Algorithms, Spring 2012-13, Homework 7
due Wednesday 1 May 2013, 10:00

For Problems 1 and 2 include the pseudocode of your algorithm and a short verbal description. Briefly argue your algorithm's correctness and explain its running time.

Problem 1

Let $G = (V, E, w)$ be a positively weighted directed graph. Give an $O(n^3)$ or an $O(nm \log n)$ algorithm that finds the minimum distance from u to v for every pair of vertices $u, v \in V$. Moreover, output the number of distinct paths from u to v of length equal to the shortest distance.

Problem 2

Let $G = (V, E, w)$ be a weighted directed graph and s be its vertex. Give an $O(mn)$ algorithm that finds all vertices u for which no path from s to u encounters (i.e., shares a vertex with) a negative weight cycle.

Problem 3

The Edmonds-Karp algorithm refines the idea of Ford and Fulkerson in the following way: in every iteration, the algorithm chooses the augmenting path that uses the smallest number of edges (if there are more such paths, it chooses one arbitrarily). Find a graph for which in some iteration the Edmonds-Karp algorithm has to choose a path that uses a backward edge. Run the algorithm on your graph – more precisely, for every iteration draw the residual graph and show the augmenting path taken by the algorithm as well as the flow after adding the augmenting path.

Do not make your graph overly complicated – if you do, you might have a very unhappy housemate :)