Title: Electricity and Magnetism I. 3 units

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Lecture Outlines

(i) Vector algebra,
(ii) Electric force and electric field,
(iii) Electric flux and Gauss' law;,
(iv) Electric potentials,
(v) Capacitance and dielectric,
(vi) Ohm's law & d. c. circuits, Kirchhoffs laws.
(vii) Measurement of resistance and potential difference.
(viii) The magnetic field, Lorentz force;
(ix) Biot-Savart & Ampere's laws;
(x) Magnetic field due to conductors;
(xi) Faraday's law of electromagnetic induction.
(xii) Modern physics.

Vector Algebra

A **scalar** quantity is characterized by its magnitude. Temperature, charge and voltage are scalars. Ordinary type is used for scalars, e.g, V for voltage. A **vector** has both magnitude and direction. Velocity and the electric field are vectors. Boldface type is used for vectors, e.g, $\vec{E}$ for electric field. A vector A may be represented as a directed line segment or arrow (Figure 1). The length of the arrow represents the magnitude of the vector, denoted by $|A|$ or A, and the arrow head indicates its direction. Switching the head and tail of arrow changes A to $-A$.
A vector is changed by rotation, but not by translation.

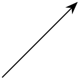


Fig.1 Vector $\vec{A}$ as a directed line segment.

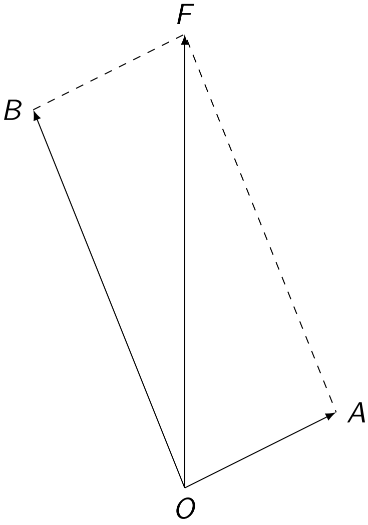
Vector Addition

To form the vectorial sum **A + B**, place the tail of B at the head of A
The sum **A + B** is the arrow from the tail of A to the head of B.
One may also reverse the order and place the tail of A at the head of B. Note that both procedures yield the same result and one which is also identical to the parallelogram method see next slide

$$\vec{OF} = \vec{OA} + \vec{OB}. \tag{1}$$

$$\vec{F} = \vec{A} + \vec{B}. \tag{2}$$

Vector Addition Cont...



Coulomb's law Example

The protons can be treated as charged particles, so the magnitude of the electrostatic force on one from the other is given by Coulomb's law.What is the magnitude of the repulsive electrostatic force between two of the protons that are separated by 4.0×10^{-15} m? Since the charge of a proton is $+e = q_1 = q_2$ then $F = \frac{1}{4\pi\epsilon_o}\frac{e^2}{r^2}$ that is $F = \frac{1}{4\pi\epsilon_o}\frac{(1.602x10^{-19})^2}{4.0\times10^{-15}} = 14N$

Exercises:

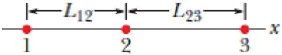
. 

fig 1

1.In the above fig 1, three charged particles lie on an x axis. Particles 1 and 2 are fixed in place. Particle 3 is free to move, but the net electrostatic force on it from particles 1 and 2 happens to be zero. If $L_{23} = L_{12}$, what is the ratio q_1/q_2?

2. In the figure above, particles 1 and 2 are fixed in place, but particle 3 is free to move. If the net electrostatic force on particle 3 due to particles 1 and 2 is zero and $L_{23} = 2.00L_{12}$, what is the ratio q_1/q_2?

Coulomb's law Example cont....

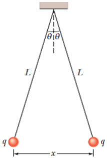


fig 2

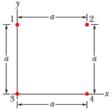
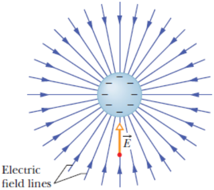


fig 3

3. In Fig 2., two tiny conducting balls of identical mass$m$ and identical charge q hang from non-conducting threads of length L. Assume that θ is so small that$tan\theta$ can be replaced by its approximate equal, $sin\theta$. (a) Show that $x = \left(\frac{q^2 L}{2\pi\epsilon_o mg}\right)^{1/3}$ gives the equilibrium separation x of the balls.
(b) If $L = 120cm$, $m = 10g$, and $x = 5.0cm$, what is $|q|$?
4.In Fig.3, four particles form a square. The charges are $q_1 = q_4 = Q$ and$q_2 = q_3 = q$. (a) What is Q/q if the net electrostatic force on particles 1 and 4 is zero? (b) Is there any value of q that makes the net electrostatic force on each of the four particles zero? Explain

The Electric Field

The electric field $\vec{E}$ at any point is defined in terms of the electrostatic force $\vec{F}$ that would be exerted on a positive test chargeq_0 placed there:
$\vec{E} = \frac{\vec{F}}{q_0}$ (electric field).



For a point charge $\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}$

The torque on an electric dipole of dipole moment $\vec{p}$ when placed in an external electric field $\vec{E}$ is given by a cross product: $\vec{\tau} = \vec{p} \times \vec{E}$

A potential energy U is associated with the orientation of the dipole moment in the field, as given by a dot product: $U = -\vec{p} \cdot \vec{E}$

The Electric Field cont...

Microwave Cooking: Food can be warmed and cooked in a microwave oven if the food contains water because water molecules are electric dipoles. When you turn on the oven, the microwave source sets up a rapidly oscillating electric field E $\rightarrow$ within the oven and thus also within the food. From above equations for$\vec{\tau}$ and U , we see that any electric field $\vec{E}$ produces a torque on an electric dipole moment $\vec{p}$ to align $\vec{p}$ with $\vec{E}$. Because the oven's $\vec{E}$ oscillates, the water molecules continuously flip-flop in a frustrated attempt to align with $\vec{E}$. The potential field of a point charge is $V = \frac{kq}{r}$ it is a scalar quantity. Another definition of Electric field is therefore: **potential gradient** that is

$$\boxed{E = -\nabla V}$$

where the operator, $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ in rectangular form. or $\nabla = \hat{r}\frac{\partial}{\partial r}$ in radial component form

Calculation of the potential produced by a dipole:

Consider two electrical charges $+q$ and $-q$, separated by a length L, then calculate the potential produced at point O at a distance r which is far greater than L See figure below.

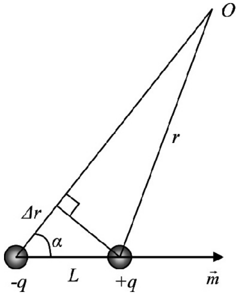


Figure: Potential and dipole moment

The potential at point O is given by

$V = \frac{q}{4\pi\epsilon_o r} - \frac{q}{4\pi\epsilon_o (r+\Delta r)}$

$V = \frac{q\Delta r}{4\pi\epsilon_o (r^2+r\Delta r)}$

$V = \frac{q\Delta r}{4\pi\epsilon_o r^2}$ where $(r >> \Delta r)$

$V = \frac{qLcos(\alpha)}{4\pi\epsilon_o r^2}$ where $(\Delta r = Lcos(\alpha))$

$V = \frac{m \cdot cos(\alpha)}{4\pi\epsilon_o r^2}$ where ($m = qL$: *elctric dipole moment*)

Gauss' Law:Electric Flux

Guass' law relates the electric field at points on a (closed) Gaussian surface to the net charge enclosed by that surface. $\Phi_E = \int_S \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_o}$ if we apply divergence theorem to this equation, we can write as
$\int_S \vec{E} \cdot d\vec{A} = \int_V \nabla \cdot \vec{E} dV = \frac{1}{\epsilon_o} \int \rho dV$ so that we can write:
The integral and differential form of Gauss law are respectively:

$$\int_S \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_o} \tag{3}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_o} \tag{4}$$

The electric field at a point near an infinite line of charge (or charged rod) with uniform linear charge density λ is perpendicular to the line and has magnitude $E = \dfrac{\lambda}{2\pi\epsilon_o r}$ where r is the perpendicular distance from the line to the point.

Magnetic Effects of Current

1. Magnetic field: It is the space around the magnet in which any other magnetic material place will feel the force of attraction or force of repulsion.
2. Orested Experiment: Acc. To Orested there is present magnetic field around the current carrying conductor.
3. Biot Savart's law: Acc to this law magnetic field due to current carrying conductor is given by $dB = \frac{\mu_0}{4\pi r^2} Idl\ sin\theta$
4. For the long straight conductor: $B = \frac{\mu_0 I}{4\pi a}(sin\theta_1 + sin\theta_2)$