JEE Type Solved Examples : Single Option Correct Type Questions

- This section contains **10 multiple choice examples.** Each example has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

• Ex. 1 *If* $\binom{2n+1}{0}+\binom{2n+1}{3}+\binom{2n+1}{6}+\ldots=170$, *then n equals*

(a) 2 (b) 4 (c) 6 (d) 8

Sol. (b) $\because\ (1+x)^{2n+1}={}^{2n+1}C_0+{}^{2n+1}C_1x+{}^{2n+1}C_2x^2+{}^{2n+1}C_3x^3+{}^{2n+1}C_4x^4+{}^{2n+1}C_5x^5+{}^{2n+1}C_6x^6+\ldots$

Putting $x=1,\omega,\omega^2$ (where ω is cube root of unity) and adding, we get

$$2^{2n+1}+(1+\omega)^{2n+1}+(1+\omega^2)^{2n+1}=3({}^{2n+1}C_0+{}^{2n+1}C_3+{}^{2n+1}C_6+\ldots$$

$$\Rightarrow\ 2^{2n+1}-\omega^{2(2n+1)}-\omega^{2n+1}=3({}^{2n+1}C_0+{}^{2n+1}C_3+{}^{2n+1}C_6+\ldots)\ [\because 1+\omega+\omega^2=0]$$

$$\Rightarrow\ {}^{2n+1}C_0+{}^{1+2n}C_3+{}^{2n+1}C_6+\ldots=\frac{1}{3}(2^{2n+1}-\omega^{2(2n+1)}-\omega^{2n+1})$$

$$\Rightarrow\ \binom{2n+1}{0}+\binom{2n+1}{3}+\binom{2n+1}{6}+\ldots=\frac{1}{3}(2^{2n+1}-\omega^{2(2n+1)}-\omega^{2n+1})$$

$$\Rightarrow\ 170=\frac{1}{3}(2^{2n+1}-\omega^{2(2n+1)}-\omega^{2n+1})$$

For $n=4,\ 170=\frac{1}{3}(512-1-1)=170$ $[\because \omega^3=1]$

Hence, $n=4$

• Ex. 2 $({}^mC_0+{}^mC_1-{}^mC_2-{}^mC_3)+({}^mC_4+{}^mC_5-{}^mC_6-{}^mC_7)+\ldots=0$ *if and only if for some positive integer k, m is equal to*

(a) $4k$ (b) $4k+1$ (c) $4k-1$ (d) $4k+2$

Sol. (c) If $\theta\in R$ and $i=\sqrt{-1}$, then $(\cos\theta+i\sin\theta)^m$

$$={}^mC_0(\cos\theta)^m+{}^mC_1(\cos\theta)^{m-1}(i\sin\theta)+{}^mC_2(\cos\theta)^{m-2}(i\sin\theta)^2+\ldots+{}^mC_m(i\sin\theta)^m$$

$$(\cos m\theta+i\sin m\theta)=[{}^mC_0(\cos\theta)^m-{}^mC_2(\cos\theta)^{m-2}\cdot\sin^2\theta+{}^mC_4(\cos\theta)^{m-4}\sin^4\theta-\ldots]+i[{}^mC_1(\cos\theta)^{m-1}\cdot\sin\theta-{}^mC_3(\cos\theta)^{m-3}\sin^3\theta+\ldots]$$

[using Demoivre's theorem]

Comparing real and imaginary parts, we get

$$\cos m\theta={}^mC_0(\cos\theta)^m-{}^mC_2(\cos\theta)^{m-2}\sin^2\theta+{}^mC_4(\cos\theta)^{m-4}\sin^4\theta-\ldots\ \ldots\text{(i)}$$

$$\sin m\theta={}^mC_1(\cos\theta)^{m-1}\cdot\sin\theta-{}^mC_3(\cos\theta)^{m-3}\cdot\sin^3\theta+\ldots\ \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$\cos m\theta+\sin m\theta={}^mC_0(\cos\theta)^m+{}^mC_1(\cos\theta)^{m-1}\cdot\sin\theta-{}^mC_2(\cos\theta)^{m-2}\sin^2\theta-{}^mC_3(\cos\theta)^{m-3}\sin^3\theta+{}^mC_4(\cos\theta)^{m-4}\sin^4\theta+\ldots\sin\left(m\theta+\frac{\pi}{4}\right)$$

$$=(\cos\theta)^m\left\{\begin{array}{l}{}^mC_0+{}^mC_1\tan\theta-{}^mC_2\tan^2\theta-{}^mC_3\tan^3\theta\\+{}^mC_4\tan^4\theta+{}^mC_5\tan^5\theta-\ldots\end{array}\right\}$$

Putting $\theta=\frac{\pi}{4}$, $\sqrt{2}\sin\left(\frac{(m+1)\pi}{4}\right)=\frac{1}{2^{m/2}}$

$$\left\{\begin{array}{l}({}^mC_0+{}^mC_1-{}^mC_2-{}^mC_3)+({}^mC_4+{}^mC_5-{}^mC_6-{}^mC_7)\\+\ldots+({}^mC_{m-3}+{}^mC_{m-2}-{}^mC_{m-1}-{}^mC_m)\end{array}\right\}$$

$\because ({}^mC_0+{}^mC_1-{}^mC_2-{}^mC_3)+({}^mC_4+{}^mC_5-{}^mC_6-{}^mC_7)+\ldots=0$ [given]

$$\therefore\ \sin\left(\frac{(m+1)\pi}{4}\right)=0\Rightarrow\frac{(m+1)\pi}{4}=k\pi$$

or $m=4k-1,\forall k\in I$

• Ex. 3 *If coefficient of* x^n *in the expansion of* $(1+x)^{101}(1-x+x^2)^{100}$ *is non-zero, then n cannot be of the form*

(a) $3\lambda+1$ (b) 3λ (c) $3\lambda+2$ (d) $4\lambda+1$

Sol. (c) $\because (1+x)^{101}(1-x+x^2)^{100}=(1+x)((1+x)(1-x+x^2))^{100}$

$$=(1+x)(1+x^3)^{100}$$

$$=(1+x)(1+{}^{100}C_1x^3+{}^{100}C_2x^6+{}^{100}C_3x^9+\ldots+\ldots+{}^{100}C_{10}x^{300})$$

Clearly, in this expression x^3 will present if $n=3\lambda$ or $n=3\lambda+1$. So, n cannot be of the form $3\lambda+2$.

• Ex. 4 *The sum* $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, *(where* $\frac{p}{q}=0$, *if* $p<q$*) is maximum when m is*

(a) 5 (b) 10 (c) 15 (d) 20

Sol. (c) $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}=\sum_{i=0}^{m}{}^{10}C_i\,{}^{20}C_{m-i}$

$$={}^{10}C_0\cdot{}^{20}C_m+{}^{10}C_1\cdot{}^{20}C_{m-1}+{}^{10}C_2\cdot{}^{20}C_{m-2}+\ldots+{}^{10}C_m\cdot{}^{20}C_0$$

= Coefficient of x^m in the expansion of product $(1+x)^{10}(1+x)^{20}$

= Coefficient of x^m in the expansion of $(1+x)^{30}={}^{30}C_m$

To get maximum value of the given sum, ${}^{30}C_m$ should be maximum. Which is so, when $m=\frac{30}{2}=15$

Ex. 5 *If* ${}^{n-1}C_r=(k^2-3)\cdot{}^nC_{r+1}$ *then k belongs to*

(a) $(-\infty,-2]$ (b) $[2,\infty)$
(c) $[-\sqrt{3},\sqrt{3}]$ (d) $(\sqrt{3},2]$

Sol. (d) $\because\ {}^{n-1}C_r=(k^2-3)\cdot{}^nC_{r+1}$

$$\Rightarrow\quad k^2-3=\frac{{}^{n-1}C_r}{{}^nC_{r+1}}=\frac{r+1}{n}\qquad\text{...(i)}$$

$$\Rightarrow\quad 0\le r\le n-1$$

$$\Rightarrow\quad 1\le r+1\le n$$

$$\Rightarrow\quad \frac{1}{n}\le\frac{r+1}{n}\le 1$$

$$\Rightarrow\quad \frac{1}{n}\le(k^2-3)\le 1$$

$$\Rightarrow\quad 3+\frac{1}{n}\le k^2\le 4\quad\text{or}\quad 3<k^2\le 4\qquad[\text{here, } n\ge 2]$$

$$\therefore\quad k\in[-2,\sqrt{3})\cup(\sqrt{3},2]$$

Ex. 6 *If* $\left(x+\frac{1}{x}+1\right)^6=a_0+\left(a_1x+\frac{b_1}{x}\right)+\left(a_2x^2+\frac{b_2}{x^2}\right)+\dots+\left(a_6x^6+\frac{b_6}{x^6}\right)$, *the value of* a_0 *is*

(a) 121 (b) 131
(c) 141 (d) 151

Sol. (c) $\because\left(x+\frac{1}{x}+1\right)^6=\sum_{r=0}^{6}{}^6C_r\left(x+\frac{1}{x}\right)^r$ for constant term r must be even integer.

$\therefore\ a_0={}^6C_0+{}^6C_2\times{}^2C_1+{}^6C_4\times{}^4C_2+{}^6C_6\times{}^6C_3$

$=1+30+90+20=141$

Ex. 7 *The coefficient of* x^{50} *in the series*

$$\sum_{r=1}^{101}rx^{r-1}(1+x)^{101-r}\ \textit{is}$$

(a) ${}^{100}C_{50}$ (b) ${}^{101}C_{50}$
(c) ${}^{102}C_{50}$ (d) ${}^{103}C_{50}$

Sol. (c) Let $S=\sum_{r=1}^{101}rx^{r-1}(1+x)^{101-r}$

$=(1+x)^{100}+2x(1+x)^{99}+3x^2(1+x)^{98}+\dots+101x^{100}$

$$S=(1+x)^{100}\left\{1+2\left(\frac{x}{1+x}\right)+3\left(\frac{x}{1+x}\right)^2+\dots+101\left(\frac{x}{1+x}\right)^{100}\right\}\qquad\text{...(i)}$$

$$\therefore\ \frac{Sx}{(1+x)}=(1+x)^{100}\left\{\left(\frac{x}{1+x}\right)+2\left(\frac{x}{1+x}\right)^2+3\left(\frac{x}{1+x}\right)^3+\dots+101\left(\frac{x}{1+x}\right)^{101}\right\}\qquad\text{...(ii)}$$

On subtracting Eq. (ii) from Eq. (i), then we get

$$\frac{S}{(1+x)}=(1+x)^{100}\left\{1+\left(\frac{x}{1+x}\right)+\left(\frac{x}{1+x}\right)^2+\dots+\left(\frac{x}{1+x}\right)^{100}-101\left(\frac{x}{1+x}\right)^{101}\right\}$$

$$=(1+x)^{100}\left\{\frac{1\cdot\left(1-\left(\frac{x}{1+x}\right)^{101}\right)}{1-\left(\frac{x}{1+x}\right)}-101\left(\frac{x}{1+x}\right)^{101}\right\}$$

$$\therefore\quad S=(1+x)^{102}-x^{101}(1+x)-101x^{101}$$

and coefficient of x^{50} in $S={}^{102}C_{50}$.

Ex. 8 *The largest integer* λ *such that* 2^λ *divides* $3^{2^n}-1$, $n\in N$ *is* by putting value n=1

(a) $n-1$ (b) n (c) $n+1$ (d) $n+2$

Sol. (d) $\because\ 3^{2^n}-1=(4-1)^{2^n}-1$

$=(4^{2^n}-{}^{2^n}C_1\cdot4^{2^n-1}+{}^{2^n}C_2\cdot4^{2^n-2}-\dots-{}^{2^n}C_{2^n-1}\cdot4+1)-1$

$=4^{2^n}-2^n\cdot4^{2^n-1}+\frac{2^n(2^n-1)}{2}\cdot4^{2^n-2}-\dots-2^n\cdot4$

$=2^{n+2}(2^{2^{n+1}-n-2}-2^{2^{n+1}-4}+\dots-1)=2^{n+2}$(Integer)

Hence, $3^{2^n}-1$ is divisible by 2^{n+2}. $\lambda=n+2$

Ex. 9 The last term in the binomial expansion of $\left(\sqrt[3]{2}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3\sqrt[3]{9}}\right)^{\log_3 8}$, the 5th term from beginning is

(a) ${}^{10}C_6$ (b) $2\,{}^{10}C_4$
(c) $\frac{1}{2}\cdot{}^{10}C_4$ (d) None of the above

Sol. (a) Since, last term in the expansion of $\left(\sqrt[3]{2}-\frac{1}{\sqrt{2}}\right)^n$

$$=\left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}\Rightarrow{}^nC_n\cdot\left(-\frac{1}{\sqrt{2}}\right)^n=\left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}$$

$$\Rightarrow\quad(-1)^n\cdot\left(\frac{1}{2}\right)^{n/2}=\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}=(3^{-5/3})^{\log_3 2^3}$$

$$=3^{-\frac{5}{3}\times3\times\log_3 2}=3^{-5\log_3 2}=3^{\log_3 2^{-5}}=2^{-5}=\left(\frac{1}{2}\right)^5$$

$$\Rightarrow\quad(-1)^n\cdot\left(\frac{1}{2}\right)^{n/2}=\left(\frac{1}{2}\right)^5\ \therefore\ n=10$$

Now, 5th term from beginning $={}^{10}C_4(\sqrt[3]{2})^6\left(-\frac{1}{\sqrt{2}}\right)^4$

$$={}^{10}C_4\cdot2^2\cdot\frac{1}{2^2}={}^{10}C_4={}^{10}C_6$$

Ex. 10 If $f(x) = \sum_{r=1}^{n}\{r^2({}^nC_r - {}^nC_{r-1}) + (2r+1){}^nC_r\}$ and $f(30) = 30(2)^{\lambda}$, then the value of λ is

(a) 3 (b) 4 (c) 5 (d) 6

Sol. (c) Here, $f(x) = \sum_{r=1}^{n}\{r^2({}^nC_r - {}^nC_{r-1}) + (2r+1){}^nC_r\}$

$$= \sum_{r=1}^{n}(r^2+2r+1){}^nC_r - r^2\cdot{}^nC_{r-1}$$

$$= \sum_{r=1}^{n}((r+1)^2\cdot{}^nC_r - r^2\cdot{}^nC_{r-1})$$

$$= (n+1)^2\cdot{}^nC_n - 1^2\cdot{}^nC_0$$

$$= (n+1)^2 - 1 = (n^2+2n)$$

$\therefore$ $f(30) = (30)^2 + 2(30) = 960$

$$= 30\times 32 = 30(2)^5 = 30(2)^{\lambda} \quad \text{[given]}$$

Hence, $\lambda = 5$

JEE Type Solved Examples : More than One Correct Option Type Questions

- This section contains **5 multiple choice examples.** Each example has four choices (a), (b), (c) and (d) out of **which more than one** may be correct.

Ex. 11 Let $a_n = \left(1+\frac{1}{n}\right)^n$. Then for each $n \in N$

(a) $a_n \ge 2$ (b) $a_n < 3$ (c) $a_n < 4$ (d) $a_n < 2$

Sol. (a, b, c)

$$\because a_n = \left(1+\frac{1}{n}\right)^n = {}^nC_0 + {}^nC_1\cdot\left(\frac{1}{n}\right) + \sum_{r=2}^{n}{}^nC_r\left(\frac{1}{n}\right)^2$$

$$= 2 + \sum_{r=2}^{n}{}^nC_r\left(\frac{1}{n}\right)^2$$

$\therefore$ $a_n \ge 2$ for all $n \in N$

Also, $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e = 2.7182\ldots$

$\therefore$ $a_n < e$

Finally, $2 \le a_n < e$

Use this way.

Ex. 12 Let $S_n(x) = \sum_{k=0}^{n}{}^nC_k\sin(kx)\cos((n-k)x)$ then

(a) $S_5\left(\frac{\pi}{2}\right) = 16$ (b) $S_7\left(\frac{-\pi}{2}\right) = 64$

(c) $S_{50}(\pi) = 0$ (d) $S_{51}(-\pi) = -2^{50}$

Sol. (a, b, c)

$$\because \quad S_n(x) = \sum_{k=0}^{n}{}^nC_k\sin(kx)\cos((n-k)x) \quad \ldots\text{(i)}$$

Replace k by $n-k$ in Eq. (i), then

$$S_n(x) = \sum_{k=0}^{n}{}^nC_{n-k}\sin((n-k)x)\cos(kx)$$

or

$$S_n(x) = \sum_{k=0}^{n}{}^nC_k\sin((n-k)x)\cos(kx) \quad \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2S_n(x) = \sum_{k=0}^{n}{}^nC_k\cdot\sin(nx) = 2^n\cdot\sin(nx)$$

$$\Rightarrow \quad S_n(x) = 2^{n-1}\cdot\sin(nx)$$

$$\therefore \quad S_5\left(\frac{\pi}{2}\right) = 2^4\cdot\sin\left(\frac{5\pi}{2}\right) = 16,$$

$$S_7\left(-\frac{\pi}{2}\right) = 2^6\cdot\sin\left(-\frac{7\pi}{2}\right) = 2^6\times -1\times -1 = 64$$

$$S_{50}(\pi) = 2^{49}\cdot\sin(50\pi) = 0$$

and $S_{51}(-\pi) = 2^{50}\cdot\sin(-51\pi) = 0$

Ex. 13 If $a+b=k$, when $a,b>0$ and $S(k,n) = \sum_{r=0}^{n}r^2({}^nC_r)a^r\cdot b^{n-r}$, then

(a) $S(1,3) = 3(3a^2+ab)$ (b) $S(2,4) = 16(4a^2+ab)$

(c) $S(3,5) = 25(5a^2+ab)$ (d) $S(4,6) = 36(6a^2+ab)$

Sol. (a, b)

$$\because \quad S(k,n) = \sum_{r=0}^{n}r^2\cdot({}^nC_r)a^r\cdot b^{n-r}$$

$$= b^n\sum_{r=0}^{n}r^2\cdot\left(\frac{n}{r}\cdot{}^{n-1}C_{r-1}\right)\cdot\left(\frac{a}{b}\right)^r$$

$$= nb^n\sum_{r=0}^{n}((r-1)+1)^{n-1}C_{r-1}\cdot\left(\frac{a}{b}\right)^r$$

$$= nb^n\sum_{r=0}^{n}((n-1)\cdot{}^{n-2}C_{r-2} + {}^{n-1}C_{r-1})\left(\frac{a}{b}\right)^r$$

$$= nb^n\cdot(n-1)\cdot\left(\frac{a}{b}\right)^2\sum_{r=0}^{n}{}^{n-2}C_{r-2}\left(\frac{a}{b}\right)^{r-2} + nb^n\cdot\left(\frac{a}{b}\right)\sum_{r=0}^{n}{}^{n-1}C_{r-}\left(\frac{a}{b}\right)^{r-1}$$

$$= nb^n\cdot(n-1)\left(\frac{a}{b}\right)^2\left(1+\frac{a}{b}\right)^{n-2} + nb^n\cdot\left(\frac{a}{b}\right)\left(1+\frac{a}{b}\right)^{n-1}$$

$$= n(n-1)a^2k^{n-2} + nak^{n-1}$$

$$= n^2a^2k^{n-2} + nak^{n-2}(k-a) = n^2a^2k^{n-2} + nabk^{n-2}$$

$$\therefore S(1,3) = 9a^2 + 3ab = 3(3a^2+ab) \quad [\because a+b=k]$$

$$S(2,4) = 16(4a^2+ab)$$

$$S(3,5) = 135(5a^2+ab)$$

$$S(4,6) = 1536(6a^2+ab)$$

● **Ex. 14** *The value of x, for which the ninth term in the expansion of* $\left\{\dfrac{\sqrt{10}}{(\sqrt{x})^{5\log_{10} x}} + x \cdot x^{\frac{1}{2\log_{10} x}}\right\}^{10}$ *is 450 is equal to*

(a) 10 (b) 10^2 (c) $\sqrt{10}$ (d) $10^{-2/5}$

Sol. (b, d) Let $\log_{10} x = \lambda \Rightarrow x = 10^{\lambda}$ ...(i)

Given, $T_9 = 450$

$$\Rightarrow {}^{10}C_8 \cdot \left(\frac{\sqrt{10}}{10^{\frac{5\lambda^2}{2}}}\right)^2 \cdot (10^{\lambda} \cdot 10^{1/2})^8 = 450$$

$$\Rightarrow {}^{10}C_2 \cdot \frac{10}{10^{5\lambda^2}} \cdot 10^{8\lambda} \cdot 10^4 = 450$$

$$\Rightarrow 10^{8\lambda + 4 - 5\lambda^2} = 1 = 10^0$$

$$\Rightarrow 8\lambda + 4 - 5\lambda^2 = 0$$

$$\Rightarrow 5\lambda^2 - 8\lambda - 4 = 0$$

$$\Rightarrow \lambda = 2, -2/5$$

$$\Rightarrow x = 10^2, 10^{-2/5} \quad \text{[from Eq. (i)]}$$

● **Ex. 15** *For a positive integer n, if the expansion of* $\left(\dfrac{5}{x^2} + x^4\right)^n$ *has a term independent of x, then n can be*

(a) 18 (b) 27 (c) 36 (d) 45

Sol. (a, b, c, d) Let $(r+1)$th term of $\left(\dfrac{5}{x^2} + x^4\right)^n$ be independent of x. We have, $T_{r+1} = {}^nC_r \left(\dfrac{5}{x^2}\right)^{n-r} (x^4)^r = {}^nC_r \cdot 5^{n-r} \cdot x^{6r-2n}$

For this term to be independent of x,

$$6r - 2n = 0 \text{ or } n = 3r$$

For $r = 6, 9, 12, 15,$

$$n = 18, 27, 36, 45.$$

JEE Type Solved Examples : Passage Based Questions

- This section contains **2 solved passages** based upon each of the passage **3 multiple choice** examples have to be answered. Each of these examples has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Passage I (Ex. Nos. 16 to 18)

Consider $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, *where* $a_0, a_1, a_2, \ldots, a_{2n}$ *are real numbers and n is a positive integer.*

16. The value of $\sum_{r=0}^{n-1} a_{2r}$ is

(a) $\dfrac{9^n - 2a_{2n} - 1}{4}$ (b) $\dfrac{9^n - 2a_{2n} + 1}{4}$

(c) $\dfrac{9^n + 2a_{2n} - 1}{4}$ (d) $\dfrac{9^n + 2a_{2n} + 1}{4}$

17. The value of $\sum_{r=1}^{n} a_{2r-1}$ is

(a) $\dfrac{9^n - 1}{2}$ (b) $\dfrac{9^n - 1}{4}$ (c) $\dfrac{9^n + 1}{2}$ (d) $\dfrac{9^n + 1}{4}$

18. The value of a_2 is

(a) ${}^{4n+1}C_2$ (b) ${}^{3n+1}C_2$

(c) ${}^{2n+1}C_2$ (d) ${}^{n+1}C_2$

Sol.

We have, $(1 + x + x^2)^{2n} = \sum_{r=0}^{4n} a_r x^r$ ...(i)

Replacing x by $\dfrac{1}{x}$ in Eq. (i), we get

$$\left(1 + \frac{1}{x} + \frac{1}{x^2}\right)^{2n} = \sum_{r=0}^{4n} a_r \left(\frac{1}{x}\right)^r$$

$$\Rightarrow (1 + x + x^2)^{2n} = \sum_{r=0}^{4n} a_r x^{4n-r} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $\sum_{r=0}^{4n} a_r x^r = \sum_{r=0}^{4n} a_r x^{4n-r}$

Equating the coefficient of x^{4n-r} on both sides, we get

$$a_{4n-r} = a_r \text{ for } 0 \le r \le 4n$$

Hence, $a_r = a_{4n-r}$

Putting $x = 1$ in Eq. (i), then

$$\sum_{r=0}^{4n} a_r = 3^{2n} = 9^n \quad \text{...(iii)}$$

Putting $x = -1$ in Eq. (i), then $\sum_{r=0}^{4n} (-1)^r a_r = 1$ ...(iv)

16. (b) On adding Eqs. (iii) and (iv), we get

$$2(a_0 + a_2 + a_4 + \ldots + a_{2n-2} + a_{2n} + \ldots + a_{4n}) = 9^n + 1$$

$$\Rightarrow 2[2(a_0 + a_2 + a_4 + \ldots + a_{2n-2}) + a_{2n}) = 9^n + 1$$

$$[\because a_r = a_{4n-r}]$$

$$\therefore a_0 + a_2 + a_4 + \ldots + a_{2n-2} = \frac{9^n - 2a_{2n} + 1}{4}$$

$$\Rightarrow \sum_{r=0}^{n-1} a_{2r} = \frac{9^n - 2a_{2n} + 1}{4}$$

17. (b) On subtracting Eq. (iv) from Eq. (iii), we get

$$2(a_1 + a_3 + a_5 + \ldots + a_{2n-1} + a_{2n+1} + \ldots + a_{4n-1}) = 9^n - 1$$

$$\Rightarrow 2[2(a_1 + a_3 + a_5 + \ldots + a_{2n-1}] = 9^n - 1 \quad [\because a_r = a_{4n-r}]$$

$\therefore\ a_1 + a_3 + a_5 + \ldots + a_{2n-1} = \dfrac{9^n - 1}{4}$

$\Rightarrow\ \sum_{r=1}^{n} a_{2r-1} = \dfrac{9^n - 1}{4}$

18. (c) $\because a_2 =$ Coefficient of x^2 in $(1 + x + x^2)^{2n}$

$$\therefore (1 + x + x^2)^{2n} = \sum_{\alpha+\beta+\gamma=2n} \frac{2n!}{\alpha!\beta!\gamma!}(1)^{\alpha}(x)^{\beta}(x^2)^{\gamma}$$

$$= \sum_{\alpha+\beta+\gamma=2n} \frac{2n!}{\alpha!\beta!\gamma!} x^{\beta+2\gamma}$$

For a_2, $\beta + 2\gamma = 2$

Possible values of α, β, γ are $(2n-2, 2, 0)$ and $(2n-1, 0, 1)$.

$$\therefore\ a_2 = \frac{2n!}{(2n-2)!2!0!} + \frac{2n!}{(2n-1)!0!1!}$$

$$= {}^{2n}C_2 + {}^{2n}C_1 = {}^{2n+1}C_2$$

Passage II

(Ex. Nos. 19 to 21)

Let $S = \sum_{r=1}^{30} \dfrac{{}^{30+r}C_r\,(2r-1)}{{}^{30}C_r\,(30+r)}$, $K = \sum_{r=0}^{30} ({}^{30}C_r)^2$

and $G = \sum_{r=0}^{60} (-1)^r ({}^{60}C_r)^2$

19. The value of $(G - S)$ is

(a) 0 (b) 1 (c) 2^{30} (d) 2^{60}

20. The value of $(SK - SG)$ is

(a) 0 (b) 1
(c) 2^{30} (d) 2^{60}

21. The value of $K + G$ is

(a) $2S - 2$ (b) $2S - 1$
(c) $2S + 1$ (d) $2S + 2$

Sol.

$$\because S = \sum_{r=1}^{30} \frac{{}^{30+r}C_r\,(2r-1)}{{}^{30}C_r\,(30+r)} = \sum_{r=1}^{30} \frac{{}^{30+r}C_r}{{}^{30}C_r}\left(1 - \frac{30-r+1}{30+r}\right)$$

$$= \sum_{r=1}^{30}\left[\frac{{}^{30+r}C_r}{{}^{30}C_r} - \frac{{}^{30+r}C_r}{{}^{30}C_r}\cdot\frac{(30-r+1)}{(30+r)}\right]$$

$$= \sum_{r=1}^{30}\left[\frac{{}^{30+r}C_r}{{}^{30}C_r} - \frac{\frac{(30+r)}{r}\cdot{}^{29+r}C_{r-1}}{{}^{30}C_r}\cdot\frac{(31-r)}{30+r}\right]$$

$$= \sum_{r=1}^{30}\left[\frac{{}^{30+r}C_r}{{}^{30}C_r} - \frac{{}^{29+r}C_{r-1}}{{}^{30}C_{r-1}}\right] \qquad \left[\because \frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}\right]$$

For $n = 30\left(\dfrac{31-r}{r}\cdot{}^{30}C_r = {}^{30}C_{r-1}\right)$

$$= \frac{{}^{30+30}C_{30}}{{}^{30}C_{30}} - \frac{{}^{29+1}C_0}{{}^{30}C_0} = {}^{60}C_{30} - 1$$

$$K = \sum_{r=0}^{30} ({}^{30}C_r)^2 = {}^{60}C_{30} \text{ and } G = \sum_{r=0}^{60} (-1)^r ({}^{60}C_r)^2$$

$$= ({}^{60}C_0)^2 - ({}^{60}C_1)^2 + ({}^{60}C_2)^2 - \ldots + ({}^{60}C_{60})^2 = {}^{60}C_{30}$$

$[\because n = 60$ is even$]$

19.(b) $G - S = {}^{60}C_{30} - ({}^{60}C_{30} - 1) = 1$

20. (a) $SK - SG = S(K - G) = S(G - G) = 0 \qquad [\because K = G]$

21. (d) $K + G = 2\cdot{}^{60}C_{30} = 2(S + 1) = 2S + 2$

JEE Type Solved Examples : Single Integer Answer Type Questions

- This section contains **2 examples.** The answer to each example is a **single digit integer** ranging from **0** to **9** (both inclusive).

• **Ex. 22** *The digit at unit's place in* $2^{9^{100}}$ *is*

Sol. (2) $\because 9^{100} = (2\cdot 4 + 1)^{100} = 4n + 1$ [say]

[where n is positive integer]

$\therefore\ 2^{9^{100}} = 2^{4n+1} = 2^{4n}\cdot 2 = (16)^n\cdot 2$

The digit at unit's place in $(16)^n = 6$.

$\therefore$ The digit at unit's place in $(16)^n \cdot 2 = 2$

• **Ex. 23** *If* $(1+x)^n = \sum_{r=0}^{n} a_r x^r$, $b_r = 1 + \dfrac{a_r}{a_{r-1}}$ *and* $\prod_{r=1}^{n} b_r = \dfrac{(101)^{100}}{100!}$, *then the value of* $\dfrac{n}{20}$ *is*

Sol. (5) Here, $a_r = {}^nC_r$

$$\therefore\ b_r = 1 + \frac{a_r}{a_{r-1}} = 1 + \frac{{}^nC_r}{{}^nC_{r-1}}$$

$$= 1 + \frac{n-r+1}{r} = \frac{(n+1)}{r}$$

$$\Rightarrow \prod_{r=1}^{n} b_r = \prod_{r=1}^{n} \frac{(n+1)}{r}$$

$$= \frac{(n+1)}{1}\cdot\frac{(n+1)}{2}\cdot\frac{(n+1)}{3}\cdots\frac{(n+1)}{n} = \frac{(n+1)^n}{n!}$$

$$= \frac{(101)^{100}}{100!} \qquad \text{[given]}$$

$\therefore\ n = 100 \Rightarrow \dfrac{n}{20} = 5$

JEE Type Solved Examples : Matching Type Questions

- This section contains **2 examples.** Examples 24 and 25 have three statements (A, B and C) given in Column I and four statements (p, q, r and s) in **Column II.** Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II.**

• Ex. 24

	Column I		Column II
(A)	If m and n are the numbers of rational terms in the expansions of $(\sqrt{2}+3^{1/5})^{10}$ and $(\sqrt{3}+5^{1/8})^{256}$ respectively, then	(p)	$n-m=6$
(B)	If m and n are the numbers of irrational terms in the expansions of $(2^{1/2}+3^{1/5})^{40}$ and $(5^{1/10}+2^{1/6})^{100}$ respectively, then	(q)	$m+n=20$
(C)	If m and n are the numbers of rational terms in the expansions of $(1+\sqrt{2}+3^{1/3})^6$ and $(1+\sqrt[3]{2}+\sqrt[5]{3})^{15}$ respectively, then	(r)	$n-m=31$
		(s)	$m+n=35$
		(t)	$n-m=39$

Sol. (A) → (r, s); (B) → (t); (C) → (p, q)

(A) $\because\ (\sqrt{2}+3^{1/5})^{10}=(2^{1/2}+3^{1/5})^{10}$

$$\therefore\quad T_{r+1}={}^{10}C_r\cdot 2^{\frac{10-r}{2}}\cdot 3^{\frac{r}{5}}$$

For rational terms, $r=0, 10$ $\quad[\because 0\le r\le 10]$

$\therefore$ Number of rational terms $=2$

i.e., $m=2$ and $(\sqrt{3}+5^{1/8})^{256}=(3^{1/2}+5^{1/8})^{256}$

$$\therefore\ T_{R+1}={}^{256}C_R\cdot 3^{\frac{256-R}{2}}\cdot 5^{R/8}$$

For rational terms, $r=0, 8, 16, 24, 32, \ldots, 256$ $[\because 0\le r\le 256]$

$\therefore$ Number of rational terms $=1+32=33$

i.e., $n=33 \Rightarrow m+n=35$ (s) and $n-m=31$

(B) T_{r+1} in $(2^{1/3}+3^{1/5})^{40}={}^{40}C_r\cdot 2^{\frac{40-r}{3}}\cdot 3^{r/5}$

For rational terms, $r=10, 25, 40$ $[\because 0\le r\le 40]$

$\because$ Number of rational terms $=3$

$\therefore$ Number of irrational terms

$=$ Total terms $-$ Number of rational terms

$=41-3=38$ i.e. $m=38$

and T_{R+1} in $(5^{1/10}+2^{1/6})^{100}={}^{100}C_R\cdot 5^{\frac{100-R}{10}}\cdot 2^{R/6}$

rational terms, $R=0, 30, 60, 90$ $\quad[\because 0\le R\le 100]$

$\because$ Number of rational terms $=4$

$\therefore$ Number of irrational terms $=101-4=97$

i.e. $n=97 \Rightarrow m+n=100, n-m=97-38=39$

(C) $\because\ (1+\sqrt{2}+3^{1/3})^6=(1+2^{1/2}+3^{1/3})^6$

$$=\sum_{\alpha+\beta+\gamma=6}\frac{6!}{\alpha!\beta!\gamma!}(1)^{\alpha}(2^{1/2})^{\beta}(3^{1/3})^{\gamma}$$

$$=\sum_{\alpha+\beta+\gamma=6}\frac{6!}{\alpha!\beta!\gamma!}2^{\beta/2}\cdot 3^{\gamma/3}$$

Values of (α,β,γ) for rational terms are $(0,0,6)$, $(1,2,3),(3,0,3),(0,6,0),(2,4,0),(4,2,0),(6,0,0)$.

$\therefore$ Number of rational terms $=7$ i.e., $m=7$

and $(1+\sqrt[3]{2}+\sqrt[5]{3})^{15}=(1+2^{1/3}+3^{1/5})^{15}$

$$=\sum_{\alpha+\beta+\gamma=15}\frac{15!}{\alpha!\beta!\gamma!}(1)^{\alpha}(2^{1/3})^{\beta}(3^{1/5})^{\gamma}$$

$$=\sum_{\alpha+\beta+\gamma=15}\frac{15!}{\alpha!\beta!\gamma!}2^{\beta/3}\cdot 3^{\gamma/5}$$

of (α,β,γ) for rational terms are $(5,0,10),(2,3,10),(10,0,5),(7,3,5),(4,6,5),(1,9,5),(15,0,0),(12,3,0),(9,6,0),(6,9,0),(3,12,0),(15,0,0)$.

$\therefore$ Number of rational terms $=13$ i.e. $n=13$

Hence, $\quad m+n=20$ and $n-m=6$

• Ex. 25 *If* $(1+x)^n=\sum_{r=0}^{n}C_r x^r$, *match the following.*

	Column I		Column II
(A)	If $S=\sum_{r=0}^{n}\lambda C_r$ and values of S are a, b, c for $\lambda=1, r, r^2$ respectively, then	(p)	$a=b+c$
(B)	If $S=\sum_{r=0}^{n}(-1)^r\lambda C_r$ and values of S are a,b,c for $\lambda=1,r,r^2$ respectively, then	(q)	$a+b=c+2$
(C)	If $S=\sum_{r=0}^{n}\frac{\lambda C_r}{(r+1)}$ and values of S are a, b, c for $\lambda=1, r, r^2$ respectively, then	(r)	$a^3+b^3+c^3=3abc$
		(s)	$b^{c-a}+(c-a)^b=1$
		(t)	$a+c=4b$

Sol. (A) → (p, q); (B) → (p, r, t); (C) →(s, t)

(A) For $\lambda=1, a=\sum_{r=0}^{n}C_r=2^n$

For $\lambda=r, b=\sum_{r=0}^{n}r\,C_r=\sum_{r=0}^{n}r\cdot\frac{n}{r}\cdot{}^{n-1}C_{r-1}$

$$=n\sum_{r=0}^{n}{}^{n-1}C_{r-1}=n\cdot 2^{n-1}$$

and for $\lambda=r^2$, $c=\sum_{r=0}^{n}r^2\,C_r=\sum_{r=0}^{n}r^2\cdot\frac{n}{r}\cdot{}^{n-1}C_{r-1}$

$$=n\sum_{r=0}^{n}r\cdot{}^{n-1}C_{r-1}=n\sum_{r=1}^{n}r\cdot{}^{n-1}C_{r-1}$$

$$= n\left[\sum_{r=1}^{n} \{(r-1)+1\}^{n-1}C_{r-1}\right]$$

$$= n\left[\sum_{r=1}^{n} (r-1)\cdot {}^{n-1}C_{r-1} + \sum_{r=1}^{n} {}^{n-1}C_{r-1}\right]$$

$$= n\left[\sum_{r=1}^{n} (r-1)\frac{(n-1)}{(r-1)}\cdot {}^{n-2}C_{r-2} + 2^{n-1}\right]$$

$$= n\left[(n-1)\sum_{r=1}^{n} {}^{n-2}C_{r-2} + 2^{n-1}\right]$$

$$= n\left[(n-1)\cdot 2^{n-2} + 2^{n-1}\right] = n(n+1)2^{n-2}$$

For $n = 1, a = 2, b = 1, c = 1$ and for $n = 2, a = 4, b = 4, c = 6$ $\left.\right\}$ $a = b + c$, $a + b = c + 2$

(B) For $\lambda = 1,\ a = \sum_{r=0}^{n} (-1)^r \cdot C_r = 0$

For $\lambda = r$,

$$b = \sum_{r=0}^{n} (-1)^r \cdot r \cdot C_r = \sum_{r=0}^{n} (-1)^r \cdot r \cdot \frac{n}{r}\, {}^{n-1}C_{r-1}$$

$$= n\sum_{r=1}^{n} (-1)^r \cdot {}^{n-1}C_{r-1} = n(1-1)^{n-1} = 0$$

and for $\lambda = r^2, c = \sum_{r=0}^{n} (-1)^r \cdot r^2 \cdot C_r$

$$= \sum_{r=0}^{n} (-1)^r \cdot r^2 \cdot \frac{n}{r}\cdot {}^{n-1}C_{r-1}$$

$$= n\sum_{r=0}^{n} (-1)^r \cdot r \cdot {}^{n-1}C_{r-1}$$

$$= n\sum_{r=0}^{n} (-1)^r \{(r-1)+1\}\, {}^{n-1}C_{r-1}$$

$$= n\sum_{r=0}^{n} (-1)^r (r-1)\, {}^{n-1}C_{r-1} + n\sum_{r=0}^{n} (-1)^r \cdot {}^{n-1}C_{r-1}$$

$$= 0 + 0 = 0$$

$\therefore a = b = c = 0 \Rightarrow a = b + c$

$\Rightarrow a^3 + b^3 + c^3 = 3abc \Rightarrow a + c = 4b$

(C) For $\lambda = 1,\ a = \sum_{r=0}^{n} \frac{C_r}{(r+1)} = \frac{1}{(n+1)}\sum_{r=0}^{n}\left(\frac{n+1}{r+1}\right)\cdot {}^{n}C_r$

$$= \frac{1}{(n+1)}\sum_{r=0}^{n} {}^{n+1}C_{r+1} = \frac{1}{n+1}(2^{n+1}-1)$$

$$= \frac{2^{n+1}-1}{n+1}$$

For $\lambda = r, b = \sum_{r=0}^{n} \frac{r\cdot C_r}{(r+1)} = \sum_{r=0}^{n}\left(1 - \frac{1}{r+1}\right)C_r$

$$= 2^n - \left(\frac{2^{n+1}-1}{n+1}\right) = \frac{(n-1)2^n + 1}{n+1}$$

For $\lambda = r^2, c = \sum_{r=0}^{n} \frac{r^2\cdot C_r}{(r+1)} = \sum_{r=0}^{n}\left((r-1) + \frac{1}{r+1}\right)C_r$

$$= \sum_{r=0}^{n} r\cdot C_r - \sum_{r=0}^{n} C_r + \sum_{r=0}^{n}\frac{C_r}{r+1}$$

$$= n\cdot 2^{n-1} - 2^n + \frac{2^{n+1}-1}{n+1}$$

$$= \frac{(n^2 - n + 2)2^{n-1} - 1}{(n+1)}$$

For $n = 1, a = \frac{3}{2}, b = \frac{1}{2}, c = \frac{1}{2}$ and for $n = 2, a = \frac{7}{3}, b = \frac{5}{3}, c = \frac{7}{3}$; $\left.\right\}$ $a + c = 4b$, $b^{c-a} + (c-a)^b = 1$

JEE Type Solved Examples : Statement I and II Type Questions

- **Directions** Example numbers 26 and 27 are Assertion-Reason type examples. Each of these examples contains two statements:

Statement-1 (Assertion) and **Statement-2** (Reason)

Each of these examples also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

- **Ex. 26 Statement-1** *$(7^9 + 9^7)$ is divisible by 16*

Statement-2 *$(x^y + y^x)$ is divisible by $(x + y)$, $\forall\ x, y$.*

Sol. (c) $7^9 + 9^7 = (8-1)^9 + (8+1)^7$

$= (8^9 - {}^9C_1\cdot 8^8 + {}^9C_2\cdot 8^7 - {}^9C_3\cdot 8^6 + \ldots + {}^9C_8\cdot 8 - 1)$

$+ (8^7 + {}^7C_1\cdot 8^6 + {}^7C_2\cdot 8^5 + \ldots + {}^7C_6\cdot 8 + 1)$

$= 8^9 - 9\cdot 8^8 + 8^7\cdot({}^9C_2 + 1) + 8^6(-{}^9C_3 + 7)$

$+ 8^5({}^9C_4 + {}^7C_2) + \ldots + 8({}^9C_8 + {}^7C_6)$

$= 64\lambda$ [λ is an integer]

$\therefore 7^9 + 9^7$ is divisible by 16.

$\therefore$ Statement-1 is true. Statement-2 is false.

- **Ex. 27. Statement-1** *Number of distinct terms in the sum of expansion $(1+ax)^{10} + (1-ax)^{10}$ is 22.*

Statement-2 *Number of terms in the expansion of $(1+x)^n$ is $n+1, \forall\ n \in N$.*

Sol. (d) $\because$ $(1+ax)^{10} + (1-ax)^{10} = 2\{1 + {}^{10}C_2(ax)^2$

$+ {}^{10}C_4(ax)^4 + {}^{10}C_6(ax)^6 + {}^{10}C_8(ax)^8 + {}^{10}C_{10}(ax)^{10}\}$

$\therefore$ Number of distinct terms = 6

$\Rightarrow$ Statement-1 is false but Statement-2 is obviously true.

99. Prove that, if p is a prime number greater than 2, the difference $[(2+\sqrt{5})^p] - 2^{p+1}$ is divisible by p, where [.] denotes greatest integer.

100. If ((x)) represents the least integer greater than x, prove that $((\{(\sqrt{3}+1)^{2n}\}))$, $n \in N$ is divisible by 2^{n+1}.

101. Solve the equation
$$^{11}C_1 x^{10} - {}^{11}C_3 x^8 + {}^{11}C_5 x^6 - {}^{11}C_7 x^4 + {}^{11}C_9 x^2 - {}^{11}C_{11} = 0.$$

102. If $g(x) = \sum_{r=0}^{200} \alpha_r \cdot x^r$ and $f(x) = \sum_{r=10}^{200} \beta_r x^4, \beta_r = 1$ for $r \geq 100$ and $g(x) = f(1+x)$, show that the greatest coefficient in the expansion of $(1+x)^{201}$ is α_{100}.

103. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, find the value of
$$\sum_{0 \leq i < j \leq n}\sum (i+j)(C_i + C_j + C_i C_j).$$

104. Evaluate $\sum_{0 \leq i \neq j \leq 10}\sum {}^{21}C_i \cdot {}^{21}C_j$.

105. Find the coefficients of x^4 in the expansions of
(i) $(1 + x + x^2 + x^3)^{11}$.
(ii) $(2 - x + 3x^2)^6$.

106. Prove the identity
$$\frac{1}{^{2n+1}C_r} + \frac{1}{^{2n+1}C_{r+1}} = \frac{2n+2}{2n+1} \cdot \frac{1}{^{2n}C_r},$$
use it to prove $\sum_{r=1}^{r=2n-1} \frac{(-1)^{r-1} r}{^{2n}C_r} = \frac{n}{n+1}$.

107. Let $a_0, a_1, a_2, \ldots$ are the coefficients in the expansion of $(1+x+x^2)^n$ arranged order of x. Find the value of $a_r - {}^nC_1 a_{r-1} + {}^nC_r a_{r-2} - \ldots + (-1)^r\, {}^nC_r a_0$, where r is not divisible by 3.

108. If for z as real or complex.
$(1+z^2+z^4)^8 = C_0 + C_1 z^2 + C_2 z^4 + \ldots + C_{16} z^{32}$, prove that
(i) $C_0 - C_1 + C_2 - C_3 + \ldots + C_{16} = 1$
(ii) $C_0 + C_3 + C_6 + C_9 + C_{12} + C_{15} + (C_2 + C_5 + C_8 + C_{11} + C_{14})\omega + (C_1 + C_4 + C_7 + C_{10} + C_{13} + C_{16})\omega^2 = 0$,
where ω is a cube root of unity.

109. Let $f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_{2n} x^{2n}$ and $g(x) = b_0 + b_1 x + b_2 x^2 + \ldots + b_{n-1} x^{n-1} + x^n + x^{n+1} + \ldots + x^{2n}$.
If $f(x) = g(x+1)$, find a_n in terms of n.

110. If $a_0, a_1, a_2, \ldots$ are the coefficients in the expansion of $(1+x+x^2)^n$ in ascending powers of x, prove that
(i) $a_0 a_1 - a_1 a_2 + a_2 a_3 - \ldots = 0$
(ii) $a_0 a_2 - a_1 a_3 + a_2 a_4 - \ldots + a_{2n-2} a_{2n} = a_{n+1}$
(iii) if $E_1 = a_0 + a_3 + a_6 + \ldots; E_2 = a_1 + a_4 + a_7 + \ldots$ and $E_3 = a_2 + a_5 + a_8 + \ldots$, then $E_1 = E_2 = E_3 = 3^{n-1}$

111. Prove that $(n-1)^2 C_1 + (n-3)^2 C_3 + (n-5)^2 C_5 + \ldots = n(n+1)2^{n-3}$, where C_r stands for ^nC_r.

112. Show that $\frac{C_0}{1} - \frac{C_1}{4} + \frac{C_2}{7} - \ldots + (-1)^n \frac{C_n}{3n+1} = \frac{3^n \cdot n!}{1 \cdot 4 \cdot 7 \ldots (3n+1)}$, where C_r stands for ^nC_r.

Binomial Theorem Exercise 8 :
Questions Asked in Previous 13 Year's Exams

- This section contains questions asked in **IIT-JEE, AIEEE, JEE Main & JEE Advanced** from year **2005** to year **2017**.

113. The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} + \ldots + \binom{30}{20}\binom{30}{30}$ is **[IIT JEE 2005, 3M]**
(a) $^{60}C_{20}$ (b) $^{30}C_{10}$ (c) $^{60}C_{30}$ (d) $^{40}C_{30}$

114. If the coefficients of pth, $(p+1)$th and $(p+2)$th terms in expansion of $(1+x)^n$ are in AP, then **[AIEEE 2005, 3M]**
(a) $n^2 - 2np + 4p^2 = 0$
(b) $n^2 - n(4p+1) + 4p^2 - 2 = 0$
(c) $n^2 - n(4p+1) + 4p^2 = 0$
(d) None of the above

115. If the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$, then ab is equal to **[AIEEE 2005, 3M]**
(a) 1 (b) 1 / 2
(c) 2 (d) 3

N

116. For natural numbers m and n, if $(1-y)^m (1+y)^n = 1 + a_1 y + a_2 y^2 + \dots$ and $a_1 = a_2 = 10$, then (m, n) is **[AIEEE 2006, 3M]**

(a) (20, 45) (b) (35, 20)
(c) (45, 35) (d) (35, 45)

117. In the binomial expansion of $(a-b)^n$, $n \ge 5$, the sum of 5th and 6th terms is zero, $\frac{a}{b}$ equals **[AIEEE 2007, 3M]**

(a) $\frac{5}{n-4}$ (b) $\frac{6}{n-5}$
(c) $\frac{n-5}{6}$ (d) $\frac{n-4}{5}$

118. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots + {}^{20}C_{10}$ is **[AIEEE 2007, 3M]**

(a) $-{}^{20}C_{10}$ (b) $\frac{1}{2}\,{}^{20}C_{10}$
(c) 0 (d) ${}^{20}C_{10}$

119. **Statement-1** $\sum_{r=0}^{n} (r+1)\,{}^nC_r = (n+2)\cdot 2^{n-1}$

Statement-2 $\sum_{r=0}^{n} (r+1)\,{}^nC_r\, x^r = (1+x)^n + nx(1+x)^{n-1}$. **[AIEEE 2007]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

120. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9, is **[AIEEE 2009, 4M]**

(a) 8 (b) 0 (c) 2 (d) 7

121. For $r = 0, 1, 2, \dots, 10$, let A_r, B_r and C_r denote respectively, the coefficients of x^r in the expansion of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$, $\sum_{r=1}^{10} A_r (B_{10} B_r - C_{10} A_r)$ is equal to **[IIT-JEE 2010, 5M]**

(a) $B_{10} - C_{10}$ (b) $A_{10}(B_{10} - C_{10}A_{10})$
(c) 0 (d) $C_{10} - B_{10}$

122. Let $S_1 = \sum_{j=1}^{10} j(j-1)\cdot {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\cdot {}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2 \cdot {}^{10}C_j$ **[IIT-JEE 2010]**

Statement-1 $S_3 = 55 \times 2^9$

Statement-2 $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

123. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$, is **[AIEEE 2011, 4M]**

(a) -132 (b) -144
(c) 132 (d) 144

124. If n is a positive integer, then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is **[AIEEE 2012, 4M]**

(a) an odd positive integer
(b) an even positive integer
(c) a rational number other than positive integer
(d) an irrational number

125. The term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$ is **[JEE Main, 2013, 4M]**

(a) 120 (b) 210
(c) 310 (d) 4

126. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, the value of n is **[JEE Advanced 2013M]**

127. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to **[JEE Main 2014, 3M]**

(a) $\left(14, \frac{272}{3}\right)$ (b) $\left(16, \frac{272}{3}\right)$
(c) $\left(14, \frac{251}{3}\right)$ (d) $\left(16, \frac{251}{3}\right)$

128. Coefficient of x^{11} in the expansion of $(1+x^2)^4 (1+x^3)^7 (1+x^4)^{12}$ is **[JEE Advanced 2014, 3M]**

(a) 1051 (b) 1106 (c) 1113 (d) 1120

129. The sum of coefficients of integral powers of x in the binomial expansion of $(1 - 2\sqrt{x})^{50}$, is **[JEE Main 2015, 4M]**

(a) $\frac{1}{2}(2^{50}+1)$ (b) $\frac{1}{2}(3^{50}+1)$
(c) $\frac{1}{2}(3^{50})$ (d) $\frac{1}{2}(3^{50}-1)$

130. The coefficients of x^9 in the expansion of $(1+x)(1+x^2)(1+x^3)\dots(1+x^{100})$ is **[JEE Advanced 2015, 4M]**

131. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \ne 0$ is 28, then the sum of the coefficients of all the terms in this expansion, is **[JEE Main 2016, 4M]**

(a) 243 (b) 729 (c) 64 (d) 2187

must

132. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2+(1+x)^3+......+(1+x)^{49}+(1+mx)^{50}$ is $(3n+1)^{51}C_3$ for some positive integer n. Then the value of n is **[JEE Advanced 2016, 3M]**

133. The value of $({}^{21}C_1 - {}^{10}C_1)+({}^{21}C_2 - {}^{10}C_2)+({}^{21}C_3 - {}^{10}C_3)+({}^{21}C_4 - {}^{10}C_4)+......+({}^{21}C_{10} - {}^{10}C_{10})$ is **[JEE Advanced 2017, 4M]**

(a) $2^{20}-2^{10}$ (b) $2^{21}-2^{11}$
(c) $2^{21}-2^{10}$ (d) $2^{20}-2^{9}$

Answers

Exercise for Session 1

1. (c) 2. (a) 3. (c) 4. (c) 5. (b) 6. (b)
7. (c) 8. (d)

Exercise for Session 2

1. (b) 2. (c) 3. (d) 4. (b) 5. (c) 6. (c)
7. (c) 8. (b) 9. (a) 10. (d)

Exercise for Session 3

1. (a) 2. (c) 3. (d) 4. (b) 5. (c) 6. (c)
7. (a) 8. (c) 9. (a) 10. (c)

Exercise for Session 4

1. (c) 2. (b) 3. (c) 4. (a) 5. (a) 6. (c)
7. (b) 8. (c) 9. (b) 10. (b) 11. (a) 12. (d)
13. (a) 14. (b)

Chapter Exercises

1. (d) 2. (d) 3. (a) 4. (b) 5. (d) 6. (b)
7. (b) 8. (c) 9. (a) 10. (c) 11. (b) 12. (a)
13. (b) 14. (c) 15. (a) 16. (b) 17. (a) 18. (a)
19. (c) 20. (d) 21. (d) 22. (d) 23. (b) 24. (d)
25. (b) 26. (b) 27. (b) 28. (a) 29. (d) 30. (a)
31. (c,d) 32. (a,b) 33. (a,d) 34. (a,d) 35. (c,d) 36. (b,c)
37. (b,c) 38. (a,d) 39. (b,c) 40. (a,b,d) 41. (c,d) 42. (a,d)
43. (a,b,c,d) 44. (a,c) 45. (a,d)
46. (b) 47. (d) 48. (b) 49. (c) 50. (b) 51. (c)
52. (c) 53. (d) 54. (d) 55. (b) 56. (a) 57. (b)
58. (b) 59. (b) 60. (d) 61. (c) 62. (a) 63. (d)
64. (b) 65. (c) 66. (d)
67. (0) 68. (3) 69. (3) 70. (8) 71. (6) 72. (4)
73. (0) 74. (3) 75. (4) 76. (9)
77. (A) $\rightarrow$ (q, r); (B) $\rightarrow$ (p, q, t); (C) $\rightarrow$ (s)
78. (A) $\rightarrow$ (r, s, t); (B) $\rightarrow$ (s, t); (C) $\rightarrow$ (p, q, r, s, t)
79. (A) $\rightarrow$ (p, q, r, s); (B) $\rightarrow$ (p, q, r, s, t); (C) $\rightarrow$ (p, q, r, s, t)
80. (A) $\rightarrow$ (q, s); (B) $\rightarrow$ (p, q, r, s); (C) $\rightarrow$ (q, s); (D) $\rightarrow$ (r, s)
81. (A) $\rightarrow$ (p, r); (B) $\rightarrow$ (q); (C) $\rightarrow$ (s); (D) $\rightarrow$ (p, r)
82. (d) 83. (c) 84. (b) 85. (b) 86. (d) 87. (c)
88. (a)
89. $x = 10$ or $10^{-5/2}$ 90. 1 91. 210 94. 9 95. 4,2
97. 10 98. ${}^nC_r(3^{n-r}-2^{n-r})$ 101. $x=\cot\left(\frac{r\pi}{11}\right)$, $r=\pm 1, \pm 2, \ldots, \pm 5$
103. $n^2 \cdot 2^n + n\left\{2^{2n-1}-\frac{2n!}{2(n!)^2}\right\}$ 104. $\frac{1}{2}\left[2^{40}-\frac{42!}{2(21!)^2}\right]$
105. (i) 990 (ii) 3660 107. 0 109. ${}^{2n+1}C_{n+1}$
113. (b) 114. (b) 115. (a) 116. (d) 117. (d) 118. (b)
119. (a) 120. (c) 121. (d) 122. (b) 123. (b) 124. (d)
125. (b) 126. (6) 127. (b) 128. (c) 129. (b) 130. (8)
131. (b) 132. (5) 133. (a)