Algorithms, Spring 2012-13, Homework 5
due Wednesday, 17 April 2013, 10:00

Problem 1

Given is a convex polygon with n vertices $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ (the vertices are listed in a clockwise order). A triangulation of a convex polygon is a set of $n-3$ non-intersecting edges, where each edge connects two non-consecutive vertices (the overall picture consists of $n-2$ triangles that together form the original polygon). We will define the *length* of a triangulation as the sum of the lengths of these $n-3$ edges. Give an $O(n^3)$ algorithm that finds the minimum possible length of a triangulation of the given polygon.

Hint: Use a 2D dynamic programming array. Do not forget to specify all three parts of the heart of the solution.

Problem 2

Given is an undirected graph $G = (V, E)$. Give an $O(n + m)$ algorithm that determines whether G is cyclic. If yes, within the same running time the algorithm should find a sequence of vertices that forms a cycle.

Problem 3

Given are n courses and for each course given are its prerequisites. Let P_i be the set of prerequisite courses for the i-th course and let $m = |P_1| + |P_2| + \ldots + |P_n|$. Give an $O(n+m)$ algorithm that finds the size of the longest prerequisite chain, i.e., the longest sequence of courses for which for every element in the sequence the previous element is its prerequisite. You may assume that the data is consistent, i.e., there are no "prerequisite loops."