Exercise 8.4

Consider the filtration data of Data Set G. Open the Excel workbook **Exe8.4G.xlsx** which contains these data from the Exercises folder.

Assuming the data to be suitably distributed, complete a two-tailed test of whether the population mean impurity differs between the two filtration agents, and interpret your findings.

Data

Batch	Agent1	Agent2
1	7.7	8.5
2	9.2	9.6
3	6.8	6.4
4	9.5	9.8
5	8.7	9.3
6	6.9	7.6
7	7.5	8.2
8	7.1	7.7
9	8.7	9.4
10	9.4	8.9
11	9.4	9.7
12	8.1	9.1

Two-tailed Test

t-Test: Paired Two Sample for Means

	Agent1	*Agent2*
Mean	8.25	8.683333333
Variance	1.059090909	1.077878788
Observations	12	12
Pearson Correlation	0.901055812	
Hypothesized Mean Difference	0	
df	11	
t Stat	-3.263938591	
P(T<=t) one-tail	0.003772997	
t Critical one-tail	1.795884819	
P(T<=t) two-tail	0.007545995	
t Critical two-tail	2.20098516	

Difference in Means	-0.433333333

Analysis

Null Hypothesis (H0): The population mean impurity is the same for both filtration agents ($\mu1 - \mu2 = 0$).
Alternative Hypothesis (H1): The population mean impurity is different for the two filtration agents ($\mu1 - \mu2 \neq 0$).

Where:

$\mu 1$ represents the population mean impurity for Agent1.
$\mu 2$ represents the population mean impurity for Agent2.

Test Statistic and Critical Values:

The calculated t-statistic for Agent 1 is -3.264, and the degrees of freedom (df) is 11.
The one-tailed critical t-value at a significance level of 0.05 is 1.796, and the two-tailed critical t-value is 2.201.

P-Values:

The one-tailed p-value for Agent 1 is 0.004, and the two-tailed p-value is 0.008.

Comparison and Interpretation:

For Agent 1, the calculated t-statistic (-3.264) is much smaller in absolute value than the one-tailed critical t-value (1.796), and the two-tailed p-value (0.008) is less than the significance level (α = 0.05). This indicates strong evidence against the null hypothesis (H0) that the population mean impurity for Agent 1 is the same as Agent 2.
For Agent 2, there is no a specific t-statistic, but we can see that the difference in means between the two agents is -0.433.

Conclusion:

Based on the statistical analysis, we should reject the null hypothesis (H0) that the population mean impurity is the same between the two filtration agents. There is strong evidence to suggest that the population mean impurity differs between Agent 1 and Agent 2.

In summary, Agent 1 appears to have a significantly different mean impurity compared to Agent 2, with Agent 1 having a lower mean impurity. The negative difference in means (-0.433) suggests that Agent 1's impurity is, on average, lower than that of Agent 2.

Exercise 8.5

Recall that in Exercise 8.4, a two-tailed test was undertaken of whether the population mean impurity differs between the two filtration agents in Data Set G.

Suppose instead a one-tailed test had been conducted to determine whether Filter Agent 1 was the more effective. What would your conclusions have been?

Analysis

Null Hypothesis (H0): The population mean impurity for Filter Agent 1 is less than or equal to the population mean impurity for Filter Agent 2 ($\mu1 \leq \mu2$).

Alternative Hypothesis (H1): The population mean impurity for Filter Agent 1 is greater than the population mean impurity for Filter Agent 2 ($\mu1 > \mu2$).

The calculated one-tailed t-statistic for Agent 1 is -3.264.
The one-tailed critical t-value at a significance level of 0.05 is 1.796.

<u>Conclusion:</u>
Since this is a one-tailed test comparing whether Filter Agent 1 is more effective than Filter Agent 2, we would compare the calculated one-tailed t-statistic to the critical t-value for the upper tail of the distribution.

In this case, the calculated t-statistic (-3.264) is much smaller in absolute value than the one-tailed critical t-value (1.796). This indicates strong evidence against the null hypothesis (H0). Therefore, we would reject the null hypothesis and conclude that there is strong statistical evidence that Filter Agent 1 is more effective at reducing impurity compared to Filter Agent 2.

So, in a one-tailed test to determine whether Filter Agent 1 is more effective, the conclusion would be that Filter Agent 1 is indeed more effective at reducing impurity compared to Filter Agent 2.

Exercise 8.6

Consider the bank cardholder data of Data Set C. Open the Excel workbook **Exe8.6C.xlsx** which contains this data from the Exercises folder.

Assuming the data to be suitably distributed, complete an appropriate test of whether the population mean income for males exceeds that of females and interpret your findings. What assumptions underpin the validity of your analysis, and how could you validate them?

F-Test Two-Sample for Variances

	Males	*Females*
Mean	52.91333333	44.23333333
Variance	233.1289718	190.1758192
Observations	60	60
df	59	59
F	1.225860221	
P(F<=f) one-tail	0.21824624	
F Critical one-tail	1.539956607	
p2	0.43649248	

t-Test: Two-Sample Assuming Equal Variances

	Males	*Females*
Mean	52.91333333	44.23333333
Variance	233.1289718	190.1758192
Observations	60	60
Pooled Variance	211.6523955	
Hypothesized Mean Difference	0	
df	118	
t Stat	3.267900001	
P(T<=t) one-tail	0.000709735	
t Critical one-tail	1.657869522	
P(T<=t) two-tail	0.00141947	
t Critical two-tail	1.980272249	
Difference in Means	8.68	

Analysis

F-Test for Variances:

For males: The variance of male income is 233.13.
For females: The variance of female income is 190.18.
The F-statistic is 1.2259.
The one-tailed p-value for the F-test is 0.218.

The F-test is used to compare the variances of the two groups. In this case, it does not indicate a significant difference in variances between males and females. The p-value is greater than the commonly used significance level of 0.05, which suggests that the variances are not significantly different.

t-Test Assuming Equal Variances:

For males: The t-statistic is 3.268 with a one-tailed p-value of 0.0007 (two-tailed p-value of 0.0014).

For females: The t-statistic indicates a difference in means of 8.68.

The t-test is used to compare the means of the two groups. In this case, the t-statistic for males is 3.268, and the p-value is very low (0.0007 for one-tailed and 0.0014 for two-tailed). This suggests that there is a statistically significant difference in the mean incomes between males and females. The difference in means is 8.68, indicating that, on average, males have a higher income than females.

Interpretation:

Based on the analysis, it can be concluded that the mean income for males is significantly higher than that of females. This is supported by the low p-value in the t-test, which indicates a significant difference in means.

Assumptions Underpinning Validity:

Random Sampling: The analysis assumes that the samples of males and females were selected randomly from the population.
Independence: It is assumed that the income data for males and females are independent of each other.
Equal Variances (for t-test): The t-test assumes that the variances of the two groups (males and females) are equal. The F-test is conducted to check this assumption, and in this case, it does not suggest a significant difference in variances.

Validation of Analysis: To validate the analysis, one can consider the following steps:

Collect Additional Data: Collect data from a larger and more representative sample, if possible, to increase the reliability of the findings.
Sensitivity Analysis: Perform sensitivity analysis with different significance levels to ensure the results are robust.
Bootstrap Sampling: Use bootstrap resampling techniques to assess the stability of the results and calculate confidence intervals.
Check Assumptions: Ensure that the assumptions of random sampling, independence, and equal variances are met. If not, apply appropriate statistical methods or transformations.
External Validation: Compare the results with external data or studies on income disparities between genders to see if the findings align with existing research.