JEE Type Solved Examples : Single Option Correct Type Questions

- This section contains **10 multiple choice examples.** Each example has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

• Ex. 1 *If $b-c$, $2b-\lambda$, $b-a$ are in HP, then $a-\dfrac{\lambda}{2}$, $b-\dfrac{\lambda}{2}$, $c-\dfrac{\lambda}{2}$ are is*

(a) AP (b) GP
(c) HP (d) None of these

Sol. (b) $(2b-\lambda)=\dfrac{2(b-c)(b-a)}{(b-c)+(b-a)}$

$$\Rightarrow \quad (2b-\lambda)=(2b-(a+c))=2[b^2-(a+c)b+ac]$$

$$\Rightarrow \quad 2b^2-2b\lambda+\lambda(a+c)-2ac=0$$

$$\Rightarrow \quad b^2-b\lambda+\frac{\lambda}{2}(a+c)-ac=0$$

$$\Rightarrow \left(b-\frac{\lambda}{2}\right)^2-\frac{\lambda^2}{4}+\frac{\lambda}{2}(a+c)-ac=0$$

$$\Rightarrow \quad \left(b-\frac{\lambda}{2}\right)^2=\frac{\lambda^2}{4}-\frac{\lambda}{2}(a+c)+ac$$

$$\Rightarrow \quad \left(b-\frac{\lambda}{2}\right)^2=\left(a-\frac{\lambda}{2}\right)\left(c-\frac{\lambda}{2}\right)$$

Hence, $a-\dfrac{\lambda}{2}, b-\dfrac{\lambda}{2}, c-\dfrac{\lambda}{2}$ are in GP.

• Ex. 2 *Let $a_1, a_2, a_3, \ldots, a_{10}$ are in GP with $a_{51}=25$ and $\displaystyle\sum_{i=1}^{101} a_i=125$, then the value of $\displaystyle\sum_{i=1}^{101}\left(\frac{1}{a_i}\right)$ equals*

(a) 5 (b) $\dfrac{1}{5}$ (c) $\dfrac{1}{25}$ (d) $\dfrac{1}{125}$

Sol. (b) Let 1st term be a and common ratio be r, then

$$\sum_{i=1}^{101}\frac{1}{a_i}=125$$

$$\Rightarrow \quad (a_1+a_1r+a_1r^2+\ldots+a_1r^{100})=125$$

$$\Rightarrow \quad \frac{a_1(1-r^{101})}{(1-r)}=125 \qquad [\text{let } 0<r<1] \ldots\text{(i)}$$

$$\therefore \quad \sum_{i=1}^{101}\frac{1}{a_i}=\frac{1}{a_1}+\frac{1}{a_1r}+\frac{1}{a_1r^2}+\ldots+\frac{1}{a_1r^{100}}=\frac{\frac{1}{a_1}\left[\left(\frac{1}{r}\right)^{101}-1\right]}{\left(\frac{1}{r}-1\right)}$$

$$\left[\text{here } \frac{1}{r}>1\right]$$

$$=\frac{(1-r^{101})}{a_1r^{100}(1-r)}=\frac{1}{a_1r^{100}}\times\frac{125}{a_1} \qquad [\text{from Eq. (i)}]$$

$$=\frac{125}{(a_1r^{50})^2}=\frac{125}{(a_{51})^2}=\frac{125}{(25)^2}=\frac{1}{5}$$

• Ex. 3 *If $x=111\ldots1$ (20 digits), $y=333\ldots3$ (10 digits) and $z=222\ldots2$ (10 digits), then $\dfrac{x-y^2}{z}$ equals*

(a) $\dfrac{1}{2}$ (b) 1 (c) 2 (d) 4

Sol. (b) $\because$ $x=\dfrac{1}{9}(999\ldots9)=\dfrac{1}{9}(10^{20}-1)$,

$$y=\frac{1}{3}(999\ldots9)=\frac{1}{3}(10^{10}-1)$$

and
$$z=\frac{2}{9}(999\ldots9)=\frac{2}{9}(10^{10}-1)\,]$$

$$\therefore \quad \frac{x-y^2}{z}=\frac{\frac{1}{9}(10^{20}-1)-\frac{1}{9}(10^{10}-1)^2}{\frac{2}{9}(10^{10}-1)}$$

$$=\frac{10^{10}+1-(10^{10}-1)}{2}=1$$

• Ex. 4 *Consider the sequence 1, 2, 2, 3, 3, 3, ..., where n occurs n times. The number that occurs as 2011th terms is*

(a) 61 (b) 62
(c) 63 (d) 64

Sol. (c) The last 4 occurs as $1+2+3+4=10$th term. The last n occurs as $\left(\dfrac{n(n+1)}{2}\right)^{\text{th}}$ term, the last 62 occurs as

$$\left(\frac{62\times 63}{2}\right)^{\text{th}}=1953\text{rd term and the last 63 occurs as}$$

$$\left(\frac{63\times 64}{2}\right)^{\text{th}}=2016\text{th term.}$$

$\therefore$ 63 occurs from 1954th term to 2016th term.

Hence, $(2011)^{\text{th}}$ term is 63.

• Ex. 5 *Let $S=\displaystyle\sum_{r=1}^{117}\frac{1}{2[\sqrt{r}]+1}$, when $[\cdot]$ denotes the greatest integer function and if $S=\dfrac{p}{q}$, when p and q are co-primes, the value of $p+q$ is*

(a) 20 (b) 76 (c) 19 (d) 69

Sol. (b) $\because$ $S=\displaystyle\sum_{r=1}^{117}\frac{1}{2[\sqrt{r}]+1}$

$$=\frac{3}{2\cdot1+1}+\frac{5}{2\cdot2+1}+\frac{7}{2\cdot3+1}+\ldots+\frac{19}{2\cdot9+1}+\frac{18}{2\cdot10+1}$$

$$=9+\frac{18}{21}=9+\frac{6}{7}=\frac{69}{7}$$

$$\therefore \quad p=69 \text{ and } q=7 \Rightarrow p+q=69+7=76$$

• Ex. 6 *If a, b, c are non-zero real numbers, then the minimum value of the expression $\dfrac{(a^8+4a^4+1)(b^4+3b^2+1)(c^2+2c+2)}{a^4b^2}$ equals*

(a) 12 (b) 24 (c) 30 (d) 60

Sol. (c) Let $P=\dfrac{(a^8+4a^4+1)(b^4+3b^2+1)(c^2+2c+2)}{a^4b^2}$

$$=\left(a^4+4+\frac{1}{a^4}\right)\left(b^2+3+\frac{1}{b^2}\right)\{(c+1)^2+1\}$$

$$\because \; =a^4+4+\frac{1}{a^4}\geq 6,\; b^2+3+\frac{1}{b^2}\geq 5 \text{ and } (c+1)^2+1\geq 1$$

$$\left[\because x+\frac{1}{x}\geq 2 \text{ for } x>0\right]$$

$$\therefore \quad P\geq 6\cdot5\cdot1=30 \Rightarrow P\geq 30$$

Hence, the required minimum value is 30.

• Ex. 7 *If the sum of m consecutive odd integers is m^4, then the first integer is*

(a) m^3+m+1 (b) m^3+m-1
(c) m^3-m-1 (d) m^3-m+1

Sol. (d) Let $2a+1, 2a+3, 2a+5, \ldots$ be the AP, then

$$m^4=(2a+1)+(2a+3)+(2a+5)+\ldots \text{ upto } m \text{ terms}$$

$$=\frac{m}{2}\{2(2a+1)+(m-1)\cdot 2\}=m(2a+1+m-1)$$

$$\Rightarrow \quad m^3=(2a+1)+m-1$$

$$\therefore \quad 2a+1=m^3-m+1$$

• Ex. 8 *The value of $\displaystyle\sum_{r=1}^{\infty}\frac{(4r+5)5^{-r}}{r(5r+5)}$ is*

(a) $\dfrac{1}{5}$ (b) $\dfrac{2}{5}$ (c) $\dfrac{1}{25}$ (d) $\dfrac{2}{125}$

Sol. (a) $\displaystyle\sum_{r=1}^{\infty}\frac{(4r+5)5^{-r}}{r(5r+5)}=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{(5r+5)-r}{r(5r+5)}\right)\cdot\frac{1}{5^r}$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{5r+5}\right)\frac{1}{5^r}$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r\cdot5^{r}}-\frac{1}{(r+1)5^{r+1}}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{5}-\frac{1}{(n+1)5^{n+1}}\right)=\frac{1}{5}-0=\frac{1}{5}$$

• Ex. 9 *Let λ be the greatest integer for which $5p^2-16$, $2p\lambda$, λ^2 are distinct consecutive terms of an AP, where $p\in R$. If the common difference of the AP is $\left(\dfrac{m}{n}\right)$, $m, n\in N$ and m, n are relative prime, the value of $m+n$ is*

(a) 133 (b) 138 (c) 143 (d) 148

Sol. (c) $\because$ $5p^2-16, 2p\lambda, \lambda^2$ are in AP, then

$$4p\lambda=5p^2-16+\lambda^2$$

$$\Rightarrow \quad 5p^2-4p\lambda+\lambda^2-16=0 \qquad \ldots\text{(i)}$$

$$\therefore \quad B-4AC\geq 0 \qquad [\because p\in R]$$

$$\Rightarrow \quad 16\lambda^2-4\cdot5\cdot(\lambda^2-16)\geq 0$$

$$\Rightarrow \quad -\lambda^2+80\geq 0 \text{ or } \lambda^2\geq 80$$

$$\Rightarrow \quad -\sqrt{80}\leq\lambda\leq\sqrt{80}$$

$$\therefore \quad \lambda=8 \qquad [\text{greatest integer}]$$

From Eq. (i), $\quad 5p^2-32p+48=0$

$$\Rightarrow \quad (p-4)(5p-12)=0$$

$$\therefore \quad p=4,\; p=\frac{12}{5}$$

$$\Rightarrow \quad p=\frac{12}{5},\; p\neq 4$$

[for $p=4$ all terms are equal]

Now, common difference $=\lambda^2-2p\lambda$

$$=64-16\times\frac{12}{5}=64\left(1-\frac{3}{5}\right)=\frac{128}{5}=\frac{m}{n} \qquad [\text{given}]$$

$$\therefore \quad m=128 \text{ and } n=5$$

Hence, $\quad m+n=133$

• Ex. 10 *If 2λ, λ and $[\lambda^2-14]$, $\lambda\in R-\{0\}$ and $[.]$ denotes the greatest integer function are the first three terms of a GP in order, then the 51th term of the sequence, $1, 3\lambda, 6\lambda, 10\lambda, \ldots$ is*

(a) 5104 (b) 5304
(c) 5504 (d) 5704

Sol. (b) $\because$ $2\lambda, \lambda, [\lambda^2-14]$ are in GP, then

$$\lambda^2=2\lambda[\lambda^2-14]$$

$$\Rightarrow \quad \frac{\lambda}{2}=[\lambda^2-14]$$

$\therefore$ λ must be an even integer

Hence, $\quad \lambda=4$

Now, required sequence 1, 12, 24, 40, ...

or 1, $4(1+2)$, $4(1+2+3)$, $4(1+2+3+4)$, ...

$$\therefore \quad 51\text{th term}=4(1+2+3+\ldots+51)$$

$$=4\cdot\frac{51}{2}(1+51)=4\cdot51\cdot26=5304$$

JEE Type Solved Examples : More than One Correct Option Type Questions

- This section contains **5 multiple choice examples.** Each example has four choices (a), (b), (c) and (d) out of **which more than one** may be correct.

• Ex. 11 *The first three terms of a sequence are 3, −1, −1. The next terms are*

(a) 2 (b) 3 (c) $-\frac{5}{27}$ (d) $-\frac{5}{9}$

Sol. (b, d) The given sequence is not an AP or GP or HP. It is an AGP, $3, (3+d)r, (3+2d)r^2, \ldots$

$\Rightarrow \quad (3+d)r = -1, (3+2d)r^2 = -1$

Eliminating r, we get $\quad (3+d)^2 = -(3+2d)$

$\Rightarrow \quad d^2 + 8d + 12 = 0 \Rightarrow d = -2, -6,$

then $\quad r = -1, \frac{1}{3}$

$\therefore$ Next term is $(3+3d)r^3 = 3, -\frac{5}{9}$

• Ex. 12 *There are two numbers a and b whose product is 192 and the quotient of AM by HM of their greatest common divisor and least common multiple is $\frac{169}{48}$. The smaller of a and b is*

(a) 2 (b) 4 (c) 6 (d) 12

Sol. (b, d) If G = GED of a and b, L = LCM of a and b, we have $GL = ab = 192$ ...(i)

$\frac{\text{AM}}{\text{HM}}$ of G and L is $\left(\frac{G+L}{2}\right)\left(\frac{G+L}{2GL}\right) = \frac{169}{48}$

$\Rightarrow \quad (G+L)^2 = \frac{169}{12}GL = \frac{169}{12} \times 192 = 13^2 \cdot 4^2$

$\Rightarrow \quad G + L = 52$ but $GL = 192$

$\Rightarrow \quad G = 4, L = 48 \Rightarrow a = 4, b = 48$ or $a = 12, b = 16$

• Ex. 13 *Consider a series* $\frac{1}{2} + \frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \frac{5}{2^5} + \ldots + \frac{\lambda n}{2^n}$. *If S_n denotes its sum to n terms, then S_n cannot be*

(a) 2 (b) 3 (c) 4 (d) 5

Sol. (a, b, c, d)

$\because \quad Sn = \frac{1}{2} + \frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \frac{5}{2^5} + \ldots + \frac{\lambda_n}{2^n}$

$= \frac{3}{4} + \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \frac{5}{2^5} + \ldots + \frac{\lambda_n}{2^n}\right)$

$+ \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{\lambda_n}{2^n}\right) - \frac{1}{4} - \frac{\lambda_n}{2^{n+2}} - \frac{\lambda_n}{2^{n+1}}$

$\Rightarrow \quad S_n = \frac{3}{4} + \frac{1}{4}S_n + \frac{1}{2}S_n - \frac{1}{4} - \frac{\lambda_n}{2^{n+2}} - \frac{\lambda_n}{2^{n+1}}$

$\Rightarrow \frac{1}{4}S_n = \frac{1}{2} - \frac{\lambda_n}{2^{n+2}} - \frac{\lambda_n}{2^{n+1}} \Rightarrow S_n = 2 - \frac{\lambda_n}{2^{n+1}} - \frac{\lambda_n}{2^{n-1}} < 2$

• Ex. 14 *If* $S_r = \sqrt{r + \sqrt{r + \sqrt{r + \sqrt{\ldots \infty}}}}, r > 0$ *then which the following is/are correct.*

(a) S_r, S_6, S_{12}, S_{20} are in AP

(b) S_4, S_9, S_{16} are irrational

(c) $(2S_4 - 1)^2, (2S_5 - 1)^2 (2S_6 - 1)^2$ are in AP

(d) S_2, S_{12}, S_{56} are in GP

Sol. (a, b, c, d)

$\because S_r = \sqrt{r + \sqrt{r + \sqrt{+\sqrt{\ldots \infty}}}} = \sqrt{r + S_r}$

$\Rightarrow \quad S_r^2 - S_r - r = 0$

$\therefore \quad S_r = \frac{1 + \sqrt{(1+4r)}}{2} \quad [\because r > 0]$

Alternate (a) S_2, S_6, S_{12}, S_{20} i.e., 2, 3, 4, 5 are in AP.

Alternate (b) S_4, S_9, S_{16} i.e., $\frac{1+\sqrt{17}}{2}, \frac{1+\sqrt{37}}{2}, \frac{1+\sqrt{65}}{2}$ are irrationals.

Alternate (c) $(2S_{4} - 1)^2, (2S_5 - 1)^2, (2S_6 - 1)^2$ i.e., 17, 21, 25 are in AP

Alternate (d) S_2, S_{12}, S_{56} i.e., 2, 4, 8 are in GP.

• Ex. 15 *If* $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ *are in AP and a, b, −2c are in GP, where a, b, c are non-zero, then*

(a) $a^3 + b^3 + c^3 = 3abc$ (b) $-2a, b, -2c$ are in AP

(c) $-2a, b, -2c$ are in GP (d) $a^2, b^2, 4c^2$ are in GP

Sol. (a, b, d)

$\because \quad \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP $\Rightarrow a, b, c$ are in HP

$\therefore \quad b = \frac{2ab}{a+c}$ ...(i)

and $a, b, -2c$ are in GP, then $b^2 = -2ac$ ...(ii)

From Eqs. (i) and (ii), we get

$b = \frac{-b^2}{a+c} \Rightarrow a + b + c = 0 \quad [\because b \neq 0]$

$\therefore \quad a^3 + b^3 + c^3 = 3abc$ and $a, b, -2c$ are in GP

$\Rightarrow \quad a^2, b^2, 4c^2$ are also in GP and $a + b + c = 0$

$\Rightarrow \quad 2b = -2a - 2c$

$\therefore \quad -2a, b, -2c$ are in AP.

JEE Type Solved Examples : Passage Based Questions

- This section contains **3 solved passages** based upon each of the passage **3 multiple choice** examples have to be answered. Each of these examples has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Passage I

(Ex. Nos. 16 to 18)

Consider a sequence whose sum to n terms is given by the quadratic function $S_n = 3n^2 + 5n$.

16. *The nature of the given series is*

(a) AP (b) GP (c) HP (d) AGP

Sol. (a) $\because \quad S_n = 3n^2 + 5n$

$\therefore \quad T_n = S_n - S_{n-1}$

$= (3n^2 + 5n) - [3(n-1)^2 + 5(n-1)]$

$= 3(2n - 1) + 5 = 6n + 2$

The nth term is a linear function in n. Hence, sequence must be an AP.

17. *For the given sequence, the number 5456 is the*

(a) 153 th term (b) 932 th term

(c) 707 th term (d) 909 th term

Sol. (d) Given, $\quad T_n = 5456$

$\Rightarrow \quad 6n + 2 = 5456 \Rightarrow 6n = 5454$

$\therefore \quad n = 909$

$\therefore$ The number 5456 is the 909 th term.

18. *Sum of the squares of the first 3 terms of the given series is*

(a) 1100 (b) 660 (c) 799 (d) 1000

Sol. (b) $T_1^2 + T_2^2 + T_3^2 = 8^2 + 14^2 + 20^2 = 64 + 196 + 400 = 660$

Passage II

(Ex. Nos. 19 to 21)

Let r be the number of identical terms in the two AP's. Form the sequence of identical terms, it will be an AP, then the rth term of this AP make $t_r \leq$ the smaller of the last term of the two AP's.

19. *The number of terms common to two AP's 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is*

(a) 14 (b) 21 (c) 28 (d) 35

Sol. (a) Sequence 3, 7, 11, ... , 407 has common difference = 4 and sequence 2, 9, 16, ... , 709 has common difference = 7.

Hence, the sequence with common terms has common difference LCM of 4 and 7 which is 28.

The first common term is 23.

Hence, the sequence is 23, 51, 79, ... , 387 which has 14 terms.

Aliter By inspection, first common term to both the series is 23, second common term is 51, third common term is 79 and so on. These numbers form an AP 23, 51, 79, ...

Since, $\quad T_{14} = 23 + 13(28) = 387 < 407$

and $\quad T_{15} = 23 + 14(28) = 415 > 407$

Hence, number of common terms = 14

20. *The 10th common term between the series* $3 + 7 + 11 + \ldots$ *and* $1 + 6 + 11 + \ldots$ *is*

(a) 189 (b) 191 (c) 211 (d) 213

Sol. (b) Series $3 + 7 + 11 + \ldots$ has common difference = 4 and series $1 + 6 + 11 + \ldots$ has common difference = 5

Hence, the series with common terms has common difference LCM of 4 and 5 which is 20.

The first common terms is 11.

Hence, the series is $11 + 31 + 51 + 71 + \ldots$

$\therefore \quad t_{10} = 11 + (10-1)(20) = 191$

Aliter t_n for $3 + 7 + 11 + \ldots = 3 + (n-1)(4) = 4n - 1$

and t_m for $1 + 6 + 11 + \ldots = 1 + (m-1)(5) = 5m - 4$

For a common term, $4n - 1 = 5m - 4$ i.e., $4n = 5m - 3$

For $m = 3, n = 3$ gives the first common term i.e., 11.

For $m = 7, n = 8$ gives the second common term i.e., 31.

For $m = 11, n = 13$ gives the third common term i.e., 51.

Hence, the common term series is $11 + 31 + 51 + \ldots$

$\therefore \quad t_{10} = 11 + (10-1)20 = 191$

21. *The value of largest term common to the sequences 1, 11, 21, 31, ... upto 100 terms and 31, 36, 41, 46, ... upto 100 terms, is*

(a) 281 (b) 381 (c) 471 (d) 521

Sol. (d) Sequence 1, 11, 21, 31, ... has common difference = 10 and sequence 31, 36, 41, 46, ... has common difference = 5. Hence, the sequence with common terms has common difference LCM of 10 and 5 which is 10.

The first common term is 31.

Hence, the sequence is 31, 41, 51, 61, 71, ...(i)

Now, t_{100} of first sequence $= 1 + (100 - 1)10 = 991$

and t_{100} of second sequence $= 31 + (100 - 1)5 = 526$

Value of largest common term < 526

$\therefore t_n$ of Eq. (i) is $31 + (n-1)10 = 10n + 21$

$t_{50} = 10 \times 50 + 21 = 521$

is the value of largest common term.

Aliter Let mth term of the first sequence be equal to the nth term of the second sequence, then

$1 + (m-1)10 = 31 + (n-1)5$

$\Rightarrow \quad 10m - 9 = 5n + 26 \Rightarrow 10m - 35 = 5n$

$\Rightarrow \quad 2m - 7 = n \le 100 \Rightarrow 2m \le 107$

$\Rightarrow \quad m \le 53\frac{1}{2}$

$\therefore$ Largest value of $m = 53$

$\therefore$ Value of largest term $= 1 + (53 - 1)\,10 = 521$

Passage III

(Ex. Nos. 22 to 24)

We are giving the concept of arithmetic mean of mth power. Let $a_1, a_2, a_3, \ldots, a_n$ be n positive real numbers (not all equal) and m be a real number. Then,

$$\frac{a_1^m + a_2^m + a_3^m + \ldots + a_n^m}{n} > \left(\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}\right)^m,$$

if $m \in R \sim [0, 1]$

However, if $m \in (0, 1)$, then

$$\frac{a_1^m + a_2^m + a_3^m + \ldots + a_n^m}{n} < \left(\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}\right)^m$$

Obviously, if $m = \{0, 1\}$, then

$$\frac{a_1^m + a_2^m + a_3^m + \ldots + a_n^m}{n} = \left(\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}\right)^m.$$

22. *If $x > 0, y > 0, z > 0$ and $x + y + z = 1$, the minimum value of $\frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z}$, is*

(a) 0.2 (b) 0.4
(c) 0.6 (d) 0.8

Sol. (c) Since, AM of (-1)th powers $\ge (-1)$th powers of AM

$$\therefore \frac{(2-x)^{-1} + (2-y)^{-1} + (2-z)^{-1}}{3} \ge \left(\frac{2-x+2-y+2-z}{3}\right)^{-1}$$

$$= \left[\frac{6-(x+y+z)}{3}\right]^{-1} = \left(\frac{6-1}{3}\right)^{-1} = \frac{3}{5} \quad [\because x + y + z = 1]$$

$$\Rightarrow \frac{(2-x)^{-1} + (2-y)^{-1} + (2-z)^{-1}}{3} \ge \frac{3}{5}$$

or $\quad \frac{1}{3}\left[\frac{1}{2-x} + \frac{1}{2-y} + \frac{1}{2-z}\right] \ge \frac{3}{5}$

$\Rightarrow \quad \frac{1}{2-x} + \frac{1}{2-y} + \frac{1}{2-z} \ge \frac{9}{5}$

or $\quad \frac{2}{2-x} + \frac{2}{2-y} + \frac{2}{2-z} \ge \frac{18}{5}$

or $\quad 1 + \frac{x}{2-x} + 1 + \frac{y}{2-y} + 1 + \frac{z}{2-z} \ge \frac{18}{5}$

or $\quad \frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z} \ge \frac{18}{5} - 3$

Hence, $\quad \frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z} \ge \frac{3}{5} = 0.6$

$\Rightarrow \quad \frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z} \ge 0.6$

Thus, minimum value of $\frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z}$ is 0.6.

23. *If $\sum_{i=1}^{n} a_i^2 = \lambda, \forall\, a_i \ge 0$ and if greatest and least values of $\left(\sum_{i=1}^{n} a_i\right)^2$ are λ_1 and λ_2 respectively, then $(\lambda_1 - \lambda_2)$ is*

(a) $n\lambda$ (b) $(n-1)\lambda$
(c) $(n+2)\lambda$ (d) $(n+1)\lambda$

Sol. (b) $\because$ AM of 2nd powers $\ge$ 2nd power of AM

$$\therefore \frac{a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2}{n} \ge \left(\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}\right)^2$$

$$\Rightarrow \frac{\lambda}{n} \ge \left(\frac{\sum_{i=1}^{n} a_i}{n}\right)^2 \qquad \therefore \left(\sum_{i=1}^{n} a_i\right)^2 \le n\lambda \qquad \ldots(\text{i})$$

Also, $(a_1 + a_2 + a_3 + \ldots + a_n)^2 = a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2 + 2\sum a_1 a_2$

$$= \lambda + 2\sum a_1 a_2 \ge \lambda$$

$$\therefore \left(\sum_{i=1}^{n} a_i\right)^2 \ge \lambda \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\lambda \le \left(\sum_{i=1}^{n} a_i\right)^2 \le n\lambda$$

$\therefore \quad \lambda_1 = n\lambda$ and $\lambda_2 = \lambda$

Then, $\quad \lambda_1 - \lambda_2 = (n-1)\lambda$

24. *If sum of the mth powers of first n odd numbers is λ, $\forall\, m > 1$, then*

(a) $\lambda < n^m$ (b) $\lambda > n^m$ (c) $\lambda < n^{m+1}$ (d) $\lambda > n^{m+1}$

Sol. (d) $\because m > 1$

$$\therefore \frac{1^m + 3^m + 5^m + \ldots + (2n-1)^m}{n} > \left(\frac{1 + 3 + 5 + \ldots + (2n-1)}{n}\right)^m = \left(\frac{\frac{n}{2}(1 + 2n - 1)}{n}\right)^m = n^m$$

$\therefore 1^m + 3^m + 5^m + \ldots + (2n-1)^m > n^{m+1}$

Hence, $\quad \lambda > n^{m+1}$

JEE Type Solved Examples : Single Integer Answer Type Questions

- This section contains **2 examples.** The answer to each example is a **single digit integer** ranging from **0** to **9** (both inclusive).

- **Ex. 25** *A sequence of positive terms $A_1, A_2, A_3, \ldots, A_n$ satisfies the relation $A_{n+1} = \frac{3(1 + A_n)}{(3 + A_n)}$. Least integral value of A_1 for which the sequence is decreasing can be*

Sol. (2) $\because A_{n+1} = \frac{3(1 + A_n)}{(3 + A_n)}$. For $n = 1$, $A_2 = \frac{3(1 + A_1)}{(3 + A_1)}$

For $n = 2$, $A_3 = \frac{3(1 + A_2)}{(3 + A_2)}$

$$= \frac{3\left(1 + \frac{3(1 + A_1)}{(3 + A_1)}\right)}{3 + \frac{3(1 + A_1)}{(3 + A_1)}} = \frac{6 + 4A_1}{4 + 2A_1} = \frac{3 + 2A_1}{2 + A_1}$$

$\because$ Given, sequence can be written as

$$A_1, \frac{3(1 + A_1)}{(3 + A_1)}, \frac{(3 + 2A_1)}{(2 + A_1)}, \ldots$$

Given, $A_1 > 0$ and sequence is decreasing, then

$$A_1 > \frac{3(1 + A_1)}{(3 + A_1)}, \frac{3(1 + A_1)}{(3 + A_1)} > \frac{(3 + 2A_1)}{(2 + A_1)}$$

$\Rightarrow \quad A_1^2 > 3$ or $A_1 > \sqrt{3}$

$\therefore \quad A_1 = 2 \qquad$ [least integral value of A_1]

- **Ex. 26** *When the ninth term of an AP is divided by its second term we get 5 as the quotient, when the thirteenth term is divided by sixth term the quotient is 2 and the remainder is 5, then the second term is*

Sol. (7) Let a be the first term and d be the common difference, then $T_9 = 5\,T_2$

$\Rightarrow \quad (a + 8d) = 5(a + d)$

$\therefore \quad 4a = 3d \qquad \ldots(\text{i})$

and $\quad T_{13} = T_6 \times 2 + 5$

$\Rightarrow \quad a + 12d = 2(a + 5d) + 5$

$\Rightarrow \quad 2d = a + 5 \qquad \ldots(\text{ii})$

From Eqs. (i) and (ii), we get

$a = 3$ and $d = 4$

$\therefore \quad T_2 = a + d = 7$

JEE Type Solved Examples : Matching Type Questions

- This section contains **2 examples.** Examples 27 has three statements (A, B and C) given in **Column I** and four statements (p, q, r and s) in **Column II** example 28 has four statements (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II.** Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II.**

- **Ex. 27**

	Column I		Column II
(A)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_6 + a_{10} + a_{21} + a_{25} + a_{30} = 120$, then $\sum_{i=1}^{30} a_i$ is	(p)	400
(B)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_5 + a_9 + a_{13} + a_{17} + a_{21} + a_{25} = 112$, then $\sum_{i=1}^{25} a_i$ is	(q)	600
(C)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 375$, then $\sum_{i=1}^{16} a_i$ is	(r)	800
		(s)	1000

Sol. (A) $\to$ (q); (B) $\to$ (p); (C) $\to$ (s)

(A) $\because a_1, a_2, a_3, \ldots$ are in AP.

$\therefore \quad a_1 + a_{30} = a_6 + a_{25} = a_{10} + a_{21} = \lambda \qquad$ [say]

$\because \quad a_1 + a_6 + a_{10} + a_{21} + a_{25} + a_{30} = 120$

$\therefore \quad 3\lambda = 120$

$\Rightarrow \quad \lambda = 40$

Then, $\quad \sum_{i=1}^{30} a_i = \frac{30}{2}(a_1 + a_{30}) = 15 \times \lambda = 15 \times 40 = 600$

(B) $\because a_1, a_2, a_3, \ldots$ are in AP.

$\therefore \quad a_1 + a_{25} = a_5 + a_{21}$

$= a_9 + a_{17} = a_{13} + a_{13} = \lambda \qquad$ [say]

$\because \quad a_1 + a_5 + a_9 + a_{13} + a_{17} + a_{21} + a_{25} = 112$

$\therefore \quad 3\lambda + \frac{\lambda}{2} = 112$

$\Rightarrow \quad \frac{7\lambda}{2} = 112$

$\Rightarrow \quad \lambda = 32$

Then, $\quad \sum_{i=1}^{25} a_i = \frac{25}{2}(a_1 + a_{25}) = \frac{25}{2} \times 32 = 400$

(C) $\because a_1, a_2, a_3, \ldots$ are in AP.

$\therefore \quad a_1 + a_{16} = a_4 + a_{13} = a_7 + a_{10} = \lambda$ [say]

$\because \quad a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 375$

$\therefore \quad 3\lambda = 375 \quad \therefore \quad \lambda = 125$

Then, $\sum_{i=1}^{16} a_i = \frac{16}{2}(a_1 + a_{16})$

$= 8 \times \lambda = 8 \times 125 = 1000$

- ***Ex. 28***

Column I		Column II
(A) If $a > 0, b > 0, c > 0$ and the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$ is λabc, then λ is	(p)	2
(B) If a, b, c are positive, $a + b + c = 1$ and the minimum value of $\left(\frac{1}{a} - 1\right)\left(\frac{1}{b} - 1\right)\left(\frac{1}{c} - 1\right)$ is λ, then λ is	(q)	4
(C) If $a > 0, b > 0, c > 0, s = a + b + c$ and the minimum value of $\frac{2s}{s-a} + \frac{2s}{s-b} + \frac{2s}{s-c}$ is $(\lambda - 1)$, then λ is	(r)	6
(D) If $a > 0, b > 0, c > 0$, a, b, c are in GP and the the minimum value of $\left(\frac{a}{b}\right)^\lambda + \left(\frac{c}{b}\right)^\lambda$ is 2, then λ is	(s)	8
	(t)	10

Sol. (A) → (r); (B) → (s); (C) → (t); (D) → (p, q, r, s, t)

(A) $\because$ AM ≥ GM

$\therefore \quad \frac{ab^2 + ac^2 + bc^2 + ba^2 + ca^2 + cb^2}{6} \geq (ab^2 \cdot ac^2 \cdot bc^2 \cdot ba^2 \cdot ca^2 \cdot cb^2)^{1/6} = abc$

$\therefore \quad a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2) \geq 6\,abc$

$\Rightarrow \quad \lambda = 6$

(B) $\because$ AM ≥ GM

For b, c, we get $\frac{(b + c)}{2} \geq \sqrt{bc}$

$\Rightarrow \quad (b + c) \geq 2\sqrt{bc}$ ...(i)

For c, a, we get $\frac{(c + a)}{2} \geq \sqrt{ca}$

$\Rightarrow \quad (c + a) \geq 2\sqrt{ca}$

and for a, b, we get $\frac{(a + b)}{2} \geq \sqrt{ab}$

$\Rightarrow \quad (a + b) \geq 2\sqrt{ab}$

On multiplying Eqs. (i), (ii) and (iii), we get

$(b + c)(c + a)(a + b) \geq 8abc$

$\Rightarrow \quad (1 - a)(1 - b)(1 - c) \geq 8abc \quad [\because a + b + c = 1]$

$\Rightarrow \quad \left(\frac{1}{a} - 1\right)\left(\frac{1}{b} - 1\right)\left(\frac{1}{c} - 1\right) \geq 8$

$\therefore \quad \lambda = 8$

(C) $\because$ AM ≥ HM

$\frac{(s - a) + (s - b) + (s - c)}{3} \geq \frac{3}{\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c}}$

$\Rightarrow \quad \frac{3s - (a + b + c)}{3} \geq \frac{3}{\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c}}$

$\Rightarrow \quad \frac{3s - s}{3} \geq \frac{3}{\left(\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c}\right)}$

$\Rightarrow \quad \frac{2s}{s-a} + \frac{2s}{s-b} + \frac{2s}{s-c} \geq 9$

Here, $\lambda - 1 = 9$

$\therefore \quad \lambda = 10$

(D) If a, b, c are in GP.

Then, $a^\lambda, b^\lambda, c^\lambda$ are also in GP.

Then, AM ≥ GM

$\frac{a^\lambda + c^\lambda}{2} \geq b^\lambda$

$\Rightarrow \quad a^\lambda + c^\lambda \geq 2b^\lambda$

$\Rightarrow \quad \left(\frac{a}{b}\right)^\lambda + \left(\frac{c}{b}\right)^\lambda \geq 2$

$\therefore \quad \lambda \in R$

Hence, $\lambda = 2, 4, 6, 8, 10$

JEE Type Solved Examples : Statement I and II Type Questions

- **Directions** Example numbers 29 to 32 are Assertion-Reason type examples. Each of these examples contains two statements:

Statement-1 (Assertion) and **Statement-2** (Reason)

Each of these examples also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

- ***Ex. 29.* Statement 1** *The sum of first n terms of the series $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \ldots$ can be $= \pm \frac{n(n+1)}{2}$.*

Statement 2 *Sum of first n natural numbers is $\frac{n(n+1)}{2}$.*

Sol. (a) Clearly, nth term of the given series is negative or positive according as n is even or odd, respectively.

Case I When n is even, in this case the given series is

$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots + (n - 1)^2 - n^2$

$= (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \ldots + [(n - 1)^2 - n^2]$

$= (1 - 2)(1 + 2) + (3 - 4)(3 + 4) + (5 - 6)(5 + 6) + \ldots + (n - 1 - n)(n - 1 + n)$

$= -(1 + 2 + 3 + 4 + 5 + 6 + \ldots + (n - 1) + n) = -\frac{n(n + 1)}{2}$

Case II When n is odd, in this case the given series is

$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots + (n - 2)^2 - (n - 1)^2 + n^2$

$= (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \ldots + [(n - 2)^2 - (n - 1)^2] + n^2$

$= (1 - 2)(1 + 2) + (3 - 4)(3 + 4) + (5 - 6)(5 + 6) + \ldots + [(n - 2) - (n - 1)][(n - 2) + (n - 1)] + n^2$

$= -[1 + 2 + 3 + 4 + 5 + 6 + \ldots + (n - 2) + (n - 1)] + n^2$

$= -\frac{(n - 1)(n - 1 + 1)}{2} + n^2 = \frac{n(n + 1)}{2}$

It is clear that Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1.

- ***Ex. 30* Statement 1** *If a, b, c are three positive numbers in GP, then $\left(\frac{a + b + c}{3}\right)\left(\frac{3abc}{ab + bc + ca}\right) = (abc)^{2/3}$.*

Statement-2 *(AM) (HM)=(GM)^2 is true for positive numbers.*

Sol. (c) If a, b be two real, positive and unequal numbers, then

AM $= \frac{a + b}{2}$, GM $= \sqrt{ab}$ and HM $= \frac{2ab}{a + b}$

$\therefore$ AM) (HM) = (GM)^2

This result will be true for n numbers, if they are in GP.

Hence, Statement-1 is true and Statement-2 is false.

- ***Ex. 31*** *Consider an AP with a as the first term and d is the common difference such that S_n denotes the sum to n terms and a_n denotes the nth term of the AP. Given that for some $m, n \in N$, $\frac{S_m}{S_n} = \frac{m^2}{n^2}$ $(m \neq n)$.*

Statement 1 *$d = 2a$ because*

Statement 2 *$\frac{a_m}{a_n} = \frac{2m + 1}{2n + 1}$*

Sol. (c) $\because \quad \frac{S_m}{S_n} = \frac{m^2}{n^2}$

Let $\quad S_m = m^2k, S_n = n^2k$

$\therefore \quad a_m = S_m - S_{m-1} = m^2k - (m - 1)^2k$

$\Rightarrow \quad a_m = (2m - 1)k$

Similarly, $\quad a_n = (2n - 1)k \quad \therefore \quad \frac{a_m}{a_n} = \frac{2m - 1}{2n - 1}$

Statement-2 is false.

Also, $\because \quad a_1 = k, a_2 = 3k, a_3 = 5k, \ldots$

Given, $\quad a_1 = a = k$

$\therefore \quad a_1 = a, a_2 = 3a, a_3 = 5a, \ldots$

$\therefore$ Common difference $d = a_2 - a_1 = a_3 - a_2 = \ldots$

$\Rightarrow \quad d = 2a$

$\therefore$ Statement-1 is true.

- ***Ex. 32* Statement-1** *1, 2, 4, 8, ... is a GP, 4, 8, 16, 32, ... is a GP and 1 + 4, 2 + 8, 4 + 16, 8 + 32, ... is also a GP.*

Statement-2 *Let general term of a GP with common ratio r be T_{k+1} and general term of another GP with common ratio r be T'_{k+1}, then the series whose general term $T''_{k+1} = T_{k+1} + T'_{k+1}$ is also a GP with common ratio r.*

Sol. (a) 1, 2, 4, 8, ...

Common ratio $r = 2$

$\therefore \quad T_{k+1} = 1 \cdot (2)^{k+1-1} = 2^k$

and $\quad$ 4, 8, 16, 32, ...

Common ratio, $r = 2$

$\therefore \quad T'_{k+1} = 4 \cdot (2)^{k+1-1} = 4 \cdot 2^k$

Then, $\quad T_{k+1} + T'_{k+1} = 5 \cdot 2^k = T''_{k+1}$

Common ratio of $T''_{k+1} = \frac{5 \cdot 2^k}{5 \cdot 2^{k-1}} = 2$, which is true.

Hence, Statement-1 and Statement-2 both are true and Statement-2 is the correct explanation of Statement-1.

Sequences and Series Exercise 1:
Single Option Correct Type Questions

- This section contains **30 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct

1. If the numbers x, y, z are in HP, then $\frac{\sqrt{yz}}{\sqrt{y}+\sqrt{z}}, \frac{\sqrt{xz}}{\sqrt{x}+\sqrt{z}}, \frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}$ are in
(a) AP (b) GP
(c) HP (d) None of these

2. If $a_1, a_2, \ldots$ are in HP and $f_k = \sum_{r=1}^{n} a_r - a_k$, then $2^{\alpha_1}, 2^{\alpha_2}, 2^{\alpha_3}, 2^{\alpha_4}, \ldots$ are in $\left\{\text{where } \alpha_1 = \frac{a_1}{f_1}, \alpha_2 = \frac{a_2}{f_2}, \alpha_3 = \frac{a_3}{f_3}, \ldots\right\}$
(a) AP (b) GP
(c) HP (d) None of these

3. ABC is a right angled triangle in which $\angle B = 90°$ and $BC = a$. If n points $L_1, L_2, \ldots, L_n$ on AB are such that AB is divided in $n+1$ equal parts and $L_1M_1, L_2M_2, \ldots, L_nM_n$ are line segments parallel to BC and $M_1, M_2, \ldots, M_n$ are on AC, the sum of the lengths of $L_1M_1, L_2M_2, \ldots, L_nM_n$ is
(a) $\frac{a(n+1)}{2}$ (b) $\frac{a(n-1)}{2}$
(c) $\frac{an}{2}$
(d) impossible to find from the given data

4. Let S_n $(1 \le n \le 9)$ denotes the sum of n terms of the series $1 + 22 + 333 + \ldots + \underbrace{999\ldots9}_{9\text{ times}}$, then for $2 \le n \le 9$
(a) $S_n - S_{n-1} = \frac{1}{9}(10^n - n^2 + n)$
(b) $S_n = \frac{1}{9}(10^n - n^2 + 2n - 2)$
(c) $9(S_n - S_{n-1}) = n(10^n - 1)$
(d) None of the above

5. If a, b, c are in GP, then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in
(a) AP (b) GP
(c) HP (d) None of these

6. Sum of the first n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots$ is equal to
(a) $2^n - n - 1$ (b) $1 - 2^{-n}$
(c) $n + 2^{-n} - 1$ (d) $2^n - 1$

7. If in a ΔPQR, $\sin P, \sin Q, \sin R$ are in AP, then
(a) the altitudes are in AP (b) the altitudes are in HP
(c) the medians are in GP (d) the medians are in AP

8. Let $a_1, a_2, \ldots, a_{10}$ be in AP and $h_1, h_2, \ldots, h_{10}$ be in HP. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then a_4h_7 is
(a) 2 (b) 3
(c) 5 (d) 6

9. If $I_n = \int_0^{\pi} \frac{1 - \sin 2nx}{1 - \cos 2x} dx$, then $I_1, I_2, I_3, \ldots$ are in
(a) AP (b) GP
(c) HP (d) None of these

10. If $a(b-c)x^2 + b(c-a)xy + c(a-b)y^2$ is a perfect square, the quantities a, b, c are in
(a) AP (b) GP
(c) HP (d) None of these

11. The sum to infinity of the series, $1 + 2\left(1 - \frac{1}{n}\right) + 3\left(1 - \frac{1}{n}\right)^2 + \ldots$ is
(a) n^2 (b) $n(n+1)$
(c) $n\left(1 + \frac{1}{n}\right)^2$ (d) None of these

12. If $\log_3 2, \log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in AP, x is equal to
(a) 2 (b) 3
(c) 4 (d) 2, 3

13. Let a, b, c be three positive prime numbers. The progression in which $\sqrt{a}, \sqrt{b}, \sqrt{c}$ can be three terms (not necessarily consecutive), is
(a) AP (b) GP
(c) HP (d) None of these

14. If n is an odd integer greater than or equal to 1, the value of $n^3 - (n-1)^3 + (n-2)^3 - \ldots + (-1)^{n-1} 1^3$ is
(a) $\frac{(n+1)^2(2n-1)}{4}$ (b) $\frac{(n-1)^2(2n-1)}{4}$
(c) $\frac{(n+1)^2(2n+1)}{4}$ (d) None of these

15. If the sides of a right angled triangle form an AP, the sines of the acute angles are
(a) $\frac{3}{5}, \frac{4}{5}$ (b) $\sqrt{3}, \frac{1}{3}$
(c) $\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}+1}{2}}$ (d) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

16. The sixth term of an AP is equal to 2. The value of the common difference of the AP which makes the product $a_1 a_4 a_5$ least, is given by
(a) $\frac{8}{5}$ (b) $\frac{5}{4}$
(c) $\frac{2}{3}$ (d) None of these

17. If the arithmetic progression whose common difference is non-zero, the sum of first $3n$ terms is equal to the sum of the next n terms. The ratio of the sum of the first $2n$ terms to the next $2n$ terms is
(a) $\frac{1}{5}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) None of these

18. The coefficient of x^{n-2} in the polynomial $(x-1)(x-2)(x-3)\ldots(x-n)$, is
(a) $\frac{n(n^2+2)(3n+1)}{24}$
(b) $\frac{n(n^2-1)(3n+2)}{24}$
(c) $\frac{n(n^2+1)(3n+4)}{24}$
(d) None of the above

19. Consider the pattern shown below:
Row 1 1
Row 2 3 5
Row 3 7 9 11
Row 4 13 15 17, 19, etc.
The number at the end of row 60 is
(a) 3659 (b) 3519
(c) 3681 (d) 3731

20. Let a_n be the nth term of an AP. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, the common difference of the AP is
(a) $\alpha - \beta$ (b) $\beta - \alpha$
(c) $\frac{\alpha - \beta}{2}$ (d) None of these

21. If a_1, a_2, a_3, a_4, a_5 are in HP, then $a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5$ is equal to
(a) $2a_1a_5$ (b) $3a_1a_5$
(c) $4a_1a_5$ (d) $6a_1a_5$

22. If a, b, c and d are four positive real numbers such that $abcd = 1$, the minimum value of $(1+a)(1+b)(1+c)(1+d)$ is
(a) 1 (b) 4
(c) 16 (d) 64

23. If a, b, c are in AP and $(a + 2b - c)(2b + c - a)(c + a - b) = \lambda\, abc$, then λ is
(a) 1 (b) 2
(c) 4 (d) None of these

24. If $a_1, a_2, a_3, \ldots$ are in GP with first term a and common ratio r, then $\frac{a_1a_2}{a_1^2 - a_2^2} + \frac{a_2a_3}{a_2^2 - a_3^2} + \frac{a_3a_4}{a_3^2 - a_4^2} + \ldots + \frac{a_{n-1}a_n}{a_{n-1}^2 - a_n^2}$ is equal to
(a) $\frac{nr}{1-r^2}$ (b) $\frac{(n-1)r}{1-r^2}$ (c) $\frac{nr}{1-r}$ (d) $\frac{(n-1)r}{1-r}$

25. The sum of the first ten terms of an AP is four times the sum of the first five terms, the ratio of the first term to the common difference is
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{4}$ (d) 4

26. If $\cos(x-y)$, $\cos x$ and $\cos(x+y)$ are in HP, the $\cos x \sec\left(\frac{y}{2}\right)$ is equal to
(a) $\pm\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $-\frac{1}{\sqrt{2}}$ (d) None of these

27. If 11 AM's are inserted between 28 and 10, the number of integral AM's is
(a) 5 (b) 6
(c) 7 (d) 8

28. If x, y, z are in GP $(x, y, z > 1)$, then $\frac{1}{2x + \ln x}, \frac{1}{4x + \ln y}, \frac{1}{6x + \ln z}$ are in
(a) AP (b) GP
(c) HP (d) None of these

29. The minimum value of the quantity $\frac{(a^2 + 3a + 1)(b^2 + 3b + 1)(c^2 + 3c + 1)}{abc}$, where $a, b, c \in R^+$, is
(a) $\frac{11^3}{2^3}$ (b) 125
(c) 25 (d) 27

30. Let $a_1, a_2, \ldots$ be in AP and $q_1, q_2, \ldots$ be in GP. If $a_1 = q_1 = 2$ and $a_{10} = q_{10} = 3$, then
(a) $a_7 q_{19}$ is not an integer (b) $a_{19} q_7$ is an integer
(c) $a_7 q_{19} = a_{19} q_{10}$ (d) None of these

Sequences and Series Exercise 2 : More than One Correct Option Type Questions

- This section contains **15 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **MORE THAN ONE** may be correct.

31. If $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{2^n - 1}$, then

(a) $a(100) < 100$ (b) $a(100) > 100$
(c) $a(200) > 100$ (d) $a(200) < 100$

32. If the first and $(2n-1)$ th term of an AP, GP and HP are equal and their nth terms are a, b and c respectively, then

(a) $a = b = c$ (b) $a \geq b \geq c$
(c) $a + c = b$ (d) $ac - b^2 = 0$

33. For $0 < \phi < \frac{\pi}{2}$, if $x = \sum_{n=0}^{\infty} \cos^{2n} \phi$, $y = \sum_{n=0}^{\infty} \sin^{2n} \phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n} \phi \sin^{2n} \phi$, then

(a) $xyz = xz + y$ (b) $xyz = xy + z$
(c) $xyz = x + y + z$ (d) $xyz = yz + x$

34. If a, b, c are in AP and a^2, b^2, c^2 are in HP, then which of the following could hold true?

(a) $-\frac{a}{2}, b, c$ are in GP (b) $a = b = c$
(c) a^3, b^3, c^3 are in GP (d) None of these

35. The next term of the GP $x, x^2 + 2, x^3 + 10$ is

(a) 0 (b) 6 (c) $\frac{729}{16}$ (d) 54

36. If the sum of n consecutive odd numbers is $25^2 - 11^2$, then

(a) $n = 14$ (b) $n = 16$
(c) first odd number is 23 (d) last odd number is 49

37. The GM of two positive numbers is 6. Their AM is A and HM is H satisfy the equation $90A + 5H = 918$, then A may be equal to

(a) $\frac{1}{5}$ (b) 5 (c) $\frac{5}{2}$ (d) 10

38. If the sum to n terms of the series $\frac{1}{1 \cdot 3 \cdot 5 \cdot 7} + \frac{1}{3 \cdot 5 \cdot 7 \cdot 9} + \frac{1}{5 \cdot 7 \cdot 9 \cdot 11} + \ldots$ is $\frac{1}{90} - \frac{\lambda}{f(n)}$, then

(a) $f(0) = 15$ (b) $f(1) = 105$
(c) $f(\lambda) = \frac{640}{27}$ (d) $\lambda = \frac{1}{3}$

39. For the series,

$$S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + \ldots$$

(a) 7th term is 16 (b) 7th term is 18
(c) sum of first 10 terms is $\frac{505}{4}$ (d) sum of first 10 terms is $\frac{405}{4}$

40. Let $E = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$, then

(a) $E < 3$ (b) $E > \frac{3}{2}$ (c) $E < 2$ (d) $E > 2$

41. Let S_n $(n \geq 1)$ be a sequence of sets defined by $S_1 = \{0\}$, $S_2 = \left\{\frac{3}{2}, \frac{5}{2}\right\}$, $S_3 = \left\{\frac{8}{3}, \frac{11}{3}, \frac{14}{3}\right\}$, $S_4 = \left\{\frac{15}{4}, \frac{19}{4}, \frac{23}{4}, \frac{27}{4}\right\}$, ..., then

(a) third element in S_{20} is $\frac{439}{20}$
(b) third element in S_{20} is $\frac{431}{20}$
(c) sum of the elements in S_{20} is 589
(d) sum of the elements in S_{20} is 609

42. Which of the following sequences are unbounded?

(a) $\left(1 + \frac{1}{n}\right)^n$ (b) $\left(\frac{2n+1}{n+2}\right)$ (c) $\left(1 + \frac{1}{n}\right)^{n^2}$ (d) $\tan n$

43. Let a sequence $\{a_n\}$ be defined by $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{3n}$, then

(a) $a_2 = \frac{11}{12}$ (b) $a_2 = \frac{19}{20}$
(c) $a_{n+1} - a_n = \frac{(9n+5)}{(3n+1)(3n+2)(3n+3)}$
(d) $a_{n+1} - a_n = \frac{-2}{3(n+1)}$

44. Let $S_n(x) = \left(x^{n-1} + \frac{1}{x^{n-1}}\right) + 2\left(x^{n-2} + \frac{1}{x^{n-2}}\right) + \ldots + (n-1)\left(x + \frac{1}{x}\right) + n$, then

(a) $S_1(x) = 1$ (b) $S_1(x) = x + \frac{1}{x}$
(c) $S_{100}(x) = \frac{1}{x^{99}}\left(\frac{x^{100}-1}{x-1}\right)^2$ (d) $S_{100}(x) = \frac{1}{x^{100}}\left(\frac{x^{100}-1}{x-1}\right)^2$

45. All the terms of an AP are natural numbers and the sum of the first 20 terms is greater than 1072 and less than 1162. If the sixth term is 32, then

(a) first term is 7 (b) first term is 12
(c) common difference is 4 (d) common difference is 5

Sequences and Series Exercise 3 : Passage Based Questions

- This section contains **8 passages**. Based upon each of the passage **3 multiple choice questions** have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Passage I
(Q. Nos. 46 to 48)

S_n be the sum of n terms of the series $\frac{8}{5} + \frac{16}{65} + \frac{24}{325} + \ldots$

46. The value of $\lim_{n \to \infty} S_n$ is

(a) 0 (b) $\frac{1}{2}$ (c) 2 (d) 4

47. The seventh term of the series is

(a) $\frac{56}{2505}$ (b) $\frac{56}{6505}$ (c) $\frac{56}{5185}$ (d) $\frac{56}{9605}$

48. The value of S_8, is

(a) $\frac{288}{145}$ (b) $\frac{1088}{545}$ (c) $\frac{81}{41}$ (d) $\frac{107}{245}$

Passage II
(Q. Nos. 49 to 51)

Two consecutive numbers from 1, 2, 3, ..., n are removed. The arithmetic mean of the remaining numbers is $\frac{105}{4}$.

49. The value of n lies in

(a) (41, 51) (b) (52, 62) (c) (63, 73) (d) (74, 84)

50. The removed numbers

(a) are less than 10 (b) lies between 10 to 30
(c) lies between 30 to 70 (d) greater than 70

51. Sum of all numbers is

(a) less than 1000 (b) lies between 1200 to 1500
(c) greater than 1500 (d) None of these

Passage III
(Q. Nos. 52 to 54)

There are two sets A and B each of which consists of three numbers in AP whose sum is 15 and where D and d are the common differences such that $D = 1 + d$, $d > 0$. If $p = 7(q - p)$, where p and q are the product of the numbers respectively in the two series.

52. The value of p is

(a) 105 (b) 140 (c) 175 (d) 210

53. The value of q is

(a) 200 (b) 160 (c) 120 (d) 80

54. The value of $7D + 8d$ is

(a) 37 (b) 22 (c) 67 (d) 52

Passage IV
(Q. Nos. 55 to 57)

There are two sets A and B each of which consists of three numbers in GP whose product is 64 and R and r are the common ratios such that $R = r + 2$. If $\frac{p}{q} = \frac{3}{2}$, where p and q are sum of numbers taken two at a time respectively in the two sets.

55. The value of p is

(a) 66 (b) 72 (c) 78 (d) 84

56. The value of q is

(a) 54 (b) 56 (c) 58 (d) 60

57. The value of $r^R + R^r$ is

(a) 5392 (b) 368 (c) 32 (d) 4

Passage V
(Q. Nos. 58 to 60)

The numbers 1, 3, 6, 10, 15, 21, 28, ... are called triangular numbers. Let t_n denotes the nth triangular number such that $t_n = t_{n-1} + n$, $\forall\, n \geq 2$.

58. The value of t_{50} is

(a) 1075 (b) 1175 (c) 1275 (d) 1375

59. The number of positive integers lying between t_{100} and t_{101} are

(a) 99 (b) 100 (c) 101 (d) 102

60. If $(m+1)$ is the nth triangular number, then $(n - m)$ is

(a) $1 + \sqrt{(m^2 + 2m)}$ (b) $1 + \sqrt{(m^2 + 2)}$
(c) $1 + \sqrt{(m^2 + m)}$ (d) None of these

Passage VI
(Q. Nos. 61 to 63)

Let $A_1, A_2, A_3, \ldots, A_m$ be arithmetic means between -3 and 828 and $G_1, G_2, G_3, \ldots, G_n$ be geometric means between 1 and 2187. Product of geometric means is 3^{35} and sum of arithmetic means is 14025.

61. The value of n is

(a) 45 (b) 30 (c) 25 (d) 10

62. The value of m is

(a) 17 (b) 34 (c) 51 (d) 68

63. The value of $G_1 + G_2 + G_3 + \ldots + G_n$ is

(a) 2044 (b) 1022
(c) 511 (d) None of these

Passage VII

(Q. Nos. 64 to 66)

Suppose α, β are roots of $ax^2 + bx + c = 0$ and γ, δ are roots of $Ax^2 + Bx + C = 0$.

64. If $\alpha, \beta, \gamma, \delta$ are in AP, then common difference of AP is

(a) $\frac{1}{4}\left(\frac{b}{a} - \frac{B}{A}\right)$ (b) $\frac{1}{3}\left(\frac{b}{a} - \frac{B}{A}\right)$

(c) $\frac{1}{2}\left(\frac{c}{a} - \frac{B}{A}\right)$ (d) $\frac{1}{3}\left(\frac{c}{a} - \frac{C}{A}\right)$

65. If a, b, c are in GP as well as $\alpha, \beta, \gamma, \delta$ are in GP, then A, B, C are in

(a) AP only (b) GP only

(c) AP and GP (d) None of these

66. If $\alpha, \beta, \gamma, \delta$ are in GP, then common ratio of GP is

(a) $\sqrt{\left(\frac{bA}{aB}\right)}$ (b) $\sqrt{\left(\frac{aB}{bA}\right)}$

(c) $\sqrt{\left(\frac{bC}{cB}\right)}$ (d) $\sqrt{\left(\frac{cB}{bC}\right)}$

Passage VIII (Q. Nos. 67 to 69)

Suppose p is the first of n $(n > 1)$ arithmetic means between two positive numbers a and b and q the first of n harmonic means between the same two numbers.

67. The value of p is

(a) $\frac{na+b}{n+1}$ (b) $\frac{nb+a}{n+1}$ (c) $\frac{na-b}{n+1}$ (d) $\frac{nb-a}{n+1}$

68. The value of q is

(a) $\frac{(n-1)ab}{nb+a}$ (b) $\frac{(n+1)ab}{nb+a}$ (c) $\frac{(n+1)ab}{na+b}$ (d) $\frac{(n-1)ab}{na+b}$

69. Final conclusion is

(a) q lies between p and $\left(\frac{n+1}{n-1}\right)^2 p$

(b) q lies between p and $\left(\frac{n+1}{n-1}\right)p$

(c) q does not lie between p and $\left(\frac{n+1}{n-1}\right)^2 p$

(d) q does not lie between p and $\left(\frac{n+1}{n-1}\right)p$

Sequences and Series Exercise 4 : Single Integer Answer Type Questions

- This section contains **10 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

70. Let a, b, c, d be positive real numbers with $a < b < c < d$. Given that a, b, c, d are the first four terms of an AP and a, b, d are in GP. The value of $\frac{ad}{bc}$ is $\frac{p}{q}$, where p and q are prime numbers, then the value of q is

71. If the coefficient of x in the expansion of $\prod_{r=1}^{110}(1+rx)$ is $\lambda(1+10)(1+10+10^2)$, then the value of λ is

72. A 3-digit palindrome is a 3-digit number (not starting with zero) which reads the same backwards as forwards *For example*, 242. The sum of all even 3-digit palindromes is $2^{n_1} \cdot 3^{n_2} \cdot 5^{n_3} \cdot 7^{n_4} \cdot 11^{n_5}$, alue of $n_1 + n_2 + n_3 + n_4 + n_5$ is

73. If n is a positive integer satisfying the equation $2 + (6 \cdot 2^2 - 4 \cdot 2) + (6 \cdot 3^2 - 4 \cdot 3) + \ldots + (6 \cdot n^2 - 4 \cdot n) = 140$, then the value of n is

74. Let $S(x) = 1 + x - x^2 - x^3 + x^4 + x^5 - x^6 - x^7 + \ldots + \infty$, where $0 < x < 1$. If $S(x) = \frac{\sqrt{2}+1}{2}$, then the value of $(x+1)^2$ is

75. The sequence $a_1, a_2, a_3, \ldots$ is a geometric sequence with common ratio r. The sequence $b_1, b_2, b_3, \ldots$ is also a geometric sequence. If $b_1 = 1, b_2 = \sqrt[4]{7} - \sqrt[4]{28} + 1, a_1 = \sqrt[4]{28}$ and $\sum_{n=1}^{\infty} \frac{1}{a_n} = \sum_{n=1}^{\infty} b_n$, then the value of $(1 + r^2 + r^4)$ is

76. Let (a_1, b_1) and (a_2, b_2) are the pair of real numbers such that $10, a, b, ab$ constitute an arithmetic progression. Then, the value of $\left(\frac{2a_1a_2 + b_1b_2}{10}\right)$ is

77. If one root of $Ax^3 + Bx^2 + Cx + D = 0, A \neq 0$, is the arithmetic mean of the other two roots, then the relation $2B^3 + \lambda ABC + \mu A^2 D = 0$ holds good. Then, the value of $2\lambda + \mu$ is

78. If $|x| > 1$, then sum of the series $\frac{1}{1+x} + \frac{2}{1+x^2} + \frac{2^2}{1+x^4} + \frac{2^3}{1+x^8} + \ldots$ upto ∞ is $\frac{1}{x-\lambda}$, then the value of λ is

79. Three non-zero real numbers form an AP and the squares of these numbers taken in same order form a GP. If the possible common ratios are $(3 \pm \sqrt{k})$ where $k \in N$, then the value of $\left[\frac{k}{8} - \frac{8}{k}\right]$ is (where [ ] denotes the greatest integer function).

Sequences and Series Exercise 5 : Matching Type Questions

- This section contains **4 questions**. Questions 80, 81 and 82 have three statements (A, B and C) and question 83 has four statements (A, B, C and D) given in **Column I** and questions 80 and 81 have four statements (p, q, r and s), question 82 has five statements (p, q, r, s and t) and question 83 has three statements (p, q and r) in **Column II**, respectively. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**.

80.

	Column I		Column II
(A)	a, b, c, d are in AP, then	(p)	$a + d > b + c$
(B)	a, b, c, d are in GP, then	(q)	$ad > bc$
(C)	a, b, c, d are in HP, then	(r)	$\frac{1}{a} + \frac{1}{d} > \frac{1}{b} + \frac{1}{c}$
		(s)	$ad < bc$

81.

	Column I		Column II
(A)	For an AP $a_1, a_2, a_3, \ldots, a_n, \ldots$; $a_1 = \frac{5}{2}$; $a_{10} = 16$. If $a_1 + a_2 + \ldots + a_n = 110$, then '$n$' equals	(p)	9
(B)	The interior angles of a convex non-equiangular polygon of 9 sides are in AP. The least positive integer that limits the upper value of the common difference between the measures of the angles in degrees is	(q)	10
(C)	For an increasing GP, $a_1, a_2, a_3, \ldots, a_n, \ldots$; $a_6 = 4a_4$; $a_9 - a_7 = 192$, if $a_4 + a_5 + a_6 + \ldots + a_n = 1016$, then n equals	(r)	11
		(s)	12

82.

	Column I		Column II
(A)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_4 + a_7 + a_{14} + a_{17} + a_{20} = 165$, $\alpha = a_2 + a_6 + a_{15} + a_{19}$ and $\beta = 2(a_9 + a_{12}) - (a_3 + a_{18})$, then	(p)	$\alpha = 2\beta$
(B)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 195$, $\alpha = a_2 + a_7 + a_{18} + a_{23}$ and $\beta = 2(a_3 + a_{22}) - (a_8 + a_{17})$, then	(q)	$\alpha + 2\beta = 260$
(C)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_7 + a_{10} + a_{21} + a_{24} + a_{30} = 225$, $\alpha = a_2 + a_7 + a_{24} + a_{29}$ and $\beta = 2(a_{10} + a_{21}) - (a_3 + a_{28})$, then	(r)	$\alpha + 2\beta = 220$
		(s)	$\alpha - \beta = 5\lambda, \lambda \in I$
		(t)	$\alpha + \beta = 15\mu, \mu \in I$

83.

	Column I		Column II
(A)	If $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$, where a, b, c are non-zero numbers, then a, b, c are in	(p)	AP
(B)	If $17a^2 + 13b^2 + 5c^2 = (3ab + 15bc + 5ca)$, where a, b, c are non-zero numbers, then a, b, c are in	(q)	GP
(C)	If $a^2 + 9b^2 + 25c^2 = abc\left(\frac{15}{a} + \frac{5}{b} + \frac{3}{c}\right)$, where a, b, c are non-zero numbers, then a, b, c are in	(r)	HP
(D)	If $(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + ca) + (a^2 + b^2 + c^2) \leq 0$, where a, b, c, p are non-zero numbers, then a, b, c are in		

Sequences and Series Exercise 6 :
Statement I and II Type Questions

- **Directions** (Q. Nos. 84 to 90) are Assertion-Reason type questions. Each of these questions contains two statements:

Statement-1 (Assertion) and **Statement-2** (Reason) Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

84. Statement 1 4, 8, 16 are in GP and 12, 16, 24 are in HP.
Statement 2 If middle term is added in three consecutive terms of a GP, resultant will be in HP.

85. Statement 1 If the nth term of a series is $2n^3 + 3n^2 - 4$, then the second order differences must be an AP.
Statement 2 If nth term of a series is a polynomial of degree m, then mth order differences of series are constant.

86. Statement 1 The sum of the products of numbers $\pm a_1, \pm a_2, \pm a_3, \ldots, \pm a_n$ taken two at a time is $-\sum_{i=1}^{n} a_i^2$.
Statement 2 The sum of products of numbers $a_1, a_2, a_3, \ldots, a_n$ taken two at a time is denoted by $\sum\sum_{1 \le i < j \le n} a_i a_j$.

87. Statement 1 $a + b + c = 18 \ (a, b, c > 0)$, then the maximum value of abc is 216.
Statement 2 Maximum value occurs when $a = b = c$

88. Statement 1 If $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$, where a, b, c are non-zero real numbers, then a, b, c are in GP.
Statement-2 If $(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_1)^2 = 0$, then $a_1 = a_2 = a_3, \forall \ a_1, a_2, a_3 \in R$.

89. Statement 1 If a and b be two positive numbers, where $a > b$ and $4 \times \text{GM} = 5 \times \text{HM}$ for the numbers. Then, $a = 4b$.
Statement 2 $(\text{AM})(\text{HM}) = (\text{GM})^2$ is true for positive numbers.

90. Statement1 The difference between the sum of the first 100 even natural numbers and the sum of the first 100 odd natural numbers is 100.
Statement 2 The difference between the sum of the first n even natural numbers and sum of the first n odd natural numbers is n.

Sequences and Series Exercise 7 :
Subjective Type Questions

- In this section, there are **24 subjective** questions.

91. The p th, $(2p)$ th and $(4p)$ th terms of an AP, are in GP, then find the common ratio of GP.

92. Find the sum of n terms of the series
$(a + b) + (a^2 + ab + b^2) + (a^3 + a^2b + ab^2 + b^3) + \ldots,$
where $a \ne 1, b \ne 1$ and $a \ne b$.

93. The sequence of odd natural numbers is divided into groups 1; 3, 5; 7, 9, 11; ... and so on. Show that the sum of the numbers in nth group is n^3.

94. Let a, b, c are respectively the sums of the first n terms, the next n terms and the next n terms of a GP. Show that a, b, c are in GP.

95. If the first four terms of an arithmetic sequence are $a, 2a, b$ and $(a - 6 - b)$ for some numbers a and b, find the sum of the first 100 terms of the sequence.

96. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \text{upto } \infty = \frac{\pi^2}{6}$, find

(i) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \text{upto } \infty$

(ii) $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \text{upto } \infty$

97. If the arithmetic mean of $a_1, a_2, a_3, \ldots, a_n$ is a and $b_1, b_2, b_3, \ldots, b_n$ have the arithmetic mean b and $a_i + b_i = 1$ for $i = 1, 2, 3, \ldots, n$, prove that
$$\sum_{i=1}^{n} (a_i - a)^2 + \sum_{i=1}^{n} a_i b_i = nab$$

98. If $a_1, a_2, a_3, \ldots$ is an arithmetic progression with common difference 1 and $a_1 + a_2 + a_3 + \ldots + a_{98} = 137$, then find the value of $a_2 + a_4 + a_6 + \ldots + a_{98}$.

99. If $t_1 = 1, t_r - t_{r-1} = 2^{r-1}, r \ge 2$, find $\sum_{r=1}^{n} t_r$.

100. Prove that $I_1, I_2, I_3, \ldots$ form an AP, if

(i) $I_n = \int_0^{\pi} \frac{\sin 2nx}{\sin x}\, dx$ (ii) $I_n = \int_0^{\pi} \left(\frac{\sin nx}{\sin x}\right)^2 dx$

101. Consider the sequence $S = 7 + 13 + 21 + 31 + \ldots + T_n$, find the value of T_{70}.

102. Find value of $\left(x + \frac{1}{x}\right)^3 + \left(x^2 + \frac{1}{x^2}\right)^3 + \ldots + \left(x^n + \frac{1}{x^n}\right)^3$.

103. If a_m be the mth term of an AP, show that
$$a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2n-1}^2 - a_{2n}^2 = \frac{n}{(2n-1)}(a_1^2 - a_{2n}^2).$$

104. If three unequal numbers are in HP and their squares are in AP, show that they are in the ratio $1 + \sqrt{3} : -2 : 1 - \sqrt{3}$ or $1 - \sqrt{3} : -2 : 1 + \sqrt{3}$.

105. If $a_1, a_2, a_3, \ldots, a_n$ are in AP with $a_1 = 0$, prove that
$$\frac{a_3}{a_2} + \frac{a_4}{a_3} + \ldots + \frac{a_n}{a_{n-1}} - a_2\left(\frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_{n-2}}\right) = \frac{a_{n-1}}{a_2} + \frac{a_2}{a_{n-1}}$$

106. Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row of two balls and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of the sides, then contains 8 balls less than each side of the triangle. Determine the initial number of balls.

107. If $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are in AP whose common difference is d, then show that
$$\sin d \{\sec \theta_1 \sec \theta_2 + \sec \theta_2 \sec \theta_3 + \ldots + \sec \theta_{n-1} \sec \theta_n\} = \tan \theta_n - \tan \theta_1.$$

108. Show that,
$$(1 + 5^{-1})(1 + 5^{-2})(1 + 5^{-4})(1 + 5^{-8}) \ldots (1 + 5^{-2^n}) = \frac{5}{4}(1 - 5^{-2^{n+1}})$$

109. Evaluate $S = \sum_{n=0}^{n} \frac{2^n}{(a^{2^n} + 1)}$ (where $a > 1$).

110. Find the sum to infinite terms of the series
$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \ldots + \tan^{-1}\left(\frac{2^{n-1}}{1 + 2^{2n-1}}\right) + \ldots$$

111. Find the sum to n terms, whose nth term is $\tan[\alpha + (n-1)\beta] \tan(\alpha + n\beta)$.

112. If $\sum_{r=1}^{n} T_r = \frac{n}{8}(n+1)(n+2)(n+3)$, find $\sum_{r=1}^{n} \frac{1}{T_r}$.

113. If S_1, S_2, S_3 denote the sum of n terms of 3 arithmetic series whose first terms are unity and their common difference are in HP, prove that
$$n = \frac{2S_3S_1 - S_1S_2 - S_2S_3}{S_1 - 2S_2 + S_3}.$$

114. Three friends whose ages form a GP divide a certain sum of money in proportion to their ages. If they do that three years later, when the youngest is half the age of the oldest, then he will receive ₹ 105 more than he gets now and the middle friend will get ₹ 15 more than he gets now. Find the ages of the friends.

Sequences and Series Exercise 8 :
Questions Asked in Previous 13 Year's Exam

- This section contains questions asked in **IIT-JEE, AIEEE, JEE Main & JEE Advanced** from year **2005** to year **2017**.

115. If a, b, c are in AP and $|a|, |b|, |c| < 1$ and

$x = 1 + a + a^2 + \ldots + \infty$

$y = 1 + b + b^2 + \ldots + \infty$

$z = 1 + c + c^2 + \ldots + \infty$

by $\frac{a-b}{b-c} = \frac{a}{c}$

Then, x, y, z will be in **[AIEEE 2005, 3M]**

(a) AP (b) GP (c) HP
(d) None of these

116. If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$, then find the least natural number n_0 such that $b_n > a_n, \forall \ n \ge n_0$. **[IIT-JEE 2006, 6M]**

117. Let $a_1, a_2, a_3, \ldots$ be terms are in AP, if $\frac{a_1 + a_2 + \ldots + a_p}{a_1 + a_2 + \ldots + a_q} = \frac{p^2}{q^2}, p \ne q$, then $\frac{a_6}{a_{21}}$ equals **[AIEEE 2006, 4.5M]**

(a) $\frac{41}{11}$ (b) $\frac{7}{2}$
(c) $\frac{2}{7}$ (d) $\frac{11}{41}$

118. If $a_1, a_2, \ldots, a_n$ are in HP, then the expression $a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n$ is equal to **[AIEEE 2006, 6M]**

(a) $n(a_1 - a_n)$
(b) $(n-1)(a_1 - a_n)$
(c) na_1a_n
(d) $(n-1)a_1a_n$

119. Let V_r denotes the sum of the first r terms of an arithmetic progression whose first term is r and the common difference is $(2r-1)$. Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots$ **[IIT-JEE 2007, 4+4+4M]**

(i) The sum $V_1 + V_2 + \ldots + V_n$ is

(a) $\frac{1}{12}n(n+1)(3n^2-n+1)$

(b) $\frac{1}{12}n(n+1)(3n^2+n+2)$

(c) $\frac{1}{2}n(2n^2-n+1)$

(d) $\frac{1}{3}(2n^3-2n+3)$

(ii) T_r is always

C (a) an odd number (b) an even number

(c) a prime number (d) a composite number

(iii) Which one of the following is a correct statement?

(a) $Q_1, Q_2, Q_3, \ldots$ are in AP with common difference 5

(b) $Q_1, Q_2, Q_3, \ldots$ are in AP with common difference 6

(c) $Q_1, Q_2, Q_3, \ldots$ are in AP with common difference 11

(d) $Q_1 = Q_2 = Q_3 = \ldots$

120. Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1}, G_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n, respectively. **[IIT-JEE 2007, 4+4+4M]**

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(i) Which one of the following statement is correct?

(a) $G_1 > G_2 > G_3 > \ldots$ (b) $G_1 < G_2 < G_3 < \ldots$

(c) $G_1 = G_2 = G_3 = \ldots$

(d) $G_1 < G_3 < G_5 < \ldots$ and $G_2 > G_4 > G_6 > \ldots$

(ii) Which of the following statement is correct?

(a) $A_1 > A_2 > A_3 > \ldots$

(b) $A_1 < A_2 < A_3 < \ldots$

(c) $A_1 > A_3 > A_5 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$

(d) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

(iii) Which of the following statement is correct?

(a) $H_1 > H_2 > H_3 > \ldots$

(b) $H_1 < H_2 < H_3 < \ldots$

(c) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 < \ldots$

(d) $H_1 < H_3 < H_5 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

121. If a geometric progression consisting of positive terms, each term equals the sum of the next two terms, then the common ratio of this progression equals **[AIEEE 2007, 3M]**

(a) $\frac{1}{2}(1-\sqrt{5})$ (b) $\frac{1}{2}\sqrt{5}$

(c) $\sqrt{5}$ (d) $\frac{1}{2}(\sqrt{5}-1)$

122. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in GP. Let $b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

Statement 1 The numbers b_1, b_2, b_3, b_4 are neither in AP nor in GP.

Statement 2 The numbers b_1, b_2, b_3, b_4 are in HP. **[IIT-JEE 2008, 3M]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

123. The first two terms of a geometric progression add upto 12 the sum of the third and the fourth terms is 48, if the terms of the geometric progression are alternately positive and negative, then the first term is **[AIEEE 2008, 3M]**

(a) -12 (b) 12 (c) 4 (d) -4

124. If the sum of first n terms of an AP is cn^2, then the sum of squares of these n terms is **[IIT-JEE 2009, 3M]**

(a) $\frac{n(4n^2-1)c^2}{6}$ (b) $\frac{n(4n^2+1)c^2}{3}$

(c) $\frac{n(4n^2-1)c^2}{3}$ (d) $\frac{n(4n^2+1)c^2}{6}$

125. The sum to infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots$ is **[AIEEE 2009, 4M]**

(a) 6 (b) 2 (c) 3 (d) 4

126. Let $S_k, k = 1, 2, \ldots, 100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and common ratio is $\frac{1}{k}$. Then, the value of $\frac{100^2}{100!} + \sum_{k=2}^{100} |(k^2 - 3k + 1) S_k|$ is **[IIT-JEE 2010, 3M]**

127. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to **[IIT-JEE 2010, 3M]**

128. A person is to count 4500 currency notes. Let a_n denotes the number of notes he counts in the nth minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in AP with common difference -2, then the time taken by him to count all notes is **[AIEEE 2010, 8M]**

(a) 34 min (b) 125 min

(c) 135 min (d) 24 min

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129. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is **[IIT-JEE 2011, 4M]**

130. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after **[AIEEE 2011, 4M (Paper I)]**

(a) 19 months (b) 20 months

(c) 21 months (d) 18 months

131. Let a_n be the nth term of an AP, if $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the AP is **[AIEEE 2011, 4M (Paper II)]**

(a) $\frac{\alpha-\beta}{200}$ (b) $\alpha-\beta$

(c) $\frac{\alpha-\beta}{100}$ (d) $\beta-\alpha$

132. If $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is **[IIT-JEE 2012, 3M]**

(a) 22 (b) 23

(c) 24 (d) 25

133. Statement 1 The sum of the series $1 + (1+2+4) + (4+6+9) + (9+12+16) + \ldots + (361+380+400)$ is 8000.

Statement 2 $\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$ for any natural number n. **[AIEEE 2012, 4M]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

134. If 100 times the 100th term of an AP with non-zero common difference equals the 50 times its 50th term, then the 150th term of this AP is **[AIEEE 2012, 4M]**

(a) 150 times its 50th term (b) 150

(c) zero (d) -150

135. If x, y, z are in AP and $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are also in AP, then **[JEE Main 2013, 4M]**

(a) $2x = 3y = 6z$ (b) $6x = 3y = 2z$

(c) $6x = 4y = 3z$ (d) $x = y = z$

136. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, …, is **[JEE Main 2013, 4M]**

(a) $\frac{7}{9}(99 - 10^{-20})$ (b) $\frac{7}{81}(179 + 10^{-20})$

(c) $\frac{7}{9}(99 + 10^{-20})$ (d) $\frac{7}{81}(179 - 10^{-20})$

C

137. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} \cdot k^2$, then S_n can take value(s) **[JEE Advanced 2013, 4M]**

(a) 1056 (b) 1088 (c) 1120 (d) 1332

138. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k - 20$ is equal to **[JEE Advanced 2013, 4M]**

139. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots + (10)(11)^9 = k(10)^9$, then k is equal to **[JEE Main 2014, 4M]**

(a) 100 (b) 110 (c) $\frac{121}{10}$ (d) $\frac{441}{100}$

140. Three positive numbers form an increasing GP. If the middle terms in this GP is doubled, the new numbers are in AP. Then, the common ratio of the GP is **[JEE Main 2014, 4M]**

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$ (c) $\sqrt{2}+\sqrt{3}$ (d) $3+\sqrt{2}$

141. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b+2$, the value of $\frac{a^2+a-14}{a+1}$ is **[JEE Advanced 2014, 3M]**

142. The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$ is **[JEE Main 2015, 4M]**

(a) 192 (b) 71 (c) 96 (d) 142

143. If m is the AM of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals **[JEE Main 2015, 4M]**

(a) $4l^2m^2n^2$ (b) $4l^2mn$

(c) $4lm^2n$ (d) $4lmn^2$

144. Suppose that all the terms of an arithmetic progression (AP) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies between 130 and 140, then the common difference of this AP is **[JEE Main 2015, 4M]**

145. If the 2nd, 5th and 9th terms of a non-eustant AP are in GP, then the common ratio of this GP is **[JEE Main 2016, 4M]**

(a) 1 (b) $\frac{7}{4}$ (c) $\frac{8}{5}$ (d) $\frac{4}{3}$

146. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots$ is $\frac{16}{5}m$, then m equal to **[JEE Main 2016, 4M]**

(a) 100 (b) 99 (c) 102 (d) 101

147. Let $b_i > 1$ for $i = 1, 2, \ldots, 101$. Suppose $\log_e b_1, \log_e b_2, \log_e b_3, \ldots, \log_e b_{101}$ are in Arithmetic Progression (AP) with the common difference $\log_e 2$. Suppose $a_1, a_2, a_3, \ldots, a_{101}$ are in AP. Such that, $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \ldots + b_{51}$ and $s = a_1 + a_2 + \ldots + a_{51}$, then **[JEE Advanced 2016, 3M]**

(a) $s > t$ and $a_{101} > b_{101}$ (b) $s > t$ and $a_{101} < b_{101}$
(c) $s < t$ and $a_{101} > b_{101}$ (d) $s < t$ and $a_{101} < b_{101}$

148. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then **[JEE Main 2017, 4M]**

(a) a, b and c are in GP
(b) b, c and a are in GP
(c) b, c and a are in AP
(d) a, b and c are in AP

Answers

Exercise for Session 1

1. (c) **2.** (d) **3.** (b) **4.** (c) **5.** (a)

Exercise for Session 2

1. (b) **2.** (a) **3.** (a) **4.** (b) **5.** (c) **6.** (c)

Exercise for Session 3

1. (b) **2.** (d) **3.** (b) **4.** (c) **5.** (d)

Exercise for Session 4

1. (c) **2.** (c) **3.** (c) **4.** (d) **5.** (a) **6.** (a)

Exercise for Session 5

1. (c) **2.** (a) **3.** (a) **4.** (c) **5.** (b) **6.** (b)
7. (b) **8.** (b) **9.** (a) **10.** (b)

Exercise for Session 6

1. (b) **2.** (d) **3.** (b) **4.** (c) **5.** (a) **6.** (a)
7. (a) **8.** (c) **9.** (b) **10.** (c)

Exercise for Session 7

1. (a) **2.** (d) **3.** (b) **4.** (d) **5.** (c) **6.** (c)
7. (a) **8.** (a)

Exercise for Session 8

1. (c) **2.** (c) **3.** (b) **4.** (a) **5.** (c) **6.** (b)
7. (c) **8.** (d) **9.** (a) **10.** (a)

Exercise for Session 9

1. (d) **2.** (c) **3.** (d) **4.** (a) **5.** (b) **6.** (a)
7. (c)

Chapter Exercises

1. (a) **2.** (d) **3.** (c) **4.** (c) **5.** (a) **6.** (c)
7. (b) **8.** (d) **9.** (a) **10.** (c) **11.** (a) **12.** (b)
13. (d) **14.** (a) **15.** (a) **16.** (c) **17.** (a) **18.** (b)
19. (a) **20.** (d) **21.** (c) **22.** (c) **23.** (c) **24.** (b)
25. (a) **26.** (a) **27.** (a) **28.** (c) **29.** (b) **30.** (c)
31. (a,c) **32.** (b,d) **33.** (b,c) **34.** (a,b) **35.** (c,d) **36.** (a,c,d)
37. (a,d) **38.** (a,b,c) **39.** (a,c) **40.** (b,c) **41.** (a,c) **42.** (c,d)
43. (b,c) **44.** (a,c) **45.** (a,d)
46. (c) **47.** (d) **48.** (a) **49.** (a) **50.** (a) **51.** (b)
52. (a) **53.** (c) **54.** (b) **55.** (d) **56.** (b) **57.** (c)
58. (c) **59.** (b) **60.** (d) **61.** (d) **62.** (b) **63.** (d)
64. (a) **65.** (b) **66.** (b) **67.** (a) **68.** (b) **69.** (c)
70. (3) **71.** (5) **72.** (8) **73.** (4) **74.** (2) **75.** (7)
76. (3) **77.** (9) **78.** (1) **79.** (0)
80. (A) $\to$ (r, s); (B) $\to$ (p, r); (C) $\to$ (p, q)
81. (A) $\to$ (r); (B) $\to$ (p); (C) $\to$ (q)
82. (A) $\to$ (p,r,s,t); (B) $\to$ (p,q,s,t); (C) $\to$ (p,s,t)
83. (A) $\to$ (r); (B) $\to$ (p); (C) $\to$ (r); (D) $\to$ (q)
84. (a) **85.** (a) **86.** (b) **87.** (a) **88.** (d) **89.** (c)
90. (a)

91. 2. **92.** $\dfrac{1}{(a-b)}\left\{\dfrac{a^2(1-a^n)}{(1-a)} - \dfrac{b^2(1-b^n)}{(1-b)}\right\}$

95. -5050 **96.** (i) $\dfrac{\pi^2}{8}$ (ii) $\dfrac{\pi^2}{12}$ **98.** 93

99. $2^{n+1} - n - 2$ **101.** 5113

102. $\dfrac{x^3(1-x^{3n})}{1-x^3} + \dfrac{(1-x^{3n})}{x^{3n}(1-x^3)} + \dfrac{3x(1-x^n)}{(1-x)} + \dfrac{3(1-x^n)}{x^n(1-x)}$

106. 1540 **109.** $\dfrac{1}{a-1}$

110. $\dfrac{\pi}{4}$ **111.** $\dfrac{\dfrac{\sin n\beta}{\cos(\alpha + n\beta)\cos\alpha} - n\tan\beta}{\tan\beta}$

112. $\dfrac{n(n+3)}{2(n+1)(n+2)}$ **114.** 12, 18, 27 **115.** (c)

116. (7) **117.** (d) **118.** (d) **119.** (i) (b), (ii) (d), (iii) (b)
120. (i) (c), (ii) (a), (iii) (b) **121.** (d) **122.** (c) **123.** (a) **124.** (c)
125. (c) **126.** (3) **127.** (0) **128.** (a) **129.** (8) **130.** (c)
131. (c) **132.** (d) **133.** (a) **134.** (c) **135.** (d) **136.** (b)
137. (a,d) **138.** (5) **139.** (a) **140.** (b) **141.** (4) **142.** (c)
143. (c) **144.** (a) **145.** (d) **146.** (d) **147.** (b) **148.** (c)