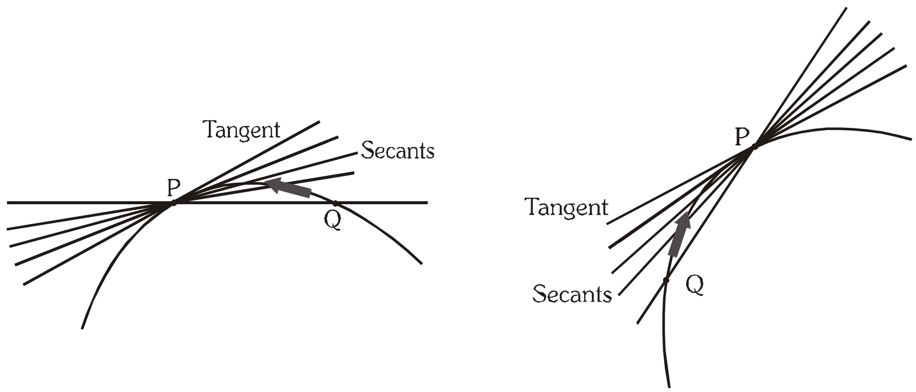
TANGENT & NORMAL

1. TANGENT TO THE CURVE AT A POINT :

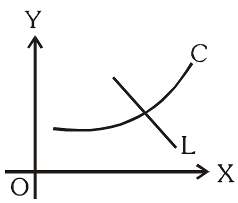
The tangent to the curve at 'P' is the line through P whose slope is limit of the secant slopes as Q $\rightarrow$ P from either side.



2. MYTHS ABOUT TANGENT :

(a) **Myth** : A line meeting the curve only at one point is a tangent to the curve.

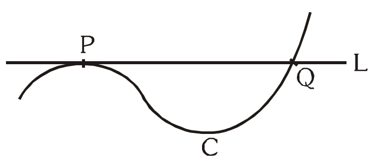
Explanation : A line meeting the curve in one point is not necessarily tangent to it.



Here L is not tangent to C

(b) **Myth** : A line meeting the curve at more than one point is not a tangent to the curve.

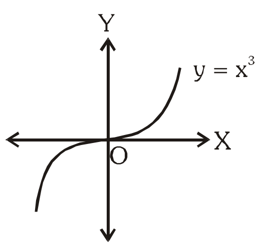
Explanation : A line may meet the curve at several points and may still be tangent to it at some point



Here L is tangent to C at P, and cutting it again at Q.

(c) **Myth** : Tangent at a point to the curve can not cross it at the same point.

Explanation : A line may be tangent to the curve and also cross it.



Here X-axis is tangent to $y = x^3$ at origin.

3. NORMAL TO THE CURVE AT A POINT :

A line which is perpendicular to the tangent at the point of contact is called normal to the curve at that point.

4. THINGS TO REMEMBER :

(a) The value of the derivative at $P(x_1, y_1)$ gives the slope of the tangent to the curve at P. Symbolically

$$f'(x_1) = \left.\frac{dy}{dx}\right]_{(x_1, y_1)} = \text{Slope of tangent at } P(x_1, y_1) = m(\text{say}).$$

(b) Equation of tangent at (x_1, y_1) is ; $y - y_1 = \left.\frac{dy}{dx}\right]_{(x_1, y_1)} (x - x_1)$

(c) Equation of normal at (x_1, y_1) is ; $y - y_1 = -\frac{1}{\left.\frac{dy}{dx}\right]_{(x_1, y_1)}} (x - x_1)$.

Note :

(i) The point P (x_1, y_1) will satisfy the equation of the curve & the equation of tangent & normal line.

(ii) If the tangent at any point P on the curve is parallel to the axis of x then $dy/dx = 0$ at the point P.

(iii) If the tangent at any point on the curve is parallel to the axis of y, then dy/dx not defined or $dx/dy = 0$.

(iv) If the tangent at any point on the curve is equally inclined to both the axes then $dy/dx = \pm 1$.

(v) If a curve passing through the origin be given by a rational integral algebraic equation, then the equation of the tangent (or tangents) at the origin is obtained by equating to zero the terms of the lowest degree in the equation. e.g. If the equation of a curve be $x^2 - y^2 + x^3 + 3x^2y - y^3 = 0$, the tangents at the origin are given by $x^2 - y^2 = 0$ i.e. $x + y = 0$ and $x - y = 0$

Illustration 1 : Find the equation of the tangent to the curve $y = (x^3 - 1)(x - 2)$ at the points where the curve cuts the x-axis.

Solution : The equation of the curve is $y = (x^3 - 1)(x - 2)$ .... (i)

It cuts x-axis at $y = 0$. So, putting $y = 0$ in (i), we get $(x^3 - 1)(x - 2) = 0$

$\Rightarrow (x-1)(x-2)(x^2+x+1) = 0 \Rightarrow x - 1 = 0, x - 2 = 0 \quad [\because x^2 + x + 1 \neq 0]$

$\Rightarrow x = 1, 2$.

Thus, the points of intersection of curve (i) with x-axis are (1, 0) and (2, 0). Now,

$y = (x^3 - 1)(x - 2) \quad \Rightarrow \frac{dy}{dx} = 3x^2(x-2) + (x^3 - 1) \quad \Rightarrow \left(\frac{dy}{dx}\right)_{(1,0)} = -3$ and $\left(\frac{dy}{dx}\right)_{(2,0)} = 7$

The equations of the tangents at (1, 0) and (2, 0) are respectively

$y - 0 = -3(x - 1)$ and $y - 0 = 7(x - 2) \Rightarrow y + 3x - 3 = 0$ and $7x - y - 14 = 0$ **Ans.**

Illustration 2 : The equation of the tangent to the curve $x = a\cos^3 t, y = a\sin^3 t$ at 't' point is

(A) $x\sec t - y\operatorname{cosec} t = a$ (B) $x\sec t + y\operatorname{cosec} t = a$

(C) $x\operatorname{cosec} t - y\sec t = a$ (D) $x\operatorname{cosec} t + y\sec t = a$

Solution : $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = -\frac{3a\sin^2 t\cos t}{3a\cos^2 t\sin t} = -\frac{\sin t}{\cos t}$

which is the slope of the tangent at 't' point. Hence equation of the tangent at 't' point is

$y - a\sin^3 t = -\frac{\sin t}{\cos t}(x - a\cos^3 t) \Rightarrow \frac{y}{\sin t} - a\sin^2 t = -\frac{x}{\cos t} + a\cos^2 t$

$\Rightarrow x\sec t + y\operatorname{cosec} t = a$ **Ans. (B)**

Illustration 3 : The equation of the normal to the curve $y = x + \sin x \cos x$ at $x = \frac{\pi}{2}$ is -

(A) $x = 2$ (B) $x = \pi$ (C) $x + \pi = 0$ (D) $2x = \pi$

Solution : $\because$ $x = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2} + 0 = \frac{\pi}{2}$, so the given point $= \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

Now from the given equation $\frac{dy}{dx} = 1 + \cos^2 x - \sin^2 x \Rightarrow \left(\frac{dy}{dx}\right)_{\left(\frac{\pi}{2},\frac{\pi}{2}\right)} = 1 + 0 - 1 = 0$

$\Rightarrow$ The curve has vertical normal at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

The equation to this normal is $x = \frac{\pi}{2}$

$\Rightarrow x - \frac{\pi}{2} = 0 \Rightarrow 2x = \pi$ **Ans. (D)**

Illustration 4 : The equation of normal to the curve $x + y = x^y$, where it cuts x-axis is -

(A) $y = x + 1$ (B) $y = -x + 1$ (C) $y = x - 1$ (D) $y = -x - 1$

Solution : Given curve is $x + y = x^y$ ..... (i)

at x-axis y=0,

$\therefore x + 0 = x^0 \Rightarrow x = 1$

$\therefore$ Point is A(1, 0)

Now to differentiate $x + y = x^y$ take log on both sides

$\Rightarrow \log(x+y) = y \log x \quad \therefore \frac{1}{x+y}\left\{1 + \frac{dy}{dx}\right\} = y.\frac{1}{x} + (\log x)\frac{dy}{dx}$

Putting $x = 1, y = 0$ $\left\{1 + \frac{dy}{dx}\right\} = 0 \Rightarrow \left(\frac{dy}{dx}\right)_{(1,0)} = -1$

$\therefore$ slope of normal = 1

Equation of normal is, $\frac{y-0}{x-1} = 1 \Rightarrow y = x - 1$ **Ans. (C)**

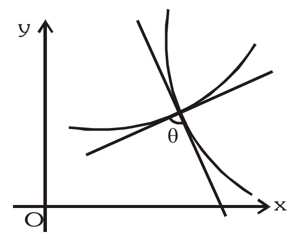
Do yourself - 1 :

(i) Find the distance between the point (1,1) and the tangent to the curve $y = e^{2x} + x^2$ drawn from the point, where the curve cuts y-axis.

(ii) Find the equation of a line passing through (–2,3) and parallel to tangent at origin for the circle $x^2 + y^2 + x - y = 0$.

5. ANGLE OF INTERSECTION BETWEEN TWO CURVES :

Angle of intersection between two curves is defined as the angle between the two tangents drawn to the two curves at their point of intersection.



Orthogonal curves :

If the angle between two curves at each point of intersection is 90 then they are called **orthogonal curves.** For example, the curves $x^2 + y^2 = r^2$ & $y = mx$ are orthogonal curves.

Illustration 5 : The angle of intersection between the curve $x^2 = 32y$ and $y^2 = 4x$ at point (16, 8) is -

(A) 60 (B) 90 (C) $\tan^{-1}\left(\frac{3}{5}\right)$ (D) $\tan^{-1}\left(\frac{4}{3}\right)$

Solution : $x^2 = 32y \Rightarrow \frac{dy}{dx} = \frac{x}{16} \Rightarrow y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

$\therefore$ at $(16, 8)$, $\left(\frac{dy}{dx}\right)_1 = 1$, $\left(\frac{dy}{dx}\right)_2 = \frac{1}{4}$

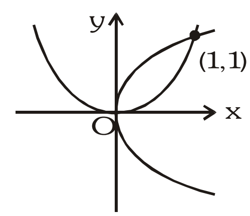
So required angle $= \tan^{-1}\left(\frac{1-\frac{1}{4}}{1+1\left(\frac{1}{4}\right)}\right) = \tan^{-1}\left(\frac{3}{5}\right)$ **Ans. (C)**

Illustration 6 : Check the orthogonality of the curves $y^2 = x$ & $x^2 = y$.

Solution : Solving the curves simultaneously we get points of intersection as (1, 1) and (0, 0).

At (1,1) for first curve $2y\left(\frac{dy}{dx}\right)_1 = 1 \Rightarrow m_1 = \frac{1}{2}$

& for second curve $2x = \left(\frac{dy}{dx}\right)_2 \Rightarrow m_2 = 2$



$m_1 m_2 \neq -1$ at (1,1).

But at (0, 0) clearly x-axis & y-axis are their respective tangents hence they are orthogonal at (0,0) but not at (1,1). Hence these curves are not said to be orthogonal.

Illustration 7 : If curve $y = 1 - ax^2$ and $y = x^2$ intersect orthogonally then the value of a is -

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) 2 (D) 3

Solution : $y = 1 - ax^2 \Rightarrow \frac{dy}{dx} = -2ax$ $\quad y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

Two curves intersect orthogonally if $\left(\frac{dy}{dx}\right)_1 \left(\frac{dy}{dx}\right)_2 = -1$

$\Rightarrow (-2ax)(2x) = -1$ $\quad \Rightarrow 4ax^2 = 1$ ..... (i)

Now eliminating y from the given equations we have $1 - ax^2 = x^2$

$\Rightarrow (1+a)x^2 = 1$ ..... (ii)

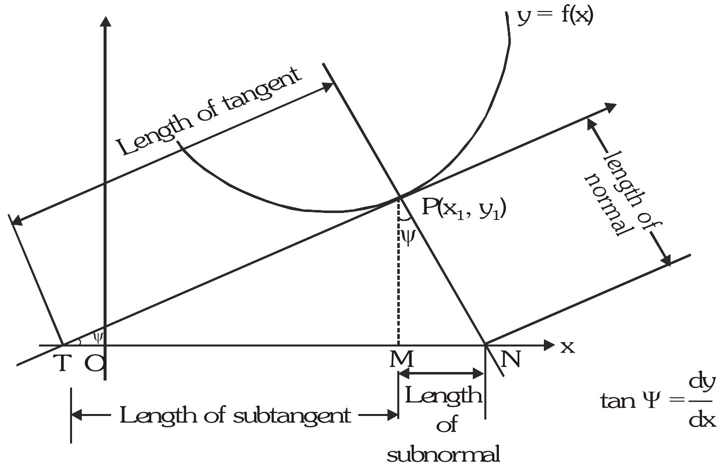
Eliminating x^2 from (i) and (ii) we get $\frac{4a}{1+a} = 1$ $\Rightarrow a = \frac{1}{3}$ **Ans. (B)**

Do yourself -2 :

(i) If two curves $y = a^x$ and $y = b^x$ intersect at an angle α, then find the value of $\tan\alpha$.

(ii) Find the angle of intersection of curves $y = 4 - x^2$ and $y = x^2$.

6. LENGTH OF TANGENT, SUBTANGENT, NORMAL & SUBNORMAL :



(a) Length of the tangent (PT) = $\left|\dfrac{y_1\sqrt{1+[f'(x_1)]^2}}{f'(x_1)}\right|$

(b) Length of Subtangent (MT) = $\left|\dfrac{y_1}{f'(x_1)}\right|$

(c) Length of Normal (PN) = $\left|y_1\sqrt{1+\left[f'(x_1)\right]^2}\right|$

(d) Length of Subnormal (MN) = $|y_1 f'(x_1)|$

Illustration 8 : The length of the normal to the curve $x = a(\theta + \sin\theta),\ y = a(1-\cos\theta)$ at $\theta = \dfrac{\pi}{2}$ is -

(A) $2a$ (B) $\dfrac{a}{2}$ (C) $\sqrt{2}a$ (D) $\dfrac{a}{\sqrt{2}}$

Solution : $\dfrac{dy}{dx} = \dfrac{\left(\dfrac{dy}{d\theta}\right)}{\left(\dfrac{dx}{d\theta}\right)} = \dfrac{a\sin\theta}{a(1+\cos\theta)} = \tan\dfrac{\theta}{2} \Rightarrow \left(\dfrac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \tan\left(\dfrac{\pi}{4}\right) = 1$

Also at $\theta = \dfrac{\pi}{2},\ y = a\left(1-\cos\dfrac{\pi}{2}\right) = a$

$\therefore$ required length of normal $= y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2} = a\sqrt{1+1} = \sqrt{2}a$ **Ans. (C)**

Illustration 9 : The length of the tangent to the curve $x = a\left(\cos t + \log\tan\dfrac{t}{2}\right),\ y = a\sin t$ is

(A) ax (B) ay (C) a (D) xy

Solution : $\dfrac{dy}{dx} = \left(\dfrac{dy}{dt}\right)\Big/\left(\dfrac{dx}{dt}\right) = \dfrac{a\cos t}{a\left(-\sin t + \dfrac{1}{\sin t}\right)} = \tan t$

$\therefore$ length of the tangent $= y\dfrac{\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}}{\left(\dfrac{dy}{dx}\right)} = a\sin t\dfrac{\sqrt{1+\tan^2 t}}{\tan t} = a\sin t\left(\dfrac{\sec t}{\tan t}\right) = a$ **Ans. (C)**

Do yourself - 3 :

(i) Prove that at any point of a curve (length of sub tangent) (length of sub normal) is equal to square of the ordinates of point of contact.

(ii) Find the length of sub tangent to the curve $x^2 + y^2 + xy = 7$ at the point $(1, -3)$.

7. APPROXIMATION :

In order to calculate the approximate value of a function, differentials may be used where the differential of a function is equal to its derivative multiplied by the differential of the independent variable. Thus if, $y = \tan x$ then $dy = \sec^2 x\, dx$.

In general $dy = f'(x)dx$ or $df(x) = f'(x)dx$

Note :

(i) For the independent variable 'x', increment Δx and differential dx are equal but this is not the case with the dependent variable 'y' i.e. $\Delta y \neq dy$.

$\therefore$ Approximate value of y when increment Δx is given to independent variable x in $y = f(x)$ is

$$y + \Delta y = f(x + \Delta x) = f(x) + \frac{dy}{dx}.\Delta x$$

(ii) The relation $dy = f'(x)\, dx$ can be written as $\frac{dy}{dx} = f'(x)$; thus the quotient of the differentials of 'y' and 'x' is equal to the derivative of 'y' w.r.t. 'x'.

Illustration 10 : Find the approximate value of square root of 25.2.

Solution : Let $f(x) = \sqrt{x}$

Now, $f(x + \Delta x) - f(x) = f'(x)\,.\,\Delta x = \dfrac{\Delta x}{2\sqrt{x}}$

we may write, $25.2 = 25 + 0.2$

Taking $x = 25$ and $\Delta x = 0.2$, we have

$$f(25.2) - f(25) = \frac{0.2}{2\sqrt{25}}$$

or $f(25.2) - \sqrt{25} = \dfrac{0.2}{10} = 0.02 \quad \Rightarrow \quad f(25.2) = 5.02$

or $\sqrt{(25.2)} = 5.02$

Do yourself - 4 :

(i) Find the approximate value of $(0.009)^{1/3}$.

8. RATE MEASUREMENT :

Whenever one quantity y varies with another quantity x, satisfying some rule $y = f(x)$, then $\frac{dy}{dx}$ (or $f'(x)$) represents the rate of change of y with respect to x and $\left.\frac{dy}{dx}\right]_{x=a}$ (or $f'(a)$) represents the rate of change of y with respect to x at x = a.

Illustration 11 : The volume of a cube is increasing at a rate of $9cm^3/s$. How fast is the surface area increasing when the length of an edge is 10cm ?

Solution : Let x be the length of side, V be the volume and S be the surface area of the cube. Then $V= x^3$ and $S = 6x^2$, where x is a function of time t.

$$\frac{dV}{dt} = 9cm^3/s = \frac{d}{dt}(x^3) = 3x^2\frac{dx}{dt}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{3}{x^2}$$

$$\frac{dS}{dt} = \frac{d}{dt}(6x^2) = 12x\left(\frac{3}{x^2}\right) = \frac{36}{x}$$

$$\left.\frac{dS}{dt}\right]_{x=10cm} = 3.6\ cm^2/s.$$

Illustration 12: x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of the area of the second square with respect to the first square.

Solution : Given x and y are sides of two squares. Thus the area of two squares are x^2 and y^2

We have to obtain $\frac{d(y^2)}{d(x^2)} = \frac{2y\frac{dy}{dx}}{2x} = \frac{y}{x}.\frac{dy}{dx}$ .... (i)

where the given curve is, $y = x - x^2 \Rightarrow \frac{dy}{dx} = 1 - 2x$ .... (ii)

Thus, $\frac{d(y^2)}{d(x^2)} = \frac{y}{x}(1-2x)$ [from (i) and (ii)]

or $\frac{d(y^2)}{d(x^2)} = \frac{(x-x^2)(1-2x)}{x} \Rightarrow \frac{d(y^2)}{d(x^2)} = (2x^2 - 3x + 1)$

The rate of change of the area of second square with respect to first square is $(2x^2 - 3x + 1)$

Do yourself - 5 :

(i) What is the rate of change of the area of a circle with respect to its radius r at r = 6cm.

(ii) A stone is dropped into a quiet lake and waves move in circles at the speed of 5cm/s. At the instant when the radius of the circular wave is 8cm, how fast is the enclosed area increasing ?

<u>Miscellaneous Illustrations</u> :

Illustration 13 : If the relation between subnormal SN and subtangent ST at any point S on the curve $by^2 = (x + a)^3$ is $p(SN) = q(ST)^2$, then find value of $\frac{p}{q}$ in terms of b and a.

Solution : $by^2 = (x + a)^3$

$$b\ .\ 2y\frac{dy}{dx} = 3(x+a)^2 \Rightarrow \frac{dy}{dx} = \frac{3(x+a)^2}{2by}$$

Given that $p(SN) = q(ST)^2$

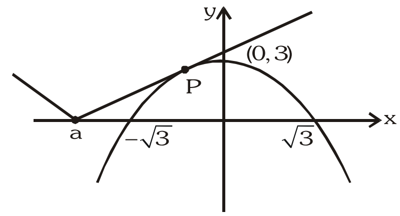
$$\Rightarrow \quad py\frac{dy}{dx} = q\frac{y^2}{\left(\frac{dy}{dx}\right)^2} \Rightarrow \frac{p}{q} = \frac{y}{\left(\frac{dy}{dx}\right)^3} = \frac{y8b^3y^3}{27(x+a)^6} = \frac{8}{27}\frac{b^3(x+a)^6}{b^2(x+a)^6} = \frac{8}{27}b \qquad \textbf{Ans.}$$

Illustration 14 : Find the possible values of 'a' such that the inequality $3 - x^2 > |x - a|$ has atleast one negative solution.

Solution : $3 - x^2 > |x - a|$

Case (i) if $a < 0$ and $y = x - a$ is tangent of $y = 3 - x^2$

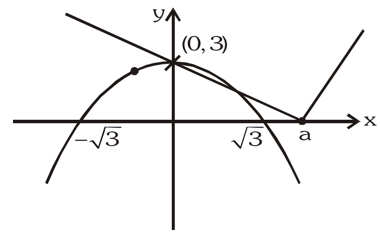
$\Rightarrow -2x = 1 \Rightarrow \quad x = -\dfrac{1}{2} \quad \Rightarrow \quad p\left(-\dfrac{1}{2}, \dfrac{11}{4}\right)$



Since $y = x - a$ passes the $\left(-\dfrac{1}{2}, \dfrac{11}{4}\right) \quad \Rightarrow \quad a = x - y$

$= -\left(\dfrac{11}{4} + \dfrac{1}{2}\right) = -\dfrac{13}{4}$ (minimum value of a)

Case (ii) $a > 0$ and $y = -x + a$ passes through (0, 3), then $a = 3$ (maximum value of a)



$\Rightarrow \quad a \in \left(-\dfrac{13}{4}, 3\right)$

Illustration 15 : Find the angle of intersection of curves, $y = [|\sin x| + |\cos x|]$ and $x^2 + y^2 = 5$ where [.] denotes greatest integral function.

Solution : We know that, $\quad 1 \le |\sin x| + |\cos x| \le \sqrt{2}$

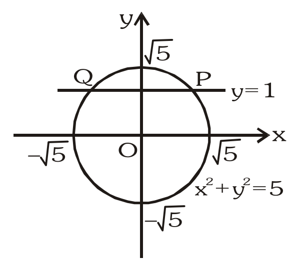
$\therefore \quad y = [|\sin x| + |\cos x|] = 1$

Let P and Q be the points of intersection of given curves.

Clearly the given curves meet at points where $y = 1$ so, we get

$x^2 + 1 = 5$

$x = \pm 2$



Now, $\quad$ P(2, 1) and Q(–2, 1)

Now, $\quad x^2 + y^2 = 5$

Differentiating the above equation w.r.t. x,

we get $\quad 2x + 2y\dfrac{dy}{dx} = 0 \quad \Rightarrow \quad \dfrac{dy}{dx} = -\dfrac{x}{y}$

$\left(\dfrac{dy}{dx}\right)_{(2,\,1)} = -2$

and $\quad \left(\dfrac{dy}{dx}\right)_{(-2,\,1)} = 2$

Clearly the slope of line $y = 1$ is zero and the slope of the tangents at P and Q are (–2) and (2) respectively.

Thus, the angle of intersection is $\tan^{-1}(2)$

ANSWERS FOR DO YOURSELF

1 :	**(i)**	$\dfrac{2}{\sqrt{5}}$	**(ii)**	$x - y + 5 = 0$
2 :	**(i)**	$\left\lvert\dfrac{\ell na - \ell nb}{1 + \ell na\,\ell nb}\right\rvert$	**(ii)**	$\tan^{-1}\left(\dfrac{4\sqrt{2}}{7}\right)$
3 :	**(ii)**	15		
4 :	**(i)**	0.208		
5 :	**(i)**	12π cm	**(ii)**	$80\ \pi cm^2/s]$

EXERCISE - 01

CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. If a variable tangent to the curve $x^2y = c^3$ makes intercepts a, b on x and y axis respectively, then the value of a^2b is -

(A) $27c^3$ (B) $\frac{4}{27}c^3$ (C) $\frac{27}{4}c^3$ (D) $\frac{4}{9}c^3$

2. The number of values of c such that the straight line $3x + 4y = c$ touches the curve $\frac{x^4}{2} = x + y$ is -

(A) 0 (B) 1 (C) 2 (D) 4

3. Let $f(x) = x^3 + ax + b$ with $a \neq b$ and suppose the tangent lines to the graph of f at $x = a$ and $x = b$ have the same gradient. Then the value of $f(1)$ is equal to -

(A) 0 (B) 1 (C) $-\frac{1}{3}$ (D) $\frac{2}{3}$

4. The tangent to the curve $3xy^2 - 2x^2y = 1$ at (1,1) meets the curve again at the point -

(A) $\left(\frac{16}{5}, \frac{1}{20}\right)$ (B) $\left(-\frac{16}{5}, -\frac{1}{20}\right)$ (C) $\left(\frac{1}{20}, \frac{16}{5}\right)$ (D) $\left(-\frac{1}{20}, \frac{16}{5}\right)$

5. The curve $y - e^{xy} + x = 0$ has a vertical tangent at -

(A) (1,1) (B) (0,1) (C) (1,0) (D) no point

6. Suppose f and g both are linear function with $f(x) = -2x + 1$ and $f(g(x)) = 6x - 7$, then slope of line $y = g(x)$ is -

(A) 3 (B) –3 (C) 6 (D) –2

7. A curve with equation of the form $y = ax^4 + bx^3 + cx + d$ has zero gradient at the point (0,1) and also touches the x-axis at the point (–1,0) then the values of x for which the curve has a negative gradient are -

(A) $x > -1$ (B) $x < 1$ (C) $x < -1$ (D) $-1 \leq x \leq 1$

8. The line which is parallel to x-axis and crosses the curve $y = \sqrt{x}$ at an angle of $\frac{\pi}{4}$ is -

(A) $y = -\frac{1}{2}$ (B) $x = \frac{1}{2}$ (C) $y = \frac{1}{4}$ (D) $y = \frac{1}{2}$

9. The lines tangent to the curves $y^3 - x^2y + 5y - 2x = 0$ and $x^4 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

10. The angle of intersection of the curves $2y = x^3$ and $y^2 = 32x$ at origin is -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

11. The angle of intersection of $x = \sqrt{y}$ and $x^3 + 6y = 7$ at (1, 1) is -

(A) $\frac{\pi}{5}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

12. At any point of a curve $\sqrt{\frac{\text{subnormal}}{\text{sub tan gent}}}$ is equal to -

(A) the abscissa of that point
(B) the ordinate of that point
(C) slope of the tangent at that point
(D) slope of the normal at that point

13. The length of the tangent to the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ at θ points is -

(A) $2a\sin\frac{\theta}{2}$ (B) $a\sin\theta$ (C) $2a\sin\theta$ (D) $a\cos\theta$

14. The length of the subnormal of the curve $y^2 = 8ax$ $(a > 0)$ is -

(A) 2a (B) 4a (C) 6a (D) 8a

15. A 13 ft. ladder is leaning against a wall when its base starts to slide away. At the instant when the base is 12 ft. away from the wall, the base is moving away from the wall at the rate of 5 ft/sec. The rate at which the angle θ between the ladder and the ground is changing is -

(A) $-\frac{12}{13}$ rad/sec. (B) -1 rad/sec. (C) $-\frac{13}{12}$ rad/sec. (D) $-\frac{10}{13}$ rad/sec.

16. Water is poured into an inverted conical vessel of which the radius of the base is 2m and height 4m, at the rate of 77 litre/minute. The rate at which the water level is rising at the instant when the depth is 70 cm is - (use $\pi = 22/7$)

(A) 10 cm/min (B) 20 cm/min (C) 40 cm /min (D) none

17. A point is moving along the curve $y^3 = 27x$. The interval in which the abscissa changes at slower rate than ordinate, is -

(A) $(-3, 3)$ (B) $(-\infty,\infty)$ (C) $(-1, 1)$ (D) $(-\infty, -3)\cup(3, \infty)$

18. A particle moves along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units per second. The value of $\frac{dx}{dt}$ when $x = 3$ is -

(A) 4 (B) $\frac{9}{2}$ (C) $\frac{3\sqrt{3}}{2}$ (D) none of these

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

19. Which of the following pair(s) of curves is/are orthogonal.

(A) $y^2 = 4ax$; $y = e^{-x/2a}$
(B) $y^2 = 4ax$; $x^2 = 4ay$
(C) $xy = a^2$; $x^2 - y^2 = b^2$
(D) $y = ax$; $x^2 + y^2 = c^2$

20. If $\frac{x}{a}+\frac{y}{b}=1$ is a tangent to the curve $x = Kt$, $y=\frac{K}{t}$, $K>0$ then :

(A) $a > 0, b > 0$ (B) $a > 0, b < 0$ (C) $a < 0, b > 0$ (D) $a < 0, b < 0$

21. The coordinates of the point(s) on the graph of the function, $f(x)=\frac{x^3}{3}-\frac{5x^2}{2}+7x-4$ where the tangent drawn cut off intercepts from the coordinate axes which are equal in magnitude but opposite in sign, is -

(A) (2,8/3) (B) (3,7/2) (C) (1,5/6) (D) none

22. For the curve represented parametrically by the equations, $x = 2\ \ell n \cot t + 1$ and $y = \tan t + \cot t$

(A) tangent at $t = \pi/4$ is parallel to x-axis

(B) normal at $t = \pi/4$ is parallel to y-axis

(C) tangent at $t = \pi/4$ is parallel to the line $y = x$

(D) normal at $t = \pi/4$ is parallel to the line $y = x$

23. Consider the curve $f(x) = x^{1/3}$, then -

(A) the equation of tangent at (0, 0) is $x = 0$

(B) the equation of normal at (0, 0) is $y = 0$

(C) normal to the curve does not exist at (0, 0)

(D) f(x) and its inverse meet at exactly 3 points.

24. Equation of common tangent(s) of $x^2 - y^2 = 12$ and $xy = 8$ is (are) -

(A) $y = 3x + 4\sqrt{6}$ (B) $y = -3x + 4\sqrt{6}$ (C) $3y = x + 4\sqrt{6}$ (D) $y = -3x - 4\sqrt{6}$

CHECK YOUR GRASP — ANSWER KEY — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	B	B	B	C	B	C	D	D	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	D	C	A	B	B	B	C	A	A,C,D	A,D
Que.	21	22	23	24						
Ans.	A,B	A,B	A,B,D	B,D						

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. The angle at which the curve $y = Ke^{Kx}$ intersects the y-axis is -

(A) $\tan^{-1}k^2$ (B) $\cot^{-1}(k^2)$ (C) $\sin^{-1}\left(\frac{1}{\sqrt{1+k^4}}\right)$ (D) $\sec^{-1}\left(\sqrt{1+k^4}\right)$

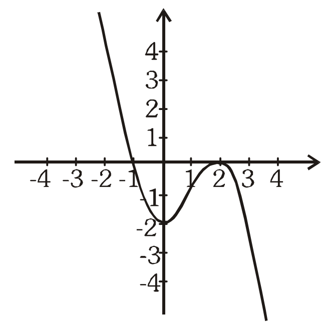
2. The coordinates of points(s) at each of which the tangents to the curve $y = x^3 - 3x^2 - 7x + 6$ cut off on the positive semi axis OX a line segment half that on the negative semi axis OY is/are given by :

(A) (–1,9) (B) (3,–15) (C) (1,–3) (D) none

3. The abscissa of the point on the curve $\sqrt{xy} = a + x$, the tangent at which cuts off equal intercepts from the co-ordinate axes is (a > 0)

(A) $\frac{a}{\sqrt{2}}$ (B) $-\frac{a}{\sqrt{2}}$ (C) $a\sqrt{2}$ (D) $-a\sqrt{2}$

4. A cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ has a graph which is tangent to the x-axis at 2, has another x-intercept at –1, and has y-intercept at –2 as shown. The values of, a + b + c + d equals-



(A) –2

(B) –1

(C) 0

(D) 1

5. Equation of a tangent to the curve $y\cot x = y^3\tan x$ at the point where the abscissa is $\frac{\pi}{4}$ is -

(A) $4x + 2y = \pi + 2$ (B) $4x - 2y = \pi + 2$ (C) x = 0 (D) y = 0

6. Consider the curve represented parametrically by the equation

$$x = t^3 - 4t^2 - 3t \text{ and } y = 2t^2 + 3t - 5 \text{ where } t \in R$$

If H denotes the number of point on the curve where the tangent is horizontal and V the number of point where the tangent is vertical then-

(A) H = 2 and V = 1 (B) H = 1 and V = 2 (C) H = 2 and V = 2 (D) H = 1 and V = 1

7. If y = f(x) be the equation of a parabola which is touched by the line y = x at the point where x = 1. Then -

(A) $f'(1) = 1$ (B) $f'(0) = f'(1)$ (C) $2f(0) = 1 - f'(0)$ (D) $f(0) + f'(0) + f''(0) = 1$

8. At the point $P(a, a^n)$ on the graph of $y = x^n (n \in N)$ in the first quadrant a normal is drawn. The normal intersects the y-axis at the point (0, b), If $\lim_{a\to 0} b = \frac{1}{2}$, then n equals -

(A) 1 (B) 3 (C) 2 (D) 4

9. A horse runs along a circle with a speed of 20 km/hr. A lantern is at the centre of the circle. A fence is along the tangent to the circle at the point at which the horse starts. The speed with which the shadow of the horse move along the fence at the moment when it covers 1/8 of the circle in km/hr is -

(A) 20 (B) 60 (C) 30 (D) 40

10. Equation of the line through the point (1/2,2) and tangent to the parabola $y=\frac{-x^2}{2}+2$ and secant to the curve $y=\sqrt{4-x^2}$ is -

(A) $2x + 2y - 5 = 0$ (B) $2x + 2y - 9 = 0$
(C) $y - 2 = 0$ (D) none

11. If the tangent at P of the curve $y^2 = x^3$ intersects the curve again at Q and the straight lines OP,OQ make angles α, β with the x-axis where 'O' is the origin then $\tan\alpha/\tan\beta$ has the value equal to -

(A) -1 (B) -2 (C) 2 (D) $\sqrt{2}$

12. Let $f(x)$ be a nonzero function whose all successive derivative exist and are nonzero. If $f(x)$, $f'(x)$ and $f''(x)$ are in G.P. and $f(0) = 1$, $f'(0) = 1$, then -

(A) $f'(x) < 0 \ \forall x \in R$ (B) $f''(x) < 1 \ \forall x \in R$
(C) $f''(0) \neq f'''(0)$ (D) $f''(x) > 0 \ \forall x \in R$

13. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then - [JEE 1986]

(A) $a > 0, b > 0$ (B) $a > 0, b < 0$ (C) $a < 0, b > 0$ (D) $a < 0, b < 0$

BRAIN TEASERS			ANSWER KEY					EXERCISE-2	
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B,C	B	A,B	B	A,B,D	B	A,C	C	D	A
Que.	11	12	13							
Ans.	B	D	B,C							

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.

	Column-I		Column-II
(A)	The angle of intersection of $y^2 = 4x$ and $x^2 = 4y$ is 90 and $\tan^{-1}\left(\frac{m}{n}\right)$ then $\|m + n\|$ is equal to (m and n are coprime)	(p)	0
		(q)	$\frac{1}{2}$
(B)	The area of triangle formed by normal at the point (1, 0) on the curve $x = e^{\sin y}$ with axes is	(r)	7
(C)	If the angle between curves $x^2y = 1$ and $y = e^{2(1-x)}$ at the point (1, 1) is θ then $\tan\theta$ is equal to		
(D)	The length of sub-tangent at any point on the curve $y = be^{x/3}$ is equal to	(s)	3

2.

	Column-I		Column-II
(A)	The slope of the curve $2y^2=ax^2+b$ at (1, –1) is –1, then	(p)	$a - b = 2$
(B)	If (a, b) be the point on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes, then	(q)	$a - b = 7/2$
(C)	If the tangent at a point (1, 2) on the curve $y = ax^2+ bx+\frac{7}{2}$ be parallel to the normal at (–2, 2) on the curve $y = x^2 + 6x + 10$, then	(r)	$a - b = 4/3$
(D)	If the tangent to the curve $xy + ax + by = 0$ at (1, 1) is inclined at an angle $\tan^{-1} 2$ with x-axis, then	(s)	$a - b = 3$

ASSERTION & REASON

These questions contain, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I** : The ratio of length of tangent to length of normal is proportional to the ordinate of the point of tangency at the curve $y^2 = 4ax$.

Because

Statement-II : Length of normal and tangent to a curve $y = f(x)$ is $\left|y\sqrt{1+m^2}\right|$ and $\left|\frac{y\sqrt{1+m^2}}{m}\right|$, where $m = \frac{dy}{dx}$.

(A) A (B) B (C) C (D) D

2 **Statement-I** : The product of the ordinates to the point of tangency to the curve $(1 + x^2)y = 2 - x$, where the tangent makes equal intercept with coordinate axes is equal to 1.

Because

Statement-II : Slope of straight line making equal intercept with coordinate axis is equal to 1.

(A) A (B) B (C) C (D) D

3. **Statement-I** : Any tangent to the curve $y = x^7 + 8x^3 + 2x + 1$ makes an acute angle with the positive x-axis.
Because
Statement-II : Any tangent to the curve $y = a_0x^{2n+1} + a_1x^{2n-1} + a_2x^{2n-3} + \ldots\ldots + a_nx + 1$ makes an acute angle with the positive x-axis where $a_1, \ldots\ldots, a_{n-1} \geq 0$; $a_0, a_n > 0$ and $n \in N$.
(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

Consider the function $f(x) = x^2 f(1) - xf'(2) + f''(3)$ such that $f(0) = 2$

On the basis of above information, answer the following questions :

1. The values of $f'(1)$ is -
(A) 0 (B) 1 (C) 2 (D) 1

2. Equation of tangent to $y = f(x)$ at $x = 3$ is -
(A) $y = x - 7$ (B) $y = \frac{x}{4} - 7$ (C) $y = 4x - 7$ (D) none of these

3. The angle of intersection of $y = f(x)$ and $y = 2e^{2x}$ is -
(A) $\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{4}{3}\right)$ (C) 0 (D) $\tan^{-1}\left(\frac{6}{7}\right)$

Comprehension # 2

Let $y = f(x)$ be a differentiable function which satisfies $f'(x) = f^2(x)$ and $f(0) = -\frac{1}{2}$. The graph of the differentiable function $y = g(x)$ contains the point (0, 2) and has the property that for each number 'P', the line tangent to $y = g(x)$ at (P, g(P)) intersects x-axis at P + 2.

On the basis of above information, answer the following questions :

1. If the tangent is drawn to the curve $y = f(x)$ at a point where it crosses the y-axis then its equation is -
(A) $x - 4y = 2$ (B) $x + 4y = 2$ (C) $x + 4y + 2 = 0$ (D) none of these

2. The function $y = g(x)$ is given by -
(A) $\frac{e^{-x/2}}{2}$ (B) $e^{-x/2}$ (C) $2.e^{-x/2}$ (D) $e^{-x/2} + 2$

3. The number of point of intersection of $y = f(x)$ and $y = g(x)$ -
(A) 4 (B) 0 (C) 2 (D) 1

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE -3

- ***Match the Column***
  1. (A) → (r); (B) → (q); (C) → (p); (D) → (s) **2.** (A) → (p); (B) → (r); (C) → (q); (D) → (s)
- ***Assertion & Reason***
  1. A 2. C 3. A
- ***Comprehension Based Questions***
  Comprehension # 1 : 1. A 2. C 3. D
  Comprehension # 2 : 1. A 2. C 3. D

EXERCISE - 04 [A]

CONCEPTUAL SUBJECTIVE EXERCISE

1. Find the equations of the tangents drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1, 2).

2. Find the equation of normal to the curve $x^2 = 4y$ passing through the point (1, 2)

3. Prove that the length intercepted by the coordinate axes on any tangent to the curve, $x^{2/3} + y^{2/3} = c^{2/3}$ is constant.

4. If tangent to the curve $y = x^2 - 5x + 6$ passes through the point M(a, 6), find the set of values of 'a'

5. Find all the tangents to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$, that are parallel to the line $x + 2y = 0$.

6. Find the equation of the normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$.

7. Prove that the segment of the normal to the curve $x = 2a\sin t + a\sin t\cos^2 t$; $y = -a\cos^3 t$ contained between the co-ordinate axes is equal to 2a.

8. A function is defined parametrically by the equations

$$x = \begin{bmatrix} 2t + t^2 \sin\dfrac{1}{t} & \text{if } t \ne 0 \\ 0 & \text{if } t = 0 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} \dfrac{1}{t}\sin t^2 & \text{if } t \ne 0 \\ 0 & \text{if } t = 0 \end{bmatrix}$$

Find the equation of the tangent and normal at the point for $t = 0$ if they exist.

9. Find the point of intersection of the tangents drawn to the curve $x^2y = 1 - y$ at the points where it is intersected by the curve $xy = 1 - y$.

10. (a) Find the angle of intersection between the curves $y^2 = \dfrac{2x}{\pi}$ and $y = \sin x$ at $x = \dfrac{\pi}{2}$

(b) Find the angle of intersection between the curves $y^2 = 4x$ and $x^2 + y^2 = 5$.

11. Show that the angle between the tangent at any point 'A' of the curve $\ell n(x^2 + y^2) = C\tan^{-1}\dfrac{y}{x}$ and the line joining A to the origin is independent of the position of A on the curve.

12. (a) Find the condition that the curves $\dfrac{x^2}{a} + \dfrac{y^2}{b} = 1$ & $\dfrac{x^2}{a'} + \dfrac{y^2}{b'} = 1$ may cut orthogonally.

(b) Show that the curves $\dfrac{x^2}{a^2 + K_1} + \dfrac{y^2}{b^2 + K_1} = 1$ & $\dfrac{x^2}{a^2 + K_2} + \dfrac{y^2}{b^2 + K_2} = 1$ intersect orthogonally $(K_1 \ne K_2)$.

13. (a) Find the value of n so that the subnormal at any point on the curve $xy^n = a^{n+1}$ may be constant.

(b) Show that in the curve $y = a.\ell n(x^2 - a^2)$, sum of the length of tangent and subtangent varies as the product of the coordinates of the point of contact.

14. Water is flowing out at the rate of 6 m^3/min from a reservoir shaped like a hemispherical bowl of radius R = 13 m. The volume of water in the hemispherical bowl is given by $V = \frac{\pi}{3}.y^2(3R - y)$ when the water is y meter deep. Find

(a) At what rate is the water level changing when the water is 8 m deep.

(b) At what rate is the radius of the water surface changing when the water is 8 m deep.

15. Sand is pouring from a pipe at the rate of 12cc/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always $1/6^{th}$ of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm.

16. If in a triangle ABC, the side 'c' and the angle 'C' remain constant, while the remaining elements are changed slightly, show that $\frac{da}{\cos A} + \frac{db}{\cos B} = 0$

17. (a) Use differentials to a approximate the values of ; (i) $\sqrt{36.6}$ and (ii) $\sqrt[3]{26}$

(b) If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

18. A man 1.5 m tall walks away from a lamp post 4.5 m high at the rate of 4 km/hr.

(a) How fast is the farther end of the shadow moving on the pavement ?

(b) How fast is his shadow lengthening ?

CONCEPTUAL SUBJECTIVE EXERCISE — **ANSWER KEY** — EXERCISE-4(A)

1. $2\sqrt{3}x - y = 2\left(\sqrt{3} - 1\right)$ or $2\sqrt{3}x + y = 2\left(\sqrt{3} + 1\right)$ **2.** x + y = 3 **4.** $a \in (-\infty , 0] \cup [5 , \infty)$

5. $x + 2y = \pi/2$ & $x + 2y = -3\pi/2$ **6.** x + y −1 = 0

8. Tangent ; 2y − x = 0; Normal : 2x + y = 0 **9.** (0, 1) **10.** (a) $\cot^{-1}\pi$ (b) $\tan^{-1}3$

11. $\theta = \tan^{-1}\frac{2}{C}$ **12.** (a) a − b = a' − b' **13.** (a) n = −2 **14.** (a) $-\frac{1}{24\pi}$ m/min., (b) $-\frac{5}{288\pi}$ m/min.

15. $1/48\pi$ cm/s **17.** (a) (i) 6.05, (ii) $\frac{80}{27}$; (b) $9.72\pi\ cm^3$ **18.** (a) 6 km/h; (b) 2 km/hr

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. The curve $y = ax^3 + bx^2 + cx + 5$, touches the x-axis at P(–2, 0) and cuts the y axis at a point Q, where its gradient is 3. Find a, b, c. [JEE 1994]

2. At time t > 0, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At t = 0, the radius of the sphere is 1 unit and at t = 15 the radius is 2 units.
   (a) Find the radius of the sphere as a function of time t.
   (b) At what time t will the volume of the sphere be 27 times its volume at t = 0

3. If p_1 and p_2 be the lengths of the perpendiculars from the origin on the tangent and normal respectively at any point (x, y) on a curve, then show that $\left.\begin{matrix} p_1 = |x\sin\Psi - y\cos\Psi| \\ p_2 = |x\cos\Psi + y\sin\Psi| \end{matrix}\right]$ where $\tan\Psi = \frac{dy}{dx}$. If in the above case, the curve be $x^{2/3} + y^{2/3} = a^{2/3}$ then show that : $4p_1^2 + p_2^2 = a^2$

4. A and B are points of the parabola $y = x^2$. The tangents at A and B meet at C. The median of the triangle ABC from C has length 'm' units. Find the area of the triangle in terms of 'm'.

5. Tangent at a point P_1 {other than (0, 0)} on the curve $y = x^3$ meets the curve again at P_2. The tangent at P_2 meets the curve at P_3, and so on. Show that the abscissae of $P_1, P_2, P_3, \ldots, P_n$, form a G.P. Also find the ratio [area $(\Delta P_1P_2P_3)$]/[area $(\Delta P_2P_3P_4)$]. [JEE 1993]

6. The chord of the parabola $y = -a^2x^2 + 5ax - 4$ touches the curve $y = \frac{1}{1-x}$ at the point x = 2 and is bisected by that point. Find 'a'.

7. Prove that the segment of the tangent to the curve $y = \frac{a}{2}\ \ell n\frac{a+\sqrt{a^2-x^2}}{a-\sqrt{a^2-x^2}} - \sqrt{a^2-x^2}$ contained between the y-axis & the point of tangency has a constant length.

8. If the tangent at the point (x_1, y_1) to the curve $x^3 + y^3 = a^3$ meets the curve again in (x_2, y_2) then show that $\frac{x_2}{x_1} + \frac{y_2}{y_1} = -1$.

9. A variable ΔABC in the xy plane has its orthocentre at vertex 'B', a fixed vertex 'A' at the origin and the third vertex 'C' restricted to lie on the parabola $y = 1 + \frac{7x^2}{36}$. The point B starts at the point (0, 1) at time t = 0 and moves upward along the y-axis at a constant velocity of 2 cm/sec. How fast is the area of the triangle increasing when $t = \frac{7}{2}$ sec.

10. What normal to the curve $y = x^2$ forms the shortest chord ? [JEE 1992]

BRAIN STORMING SUBJECTIVE EXERCISE **ANSWER KEY** EXERCISE-4(B)

1. $a = -\frac{1}{2}$, $b = \frac{-3}{4}$, c = 3 2. (a) $r = (1+t)^{1/4}$, (b) t = 80 4. $\frac{m\sqrt{m}}{\sqrt{2}}$ 5. 1 : 64
6. a = 1 9. $\frac{66}{7}$ 10. $x + \sqrt{2}y = \sqrt{2}$ or $x - \sqrt{2}y = -\sqrt{2}$

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. The normal to the curve $x = a(1 + \cos\theta)$, $y = a\sin\theta$ at 'θ' always passes through the fixed point- [AIEEE-2004]

(1) (a, 0) (2) (0, a) (3) (0, 0) (4) (a, a)

2. The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point θ is such that-

(1) it passes through the origin

(2) it makes angle $\left(\frac{\pi}{2}+\theta\right)$ with the x-axis

(3) it passes through $\left(a\frac{\pi}{2},-a\right)$

(4) it is a constant distance from the origin

3. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is- [AIEEE-2006]

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

4. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is :- [AIEEE-2010]

(1) y = 0 (2) y = 1 (3) y = 2 (4) y = 3

5. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\, dt$, $x \in R$, which are parallel to the line $y = 2x$, are equal to [JEE-MAIN 2013]

(1) ± 1 (2) ± 2 (3) ± 3 (4) ± 4

PREVIOUS YEARS QUESTIONS	ANSWER KEY	EXERCISE-5 [A]

1. 1 2. 2,4 3. 1 4. 4 5. 1

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. If the normal to the curve, $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x-axis. Then $f'(3)$ **[JEE 2000 (Screening) 1M out of 35]**

(A) -1 (B) $-\frac{3}{4}$ (C) $\frac{4}{3}$ (D) 1

2. The point(s) on the curve $y^3+3x^2 = 12y$ where the tangent is vertical, is(are) -

(A) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (B) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$ (C) (0, 0) (D) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

[JEE 2002 (Screening), 3M]

3. Tangent to the curve $y = x^2 + 6$ at a point P(1, 7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then the coordinates of Q are - **[JEE 2005 (Mains), 2M out of 60]**

(A) (–6, –11) (B) (–9, –13) (C) (–10, –15) (D) (–6, –7)

4. If $|f(x_1) - f(x_2)| < (x_1 - x_2)^2$, for all $x_1, x_2 \in R$. Find the equation of tangent to the curve $y = f(x)$ at the point (1, 2). **[JEE 2005 (Mains), 2M out of 60]**

5. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$:- **[JEE 2007, 3M]**

(A) on the left of $x = c$ (B) on the right of $x = c$

(C) at no point (D) at all points

PREVIOUS YEARS QUESTIONS		ANSWER KEY		EXERCISE-5 [B]
1. D	2. D	3. D	4. $y - 2 = 0$	5. A