Let the length $AB = L_1$
Let the length $AC = L_2$
p.d across $AB = E_1$
p.d across $AD = E_2$
The ratio of p.d across AB to p.d across AB is

$$\frac{E_1}{E_2}$$

$$\text{p.d across AD} = \frac{L_2}{L_1} \times \text{p.d across AB.}$$

$$\therefore E_2 \times L_1 = L_2 \times E_1$$

$$\text{hence, } \frac{E_2}{E_1} = \frac{L_2}{L_1}$$

Worked example 8.6

The balance length of a potentiometer wire for a cell of e.m.f 1.62V is 90cm. If the cell is replaced by another one of e.m.f 1.08V, calculate its new balance.

Solution

$$\frac{E_1}{L_1} = \frac{E_2}{L_2}$$

$$\frac{1.62}{90} = \frac{1.08}{L_2}$$

$$L_2 = \frac{90 \times 1.08}{1.62} = 60\text{cm}$$

Advantages of potentiometer over voltmeter

(i) It does not draw current from the circuit at a balance point in error due to internal resistance.
(ii) It has no zero scale error.
(iii) It gives an accurate reading for p.d than the voltmeter.

8.10 Resistance in Series

Resistances are in series when two or more resistances are connected end to end, so that the same current flows through all of them, though they have different voltages as shown in Figure 8.6 below.

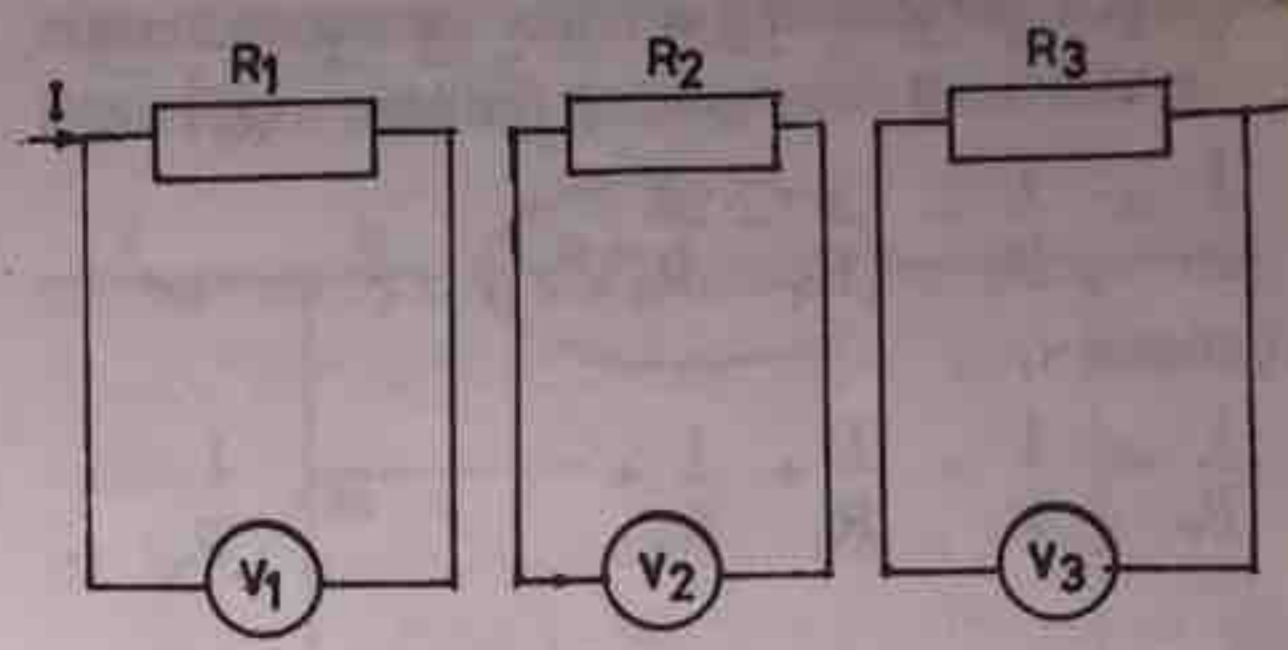

Fig. 8.6 Resistance in series

Using $V = IR$, $V = V_1 + V_2 + V_3$
$\therefore IR = IR_1 + IR_2 + IR_3$
divide throughout by I

$$R = R_1 + R_2 + R_3$$

In general,

$$R_c = R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots + R_n$$

NB: Total ≡ combined, effective, resultant, net, etc.

8.11 Resistance in Parallel

Resistors are said to be in parallel when two or more resistances or conductors are connected to common terminals so that the potential difference across conductor is the same but different current flow through them.

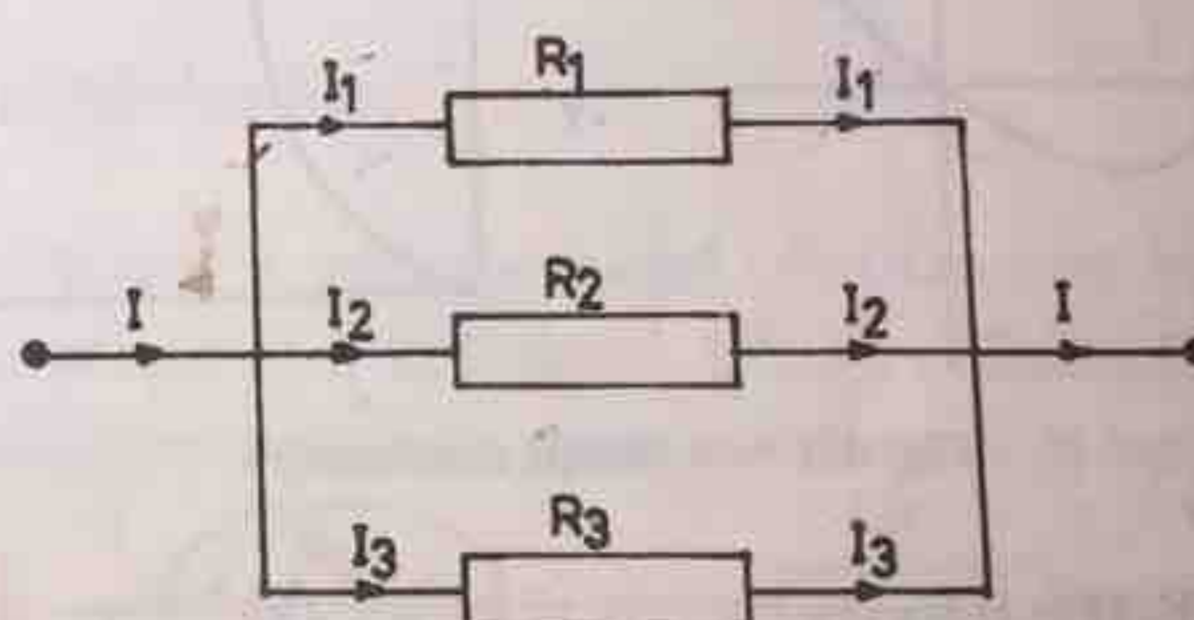

Fig. 8.7 Resistance in parallel

Applying Ohm's law

$$V = IR \Rightarrow I = \frac{V}{R}$$

$$I = I_1 + I_2 + I_3$$

$$\therefore \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

multiply throughout by $\frac{1}{V}$

$$\frac{V}{R} \times \frac{1}{V} = \frac{V}{R_1} \times \frac{1}{V} + \frac{V}{R_2} \times \frac{1}{V} + \frac{V}{R_3} \times \frac{1}{V}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\cdots\cdots \frac{1}{R_n}$$

8.12 Ohm's Law

Ohm's law states that the electric current in a given metallic conductor is directly proportional to the potential difference applied between its ends, provided that the temperature of the conductor and other physical factors such as length and cross-sectional area remain constant.

Conductors that obey Ohm's law are called *Ohmic conductors,* e.g. metals and alloys. Conductors which do not obey Ohm's law are called non-ohmic conductors such as metal rectifiers, divide value, voltage-dependent resistors, vacuum triode, crystal gold, thermostat, etc.

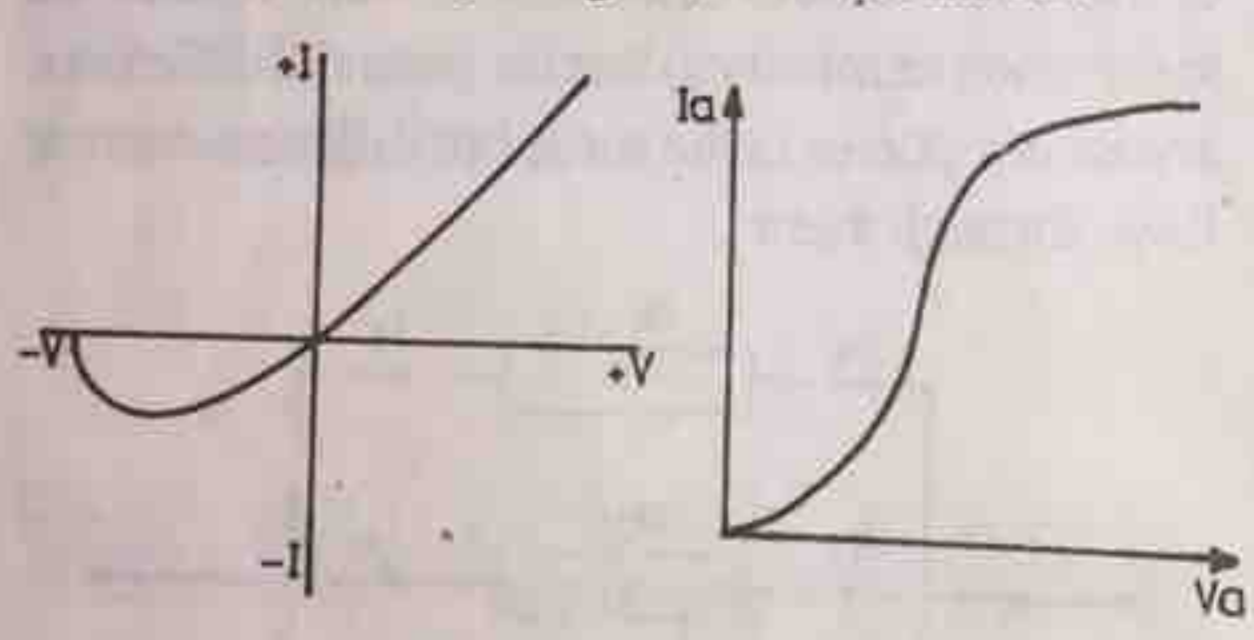

Fig. 8.8 Non-ohmic conductors

8.13 Resistivity

Resistivity is defined as the resistance per unit length per unit cross-sectional area of the material concerned. It is denoted by P and measured in Ωm- SI unit.

The resistance of a conductor is directly proportional to the length of the conductor and inversely proportional to the cross-sectional area of the conductor.

Mathematically,

$R \; \alpha \; L$ .... (i) (direct variation)

$R \; \alpha \; \frac{1}{A}$ ....(ii) (inverse variation)

$R \; \alpha \; \frac{L}{A}$ (joint variation)

$$R = \frac{KL}{A}$$

K = constant = ρ (resistivity)

$$R = \frac{\rho L}{A}$$

If $A = \pi r^2 = \frac{\pi d^2}{4}$

Since $2r = d$

$$R = \frac{\rho L}{\pi r^2}$$

Similarly,

$$R = \frac{\rho L}{\pi d^2/_4} = \frac{4\rho L}{\pi d^2}$$

Where $\pi = \frac{22}{7} = 3.142$

N.B: (i) $1 \times 10^4 cm^2 = 1m^2$

$1 \times 10^6 mm^2 = 1m^2$

(ii) Resistivity is the degree of hindrance at which the flow of current through a conductor is determined.

Electrical conductivity: Electrical conductivity is defined as the reciprocal of resistivity. It is denoted by K and measured in $\Omega^{-1}m^{-1}$ or $(\Omega m)^{-1}$

Thus,

$$K = \frac{1}{\rho} = \frac{1}{RA/_L} = \frac{L}{RA}$$

Worked example 8.7

An electric heating element which is to dissipate 440w on a 220v mains is to be made from a wire of cross-sectional area $0.5 \times 10^{-7} m^2$ and length 5m. Determine the resistivity of the wire.

Solution

$P = 440W$, $A = 0.5 \times 10^{7} m^2$, $L = 5m$, $V = 220V$

$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$$

$$\therefore R = \frac{(220)^2}{440} = 110\Omega$$

$$R = \frac{pL}{A} \Rightarrow P = \frac{RA}{L}$$

$$= \frac{110 \times 0.5 \times 10^{7}}{5}$$

$$\rho = 1.1 \times 10^{-6}\Omega m$$

Worked example 8.8

If a wire has a resistance of 1.32Ω, a length of 110cm and an area of cross-sectional of 0.00415cm², find the resistivity of the material of which it is made.

Solution

$$L = 110cm = 1.1m, \quad R = 1.32\Omega$$

$$A = 4.15 \times 10^{-3}cm^2 = 4.15 \times 10^{-7}m^2, \quad \rho = ?$$

$$R = \frac{\rho L}{A} \Rightarrow \rho = \frac{RA}{L}$$

$$\therefore \rho = \frac{1.32 \times 4.15 \times 10^{-7}}{1.1}$$

$$\rho = 4.98 \times 10^{-7}\Omega m$$

Worked example 8.9

A wire 40cm long and of diameter 0.60mm has a resistance of 1.5Ω. What is the resistivity of the material of which it is made. [$\pi = 3.142$]

Solution

$$L = 40cm = 0.4m, \quad R = 1.5\Omega$$

$$d = 0.6mm = 6 \times 10^{-4}m$$

$$R = \frac{4\rho L}{4d^2} \Rightarrow P = \frac{R\pi d^2}{4L}$$

$$\rho = \frac{1.5 \times 1.42 \times (6 \times 10^{-4})^4}{4 \times 0.4}$$

$$\rho = \frac{1.5 \times 3.142 \times 36 \times 10^{-8}}{1.6} = 1.06 \times 10^{-6}\Omega m$$

8.14 Shunt

When a galvanometer (milliammeter) is converted to an ammeter, a low shunt resistance is placed in parallel with the cell. The current is reduced through the galvanometer connecting a low resistance parallel to the galvanometer, so that current passes through the galvanometer, and from there to the parallel resistor called the *shunt*.

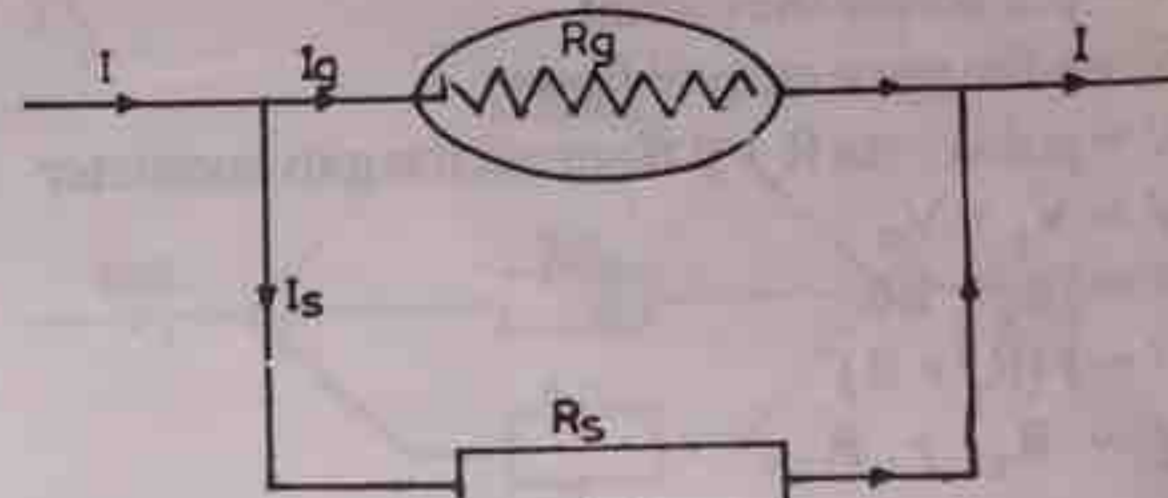

Fig. 8.9 The shunt

R_s = shunt resistance

R_g = galvanometer resistance

I_g = current flowing through the galvanometer

I_s = shunt current

I = measured current flowing in the circuit.

R_g and R_s are in parallel ⇒ same p.d across them.

i.e, $V_g = V_s$

$$I_gR_g = I_sR_s$$

but $I = I_g + I_s$

hence, $I_s = I - I_g$

$$\therefore I_gR_g = (I - I_g)R_s$$

$$R_s = \frac{I_gR_s}{I - I_g}$$

8.15 Multiplier

When a galvanometer (milliammeter) is converted to a voltmeter, a high resistance or multiplier is placed n series with the cell or battery.

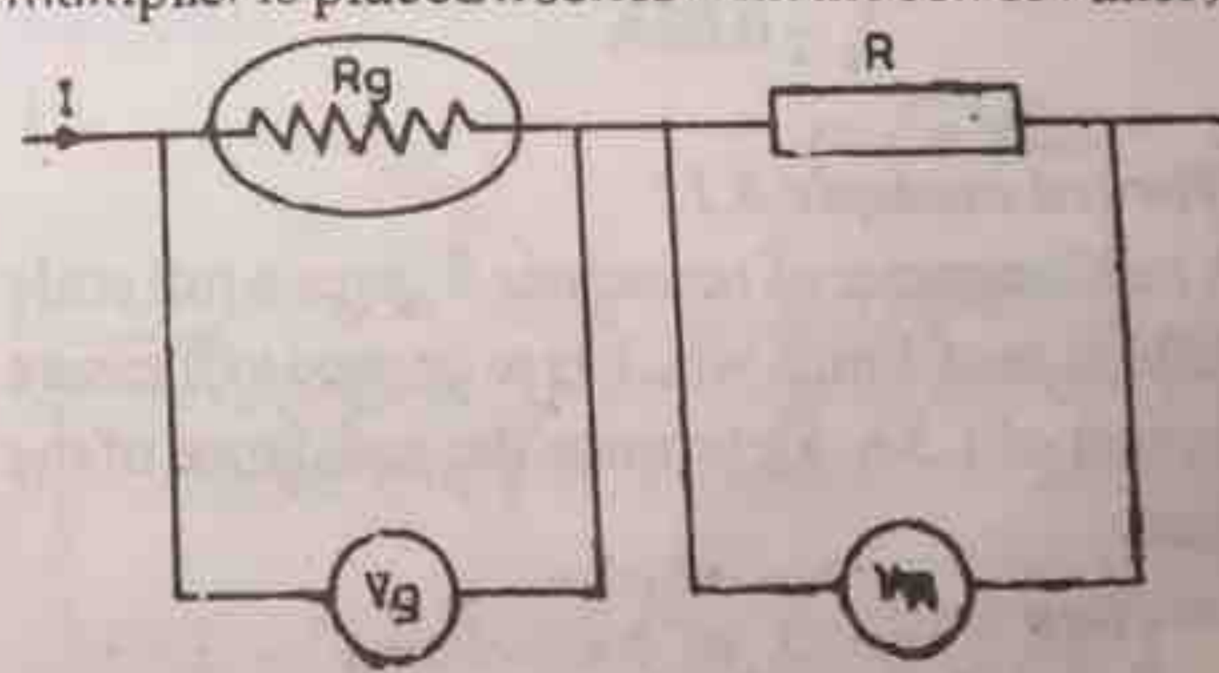

Fig. 8.10 Multiplier

R = multiplier (resistor)

R_g = resistance of the galvanometer

V_R = p.d across multiplier (resistor)
V_g = p.d across the galvanometer
V = p.d across the circuit
Since the same current flows,
V = p.d across R + p.d across the galvanometer

$$V = V_g + V_R$$
$$V = IR_g + IR$$
$$V = I(R_g + R)$$
$$\frac{V}{I} = R_g + R$$
$$\therefore R = \frac{V}{I} - R_g$$

Worked example 8.10

A galvanometer has a resistance of 5Ω. By using a shunt wire of resistance 0.05Ω, the galvanometer could be converted to an ammeter, capable of reading 2A. What is the current through the galvanometer?

Solution

$$I = 2A,\ R_g = 5\Omega,\ R_s = 0.05\Omega,\ I_g = ?$$
$$R_s = \frac{R_g I_g}{I - I_g}$$
$$0.05 = \frac{5I_g}{(2 - I_g)}$$
$$0.05(2 - I_g) = 5I_g$$
$$0.1 - 0.05I_g = 5I_g$$
$$0.1 = 0.05I_g + 5I_g$$
$$I_g = \frac{0.1}{5.05} = 0.0198A$$
$$I_g \simeq 0.02A$$

Worked example 8.11

A milliammeter of resistance 5 gives a full scale deflection of 15mA which is to be used to measure current of 1.5A. Determine the resistance of the shunt.

Solution

$$R_g = 5\Omega,\ I_g = 15mA = 15 \times 10^{-3}A = 0.015A$$
$$I = 1.5A$$
$$R_2 = \frac{I_g R_g}{I - I_g}$$
$$= \frac{0.015 \times 5}{1.5 - 0.015}$$
$$= \frac{0.075}{1.485}$$
$$R_s = 0.0505\Omega$$

Worked example 8.12

A galvanometer of internal resistance 50Ω has full scale deflection for a current of 5mA. What is the resistance required to convert it to voltmeter with full scale deflection of 10V?

Solution

$$R = ?,\ V = 10V,\ I_g = 5mA = 5 \times 10^{-3}A$$
$$R_g = 50\Omega.$$
$$R = \frac{V - IR_g}{I}$$
$$= \frac{10 - 0.005 \times 50}{0.005}$$
$$= \frac{10 - 0.25}{0.005}$$
$$= \frac{9.75}{0.005}$$
$$\therefore R = 1950\Omega$$

Worked example 8.13

Find the resistance of a multiplier connected with milliammeter of resistance 10Ω with a deflection of 10mA and the voltage of the circuit is 15V.

Solution

$$R_g = 10\Omega,\ V = 15V,$$
$$I = I_g = 10mA = 0.01A$$
$$R = \frac{V}{I} - R_g$$
$$= \frac{15}{0.01} - 10$$
$$= 1500 - 10$$
$$= 1490\Omega$$

8.16 Measuring Instruments

(i) Galvanometer: This is an instrument used in detecting and indicating current in an electrical circuit. It is sensitive and accurate. The detection is directly proportional to the current measured. It

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9. ELECTRICAL ENERGY AND POWER

9.1 Electrical Energy

Electrical energy is the work done when a quantity of charge moves between two points of potential differences measured in joules.
Work done = Quantity of charge x p.d

$$V = \frac{W}{Q}$$

$\therefore$ W = QV ....(i)

W = Work done
Q = Quantity of charge
V = p.d. across the terminals
and, we already know that Q = It

$\therefore$ W = (It)v
W = IVt ....(ii)

Where I = Current
V = voltage
T = time

W = IVt
Since V = IR
W = I (IR)t
= I^2Rt ....(iii)

Since I = V/R

$$W = \frac{V^2}{R^2} \times Rt$$

$$W = \frac{V^2t}{R} \quad \text{..............................(iv)}$$

Worked example 9.1
Calculate the electrical energy produced by a heater with a voltage supply of 220v, when a current of 10 amps passed through it for 5 minutes.

Solution
I = 10amps
V = 220V
t = 5 x 60 = 300
$\therefore$ W = Ivt = 10 x 220 x 300
= 660000 joules (J)
= 660KJ

Worked example 9.2
Calculate the energy in a lamp with a resistance of 4 ohms when a current of 5 amps passes through it in 150 seconds.

Solution
W = I^2Rt
I = 5A, R = 4Ω, t = 150 sec
W = $(5)^2$ x 4 x 150
= 25 x 4 x 150
= 15000 Joules = 15KJ

9.2 Heating Effect of Electrical Energy

When current passes through a wire or a conductor, electrical energy is converted entirely into heat energy. Thus, Joule's law of electrical heating states that: The heat developed in a wire is directly proportional to:

(i) Time: for a given resistance and current, i.e, W α t (R, I are constant.

(ii) The square of the current: for a given resistance and time, i.e, W α I^2 (R, t are constant)

(iii) The resistance of the wire: for a given constant current and time, i.e W α R (I and t are constant)

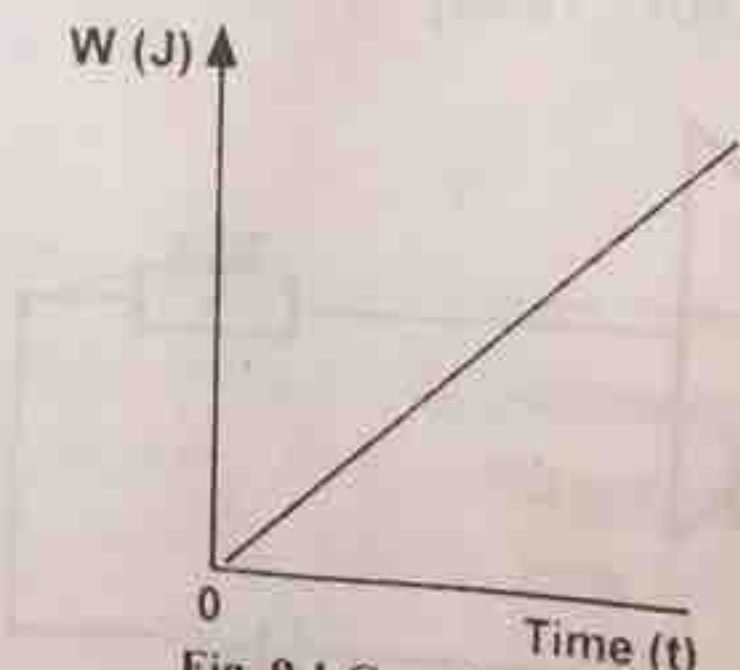

Fig. 9.1 Graph of W against t

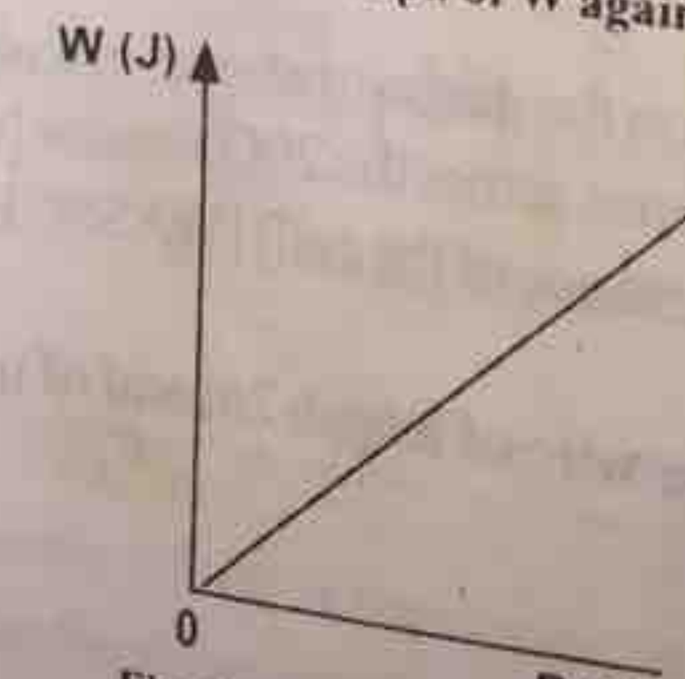

Fig. 9.2 Graph of W against R