(e^{-\frac{5}{\sqrt{m}}} - \frac{5u}{a\sqrt{m}}\right) = F^{-1}(U)$, where u follows U(0,1).

The plot of the pdf corresponding to Class-I matrix is given below which verified our pdf which contains exponential term with negative power.



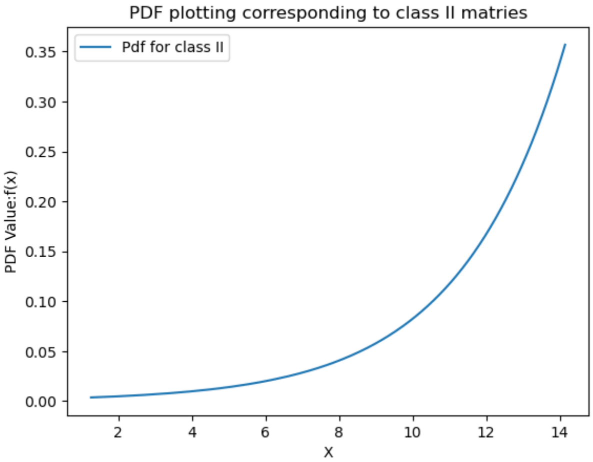
Class II matrices

Here, the pdf $f(x) = ae^{\frac{5x}{\sqrt{m}}}$

$$\int_1^{\sqrt{m}} f(x)\,dx = 1 \implies \int_1^{\sqrt{m}} ae^{\frac{5x}{\sqrt{m}}}\,dx = 1 \implies a = \frac{5}{\sqrt{m}(e^5 - e^{5/\sqrt{m}})}$$

Now, the cumulative distribution function (CDF) is given by $u = F(x) = \int_1^x ae^{\frac{5x}{\sqrt{m}}}, dx$. Solving for x, we get $x = \frac{\sqrt{m}}{5}\ln(\frac{5u_2}{a\sqrt{m}} + e^{\frac{5}{\sqrt{m}}})$, where u follows a uniform distribution on the interval [0,1].

The plot of the PDF corresponding to Class-II matrices is given below, which verifies that our PDF contains an exponential term with a positive power.



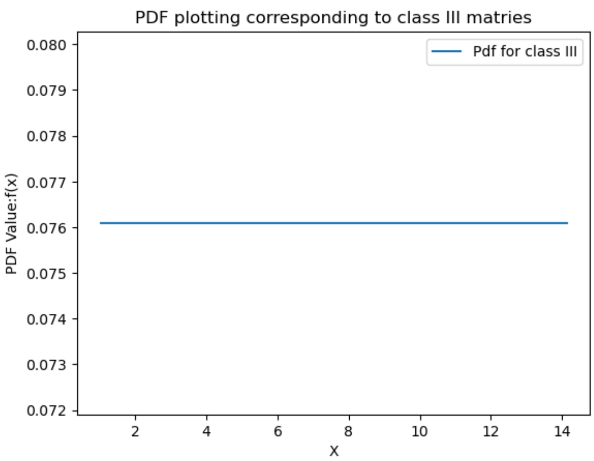
Class III matrices

Here, the pdf $f(x) = \frac{1}{\sqrt{m}-1}$

Now CDF is

$$u = F(x) = \int_1^x \frac{1}{\sqrt{m}-1}\,dx \implies x = 1 + u(\sqrt{m}-1)$$

where u follows U(0,1).The plot of the PDF corresponding to Class-III matrices is given below, which verifies that our PDF is uniform.



Error analysis

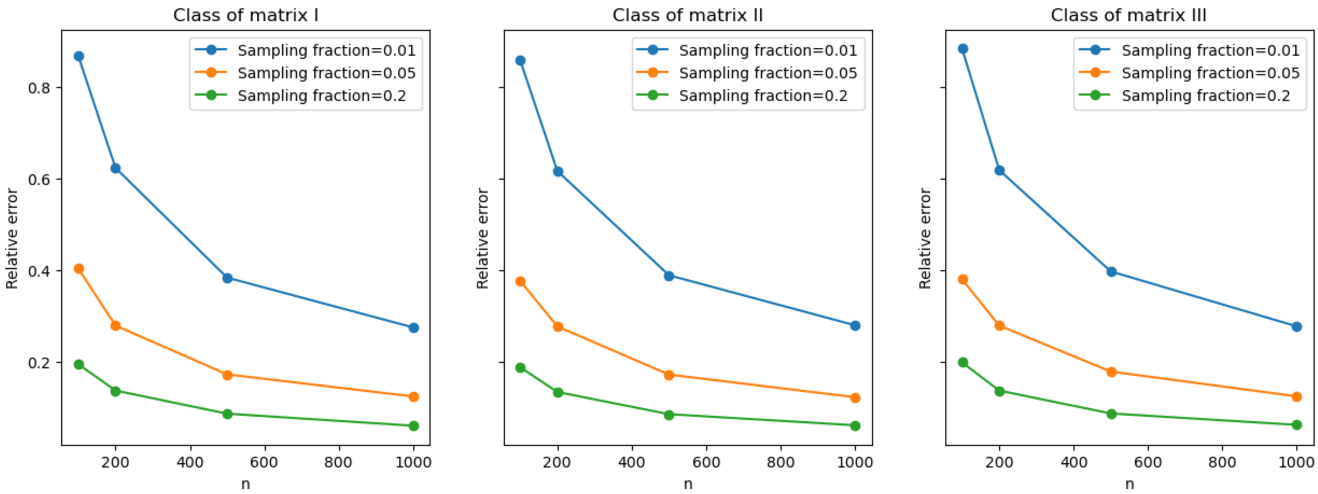
Increasing the size of the matrix, represented by the variable n, increases the number of computations required to perform the matrix multiplication. As a result, increasing n often decreases the relative Frobenius norm error, as the computation becomes more accurate with larger matrix dimensions.

Similarly, increasing the sampling fraction c in the randomized matrix multiplication algorithm leads to more accurate results, as the algorithm samples a larger subset of the matrix. Therefore, increasing c often leads to a decrease in the relative Frobenius norm error.

The theoretical error bound for the relative error E is given by:

$$E <= \frac{C}{\sqrt{c}} \times ((||A||_F * ||B||_F))$$

where C is a constant and $||.||_F$ denotes the Frobenius norm of a matrix. This bound suggests that the relative error decreases as the sampling fraction c increases. It also indicates that the error decreases as the Frobenius norms of A and B increase.



Also, i have given the error table below

Table 1: Relative error array for different matrix types, c_n, and n

Matrix Type	sampling coefficient	n	Relative Error E
Class I	0.01	100	0.86159636
		200	0.61377416
		500	0.38905074
		1000	0.27643675
Class I	0.05	100	0.38507025
		200	0.27708192
		500	0.1755156
		1000	0.12419114
Class I	0.2	100	0.19669564
		200	0.13877824
		500	0.08731363
		1000	0.0616853
Class II	0.01	100	0.86269336
		200	0.6325091
		500	0.39209218
		1000	0.2785395
Class II	0.05	100	0.38615535
		200	0.27201391
		500	0.17220702
		1000	0.1249235
Class II	0.2	100	0.19449027
		200	0.14592841
		500	0.08879419
		1000	0.0624546
Class III	0.01	100	0.87302436
		200	0.60817083
		500	0.39942941
		1000	0.27759464
Class III	0.05	100	0.40288535
		200	0.28107871
		500	0.17317941
		1000	0.12471027
Class III	0.2	100	0.18939274
		200	0.13715452
		500	0.0874742
		1000	0.06248316