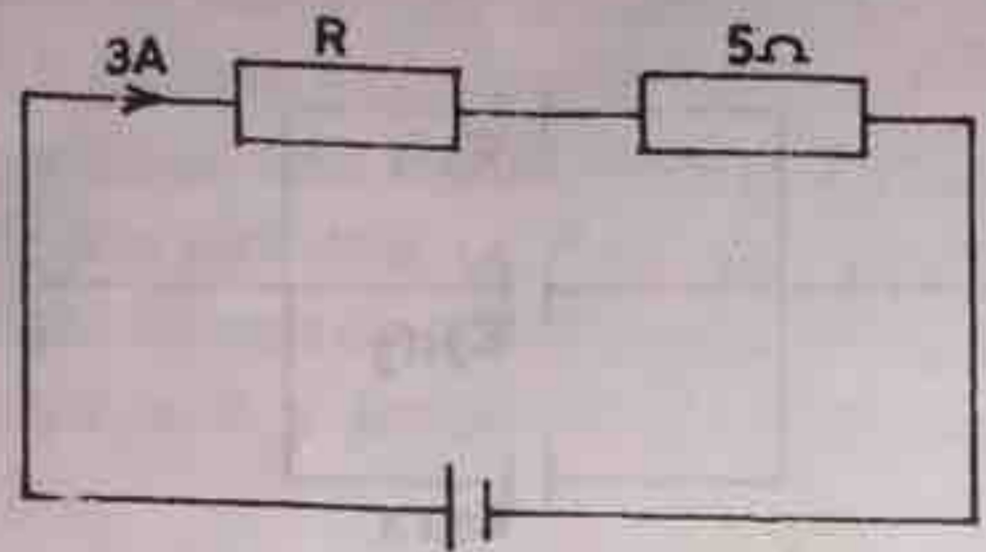
Worked example 10.2

Two resistors of resistance R and 5Ω are connected in a series to a battery of e.m.f 24V and internal resistance 1Ω. If the current through the circuit is 3Ω, find the value of R.

Solution



$E_c = 24V,\ r_c = 1,\ R_c = (R+5)\Omega$

$$I = \frac{E_c}{R+r}$$

$$\therefore\ 3 = \frac{24}{1+(R+5)} = \frac{24}{6+R}$$

$$3(6+R) = 24$$

$$18 + 3R = 24$$

$$3R = 24 - 18 = 6$$

$$R = \frac{6}{3} = 2\Omega$$

OR

$$3(6+R) = 24$$

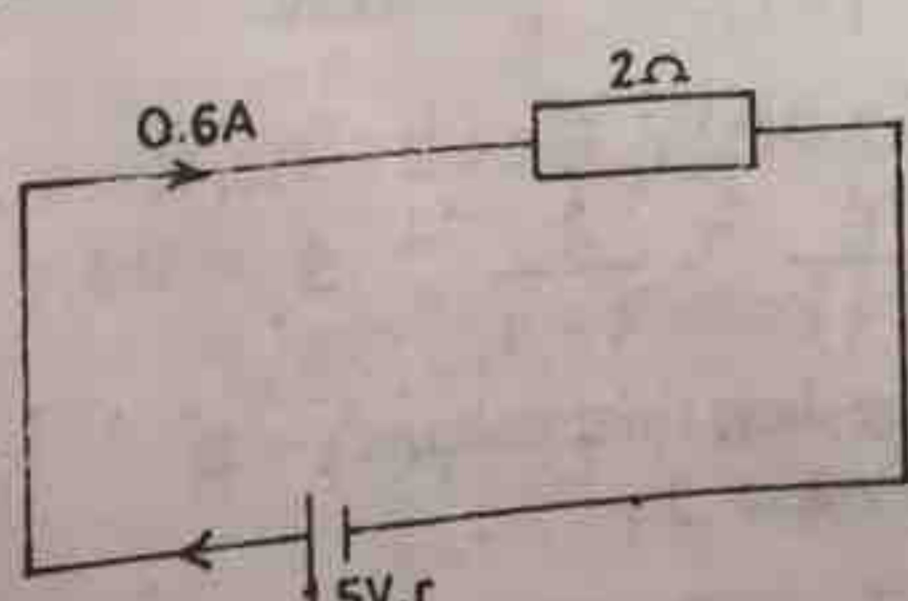
$$6 + R = \frac{24}{8} = 8$$

$$R = 8 - 6 = 2\Omega$$

Worked example 10.3

A cell of e.m.f 1.5volts supplies a current of 0.6A through a cell of resistance 2Ω. Calculate the internal resistance of the cell.

Solution



$$E = IR + Ir = I(R+r)$$

$$1.5 = 0.6(2+r)$$

$$\frac{1.5}{0.6} = 2 + r$$

$$2.5 = 2 + r$$

$$r = 2.5 - 2$$

$$\therefore r = 0.5\Omega$$

Worked example 10.4

A battery of the three cells in a series, each of e.m.f 2v and internal resistance 0.5Ω is connected to 2Ω resistor in a series with a parallel combination of two 3Ω resistors. Calculate the:

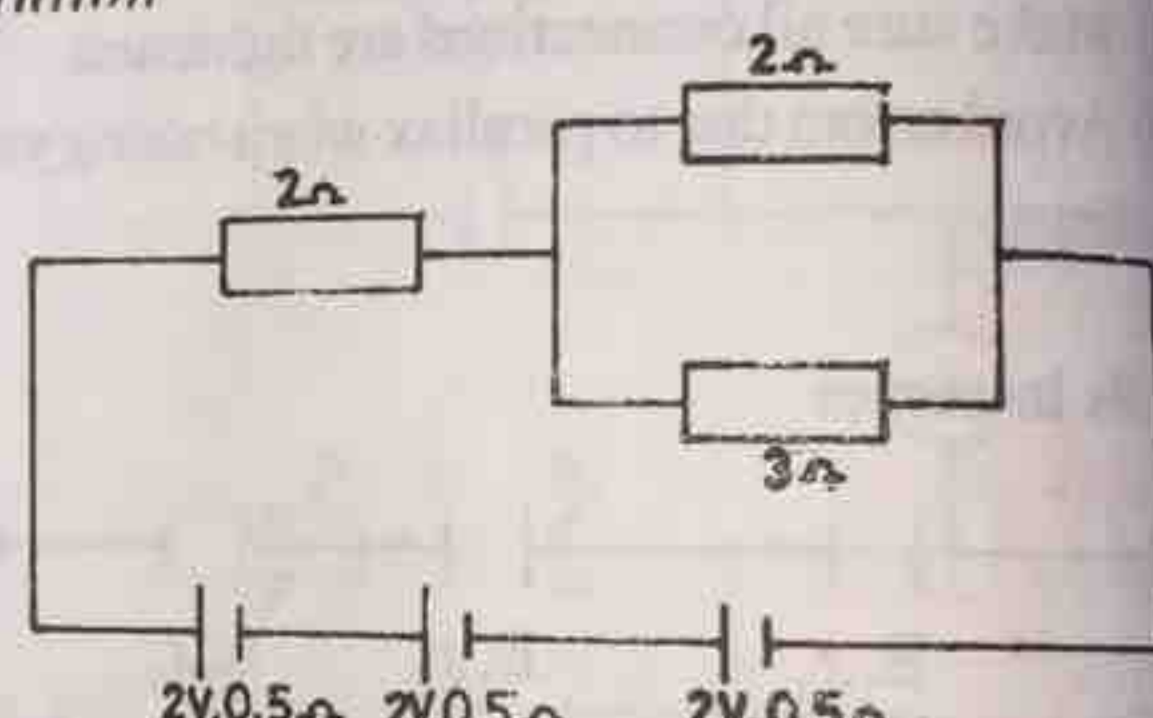
(i) effective external resistance

(ii) current in the circuit

(iii) lost volts in the battery

(iv) current in one of the 3Ω resistors

Solution



(i) $R_c = 2 + \frac{3 \times 3}{3+3} = 2 + \frac{9}{6}$

$= 2 + 1.5 = 3.5\Omega$

(ii) $E_c = 2 + 2 + 2 = 6v,$

$r_c = 0.5 + 05 + 0.5 = 1.5,\ R_c = 3.5\Omega$

$$I = \frac{E}{R+r} = \frac{6}{3.5+1.5} = \frac{6}{5} = 1.2A$$

(iii) lost voltage $= 1.2 \times 1.5 = 1.8V$

(iv) The p.d across the terminals $= IR_C$

$= 1.2 \times 3.5 = 4.2V$

OR

The p.d across the terminals = E - IRC

$= 6 - 1.8 = 4.2V$

The p.d across the 3Ω resistor = IR1

$= 1.2 \times 3 = 3.6V$

∴ The p.d of two 3Ω resistors

in parallel $= (4.2 - 3.6)V = 0.6V$

Thus, the current flowing in one 3Ω resistor =

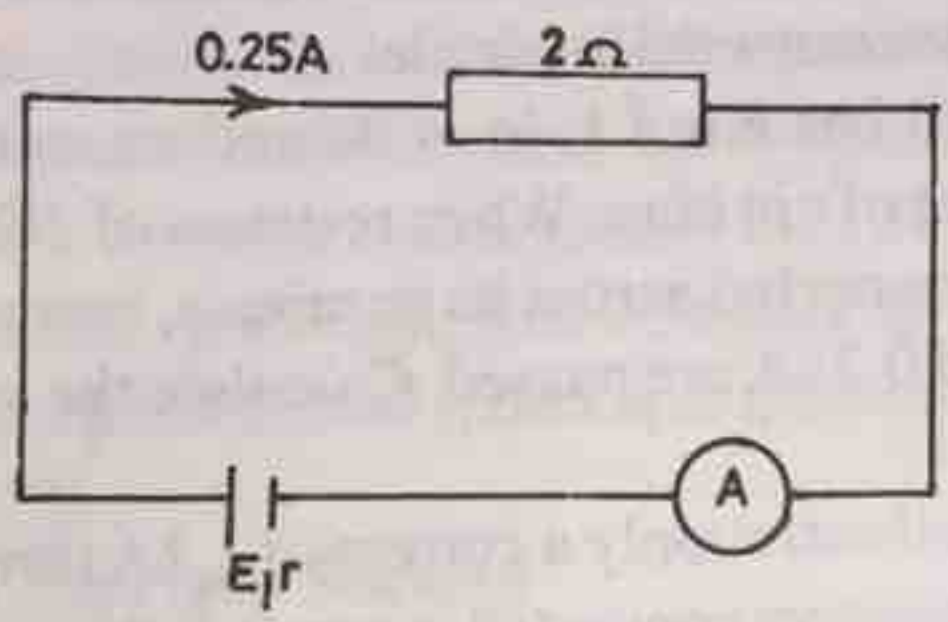
$$\frac{V}{R} = \frac{0.6}{3} = 0.2A$$

NB: The two 3Ω resistors have the same p.d because they are in parallel.

Worked example 10.5

A cell is joined in a series with a resistance of 2Ω and a current of 0.25 flows through it. When a second resistance of 2Ω is connected in parallel to the first, the current in the cell increases to 0.30. What is: (i) The internal resistance of the cell (ii) The e.m.f.

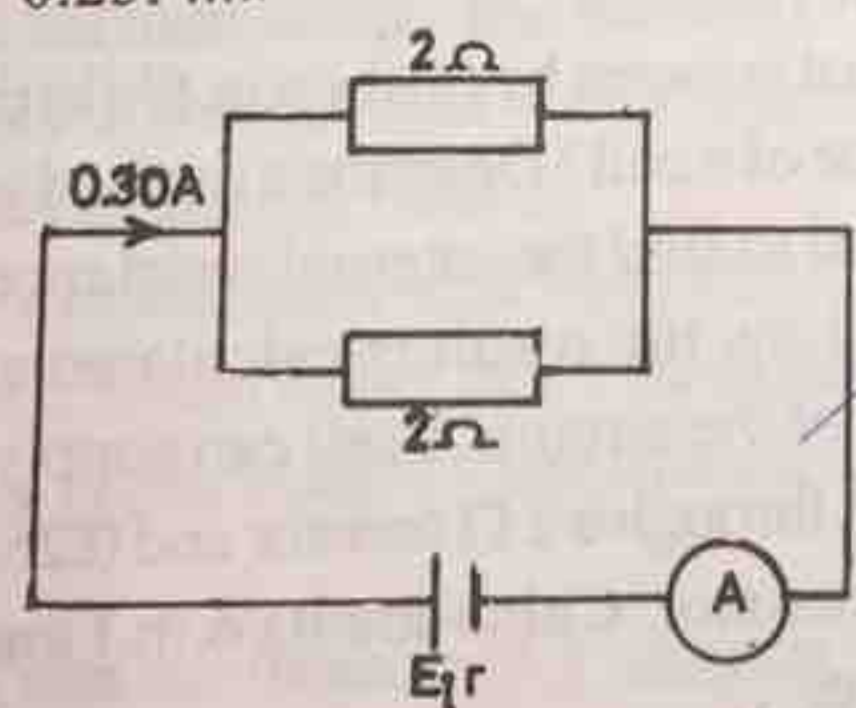
Solution

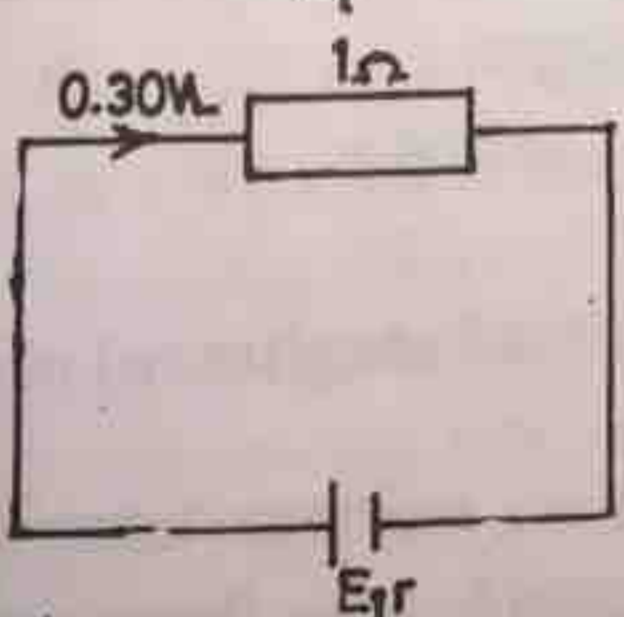


$E = IR = Ir$

$E = (0.25 \times 2) + 0.25 \times r$

$E = 0.5 + 0.25r$ ....





$$R_c = \frac{2 \times 2}{2 \times 2} = \frac{4}{4} = 1\Omega$$

$E = IR + Ir$

$E = 0.3 \times 1 + 0.3 \times r$

$E = 0.3 + 0.3r$ ....(ii)

Equating equations (i) and (ii)

$0.3 + 0.3r = 0.5 + 0.25r$

$0.3r - 0.25r = 0.5 - 0.3$

$0.05r = 0.2$

$$r = \frac{0.2}{0.05} = 4\Omega$$

$E = 0.5 + 0.25r$

$= 0.5 + (0.25 \times 4)$

$E = 0.5 + 1 = 1.5$

OR

$E = 0.3 + 0.3r$

$= 0.3 + 0.3 \times 4$

$= 0.3 + 1.2$

$= 1.5V$

Revision exercise

1. What is meant by the electromotive force of a cell? [SSCE, Aug. 1991]
2. Explain why the e.m.f of a cell is usually greater than the p.d across its terminals when the cell is used to supply current to an external circuit. [SSCE, Nov. 1990]
3. Four identical cells, each of e.m.f E and internal resistance are connected in series with a resistance R. Write down the formula for the current in the circuit. [SSCE, June 1989]
4. State two sources of e.m.f other than chemical cell. [SSCE, June 1996]
5. Explain why a battery of eight dry Lechanches cells, each of e.m.f 1.5V is not normally used in place of a motor-car battery of 12V to start a car. [WASSCE, June 1999]
6. Distinguish between e.m.f and p.d. Give one similarity between them.
7. An electric bell takes a current of 0.2A from a battery of two dry cells connected in a series. Each cell has e.m.f of 1.5V and an internal resistance of 1.0Ω (S). (i) Calculate the effective resistance of the cell. (ii) What current would the bell take if the cells were arranged in parallel? [SSCE, June 1997]
8. Describe an experiment to determine the

$M_1 = 10^{12}$kg, $M_2 = 10^{13}$kg

Using $F = \frac{GM_1M_2}{r^2} = \frac{6.67 \times 10^{-11} \times 10^{12} \times 10^{13}}{r^2}$

$= 6.7$

$r^2 = \frac{6.67 \times 10^{14}}{6.7} = 9.95 \times 10^{13}$

$r = \sqrt{9.95 \times 10^{13}}$

$= 3.16 \times 10^{13}$ m

$r = 3.16 \times 10^{10}$ km

Worked example 37.4

A rocket of mass 150kg is fired from the earth's surface so that it just escapes from the gravitational influence of the earth. Calculate the velocity with which it escapes where $G = 6.67 \times 10^{-11}Nm^2/kg^2$, $R = 6.4 \times 10^6$m and mass of the earth is 6.0×10^{24}kg

Solution

Given $M_1 = 150$kg, $G = 6.67 \times 10^{11}Nm^2/kg^2$

$M_2 = 6.0 \times 10^{24}$kg, $r = 6.4 \times 10^6$m

Using

$V = \sqrt{\frac{2GM}{R}}$

$= \frac{2 \times 6.67 \times 10^{-11} \times 150}{6.4 \times 10^6}$

$= \sqrt{3.127 \times 10^{-15}}$

$= 5.59 \times 10^{-8}$m/s.

Electric field

Electric field is defined as any region where a charge experiences a force of electrical origin.

Electric lines of force are imaginary lines drawn in such a way that the direction at any point of the tangent is the same as the direction of the field at the same point.

37.11 Lines of Force

The path taken by a unit positive (+ve) charge in an electrostatic field is called *an electrostatic line of force*.

Some properties of lines of force are as follows:

(i) Two lines of force never intersect each other

(ii) The tangent to the curve at any point gives the direction of the electric intensity at that point.

(iii) Lines of force exert a lateral pressure.

(iv) It has no continuity inside the body of the conductor except in case of isotopic dielectrics.

(v) They are continuous curves in an electric field starting from the positively charged body to the negatively charged.

37.12 Electric Force between Point Charges

Force **F** exists between two charges q_1 and q_2. The force between two charges, q_1 and q_2 is proportional to their product and inversely proportional to their distance R apart.

Force is a free space represented by F, thus,

$F \alpha \frac{q_1q_2}{R_2}$

$F = \frac{kq_1q_2}{R_2}$

where k = costant,

$k = \frac{1}{4\pi\varepsilon_o}, = 9 \times 10^9 \frac{m}{f}$

ε_o = permittivity of free space (vacuum)

37.13 Coulomb's Law

Coulomb investigated the magnitude of the force between two electrically charged particles. He stated the law of electrostatic force of attraction called *Coulomb's law*. Coulomb's law states that the force of attraction between two charged particles, Q and q is directly proportional to the product of their charges and inversely proportional to the square of their distance (R) apart.

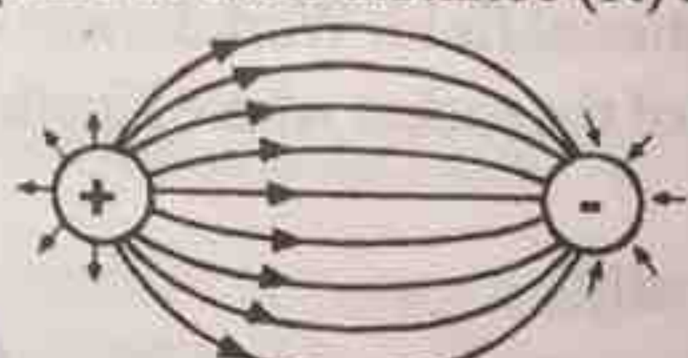


Fig. 37.1 Unlike Charges

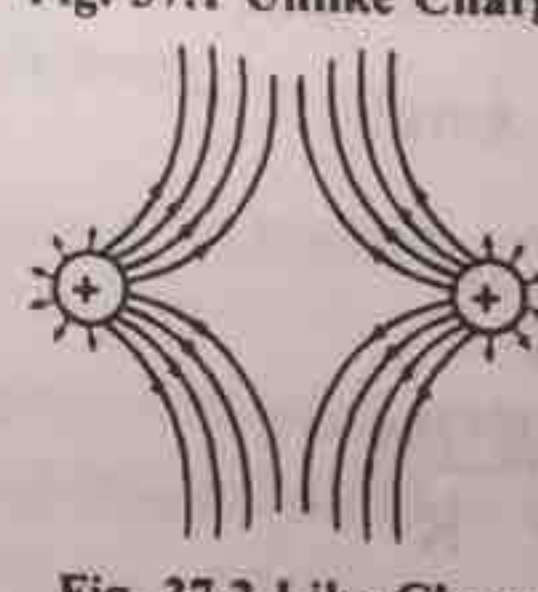


Fig. 37.2 Like Charges

N.B: (i) Like charges repel

(ii) Unlike charges attract

Mathematically,

$$F \propto \frac{Qq}{R^2}$$

$$F = \frac{kQq}{R^2}$$

Where k is the permittivity of free space (vacuum).

$$K = \frac{1}{4\pi\varepsilon_o} = 9.00 \times 10^9 \text{m/F}.$$

where $\varepsilon_o = 8.85 \times 10^{-12}\,Fm^{-1}$.

Worked example 37.5

Two equal charges are placed in a vacuum, 100m apart. If the force of attraction between them is 0.2N, calculate the value of the two equal charges.

Taking $\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9$ m/f

Solution

Given that $F = \frac{kq \times q_2}{r^2}$

where $k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9$ m/f

$$0.2 = \frac{9 \times 10^9 \times q \times q}{(100)^2}$$

$$0.2 \times (100)^2 = 9 \times 10^9 \times (q)^2$$

$$q^2 = \frac{0.2 \times 10000}{9 \times 10^9}$$

$$= \frac{2000}{9 \times 10^9}$$

$$q = \sqrt{2.22 \times 10^{-7}}$$

$$\therefore q = 4.71 \times 10^{-4}\text{C}.$$

Worked example 37.6

Calculate the electrostatic force of attraction between two charges, if the distance apart is 5 x 10^{10}m given that Q and q are 1.65×10^{-10}C and 1.55×10^{-10}C respectively. ($K = 9 \times 10^9$m/f)

Solution

Given $Q = 1.65 \times 10^{-10}$C, $q = 1.55 \times 10^{-10}$C

$R = 5 \times 10^{-10}$m, $K = 9 \times 10^9$ m/f

$F = ?$

Using $F = \frac{KQq}{r^2} = \frac{9 \times 10^9 \times 1.65 \times 10^{-10} \times 1.55 \times 10^{-10}}{(5 \times 10^{-10})^2}$

$$\frac{9 \times 1.65 \times 1.55 \times 10^{9-10-10}}{(5 \times 10^{-10})^2}$$

$$= \frac{23.0175 \times 10^{-11}}{(5 \times 10^{-10})^2}$$

$$= \frac{23.0175 \times 10^{-11}}{25 \times 10^{-20}}$$

$$= 9.207 \times 10^8\text{N}$$

37.14 Electric Field Intensity

Electric field strength or intensity (E) at a point, is defined as the force per coulomb in a small charge particle placed at a point, which does not influence the field. This field is represented by electric line of force which shows the direction of the field and their intensity or strength. Field intensity (E) is placed in a distance (R) from charge q and Q.

$$\text{Force} = \frac{kQq}{R^2} \text{ where } k = \frac{1}{4\pi\varepsilon_o}$$

$$F = \frac{Qq}{4\pi\varepsilon_o R^2} \text{ ....(1)}$$

since electric field strength or intensity is the force per unit coulomb.

$$E = \frac{F}{q} \text{ ....(2)}$$

Substitute for F in equation (2)

$$E = \frac{F}{q} = \frac{Qq}{4\pi\varepsilon_o R^2 q}$$

$$E = \frac{Q}{4\pi\varepsilon_o R^2}$$

Worked example 37.7

Given the following:

(a) Charge $q = 2 \times 10^{+15}$C

(b) Distance apart = 10m

(c) (Electrostatic constant = 6.67×10^{-11} Nm^{2}kg^{-2})

Find the electrostatic field intensity

Solution

$$E = \frac{KQ}{r^2} \text{ where } k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9\text{m/f}$$

$$= \frac{9 \times 10^{9} \times 2 \times 10^{+15}}{(10)^{2}}$$

$$= \frac{9 \times 2 \times 10^{9} \times 10^{+15}}{(10)^{2}}$$

$$= \frac{18 \times 10^{9+15}}{100}$$

$$= 0.18 \times 10^{24}$$

$$= 18 \times 10^{22} NC^{-1}$$

37.15 Force between Protons

The force of attraction between two protons, each carrying a positive charge is represented as q.

$$F = \frac{Qq}{4\pi\varepsilon_o R^2}$$

Since Q = q (charge is the same)

$$F = \frac{Qq}{4\pi\varepsilon_o R^2} = \frac{q^2}{4\pi\varepsilon_o R^2}$$

Assuming the proton charge (q) = + 1.6×10^{-19}C has a distance of 10^{-10}m apart

$$F = \frac{Q^2}{4\pi\varepsilon_o R^2} = \frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^{9}}{(10^{-10})^2}$$

$$F = 1.44 \times 10^{11} N$$

37.16 Electric Potential

Work is said to be done when a force acts on charges from an electric point to another. Electric potential, V, at a point is defined as the work done per coulomb in moving a positive charge from infinity to a point.

Since V = ER, where R is the distance

$$V = \frac{Q}{4\pi\varepsilon_o R^2} \times R$$

where $E = \frac{Q}{4\pi\varepsilon_o R^2}$

$$V = \frac{Q}{4\pi\varepsilon_o R^2} \times R = \frac{Q}{4\pi\varepsilon_o R}$$

$$\therefore \quad V = \frac{Q}{4\pi\varepsilon_o R}$$

37.17 Relationship Between E and V

Let us assume that a positive charge q moved from one point A to point B between plates opposite the direction of electric field strength or intensity E. The distance apart is d, then work = force x distance

$= Eq \times d$ .... (1)

Since work done = Vq ....(2)

equating the equations (1) and (2),

workdone = EqV

workdone = qv

Eqd = Vq

Ed = V

$E = \frac{V}{d}$ ....(1)

or V = Ed .... (2)

hence, E is directly proportional to V.

Revision exercise

1. What is meant by the statement: *The gravitational field about a region?*
2. State the inverse square or Newton's law of gravitation, showing the derivation of the equation.
3. Find the force of attraction or repulsion between two balls of mass 30.3kg and 40.17kg, if their distance of separation is 0.5m, [assuming inverse gravitational constant is 6.7×10^{-11} Nm^{2}kg^{-2}.]
4. Find the dimension of the gravitational constant.
5. Find the value of the acceleration due to gravity of a body of mass 0.6kg and at a distance of 6×10^{6}m from the moon [G = 6.7×10^{-11}Nm^{2}kg^{-2}]
6. Find the gravitational potential of a body on earth of mass 600 kg and of radius 6.4×10^{-2}m assuming G is 6.7×10^{-11}Nm^{2}kg^{-2}.
7. Calculate the escape velocity of a rocket from the earth's surface of radius 6.4×10^{6}m.
8. Show that the velocity of escape of a body given by $V = \frac{\sqrt{2GM}}{R}$ can be reduced to $V = \sqrt{2gr}$
9. The mass of two like bodies are attracted with a