Worked example 31.2

A ray of light is incidented in water at angle 30°, from water to air plane surface. Find the angle of refraction in the air (μ of water = 1.33).

Solution

$$\mu = \frac{\sin i}{\sin r}$$

$$\frac{4}{3} = \frac{\sin 30^\circ}{\sin r}$$

$$\sin r = \frac{0.5}{\frac{4}{3}}$$

$$\sin r = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$$= 0.375$$

$$r = \sin^{-1}(0.375)$$

$$r = 22.02^\circ$$

$$r \simeq 22^\circ$$

Worked example 31.3

A ray of light is incidented at angle 60° in air on an air-glass plane surface. Find the angle of refraction of the glass (${}_a\mu_g = 1.5$)

Solution

$${}_a\mu_g = 1.5$$

$${}_a\mu_g = \frac{\sin i}{\sin r}$$

$$1.5 = \frac{\sin 60^\circ}{\sin r}$$

$$1.5 = \frac{0.8660}{\sin r}$$

$$1.5 \times \sin r = 0.8660$$

$$\sin r = \frac{0.8660}{1.5}$$

$$\sin r = 0.5773$$

$$r = \sin^{-1} 0.5773$$

$$r = 35.2^\circ$$

31.7 Measurement of Refractive Index in Liquid

Refractive index of a medium can be determined using three methods:

(i) Construction method [using glass prism]

(ii) Solid method

(iii) Liquid method

Liquid method

Aim: To determine the refractive index of liquid

Apparatus: A retort stand, object, pin, beaker and a measuring tape

Reagent: Water

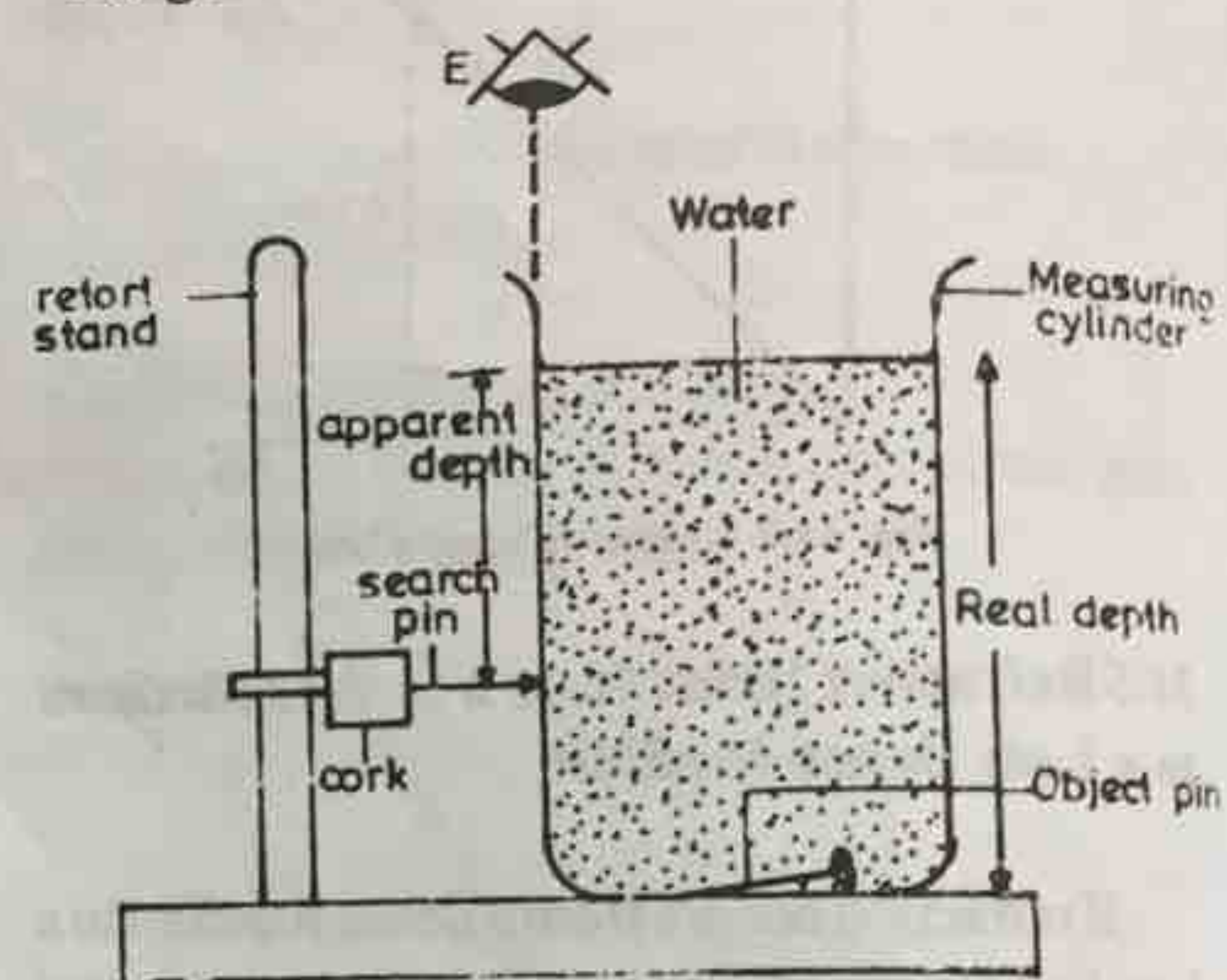

Fig. 31.4 To determine the refractive index of liquid

Method: The apparatus is set up as shown above. In the beaker, a search pin was placed at the bottom of the beaker. The distance of the beaker or the height is noted from the top to the closed end. A liquid substance is poured to fill the beaker completely. It will be observed that the position of the pin has been shifted upward towards the position of the observer. This new location is caused as a result of refraction. The distance from the new position is measured to the level of the water in the beaker known as *apparent depth*. From the tip of the beaker to the bottom is known as the *real depth*. The distance from the new position of the pin to the initial position of the object is known as *displacement*.

By calculation,

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

The relationship combines the displacement of the pin, the thickness of the media and the refractive index of the medium such that,

$$D = t\left(1 - \frac{1}{\mu}\right)$$

Where D is displacement, t is the thickness and μ the refractive index.

Practical application of refractive index

(1) To confirm the identity of a substance, refractive index may be employed. (2) In removing chromatic aberation by using a double lenses made of materials of different refractive indices. (3) In analysis mixtures, a refractive index may be employed.

Worked example 31.4

A coin is placed at the bottom of a pond, 3m deep. Calculate the apparent displacement of the coin when it is viewed directly from top [μ of air to water = $^4/_3$

Solution

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\text{Apparent depth} = \frac{\text{Real depth}}{\mu}$$

$$\mu = \frac{3}{4/3} = \frac{3}{1} \times \frac{3}{4}$$

$$\mu = \frac{9}{4} = 2.25$$

$\therefore$ Apparent depth = 2.25

Worked example 31.5

The real depth of a swimming pool is 2.8m. What is the apparent depth of the pool when viewed from a screen board vertically above? [μ of water = 1-33]

Solution

Real depth = 2.8m

$$\text{Apparent depth} = \frac{\text{Real depth}}{\mu \text{ of water}}$$

$$= \frac{2.8}{1.33}$$

$$= 2.10\text{m}$$

Apparent depth = 2.10m

Worked example 31.6

A microscope is focused on a table when a mark is covered by a plate of glass, 3cm thick. The microscope has to be placed 1.18cm for the mark to be on a small focus. Calculate the refractive index of the medium.

Solution

D = 1.18cm, t = 3cm

using $D = t\left(1 - \frac{1}{\mu}\right)$

$$D = \left(t - \frac{t}{\mu}\right)$$

$$D = \frac{\mu t - t}{\mu}$$

$$D\mu = \mu t - t$$

$$D\mu - t\mu = -t$$

$$\mu(D - t) = -t$$

$$\mu = \frac{-t}{D - t} = \frac{-3}{1.18 - 3}$$

$$= \frac{3}{1.82} = \simeq 1.65\text{cm}$$

Worked example 31.7

A fish pond appears to be 1m deep but when viewed vertically above the refractive index of the water on the pond it is 1.5. Calculate the real depth formed.

Solution

μ of water = 1.5

Apparent depth = 1m

$$\mu \text{ of water} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

Real depth = μ of water x Apparent depth

= 1.5 x 1

Real depth = 1.5m

Critical angle

Critical angle is the angle of incidence in the denser medium when the angle of refraction in the less dense medium is 90°. It is denoted by C.

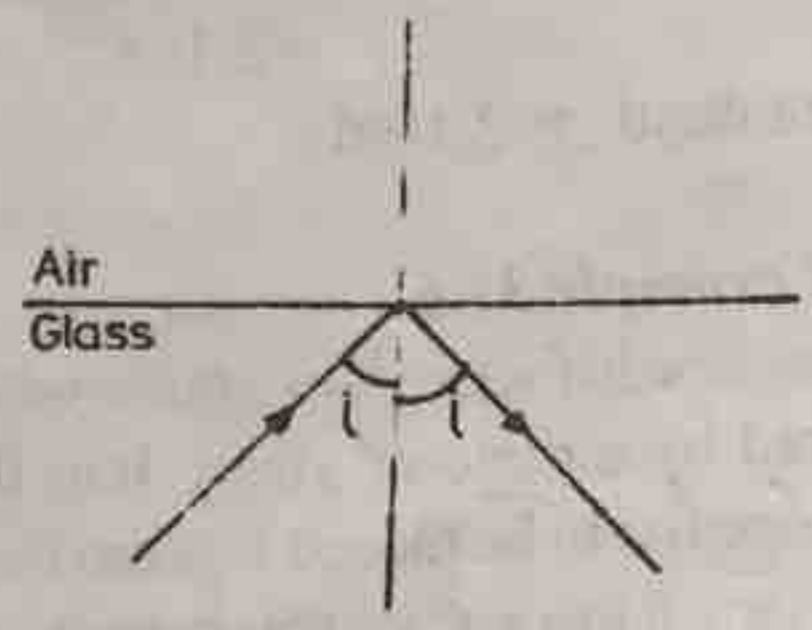

Fig. 31.5 Critical angle

The refractive index of air to glass = $\frac{\sin i}{\sin r}$

Since $\sin i = \sin c$, $\sin r = \sin 90^\circ$

$${}_a\mu_g = \frac{\sin 90}{\sin c}, \sin 90^\circ = 1$$

$${}_a\mu_g = \frac{1}{\sin c}$$

Total internal reflection: When a ray of light passes from optically denser medium to optically less dense medium, there exists a weak internal reflection and a strong refraction. But as the angle of incidence increases, the angle of refraction also increases and at the same time, the intensity of the reflected ray gets stronger and that of the refracted ray becomes weaker.

When the angle of incidence exceeds the critical angle there exists no refraction and a total reflection occurs This phenomeon is called *total internal reflection.*

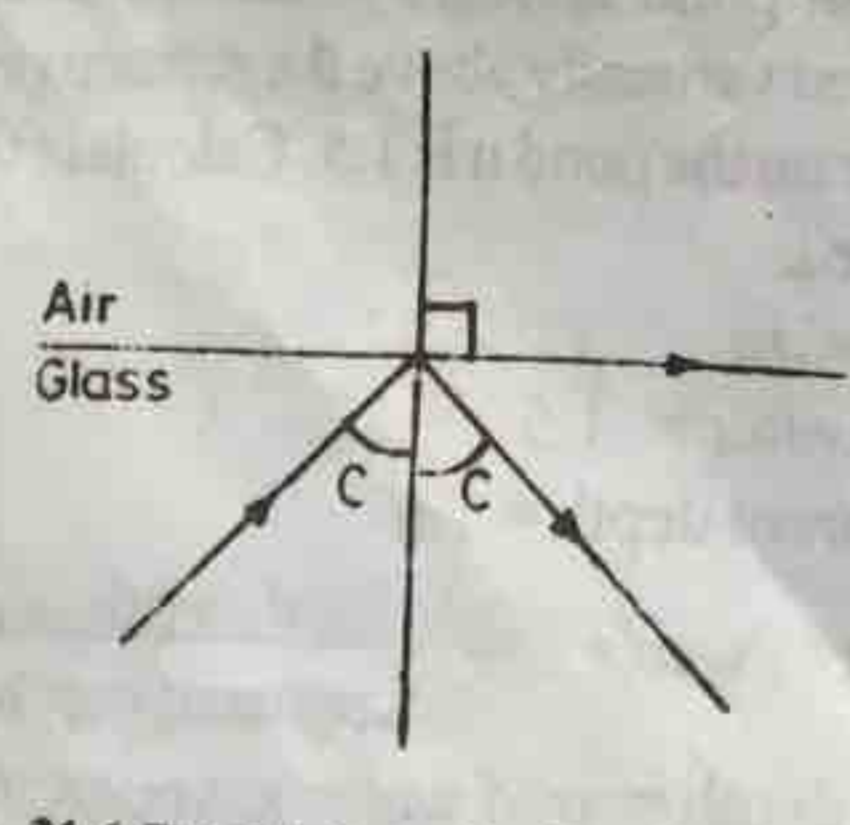

Fig. 31.6 Total internal reflection

The conditions under which total internal reflection occurs are:

(i) Light rays must travel through a denser medium to a less dense medium.

(ii) The angle of incidence must exceed critical angle.

31.8 Application of Total Internal Reflection

The principle of total internal reflection is adopted in the invention and operation of the following:

(i) Binoculars
(ii) Mirage
(iii) Refractometer
(iv) Optical fibres
(v) Total reflecting prisms are employed in periscopes, prism binoculars, projection lanterns and cameras.
(vi) Total internal reflection of radio waves is employed.

Optical fibres

Optical fibres have two parallel plane mirrors covering a transparent glass. The density of the transparent glass and the refractive index is greater than the density of the two mirrors and because of this ,the glass itself can totally and internally reflect rays from the object itself.

Optical fibres are used (i) in conveying information (ii) in medical laboratory to see deep down the throat of a patient.

Mirage

Temperature is a major determinant of mirage. The degree of hotness or coldness of a body varies from place to place. The temperature in the desert is different from that in the rain forest.

To a driver, driving on a road in a temperate region on a good sunny day, it could mean that the air very close to the ground is hot and in which the hotter air is optically less dense.

Rays from the sky passing through the cold air, on getting to the normal air layer bends easily from the incident direction into the layer of the air very close to the ground. At a point, what appears to the driver is like a pool of water in his front and anything placed beside it seems inverted.

Mirage is an optical illusion which is caused by total internal reflection of light separating two layers of air of different densities.

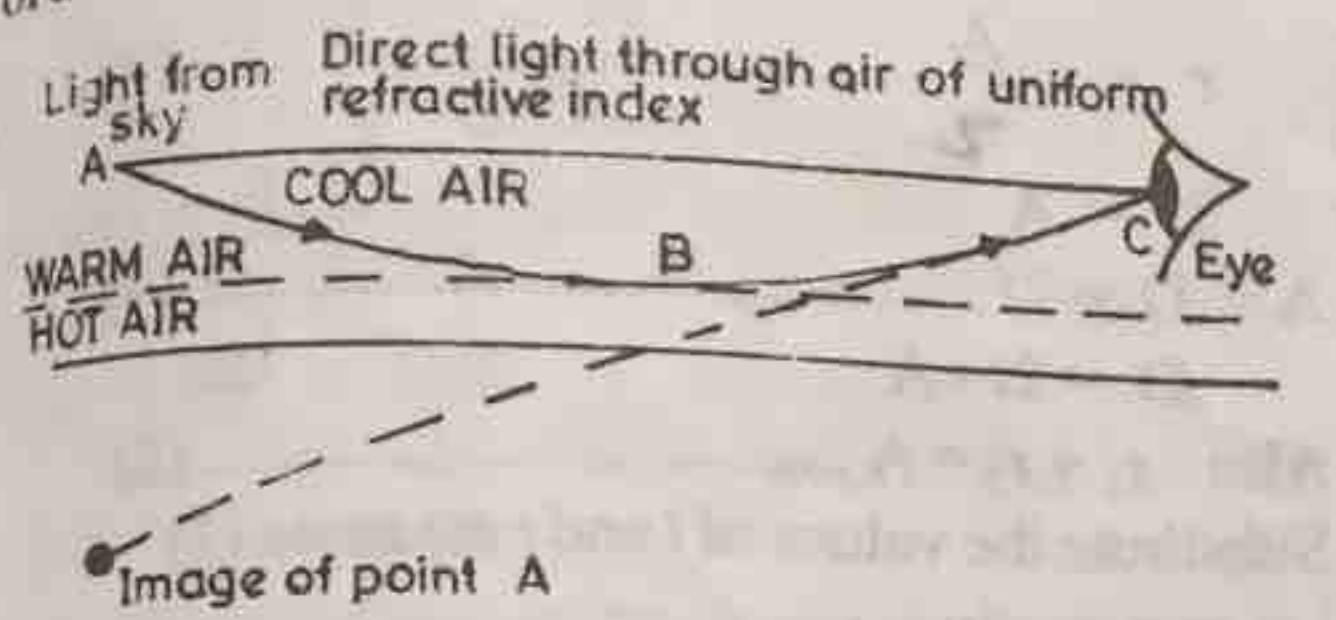

Fig. 31.7 Formation of a mirage

31.9 Field of View Under Water

A swimmer or a fish swimming with his back under water is liable to see objects above the water surface. Everything is seen from whatever depth the fish or the swimmer may be provided the water is not rippled. Rays refracted from the air angle of incidents of the rays are 90° each. The swimmer or the fish sees the reflection of objects from the sides and bottom of the water by total internal reflection.

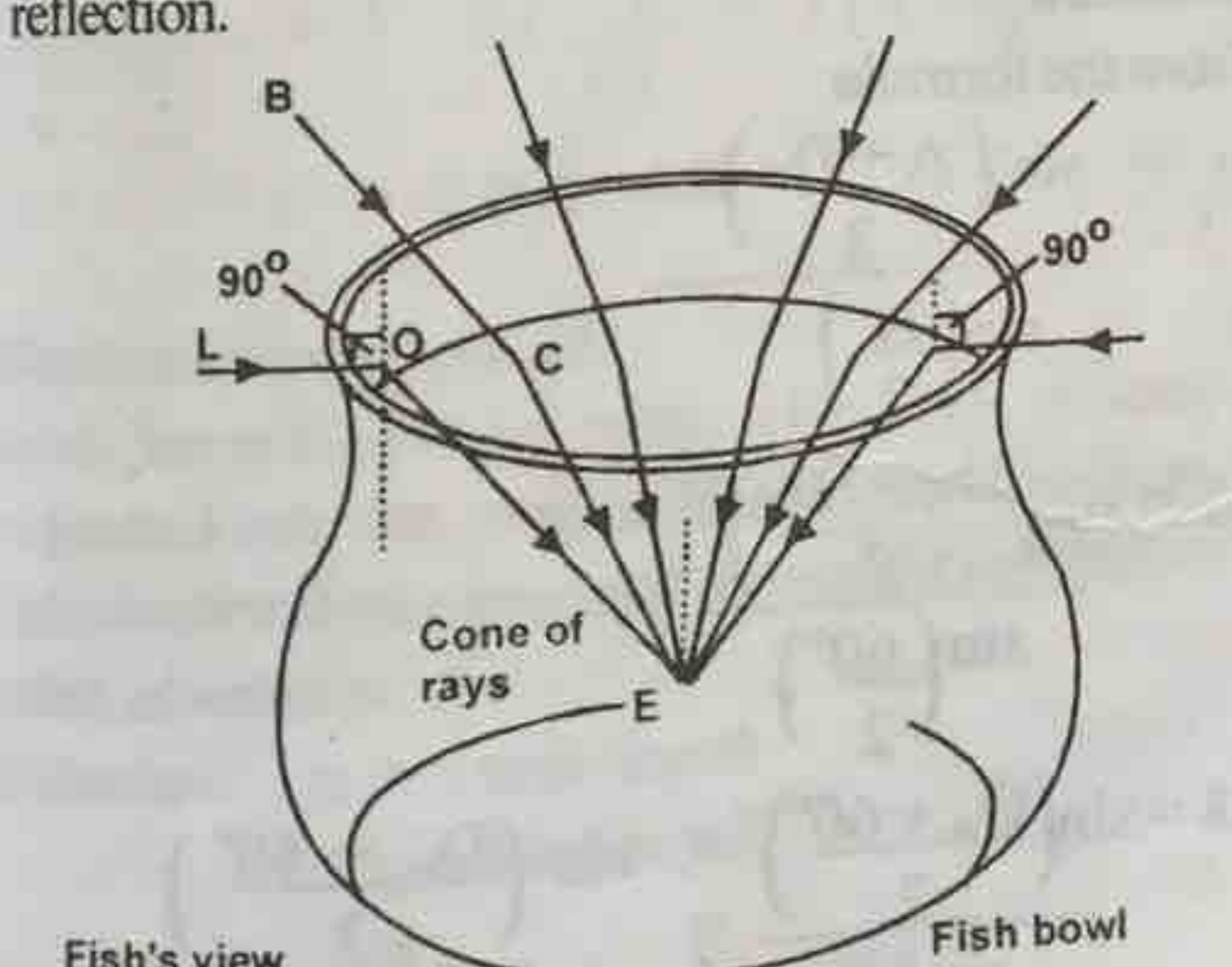

Fig. 31.8 Field of view under water

The angle of refraction of the swimmer or the fish in the water is *r* which is calculated.

The refractive index of air to water is 4/3, the incident angle is 90°.

$${}_a\mu_w = \frac{\sin i}{\sin r} \quad \text{since } {}_a\mu_w = \frac{4}{3}$$

$$\frac{4}{3} = \frac{\sin 90^\circ}{\sin r}$$

$$4 \sin r = 3 \sin 90^\circ, \quad \text{since } \sin 90^\circ = 1$$

$$4 \sin r = 3 \times (1)$$

$$\sin r = \frac{3}{4}$$

$$\sin r = 0.75$$

$$r = \sin^{-1}(0.75)$$

$$= 48.59$$

$$\simeq 49^\circ$$

Worked example 31.8

What will be the angle of incident when a ray of light passes through the air into the eye of a fish inside the river, if the angle of refraction is 49° to the normal. ${}_a\mu_w = 1.33$?

Solution

Angle of refraction $= 49^\circ$

Refractive index $= a\mu w = 1.33$

incident angle = ?

using this

$$a\mu g = \frac{\sin i}{\sin r}$$

$$1.33 = \frac{\sin i}{\sin 49^\circ} = \frac{\sin i}{0.75470}$$

$$\sin i = 1.33 \times 0.7499$$

$$= 1.00376$$

$$i = \sin^{-1}(1)$$

$$= 90^\circ$$

$$i = 90^\circ$$

31.10 Refraction Through Prism

There are two types of prisms namely;

(i) Rectangular prism

(ii) Triangular prism

Rays are produced from rectangular prism which has the angle of refraction either produced from the normal or produced to the normal, but rays transmitted through a triangular prism are always refracted towards the normal and close to the base of the triangular prism.

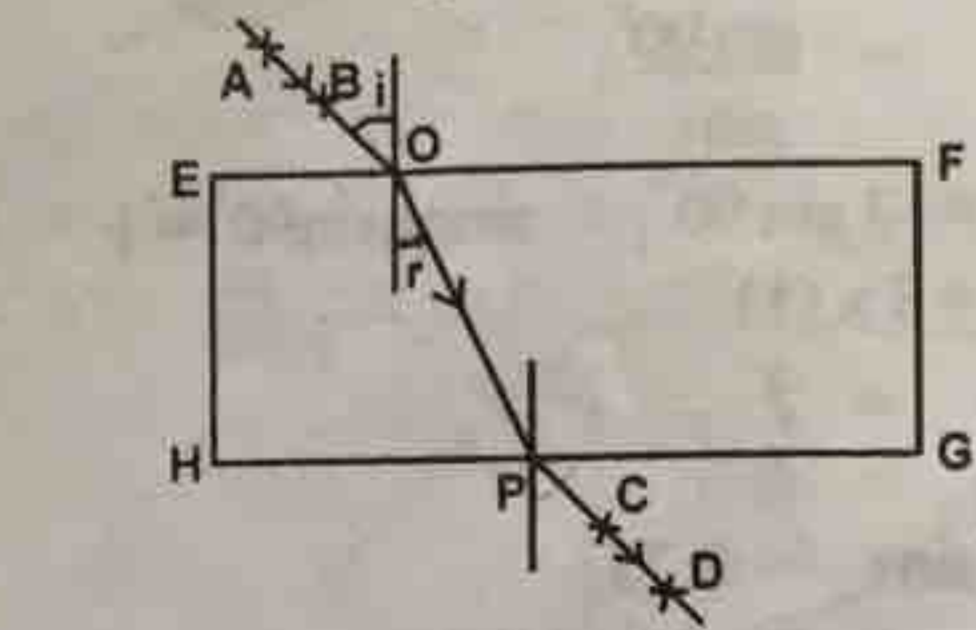

Fig. 31.9 Refraction by a rectangular prism

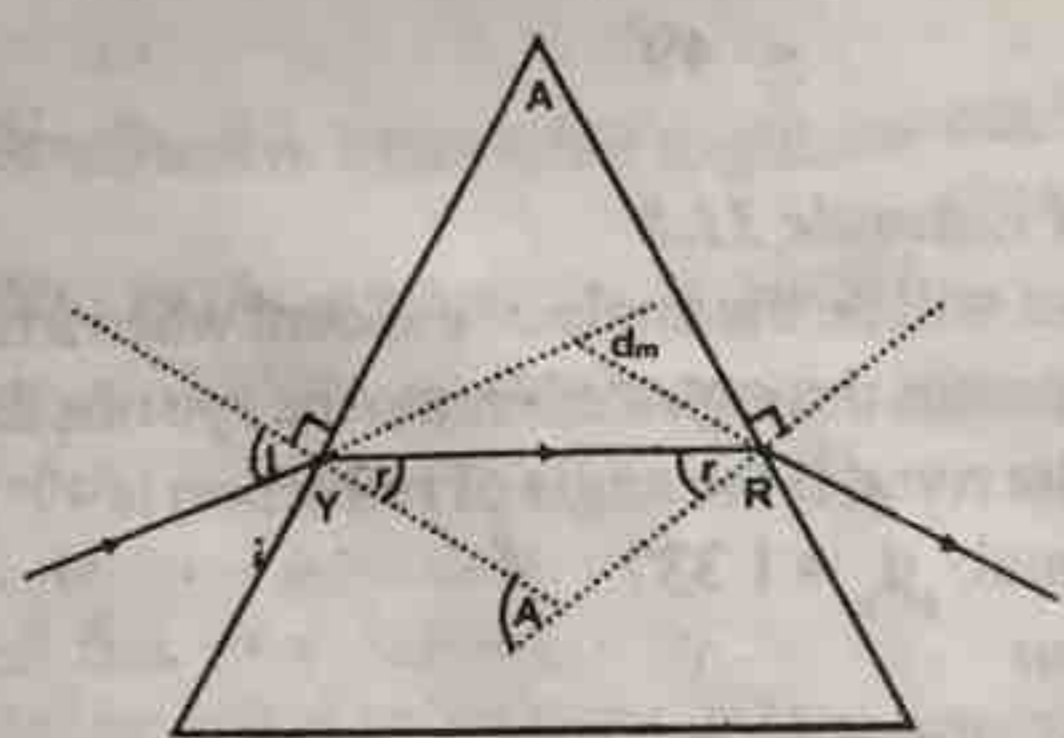

Fig. 31.10 Refraction by a triangular prism

(i) The angle of incidence, i
(ii) Angle of refraction, r
(iii) Angle of emergence, e
(iv) Minimum deviation, d_m
(v) Equilateral angle, A

$A = 60°$, if the prism is equilaterial.

NB: Minimum deviation, D_{min}, of the light occurs when the ray passes symmetrically through the prism.

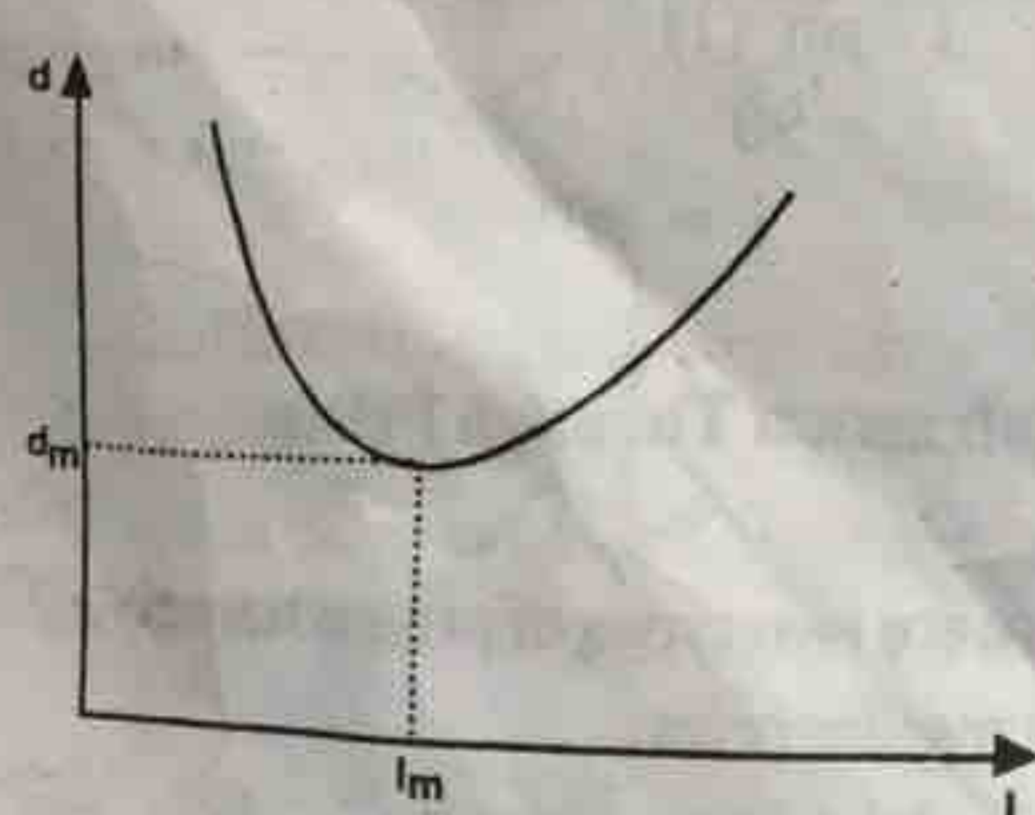

Fig. 31.11 Graph of angle of deviation against angle of incidence

Refractive index of equilateral triangular prism

$$a\mu g = \frac{\sin i}{\sin r} \quad \text{.......................(1)}$$

$$i = \frac{A + D}{2} \quad \text{.......................(2)}$$

$$r = \frac{A}{2} \quad \text{.......................(3)}$$

$$2r = A$$

$$A + D = 2i \quad \text{.......................(4)}$$

$$D = 2i - A$$

Also $r_1 + r_2 = A$ ....(5)

Substitute the values of i and r equation (1)

i.e $a\mu g = \frac{\sin i}{\sin r}$

$$_a\mu_g = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Worked example 31.9

A 60° prism of refractive index is 1.8, What is the angle of deviation (minimum)? Calculate the angle of refraction of the light passing through the prism at minimum deviation and the angle of incidence on the prism at minimum deviation.

Solution

From the formula

$$_a\mu_g = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$n = \frac{\sin\left(\frac{D_m + 60^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$1.8 = \frac{\sin\left(\frac{D_m + 60^\circ}{2}\right)}{\sin 30^\circ} = \frac{\sin\left(\frac{D_m + 60^\circ}{2}\right)}{0.5}$$

$$0.5 \times 1.8 = \sin\left(\frac{D_m}{2} + 30^\circ\right)$$

$$0.9 = \text{Sin}\left(\frac{D_m}{2} + \sin 30^\circ\right)$$