POLARISATION

Light is a **transverse electromagnetic wave.**

We have not really discussed the direction of the Electric field other that that it is **perpendicular to the direction of motion.**

If the E field points in a constant direction it is **linearly polarised**. However this direction is not always unchanging.

The Principle of Superposition

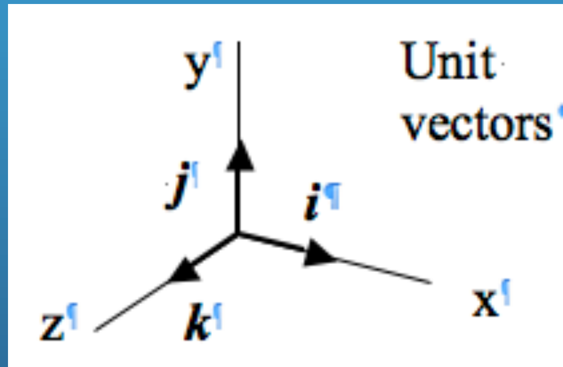
*If 2 or more waves exist at the same point in time or space, the resultant disturbance at any point is **just the vector sum of the individual waves**.*

Thus if 2, in phase, waves moving in the z (or ***k***) direction, with E fields pointing in the same direction (the ***j*** or y direction)

$$\boldsymbol{E_1} = \boldsymbol{j}E_{01}Cos(kz - \omega t)$$
$$\boldsymbol{E_2} = \boldsymbol{j}E_{02}Cos(kz - \omega t)$$

overlap in space, the resultant Electric field, E, is just the sum

$E = E_1 + E_2$ and points in the ***j*** direction



$$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2} = \boldsymbol{j}(E_{01} + E_{02})Cos(kz - \omega t)$$

What if the E fields point in different directions?

Consider 2 waves travelling in the z (***k***) direction but one has an E field pointing in the x (***i***) direction and one with E field pointing in the y (***j***) direction. Then

$$\boldsymbol{E_1} = \boldsymbol{i}E_{0x}Cos(kz - \omega t)$$
$$\boldsymbol{E_2} = \boldsymbol{j}E_{0y}Cos(kz - \omega t)$$

Adding the electric fields gives:

$$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2} = (\boldsymbol{i}E_{0x} + \boldsymbol{j}E_{0y})Cos(kz - \omega t)$$

The resultant E field is a vector pointing in a different direction but with components in the x (***i***) and y (***j***) direction. This wave is still **linearly polarised** but with a different **plane of vibration**

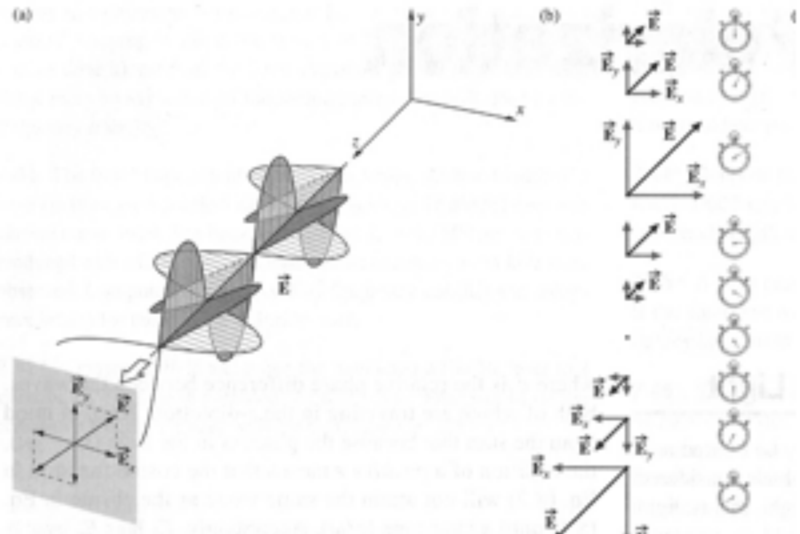


Figure 8.1 Linear light. (a) The E-field linearly polarized in the first and third quadrants. (b) The oscillating field seen head on. (c) Light linearly polarized in the second and fourth quadrants.

Circular Polarisation

The previous waves were in phase. What if there is a **phase difference**? Consider 2 equal amplitude waves polarised in ***i*** and ***j*** directions but with a **relative phase shift of +π/2** radians (90°).

$\mathbf{E_1}=\mathbf{i}E_0Cos(kz-wt)$

$\mathbf{E_2}=\mathbf{j}E_0Cos(kz-wt+\pi/2) = \mathbf{j}E_0Sin(kz-wt)$

The total electric field is then:

$\mathbf{E}=\mathbf{E_1}+\mathbf{E_2}= E_0(\mathbf{i}Cos(kz-wt) + \mathbf{j}Sin(kz-wt))$

Now the magnitude of the components in the ***i*** and ***j*** direction vary in time (and with position, z). What does this mean for the direction of the E field?
We can write it as:

$$E = iE_x + jE_y$$

$$E_x = E_0 Cos(kz - \omega t)$$

$$E_y = E_0 Sin(kz - \omega t)$$

This means that:

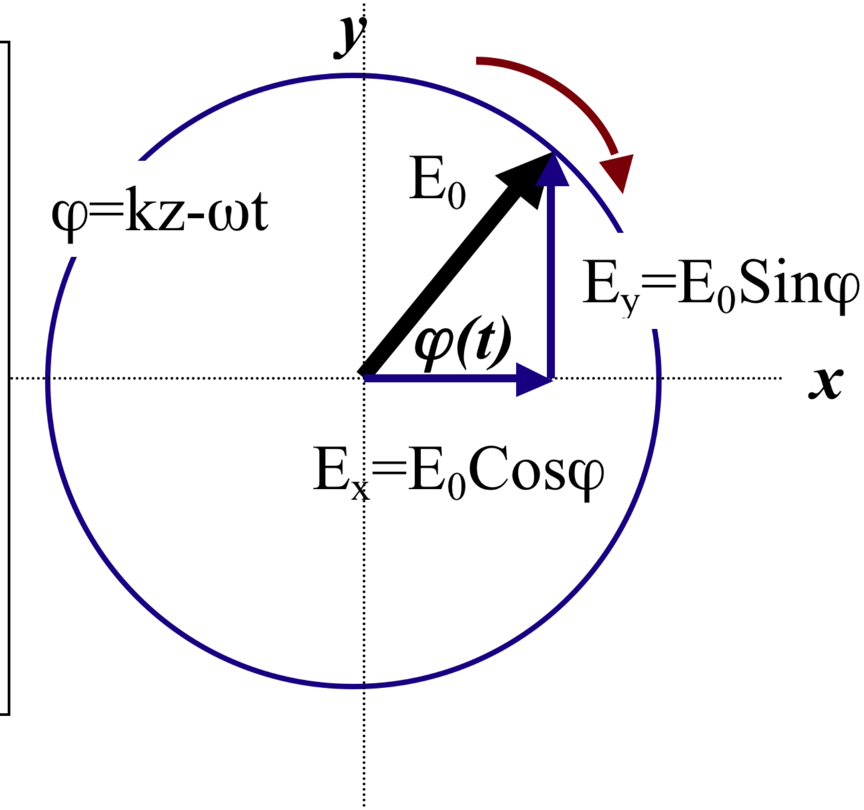
$$E_0^2 = E_x^2 + E_y^2$$

This is a circle!

$$E = iE_1 + jE_2 = E_0\left[iCos(kz-\omega t) + jSin(kz-\omega t)\right]$$

But this equation is just the equation for a vector rotating at angular frequency, ω.

This means the E field of the light rotates!



However the total magnitude $|\mathbf{E}| = \sqrt{(\mathbf{E} \bullet \mathbf{E})} = E_0$ is constant.

Elliptical Polarisation

What if the amplitudes of the individual fields are **not equal** and the waves have a phase shift **other than $\pm\pi/2$**?

This is the most general case.

In this case the electric field will rotate but the tip will trace out an **ellipse** instead of a circle.

Consider two waves:

$$\boldsymbol{E}_x = \boldsymbol{i}E_{0x}Cos(kz - \omega t)$$
$$\boldsymbol{E}_y = \boldsymbol{j}E_{0y}Cos(kz - \omega t + \varepsilon)$$

Let's work out the equation for the curve traced out by the tip of the E field. Using trig identities we can write

$$\frac{E_y}{E_{0y}} = Cos\left(kz - \omega t + \varepsilon\right)$$

Is the same as:

$$\frac{E_y}{E_{0y}} = Cos\left(kz - \omega t\right)Cos\varepsilon - Sin\left(kz - \omega t\right)Sin\varepsilon$$

But

$$\frac{E_x}{E_{0x}} = Cos(kz - \omega t)$$

For any angle $Sin^2\varphi = 1 - Cos^2\varphi$
so

This is the equation of an ellipse that makes an angle α with the E_x axis, where α is given by:

$$Tan2\alpha = \frac{2E_{0x}E_{0y}Cos\varepsilon}{E_{0x}^2 - E_{0y}^2}$$

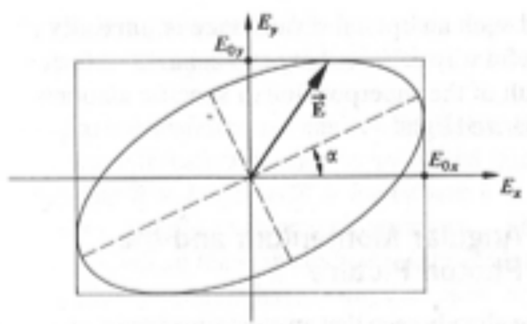


Figure 8.6 Elliptical light. The endpoint of the electric field vector sweeps out an ellipse as it rotates once around.

If $\varepsilon = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$ etc. and $E_{0x} = E_{0x} = E_0$

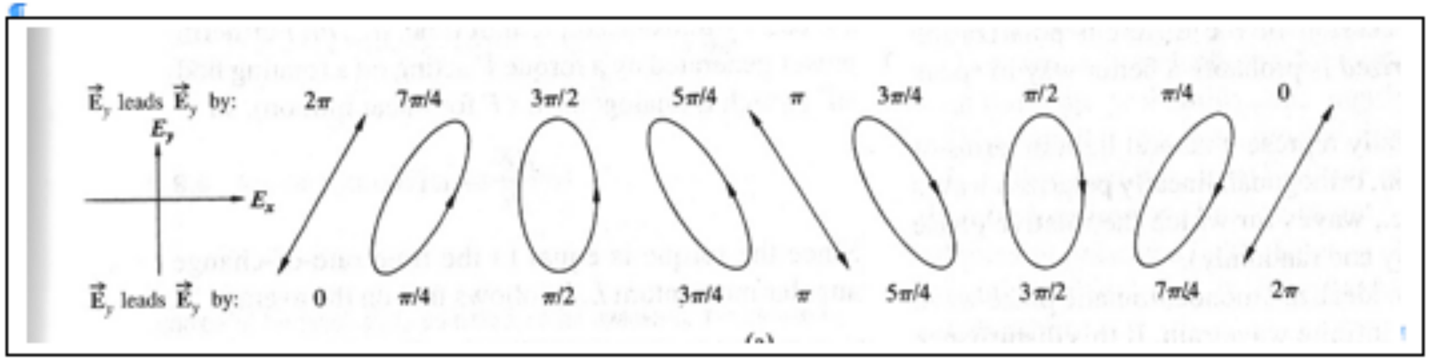
then $E_y^{\,2} + E_x^{\,2} = E_0^{\,2}$

which is a circle, ie circular polarised light.

If $\varepsilon = 0, \pm 2\pi, \pm 4\pi$ etc then

Similarly, if $\varepsilon = \pm\pi, \pm 3\pi, \pm 5\pi$, etc. then

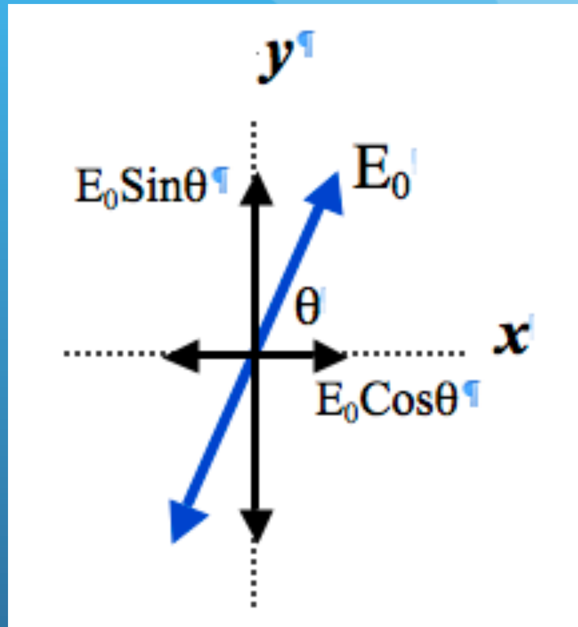
And we have linearly polarised light! For a given E_x and E_y we can have a range of different types of polarisation depending on the phase shift. Remember, we only get circular polarisation if $E_x = E_y$.



Natural Light

Natural light consists of a large number of waves with polarisation in all different directions. This is known as **unpolarised light** or more accurately **randomly polarised light.** Actually this is only partly true. Sunlight is partially polarised.

For any polarised wave we can split it into components parallel and perpendicular to some axis, ie the ***x*** axis.



Polarisers

A polariser is a device that only transmits the components of light polarised in one direction.
If natural light (random polarisation) falls on a polariser, only the component parallel to the **transmission axis** will be transmitted.

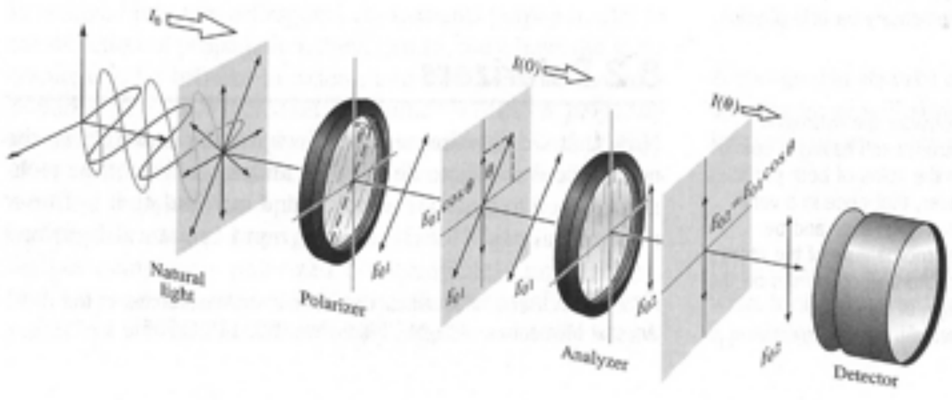


Figure 8.11 A linear polarizer and analyzer—Malus's Law. Natural light of irradiance I_0 is incident on a linear polarizer tilted at an angle θ with respect to the vertical. The irradiance leaving the first linear polarizer is $I_1 = I(0)$. The irradiance leaving the second linear polarizer (which makes an angle θ with the first) is $I(\theta)$.

$E_0 \rightarrow \quad E_{01} \rightarrow \quad E_{02}$

If a second polariser (known as an **analyser**) is added to the right of the first polariser then th
light transmitted depends on the **angle between the transmission axes of the two polariser**
The component of the polarised light parallel to the transmission axis of the analyser will be
Passed. Therefore

$$E_{02} = E_{01} Cos\theta$$

or

$$I_{02} = I_{01} Cos^2\theta$$

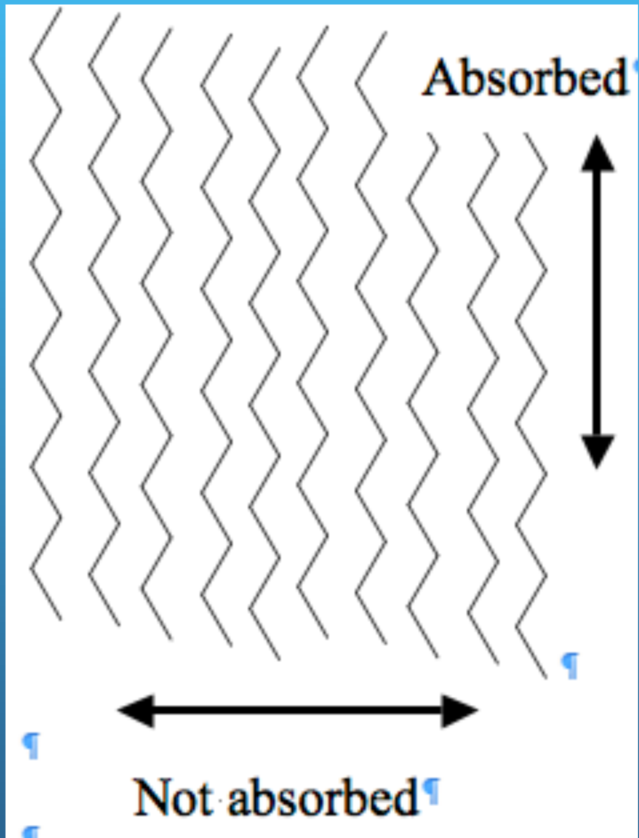
Malus's Law

Polarisers are based on one of four mechanisms

- -Dichroism (selective absorption)
- -Birefringence (double refraction)
- -Reflection
- -Scattering

Dichroism

Dichroic polarisers are made from materials such as Polaroid.
These are plastics made from aligned polymer chains.



A light Electric field **parallel** to the chains **accelerates** electrons **along** the chain.
Therefore energy is absorbed.

This is **not possible** for the E field **perpendicular** to the chain as the electrons are bound to a given chain. Therefore energy is not absorbed and light is **transmitted**.

The transmission axis is perpendicular to the direction of the polymer chains.

BIREFRINGENCE

Crystalline materials are characterised by the ordered arrangement of the atoms. Many different types of ordering exist. In some crystals such as **Calcite** ($CaCO_3$) the ordering may be such that atoms are **arranged differently in one direction** compared to the other two. This is known as a **uniaxial crystal.**

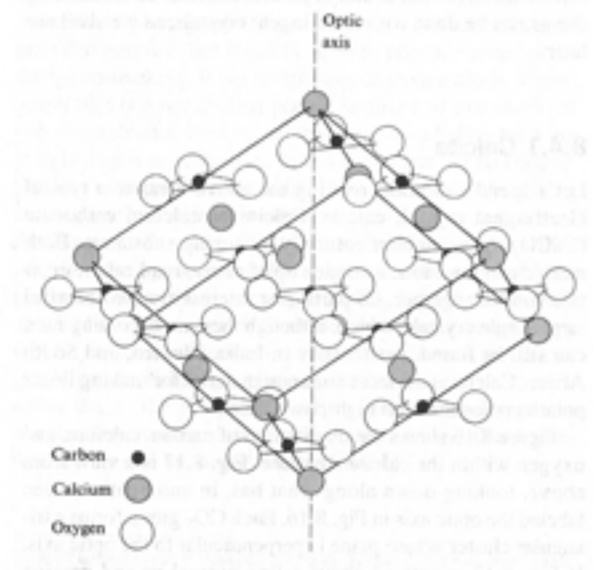
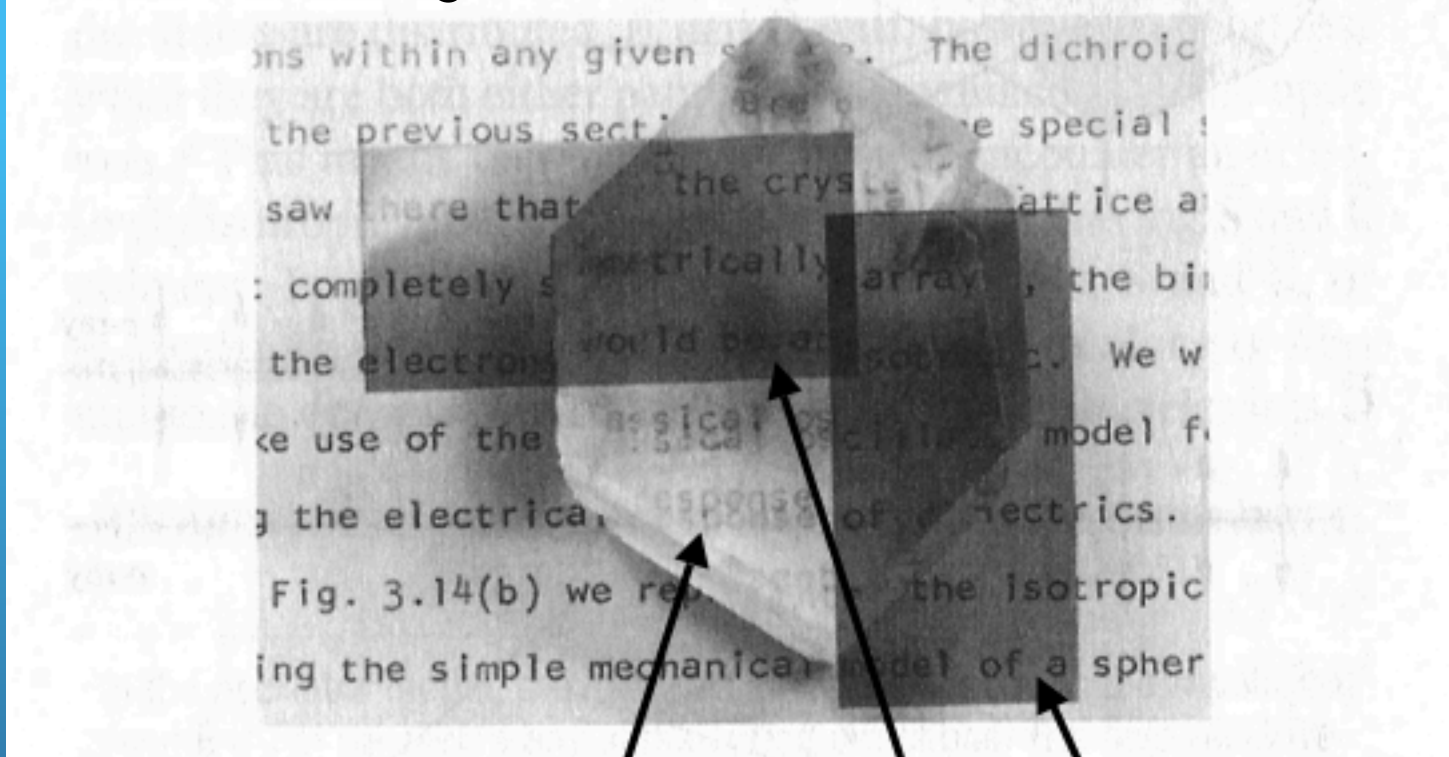


Figure 8.16 Arrangement of atoms in calcite.

This means that the **restoring force constant,** ***k*****,** for electrons can be **different in one direction** compared to the other two.

This means that $\omega_0^2=k/m$ is different in one direction. It follows that the **refractive index** is different if the electrons are accelerated in one direction compared to the other directions. ***This means that the refractive index depends on polarisation***.

One way this is manifested is that uniaxial crystals transmit double images. This is known as **Birefringence**.



A calcite crystal (blunt corner on the bottom). The transmission axes of the two polarizers are parallel to their short edges. Where the image is doubled the lower, undeflected one is the ordinary image. Take a long look: there's a lot in this one. (Photo by E.H.)

Note the **double image**. It can seen by using perpendicular polarisers that each image consists of one particular polarisation.

Note that one image is shifted while the other is in the correct position.

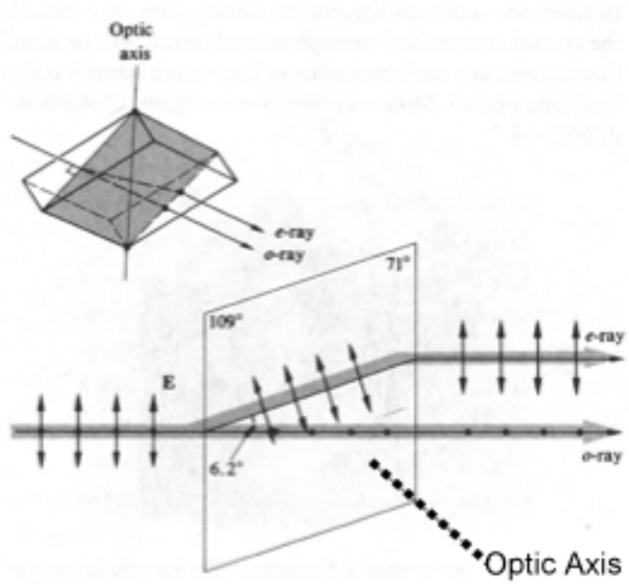


Figure 8.19 A light beam with two orthogonal field components traversing a calcite principal section.

The special direction is known as the **optic axis** of the material.
The material responds differently to electric fields with components in this direction.

Here the light polarised perpendicular to the page is also polarised **perpendicular to the optic axis**. This acts normally and is called the **ordinary ray or *o*-ray**.

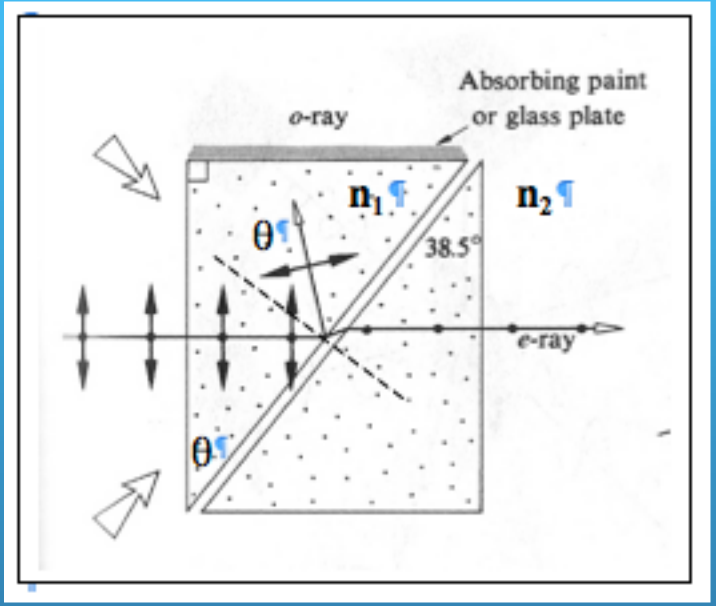
The light polarised parallel to the page has a component of E field ***parallel* to the optic axis** (as well as one perpendicular to the optic axis). Rays polarised parallel to the axis are **extraordinary rays or *e*-rays**.

O-rays and *e*-rays experience **different refractive indices**, n_o and n_e and so travel at different velocities.

For calcite, $n_o = 1.66$, while $n_e = 1.49$.

BIREFRINGENT POLARISERS

Birefringent materials can be used to polarise light.
A simple example is the **Glan-Foucault** or **Glan-Thompson** polariser.



This consists of two right angled prisms made of a birefringent material, separated by a **small gap**. The optic axis is **perpendicular** to the page in the figure.

When light is incident on the interface at the gap, it will be **totally internally reflected** if the angle of incidence is greater than the **critical angle, θ_{1c}**.
Here material 1 is the first prism while material 2 is the material in the gap.

$\sin\theta_{1c} = n_2/n_1$ therefore if

$\sin\theta > n_2/n_1$ we get TIR but if

$\sin\theta < n_2/n_1$ we get refraction (transmission)

If material 1 is birefringent, then n_1 has two values, n_o and n_e.
We can arrange things so that the **e-ray is refracted** and so transmitted and the **o-ray is reflected**. This occurs if

$$n_e < \frac{n_2}{Sin\theta} < n_o$$

This can be arranged by controlling the refractive index of the gap material, n_2, and the prism angle, θ.

If the gap consists of air this is a Glan-Foucault Polariser.
If it consists of anything else such as glycerine, then it is a Glan-Thompson polariser.

Polarisation by Scattering

Imagine a linearly polarised light wave incident on an **air molecule**. This causes the electrons in
the atom to **oscillate at the same frequency as the incident light**.

The oscillating dipoles **generate their own electric field**, again at the same frequency.
But Amperes law says this oscillating E field will generate an oscillating B field and so on.

$$\int B \bullet dl = \mu\varepsilon A \frac{dE}{dt} = \mu\varepsilon A \frac{d}{dt}\left(E_1 e^{i\omega t}\right) = i\omega\mu\varepsilon A E_1 e^{i\omega t}$$

$$\int E \bullet dl = A \frac{dB}{dt} = A \frac{d}{dt}\left(B_1 e^{i\omega t}\right) = i\omega A B_1 e^{i\omega t}$$

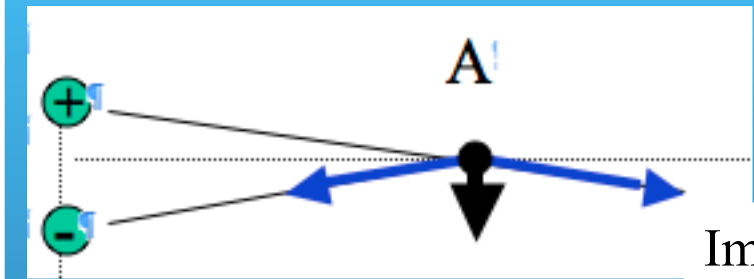
Where E_1 and B_1 are constants

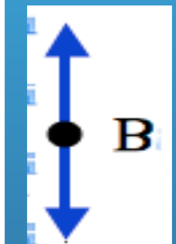
The resultant generated B and E fields are in the form of an electromagnetic wave: light!

This means that oscillating dipoles generate light!

However, this light is not emitted in all directions.

Oscillating dipoles generate light…..





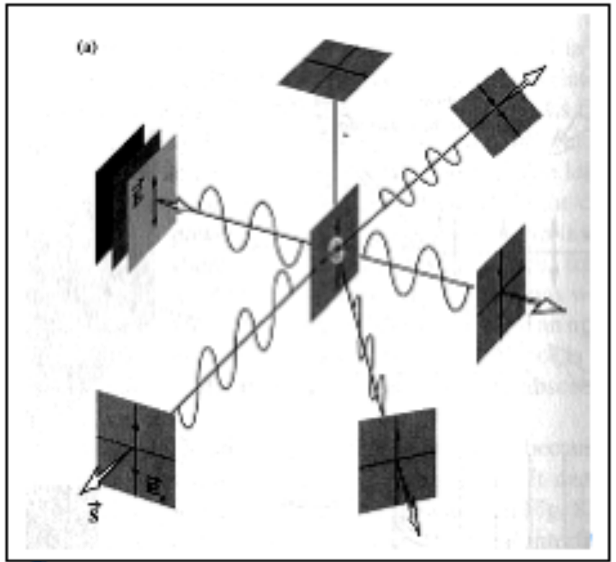
Imagine a dipole consisting of a positive and negative charge

Imagine observing the field at point A, due to the oscillating dipole. It is clear that we would see a resultant field (black arrow). However if we observe from point B, the components of the field from the two opposite charges cancels out.

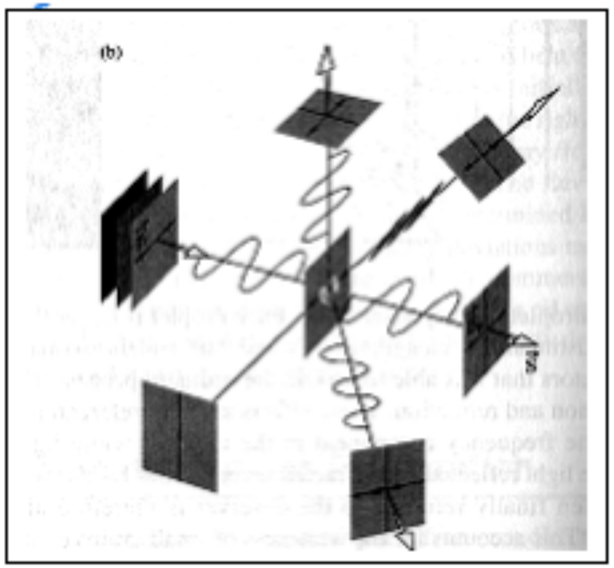
There is no field along the direction of the dipole.

This means that dipoles radiate in all directions EXCEPT in the direction of their axis.

Now consider the wave incident on an **air molecule.** If the wave is unpolarised we can represent it as a **combination of a horizontally polarised wave and a vertically polarised wave.**

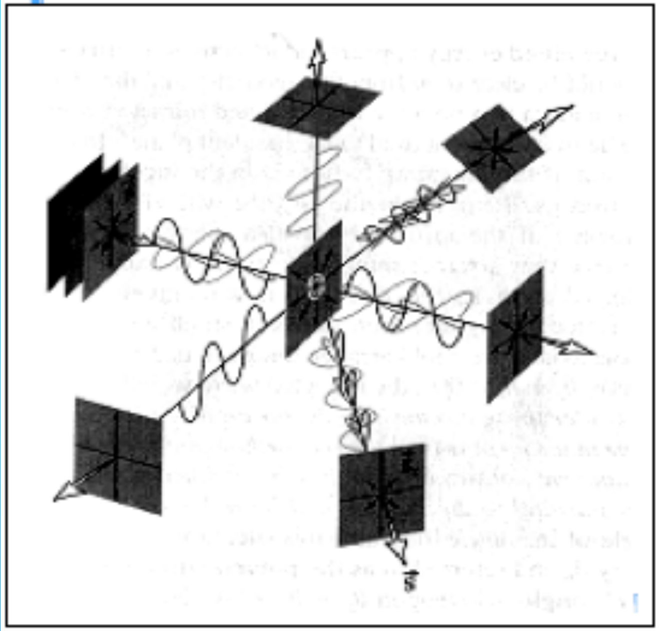


Let's look at the vertically polarised component first. **The E field causes the electrons in the atom to oscillate vertically.**
This then generates light in all directions except in the vicinity of the dipole axis ie **not vertically.**



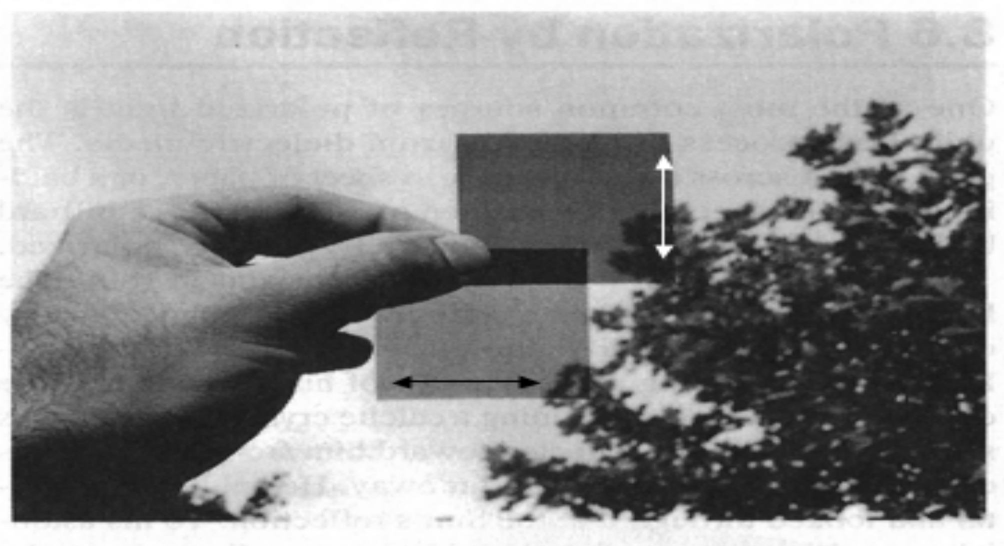
Now look at the horizontally polarised component. **The E field causes the electrons in the atom to oscillate horizontally.**
This then generates light in all directions except in the vicinity of the dipole axis ie **not horizontally.**

Combining these two polarisations as in randomly polarised light we get:



This means that when you observe **scattered light perpendicular** to the original propagation direction, it is **completely linearly polarised**.

This explains why the sunlight is polarised. If we look up we will only see the horizontally scattered component.

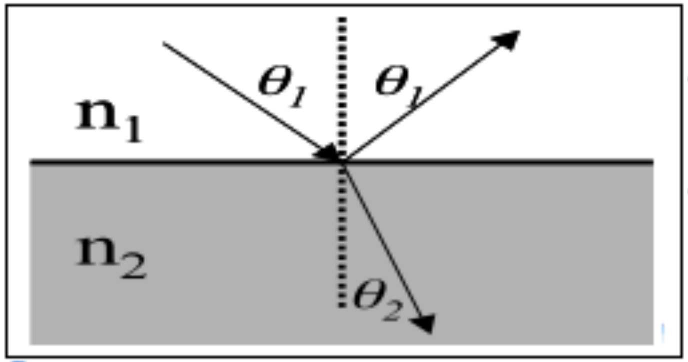


A pair of crossed polarizers. The upper polaroid is noticeably darker than the lower one, indicating the partial polarization of sky light. (Photo by E.H.)

The arrow shows the direction of the transmission axis for each polarisor.

Polarisation by Reflection

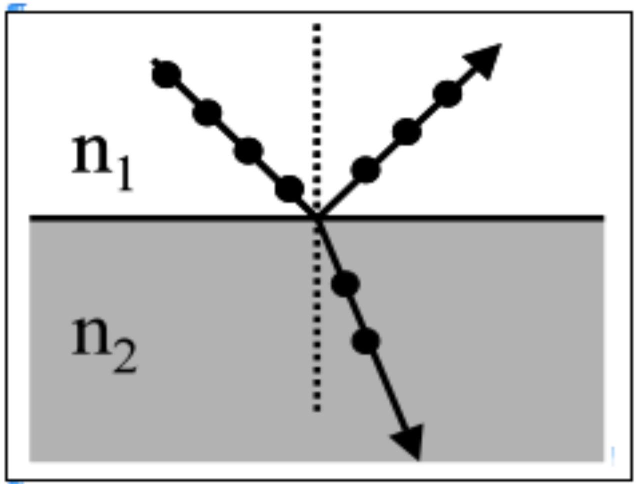
Consider a light wave incident on a glass surface.



$$n_1 < n_2$$

$$n_1 Sin\theta_1 = n_2 Sin\theta_2$$

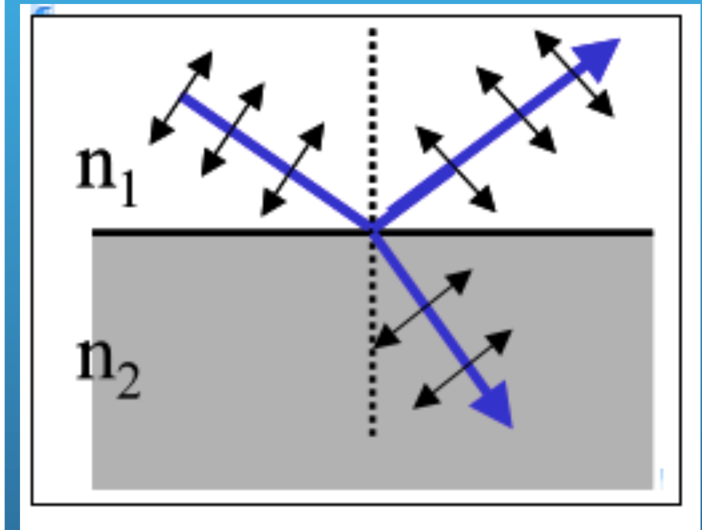
Now consider light polarised perpendicular to the page.



When the light hits the surface the E field causes the **surface atoms to oscillate** and hence **radiate** in all directions except near the dipole axis direction (into the page). What happens next is that the light radiating from all the surface atoms undergoes **interference**. It turns out that **constructive interference** only occurs in the directions shown in the diagram.

These are the directions described by the laws of reflection and refraction.
-The incident, reflected and refracted rays and the normal lie in the same plane.
-The angle of incidence and the angle of reflection are equal
-The angle of incidence and the angle of refraction are described by $n_1 \mathrm{Sin}\theta_1 = n_2 \mathrm{Sin}\theta_2$.

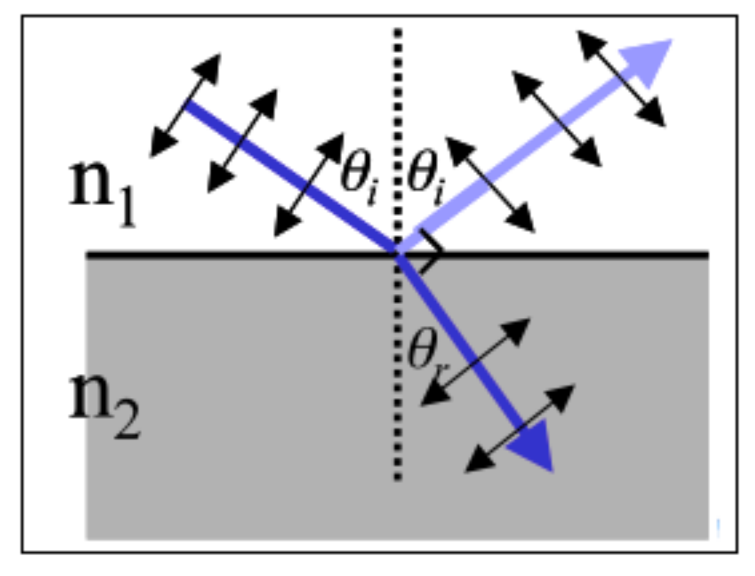
Things are a bit different if the light is polarised in the plane of the page.



When the light hits the surface the E field causes the ***surface atoms*** **to oscillate** and hence **radiate** in all directions except near the dipole axis direction.

This means that the dipoles **radiate strongly** in the direction of the **refracted wave.**

For the situation given above, the dipoles can only **radiate weakly** in the direction of motion of the **reflected wave** as this is close to the **axis direction.**



In fact if the **refracted** and **reflected** directions are **perpendicular,** *no light of this polarisation will be reflected*!

The angle of incidence at which this occurs is known as the **Brewster Angle.** This means that if randomly polarised light is incident at the Brewster angle the Reflected light will be **completely polarised perpendicular to the plane of incidence** (perpendicular to the page). To work out the Brewster angle note that this occurs when the refracted and reflected directions are perpendicular, ie.

$$\theta_{iB} + \theta_r = 90^o = \pi / 2rad$$

but

$$n_1 Sin\theta_{iB} = n_2 Sin\theta_r$$

and

$$\theta_r = 90^o - \theta_{iB}$$

so

$$n_1 Sin\theta_{iB} = n_2 Sin(90 - \theta_{iB}) = n_2 Cos\theta_{iB}$$

Therefore

$$Tan\theta_{iB} = n_2 / n_1$$

$\theta_{iB} \sim 56^0$ for glass

However the efficiency of such a device is low as much light is lost to refraction. To combat this you can make a “pile of plates polarisor”, which is just a pile of glass sheets. Here reflections from the large number of sheets build up and much more polarised light can be obtained.

REFLECTANCE

It is possible to calculate the fraction of light reflected.

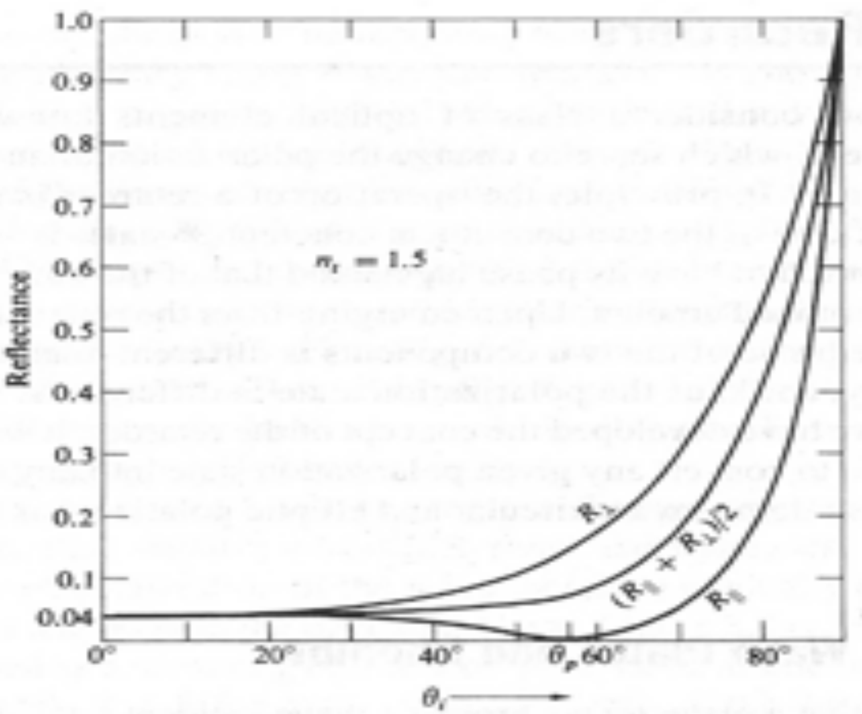


Figure 8.35 Reflectance versus incident angle.

This clearly shows that none of the in plane polarised light is reflected at the Brewster angle. It also shows that the amount of light reflected **increases as the angle of incidence is increased.**

This is why you get glare from roads, for example, when the sun is low in the sky.

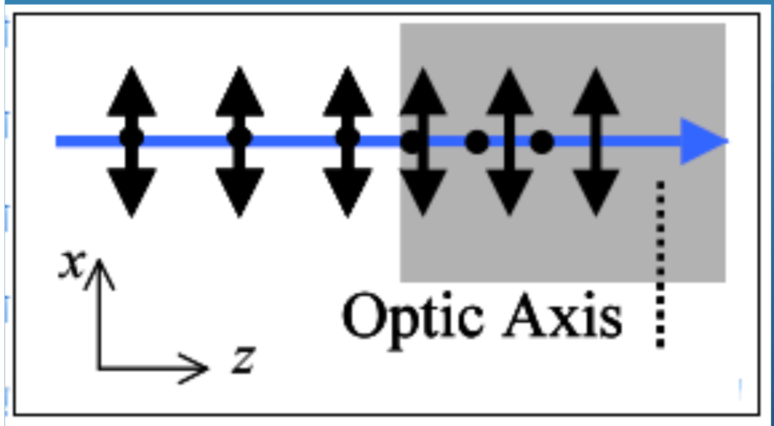
RETARDERS

A retarder is an optical element that changes the polarisation of a light wave. The simplest retarders are made from thin sheets of birefringent material such as calcite.

Imagine a sheet of calcite that has its **optic axis parallel to the front and back surfaces**, say in the ***x*** direction.

Then incident linearly polarised light will have components **parallel** (e-ray) and **Perpendicular** (o-ray) to the optic axis.

As these rays experience **different refractive indices**, n_o and n_e, they will travel at **different speeds** and hence get **out of phase**.



In calcite, $n_o > n_e$.
Phase, $\varphi = (kz - \omega t)$

At a distance *d* into the material, the phase is different by an amount $\Delta\varphi = \boldsymbol{kd}$ to the phase at the surface.

However remember, in a material: $\boldsymbol{k} = \boldsymbol{nk_0}$ where $k_0 = 2\pi/\lambda_0$

At a distance, d, into the material the phase of the **o-ray** has changed by $\Delta\varphi_o = n_o \boldsymbol{k_0 d}$ relative to the surface.

For the e-ray, the phase change is $\Delta\varphi_e = n_e \boldsymbol{k_0 d}$

Remember n_o and n_e are different so these phase shifts are different.

The relative phase shift between the o-ray and e-ray, $\Delta\varphi$ is just:

$$\Delta\varphi = \Delta\varphi_o - \Delta\varphi_e = k_0 d(n_o - n_e)$$

or

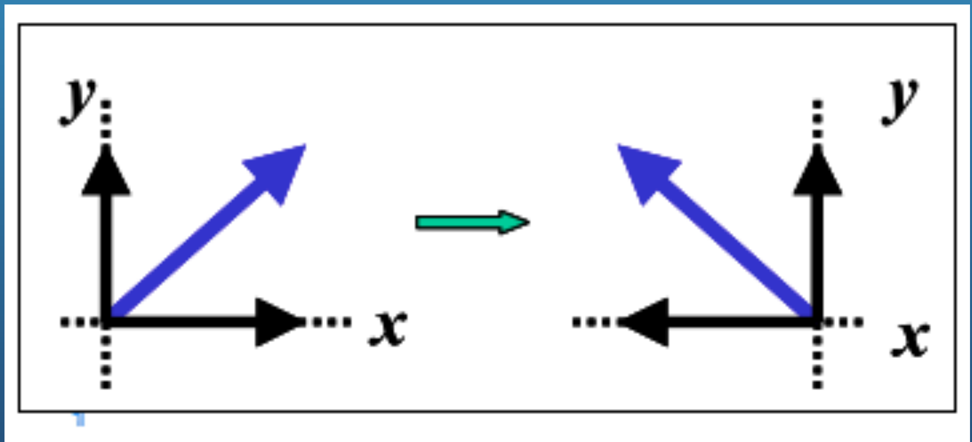
$$\Delta\varphi = \frac{2\pi}{\lambda_0} d(n_o - n_e)$$

The half wave plate

If the thickness of the sheet is arranged such that $\Delta\varphi=\pi$ (180°) we have a **half wave plate.**

When linearly polarised light passes through the half wave plate a **phase difference of π rads** will have been induced **between the o-ray and the e-ray**, ie between the component polarised in the ***x***-direction and the component polarised in the y-direction.

A shift of π **rads** is equivalent to a shift of a half wavelength. This is equivalent to reversing one of the components. If for example the ***x*** component is reversed, then:

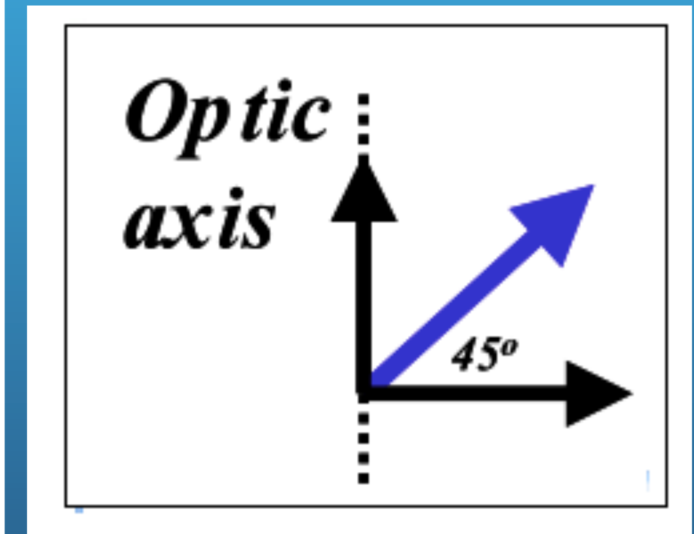


In a similar way the half wave plate can flip the orientation of elliptical light.

Quarter wave plate

If the thickness of the sheet is arranged such that $\Delta\varphi = \pi/2$ (90^o) we have a **quarter wave plate.**

A shift of ***π/2* rads** is equivalent to a shift of a quarter wavelength.



If incident light is linearly polarised with its **polarisation direction at an angle of 45° to the optic axis**, then the components of E field parallel and perpendicular to the optic axis are of **equal magnitude**.

On passing through a quarter wave plate, one of these components is shifted relative to the other by $\Delta\varphi=\pi/2$ (90°)
Then we go from **linear light to circularly polarised light.**

Similarly, if the initial polarisation angle is other than 45° we go from **linear light to elliptically polarised light.**

These processes can also happen in reverse to turn circularly polarised light into linear light or elliptically polarised light into linear light.

Angular Momentum of light

If circularly polarised light, travelling in the ***z*** direction, falls on a material, the **E fields will accelerate electrons.** As circularly polarised light consists of E-field components in the ***x*** and ***y*** direction, the electron will be **accelerated in both directions simultaneously.**

The resultant electron motion is **circular.**
Where Γ is the torque.
(this equation is equivalent to $dE/dt=vF$ in linear motion
In circular motion the power absorbed (energy per time) is given by:

$$\frac{dE}{dt} = \omega\Gamma$$

Where Γ is the torque
(this equation is equivalent to $dE/dt=vF$ in linear motion)

The torque is equal to the rate of change in angular momentum (Newtons second law) so

$$\frac{dE}{dt} = \omega \frac{dL}{dt}$$

Where L is the angular momentum

Thus if an electron absorbs ΔE energy from the light, its angular momentum will change by

$$\Delta L = \pm \frac{\Delta E}{\omega}$$

In 1905 Einstein showed that light consists of massless particles called photons. The energy of a photon is given by:

$$E = \hbar\omega$$

This means **every** photon must have an intrinsic angular momentum:

$$L = \pm\hbar$$

So what about linearly polarised light?

It turns out that linearly polarised light can be thought of as a **combination of right and left circularly polarised light** such that the **total angular momentum is zero**.