RSAConference2018
San Francisco | April 16 – 20 | Moscone Center

SESSION ID: CRYP-F02

CODES AND ISOGENIES

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REVOCABLE IDENTITY-BASED ENCRYPTION FROM CODES WITH RANK METRIC

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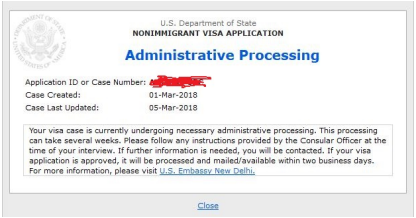
Date: 04/20/2018 Session: CRYP-F02 Time: 10:15 AM - 11:00 AM

Thanks to US Department of State for not providing Nonimmigrant Visa to Dr. Somitra Kumar Sanadhya.


U.S. Department of State
NONIMMIGRANT VISA APPLICATION

Administrative Processing

Application ID or Case Number: [redacted]
Case Created: 01-Mar-2018
Case Last Updated: 05-Mar-2018

Your visa case is currently undergoing necessary administrative processing. This processing can take several weeks. Please follow any instructions provided by the Consular Officer at the time of your interview. If further information is needed, you will be contacted. If your visa application is approved, it will be processed and mailed/available within two business days. For more information, please visit U.S. Embassy New Delhi.

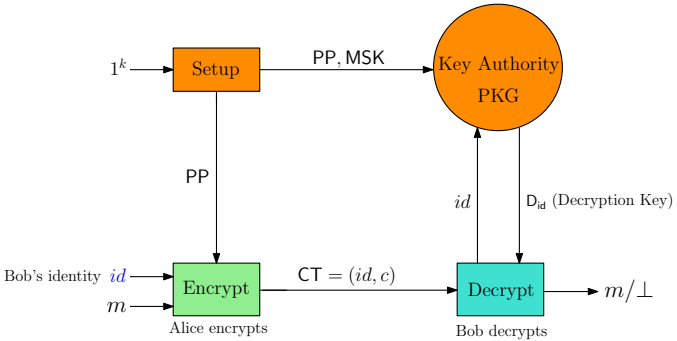
Close

Dr. Reza Azarderakhsh kindly agrees to present our work at CT-RSA 2018.

We would like to thank Dr. Reza Azarderakhsh for accepting our request.

Motivation

Identity-based Crypto: To resolve the problem of key management in standard PKE.



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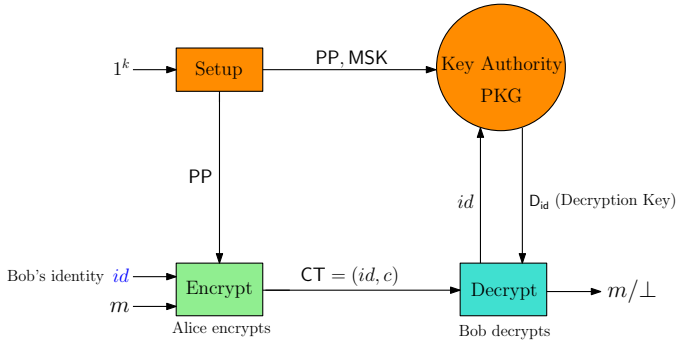
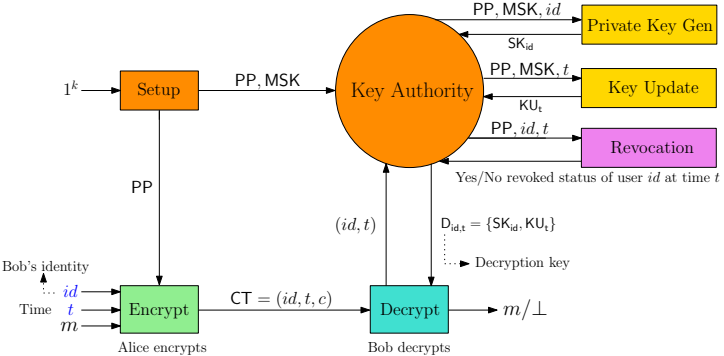


Problem: how to efficiently revoke past users in identity-based encryption (IBE) ?

Revocable IBE (RIBE)

Adding efficient revocation procedure to IBE …



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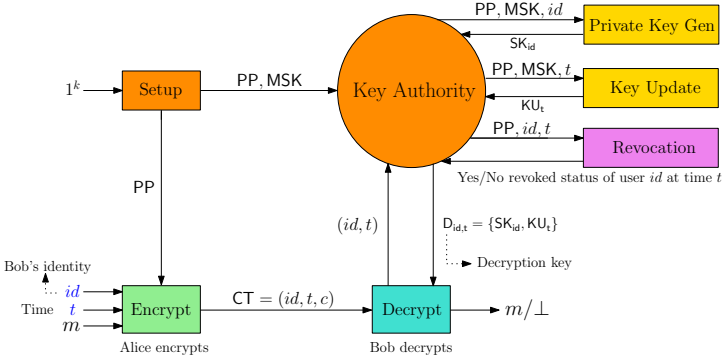
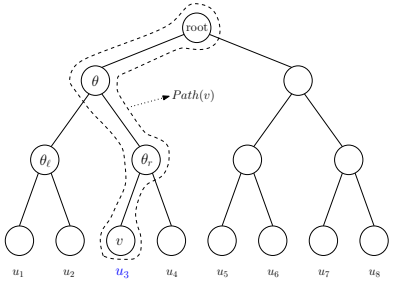


- *Initially, the key update is public to all users. At a later time, stop publishing the key updates for a revoked user !*

Key Generation on a Binary Tree

- Upon registration, the key authority provides the user u_3 with a set of distinct private keys for each node in Path(v) on a binary tree BT.



- To generate the key for a user u, one can choose a random value r
  - split r into two parts r_1 and r_2 for each node
  - generate private-key SK_{id} on (id, r_1) using *private key generation* algorithm
  - do key-updates KU_t on (t, r_2) using *key-updates* algorithm
- The decryption-key is: $\mathsf{DK}_{id,t} = \{\mathsf{SK}_{id}, \mathsf{KU}_t\}$ for a node on a path corresponding to a user u.

How to revoke a user ?

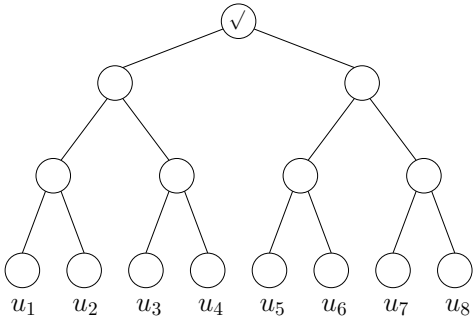
- Run the Revocation algorithm $\mathcal{R}(id^*, t^*, \mathrm{RL})$:
  - adds revoked user id^* to the revocation list RL at revoked time t^*.
- Do the key-updates for non-revoked users (the number of key-updates are logarithmic in the number of users).
- Stop sending key-updates for a revoked user for the time $t \geq t^*$.

How to do Key-Updates for a user ?

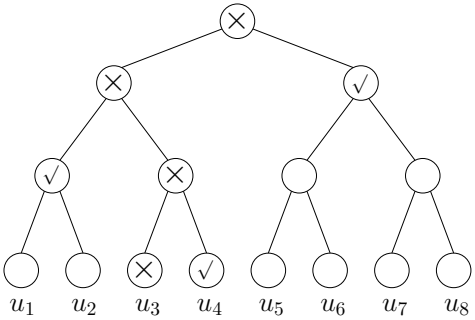
- Run the Key-Update-Nodes algorithm KUNodes(BT, RL, t) :
  - returns a minimal set $Y \subset$ BT of nodes for which key update needs to be published.

KUNodes(BT, RL, t)
$X, Y \leftarrow \phi$
$\forall(v_i, t_i) \in$ RL
if $t_i \leq t$ *then add* Path(v_i) *to* X
$\forall \theta \in X$
if $\theta_\ell \notin X$ *then add* θ_ℓ *to* Y
if $\theta_r \notin X$ *then add* θ_r *to* Y
If $Y = \phi$ *then add* root *to* Y
Return Y

Examples of Key-Updates Algorithm



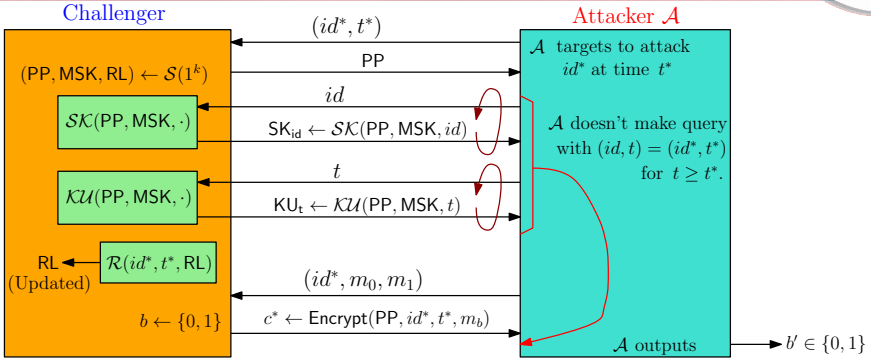
(a) No user is revoked



(b) User u_3 is revoked

✓ shows that the key-updates KU_t is published for those nodes at time t.

Selective-Revocable-ID Security of RIBE



The advantage of $\mathcal{A}$ is defined as

$$\mathsf{Adv}^{\mathsf{ind\text{-}srid\text{-}cpa}}_{\mathcal{A},\mathsf{RIBE}}(k) \quad := \quad \left| \Pr[b' = b] - \frac{1}{2} \right| .$$

A Bit of (R)IBE History

- Boneh and Franklin : *Identity-based Encryption from Weil Pairing*, CRYPTO 2001.
- Boldyreva, Goyal and Kumar : Identity-based Encryption (from Weil pairing) with Efficient Revocation, ACM CCS 2008.
- Agrawal, Boneh and Boyen : *Efficient Lattice (H)IBE in the Standard Model*, EUROCRYPT 2010.
- Chen, Lim, Ling, Wang and Nguyen : *Revocable Identity-Based Encryption from Lattices*, ACISP 2012.
- Gaborit, Hauteville, Phan and Tillich : *Identity-based Encryption from Codes with Rank Metric*, CRYPTO 2017.

Our Work

- We construct Revocable IBE (RIBE) from Codes with Rank Metric.
- Highlights of RIBE :
  - Constructed from Low Rank Parity-Check (LRPC) codes.
  - Master Secret Key (MSK) is defined as the “Trapdoor” generated through the RankSign algorithm[1].
  - Provides IND-sRID-CPA security which relies mainly on the Rank Syndrome Decoding (RSD) problem[1].
  - Binary-Tree data structure[2] is used to add efficient revocation procedure to IBE.

[1]Gaborit et al. *RankSign: An Efficient Signature Algorithm based on the Rank Metric*, PQCrypto 2014.
[2]Boldyreva, Goyal, and Kumar. *Identity-based Encryption with Efficient Revocation*, ACM CCS 2008.

Rank Metric over $\mathbb{F}_{q^m}^n$

- Let $\mathbb{F}_{q^m}$ be a m-dimensional vector space over the field $\mathbb{F}_q$ with prime q.
- Let $\boldsymbol{B} = (b_1, \ldots, b_m)$ be a basis of $\mathbb{F}_{q^m}$.
- Let $\boldsymbol{x} = (x_1, \ldots, x_n) \in \mathbb{F}_{q^m}^n$, then each coordinate x_j can be expressed in terms of basis $\boldsymbol{B}$ as :

$$x_j = \sum_{i=1}^{m} m_{ij}\, b_i \qquad \text{where } m_{ij} \in \mathbb{F}_q$$

- The $m \times n$ matrix associated with $\boldsymbol{x}$ is defined as $\boldsymbol{M}(\boldsymbol{x}) = (m_{ij})_{1 \le i \le m,\, 1 \le j \le n}$.
- *Rank weight* of $\boldsymbol{x}$ is defined as

$$\|\boldsymbol{x}\| = \mathit{Rank}\ \boldsymbol{M}(\boldsymbol{x})$$

- *Rank distance (metric)* $d(\boldsymbol{x}, \boldsymbol{y})$ between elements $\boldsymbol{x}$ and $\boldsymbol{y}$ in $\mathbb{F}_{q^m}^n$ is defined as

$$d(\boldsymbol{x}, \boldsymbol{y}) = \|\boldsymbol{x} - \boldsymbol{y}\|$$

Rank Code

- A linear code $\mathcal{C}$ of dimension k and length n is a subspace of dimension k of $\mathbb{F}_{q^m}^n$ embedded with the rank metric.
- $\mathcal{C}$ can be represented by two ways :
  - by a generator matrix $\boldsymbol{G} \in \mathbb{F}_{q^m}^{k \times n}$. Each rows of $\boldsymbol{G}$ is an element of a basis of $\mathcal{C}$,

$$\mathcal{C} = \{\boldsymbol{x}\boldsymbol{G} \mid \boldsymbol{x} \in \mathbb{F}_{q^m}^k\}$$

  - by a parity-check matrix $\boldsymbol{H} \in \mathbb{F}_{q^m}^{(n-k) \times n}$. Each rows of $\boldsymbol{H}$ determines a parity-check equation verified by its elements of $\mathcal{C}$:

$$\mathcal{C} = \{\boldsymbol{x} \in \mathbb{F}_{q^m}^n \mid \boldsymbol{H}\boldsymbol{x}^T = \boldsymbol{0}\}$$

Low Rank Parity-Check (LRPC) Codes

LRPC Codes

- Let $\boldsymbol{H} = (h_{ij})_{1 \le i \le n-k,\, 1 \le j \le n} \in \mathbb{F}_{q^m}^{(n-k) \times n}$ be a full rank matrix.
- All the elements of $\boldsymbol{H}$ generate an $\mathbb{F}_q$-subspace F of dimension d:

$$F = \langle h_{ij} \rangle_{\mathbb{F}_q}$$

- The code $\mathcal{C}[n, k, d]_{q^m}$ with parity-check matrix $\boldsymbol{H}$ is called a LRPC code of weight d.

Such a matrix $\boldsymbol{H}$ is called a homogeneous matrix of weight d.

Augmented Low Rank Parity-Check Codes

Augmented LRPC Codes

- Let $\boldsymbol{H} \in \mathbb{F}_{q^m}^{(n-k)\times n}$ be a homogeneous matrix of full rank and of weight d.
- $\boldsymbol{R} \in \mathbb{F}_{q^m}^{(n-k)\times \ell}$ be a random matrix.
- $\boldsymbol{P} \in GL_{n-k}(\mathbb{F}_{q^m})$ and $Q \in GL_{n+\ell}(\mathbb{F}_q)$ be two invertible matrices.
- $\boldsymbol{H'} = \boldsymbol{P}(\boldsymbol{R}|\boldsymbol{H})\boldsymbol{Q}$ be the parity-check matrix of a code $\mathcal{C}$ of type $[n+\ell, k+\ell]$.

We call such a code, an LRPC$^+$ code. If $\ell = 0$, then $\mathcal{C}$ is a LRPC code.

Definition (LRPC$^+$ Problem)

Given an LRPC$^+$ code, distinguish it from a random code with the same parameters.

Rank Syndrome Decoding Problem

Definition (Rank Syndrome Decoding (RSD) Problem)

Instance : a parity-check matrix $\boldsymbol{H}$ in $\mathbb{F}_{q^m}^{(n-k)\times n}$, a syndrome $\boldsymbol{s}$ in $\mathbb{F}_{q^m}^{n-k}$ and an integer w.

Question : does there exist $\boldsymbol{x} \in \mathbb{F}_{q^m}^{n}$ such that $\boldsymbol{H}.\boldsymbol{x}^{\mathsf{T}} = \boldsymbol{s}$ and $w_R(\boldsymbol{x}) \leq w$?

Definition (Decisional Rank Syndrome Decoding (DRSD) Problem)

Instance : a generator matrix $\boldsymbol{G}$ in $\mathbb{F}_{q^m}^{k\times n}$, $\boldsymbol{m} \in \mathbb{F}_{q^m}^{k}$ and $\boldsymbol{x} \in \mathbb{F}_{q^m}^{n}$ of weight w.

Question : can we distinguish the pair $(\boldsymbol{G}, \boldsymbol{mG} + \boldsymbol{x})$ from $(\boldsymbol{G}, \boldsymbol{y})$ with $\boldsymbol{y} \xleftarrow{\$} \mathbb{F}_{q^m}^{n}$?

Rank Support Learning Problem

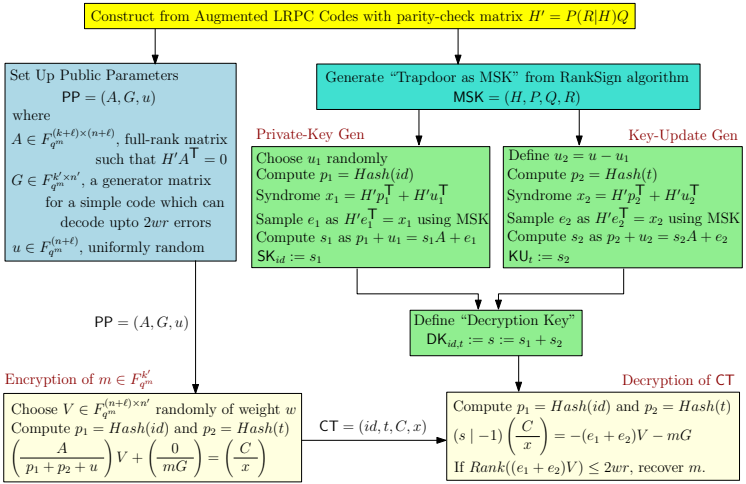
Definition (Rank Support Learning (RSL) Problem)

- Let $\boldsymbol{A}$ be a random full-rank matrix of size $(n-k) \times n$ over $\mathbb{F}_{q^m}$ and U be a subspace of $\mathbb{F}_{q^m}$ of dimension w.
- Let $\mathcal{O}$ be an oracle which gives samples of the form $(\boldsymbol{A}, \boldsymbol{Av})$, where $\boldsymbol{v} \xleftarrow{\$} U^n$.
- The RSL **problem** is to recover $\boldsymbol{V}$ given only access to the oracle. That is,

$$\Pr[\mathcal{A}(\boldsymbol{A}, \boldsymbol{AV}) = \boldsymbol{V}] \leq \epsilon, \quad \boldsymbol{V} \xleftarrow{\$} U^{n \times N}$$

  where N be the number of queries made to the oracle $\mathcal{O}$.
- The DRSL **problem** is to distinguish $(\boldsymbol{A}, \boldsymbol{AV})$ from $(\boldsymbol{A}, \boldsymbol{Y})$ with $\boldsymbol{Y} \xleftarrow{\$} \mathbb{F}_{q^m}^{(n-k) \times N}$.

Overview of RIBE from LRPC Codes



Security of RIBE

Theorem

Suppose there exists an adversary $\mathcal{A}$ against the IND-sRID-CPA security, who makes at most q_{H_1} and q_{H_2} distinct queries to the H_1 and H_2 random oracles, then the advantage of adversary $\mathcal{A}$ is given by the following expression

$$\epsilon_{\mathsf{ribe}} \leq \left(q_{H_1} + q_{H_2}\right) \cdot \left(\frac{2}{q} + \epsilon_{\mathsf{drsd}}\right) + \epsilon_{\mathsf{lrpc}^+} + \epsilon_{\mathsf{drsl}},$$

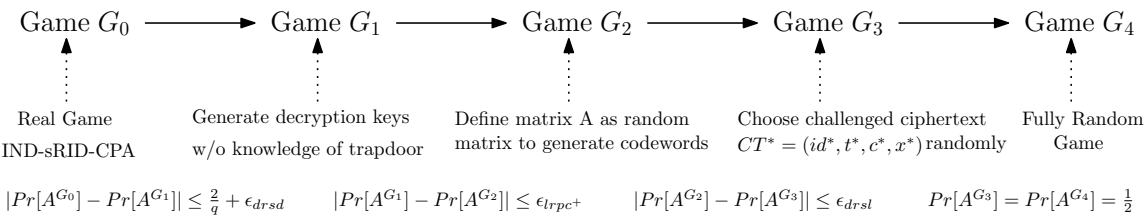
where ϵ_{ribe}, ϵ_{drsd}, ϵ_{drsl} and $\epsilon_{\mathsf{lrpc}^+}$ are respectively the bound on the advantage of the attacks against the RIBE system, the DRSD, DRSL and LRPC$^+$ problems.

Proof Sketch (Selective-Revocable-ID security of RIBE)

Suppose $\mathcal{A}$ be a probabilistic polynomial time adversary who wishes to break the security of RIBE in selective-revocable-ID security.

Goal : To show that a PPT adversary $\mathcal{A}$ has a negligible advantage in winning the original IND-sRID-CPA game.

Transitions of Games:



Practical set of Parameters

We choose the parameters of the simple code in such a way that it can decode up to $2wr$ errors and the decoding error with failure probability $\approx \frac{1}{q^{\ell'-2wr+1}}$ is small.

Scheme	n	$n-k$	m	q	d	ℓ	r	d_{GV}	d_{sign}
IBE[3]	100	20	96	2^{192}	5	12	16	11	20
RIBE	100	20	96	2^{192}	5	12	16	11	20

Scheme	Public Key Size (Bytes)	n'	k'	ℓ'	w
IBE[5]	4,239,360 of $\boldsymbol{A}$	96	9	66	4
RIBE	4,497,408 of $(\boldsymbol{A}, \boldsymbol{u})$	96	9	66	2

With the above parameters, one can achieve decoding failure probability $\approx 2^{-576}$.

[3] Gaborit et al.: *Identity-based Encryption from Codes with Rank Metric*, CRYPTO 2017.

Conclusion and Future Work

- We built the revocable IBE from codes with rank metric.
- We formally proved the selective-ID CPA security of revocable IBE in the random oracle model.

- As an extension of this work, one might think of constructing Revocable IBE with adaptive-ID CCA security, which is a strongest security model.
- Another extension could be a scheme secure in standard model.

THANK YOU
FOR YOUR
ATTENTION

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AN EXPOSURE MODEL FOR SUPERSINGULAR ISOGENY DIFFIE-HELLMAN KEY EXCHANGE

Brian Koziel, Reza Azarderakhsh, David Jao

Assistant Professor
Florida Atlantic University

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- RSA: Protected by the factorization and discrete logarithm problems
- Large-scale quantum computers with Shor's algorithm will BREAK the security assumptions for these primitives

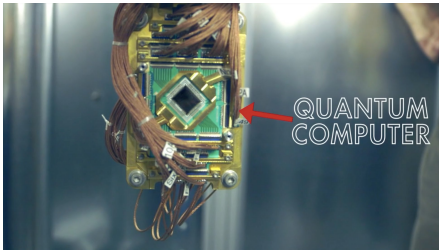

Figure: Quantum Computer (The Verge)

NIST has started a PQC standardization process

Primary Post-Quantum Cryptography (PQC) Candidates:

- Code-Based: McEliece

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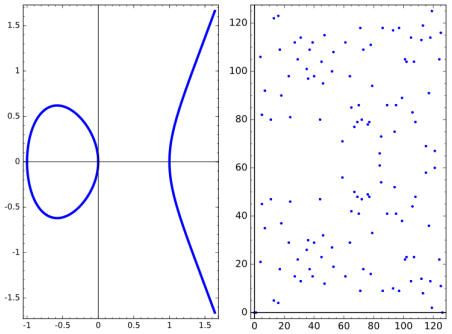

Figure: $E : y^2 = x^3 - x$ Left: $E/\mathbb{Z}$ Right: $E/\mathbb{F}_{127}$

SIDH offers the smallest key sizes of known quantum-resistant algorithms

Table: Comparison of different post-quantum key exchange and encryption algorithms at 128-bit quantum security level. Key sizes are in Bytes

Algorithm	NTRU	New Hope	McBits	SIDH	Compressed SIDH
Type	Lattice	Ring-LWE	Code	Isogeny	Isogeny
Public Key	6,130	2,048	1,046,739	576	336
Private Key	6,743	2,048	10,992	48	48
Performance	Slow	Very Fast	Slow	Very Slow	Very Slow

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- Small key sizes reduce transmission cost and storage requirement

Why would I want to use SIDH or SIKE as a quantum alternative?

Pros :)

- Very small public/private keys
- Implementations resemble ECC
- Security based on supersingular isogeny problem
- SIKE: IND-CCA KEM alternative to SIDH $\rightarrow$ static keys can be reused!
- No possibility for decryption error
- No complicated error distributions, rejection sampling, etc.
- Conservative security analysis when assuming generic attacks

Cons :(

- Newest candidate for PQC applications
- Very slow
- SIDH has security concerns if keys are reused

Isogeny-Based Cryptography History

- 1996: Couveignes - First mention of isogenies in cryptography. Published in 2006

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- 2017: Jao et al. - Supersingular Isogeny Key Encapsulation (SIKE) submitted to NIST PQC process

Isogeny-Based Cryptography underlying security

Consider two supersingular elliptic curves defined over a large prime extension field

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Supersingular Isogeny Problem

Given $P, Q \in E_1$ and $\phi(P), \phi(Q) \in E_2$, compute the secret isogeny, ϕ

Isogeny-Based Cryptography underlying security

Consider two supersingular elliptic curves defined over a large prime extension field

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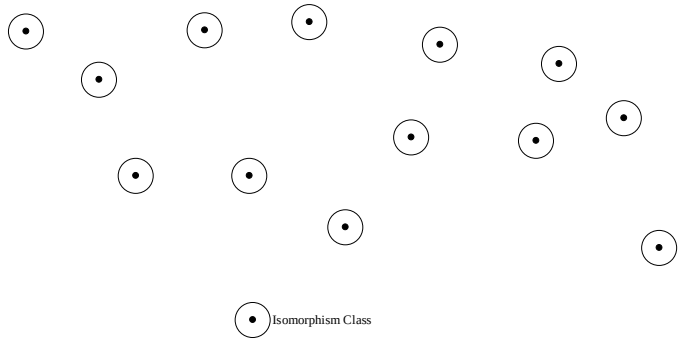
  - Quantum attack $O(p^{1/6})$

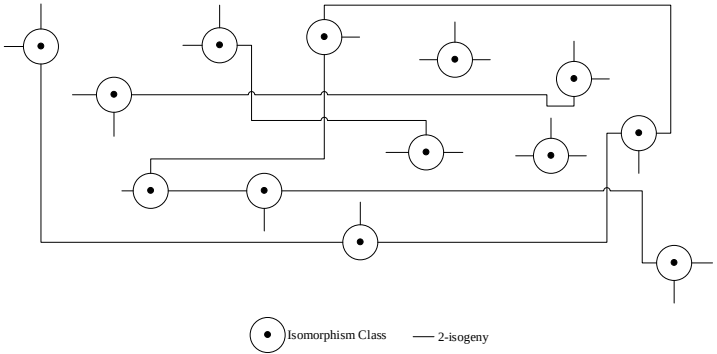
Visualizing the Supersingular Isogeny Problem

Consider a graph where each node represents supersingular isomorphism classes



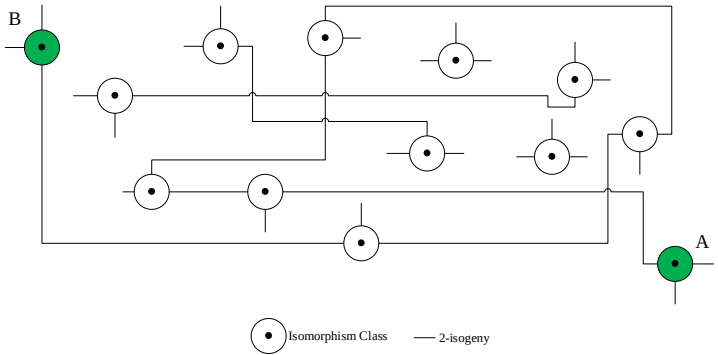
Visualizing the Supersingular Isogeny Problem

For $\ell = 2$, each node is connected by three unique 2-isogenies



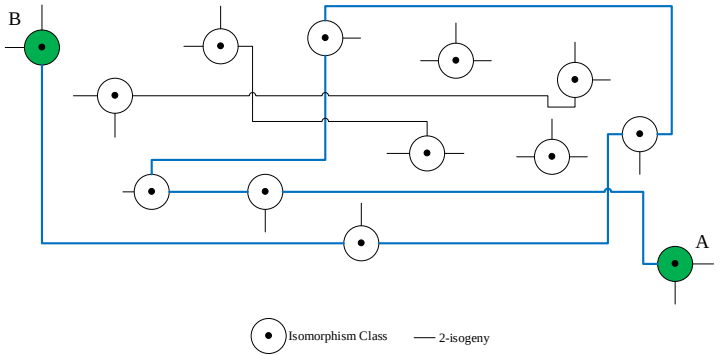
Visualizing the Supersingular Isogeny Problem

Consider finding an isogeny from Class A to Class B when $\ell = 2$



Visualizing the Supersingular Isogeny Problem

For large isogeny graphs (i.e., *p* is 512 bits or more), finding an isogeny path is HARD.



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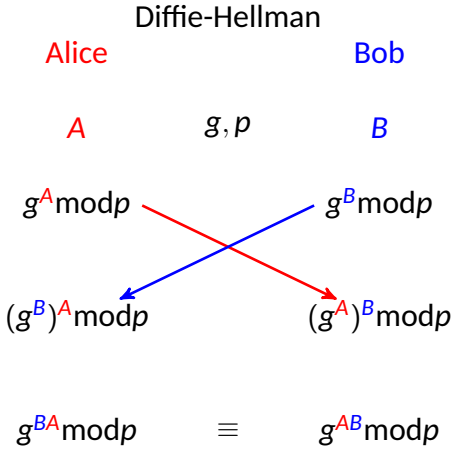

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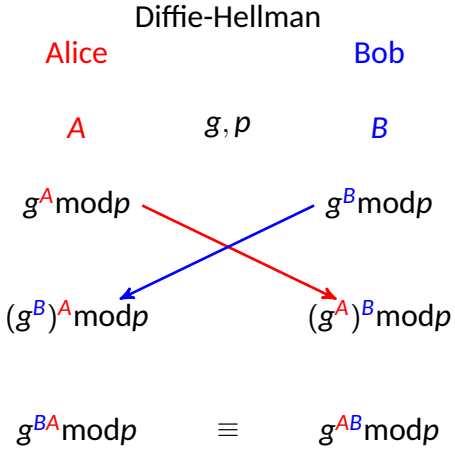

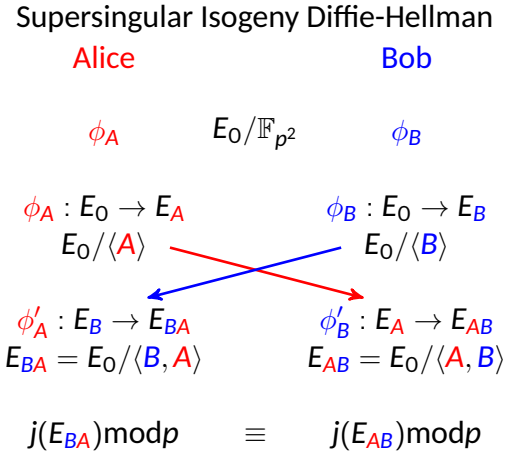

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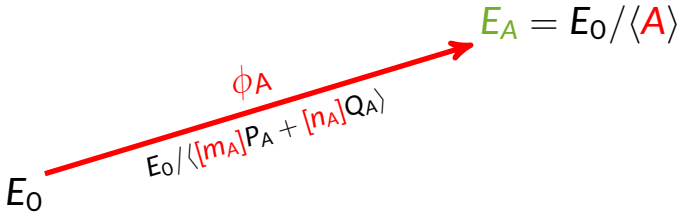

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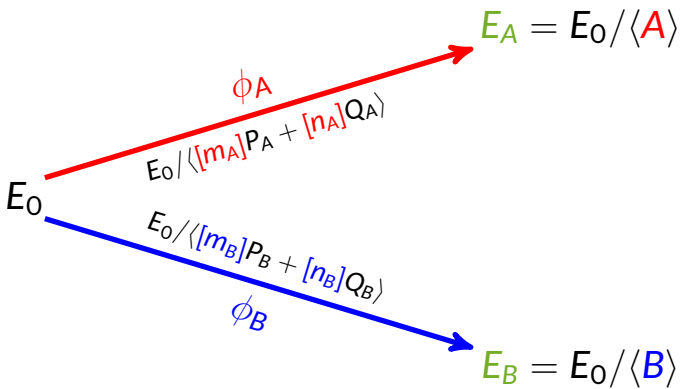

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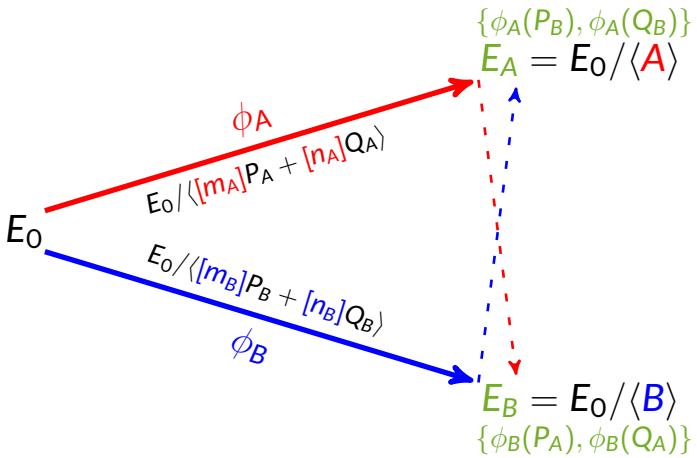

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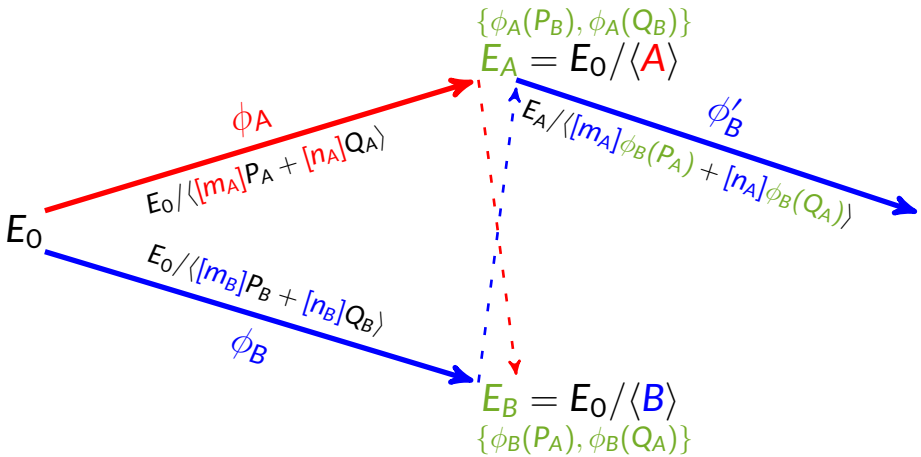

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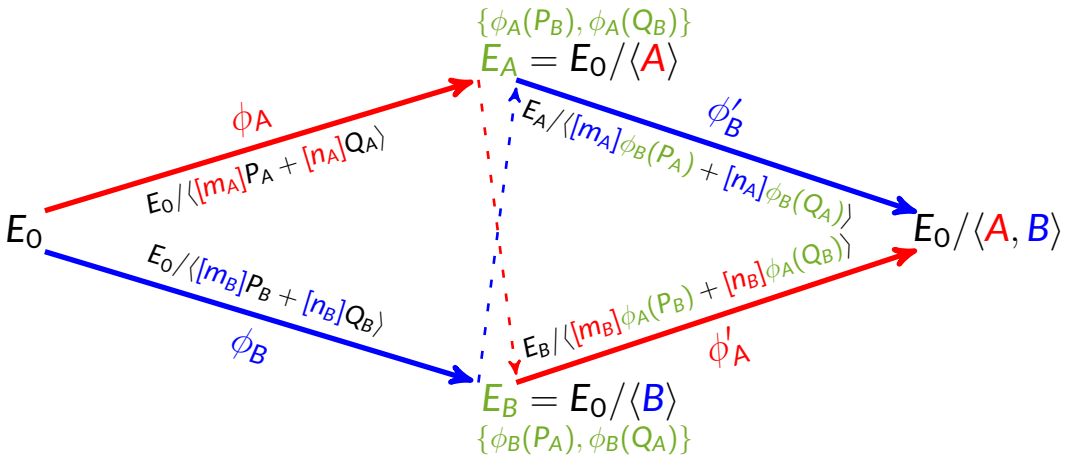


SIDH Computations

Secret Kernel Generation

- Inputs:
  - Supersingular elliptic curve $E(\mathbb{F}_{p^2})$, torsion basis $\{P,Q\}$, private keys m, n
- Compute $R = \langle [m]P + [n]Q \rangle$

Large-Degree Isogeny

- Inputs:
  - Supersingular elliptic curve E, secret kernel point R
- Compute $\phi : E \to E/\langle R \rangle$ by iteratively computing isogenies

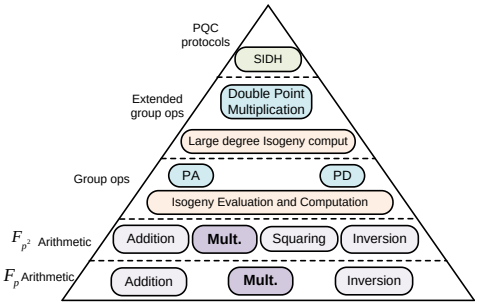
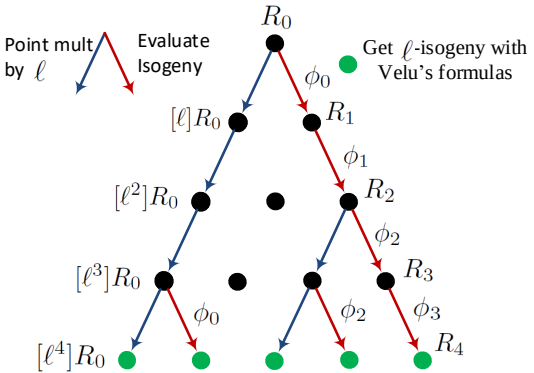


Figure: Breakdown of supersingular isogeny computations

Visualizing the large-degree isogeny computation

- Large-degree isogenies can be computed by iteratively computing small-degree isogenies
- Set $E_0 = E$ and $R_0 = ker(\phi)$
- Find kernel point $\ker(\phi_i) = \ell^{e-i-1}R_i$
- Compute ith isogeny $\phi_i : E_i \to E_i/\langle \ell^{e-i-1}R_i \rangle = E_{i+1}$
- Push kernel point to new curve $R_{i+1} = \phi_i(R_i)$



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- Necessary to account for weak implementations or new attacks
- Looking specifically at the large-degree isogeny

Why might intermediate values be exposed?

- Poor implementation
- New attacks on large-degree isogeny
- Cache prime and probe
- Spectre and Meltdown
- Intermediate values not cleared
- Unexpected reset
- etc. etc.

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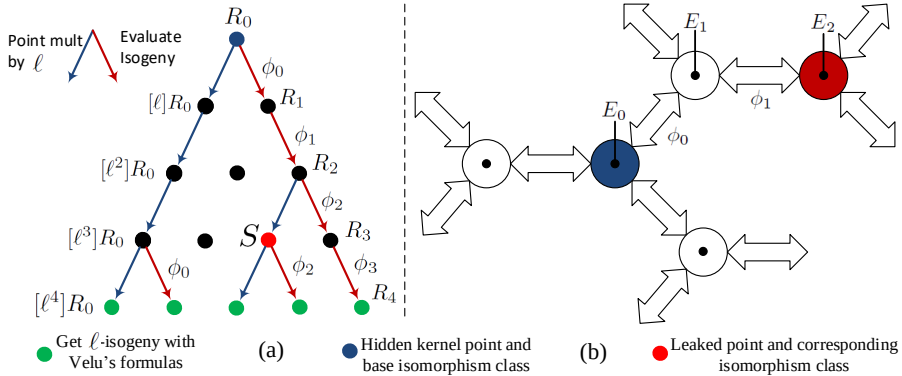
  - If corresponding curve is found, then isogeny decisions are revealed
  - After the corresponding curve has been found, this situation resembles the intermediate curve scenario

Visualization of an exposed kernel point

- With an exposed kernel point (CLASS 2), an attacker can find the corresponding curve and compute the remaining isogenies that compose the secret isogeny



An exposed kernel point is a disaster as it can be used to retrieve private keys

General attack procedure for point after k ℓ-isogenies and j point multiplications by ℓ
Intermediate kernel point is of the form $S = \phi_{k-1:0}([\ell^j m]P + [\ell^j n]Q)$ on curve E_k (CLASS 2). Original secret kernel point is of the form $R = [m]P + [n]Q$

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4. Perform exhaustive search on the point multiples j for the rest of the key (difficulty $O(\ell^j)$)
   - Remaining secret key bits is some secret multiple of the point order: $m' \equiv x \times m \bmod \ell^j$

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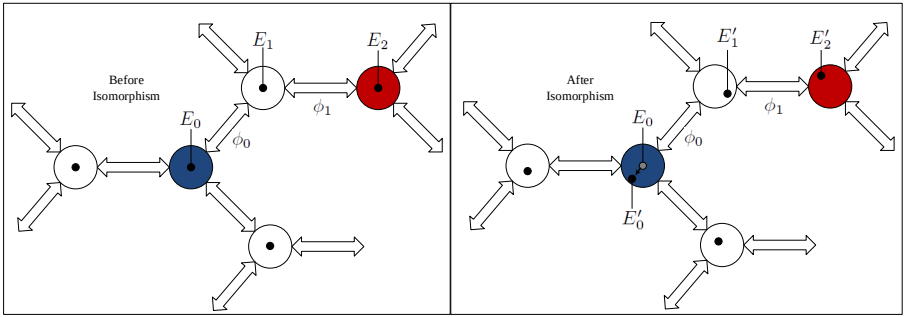
  - This scaling changes the output curves of isogenies and obfuscates any points that are exposed

Visualization of a pre-isogeny isomorphism

- The pre-isogeny isomorphism obfuscates any exposed points throughout the isogeny operation



Visualization of a pre-isogeny isomorphism

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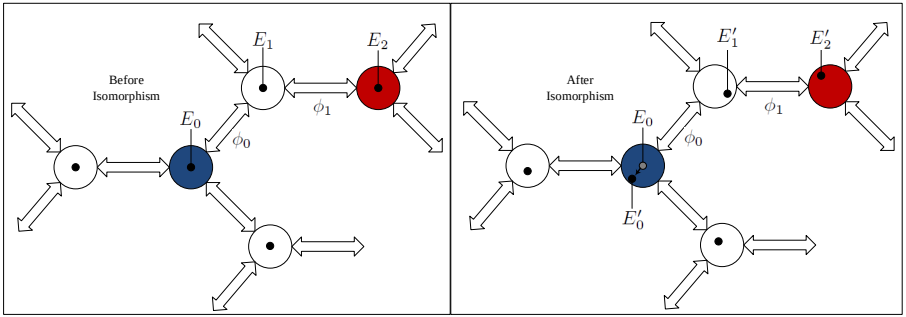
- Protects against CLASS 2 and CLASS 3 exposures



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- Major cost is generating random numbers

Table: Cost of Pre-isogeny Isomorphism

Protocol	r	δ	I	M	S
SIDH Round 1	1	0	1	9	3
SIDH Round 2	1	0	1	5	3
SIDH Indirect Key Validation (Kirkwood et al. Validiation)	1	4	1	15	5

- Let r be the cost to generate a random number, I be a finite-field inversion, M be a finite-field multiplication, S be a finite-field squaring, and δ be a finite-field comparison

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- Thank you very much for your attention. Questions?