JEE Type Solved Examples :
Single Option Correct Type Questions

- This section contains **10 multiple choice examples.** Each example has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Ex. 1 *If z_1 and z_2 be the n th root of unity which subtend right angled at the origin. Then, n must be of the form*

(a) $4k+1$ (b) $4k+2$ (c) $4k+3$ (d) $4k$

Sol. (*d*) The nth roots of unity is given by

$$\cos\left(\frac{2r\pi}{n}\right)+i\sin\left(\frac{2r\pi}{n}\right)=e^{2r\pi i/n},$$

where $r=0,1,2,\ldots,(n-1)$

So, let $z_1=e^{2\pi r_1 i/n}$ and $z_2=e^{2\pi r_2 i/n}$, where $0\le r_1, r_2<n$, $r_1\ne r_2$.

It is given that the line segment joining the points having affixes z_1 and z_2, subtends a right angled at the origin. Therefore,

$$\arg\left(\frac{z_1}{z_2}\right)=\pm\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{2\pi r_1}{n}-\frac{2\pi r_2}{n}=\pm\frac{\pi}{2}$$

$$\Rightarrow \quad n=\pm 4(r_1-r_2)$$

$$\therefore \quad n=4k, \text{ where } k=\pm(r_1-r_2)$$

Ex. 2 *If $|z|=1$ and $\omega=\dfrac{z-1}{z+1}$ (where $z\ne-1$), then $\mathrm{Re}(\omega)$ is*

(a) 0 (b) $-\dfrac{1}{|z+1|^2}$

(c) $\left|\dfrac{z}{z+1}\right|\cdot\dfrac{1}{|z+1|^2}$ (d) $\dfrac{\sqrt{2}}{|z+1|^2}$

Sol. (*a*) We have, $|z|=1$.

Let $z=e^{i\theta}$, where $\theta\in R$ and $i=\sqrt{-1}$.

Then, $\omega=\dfrac{z-1}{z+1}=\dfrac{e^{i\theta}-1}{e^{i\theta}+1}=i\tan\left(\dfrac{\theta}{2}\right)$

$\therefore \quad \mathrm{Re}(\omega)=0$

Ex. 3 *If a, b, c, a_1, b_1 and c_1 are non-zero complex numbers satisfying $\dfrac{a}{a_1}+\dfrac{b}{b_1}+\dfrac{c}{c_1}=1+i$ and $\dfrac{a_1}{a}+\dfrac{b_1}{b}+\dfrac{c_1}{c}=0$, where $i=\sqrt{-1}$, the value of $\dfrac{a^2}{a_1^2}+\dfrac{b^2}{b_1^2}+\dfrac{c^2}{c_1^2}$ is*

(a) $2i$ (b) $2+2i$ (c) 2 (d) None of these

Sol. (*a*) We have, $\dfrac{a}{a_1}+\dfrac{b}{b_1}+\dfrac{c}{c_1}=1+i$

On squaring both sides, we get

$$\frac{a^2}{a_1^2}+\frac{b^2}{b_1^2}+\frac{c^2}{c_1^2}+2\left(\frac{ab}{a_1b_1}+\frac{bc}{b_1c_1}+\frac{ca}{c_1a_1}\right)=1+i^2+2i$$

$$\Rightarrow \quad \frac{a^2}{a_1^2}+\frac{b^2}{b_1^2}+\frac{c^2}{c_1^2}+2\frac{abc}{a_1b_1c_1}\left(\frac{c_1}{c}+\frac{a_1}{a}+\frac{b_1}{b}\right)=1-1+2i$$

$$\Rightarrow \quad \frac{a^2}{a_1^2}+\frac{b^2}{b_1^2}+\frac{c^2}{c_1^2}+0=2i$$

$$\therefore \quad \frac{a^2}{a_1^2}+\frac{b^2}{b_1^2}+\frac{c^2}{c_1^2}=2i$$

Ex. 4 *Let z and ω be complex numbers. If $\mathrm{Re}(z)=|z-2|$, $\mathrm{Re}(\omega)=|\omega-2|$ and $\arg(z-\omega)=\dfrac{\pi}{3}$, the value of $\mathrm{Im}(z+\omega)$ is*

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{2}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\dfrac{4}{\sqrt{3}}$

Sol. (*d*) Let $z=x+iy,\ x,y\in R$ and $i=\sqrt{-1}$

$\because \quad \mathrm{Re}(z)=|z-2|$

$\Rightarrow \quad x=\sqrt{(x-2)^2+y^2}$

$\Rightarrow \quad y^2=4(x-1)$

$\therefore z=1+t^2+2ti$, parametric form and let $\omega=p+iq$

Similarly, $\omega=1+s^2+2si$

$\therefore \quad z-\omega=(t^2-s^2)+2i(t-s)$

$\Rightarrow \quad \arg(z-\omega)=\dfrac{\pi}{3}$ [given]

$\therefore \quad \tan^{-1}\left(\dfrac{2}{t+s}\right)=\dfrac{\pi}{3} \Rightarrow \dfrac{2}{t+s}=\sqrt{3}$

$\Rightarrow \quad (t+s)=\dfrac{2}{\sqrt{3}}$

Now, $z+\omega=2+t^2+s^2+2i(t+s)$

$\therefore \quad \mathrm{Im}(z+\omega)=2(t+s)=\dfrac{4}{\sqrt{3}}$

Ex. 5 *The mirror image of the curve $\arg\left(\dfrac{z-3}{z-i}\right)=\dfrac{\pi}{6}$, $i=\sqrt{-1}$ in the real axis, is*

(a) $\arg\left(\frac{z+3}{z+i}\right)=\frac{\pi}{6}$ (b) $\arg\left(\frac{z-3}{z+i}\right)=\frac{\pi}{6}$

(c) $\arg\left(\frac{z+i}{z+3}\right)=\frac{\pi}{6}$ (d) $\arg\left(\frac{z+i}{z-3}\right)=\frac{\pi}{6}$

Sol. (*d*) $\because$ The image of z in the real axis is $\bar{z}$.

The image is given by

$$\arg\left(\frac{\bar{z}-3}{\bar{z}-i}\right)=\frac{\pi}{6}$$

$$\Rightarrow \quad -\arg\left(\frac{z-3}{z+i}\right)=\frac{\pi}{6} \qquad [\because \arg(\bar{z})=-\arg(z)]$$

$$\Rightarrow \quad \arg\left(\frac{z+i}{z-3}\right)=\frac{\pi}{6} \qquad \left[\because \arg\left(\frac{z_1}{z_2}\right)=-\arg\left(\frac{z_2}{z_1}\right)\right]$$

• Ex. 6 *The mirror image of the curve* $\arg\left(\frac{z+i}{z-1}\right)=\frac{\pi}{4}$, $i=\sqrt{-1}$ *in the line* $x-y=0$, *is*

(a) $\arg\left(\frac{z+i}{z+1}\right)=\frac{\pi}{4}$ (b) $\arg\left(\frac{z+1}{z-i}\right)=\frac{\pi}{4}$

(c) $\arg\left(\frac{z-i}{z+1}\right)=\frac{\pi}{4}$ (d) $\arg\left(\frac{z+i}{z-1}\right)=\frac{\pi}{4}$

Sol. (*c*) $\because$ The image of z in the line $y=x$ is $i\bar{z}$.

$\therefore$ The image of the given curve is

$$\arg\left(\frac{i\bar{z}+i}{i\bar{z}-1}\right)=\frac{\pi}{4}$$

$$\Rightarrow \quad \arg\left(\frac{\bar{z}+1}{\bar{z}+i}\right)=\frac{\pi}{4}$$

$$\Rightarrow \quad -\arg\left(\frac{z+1}{z-i}\right)=\frac{\pi}{4} \qquad [\because \arg(\bar{z})=-\arg(z)]$$

$$\Rightarrow \quad \arg\left(\frac{z-i}{z+1}\right)=\frac{\pi}{4} \quad \left[\because \arg\left(\frac{z_1}{z_2}\right)=-\arg\left(\frac{z_2}{z_1}\right)\right]$$

• Ex. 7 *If* $z+\frac{1}{z}=1$ *and* $a=z^{2017}+\frac{1}{z^{2017}}$ *and b is the last digit of the number* $2^{2^n}-1$, *when the integer* $n>1$, *the value of* a^2+b^2 *is*

(a) 23 (b) 24 (c) 26 (d) 27

Sol. (c) $\because z+\frac{1}{z}=1 \Rightarrow z^2-z+1=0$

$$\therefore \quad z=\frac{-(-1)\pm\sqrt{(1-4)}}{2}=-\omega,-\omega^2$$

[ω is cube root of unity]

and $z^{2017}=(-\omega)^{2017}=-\omega$

$z^{2017}=(-\omega^2)^{2017}=-\omega^2$

$$\therefore \quad a=z^{2017}+\frac{1}{z^{2017}}=-\left(\omega+\frac{1}{\omega}\right)=-(\omega+\omega^2)=1$$

and $2^{2^n}=2^{4\cdot 2^{n-4}}=16^{2^{n-4}}$ has last digit 6.

$\therefore \quad b=6-1=5$

Hence, $a^2+b^2=1^2+5^2=26$

• Ex. 8 *If* ω *is complex cube root of unity and a, b, c are such that* $\frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}=2\omega^2$ *and* $\frac{1}{a+\omega^2}+\frac{1}{b+\omega^2}+\frac{1}{c+\omega^2}=2\omega$, *then the value of* $\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}$ *is equal to*

(a) -2 (b) -1 (c) 1 (d) 2

Sol. (*d*) $\because \frac{1}{a+\omega}+\frac{1}{b+\omega}+\frac{1}{c+\omega}=2\omega^2=\frac{2}{\omega}$

and $\frac{1}{a+\omega^2}+\frac{1}{b+\omega^2}+\frac{1}{c+\omega^2}=2\omega=\frac{2}{\omega^2}$

It is clear that, ω and ω^2 are the roots of the equation

$$\frac{1}{a+x}+\frac{1}{b+x}+\frac{1}{c+x}=\frac{2}{x} \qquad \text{...(i)}$$

$\Rightarrow \quad x\sum(b+x)(c+x)=2(a+x)(b+x)(c+x)$

$\Rightarrow \quad x^3-(ab+bc+ca)x-2abc=0$

$\because$ Coefficient of $x^2=0$, the sum of roots $=0$

$\Rightarrow \quad \alpha+\omega+\omega^2=0 \Rightarrow \alpha-1=0$

$\therefore \quad \alpha=1$

$\therefore$ Third root is 1.

From Eq. (i), we get

$$\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}=2$$

• Ex. 9 *If a, b and c are distinct integers and* $\omega(\neq 1)$ *is a cube root of unity, then the minimum value of* $|a+b\omega+c\omega^2|+|a+b\omega^2+c\omega|$, *is*

(a) $2\sqrt{3}$ (b) 3 (c) $4\sqrt{2}$ (d) 2

Sol. (*a*) Let $z=a+b\omega+c\omega^2$. Then,

$$|z|^2=z\bar{z}=(a+b\omega+c\omega^2)(a+b\bar{\omega}+c\overline{\omega^2})$$

$$=(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$$

$$=a^2+b^2+c^2-ab-bc-ca$$

$$=\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$$

$$\Rightarrow |z|^2\geq\frac{1}{2}\times 6=3 \qquad \left[\begin{array}{l}\because a\neq b\neq c\\ \therefore |a-b|\geq 1, |b-c|\geq 1\\ \text{and } |a-c|\geq 2\end{array}\right]$$

$\therefore \quad |z| \geq \sqrt{3}$

$\therefore \quad |a + b\omega + c\omega^2| + |a + b\omega^2 + c\omega|$

$$= |a + b\omega + c\omega^2| + |a + b\overline{\omega^2} + c\overline{\omega}|$$
$$= |a + b\omega + c\omega^2| + |a + b\omega + c\omega^2|$$
$$= 2|a + b\omega + c\omega^2| = 2|z| \geq 2\sqrt{3}$$

Hence, the minimum value of $|a + b\omega + c\omega^2| + |a + b\omega^2 + c\omega|$ is $2\sqrt{3}$.

Ex. 10 *If $|z - 2i| \leq \sqrt{2}$, where $i = \sqrt{-1}$, then the maximum value of $|3 - i(z - 1)|$, is*

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $2 + \sqrt{2}$ (d) $3 + 2\sqrt{2}$

Sol. (*b*) $\because \quad |3 - i(z - 1)| = |-i(z - 1 - 3i)| = |-i||z - 1 - 3i|$

$$= |z - 1 - 3i| \quad \text{...(i)}$$

and $|z - 1 - 3i| = |(z - 2i) + (-1 - i)| \leq |z - 2i| + |-1 - i|$

$[\because |z_1 + z_2| \leq |z_1| + |z_2|]$

$\therefore \quad |z - 1 - 3i| \leq |z - 2i| + \sqrt{2}$

$\Rightarrow \quad |z - 1 - 3i| \leq \sqrt{2} + \sqrt{2} \quad [\because |z - 2i| \leq \sqrt{2}]$

$\Rightarrow \quad |z - 1 - 3i| \leq 2\sqrt{2}$

From Eq. (i), we get

$$|3 - i(z - 1)| \leq 2\sqrt{2}$$

Hence, the maximum value of $|3 - i(z - 1)|$ is $2\sqrt{2}$.

JEE Type Solved Examples :
More than One Correct Option Type Questions

- This section contains **5 multiple choice examples.** Each example has four choices (a), (b), (c) and (d) out of **which more than one** may be correct.

Ex. 11 *If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $\text{Re}(z_1\bar{z}_2) = 0$, where $i = \sqrt{-1}$, then the complex numbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfy*

(a) $|\omega_1| = 1$ (b) $|\omega_2| = 1$

(c) $\text{Re}(\omega_1\bar{\omega}_2) = 0$ (d) None of these

Sol. (*a, b, c*)

$\because \quad |z_1| = 1, |z_2| = 1 \Rightarrow z_1 = CiS\,\theta, z_2 = CiS\,\phi$

$\text{Re}(z_1\bar{z}_2) = \text{Re}(CiS(\theta - \phi)) = 0$ [given]

$\Rightarrow \quad \cos(\theta - \phi) = 0 \Rightarrow \theta - \phi = \frac{\pi}{2} \Rightarrow \phi = \theta - \frac{\pi}{2}$

and $a = \cos\theta, b = \sin\theta, c = \cos\phi, d = \sin\phi$

$\therefore \quad \omega_1 = a + ic = \cos\theta + i\cos\phi = \cos\theta + i\sin\theta$

$\left[\because \phi = \theta - \frac{\pi}{2}\right]$

and $\omega_2 = b + id = \sin\theta + i\sin\phi = \sin\theta - i\cos\theta$

$\left[\because \phi = \theta - \frac{\pi}{2}\right]$

$\therefore \quad |\omega_1| = 1, |\omega_2| = 1$

and $\text{Re}(\omega_1\bar{\omega}_2) = \text{Re}(\cos\theta + i\sin\theta)(\sin\theta + i\cos\theta) = 0$

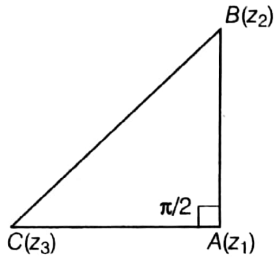
Ex. 12 *The complex numbers z_1, z_2, z_3 satisfying $(z_2 - z_3) = (1 + i)(z_1 - z_3)$, where $i = \sqrt{-1}$, are vertices of a triangle which is*

(a) equilateral (b) isosceles

(c) right angled (d) scalene

Sol. (*b,c*)

$\because \quad (z_2 - z_3) = (1 + i)(z_1 - z_3)$



$\Rightarrow \quad (z_2 - z_1) = i(z_1 - z_3)$

$\Rightarrow \quad (z_2 - z_1) = -i(z_3 - z_1)$

or $\quad (z_3 - z_1) = i(z_2 - z_1)$

$\Rightarrow \quad \frac{z_3 - z_1}{z_2 - z_1} = e^{i\pi/2}$

AC is obtained by rotating AB about A through $\frac{\pi}{2}$ anti-clockwise.

$\therefore \quad AB = AC, \angle CAB = \frac{\pi}{2}$

Hence, z_1, z_2, z_3 form isosceles right angled triangle.

Ex. 13 *If z satisfies the inequality $|z - 1| < |z + 1|$, then one has*

(a) $|z - 2 - i| < |z + 2 - i|, i = \sqrt{-1}$

(b) $|\arg(z + i)| < \frac{\pi}{2}, i = \sqrt{-1}$

(c) $\text{Re}(z) < 0$

(d) $\text{Im}(i\bar{z}) > 0, i = \sqrt{-1}$

Sol. (*a, b, d*)

On putting $z = x + iy$ in the given relation, we have

$$(x - 1)^2 + y^2 < (x + 1)^2 + y^2$$

$\Rightarrow \quad x > 0$

i.e. $\quad \text{Re}(z) > 0 \quad$ ...(i)

and on putting $z = x + iy$ in alternate (a), then

$$(x-2)^2 + (y-1)^2 < (x+2)^2 + (y-1)^2$$

$\Rightarrow \quad x > 0 \quad$ [from Eq. (i)]

which is true.

$\therefore$ Real part of $(z + i) = x > 0$,

then $\arg(z + i)$ lies between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

and hence $|\arg(z + i)| < \frac{\pi}{2}$

and $\quad \text{Im}(i\bar{z}) = \text{Im}(i(x - iy)) = \text{Im}(y + ix)$

$= x > 0 \quad$ [from Eq. (i)]

which is true.

- ***Ex. 14*** *The equation $z^2 - i|z-1|^2 = 0$, where $i = \sqrt{-1}$, has*
  (a) no real root
  (b) no purely imaginary root
  (c) all roots inside $|z| = 1$
  (d) atleast two roots

Sol. (*a, b, c*)

On putting $z = x + iy$, we have

$$(x + iy)^2 - i|x + iy - 1|^2 = 0$$

$\Rightarrow \quad x^2 - y^2 + 2ixy - i((x-1)^2 + y^2) = 0$

On comparing real and imaginary parts, we get

$$x^2 - y^2 = 0 \text{ and } 2xy = (x-1)^2 + y^2$$

***Case* I** When $y = x$, then $2xy = (x-1)^2 + y^2$ reduces to

$$2x^2 = (x-1)^2 + x^2$$

$\Rightarrow \quad 0 = -2x + 1$

$\therefore \quad x = \frac{1}{2} = y$

$\Rightarrow \quad z = x + iy = \frac{1+i}{2} \quad$ ...(i)

***Case* II** When $y = -x$, then $2xy = (x-1)^2 + y^2$ reduces to

$-2x^2 = (x-1)^2 + x^2$

$\Rightarrow (x-1)^2 + 3x^2 = 0$ which is not possible. $\quad$ ...(ii)

From Eqs. (i) and (ii), we get $z = \frac{1+i}{2}$

i.e., no real and no purely imaginary roots

and $\quad |z| = \frac{1}{\sqrt{2}} < 1$

- ***Ex. 15*** *Let z_1 and z_2 be two complex numbers represented by points on circles $|z| = 1$ and $|z| = 2$ respectively, then*
  (a) max.$|2z_1 + z_2| = 4$ (b) min.$|z_1 - z_2| = 1$
  (c) $\left|z_2 + \frac{1}{z_1}\right| \le 3$ (d) $\left|z_1 + \frac{2}{z_2}\right| \le 2$

Sol. (*a, b, c, d*)

$\because \quad |z_1| = 1$ and $|z_2| = 2$

$\therefore \quad |2z_1 + z_2| \le |2z_1| + |z_2| = 2|z_1| + |z_2|$

$\Rightarrow \quad |2z_1 + z_2| \le 2|z_1| + |z_2| = 2 + 2 = 4$

$\therefore \quad |2z_1 + z_2| \le 4$

$\Rightarrow \quad$ max.$|2z_1 + z_2| = 4 \quad$ [alternate (a)]

and $\quad |z_1 - z_2| \ge ||z_1| - |z_2|| = |1 - 2| = 1$

$\Rightarrow \quad |z_1 - z_2| \ge 1$

$\therefore \quad$ min.$|z_1 - z_2| = 1 \quad$ [alternate (b)]

$$\left|z_2 + \frac{1}{z_1}\right| \le |z_2| + \left|\frac{1}{z_1}\right| = |z_2| + \frac{1}{|z_1|} = 2 + \frac{1}{1} = 3$$

$\therefore \quad \left|z_2 + \frac{1}{z_1}\right| \le 3 \quad$ [alternate (c)]

and $\quad \left|z_1 + \frac{2}{z_2}\right| \le |z_1| + \left|\frac{2}{z_2}\right| = |z_1| + \frac{2}{|z_2|} = 1 + \frac{2}{2} = 2$

$\therefore \quad \left|z_1 + \frac{2}{z_2}\right| \le 2 \quad$ [alternate (d)]

JEE Type Solved Examples : Passage Based Questions

- This section contains **2 solved passages** based upon each of the passage **3 multiple choice** examples have to be answered. Each of these examples has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Passage I (Ex. Nos. 16 to 18)

Consider a quadratic equation $az^2 + bz + c = 0$, where a, b and c are complex numbers.

16. The condition that the equation has one purely imaginary root, is

(a) $(a\bar{b} - \bar{a}b)(b\bar{c} + \bar{b}c) + (c\bar{a} - \bar{c}a)^2 = 0$

(b) $(a\bar{b} + \bar{a}b)(b\bar{c} + \bar{b}c) + (c\bar{a} - \bar{c}a)^2 = 0$

(c) $(a\bar{b} - \bar{a}b)(b\bar{c} - \bar{b}c) + (c\bar{a} + \bar{c}a)^2 = 0$

(d) $(a\bar{b} + \bar{a}b)(b\bar{c} - \bar{b}c) + (c\bar{a} - \bar{c}a)^2 = 0$

Sol. (*b*) $\because \quad az^2 + bz + c = 0 \quad$ ...(i)

$\therefore \quad \overline{az^2 + bz + c} = \bar{0}$

$\Rightarrow \quad \bar{a}(\bar{z})^2 + \bar{b}\bar{z} + \bar{c} = 0$

For purely imaginary root,

$\bar{z} = -z$

Then, $\quad \bar{a}z^2 - \bar{b}z + \bar{c} = 0 \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{z^2}{b\bar{c}+\bar{b}c}=\frac{z}{c\bar{a}-\bar{c}a}=\frac{1}{-a\bar{b}-\bar{a}b}$$

$$\Rightarrow \quad z=\frac{b\bar{c}+\bar{b}c}{c\bar{a}-\bar{c}a}=\frac{c\bar{a}-\bar{c}a}{-a\bar{b}-\bar{a}b}$$

$$\therefore \quad (a\bar{b}+\bar{a}b)(b\bar{c}+\bar{b}c)+(c\bar{a}-\bar{c}a)^2=0$$

17. The condition that the equation has one purely real root, is

(a) $(a\bar{b}+\bar{a}b)(b\bar{c}-\bar{b}c)=(c\bar{a}+\bar{c}a)^2$

(b) $(a\bar{b}-\bar{a}b)(b\bar{c}+\bar{b}c)=(c\bar{a}+\bar{c}a)^2$

(c) $(a\bar{b}-\bar{a}b)(b\bar{c}-\bar{b}c)=(c\bar{a}-\bar{c}a)^2$

(d) $(a\bar{b}-\bar{a}b)(b\bar{c}-\bar{b}c)=(c\bar{a}+\bar{c}a)^2$

Sol. (c) $\because \quad az^2+bz+c=0 \quad$ ...(i)

$$\Rightarrow \quad \overline{az^2+bz+c}=\bar{0}$$

$$\Rightarrow \quad \bar{a}(\bar{z})^2+\bar{b}\bar{z}+\bar{c}=0$$

For purely real root, $\bar{z}=z$

Then, $\quad \bar{a}z^2+\bar{b}z+\bar{c}=0 \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{z^2}{b\bar{c}-\bar{b}c}=\frac{z}{c\bar{a}-\bar{c}a}=\frac{1}{a\bar{b}-\bar{a}b}$$

$$\Rightarrow \quad z=\frac{b\bar{c}-\bar{b}c}{c\bar{a}-\bar{c}a}=\frac{c\bar{a}-\bar{c}a}{a\bar{b}-\bar{a}b}$$

$$\Rightarrow \quad (a\bar{b}-\bar{a}b)(b\bar{c}-\bar{b}c)=(c\bar{a}-\bar{c}a)^2$$

18. The condition that the equation has two purely imaginary roots, is

(a) $\frac{a}{\bar{a}}=\frac{b}{\bar{b}}=\frac{c}{\bar{c}}$ (b) $-\frac{a}{\bar{a}}=\frac{b}{\bar{b}}=\frac{c}{\bar{c}}$

(c) $\frac{a}{\bar{a}}=\frac{b}{\bar{b}}=-\frac{c}{\bar{c}}$ (d) $\frac{a}{\bar{a}}=-\frac{b}{\bar{b}}=\frac{c}{\bar{c}}$

Sol. (d) $\because \quad az^2+bz+c=0 \quad$ ...(i)

$$\therefore \quad \overline{az^2+bz+c}=\bar{0}$$

$$\Rightarrow \quad \bar{a}(\bar{z})^2+\bar{b}\bar{z}+\bar{c}=0$$

Since, both roots are purely imaginary.

$$\therefore \quad \bar{z}=-z$$

Then, $\quad \bar{a}z^2-\bar{b}z+\bar{c}=0 \quad$ ...(ii)

Hence, Eqs. (i) and (ii) are identical.

$$\therefore \quad \frac{a}{\bar{a}}=-\frac{b}{\bar{b}}=\frac{c}{\bar{c}}$$

Passage II (Ex. Nos. 19 to 21)

Let P be a point denoting a complex number z on the complex plane.

i.e. $\quad z=\text{Re}(z)+i\,\text{Im}(z)$, where $i=\sqrt{-1}$

If $\quad \text{Re}(z)=x$ and $\text{Im}(z)=y$, then $z=x+iy$

19. If P moves such that

$$|\text{Re}(z)|+|\text{Im}(z)|=a\ (a\in R^+)$$

The locus of P is

(a) a parallelogram which is not a rhombus

(b) a rhombus which is not a square

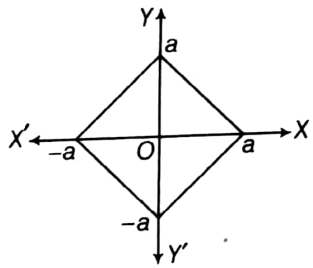
(c) a rectangle which is not a square

(d) a square

Sol. (d) $\because \quad |\text{Re}(z)|+|\text{Im}(z)|=a$

$$\Rightarrow \quad |x|+|y|=a$$

Remember the graph of this.



$\therefore$ Locus of P is a square.

20. The area of the circle inscribed in the region denoted by $|\text{Re}(z)|+|\text{Im}(z)|=10$ equals to

(a) 50π sq units (b) 100π sq units

(c) 55 sq units (d) 110 sq units

Sol. (a) From above, $a=10$

Diameter of circle = Distance between sides of square

= Length of side of square $=a\sqrt{2}=10\sqrt{2}$

or $\quad 2r=10\sqrt{2}$

$\Rightarrow \quad r=5\sqrt{2}$

$\therefore$ Area of a circle $=\pi r^2=50\pi$ sq units

21. Number of integral solutions satisfying the inequality $|\text{Re}(z)|+|\text{Im}(z)|<21$, is

(a) 841 (b) 839

(c) 840 (d) 842

Sol. (c) $\because \quad |x|+|y|<21 \Rightarrow 0\le|x|+|y|\le 20$

If $\quad x>0, y>0, 0\le x+y\le 20$

Number of solutions $={}^{21}C_2=\frac{21\cdot 20}{2}=210$

$\therefore$ Total integral solutions $=4\times 210=840$

JEE Type Solved Examples :
Single Integer Answer Type Questions

- This section contains **2 examples.** The answer to each example is a **single digit integer** ranging from **0** to **9** (both inclusive).

- **Ex. 22** *If $z_1, z_2 \in C$, $z_1^2 + z_2^2 \in R$, $z_1(z_1^2 - 3z_2^2) = 2$ and $z_2(3z_1^2 - z_2^2) = 11$, the value of $z_1^2 + z_2^2$ is*

Sol. (5) We have, $z_1(z_1^2 - 3z_2^2) = 2$ ...(i)

and $z_2(3z_1^2 - z_2^2) = 11$ ...(ii)

multiplying Eq. (ii) by $i(\sqrt{-1})$ and then adding in Eq. (i), we get

$$z_1^3 - 3z_1z_2^2 + i(3z_1^2z_2 - z_2^3) = 2 + 11i$$

$$\Rightarrow \quad (z_1 + iz_2)^3 = 2 + 11i \quad \text{...(iii)}$$

Again, multiplying Eq. (ii) by $(-i)$ and then adding in Eq. (i), we get

$$z_1^3 - 3z_1z_2^2 - i(3z_1^2z_2 - z_2^3) = 2 - 11i$$

$$\Rightarrow \quad (z_1 - iz_2)^3 = 2 - 11i \quad \text{...(iv)}$$

Now, on multiplying Eqs. (iii) and (iv), we get

$$(z_1^2 + z_2^2)^3 = 4 + 121 = 125 = 5^3$$

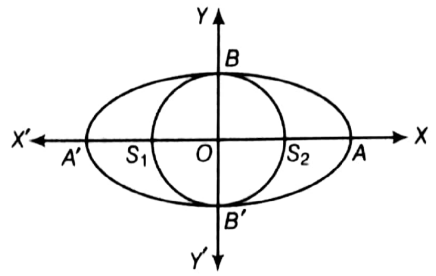
$$\therefore \quad z_1^2 + z_2^2 = 5$$

- **Ex. 23** *The number of solutions of the equations $|z - (4 + 8i)| = \sqrt{10}$ and $|z - (3 + 5i)| + |z - (5 + 11i)| = 4\sqrt{5}$, where $i = \sqrt{-1}$.*

This approach is very imp

Sol. (2) Here, $|z - (4 + 8i)| = \sqrt{10}$

represents a circle with centre (4, 8) and radius $\sqrt{10}$.



Also, $|z - (3 + 5i)| + |z - (5 + 11i)| = 4\sqrt{5}$

represents an ellipse.

$\therefore \quad |(3 + 5i) - (5 + 11i)| = \sqrt{4 + 36} = \sqrt{40} < 4\sqrt{5}$

with foci $S_1(3, 5)$ and $S_2(5, 11)$.

Distance between foci $= S_1S_2 = \sqrt{40} = 2\sqrt{10}$ = Diameter of circle

i.e., $2ae = 2\sqrt{10}$

$\Rightarrow \quad ae = \sqrt{10}$ and $2a = 4\sqrt{5} \Rightarrow a = 2\sqrt{5}$

$\therefore \quad e = \dfrac{ae}{a} = \dfrac{1}{\sqrt{2}}$

Now, $b = a\sqrt{(1 - e^2)} = 2\sqrt{5}\sqrt{\left(1 - \dfrac{1}{2}\right)} = \sqrt{10}$ = Radius of circle

$\therefore$ Centre of the ellipse = Mid-point of S_1 and S_2

$$= \frac{3 + 5i + 5 + 11i}{2} = 4 + 8i \text{ i.e., } (4, 8)$$

which coincides with the centre of the circle and length of minor-axis is equal to the radius of the circle. Hence, there are only (2) two solutions of the given equations.

JEE Type Solved Examples :
Matching Type Questions

- This section contains **2 examples.** Examples 24 and 25 have three statements (A, B and C) given in Column I and four statements (p, q, r and s) in **Column II.** Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II.**

- **Ex. 24**

	Column I		Column II
(A)	If λ and μ are the greatest and least values of $\lvert z - 1\rvert$, if $\lvert z + 2 + i\rvert \le 1$, where $i = \sqrt{-1}$, then	(p)	$\lambda + \mu$ = rational
(B)	If λ and μ are the greatest and least values of $\lvert z - 2\rvert$, if $\lvert z + i\rvert \le 1$, where $i = \sqrt{-1}$, then	(q)	$\lambda + \mu$ = irrational
(C)	If λ and μ are the greatest and least values of $\lvert z + 2i\rvert$, if $1 \le \lvert z - 1\rvert \le 3$, where $i = \sqrt{-1}$, then	(r)	$\lambda - \mu$ = rational
		(s)	$\lambda - \mu$ = irrational

Sol. (A) → (q, r); (B) → (q, r); (C) → (p, s)

(A) $\because$ $|z + 2 + i| \le 1$

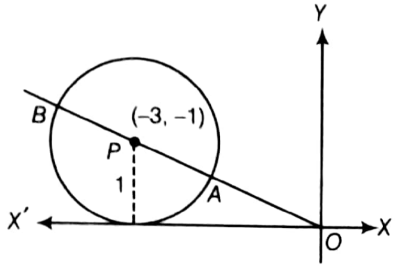
$\Rightarrow$ $|(z - 1) + (3 + i)| \le 1$

$\Rightarrow$ $|\omega + (3 + i)| \le 1$ where, $\omega = z - 1$

From the figure, the greatest value of

$$|z - 1| = |\omega| = |\omega - 0| = OB = OP + PB = \sqrt{10} + 1$$

$\therefore$ $\lambda = \sqrt{10} + 1$



and the least value of $|z - 1| = |\omega|$

$$= |\omega - 0| = OP - AP = \sqrt{10} - 1$$

$\therefore$ $\mu = \sqrt{10} - 1$

$\Rightarrow$ $\lambda + \mu = (\sqrt{10} + 1) + (\sqrt{10} - 1) = 2\sqrt{10}$ = irrational

and $\lambda - \mu = (\sqrt{10} + 1) - (\sqrt{10} - 1) = 2$ = rational

Aliter

$\because$ $|z + 2 + i| \le 1$

$\Rightarrow$ $|(z - 1) + (3 + i)| \le 1$

$\Rightarrow$ $|\omega + (3 + i)| \le 1$ ...(i)

where, $\omega = z - 1$

$\therefore$ $|\omega + (3 + i)| \ge ||\omega| - |3 + i||$

or $|\omega + (3 + i)| \ge ||\omega| - \sqrt{10}|$ ...(ii)

From Eqs. (i) and (ii), we get

$$||\omega| - \sqrt{10}| \le |\omega + 3 + i| \le 1$$

$\Rightarrow$ $||\omega| - \sqrt{10}| \le 1$

or $-1 \le |\omega| - \sqrt{10} \le 1$

or $\sqrt{10} - 1 \le |\omega| \le \sqrt{10} + 1$

$\therefore$ $\lambda = \sqrt{10} + 1$ and $\mu = \sqrt{10} - 1$

$\Rightarrow$ $\lambda + \mu = 2\sqrt{10}$ = irrational

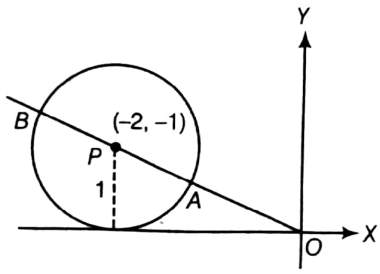
and $\lambda - \mu = 2$ = rational

(B) $\because$ $|z + i| \le 1$

$\Rightarrow$ $|(z - 2) + (2 + i)| \le 1$

$\Rightarrow$ $|\omega + (2 + i)| \le 1$

where, $\omega = z - 2$



From the figure, the greatest value of $|z - 2| = |\omega|$

$$= |\omega - 0| = OB$$

$$= OP + PB = \sqrt{5} + 1$$

$\therefore$ $\lambda = \sqrt{5} + 1$

and the least value of $|z - 2| = |\omega|$

$$= |\omega - 0| = OA = OP - AP = \sqrt{5} - 1$$

$\therefore$ $\mu = \sqrt{5} - 1$

$\Rightarrow$ $\lambda + \mu = (\sqrt{5} + 1) + (\sqrt{5} - 1) = 2\sqrt{5}$ = irrational

and $\lambda - \mu = (\sqrt{5} + 1) - (\sqrt{5} - 1) = 2$ = rational

Aliter

$\because$ $|z + i| \le 1$

$\Rightarrow$ $|(z - 2) + (2 + i)| \le 1$

$\Rightarrow$ $|\omega + (2 + i)| \le 1$

where, $\omega = z - 2$ ...(i)

$\because$ $|\omega + (2 + i)| \ge ||\omega| - |2 + i||$

or $|\omega + (2 + i)| \ge ||\omega| - \sqrt{5}|$ ...(ii)

From Eqs. (i) and (ii), we get

$$||\omega| - \sqrt{5}| \le |\omega + 2 + i| \le 1$$

$\Rightarrow$ $||\omega| - \sqrt{5}| \le 1$

or $-1 \le |\omega| - \sqrt{5} \le 1$ or $\sqrt{5} - 1 \le |\omega| \le \sqrt{5} + 1$

$\therefore$ $\lambda = \sqrt{5} + 1$ and $\mu = \sqrt{5} - 1$

$\Rightarrow$ $\lambda + \mu = 2\sqrt{5}$ = irrational

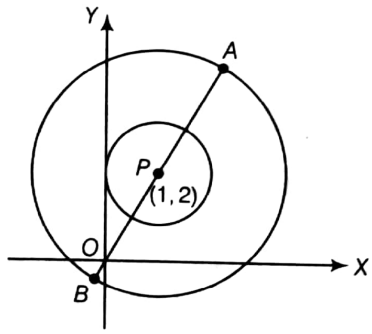
and $\lambda - \mu = 2$ = rational

(C) $\because$ $1 \le |z - 1| \le 3$

$\Rightarrow$ $1 \le |(z + 2i) - (1 + 2i)| \le 3$

$\Rightarrow$ $1 \le |\omega - (1 + 2i)| \le 3$ ...(i)

where, $\omega = z + 2i$



From the figure, the greatest value of $|z + 2i| = |\omega|$

$$= |\omega - 0| = OA = OP + PA = \sqrt{5} + 3$$

$\therefore$ $\lambda = 3 + \sqrt{5}$

and the least value of $|z + 2i| = |\omega|$

$$= |\omega - 0| = OB = PB - OP = 3 - \sqrt{5}$$

$\therefore$ $\mu = 3 - \sqrt{5}$

$\Rightarrow$ $\lambda + \mu = (3 + \sqrt{5}) + (3 - \sqrt{5}) = 6$ = rational

and $\lambda - \mu = (3 + \sqrt{5}) - (3 - \sqrt{5}) = 2\sqrt{5}$ = irrational

Ex. 25

	Column I		Column II
(A)	If $\sqrt{(3-4i)}+\sqrt{(-3-4i)}=z$, the principal value of arg (z) can be (where $i=\sqrt{-1}$)	(p)	0
(B)	If $\sqrt{(5+12i)}+\sqrt{(-5+12i)}=z$, the principal value of arg (z) can be (where $i=\sqrt{-1}$)	(q)	$\pm\frac{\pi}{4}$
(C)	If $\sqrt{(-15+8i)}+\sqrt{(-15-8i)}=z$, the principal value of arg (z) can be (where $i=\sqrt{-1}$)	(r)	$\pm\frac{\pi}{2}$
		(s)	$\pm\frac{3\pi}{4}$

Sol. (A) → (q, s); (B) → (q, s); (C) → (p, r)

$$\because \sqrt{z}=\pm\left(\sqrt{\frac{|z|+\text{Re}(z)}{2}}+i\sqrt{\frac{|z|-\text{Re}(z)}{2}}\right), \text{Im}(z)>0$$

$$=\pm\left(\sqrt{\frac{|z|+\text{Re}(z)}{2}}-i\sqrt{\frac{|z|-\text{Re}(z)}{2}}\right), \text{Im}(z)<0$$

(A) $\sqrt{(3-4i)}=\pm\left(\sqrt{\frac{5+3}{2}}-i\sqrt{\frac{5-3}{2}}\right)=\pm(2-i)$

$$\sqrt{(-3-4i)}=\pm\left(\sqrt{\frac{5-3}{2}}-i\sqrt{\frac{5+3}{2}}\right)=\pm(1-2i)$$

$$\because \quad z=\sqrt{(3-4i)}+\sqrt{(-3-4i)}$$

$$\therefore \quad z=\pm(2-i)\pm(1-2i)$$

$$=3-3i, 1+i, -1-i, -3+3i$$

$\therefore$ Principal values of arg $(z)=-\frac{\pi}{4}, \frac{\pi}{4}, -\frac{3\pi}{4}, \frac{3\pi}{4}$

(B) $\sqrt{(5+12i)}=\pm\left(\sqrt{\frac{13+5}{2}}+i\sqrt{\frac{13-5}{2}}\right)$

$$=\pm(3+2i)$$

$$\sqrt{(-5+12i)}=\pm\left(\sqrt{\frac{13-5}{2}}+i\sqrt{\frac{13+5}{2}}\right)$$

$$=\pm(2+3i)$$

$$\because \quad z=\sqrt{(5+12i)}+\sqrt{(-5+12i)}$$

$$=\pm(3+2i)\pm(2+3i)$$

$$=5+5i, 1-i, -1+i, -5-5i$$

$\therefore$ Principal values of arg $(z)=\frac{\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, -\frac{3\pi}{4}$

(C) $\sqrt{-15+8i}=\pm\left(\sqrt{\frac{17-15}{2}}+i\sqrt{\frac{17+15}{2}}\right)$

$$=\pm(1+4i)$$

and $\sqrt{-15-8i}=\sqrt{\overline{-15+8i}}=\pm(\overline{1+4i})$

$$=\pm(1-4i)$$

$$\because \quad z=\sqrt{(-15+8i)}+\sqrt{(-15-8i)}$$

$$=\pm(1+4i)\pm(1-4i)=2, 8i, -8i, -2$$

$\therefore$ Principal values of arg $(z)=0, \frac{\pi}{2}, -\frac{\pi}{2}, \pi$.

JEE Type Solved Examples : Statement I and II Type Questions

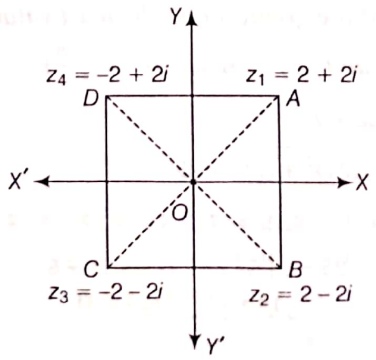
- **Directions** Example numbers 26 and 27 are Assertion-Reason type examples. Each of these examples contains two statements:
  Statement-1 (Assertion) and **Statement-2** (Reason)
  Each of these examples also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.
  (a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
  (b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
  (c) Statement-1 is true, Statement-2 is false
  (d) Statement-1 is false, Statement-2 is true

Ex. 26 *Consider four complex numbers $z_1=2+2i$, $z_2=2-2i$, $z_3=-2-2i$ and $z_4=-2+2i$, where $i=\sqrt{-1}$.*
Statement-1 *z_1, z_2, z_3 and z_4 constitute the vertices of a square on the complex plane because*
Statement-2 *The non-zero complex numbers $z, \bar{z}, -z, -\bar{z}$ always constitute the vertices of a square.*

Sol. (c) **Statement-1**

$$AB=|z_1-z_2|=|4i|=4,$$
$$BC=|z_2-z_3|=|4|=4,$$
$$CD=|z_3-z_4|=|-4i|=4$$
$$DA=|z_4-z_1|=|-4|=4$$
$$AC=|z_1-z_3|=|4+4i|=4\sqrt{2}$$

and $BD=|z_2-z_4|=|4-4i|=4\sqrt{2}$



It is clear that, $AB = BC = CD = DA$ and $AC = BD$

Hence, z_1, z_2, z_3 and z_4 are the vertices of a square.

$\therefore$ Statement-1 is true.

Statement-2 If $z = a + ib$

If $a \neq b$

Then, $AB = |z - \bar{z}| = |(a + ib) - (a - ib)| = 2|b|$

$BC = |\bar{z} - (-z)| = |\bar{z} + z| = |a - ib + a + ib| = 2|a|$

$\therefore \quad AB \neq BC$

Statement-2 is false.

- **Ex. 27** *Consider z_1 and z_2 are two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$*

Statement-1 *$amp(z_1) - amp(z_2) = 0$*

Statement-2 *The complex numbers z_1 and z_2 are collinear with origin.*

Sol. (*b*) **Statement-1**

$\because \quad |z_1 + z_2| = |z_1| + |z_2| \quad$ ...(i)

If $\text{amp}(z_1) = \theta_1$ and $\text{amp}(z_2) = \theta_2$, then

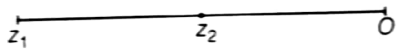
$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2| \cos(\theta_1 - \theta_2)$$

$$\Rightarrow \quad (|z_1| + |z_2|)^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2| \cos(\theta_1 - \theta_2)$$

[from Eq. (i)]

$$\Rightarrow \quad |z_1|^2 + |z_2|^2 + 2|z_1||z_2| = |z_1|^2 + |z_2|^2 + 2|z_1||z_2| \cos(\theta_1 - \theta_2)$$

$$\therefore \quad \cos(\theta_1 - \theta_2) = 1$$



$$\Rightarrow \quad \theta_1 - \theta_2 = 0$$

or $\quad \text{amp}(z_1) - \text{amp}(z_2) = 0$

$\therefore$ Statement-1 is true.

Statement-2 Since, z_1, z_2 and O (origin) are collinear, then

$$\text{amp}\left(\frac{O - z_1}{O - z_2}\right) = 0$$

$$\Rightarrow \quad \text{amp}\left(\frac{z_1}{z_2}\right) = 0$$

$$\Rightarrow \quad \text{amp}(z_1) - \text{amp}(z_2) = 0$$

$\therefore$ Statement-2 is true, which is not a correct explanation of Statement-1.

Subjective Type Examples

- In this section, there are **24 subjective** solved examples.

- **Ex. 28** *If $|z - i\,\text{Re}(z)| = |z - \text{Im}(z)|$, then prove that z lies on the bisectors of the quadrants, where $i = \sqrt{-1}$.*

Sol. Let $z = x + iy$, where $x, y \in R$ and $i = \sqrt{-1}$

$\therefore \quad \text{Re}(z) = x$ and $\text{Im}(z) = y$

Then, $|z - i\,\text{Re}(z)| = |z - \text{Im}(z)|$

$$\Rightarrow \quad |x + iy - ix| = |x + iy - y|$$

$$\Rightarrow \quad |x - i(x - y)| = |(x - y) + iy|$$

$$\Rightarrow \quad \sqrt{x^2 + (x - y)^2} = \sqrt{(x - y)^2 + y^2}$$

$$\Rightarrow \quad x^2 + (x - y)^2 = (x - y)^2 + y^2$$

$$\Rightarrow \quad x^2 = y^2 \text{ or } y = \pm x$$

Hence, z lies on the bisectors of the quadrants.

- **Ex. 29** *Find the greatest and the least values of $|z_1 + z_2|$, if $z_1 = 24 + 7i$ and $|z_2| = 6$, where $i = \sqrt{-1}$.*

Sol. $\because \quad z_1 = 24 + 7i$

$\therefore \quad |z_1| = \sqrt{(24)^2 + (7)^2} = 25$

$$||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$$

$$\Rightarrow \quad |25 - 6| \leq |z_1 + z_2| \leq 25 + 6$$

or $\quad 19 \leq |z_1 + z_2| \leq 31$

Hence, the least value of $|z_1 + z_2|$ is 19 and the greatest value of $|z_1 + z_2|$ is 31.

- **Ex. 30** *Let S denotes the real part of the complex number $z = \frac{5 + 2i}{2 - 5i} + \frac{20 + 5i}{7 + 6i} + 3i$, where $i = \sqrt{-1}$, K denotes the sum of the imaginary parts of the roots of the equation $z^2 - 8(1 - i)z + 63 - 16i = 0$ and G denotes the value of $\sum_{r=4}^{2012} i^r$, where $i = \sqrt{-1}$, find the value of $S - K + G$.*

Sol. For S,

$$z = \frac{(5 + 2i)}{(2 - 5i)} + \frac{(20 + 5i)}{(7 + 6i)} + 3i$$

$$= \frac{(5 + 2i)(2 + 5i)}{29} + \frac{(20 + 5i)(7 - 6i)}{85} + 3i$$

$$= \frac{0 + 29i}{29} + \frac{170 - 85i}{85} + 3i$$

$$= i + 2 - i + 3i = 2 + 3i$$

$\therefore \text{Re}(z) = 2 \qquad \therefore \quad S = 2$

For K,

Put $z = x + iy$ in the given equation, then

$$(x + iy)^2 - 8(1 - i)(x + iy) + 63 - 16i = 0$$

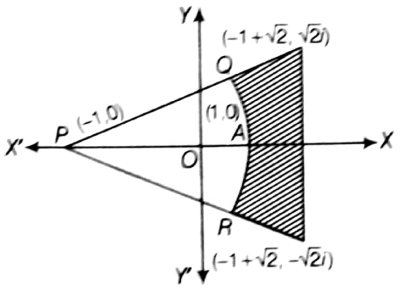
Complex Numbers Exercise 8 :
Questions Asked in Previous 13 Years' Exams

- This section contains questions asked in **IIT-JEE, AIEEE, JEE Main & JEE Advanced** from year **2005** to year **2017**.

103. If ω is a cube root of unity but not equal to 1, then minimum value of $|a + b\omega + c\omega^2|$, (where a, b and c are integers but not all equal), is **[IIT-JEE 2005, 3M]**

(a) 0 (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) 2

104. PQ and PR are two infinite rays. QAR is an arc. Point lying in the shaded region excluding the boundary satisfies **[IIT-JEE 2005, 3M]**



(a) $|z-1| > 2; |\arg(z-1)| < \frac{\pi}{4}$

(b) $|z-1| > 2; |\arg(z-1)| < \frac{\pi}{2}$

(c) $|z+1| > 2; |\arg(z+1)| < \frac{\pi}{4}$

(d) $|z+1| > 2; |\arg(z+1)| < \frac{\pi}{3}$

105. If one of the vertices of the square circumscribing the circle $|z-1| = \sqrt{2}$ is $2 + \sqrt{3}i$, where $i = \sqrt{-1}$. Find the other vertices of the square. **[IIT-JEE 2005, 4M]**

106. If z_1 and z_2 are two non-zero complex numbers, such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg(z_1) - \arg(z_2)$ is equal to **[AIEEE 2005, 3M]**

(a) $-\pi$ (b) $-\pi/2$

(c) $\pi/2$ (d) 0

107. If $1, \omega, \omega^2$ are the cube roots of unity, then the roots of the equation $(x-1)^3 + 8 = 0$ are **[AIEEE 2005, 3M]**

(a) $-1, 1+2\omega, 1+2\omega^2$ (b) $-1, 1-2\omega, 1-2\omega^2$

(c) $-1, -1, -1$ (d) None of these

108. If $\omega = \dfrac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, where $i = \sqrt{-1}$, then z lies on **[AIEEE 2005, 3M]**

(a) a straight line (b) a parabola
(c) an ellipse (d) a circle

109. If $\omega = \alpha + i\beta$, where $\beta \neq 0$, $i = \sqrt{-1}$ and $z \neq 1$, satisfies the condition that $\left(\dfrac{\omega - \bar{\omega} z}{1 - z}\right)$ is purely real, the set of values of z is **[IIT-JEE 2006, 3M]**

(a) $\{z : |z| = 1\}$ (b) $\{z : z = \bar{z}\}$
(c) $\{z : z \neq 1\}$ (d) $\{z : |z| = 1, z \neq 1\}$

110. The value of $\sum_{k=1}^{10}\left(\sin\dfrac{2k\pi}{11} + i\cos\dfrac{2k\pi}{11}\right)$ (where $i = \sqrt{-1}$) is **[AIEEE 2006, 3M]**

(a) i (b) 1
(c) -1 (d) $-i$

111. If $z^2 + z + 1 = 0$, where z is a complex number, the value of

$$\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \ldots + \left(z^6 + \frac{1}{z^6}\right)^2$$

is **[AIEEE 2006, 6M]**

(a) 18 (b) 54
(c) 6 (d) 12

112. A man walks a distance of 3 units from the origin towards the North-East (N 45° E) direction. From there, he walks a distance of 4 units towards the North-West (N 45° W) direction to reach a point P. Then, the position of P in the Argand plane, is **[IIT-JEE 2007, 3M]**

(a) $3e^{i\pi/4} + 4i$ (b) $(3 - 4i)e^{i\pi/4}$
(c) $(4 + 3i)e^{i\pi/4}$ (d) $(3 + 4i)e^{i\pi/4}$
(where $i = \sqrt{-1}$)

113. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\dfrac{z}{1 - z^2}$ lie on **[IIT-JEE 2007, 3M]**

(a) a line not passing through the origin
(b) $|z| = \sqrt{2}$
(c) the X-axis
(d) the Y-axis

114. If $|z + 4| \leq 3$, the maximum value of $|z + 1|$ is **[AIEEE 2007, 3M]**

(a) 4 (b) 10
(c) 6 (d) 0

Passage (Q. Nos. 115 to 117)

Let A, B and C be three sets of complex numbers as defined below:

$A = \{z : \text{Im}(z) \geq 1\}$

$B = \{z : |z - 2 - i| = 3\}$

$C = \{z : \text{Re}((1 - i)z) = \sqrt{2}\}$, where $i = \sqrt{-1}$

[IIT-JEE 2008, 4+4+4M]

115. The number of elements in the set $A \cap B \cap C$, is

(a) 0 (b) 1
(c) 2 (d) ∞

116. Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

(a) 25 and 29 (b) 30 and 34
(c) 35 and 39 (d) 40 and 44

117. Let z be any point in $A \cap B \cap C$ and ω be any point satisfying $|\omega - 2 - i| < 3$. Then, $|z| - |\omega| + 3$ lies between

(a) -6 and 3 (b) -3 and 6
(c) -6 and 6 (d) -3 and 9

118. A particle P starts from the point $z_0 = 1 + 2i$, $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1, the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\dfrac{\pi}{2}$ in anti-clockwise direction on a circle with centre at origin, to reach a point z_2, then the point z_2 is given by **[IIT-JEE 2008, 3M]**

(a) $6 + 7i$ (b) $-7 + 6i$
(c) $7 + 6i$ (d) $-6 + 7i$

119. If the conjugate of a complex numbers is $\dfrac{1}{i - 1}$, where $i = \sqrt{-1}$. Then, the complex number is **[AIEEE 2008, 3M]**

(a) $\dfrac{-1}{i - 1}$ (b) $\dfrac{1}{i + 1}$
(c) $\dfrac{-1}{i + 1}$ (d) $\dfrac{1}{i - 1}$

120. Let $z = x + iy$ be a complex number, where x and y are integers and $i = \sqrt{-1}$. Then, the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$, is **[IIT-JEE 2009, 3M]**

(a) 48 (b) 32
(c) 40 (d) 80

121. Let $z = \cos\theta + i\sin\theta$, where $i = \sqrt{-1}$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is **[IIT-JEE 2009, 3M]**

(a) $\dfrac{1}{\sin 2°}$ (b) $\dfrac{1}{3\sin 2°}$
(c) $\dfrac{1}{2\sin 2°}$ (d) $\dfrac{1}{4\sin 2°}$

122. If $\left|z - \dfrac{4}{z}\right| = 2$, the maximum value of $|z|$ is equal to **[AIEEE 2009, 4M]**

(a) $2 + \sqrt{2}$ (b) $\sqrt{3} + 1$
(c) $\sqrt{5} + 1$ (d) 2

123. Let z_1 and z_2 be two distinct complex numbers and $z = (1-t)z_1 + iz_2$, for some real number t with $0 < t < 1$ and $i = \sqrt{-1}$. If $\arg(w)$ denotes the principal argument of a non-zero complex number w, then **[IIT-JEE 2010, 3M]**

(a) $|z - z_1| + |z - z_2| = |z_1 - z_2|$
(b) $\arg(z - z_1) = \arg(z - z_2)$
(c) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$
(d) $\arg(z - z_1) = \arg(z_2 - z_1)$

124. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$, where $i = \sqrt{-1}$, then the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$, is equal to **[IIT-JEE 2010, 3M]**

(a) 0 (b) 1
(c) 2 (d) 3

125. Match the statements in **Column I** with those in **Column II**.

[**Note** Here, z takes values in the complex plane and Im (z) and Re (z) denote respectively, the imaginary part and the real part of z.] **[IIT- JEE 2010, 8M]**

	Column I		Column II
(A)	The set of points z satisfying $\|z - i\|z\|\| = \|z + i\|z\|\|$, where $i = \sqrt{-1}$, is contained in or equal to	(p)	an ellipse with eccentricity 4/5
(B)	The set of points z satisfying $\|z+4\| + \|z-4\| = 10$ is contained in or equal to	(q)	the set of points z satisfying Im $(z) = 0$
(C)	If $\|w\| = 2$, the set of points $z = w - \frac{1}{w}$ is contained in or equal to	(r)	the set of points z satisfying $\|\text{Im}(z)\| \le 1$
(D)	If $\|w\| = 1$, the set of points $z = w + \frac{1}{w}$ is contained in or equal to	(s)	the set of points satisfying $\|\text{Re}(z)\| \le 2$
		(t)	the set of points z satisfying $\|z\| \le 3$

126. If α and β are the roots of the equation $x^2 - x + 1 = 0$, $\alpha^{2009} + \beta^{2009}$ is equal to **[AIEEE 2010, 4M]**

(a) -1 (b) 1
(c) 2 (d) -2

127. The number of complex numbers z, such that $|z-1| = |z+1| = |z-i|$, where $i = \sqrt{-1}$, equals to **[AIEEE 2010, 4M]**

(a) 1 (b) 2
(c) ∞ (d) 0

128. If z is any complex number satisfying $|z - 3 - 2i| \le 2$, where $i = \sqrt{-1}$, then the minimum value of $|2z - 6 + 5i|$, is **[IIT-JEE 2011, 4M]**

129. The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number } |z| = 1, z \ne \pm 1\right\}$ is **[IIT-JEE 2011, 2M]**

(a) $(-\infty, -1] \cup [1, \infty)$ (b) $(-\infty, 0) \cup (0, \infty)$
(c) $(-\infty, -1) \cup (1, \infty)$ (d) $[2, \infty)$

130. The maximum value of $\left|\arg\left(\frac{1}{1-z}\right)\right|$ for $|z| = 1, z \ne 1$, is given by **[IIT-JEE 2011, 2M]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

131. Let $w = e^{i\pi/3}$, where $i = \sqrt{-1}$ and a, b, c, x, y and z be non-zero complex numbers such that

$$a + b + c = x$$
$$a + bw + cw^2 = y$$
$$a + bw^2 + cw = z.$$

The value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$, is **[IIT-JEE 2011, 4M]**

132. Let α and β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $(z) = 1$, then it is necessary that **[AIEEE 2011, 4M]**

(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$
(c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$

133. If $\omega (\ne 1)$ is a cube root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals to **[AIEEE 2011, 4M]**

(a) (1, 1) (b) (1, 0)
(c) (−1, 1) (d) (0, 1)

134. Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2 + z + 1$ is real. Then, a **cannot** take the value **[IIT-JEE 2012, 3M]**

(a) -1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

135. If $z \ne 1$ and $\frac{z^2}{z-1}$ is real, the point represented by the complex number z lies **[AIEEE 2012, 4M]**

(a) on a circle with centre at the origin
(b) either on the real axis or on a circle not passing through the origin
(c) on the imaginary axis
(d) either on the real axis or on a circle passing through the origin

136. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals to **[JEE Main 2013, 4M]**

(a) $\frac{\pi}{2}-\theta$ (b) θ

(c) $\pi-\theta$ (d) $-\theta$

137. Let complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on circles $(x-x_0)^2+(y-y_0)^2=r^2$ and $(x-x_0)^2+(y-y_0)^2=4r^2$, respectively. If $z_0=x_0+iy_0$ satisfies the equation $2|z_0|^2=r^2+2$, then $|\alpha|$ equals to **[JEE Advanced 2013, 2M]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

138. Let $w=\frac{\sqrt{3}+i}{2}$ and $P=\{w^n : n=1,2,3,\ldots\}$. Further,

$$H_1=\left\{z\in C:\text{Re}(z)>\frac{1}{2}\right\} \text{ and } H_2=\left\{z\in C:\text{Re}(z)<\left(-\frac{1}{2}\right)\right\},$$

where C is the set of all complex numbers. If $z_1\in P\cap H_1$, $z_2\in P\cap H_2$ and O represents the origin, then $\angle z_1Oz_2$ equals to **[JEE Advanced 2013, 3M]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

Passage (Q. Nos. 139 to 140)

Let $S=S_1\cap S_2\cap S_3$, where

$S_1=\{z\in C:|z|<4\}$,

$$S_2=\left\{z\in C:\text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right]>0\right\}$$

and $S_3=\{z\in C:\text{Re } z>0\}$. **[JEE Advanced 2013, 3+3M]**

139. $\min_{z\in S}|1-3i-z|$ equals to

(a) $\frac{2-\sqrt{3}}{2}$ (b) $\frac{2+\sqrt{3}}{2}$

(c) $\frac{3-\sqrt{3}}{2}$ (d) $\frac{3+\sqrt{3}}{2}$

140. Area of S equals to

(a) $\frac{10\pi}{3}$ (b) $\frac{20\pi}{3}$

(c) $\frac{16\pi}{3}$ (d) $\frac{32\pi}{3}$

141. If z is a complex number such that $|z|\geq 2$, then the minimum value of $\left|z+\left(\frac{1}{2}\right)\right|$, is **[JEE Main 2014, 4M]**

(a) is strictly greater than $\frac{5}{2}$

(b) is equal to $\frac{5}{2}$

(c) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(d) lies in the interval (1, 2)

142. Let $z_k=\cos\left(\frac{2k\pi}{10}\right)+i\sin\left(\frac{2k\pi}{10}\right)$; $k=1,2,\ldots,9$. Then, match the column.

	Column I		Column II
(A)	For each z_k there exists a z_j such that $z_k\cdot z_j=1$	(1)	True
(B)	There exists a $k\in\{1,2,\ldots,9\}$ such that $z_1\cdot z=z_k$ has no solution z in the set of complex numbers	(2)	False
(C)	$\frac{\lvert 1-z_1\rvert\lvert 1-z_2\rvert\ldots\lvert 1-z_9\rvert}{10}$ equals to	(3)	1
(D)	$1-\sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ equals to	(4)	2

[JEE Advanced 2014, 3M]

Codes

	A	B	C	D		A	B	C	D
(a)	1	2	4	3	(b)	2	1	3	4
(c)	1	2	3	4	(d)	2	1	4	3

143. A complex number z is said to be unimodular if $|z|=1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a **[JEE Main 2015, 4M]**

(a) circle of radius 2

(b) circle of radius $\sqrt{2}$

(c) straight line parallel to X-axis

(d) straight line parallel to Y-axis

144. Let $\omega\neq 1$ be a complex cube root of unity. If $(3-3\omega+2\omega^2)^{4n+3}+(2+3\omega-3\omega^2)^{4n+3}+(-3+2\omega+3\omega^2)^{4n+3}=0$, then possible value(s) of n is (are) **[JEE Advanced 2015, 2M]**

(a) 1 (b) 2

(c) 3 (d) 4

145. For any integer k, let $\alpha_k=\cos\left(\frac{k\pi}{7}\right)+i\sin\left(\frac{k\pi}{7}\right)$, where $i=\sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12}|\alpha_{k+1}-\alpha_k|}{\sum_{k=1}^{3}|\alpha_{4k-1}-\alpha_{4k-2}|}$ is **[JEE Advanced 2015, 4M]**

146. A value of θ for which $\dfrac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is **[JEE Main 2016, 4M]**

(a) $\dfrac{\pi}{6}$ (b) $\sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$

(c) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ (d) $\dfrac{\pi}{3}$

147. Let $a, b \in R$ and $a^2 + b^2 \neq 0$.

Suppose $S = \left\{z \in C : z = \dfrac{1}{a+ibt}, t \in R, t \neq 0\right\}$, where $i = \sqrt{-1}$. If $z = x + iy$ and $z \in S$, then (x, y) lies on **[JEE Advanced 2016 4M]**

(a) the circle with radius $\dfrac{1}{2a}$ and centre $\left(\dfrac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$

(b) the circle with radius $-\dfrac{1}{2a}$ and centre $\left(-\dfrac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$

(c) the X-axis for $a \neq 0, b = 0$

(d) the Y-axis for $a = 0, b \neq 0$

148. Let ω be a complex number such that $2\omega + 1 = z$, when $z = \sqrt{-3}$ if $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to **[JEE Main 2017, 4M]**

(a) 1 (b) $-z$ (c) z (d) -1

Answers

Exercise for Session 1

1. (d) 2. (c) 3. (b) 4. (b) 5. (c) 6. (b)
7. (d) 8. (a)

Exercise for Session 2

1. (b) 2. (b) 3. (b) 4. (b) 5. (b) 6. (b)
7. (d) 8. (c) 9. (b) 10. (b) 11. (a) 12. (c)
13. (c) 14. (a)

Exercise for Session 3

1. (a) 2. (b) 3. (d) 4. (a) 5. (b) 6. (a)
7. (c) 8. (b) 9. (c) 10. (d) 11. (c) 12. (a)
13. (b) 14. (b) 15. (a)

Exercise for Session 4

1. (a) 2. (d) 3. (c) 4. (d) 5. (b) 6. (b)
7. (a) 8. (d) 9. (d) 10. (b) 11. (b) 12. (a)
13. (b) 14. (c)

Chapter Exercises

1. (b) 2. (c) 3. (d) 4. (d) 5. (a) 6. (d)
7. (b) 8. (d) 9. (b) 10. (c) 11. (c) 12. (b)
13. (c) 14. (d) 15. (a) 16. (c) 17. (a) 18. (c)
19. (b) 20. (b) 21. (c) 22. (c) 23. (c) 24. (c)
25. (d) 26. (d) 27. (b) 28. (b) 29. (b) 30. (c)
31. (b,c,d) 32. (b,c,d) 33. (b,c,d) 34. (b,c,d) 35. (a,c,d) 36. (a,b)
37. (a,b,c) 38. (a,d) 39. (a,c) 40. (a,b,d) 41. (a,d)
42. (b,d) 43. (a,c,d) 44. (a,d) 45. (a,b,d)
46. (a) 47. (d) 48. (b) 49. (a) 50. (c) 51. (b)
52. (d) 53. (c) 54. (a) 55. (c) 56. (d) 57. (b)
58. (1) 59. (5) 60. (9) 61. (8) 62. (3) 63. (4)
64. (6) 65. (5) 66. (4) 67. (4)
68. A $\to$ (p, q); B $\to$ (p, r); C $\to$ (p, r, s)
69. A $\to$ (q); B $\to$ (q, r); C $\to$ (q, s)
70. A $\to$ (p, q, r, t); B $\to$ (p, s); C $\to$ (p, r)
71. A $\to$ (r); B $\to$ (p, s); C $\to$ (q, t)
72. (d) 73. (d) 74. (c) 75. (d) 76. (a) 77. (d)
78. (d)
82. 2
84. $z = c + i(-1 \pm \sqrt{(1-c^2-2c)})$ for $0 \le c \le \sqrt{2}-1$ and no solution for $c > \sqrt{2}-1$
85. $(1-3i)z + (1+3i)\bar{z} - 22 = 0$ and $(3+i)z + (3-i)\bar{z} + 14 = 0$
87. No solution
97. $\dfrac{\sum |z_1|^2 (z_2 - z_3)}{\sum \bar{z}_1 (z_2 - z_3)}$
98. $\dfrac{\sum z_1^2(\bar{z}_2 - \bar{z}_3) + \sum |z_1|^2 (z_2 - z_3)}{\sum (z_1\bar{z}_2 - z_2\bar{z}_1)}$
99. (i) $x^3 - 24x^2 + 80x - 64 = 0$
(ii) $x^3 - 21x^2 + 35x - 7 = 0$
(iii) 4
100. Roots of $z^7 + 1 = 0$ are $-1, \alpha, \alpha^3, \alpha^5, \bar{\alpha}, \bar{\alpha}^3, \bar{\alpha}^5$, where $\alpha = \cos\dfrac{\pi}{7} + i\sin\dfrac{\pi}{7}$
101. $a \in \left(\dfrac{1-\sqrt{71}}{2}, \dfrac{-1-4\sqrt{11}}{5}\right) \cup \left(\dfrac{-1+4\sqrt{11}}{5}, \dfrac{1+\sqrt{71}}{2}\right)$
102. (i) ${}^{2n+1}C_1(1-x)^n - {}^{2n+1}C_3(1-x)^{n-1}x + \ldots + (-1)^n x^n = 0$
(ii) ${}^{2n+1}C_1 x^n - {}^{2n+1}C_3 x^{n-1} + {}^{2n+1}C_5 x^{n-2} - \ldots = 0$
103. (c) 104. (c) 105. $(1-\sqrt{3}) + i, -i\sqrt{3}, (\sqrt{3}+1) - i$ 106. (d)
107. (b) 108. (a) 109. (d) 110. (d) 111. (d) 112. (d)
113. (d) 114. (c) 115. (b) 116. (c) 117. (d) 118. (d)
119. (c) 120. (a) 121. (d) 122. (c) 123. (a, c, d) 124. (b)
125. A $\to$ (q, r); B $\to$ (p); C $\to$ (p, s); D $\to$ (q, r, s, t) 126. (b) 127. (a)
128. (5) 129. (a) 130. (c) 131. (3) 132. (c) 133. (a)
134. (d) 135. (d) 136. (b) 137. (c) 138. (c) 139. (c)
140. (b) 141. (d) 142. (c) 143. (a) 144. (a, b, d)
145. (4) 146. (c) 147. (a,c,d) 148. (b)