3 Resistance variation

At the end of this chapter you should be able to:

- appreciate that electrical resistance depends on four factors
- appreciate that resistance $R = \dfrac{\rho l}{a}$, where ρ is the resistivity
- recognize typical values of resistivity and its unit
- perform calculations using $R = \dfrac{\rho l}{a}$
- define the temperature coefficient of resistance, α
- recognize typical values for α
- perform calculations using $R_\theta = R_0(1 + \alpha\theta)$

3.1 Resistance and resistivity

The resistance of an electrical conductor depends on 4 factors, these being: (a) the length of the conductor, (b) the cross-sectional area of the conductor, (c) the type of material and (d) the temperature of the material.

Resistance, R, is directly proportional to length, l, of a conductor, i.e. $R \propto l$. Thus, for example, if the length of a piece of wire is doubled, then the resistance is doubled.

Resistance, R, is inversely proportional to cross-sectional area, a, of a conductor, i.e. $R \propto 1/a$. Thus, for example, if the cross-sectional area of a piece of wire is doubled then the resistance is halved.

Since $R \propto l$ and $R \propto 1/a$ then $R \propto l/a$. By inserting a constant of proportionality into this relationship the type of material used may be taken into account. The constant of proportionality is known as the **resistivity** of the material and is given the symbol ρ (Greek rho). Thus,

resistance $\boxed{\boldsymbol{R = \dfrac{\rho l}{a}} \textbf{ ohms}}$

ρ is measured in ohm metres (Ωm)

The value of the resistivity is that resistance of a unit cube of the material measured between opposite faces of the cube.

Resistivity varies with temperature and some typical values of resistivities measured at about room temperature are given below:

Copper 1.7×10^{-8} Ωm (or 0.017 $\mu\Omega$m)

Aluminium 2.6×10^{-8} Ωm (or 0.026 $\mu\Omega$m)

Carbon (graphite) 10×10^{-8} Ωm (or 0.10 $\mu\Omega$m)

Glass 1×10^{10} Ωm (or 10^4 μΩm)

Mica 1×10^{13} Ωm (or 10^7 μΩm)

Note that good conductors of electricity have a low value of resistivity and good insulators have a high value of resistivity.

> Problem 1. The resistance of a 5 m length of wire is 600 Ω. Determine (a) the resistance of an 8 m length of the same wire, and (b) the length of the same wire when the resistance is 420 Ω.

(a) Resistance, R, is directly proportional to length, l, i.e. $R \propto l$

Hence, 600 Ω ∝ 5 m or $600 = (k)(5)$, where k is the coefficient of proportionality. Hence,

$$k = \frac{600}{5} = 120$$

When the length l is 8 m, then resistance

$R = kl = (120)(8) = \mathbf{960\ \Omega}$

(b) When the resistance is 420 Ω, $420 = kl$, from which,

$$\text{length } l = \frac{420}{k} = \frac{420}{120} = \mathbf{3.5\ m}$$

> Problem 2. A piece of wire of cross-sectional area 2 mm^2 has a resistance of 300 Ω. Find (a) the resistance of a wire of the same length and material if the cross-sectional area is 5 mm^2, (b) the cross-sectional area of a wire of the same length and material of resistance 750 Ω

Resistance R is inversely proportional to cross-sectional area, a, i.e. $R \propto \dfrac{1}{a}$

$$\text{Hence } 300\ \Omega \propto \frac{1}{2\ \text{mm}^2} \text{ or } 300 = (k)\left(\frac{1}{2}\right),$$

from which, the coefficient of proportionality, $k = 300 \times 2 = 600$

(a) When the cross-sectional area $a = 5$ mm^2 then $R = (k)\left(\frac{1}{5}\right)$

$$= (600)\left(\tfrac{1}{5}\right) = \mathbf{120\ \Omega}$$

(Note that resistance has decreased as the cross-sectional is increased.)

(b) When the resistance is 750 Ω then $750 = (k)(1/a)$, from which

$$\text{cross-sectional area, } a = \frac{k}{750} = \frac{600}{750} = \mathbf{0.8\ mm^2}$$

Problem 3. A wire of length 8 m and cross-sectional area 3 mm^2 has a resistance of 0.16 Ω. If the wire is drawn out until its cross-sectional area is 1 mm^2, determine the resistance of the wire.

Resistance R is directly proportional to length l, and inversely proportional to the cross-sectional area, a, i.e.,

i.e., $R \propto \dfrac{l}{a}$ or $R = k\left(\dfrac{l}{a}\right)$, where k is the coefficient of proportionality.

Since $R = 0.16$, $l = 8$ and $a = 3$, then $0.16 = (k)\left(\frac{8}{3}\right)$, from which

$k = 0.16 \times \frac{3}{8} = 0.06$

If the cross-sectional area is reduced to $\frac{1}{3}$ of its original area then the length must be tripled to 3×8, i.e., 24 m

$$\text{New resistance} \quad R = k\left(\frac{l}{a}\right) = 0.06\left(\frac{24}{1}\right) = \mathbf{1.44\ \Omega}$$

Problem 4. Calculate the resistance of a 2 km length of aluminium overhead power cable if the cross-sectional area of the cable is 100 mm^2. Take the resistivity of aluminium to be 0.03×10^{-6} Ωm

Length $l = 2\ km = 2000$ m; area, $a = 100\ mm^2 = 100 \times 10^{-6} m^2$; resistivity $\rho = 0.03 \times 10^{-6}\ \Omega m$

$$\text{Resistance} \quad R = \frac{\rho l}{a} = \frac{(0.03 \times 10^{-6}\ \Omega m)(2000\ m)}{(100 \times 10^{-6}\ m^2)} = \frac{0.03 \times 2000}{100}\ \Omega$$
$$= \mathbf{0.6\ \Omega}$$

Problem 5. Calculate the cross-sectional area, in mm^2, of a piece of copper wire, 40 m in length and having a resistance of 0.25 Ω. Take the resistivity of copper as 0.02×10^{-6} Ωm

$$\text{Resistance } R = \frac{\rho l}{a} \text{ hence cross-sectional area } a = \frac{\rho l}{R}$$
$$= \frac{(0.02 \times 10^{-6}\ \Omega m)(40\ m)}{0.25\ \Omega} = 3.2 \times 10^{-6}\ m^2$$
$$= (3.2 \times 10^{-6}) \times 10^{6}\ mm^2 = \mathbf{3.2\ mm^2}$$

Problem 6. The resistance of 1.5 km of wire of cross-sectional area 0.17 mm^2 is 150 Ω. Determine the resistivity of the wire.

Resistance, $R = \dfrac{\rho l}{a}$

hence, resistivity $\rho = \dfrac{Ra}{l} = \dfrac{(150\ \Omega)(0.17 \times 10^{-6}\ \text{m}^2)}{(1500\ \text{m})}$

$= \mathbf{0.017 \times 10^{-6}\ \Omega m}$ **or** $\mathbf{0.017\ \mu\Omega m}$

> Problem 7. Determine the resistance of 1200 m of copper cable having a diameter of 12 mm if the resistivity of copper is $1.7 \times 10^{-8}\ \Omega$m

Cross-sectional area of cable, $a = \pi r^2 = \pi \left(\frac{12}{2}\right)^2$

$= 36\pi\ \text{mm}^2 = 36\pi \times 10^{-6}\ \text{m}^2$

Resistance $R = \dfrac{\rho l}{a} = \dfrac{(1.7 \times 10^{-8}\ \Omega\text{m})\ (1200\ \text{m})}{(36\pi \times 10^{-6}\ \text{m}^2)}$

$= \dfrac{1.7 \times 1200 \times 10^6}{10^8 \times 36\pi}\Omega = \dfrac{1.7 \times 12}{36\pi}\Omega$

$= \mathbf{0.180\ \Omega}$

Further problems on resistance and resistivity may be found in Section 3.3, problems 1 to 7, page 29.

3.2 Temperature coefficient of resistance

In general, as the temperature of a material increases, most conductors increase in resistance, insulators decrease in resistance, whilst the resistance of some special alloys remain almost constant.

The **temperature coefficient of resistance** of a material is the increase in the resistance of a 1 Ω resistor of that material when it is subjected to a rise of temperature of 1°C. The symbol used for the temperature coefficient of resistance is α (Greek alpha). Thus, if some copper wire of resistance 1 Ω is heated through 1°C and its resistance is then measured as 1.0043 Ω then $\alpha = 0.0043\ \Omega/\Omega°\text{C}$ for copper. The units are usually expressed only as 'per °C', i.e., $\alpha = 0.0043/°\text{C}$ for copper. If the 1 Ω resistor of copper is heated through 100°C then the resistance at 100°C would be $1 + 100 \times 0.0043 = 1.43\Omega$

Some typical values of temperature coefficient of resistance measured at 0°C are given below:

Copper	0.0043/°C	Aluminium	0.0038/°C
Nickel	0.0062/°C	Carbon	−0.000 48/°C
Constantan	0	Eureka	0.000 01/°C

(Note that the negative sign for carbon indicates that its resistance falls with increase of temperature.)

If the resistance of a material at 0°C is known the resistance at any other temperature can be determined from:

$$\boxed{\boldsymbol{R_\theta = R_0(1 + \alpha_0\theta)}}$$

where R_0 = resistance at 0°C

R_θ = resistance at temperature θ°C

α_0 = temperature coefficient of resistance at 0°C

> Problem 8. A coil of copper wire has a resistance of 100 Ω when its temperature is 0°C. Determine its resistance at 70°C if the temperature coefficient of resistance of copper at 0°C is 0.0043/°C

Resistance $R_\theta = R_0(1 + \alpha_0\theta)$

Hence resistance at 70°C, $R_{70} = 100[1 + (0.0043)(70)]$

$= 100[1 + 0.301] = 100(1.301)$

$= \mathbf{130.1\ \Omega}$

> Problem 9. An aluminium cable has a resistance of 27 Ω at a temperature of 35°C. Determine its resistance at 0°C. Take the temperature coefficient of resistance at 0°C to be 0.0038/°C

Resistance at θ°C, $R_\theta = R_0(1 + \alpha_0\theta)$

Hence resistance at 0°C, $R_0 = \dfrac{R_\theta}{(1 + \alpha_0\theta)} = \dfrac{27}{[1 + (0.0038)(35)]}$

$= \dfrac{27}{1 + 0.133} = \dfrac{27}{1.133} = \mathbf{23.83\ \Omega}$

> Problem 10. A carbon resistor has a resistance of 1 kΩ at 0°C. Determine its resistance at 80°C. Assume that the temperature coefficient of resistance for carbon at 0°C is −0.0005/°C

Resistance at temperature θ°C, $R_\theta = R_0(1 + \alpha_0\ \theta)$

i.e., $R_\theta = 1000[1 + (-0.0005)(80)]$

$= 1000[1 - 0.040] = 1000(0.96) = \mathbf{960\ \Omega}$

If the resistance of a material at room temperature (approximately 20°C), R_{20}, and the temperature coefficient of resistance at 20°C, α_{20}, are known then the resistance R_θ at temperature θ°C is given by:

$$\boxed{\boldsymbol{R_\theta = R_{20}[1 + \alpha_{20}(\theta - 20)]}}$$

> Problem 11. A coil of copper wire has a resistance of 10 Ω at 20°C. If the temperature coefficient of resistance of copper at 20°C is 0.004/°C determine the resistance of the coil when the temperature rises to 100°C

Resistance at θ°C, $R = R_{20}[1 + \alpha_{20}(\theta - 20)]$

$$\begin{aligned}\text{Hence resistance at } 100^\circ\text{C}, R_{100} &= 10[1 + (0.004)(100 - 20)]\\ &= 10[1 + (0.004)(80)]\\ &= 10[1 + 0.32]\\ &= 10(1.32) = \mathbf{13.2\ \Omega}\end{aligned}$$

> Problem 12. The resistance of a coil of aluminium wire at 18°C is 200 Ω. The temperature of the wire is increased and the resistance rises to 240 Ω. If the temperature coefficient of resistance of aluminium is 0.0039/°C at 18°C determine the temperature to which the coil has risen.

Let the temperature rise to θ°

Resistance at θ°C, $R_\theta = R_{18}[1 + \alpha_{18}(\theta - 18)]$

i.e.
$$\begin{aligned}240 &= 200[1 + (0.0039)(\theta - 18)]\\ 240 &= 200 + (200)(0.0039)(\theta - 18)\\ 240 - 200 &= 0.78(\theta - 18)\\ 40 &= 0.78(\theta - 18)\\ \frac{40}{0.78} &= \theta - 18\\ 51.28 &= \theta - 18, \text{ from which, } \theta = 51.28 + 18 = 69.28^\circ\text{C}\end{aligned}$$

Hence the temperature of the coil increases to 69.28°C

If the resistance at 0°C is not known, but is known at some other temperature θ_1, then the resistance at any temperature can be found as follows:

$$R_1 = R_0(1 + \alpha_0\theta_1) \text{ and } R_2 = R_0(1 + \alpha_0\theta_2)$$

Dividing one equation by the other gives:

$$\boxed{\frac{R_1}{R_2} = \frac{1 + \alpha_0\theta_1}{1 + \alpha_0\theta_2}}$$

where R_2 = resistance at temperature θ_2

Problem 13. Some copper wire has a resistance of 200 Ω at 20°C. A current is passed through the wire and the temperature rises to 90°C. Determine the resistance of the wire at 90°C, correct to the nearest ohm, assuming that the temperature coefficient of resistance is 0.004/°C at 0°C

$R_{20} = 200\ \Omega$, $\alpha_0 = 0.004/°\text{C}$

$$\frac{R_{20}}{R_{90}} = \frac{[1 + \alpha_0(20)]}{[1 + \alpha_0(90)]}$$

Hence $$R_{90} = \frac{R_{20}[1 + 90\alpha_0]}{[1 + 20\alpha_0]} = \frac{200[1 + 90(0.004)]}{[1 + 20(0.004)]} = \frac{200[1 + 0.36]}{[1 + 0.08]}$$

$$= \frac{200(1.36)}{(1.08)} = \mathbf{251.85\ \Omega}$$

i.e., the resistance of the wire at 90°C is 252 Ω

Further problems on temperature coefficient of resistance may be found in Section 3.3, following, problems 8 to 14, page 30.

3.3 Further problems on resistance variation

Resistance and resistivity

1 The resistance of a 2 m length of cable is 2.5 Ω. Determine (a) the resistance of a 7 m length of the same cable and (b) the length of the same wire when the resistance is 6.25 Ω. [(a) 8.75 Ω (b) 5 m]

2 Some wire of cross-sectional area 1 mm^2 has a resistance of 20 Ω. Determine (a) the resistance of a wire of the same length and material if the cross-sectional area is 4 mm^2, and (b) the cross-sectional area of a wire of the same length and material if the resistance is 32 Ω. [(a) 5 Ω (b) 0.625 mm^2]

3 Some wire of length 5 m and cross-sectional area 2 mm^2 has a resistance of 0.08 Ω. If the wire is drawn out until its cross-sectional area is 1 mm^2, determine the resistance of the wire. [0.32 Ω]

4 Find the resistance of 800 m of copper cable of cross-sectional area 20 mm^2. Take the resistivity of copper as 0.02 μΩm. [0.8 Ω]

5 Calculate the cross-sectional area, in mm^2, of a piece of aluminium wire 100 m long and having a resistance of 2 Ω. Take the resistivity of aluminium as 0.03×10^{-6} Ωm. [1.5 mm^2]

6 (a) What does the resistivity of a material mean?

(b) The resistance of 500 m of wire of cross-sectional area 2.6 mm^2 is 5 Ω. Determine the resistivity of the wire in μΩm. [0.026 μΩm]

7 Find the resistance of 1 km of copper cable having a diameter of 10 mm if the resistivity of copper is 0.017×10^{-6} Ωm. [0.216 Ω]

Temperature coefficient of resistance

8 A coil of aluminium wire has a resistance of 50 Ω when its temperature is 0°C. Determine its resistance at 100°C if the temperature coefficient of resistance of aluminium at 0°C is 0.0038/°C. [69 Ω]

9 A copper cable has a resistance of 30 Ω at a temperature of 50°C. Determine its resistance at 0°C. Take the temperature coefficient of resistance of copper at 0°C as 0.0043/°C. [24.69 Ω]

10 The temperature coefficient of resistance for carbon at 0°C is −0.00048/°C. What is the significance of the minus sign? A carbon resistor has a resistance of 500 Ω at 0°C. Determine its resistance at 50°C. [488 Ω]

11 A coil of copper wire has a resistance of 20 Ω at 18°C. If the temperature coefficient of resistance of copper at 18°C is 0.004/°C, determine the resistance of the coil when the temperature rises to 98°C [26.4 Ω]

12 The resistance of a coil of nickel wire at 20°C is 100 Ω. The temperature of the wire is increased and the resistance rises to 130 Ω. If the temperature coefficient of resistance of nickel is 0.006/°C at 20°C, determine the temperature to which the coil has risen. [70°C]

13 Some aluminium wire has a resistance of 50 Ω at 20°C. The wire is heated to a temperature of 100°C. Determine the resistance of the wire at 100°C, assuming that the temperature coefficient of resistance at 0°C is 0.004/°C [64.8 Ω]

14 A copper cable is 1.2 km long and has a cross-sectional area of 5 mm^2. Find its resistance at 80°C if at 20°C the resistivity of copper is 0.02×10^{-6} Ωm and its temperature coefficient of resistance is 0.004/°C [5.952 Ω]