6 Capacitors and capacitance

At the end of this chapter you should be able to:

- describe an electrostatic field
- define electric field strength E and state its unit
- define capacitance and state its unit
- describe a capacitor and draw the circuit diagram symbol
- perform simple calculations involving $C = \dfrac{Q}{V}$ and $Q = It$
- define electric flux density D and state its unit
- define permittivity, distinguishing between ε_0, ε_r and ε
- perform simple calculations involving $D = \dfrac{Q}{A}$, $E = \dfrac{V}{D}$ and $\dfrac{D}{E} = \varepsilon_0\varepsilon_r$
- understand that for a parallel plate capacitor, $C = \dfrac{\varepsilon_0\varepsilon_r A(n-1)}{d}$
- perform calculations involving capacitors connected in parallel and in series
- define dielectric strength and state its unit
- state that the energy stored in a capacitor is given by $W = \frac{1}{2}\,\mathrm{CV}^2$ joules
- describe practical types of capacitor
- understand the precautions needed when discharging capacitors

6.1 Electrostatic field

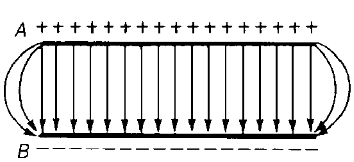

Figure 6.1 *Electrostatic field*

Figure 6.1 represents two parallel metal plates, A and B, charged to different potentials. If an electron that has a negative charge is placed between the plates, a force will act on the electron tending to push it away from the negative plate B towards the positive plate, A. Similarly, a positive charge would be acted on by a force tending to move it toward the negative plate. Any region such as that shown between the plates in Figure 1, in which an electric charge experiences a force, is called an **electrostatic field**. The direction of the field is defined as that of the force acting on a positive charge placed in the field. In Figure 6.1, the direction of the force is from the positive plate to the negative plate.

Such a field may be represented in magnitude and direction by **lines of electric force** drawn between the charged surfaces. The closeness of the lines is an indication of the field strength. Whenever a p.d. is established between two points, an electric field will always exist. Figure 6.2(a) shows a typical field pattern for an isolated point charge, and Figure 6.2(b) shows the field pattern for adjacent charges of opposite polarity. Electric lines of force (often called electric flux lines) are continuous and start and finish on point charges. Also, the lines cannot cross each other. When a charged body is placed close to an uncharged body, an induced charge of opposite sign appears on the surface of the uncharged body. This is because lines of force from the charged body terminate on its surface.

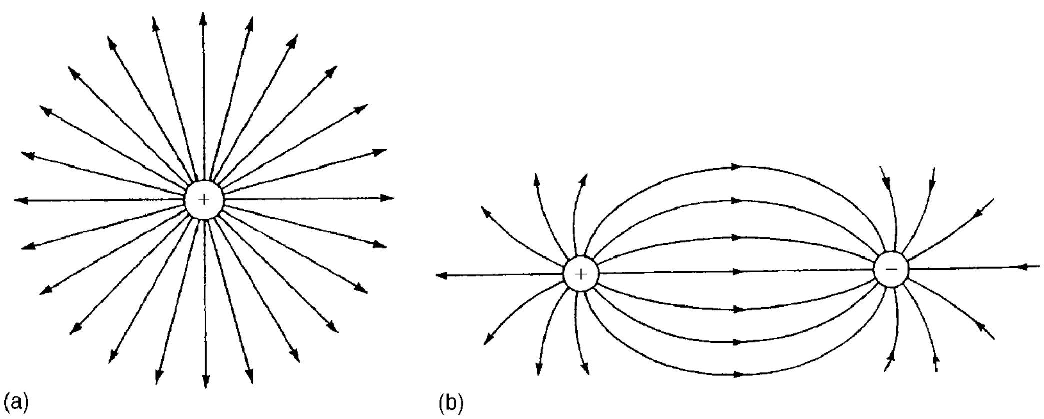

Figure 6.2 *(a) Isolated point charge; (b) adjacent charges of opposite polarity*

The concept of field lines or lines of force is used to illustrate the properties of an electric field. However, it should be remembered that they are only aids to the imagination.

The **force of attraction or repulsion** between two electrically charged bodies is proportional to the magnitude of their charges and inversely proportional to the square of the distance separating them,

i.e. force $\propto \dfrac{q_1 q_2}{d^2}$ or $\boxed{\text{force} = k\dfrac{q_1 q_2}{d^2}}$ where constant $k \approx 9 \times 10^9$ in air

This is known as **Coulomb's law**.

Hence the force between two charged spheres in air with their centres 16 mm apart and each carrying a charge of $+1.6\ \mu\text{C}$ is given by:

$$\text{force} = k\frac{q_1 q_2}{d^2} \approx (9 \times 10^9)\frac{(1.6 \times 10^{-6})^2}{(16 \times 10^{-3})^2} = \mathbf{90\ newtons}$$

6.2 Electric field strength

Figure 6.3 shows two parallel conducting plates separated from each other by air. They are connected to opposite terminals of a battery of voltage V volts.

There is therefore an electric field in the space between the plates. If the plates are close together, the electric lines of force will be straight and parallel and equally spaced, except near the edge where fringing will occur (see Figure 6.1). Over the area in which there is negligible fringing,

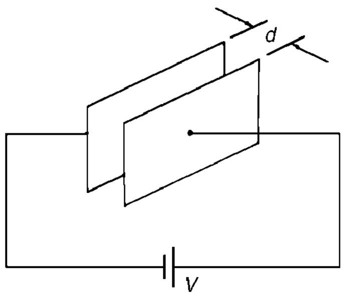

Figure 6.3

$$\textbf{Electric field strength, } E = \frac{V}{d} \textbf{ volts/metre}$$

where d is the distance between the plates. Electric field strength is also called **potential gradient**.

6.3 Capacitance

Static electric fields arise from electric charges, electric field lines beginning and ending on electric charges. Thus the presence of the field indicates the presence of equal positive and negative electric charges on the two plates of Figure 6.3. Let the charge be $+Q$ coulombs on one plate and $-Q$ coulombs on the other. The property of this pair of plates which determines how much charge corresponds to a given p.d. between the plates is called their capacitance:

$$\textbf{capacitance } C = \frac{Q}{V}$$

The **unit of capacitance** is the **farad F** (or more usually $\mu F = 10^{-6}$ F or $pF = 10^{-12}$ F), which is defined as the capacitance when a pd of one volt appears across the plates when charged with one coulomb.

6.4 Capacitors

Every system of electrical conductors possesses capacitance. For example, there is capacitance between the conductors of overhead transmission lines and also between the wires of a telephone cable. In these examples the capacitance is undesirable but has to be accepted, minimized or compensated for. There are other situations where capacitance is a desirable property.

Devices specially constructed to possess capacitance are called **capacitors** (or condensers, as they used to be called). In its simplest form a capacitor consists of two plates which are separated by an insulating material known as a **dielectric**. A capacitor has the ability to store a quantity of static electricity.

The symbols for a fixed capacitor and a variable capacitor used in electrical circuit diagrams are shown in Figure 6.4.

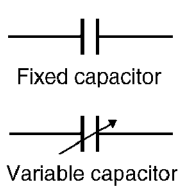

Figure 6.4

The **charge Q** stored in a capacitor is given by:

$$\boxed{Q = I \times t \text{ coulombs,}}$$

where I is the current in amperes and t the time in seconds.

> Problem 1. (a) Determine the p.d. across a 4 μF capacitor when charged with 5 mC.
> (b) Find the charge on a 50 pF capacitor when the voltage applied to it is 2 kV.

(a) $C = 4\ \mu\text{F} = 4 \times 10^{-6}$ F; $Q = 5$ mC $= 5 \times 10^{-3}$ C

Since $C = \dfrac{Q}{V}$ then $V = \dfrac{Q}{C} = \dfrac{5 \times 10^{-3}}{4 \times 10^{-6}} = \dfrac{5 \times 10^{6}}{4 \times 10^{3}} = \dfrac{5000}{4}$

Hence p.d. $=$ 1250 V or 1.25 kV

(b) $C = 50$ pF $= 50 \times 10^{-12}$ F; $V = 2$ kV $= 2000$ V

$Q = CV = 50 \times 10^{-12} \times 2000 = \dfrac{5 \times 2}{10^{8}} = 0.1 \times 10^{-6}$

Hence charge $=$ 0.1 μC

> Problem 2. A direct current of 4 A flows into a previously uncharged 20 μF capacitor for 3 ms. Determine the pd between the plates.

$I = 4$ A; $C = 20\ \mu\text{F} = 20 \times 10^{-6}$ F; $t = 3$ ms $= 3 \times 10^{-3}$ s

$Q = It = 4 \times 3 \times 10^{-3}$ C

$$V = \frac{Q}{C} = \frac{4 \times 3 \times 10^{-3}}{20 \times 10^{-6}} = \frac{12 \times 10^{6}}{20 \times 10^{3}} = 0.6 \times 10^{3} = 600 \text{ V}$$

Hence, the pd between the plates is 600 V

> Problem 3. A 5 μF capacitor is charged so that the pd between its plates is 800 V. Calculate how long the capacitor can provide an average discharge current of 2 mA.

$C = 5\ \mu\text{F} = 5 \times 10^{-6}$ F; $V = 800$ V; $I = 2$ mA $= 2 \times 10^{-3}$ A

$Q = CV = 5 \times 10^{-6} \times 800 = 4 \times 10^{-3}$ C

Also, $Q = It$. Thus, $t = \dfrac{Q}{I} = \dfrac{4 \times 10^{-3}}{2 \times 10^{-3}} = 2$ s

Hence the capacitor can provide an average discharge current of 2 mA for 2 s

Further problems on charge and capacitance may be found in Section 6.13, problems 1 to 5, page 70.

6.5 Electric flux density

Unit flux is defined as emanating from a positive charge of 1 coulomb. Thus electric flux Ψ is measured in coulombs, and for a charge of Q coulombs, the flux $\Psi = Q$ coulombs.

Electric flux density D is the amount of flux passing through a defined area A that is perpendicular to the direction of the flux:

$$\boxed{\textbf{electric flux density, } \boldsymbol{D} = \frac{\boldsymbol{Q}}{\boldsymbol{A}} \textbf{ coulombs/metre}^2}$$

Electric flux density is also called **charge density, $\boldsymbol{\sigma}$**

6.6 Permittivity

At any point in an electric field, the electric field strength E maintains the electric flux and produces a particular value of electric flux density D at that point. For a field established in **vacuum** (or for practical purposes in air), the ratio D/E is a constant ε_0, i.e.

$$\boxed{\frac{\boldsymbol{D}}{\boldsymbol{E}} = \boldsymbol{\varepsilon_0}}$$

where ε_0 is called the **permittivity of free space** or the free space constant. The value of ε_0 is 8.85×10^{-12} F/m.

When an insulating medium, such as mica, paper, plastic or ceramic, is introduced into the region of an electric field the ratio of D/E is modified:

$$\boxed{\frac{\boldsymbol{D}}{\boldsymbol{E}} = \boldsymbol{\varepsilon_0 \varepsilon_r}}$$

where ε_r, the **relative permittivity** of the insulating material, indicates its insulating power compared with that of vacuum:

$$\text{relative permittivity } \varepsilon_r = \frac{\text{flux density in material}}{\text{flux density in vacuum}}$$

ε_r has no unit. Typical values of ε_r include air, 1.00; polythene, 2.3; mica, 3–7; glass, 5–10; water, 80; ceramics, 6–1000.

The product $\varepsilon_0\varepsilon_r$ is called the **absolute permittivity**, ε, i.e.,

$$\boxed{\boldsymbol{\varepsilon = \varepsilon_0 \varepsilon_r}}$$

The insulating medium separating charged surfaces is called a **dielectric.** Compared with conductors, dielectric materials have very high resistivities. They are therefore used to separate conductors at different potentials, such as capacitor plates or electric power lines.

Problem 4. Two parallel rectangular plates measuring 20 cm by 40 cm carry an electric charge of 0.2 μC. Calculate the electric flux density. If the plates are spaced 5 mm apart and the voltage between them is 0.25 kV determine the electric field strength.

Charge $Q = 0.2\ \mu\text{C} = 0.2 \times 10^{-6} C$;

Area $A = 20\text{ cm} \times 40\text{ cm} = 800\text{ cm}^2 = 800 \times 10^{-4}\text{ m}^2$

$$\textbf{Electric flux density } \boldsymbol{D} = \frac{Q}{A} = \frac{0.2 \times 10^{-6}}{800 \times 10^{-4}} = \frac{0.2 \times 10^4}{800 \times 10^6}$$

$$= \frac{2000}{800} \times 10^{-6} = \mathbf{2.5\ \mu C/m^2}$$

Voltage $V = 0.25\text{ kV} = 250\text{ V}$; Plate spacing, $d = 5\text{ mm} = 5 \times 10^{-3}\text{ m}$

$$\textbf{Electric field strength } \boldsymbol{E} = \frac{V}{d} = \frac{250}{5 \times 10^{-3}} = \mathbf{50\ kV/m}$$

Problem 5. The flux density between two plates separated by mica of relative permittivity 5 is 2 μC/m². Find the voltage gradient between the plates.

Flux density $D = 2\ \mu\text{C/m}^2 = 2 \times 10^{-6}\text{ C/m}^2$;

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m; $\varepsilon_r = 5$.

$$\frac{D}{E} = \varepsilon_0\varepsilon_r, \text{ hence } \textbf{voltage gradient } \boldsymbol{E} = \frac{D}{\varepsilon_0\varepsilon_r}$$

$$= \frac{2 \times 10^{-6}}{8.85 \times 10^{-12} \times 5}\text{ V/m}$$

$$= \mathbf{45.2\ kV/m}$$

Problem 6. Two parallel plates having a pd of 200 V between them are spaced 0.8 mm apart. What is the electric field strength? Find also the flux density when the dielectric between the plates is (a) air, and (b) polythene of relative permittivity 2.3

$$\textbf{Electric field strength } \boldsymbol{E} = \frac{V}{D} = \frac{200}{0.8 \times 10^{-3}} = \mathbf{250\ kV/m}$$

(a) For air: $\varepsilon_r = 1$

$$\frac{D}{E} = \varepsilon_0\varepsilon_r. \quad \text{Hence}$$

electric flux density D $= E\varepsilon_0\varepsilon_r$

$= (250 \times 10^3 \times 8.85 \times 10^{-12} \times 1)\ \mathrm{C/m^2}$

$= \mathbf{2.213\ \mu C/m^2}$

(b) For polythene, $\varepsilon_r = 2.3$

Electric flux density D $= E\varepsilon_0\varepsilon_r$

$= (250 \times 10^3 \times 8.85 \times 10^{-12} \times 2.3)\ \mathrm{C/m^2}$

$= \mathbf{5.089\ \mu C/m^2}$

Further problems on electric field strength, electric flux density and permittivity may be found in Section 6.13, problems 6 to 10, page 71.

6.7 The parallel plate capacitor

For a parallel-plate capacitor, as shown in Figure 6.5(a), experiments show that capacitance C is proportional to the area A of a plate, inversely proportional to the plate spacing d (i.e., the dielectric thickness) and depends on the nature of the dielectric:

$$\boxed{\textbf{Capacitance, } C = \frac{\varepsilon_0\varepsilon_r A}{d} \textbf{ farads}}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m (constant)
ε_r = relative permittivity
A = area of one of the plates, in m^2, and
d = thickness of dielectric in m

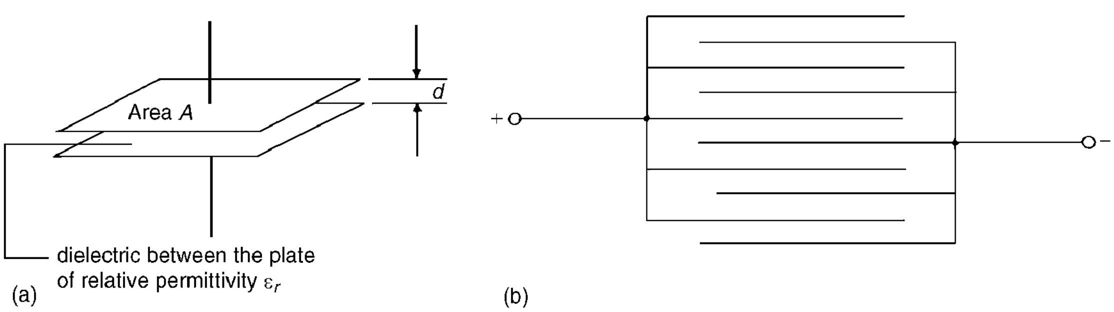

Figure 6.5

Another method used to increase the capacitance is to interleave several plates as shown in Figure 6.5(b). Ten plates are shown, forming nine capacitors with a capacitance nine times that of one pair of plates.

If such an arrangement has n plates then capacitance $C \propto (n - 1)$.

Thus capacitance $\boxed{\boldsymbol{C = \frac{\varepsilon_0 \varepsilon_r A(n-1)}{d}} \textbf{ farads}}$

Problem 7. (a) A ceramic capacitor has an effective plate area of 4 cm^2 separated by 0.1 mm of ceramic of relative permittivity 100. Calculate the capacitance of the capacitor in picofarads. (b) If the capacitor in part (a) is given a charge of 1.2 μC what will be the pd between the plates?

(a) Area A = 4 $cm^2 = 4 \times 10^{-4}$ m^2; $d = 0.1$ mm $= 0.1 \times 10^{-3}$ m;

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m; $\varepsilon_r = 100$

$$\textbf{Capacitance } \boldsymbol{C} = \frac{\varepsilon_0\ \varepsilon_r\ A}{d} \text{ farads}$$

$$= \frac{8.85 \times 10^{-12} \times 100 \times 4 \times 10^{-4}}{0.1 \times 10^{-3}} \text{ F}$$

$$= \frac{8.85 \times 4}{10^{10}} \text{ F} = \frac{8.85 \times 4 \times 10^{12}}{10^{10}} \text{ pF}$$

$$= \mathbf{3540\ pF}$$

(b) $Q = CV$ thus $\boldsymbol{V} = \dfrac{Q}{C} = \dfrac{1.2 \times 10^{-6}}{3540 \times 10^{-12}}$ $V = \mathbf{339\ V}$

Problem 8. A waxed paper capacitor has two parallel plates, each of effective area 800 cm^2. If the capacitance of the capacitor is 4425 pF determine the effective thickness of the paper if its relative permittivity is 2.5

$A = 800$ $cm^2 = 800 \times 10^{-4}$ $m^2 = 0.08$ m^2;

$C = 4425$ pF $= 4425 \times 10^{-12}$ F; $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m; $\varepsilon_r = 2.5$

Since $C = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$ then $d = \dfrac{\varepsilon_0 \varepsilon_r A}{C}$

Hence, $d = \dfrac{8.85 \times 10^{-12} \times 2.5 \times 0.08}{4425 \times 10^{-12}} = 0.0004$ m

Hence the thickness of the paper is 0.4 mm

Problem 9. A parallel plate capacitor has nineteen interleaved plates each 75 mm by 75 mm separated by mica sheets 0.2 mm thick. Assuming the relative permittivity of the mica is 5, calculate the capacitance of the capacitor.

$n = 19;\ n - 1 = 18;\ A = 75 \times 75 = 5625\ \text{mm}^2 = 5625 \times 10^{-6}\ \text{m}^2;$
$\varepsilon_r = 5;\ \varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m};\ d = 0.2\ \text{mm} = 0.2 \times 10^{-3}\ \text{m}$

$$\text{Capacitance } C = \frac{\varepsilon_0\, \varepsilon_r A(n-1)}{d}$$
$$= \frac{8.85 \times 10^{-12} \times 5 \times 5625 \times 10^{-6} \times 18}{0.2 \times 10^{-3}}\ \text{F}$$
$$= \mathbf{0.0224\ \mu F\ or\ 22.4\ nF}$$

Further problems on parallel plate capacitors may be found in Section 6.13, problems 11 to 17, page 71.

6.8 Capacitors connected in parallel and series

(a) Capacitors connected in parallel

Figure 6.6 shows three capacitors, C_1, C_2 and C_3, connected in parallel with a supply voltage V applied across the arrangement.

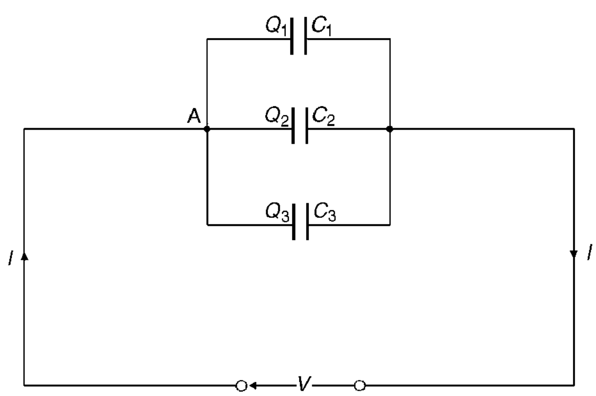

Figure 6.6

When the charging current I reaches point A it divides, some flowing into C_1, some flowing into C_2 and some into C_3. Hence the total charge $Q_T(= I \times t)$ is divided between the three capacitors. The capacitors each store a charge and these are shown as Q_1, Q_2 and Q_3 respectively. Hence

$$Q_T = Q_1 + Q_2 + Q_3$$

But $Q_T = CV,\ Q_1 = C_1V,\ Q_2 = C_2V$ and $Q_3 = C_3V$

Therefore $CV = C_1V + C_2V + C_3V$ where C is the total equivalent circuit capacitance,

i.e. $\boldsymbol{C = C_1 + C_2 + C_3}$

It follows that for n parallel-connected capacitors,

$$\boxed{C = C_1 + C_2 + C_3 \ldots + C_n,}$$

i.e. the equivalent capacitance of a group of parallel-connected capacitors is the sum of the capacitances of the individual capacitors. (Note that this formula is similar to that used for **resistors** connected in **series**)

(b) Capacitors connected in series

Figure 6.7 shows three capacitors, C_1, C_2 and C_3, connected in series across a supply voltage V. Let the p.d. across the individual capacitors be V_1, V_2 and V_3 respectively as shown.

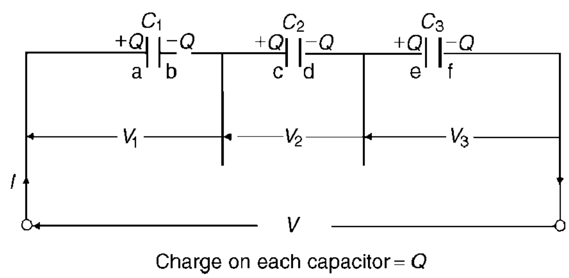

Figure 6.7

Let the charge on plate ‘a’ of capacitor C_1 be $+Q$ coulombs. This induces an equal but opposite charge of $-Q$ coulombs on plate ‘b’. The conductor between plates ‘b’ and ‘c’ is electrically isolated from the rest of the circuit so that an equal but opposite charge of $+Q$ coulombs must appear on plate ‘c’, which, in turn, induces an equal and opposite charge of $-Q$ coulombs on plate ‘d’, and so on.
Hence when capacitors are connected in series the charge on each is the same.

In a series circuit: $V = V_1 + V_2 + V_3$

Since $V = \dfrac{Q}{C}$ then $\dfrac{Q}{C} = \dfrac{Q}{C_1} + \dfrac{Q}{C_2} + \dfrac{Q}{C_3}$

where C is the total equivalent circuit capacitance,

i.e. $\dfrac{\mathbf{1}}{\mathbf{C}} = \dfrac{\mathbf{1}}{\mathbf{C_1}} + \dfrac{\mathbf{1}}{\mathbf{C_2}} + \dfrac{\mathbf{1}}{\mathbf{C_3}}$

It follows that for n series-connected capacitors:

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n},}$$

i.e. for series-connected capacitors, the reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances. (Note that this formula is similar to that used for **resistors** connected in **parallel**)

For the special case of **two capacitors in series:**

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1\, C_2}$$

$$\text{Hence } \boxed{\boldsymbol{C = \frac{C_1 C_2}{C_1 + C_2}}} \quad \left(\text{i.e. } \frac{\text{product}}{\text{sum}}\right)$$

> Problem 10. Calculate the equivalent capacitance of two capacitors of 6 μF and 4 μF connected (a) in parallel and (b) in series

(a) In parallel, equivalent capacitance $\boldsymbol{C} = C_1 + C_2 = 6\ \mu\text{F} + 4\ \mu\text{F} =$ **10 μF**

(b) In series, equivalent capacitance C is given by: $C = \dfrac{C_1 C_2}{C_1 + C_2}$

This formula is used for the special case of **two** capacitors in series.

Thus $\boldsymbol{C} = \dfrac{6 \times 4}{6 + 4} = \dfrac{24}{10} = \mathbf{2.4\ \mu F}$

> Problem 11. What capacitance must be connected in series with a 30 μF capacitor for the equivalent capacitance to be 12 μF?

Let $C = 12\ \mu\text{F}$ (the equivalent capacitance), $C_1 = 30\ \mu\text{F}$ and C_2 be the unknown capacitance.

For two capacitors in series $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

Hence $\dfrac{1}{C_2} = \dfrac{1}{C} - \dfrac{1}{C_1} = \dfrac{C_1 - C}{C C_1}$

and $\boldsymbol{C_2} = \dfrac{C C_1}{C_1 - C} = \dfrac{12 \times 30}{30 - 12} = \dfrac{360}{18} = \mathbf{20\ \mu F}$

> Problem 12. Capacitances of 1 μF, 3 μF, 5 μF and 6 μF are connected in parallel to a direct voltage supply of 100 V. Determine (a) the equivalent circuit capacitance, (b) the total charge and (c) the charge on each capacitor.

(a) The equivalent capacitance C for four capacitors in parallel is given by:

$$C = C_1 + C_2 + C_3 + C_4$$

i.e. $\boldsymbol{C} = 1 + 3 + 5 + 6 = \mathbf{15\ \mu F}$

(b) Total charge $Q_T = CV$ where C is the equivalent circuit capacitance

i.e. $\boldsymbol{Q_T} = 15 \times 10^{-6} \times 100 = 1.5 \times 10^{-3}\ C = \mathbf{1.5\ mC}$

(c) The charge on the 1 μF capacitor $\boldsymbol{Q_1} = C_1V = 1 \times 10^{-6} \times 100 = \mathbf{0.1\ mC}$

The charge on the 3 μF capacitor $\boldsymbol{Q_2} = C_2V = 3 \times 10^{-6} \times 100 = \mathbf{0.3\ mC}$

The charge on the 5 μF capacitor $\boldsymbol{Q_3} = C_3V = 5 \times 10^{-6} \times 100 = \mathbf{0.5\ mC}$

The charge on the 6 μF capacitor $\boldsymbol{Q_4} = C_4V = 6 \times 10^{-6} \times 100 = \mathbf{0.6\ mC}$

[Check: In a parallel circuit $Q_T = Q_1 + Q_2 + Q_3 + Q_4$

$Q_1 + Q_2 + Q_3 + Q_4 = 0.1 + 0.3 + 0.5 + 0.6 = 1.5\ \text{mC} = Q_T$]

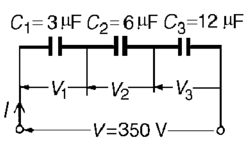

Figure 6.8

Problem 13. Capacitances of 3 μF, 6 μF and 12 μF are connected in series across a 350 V supply. Calculate (a) the equivalent circuit capacitance, (b) the charge on each capacitor and (c) the pd across each capacitor.

The circuit diagram is shown in Figure 6.8.

(a) The equivalent circuit capacitance C for three capacitors in series is given by:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

i.e. $$\frac{1}{C} = \frac{1}{3} + \frac{1}{6} + \frac{1}{12} = \frac{4+2+1}{12} = \frac{7}{12}$$

Hence the equivalent circuit capacitance $\boldsymbol{C} = \mathbf{\frac{12}{7} = 1\frac{5}{7}\ \mu F}$

(b) Total charge $Q_T = CV$,

hence $$Q_T = \frac{12}{7} \times 10^{-6} \times 350 = 600\ \mu\text{C or } 0.6\ \text{mC}$$

Since the capacitors are connected in series 0.6 mC is the charge on each of them.

(c) The voltage across the 3 μF capacitor, $V_1 = \dfrac{Q}{C_1} = \dfrac{0.6 \times 10^{-3}}{3 \times 10^{-6}} = \mathbf{200\ V}$

The voltage across the 6 μF capacitor, $V_2 = \frac{Q}{C_2} = \frac{0.6 \times 10^{-3}}{6 \times 10^{-6}}$

$= \mathbf{100\ V}$

The voltage across the 12 μF capacitor, $V_3 = \frac{Q}{C_3} = \frac{0.6 \times 10^{-3}}{12 \times 10^{-6}}$

$= \mathbf{50\ V}$

[Check: In a series circuit $V = V_1 + V_2 + V_3$

$V_1 + V_2 + V_3 = 200 + 100 + 50 = 350\ V =$ supply voltage.]

In practice, capacitors are rarely connected in series unless they are of the same capacitance. The reason for this can be seen from the above problem where the lowest valued capacitor (i.e. 3 μF) has the highest pd across it (i.e. 200 V) which means that if all the capacitors have an identical construction they must all be rated at the highest voltage.

Further problems on capacitors in parallel and series may be found in Section 6.13, problems 18 to 25, page 72.

6.9 Dielectric strength

The maximum amount of field strength that a dielectric can withstand is called the dielectric strength of the material.

Dielectric strength, $\boxed{E_m = \frac{V_m}{d}}$

Problem 14. A capacitor is to be constructed so that its capacitance is 0.2 μF and to take a p.d. of 1.25 kV across its terminals. The dielectric is to be mica which, after allowing a safety factor of 2, has a dielectric strength of 50 MV/m. Find (a) the thickness of the mica needed, and (b) the area of a plate assuming a two-plate construction. (Assume ε_r for mica to be 6)

(a) Dielectric strength, $E = \frac{V}{d}$, i.e. $d = \frac{V}{E} = \frac{1.25 \times 10^3}{50 \times 10^6}$ m

$= \mathbf{0.025\ mm}$

(b) Capacitance, $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$, hence area

$$A = \frac{Cd}{\varepsilon_0 \varepsilon_r} = \frac{0.2 \times 10^{-6} \times 0.025 \times 10^{-3}}{8.85 \times 10^{-12} \times 6}\ \text{m}^2$$

$= 0.09416\ \text{m}^2 = \mathbf{941.6\ cm^2}$

6.10 Energy stored

The energy, W, stored by a capacitor is given by

$$\boxed{W = \tfrac{1}{2} CV^2 \text{ joules}}$$

Problem 15. (a) Determine the energy stored in a 3 μF capacitor when charged to 400 V. (b) Find also the average power developed if this energy is dissipated in a time of 10 μs

(a) **Energy stored** $W = \frac{1}{2}\,\mathrm{C}\,\mathrm{V}^2$ joules

$$= \tfrac{1}{2} \times 3 \times 10^{-6} \times 400^2 = \tfrac{3}{2} \times 16 \times 10^{-2}$$

$$= \mathbf{0.24\ J}$$

(b) **Power** $= \dfrac{\text{Energy}}{\text{time}} = \dfrac{0.24}{10 \times 10^{-6}}\ \mathrm{W} = \mathbf{24\ kW}$

Problem 16. A 12 μF capacitor is required to store 4 J of energy. Find the pd to which the capacitor must be charged.

Energy stored $W = \dfrac{1}{2}\,CV^2$ hence $V^2 = \dfrac{2W}{C}$

and $V = \sqrt{\left(\dfrac{2W}{C}\right)} = \sqrt{\left(\dfrac{2 \times 4}{12 \times 10^{-6}}\right)} = \sqrt{\left(\dfrac{2 \times 10^6}{3}\right)} = \mathbf{816.5\ V}$

Problem 17. A capacitor is charged with 10 mC. If the energy stored is 1.2 J find (a) the voltage and (b) the capacitance.

Energy stored $W = \dfrac{1}{2}\,CV^2$ and $C = \dfrac{Q}{V}$

Hence $\quad W = \dfrac{1}{2}\left(\dfrac{Q}{V}\right)V^2 = \dfrac{1}{2}\,QV$

from which $\quad V = \dfrac{2W}{Q}$

$Q = 10$ mc $= 10 \times 10^{-3}\ C$ and $W = 1.2$ J

(a) Voltage $V = \dfrac{2W}{Q} = \dfrac{2 \times 1.2}{10 \times 10^{-3}} = \mathbf{0.24\ kV\ or\ 240\ V}$

(b) Capacitance $C = \dfrac{Q}{V} = \dfrac{10 \times 10^{-3}}{240}\ \mathrm{F} = \dfrac{10 \times 10^6}{240 \times 10^3}\ \mu\mathrm{F} = \mathbf{41.67\ \mu F}$

Further problems on energy stored may be found in Section 6.13, problems 26 to 30, page 73.

6.11 Practical types of capacitor

Practical types of capacitor are characterized by the material used for their dielectric. The main types include: variable air, mica, paper, ceramic, plastic, titanium oxide and electrolytic.

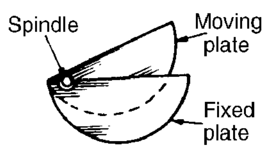

Figure 6.9

1 **Variable air capacitors.** These usually consist of two sets of metal plates (such as aluminium) one fixed, the other variable. The set of moving plates rotate on a spindle as shown by the end view of Figure 6.9.

As the moving plates are rotated through half a revolution, the meshing, and therefore the capacitance, varies from a minimum to a maximum value. Variable air capacitors are used in radio and electronic circuits where very low losses are required, or where a variable capacitance is needed. The maximum value of such capacitors is between 500 pF and 1000 pF.

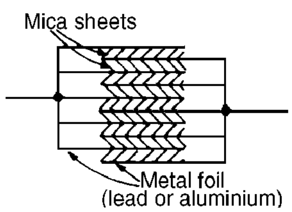

Figure 6.10

2 **Mica capacitors.** A typical older type construction is shown in Figure 6.10.

Usually the whole capacitor is impregnated with wax and placed in a bakelite case. Mica is easily obtained in thin sheets and is a good insulator. However, mica is expensive and is not used in capacitors above about 0.2 μF. A modified form of mica capacitor is the silvered mica type. The mica is coated on both sides with a thin layer of silver which forms the plates. Capacitance is stable and less likely to change with age. Such capacitors have a constant capacitance with change of temperature, a high working voltage rating and a long service life and are used in high frequency circuits with fixed values of capacitance up to about 1000 pF.

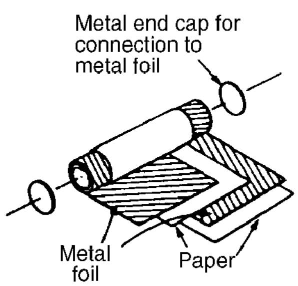

Figure 6.11

3 **Paper capacitors.** A typical paper capacitor is shown in Figure 6.11 where the length of the roll corresponds to the capacitance required. The whole is usually impregnated with oil or wax to exclude moisture, and then placed in a plastic or aluminium container for protection. Paper capacitors are made in various working voltages up to about 150 kV and are used where loss is not very important. The maximum value of this type of capacitor is between 500 pF and 10 μF. Disadvantages of paper capacitors include variation in capacitance with temperature change and a shorter service life than most other types of capacitor.

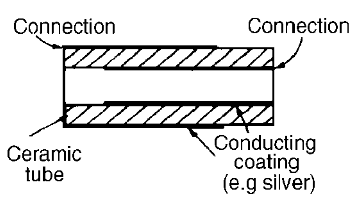

Figure 6.12

4 **Ceramic capacitors.** These are made in various forms, each type of construction depending on the value of capacitance required. For high values, a tube of ceramic material is used as shown in the cross section of Figure 6.12. For smaller values the cup construction is used as shown in Figure 6.13, and for still smaller values the disc construction shown in Figure 6.14 is used. Certain ceramic materials have a very high permittivity and this enables capacitors of high capacitance to be made which are of small physical size with a high working voltage

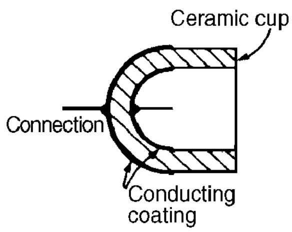

Figure 6.13

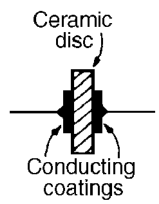

Figure 6.14

rating. Ceramic capacitors are available in the range 1 pF to 0.1 μF and may be used in high frequency electronic circuits subject to a wide range of temperatures.

5 **Plastic capacitors.** Some plastic materials such as polystyrene and Teflon can be used as dielectrics. Construction is similar to the paper capacitor but using a plastic film instead of paper. Plastic capacitors operate well under conditions of high temperature, provide a precise value of capacitance, a very long service life and high reliability.

6 **Titanium oxide capacitors** have a very high capacitance with a small physical size when used at a low temperature.

7 **Electrolytic capacitors.** Construction is similar to the paper capacitor with aluminium foil used for the plates and with a thick absorbent material, such as paper, impregnated with an electrolyte (ammonium borate), separating the plates. The finished capacitor is usually assembled in an aluminium container and hermetically sealed. Its operation depends on the formation of a thin aluminium oxide layer on the positive plate by electrolytic action when a suitable direct potential is maintained between the plates. This oxide layer is very thin and forms the dielectric. (The absorbent paper between the plates is a conductor and does not act as a dielectric.) Such capacitors **must always be used on dc** and must be connected with the correct polarity; if this is not done the capacitor will be destroyed since the oxide layer will be destroyed. Electrolytic capacitors are manufactured with working voltage from 6 V to 600 V, although accuracy is generally not very high. These capacitors possess a much larger capacitance than other types of capacitors of similar dimensions due to the oxide film being only a few microns thick. The fact that they can be used only on dc supplies limit their usefulness.

6.12 Discharging capacitors

When a capacitor has been disconnected from the supply it may still be charged and it may retain this charge for some considerable time. Thus precautions must be taken to ensure that the capacitor is automatically discharged after the supply is switched off. This is done by connecting a high value resistor across the capacitor terminals.

6.13 Further problems on capacitors and capacitance

(Where appropriate take ε_0 as 8.85×10^{-12} F/m)

Charge and capacitance

1 Find the charge on a 10 μF capacitor when the applied voltage is 250 V. [2.5 mC]

2 Determine the voltage across a 1000 pF capacitor to charge it with 2 μC. [2 kV]

3 The charge on the plates of a capacitor is 6 mC when the potential between them is 2.4 kV. Determine the capacitance of the capacitor. [2.5 μF]

4 For how long must a charging current of 2 A be fed to a 5 μF capacitor to raise the pd between its plates by 500 V. [1.25 ms]

5 A steady current of 10 A flows into a previously uncharged capacitor for 1.5 ms when the pd between the plates is 2 kV. Find the capacitance of the capacitor. [7.5 μF]

Electric field strength, electric flux density and permittivity

6 A capacitor uses a dielectric 0.04 mm thick and operates at 30 V. What is the electric field strength across the dielectric at this voltage? [750 kV/m]

7 A two-plate capacitor has a charge of 25 C. If the effective area of each plate is 5 cm^2 find the electric flux density of the electric field. [50 kC/m^2]

8 A charge of 1.5 μC is carried on two parallel rectangular plates each measuring 60 mm by 80 mm. Calculate the electric flux density. If the plates are spaced 10 mm apart and the voltage between them is 0.5 kV determine the electric field strength. [312.5 $\mu C/m^2$, 50 kV/m]

9 The electric flux density between two plates separated by polystyrene of relative permittivity 2.5 is 5 $\mu C/m^2$. Find the voltage gradient between the plates. [226 kV/m]

10 Two parallel plates having a pd of 250 V between them are spaced 1 mm apart. Determine the electric field strength. Find also the electric flux density when the dielectric between the plates is (a) air and (b) mica of relative permittivity 5. [250 kV/m (a) 2.213 $\mu C/m^2$ (b) 11.063 $\mu C/m^2$]

Parallel plate capacitor

11 A capacitor consists of two parallel plates each of area 0.01 m^2, spaced 0.1 mm in air. Calculate the capacitance in picofarads. [885 pF]

12 A waxed paper capacitor has two parallel plates, each of effective area 0.2 m^2. If the capacitance is 4000 pF determine the effective thickness of the paper if its relative permittivity is 2. [0.885 mm]

13 Calculate the capacitance of a parallel plate capacitor having 5 plates, each 30 mm by 20 mm and separated by a dielectric 0.75 mm thick having a relative permittivity of 2.3 [65.14 pF]

14 How many plates has a parallel plate capacitor having a capacitance of 5 nF, if each plate is 40 mm by 40 mm and each dielectric is 0.102 mm thick with a relative permittivity of 6. [7]

15 A parallel plate capacitor is made from 25 plates, each 70 mm by 120 mm interleaved with mica of relative permittivity 5. If the capacitance of the capacitor is 3000 pF determine the thickness of the mica sheet. [2.97 mm]

16 The capacitance of a parallel plate capacitor is 1000 pF. It has 19 plates, each 50 mm by 30 mm separated by a dielectric of thickness 0.40 mm. Determine the relative permittivity of the dielectric. [1.67]

17 A capacitor is to be constructed so that its capacitance is 4250 pF and to operate at a pd of 100 V across its terminals. The dielectric is to be polythene ($\varepsilon_r = 2.3$) which, after allowing a safety factor, has a dielectric strength of 20 MV/m. Find (a) the thickness of the polythene needed, and (b) the area of a plate.
[(a) 0.005 mm (b) 10.44 cm^2]

Capacitors in parallel and series

18 Capacitors of 2 μF and 6 μF are connected (a) in parallel and (b) in series. Determine the equivalent capacitance in each case.
[(a) 8 μF (b) 1.5 μF]

19 Find the capacitance to be connected in series with a 10 μF capacitor for the equivalent capacitance to be 6 μF [15 μF]

20 Two 6 μF capacitors are connected in series with one having a capacitance of 12 μF. Find the total equivalent circuit capacitance. What capacitance must be added in series to obtain a capacitance of 1.2 μF? [2.4 μF, 2.4 μF]

21 Determine the equivalent capacitance when the following capacitors are connected (a) in parallel and (b) in series:
(i) 2 μF, 4 μF and 8 μF
(ii) 0.02 μF, 0.05 μF and 0.10 μF
(iii) 50 pF and 450 pF
(iv) 0.01 μF and 200 pF
[(a) (i) 14 μF (ii) 0.17 μF (iii) 500 pF (iv) 0.0102 μF
(b) (i) $1\frac{1}{7}$ μF (ii) 0.0125 μF (iii) 45 pF (iv) 196.1 pF]

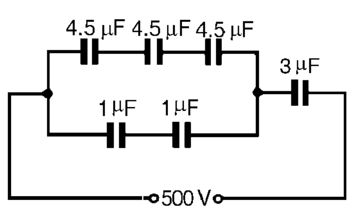

Figure 6.15

22 For the arrangement shown in Figure 6.15 find (a) the equivalent circuit capacitance and (b) the voltage across a 4.5 μF capacitor.
[(a) 1.2 μF (b) 100 V]

23 Three 12 μF capacitors are connected in series across a 750 V supply. Calculate (a) the equivalent capacitance, (b) the charge on each capacitor and (c) the pd across each capacitor.
[(a) 4 μF (b) 3 mC (c) 250 V]

24 If two capacitors having capacitances of 3 μF and 5 μF respectively are connected in series across a 240 V supply, determine (a) the p.d. across each capacitor and (b) the charge on each capacitor.
[(a) 150 V, 90 V (b) 0.45 mC on each]

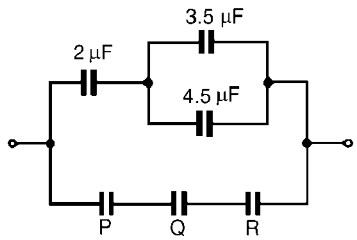

Figure 6.16

25 In Figure 6.16 capacitors P, Q and R are identical and the total equivalent capacitance of the circuit is 3 μF. Determine the values of P, Q and R [4.2 μF each]

Energy stored

26 When a capacitor is connected across a 200 V supply the charge is 4 μC. Find (a) the capacitance and (b) the energy stored. [(a) 0.02 μF (b) 0.4 mJ]

27 Find the energy stored in a 10 μF capacitor when charged to 2 kV. [20 J]

28 A 3300 pF capacitor is required to store 0.5 mJ of energy. Find the pd to which the capacitor must be charged. [550 V]

29 A capacitor, consisting of two metal plates each of area 50 cm^2 and spaced 0.2 mm apart in air, is connected across a 120 V supply. Calculate (a) the energy stored, (b) the electric flux density and (c) the potential gradient
[(a) 1.593 μJ (b) 5.31 $\mu C/m^2$ (c) 600 kV/m]

30 A bakelite capacitor is to be constructed to have a capacitance of 0.04 μF and to have a steady working potential of 1 kV maximum. Allowing a safe value of field stress of 25 MV/m find (a) the thickness of bakelite required, (b) the area of plate required if the relative permittivity of bakelite is 5, (c) the maximum energy stored by the capacitor and (d) the average power developed if this energy is dissipated in a time of 20 μs.
[(a) 0.04 mm (b) 361.6 cm^2 (c) 0.02 J (d) 1 kW]