

Light

It is a visible spectrum

It is a form of energy

It is also a transverse wave

It falls in d visible part of the spectrum

Has a wavelength between 4×10^{-7} to 8×10^{-7} m

Principle

Reflection of light from a smooth surface is called Specular reflection

Reflection from any rough surface is called difuse reflection

* A surface behaves as a smooth surface as long as the wavelengths are much smaller than the wavelength of the incident light

Characteristics of image formed by a plane mirror

* It is virtual as it can't be formed on a screen

* It is the same size as the object

* A virtual object (convergent beam) gives rise to a real image

* A real object (divergent beam) gives rise to a virtual image

* Object and image are at equal distance from the mirror

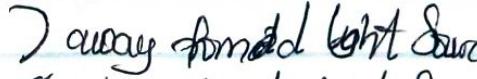
Reflection of curved mirrors

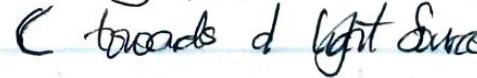
* Used in driving mirrors in cars

* Makeup and dentist mirrors are curved mirrors

* Curved mirrors are part of spherical surfaces

2. Types

* Concave - bulges inward as in 

* Convex - bulges outward as in 

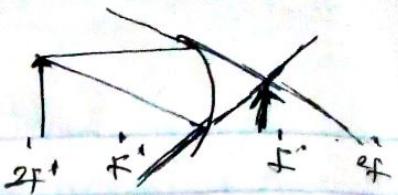
Convex

* They are not used to focus light as they reflect light outwards

* They always form a virtual image since the focal point (and the centre of curvature ($2f$)) are both imaginary points inside of mirror of which can't be reacted

As a result, such images can't be projected on a Screen

Images formed are virtual, upright, Reduced



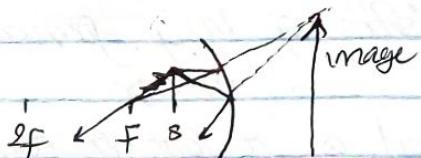
Concave

- * Has a reflecting surface that bulges ~~outward~~ (away from the incident light)
- * They are used to focus light
- * They show different image types depending on the distance b/w object and mirror

Object position = S

Focal length = F

when $S < F$ i.e.



* Image is virtual

* Image is erect / upright

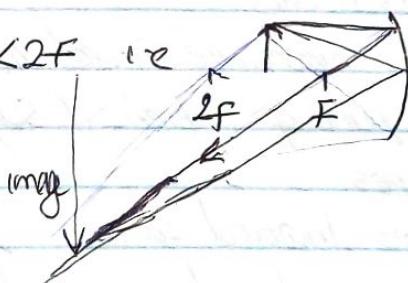
* Image is magnified

when $S = F$ i.e.



* Reflected rays are parallel so they do not meet

when $F < S$ but $S < 2F$ i.e.



* Image is real

* Image is magnified

* Image is inverted

when $S = 2F$

i.e.

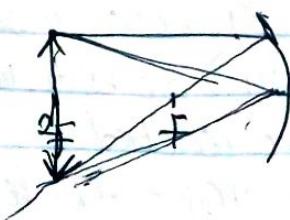


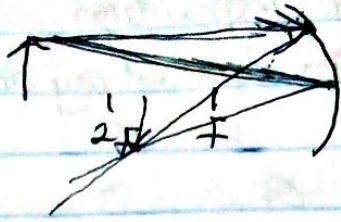
image is real

image is inverted

same size

formed at the centre of curvature

When $S > 2F$



* Image is real

* Image is diminished

* Image is inverted

* As the object distance increases, the image approaches the focal point

* In d limit where S approaches infinity, image size approaches Zero

* Where the image is formed on the mirror is known as d aperture

Sine Rule

* A real object or image distance is a positive distance.

* A virtual object or image distance is a negative distance.

* Focal length of a concave mirror is a positive distance.

* Focal length of a Convex mirror is a negative distance.

$$\frac{1}{V} + \frac{1}{U} = \frac{1}{F}; \quad U = \text{object distance}$$

$V = \text{image distance}$

Magnification (m) is the height of the image divided by the height of the object

$$m = \frac{h_i}{h_o} = \frac{V}{U}$$

~~Reflection~~ Reflection of
the in the direction of light when it passes from
one another

The incidence ray the normal ray and the reflected ray all lie
on the same plane at a point of incidence

The angle of incidence is equal to the angle of reflection

Laws of Refraction

* the incident ray, the reflected ray and the refracted ray all lie on the same plane

angle of incidence θ_1

angle of refraction θ_2

relationship $\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}$ - speed of light in 2nd medium
 v_1 - speed of light in first medium

* when light moves from a material in which its speed is high to a material in which its speed is lower the angle of refraction θ_2 is less than the angle of incidence θ_1 and the ray is bent towards a normal

Index of Refraction

* Speed of light in any material is less than its speed in vacuum

* Light travels at its maximum speed in vacuum

$$\text{Index of refraction } (n) = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{v}$$

(n) is always greater than unity (1) $n = \frac{c}{v}$
v is always less than c

* As light travels from one medium to another, its frequency does not change

Since frequency is constant

$$v_1 = \lambda_1 f; v_2 = \lambda_2 f$$

$$f = \frac{v_1}{\lambda_1}; f = \frac{v_2}{\lambda_2}$$

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

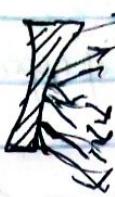
$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{c/n_1}{c/n_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

$$\therefore \lambda_1 n_1 = \lambda_2 n_2$$

Refraction Due to ~~Lenses~~

Converging lenses are thicker in the middle than at the edges - Convex
 Diverging lenses are thinner in the middle than at the edges - Concave



Concave lens or diverging lens



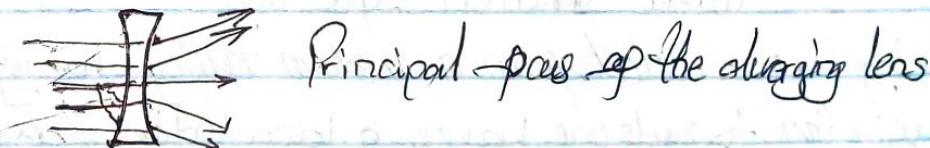
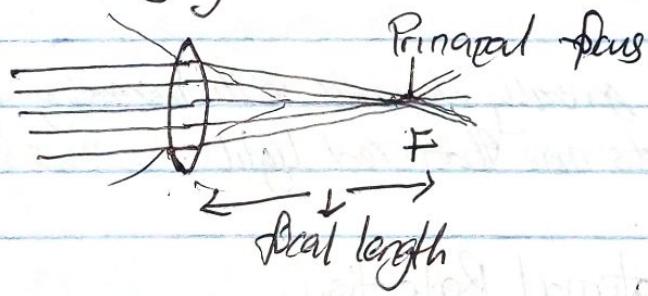
Convex lens or converging lens

* The principal axis of a lens is the line joining the centre of curvature of the surface and passing through the centre of the lens.

* The rays, after passing through a lens all converge to a point on the axis which is called the principal focus F.

* In the case of a diverging lens, the rays would spread out after passing through the lens as if diverging from a focus behind the lens.

Thus; The principal focus is real for a converging lens and virtual for a diverging lens.



* Centre of curvature: A point on which rays of light pass without being deviated.

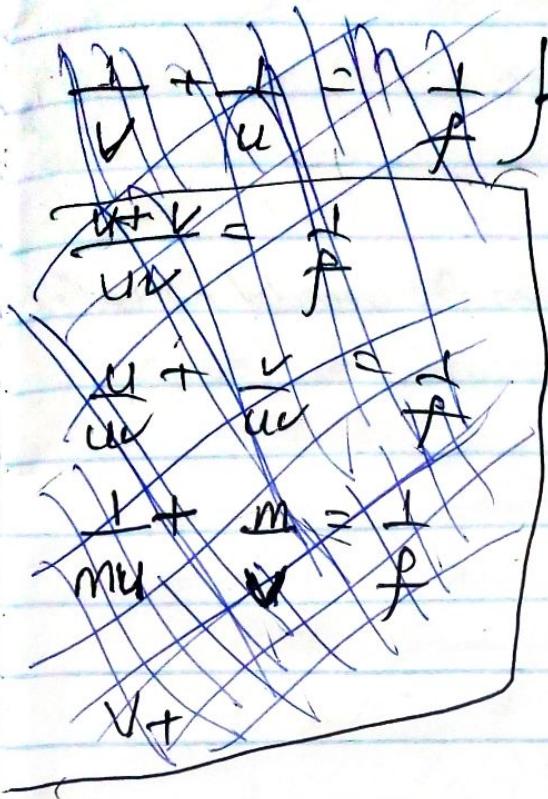
* The focal length (F) is the distance b/w the optical centre and principal focus of the lens.

Lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$m = \frac{v}{u} - \text{Image distance}$$

u - object distance



multiply 2m by v

$$v\left(\frac{1}{v} + \frac{1}{u}\right) = v\left(\frac{1}{f}\right)$$

$$2 \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$1 + \frac{1}{u} = \frac{1}{f}$$

$$1 + m = \frac{1}{f}$$

$$m = \frac{v}{u} - 1$$

* Power of lens is defined as reciprocal of focal length

$$\left[\frac{1}{P} = \frac{1}{f} \right] \quad \left[\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \right]$$

* The shorter the focal length, the larger the power

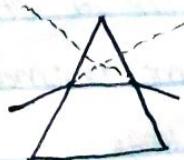
* Unit of power of a lens is dioptre

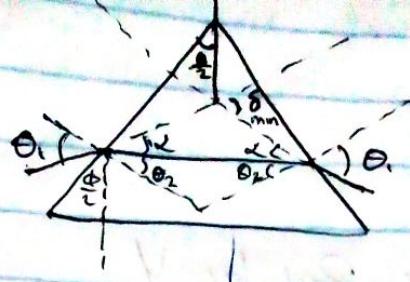
Dispersion and chromatic aberration

This is an effect resulting from

Dispersion

is the phenomenon in which the phase velocity of a wave depends on its frequency. A media having this property may be termed dispersive media. It is also defined as the separation of the colours by the prism





ϕ = apex angle

θ_1 = angle of incidence

δ_{\min} = minimum angle of deviation (occurs when the refracted ray makes d some angle with the normal)

$$\theta_2 = \frac{\phi}{2}$$

$$\delta_{\min} = 2\alpha$$

$$\theta_1 = \theta_2 + \alpha$$

$$\theta_1 = \theta_2 + \alpha = \frac{\phi}{2} + \frac{\delta_{\min}}{2} = \frac{\phi + \delta_{\min}}{2}$$

assuming θ_1 is from air

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_1 = 1.00$$

$$\sin \theta_1 = n_2 \sin \theta_2$$

$$n_2 = \frac{\sin \theta_1}{\sin \theta_2}$$

$$n_2 = \frac{\sin \left(\frac{\phi + \delta_{\min}}{2} \right)}{\sin \left(\frac{\phi}{2} \right)}$$

Production of pure spectrum

- * Newton's spectrum of sunlight is an impure spectrum as the different coloured images overlap.
- * A pure spect is one in which the different coloured images contain light of one colour only i.e. monochromatic images
- * For a monochromatic image
- * To obtain a pure spectrum, a white light must be admitted on a very narrow opening so as to assist in reduction of overlapping of the images

* Beams of coloured rays emanating from the prism must be parallel so that each

can be brought in a separate focus

A spectrometer can be used to provide a true spectrum
Achromatic lenses

When white light of an object is diffracted by a lens, a coloured image is formed. This is because the glass reflects colours such as red and blue

The formed images are formed at slightly different places. This is called Achromatic Aberration.

Condition for achromatic lenses

$$\frac{1}{f} = (n - 1) \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

for red light

$$\frac{1}{f_r} = (n_r - 1) \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

for blue light

$$\frac{1}{f_b} = (n_b - 1) \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

Ques: A convex lens whose two faces have same radius of curvature has a focal length of 16cm. The refractive index of glass is 1.52. Find the radius of curvature.

$$\frac{1}{f} = (n - 1) \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \quad n/6 : r_1 = r_2$$

$$\frac{1}{16} = (1.52 - 1) \left(\frac{1}{r_1} \right)$$

$$r_1 = r_2 = 16.64$$

Physical Optics

Optics is the branch of physics which deals with the study of propagation and properties of light

Light is a type of energy which produces the sensation of vision

Wave theory of light

Huygen's ~~posed~~ proposed that light energy from a luminous source travels by means of wave motion

The discovery of a wave characteristic; interference of light by John Young supported the Huygen's wave theory

Two types of wave front

- Spherical wavefront - formed from a homogenous light source
- Plane wavefront - coming from a point source (sunlight)

Huygen's Principle

To determine the next wavefront

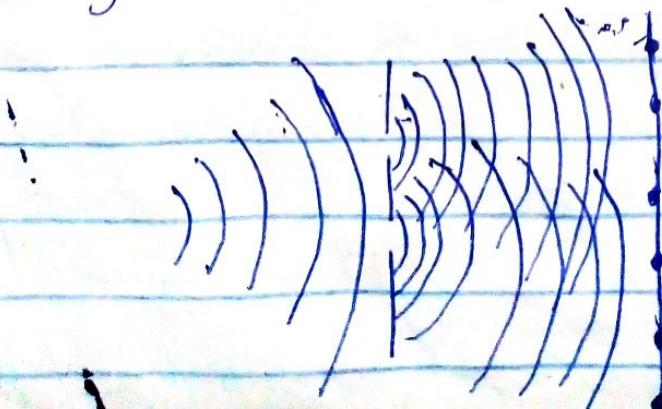
- Each point on the wavefront is known as the secondary source
Huygen's principle states that

The new position after a certain interval of time

Interference in Light Waves

- It can occur
- Constructively
- Destructively

Young's double slit experiment (on superposition)



Diffration of light
The property of bending of light around obstacle and spreading of light into shades of light is called diffraction

Interference

This is the result of superposition of light coming from the different wavefronts of same waves

Refraction

Result of interaction of light coming from different parts of same wavefront originating from same source

Polarisation

Ordinary light consists of transverse waves in which vibration takes place in all directions

Physics for scientist and engineers

Huygen's Principle

He proposed geometric method as a proof of laws of reflection and refraction

This principle is a geometric construction for using knowledge of an earlier wavefront to determine the position of a new wave front at some instant
→ All points on a given wavefront are taken as point sources for the production of spherical secondary waves called wavelets that propagate outward in a medium with speeds characteristic of waves in that medium. After some time interval has passed, the new position of the wavefront is the surface tangent to the wavelets

Dispersion

For a given material, the index of refraction varies with the wavelength of the light passing in it material. This behaviour is called dispersion. Snell's law indicates that light of different wavelengths is refracted at different angles when incident on a material

- * Index of refraction generally decreases with increasing wavelength. For example; ~~visible~~ violet light refracts more than red light when passing into a material.
- * Suppose a beam of white light is incident on a prism. Clearly, the angle of deviation δ depends on wavelength. The rays that emerge spread out in a series of colours known as the ~~Visible Light Spectrum~~ spectrum. The colours in order of decreasing wavelength are; red, orange, yellow, green, blue, indigo and violet.

Total Internal Reflection

It occurs when light is directed from a medium having a given index of refraction towards one having a lower index of refraction.

The refracted rays are bent away from the normal because $n_1 > n_2$.

At some particular angle of incidence θ_c , called the critical angle, the refracted rays are bent away from the normal and moves parallel to the boundary so that $\theta_2 = 90^\circ$

For angles of incidence greater than θ_c , the ray is entirely reflected at the boundary.

Snell's law of refraction can be used to find the critical angle

$$n_1 \sin \theta_c = n_2 \sin 90^\circ = n_2$$

$$n_1 \sin \theta_c = n_2$$

$$\frac{\sin \theta_c}{n_1} = \frac{n_2}{n_1} \quad (n_2 \text{ must be less than } n_1) \\ n_1 > n_2$$

- * The critical angle for total internal reflection is small when n_1 is considerably greater than n_2 .

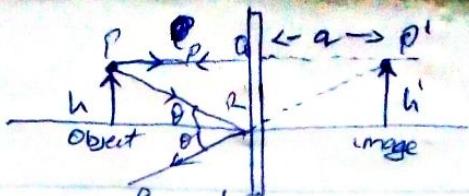
- * A flexible light pipe is called an optical fibre

Image Formation

Images Formed by Flat Mirrors

The image of an object seen in a flat mirror is always virtual.

- * Real images can be displayed on a screen but virtual images can't



* For a flat mirror, images ~~form~~ are as far from the mirror as the object is

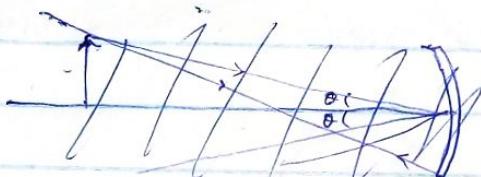
$$\therefore \text{Image distance} = \text{object distance}$$

* The images are always upright so image height is always positive
magnification $M = \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h} = +1$

* A flat mirror produces an image that has an apparent left-right reversal

Images formed by Spherical Mirrors

Concave Mirrors



Rays that diverge from the object and make a small angle with the principal axis also lie close to the axis run out of system
so all paraxial rays reflect run to image point

* Rays that are far from the principal axis converge to other points on the principal axis producing a blurred image. This effect is called spherical aberration and is present to some extent for any spherical mirror

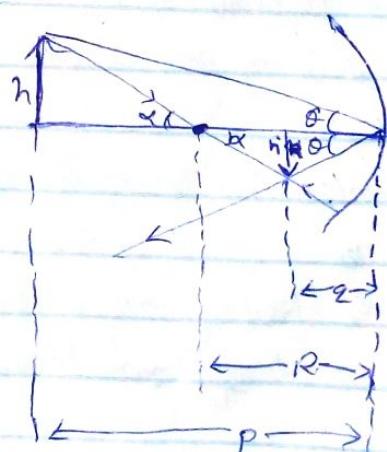
Mirror equation

$$\frac{1}{P} + \frac{1}{Q} = \frac{1}{f}$$

Often form

$$\frac{1}{P} + \frac{1}{Q} = \frac{1}{R}$$

$$\text{and } f = \frac{R}{2}$$



Convex mirror

* It is also called a diverging mirror as rays from any point on an object diverge after reflection as though they were coming from some point behind the mirror.

* The image is virtual

* The image is always upright

Sign conventions for Mirrors

Quantity	Positive when	Negative when
Object location (p)	Object is in front of mirror (real object)	Object is behind the mirror (virtual object)
Image location (p')	Image is in front of mirror (real image)	Image is behind the mirror (virtual image)
Image height (h')	Image is upright	Image is inverted
Focal length (f) and radius (R)	Mirror is concave	Mirror is convex
Magnification (M)	Image is upright	Image is inverted

Ray Diagrams for mirrors

For Concave Mirrors

* Ray 1 is drawn from the top of the object parallel to principal axis and is reflected in the focal point F

* Ray 2 is drawn from the top of the object in the focal point (or as if coming from the focal point if $P < f$) and is reflected parallel to principal axis

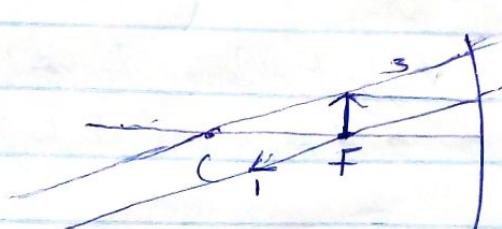
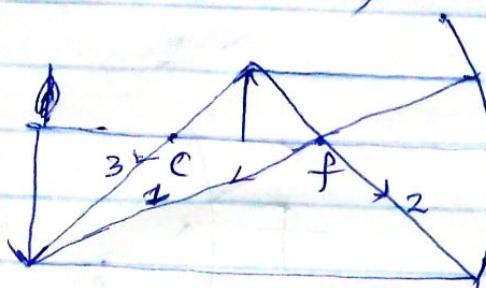
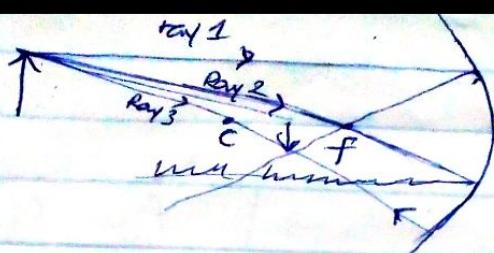
* Ray 3 is drawn from the top of the object in or centre of curvature (or as if coming from the centre if $P > f$) and is reflected back on itself

- The intersection of any two of these rays locates the image. The third ray serves as a check of the construction

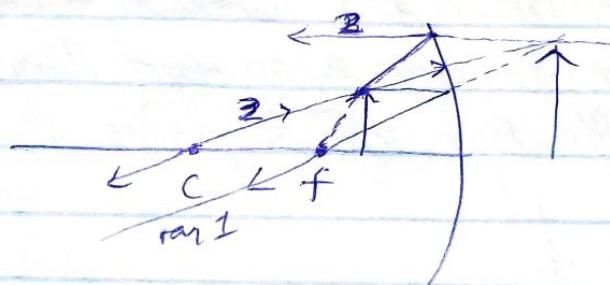
- As the object is moved closer to the mirror the real inverted image moves to the left and becomes larger as the object approaches the focal point

- When the object is at the focal point, the image is infinitely far to the left

- When the object lies between the focal point and the mirror, the image is to the right, behind the mirror, virtual, upright and enlarged

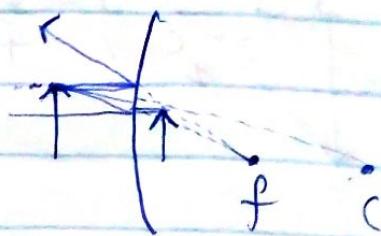
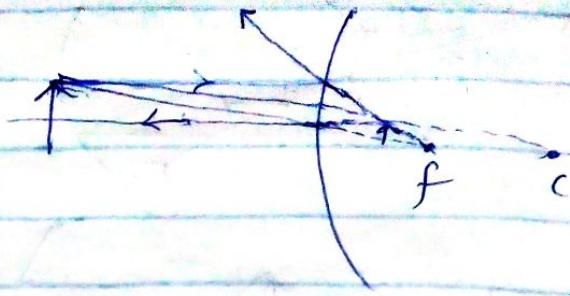


Rays are parallel
no image formed



For Convex mirrors

- * Ray 1 is drawn from the top of the object parallel to the principal axis and is reflected such that it appears to come from the focal point f
- * Ray 2 is drawn from the top of the object toward the focal point on the back side of the mirror and is reflected parallel to the principal axis
- * Ray 3 is drawn from the top of the object towards the centre of curvature C on the backside of the mirror and is reflected back on itself
- Image is always virtual, upright and reduced in size
- As the object distance decreases, the virtual image increases in size, and moves away from the focal point towards the mirror as the object approaches the mirror



→ Object distance decreased, Image moves closer
to d mirror and its height is increased as well

A spherical mirror has a focal length of +10.0cm.

- (a) Locate and describe the image for an object distance of 25.0cm
hint: because the focal length is positive, it's a concave mirror

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{q} = \frac{1}{10} - \frac{1}{25} \quad \frac{1}{q} = \frac{1}{2} - \frac{1}{25}$$

$$\frac{1}{q} = 0.1 - 0.04$$

$$\frac{1}{q} = 0.06 = \frac{6}{100}$$

$$q = \frac{100}{6} = 16.7\text{cm}$$

$$M = -\frac{q}{p} = -\frac{16.7}{25} = -0.667$$

M/B; Since the value of image distance is positive, the image is real i.e. it is formed in front of d mirror and not behind

- (b) Since the magnification is negative and is less than 1, the image is inverted and diminished respectively

b) Locate and describe the image for an object distance of 10.0 cm

$$\frac{1}{l} = \frac{1}{l} - \frac{1}{P}$$

$$\frac{1}{Q} + \frac{1}{P} = \frac{1}{f}$$

$$\frac{1}{Q} = \frac{1}{l} - \frac{1}{P} = \frac{10 - 10}{10} = \frac{0}{10} = 0$$

$$\frac{1}{Q} = \infty$$

$$Q = \infty$$

In this case the image is infinitely far away because the object is placed on the focal point.

c) Locate and describe the image for an object distance of 5.00 cm

$$\frac{1}{l} = \frac{1}{l} - \frac{1}{P}$$

$$\frac{1}{Q} + \frac{1}{P} = \frac{1}{f}$$

$$\frac{1}{Q} = \frac{1}{l} - \frac{1}{P} = \frac{1}{5} - \frac{1}{10} = \frac{1}{10}$$

$$Q = -10 \text{ cm}$$

$$M = -\frac{Q}{P} = -\frac{-10}{5} = +2$$

* From this solution we infer that

- ① Since the image distance is negative the image is virtual formed behind the mirror
- ② The magnification is positive i.e. the image is upright
- ③ The magnification is greater than unity i.e. the image is enlarged

The Image formed by a Convex Mirror

An automobile rearview mirror shows an image of a truck located 10.0 cm from the mirror. The focal length of the mirror is -0.60 m

(A) Find the position of the image of the truck

$$\frac{1}{P} + \frac{1}{Q} = \frac{1}{f}$$

$$\frac{1}{Q} = \frac{1}{P} - \frac{1}{f}$$



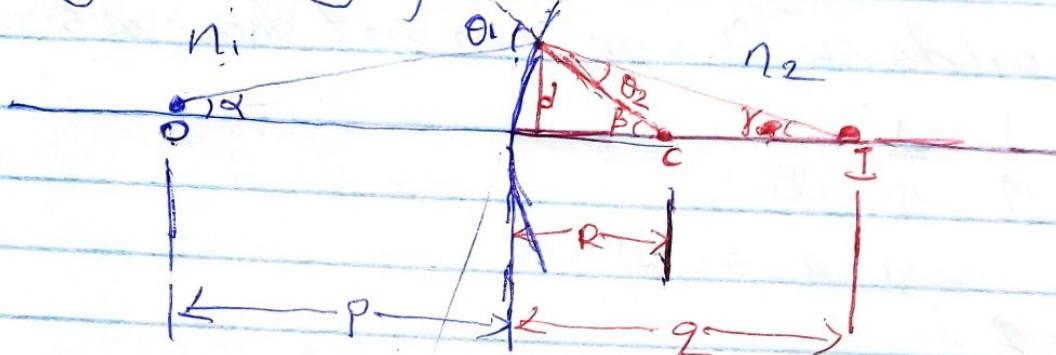
$$\frac{1}{Q} = -0.57$$



$$M = -\frac{q}{p} = -\frac{(-0.57)}{1.0} = +0.57$$

- * The negative value of q indicates ~~that~~ the image is formed behind the mirror and thus is virtual.
- * The ~~large~~ value of magnification is positive, which indicates that the image is upright.
- * The value of magnification is less than unity; This shows that the image is diminished/reduced.

Images formed by refraction



From Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Because θ_1 and θ_2 are assumed to be small, we can use the small angle approximation $\sin \theta \approx 0$

$$\text{So } n_1 \theta_1 = n_2 \theta_2$$

$$\theta_1 = \alpha + \beta$$

$$\beta = \theta_2 + \gamma$$

Combining all equations and eliminating θ_1 and θ_2

$$n_1(\alpha + \beta) = n_2(\beta + \gamma)$$

$$n_1\alpha + n_1\beta = n_2\beta + n_2\gamma$$

$$n_1\alpha + n_2\gamma = n_2\beta - n_1\beta$$

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$$

In the ~~Diagram~~ above There are three right ~~angle~~ triangles that share a common vertical length d . The horizontal legs of these triangles are p for the triangle containing α , R for the triangle containing β and $n_2 d$ for the triangle containing γ .

As a small angle approximation, $\tan \theta \approx 0$

So $\tan \alpha \approx \alpha \approx d$

$$\tan \beta = \beta \approx \frac{d}{R}$$

$$\tan \gamma \approx \gamma \approx \frac{d}{q}$$

Substituting into former equation

$$n_1 \alpha + n_2 \gamma = (n_2 - n_1) \beta$$

$$n_1 \left(\frac{d}{P} \right) + n_2 \left(\frac{d}{q} \right) = (n_2 - n_1) \left(\frac{d}{R} \right)$$

$$\frac{d}{P} \left(n_1 + \frac{n_2}{q} \right) = \frac{(n_2 - n_1)}{R} d$$

* For a fixed object distance P , the image distance q is independent of the angle the ray makes with the ~~normals~~

* All parallel rays focus at d same point

. Sign Conventions for Refracting Surfaces

Quantity	Positive when	Negative when
Object location (P)	Object is <u>in front of</u> surface (real object)	object is <u>in back of</u> surface (virtual object)
Image location	Image is <u>at d back of</u> surface (real image)	image is <u>in front of</u> surface (virtual image)
Image height	Image is upright	Image is inverted
Radius (R)	Centre of curvature is <u>in d</u> <u>back of</u> surface	Centre of curvature is <u>in front of</u> surface

* Flat refracting surfaces

* If a refracting surface is flat, then R is infinite, then we have
the equation $\left(\frac{n_1}{P} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \right)$, reduced to $\frac{n_1}{P} + \frac{n_2}{q} = 0$

$$\text{So } \frac{n_1}{P} = -\frac{n_2}{q}$$

A set of coins is embedded into a spherical plastic paper weight having a radius of 3.6cm. The index of refraction of the plastic is $n_1 = 1.5$. One coin is located 2cm from the edge of sphere. Find the position of image of a coin.

$$p = 2.0 \text{ cm} \quad n_1 = 1.5$$

$$q = ? \quad n_2 = 1.00 \text{ (refractive index of air)}$$

$$R = -3.6 \text{ cm}$$

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$$

$$1.5 + \frac{1}{q} = \frac{(1 - 1.5)}{-3.6}$$

$$\frac{1}{q} = \frac{-0.5}{3.6}$$

$$q = -7.2 \text{ cm}$$

$$\frac{1.5x + 2}{2x} = \frac{+0.5}{3}$$

$$\frac{4.5x + 6}{2x} = \frac{1.5x + 2}{3} \quad \frac{1.5x + 2}{2x} = \frac{+1}{6}$$

$$\frac{4.5x + 6}{2x} = \frac{1.5x + 2}{3}$$

$$9x + 12 = 2x$$

$$\frac{12}{-7x} = \frac{7x}{-7x}$$

$$x = -1.7 \text{ cm}$$

A small fish is swimming at a depth d below the surface of a pond. (i) what is the apparent depth q of the fish as viewed from directly overhead nearby $n_1 > n_2$ where $n_2 = 1.00$ (n of air)

Because the water is a flat surface R is infinite

so the formula $\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$, would be applied

$$p = d \text{ (depth of fish)}$$

$$q = \text{(apparent depth)}$$

$$\frac{n_1}{d} = \frac{-n_2}{q}$$

$$\frac{1.33}{d} = \frac{-1}{q}$$

Making q the subject

$$\frac{q}{-1} = \frac{d}{1.33}$$

$$q = d \times -\frac{1}{1.33}$$

$$q = -0.752d$$

B At what apparent distance does the fish see your face if your face is at a distance d above water surface

Since $R = \infty$ & $n_1 + n_2 = 0$ Standards

$$P \quad Q$$

now $\frac{1}{P} + \frac{1}{Q} = 0$ $\frac{1.33}{P} + \frac{1}{d} = 0$

remember d = distance of your eyes from water surface

$$\frac{1.33}{P} = -\frac{1}{d}$$

$$P = -d$$

$$1.33$$

$$P = -1.33d$$

Images formed by Thin lenses

$$\frac{1}{P} + \frac{1}{Q} = n^{-1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{lens makers equation} = \frac{1}{f} = n^{-1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Thin lens equation: $\frac{1}{P} + \frac{1}{Q} = \frac{1}{f}$

* Because light can travel in either direction, in a lens, each lens has two focal points

Sign Convention for thin lenses

Quantity	Positive when	Negative when
Object location	Object is in front of lens real object	object is on back of lens virtual object
Image location	Image is out of back of lens (real image)	Image is in front of lens (virtual image)
Image height	Image is upright	Image is inverted
R_1 & R_2	Centre of curvature is at a back	Centre of curvature is in front
focal length	a converging lens	a diverging lens

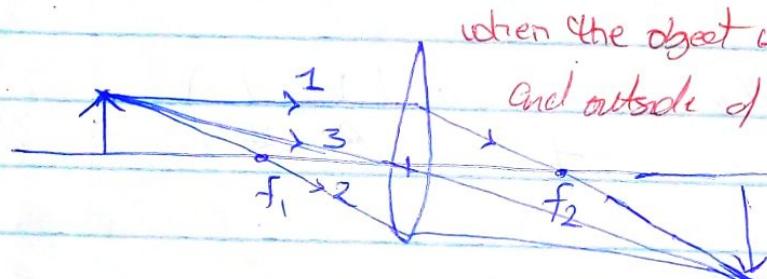
$$M = \frac{h'}{h} = -\frac{q}{p}$$

when M is positive; the image is upright and on the same side of the lens as the object. when M is negative, the image is inverted and on side of lens opposite of object

Ray diagrams for thin lenses

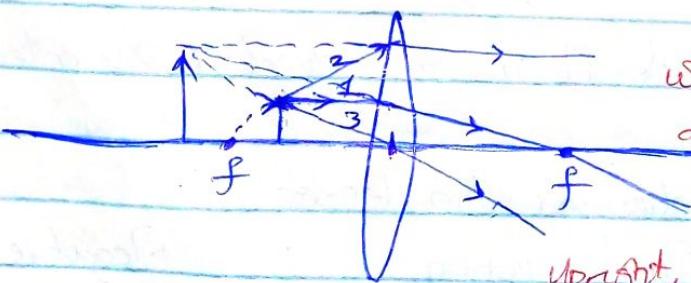
for a converging lens

- Ray 1 is drawn parallel to principal axis. after being refracted by the lens, the ray passes on a focal point on the back side of the lens
- Ray 2 is drawn on a focal point on the front side of lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis
- Ray 3 is drawn from the centre of the lens and continues in a straight line



when the object is in front of mirror

and outside of focal point, the image is real and inverted and on the back of the lens



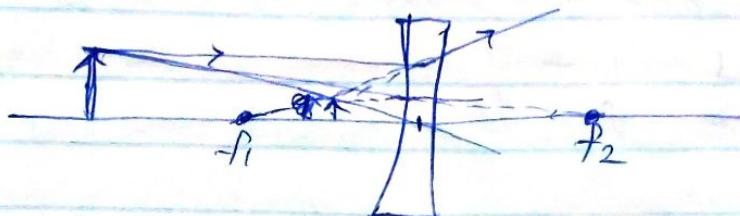
when the object is between the focal point and the lens

the image is always upright, virtual and larger than the object and also it is in front of the lens

for a Diverging lens

Ray 1 is drawn parallel to the principal axis, after being refracted by the lens, this ray emerges diverged away from the focal point on a front side of the lens

Ray 2 is drawn in the direction towards the focal point on the back side of the lens and emerges from lens parallel to the principal axis
 Ray 3 is drawn on the centre of the lens and continues in a straight line



For the Converging lens

when $p > f$, the image is real, and inverted

when $p < f$ the image is virtual and upright in this case, it acts as a magnifying lens

for a diverging lens

image is always virtual and upright regardless of where object is placed

Combination of thin lenses

If 2 lenses are used to form an image

- (a) The image formed by the first lens is located as if the second lens were not present
- (b) A ray diagram is drawn of second lens with the image formed by the first lens serving as the object for the second lens
- (c) Overall magnification of an image due to combination of lenses is a product of individual magnifications

$$M = M_1 M_2$$

- A system of 2 lenses of focal lengths f_1 and f_2 in contact with each other, if $p_1 = p$ is the object distance for the combination, application of the thin lens equation to the first lens gives

$$\frac{1}{P} + \frac{1}{q_1} = \frac{1}{f_1}$$

where q_1 is the image distance for the first lens and object of the second lens
 $\therefore P_2 = -q_1$

for the second lens

$$\frac{1}{P_2} + \frac{1}{q_2} = \frac{1}{f_2} \rightarrow -\frac{1}{q_1} + \frac{1}{q} = \frac{1}{f_2}$$

where $q = q_2$ = final image distance

adding d equations + d 2 lenses eliminates q_1 and gives

$$\frac{1}{P} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2}$$

Lens Abber

Lens ABERRATION

Analysis of mirrors and lens assumes that rays make small angles with the principal axis and lenses are thin.

In this simple model, all rays leaving a point source, focus at a single point producing a sharp image. This is not always true.

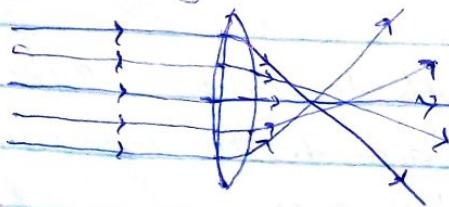
A precise analysis of image formation requires tracing each ray using Snell's law at each refracting surface and law of reflection at each reflecting surface.

This procedure shows that the rays from a point do not focus at a single point, with the result that the image is blurred.

Departures of actual images from the ideal predicted result from the simplified model are called Aberrations.

Spherical Aberration

This occurs because the focal points of rays far from the principal axis ($q \gg$ spherical lens or mirror) are different from the focal points of rays of same wavelength passing near the axis.



* Rays passing far points near the center of a lens are imaged farther from the lens than rays passing through points near the edges.

* Many cameras have an adjustable aperture to control light intensity and reduce spherical aberration. Sharper images are produced as the aperture size is reduced.

In case of mirrors, spherical aberration can be reduced. thru d use of a parabolic reflecting surface rather than a spherical surface

To parallel ^{light} rays incident on a parabolic surface focus at a common point regardless of their distance from principal axis

Chromatic Aberration

Rays of different wavelength form same light source focus at different points causing a blurred image

- * violet rays are refracted more than red rays when light passes thru a lens
- the focal length of a lens is greater for red light than for violet light
- other wavelengths have focal points intermediate between those of red light and violet light which causes a blurred image and is called ~~spherical~~ chromatic aberration

Optic Lenses

Wave Optics

Here, light is treated as a ^{wave} rather than as a ray as in ray optics

Young's Double-Slit Experiment

light waves interfere with one another

Fundamentally, all interference associated with light waves arise when the magnetic field that constitutes individual waves combine

Interference in light waves was first demonstrated by Thomas Young.

Plane light waves arrive at a barrier that contains two slits. The light from the slits produces on a viewing screen a visible pattern of bright and dark parallel bands called fringes

When the light from both slits arrive at a point on the screen such that constructive interference occurs at that location, a bright fringe appears. When the light from the 2 slits combines destructively at any location on the screen, a dark fringe results

When two waves leave the slit in phase strike a screen at d. Central point and travel the same distance they arrive at a point on the screen in phase causing a constructive interference at that point, thus, a bright fringe is observed

Also when the two waves start in phase but the lower wave has to travel one wavelength farther than the upper wave to reach d point because the lower wave falls behind the upper wave by exactly one wavelength, they still arrive in phase and a second bright fringe appears at that location

However, if the lower wave has fallen half a wavelength behind the upper wave and a crest of the upper wave overlaps a trough of the lower wave, a destructive interference occurs



If two light bulbs are placed side by side so that light from both bulbs combines, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb

The emission from the two bulbs do not maintain a constant phase relationship with each other over time. They undergo random phase changes in time interval of less than a nanosecond.

Therefore, the conditions for constructive interference, destructive interference and some intermediate state are maintained only for such short time intervals. Because the eye cannot follow such rapid changes no interference effects are observed

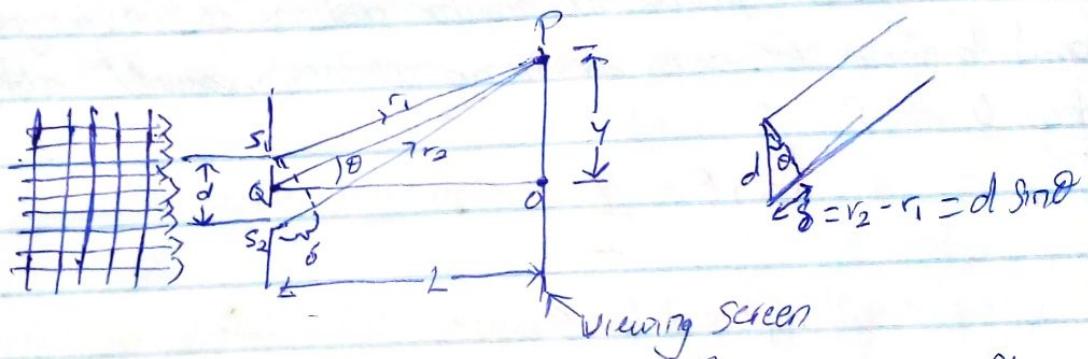
Such light sources are incoherent

To observe interference of waves from two sources, the following conditions must be met

- * The sources must be coherent i.e., they must maintain a constant phase with respect to each other
- * The sources should be monochromatic i.e., they should be of a single wavelength

For example, Single-frequency soundwaves emitted by two side-by-side loudspeakers driven by a single amplifier can interfere with each other because the two speakers are coherent. In other words, they respond to d amplifier in d same way at d same time.

Waves in interference



The viewing screen is located a perpendicular distance L from the barrier containing two slits, s_1 and s_2 . The slits are separated by a distance d and the source is monochromatic.

To reach an arbitrary point P in the upper half of the screen, a wave from the lower slit must travel farther than a wave from the upper slit. The extra distance travelled from the lower slit is the path difference δ . If we assume that the rays are parallel, which is true if L is much greater than d , then δ is $\delta = r_2 - r_1 = d \sin \theta$.

The value of δ determines whether the two waves are in phase when they arrive at P .

If δ is either 0 or some integer multiple of d wavelength, the two waves are in phase at point P and constructive interference results.

Therefore, the condition for bright fringes or constructive interference at point P is

$$d \sin \theta_{\text{bright}} = m \lambda \quad m = 0, \pm 1, \pm 2$$

m is called order number; for constructive interference, the order number is the same as the number of wavelengths that represents the path difference between the waves from the two slits.

The central bright fringe at $\theta_{\text{bright}} = 0$ is called the zeroth order maximum. The first maximum on either side where $m = \pm 1$ is called the first order maximum and so forth.

When d is an odd multiple of $\frac{\lambda}{2}$, the two waves arriving at point P are 180° out of phase and give rise to destructive interference. Therefore the condition for dark fringes, or destructive interference at point P is

$$d \sin \theta_{\text{dark}} = (m + \frac{1}{2}) \lambda \quad m = 0, \pm 1, \pm 2$$

These equations provide the angular positions of the fringes. It is also useful to obtain expressions for linear positions measured along d screen from O to P

$$\tan \theta = \frac{y}{L}$$

$$y_{\text{bright}} = L \tan \theta_{\text{bright}}$$

$$y_{\text{dark}} = L \tan \theta_{\text{dark}}$$

When the angles to the fringes are small, the positions of the fringes are linear near the centre of the pattern having $\tan \theta \approx \sin \theta$

So $y_{\text{bright}} = L \tan \theta_{\text{bright}}$ becomes

$$y_{\text{bright}} = L \frac{(m \lambda)}{d} \quad (\text{small angles})$$

This shows that y_{bright} is linear in d order number m, so the fringes are equally spaced for small angles. Similarly for dark fringes

$$y_{\text{dark}} = L \frac{(m + \frac{1}{2}) \lambda}{d} \quad (\text{small angles})$$

* Young's double slit experiment provides a method for measuring the wavelength of light

This experiment gave the wave model of light a great deal of credibility

Measuring the wavelength of a light source

A viewing screen is separated from a double slit by 4.80m. The distance between 2 slits is 0.030mm. Monochromatic light is directed toward the slit and forms an interference pattern on the screen. The first dark fringe is 4.50cm from the centreline of the screen.

(A) Determine the wavelength of the light

$$\gamma_{dark} = L \frac{(m + \frac{1}{2})\lambda}{d}$$

$$\gamma_{dark} = 4.50\text{cm}$$

$$d = 3$$

$$d = 0.03\text{mm}$$

$$L = 4.80\text{m}$$

$$\lambda = \frac{\gamma_{dark} d}{L(m + \frac{1}{2})} = \frac{4.50 \times 10^{-2} \times 0.03 \times 10^{-3}}{4.8(0 + \frac{1}{2})} \\ = 5.62 \times 10^{-7} \text{m} = 562\text{nm}$$

(B) Calculate the distance between adjacent bright fringes

$$\gamma_{m+1} - \gamma_m = L \frac{(m+1)\lambda}{d} - L \frac{m\lambda}{d}$$

$$= \frac{Lm\cancel{\lambda} + LA - Lm\cancel{\lambda}}{d}$$

$$= L \frac{\lambda}{d}$$

$$= 4.8 \times \frac{5.62 \times 10^{-7}}{3.0 \times 10^{-5}} \approx 9.00 \times 10^{-2}$$

$$\approx 9.00\text{cm}$$

Separating Double Slits fringes of Two wavelengths

A light source emits visible light of two wavelengths $\lambda = 430\text{nm}$ and $\lambda' = 510\text{nm}$. The source is used in a double slit interference experiment in which $L = 1.5\text{m}$ and $d = 0.025\text{mm}$. Find the separation distance between the third order bright fringes + d 2 wavelengths

$$\begin{aligned}\Delta y &= Y_{\text{bright}} - Y_{\text{bright.}} = \frac{L m \lambda'}{d} - \frac{L m \lambda}{d} \\ &= \frac{L m}{d} (\lambda' - \lambda) \\ &= \frac{1.5(3)}{2.5 \times 10^{-5}} (510\text{nm} - 430\text{nm}) \\ &\approx 0.0144\text{m} = 1.44\text{cm}\end{aligned}$$

Intensity Distribution of the Double-Slit Interference Pattern

Magnitude of electric field at point P due to each wave separately

$$E_1 = E_0 \sin \theta$$

$$\text{and } E_2 = E_0 \sin(\theta + \phi)$$

Although the waves are in phase at the slits, their phase difference ϕ at P depends on path difference $\delta = d_2 - d_1 = ds \sin \theta$. A path difference of λ (for constructive interference) corresponds to a phase difference of 2π rad.

A path difference of δ is the same fraction of λ as the phase difference ϕ is of 2π .

$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi}$$

$$\phi = \frac{\delta}{\lambda} 2\pi$$

$$\left. \begin{aligned} \phi &= \frac{2\pi}{\lambda} ds \sin \theta \end{aligned} \right\} \leftarrow \text{Phase difference depends on angle } \theta$$

Diffraction Patterns from Slit or Slits

It is observed that a diffraction pattern is actually an interference pattern in which the different sources of light are different portions of the single slit.

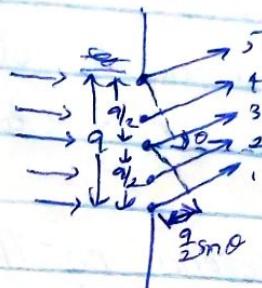
Therefore, the diffraction patterns are applications of the wave interference analysis model.

Waves from the upper half of the slit interfere destructively with waves from the lower half when

$$\sin \theta = \pm \frac{\lambda}{a}$$

Considering waves at angle θ both above and below the dashed line

$$\sin \theta = \pm \frac{\lambda}{a}$$



Dividing slit into four equal parts, the viewing screen is dark when

$$\sin \theta = \pm 2 \frac{\lambda}{a}$$

Because dividing the slit into four equal parts should not darkness occur when

$$\sin \theta = \pm 3 \frac{\lambda}{a}$$

Therefore, the general condition for destructive interference is

$$\sin \theta_{\text{dark}} = m \frac{\lambda}{a}$$

where $m = \text{No. of parts the slit is divided}$

Each bright fringe lies approximately halfway between its bordering dark fringes.

Example

Light of wavelength 580 nm is incident on a slit having a width of 0.3 mm . The viewing screen is 200 cm from the slit. Find the width of a central fringe.

$$\sin \theta = \pm \frac{\lambda}{a}$$

$$\tan \theta_{\text{dark}} = m / L$$

and because θ is small $\tan \theta \approx \sin \theta$

$$\sin \theta_{\text{bright}} = \frac{y_1}{L}$$

$$\frac{y_1}{L} = \frac{A}{a}$$

$$y_1 = L \frac{A}{a}$$

$$2y_1 = 2L \frac{\lambda}{a}$$

$$= 2 \times 2.00 \times \frac{580 \times 10^{-9}}{3 \times 10^{-4}}$$

$$= 7.73 \times 10^3 \text{ m} = 7.73 \text{ mm}$$

Diffracted Grating

The diffraction grating, a useful device for analysing sources, consists of a large number of equally spaced parallel slits.

A typical grating ruled with 6000 grooves/cm has a slit spacing $d = (1/6000) \text{ cm} = 2.00 \times 10^{-4} \text{ cm}$

A plane wave is incident from the left, normal to the plane of the grating. The pattern observed on the screen far from the grating is the result of the combined effects of interference and diffraction. Each slit produces diffraction, and the diffracted beams interfere with one another to produce the final pattern.

Conditions for maxima

$$ds \sin \theta_{\text{bright}} = m\lambda \quad m = 0, \pm 1, \pm 2, \dots$$

d = grating spacing

for small values of d θ_{bright} are large

Monochromatic light from a helium-neon laser ($\lambda = 632.8 \text{ nm}$) is incident normally on a diffraction grating containing 6000 grooves per cm. Find the angles at which the first and second order maxima are observed.

$$d = (4600) \text{ cm} = 1.667 \times 10^{-1} \text{ cm} = 1667 \text{ nm}$$

$$\sin \theta_1 = m \frac{\lambda}{d}$$

$$\sin \theta_1 = \frac{1 \times 632.8 \text{ nm}}{1667 \text{ nm}} = 0.3797$$

$$\theta_1 = 22.31^\circ$$

$$\sin \theta_2 = 2 \frac{\lambda}{d} = \frac{2(632.8)}{1667} = 0.7594$$

$$\theta_2 = +90^\circ$$

Condition for constructive interference (maximum in reflected beam) is

$$2ds \sin \theta = m\lambda \quad m = 1, 2, 3$$

Bragg's law \Rightarrow

Polarization of light waves

light and all other electromagnetic waves are transverse in nature

Polarisation is a firm evidence of this transverse nature of light

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave having some particular orientation of the electric field vector \vec{E} , corresponding to direction of atomic vibration.

The direction of polarization of each individual wave is defined to be the direction in which the electric field is vibrating. This direction happens to be along the Y axis. All individual electromagnetic waves travelling in Z direction have an \vec{E} vector parallel to the YZ plane, but it could be at any possible angle with respect to the Y axis.

* The resultant electromagnetic wave is a ^{Superposition} of waves vibrating in many different directions.

* A wave is said to be linearly polarized if the resultant of all individual waves from the field \vec{E} vibrates in the same direction at all times at a particular point. Sometimes such a wave is called plane polarized.

The plane formed by \vec{E} and the direction of propagation is called PLANE OF POLARIZATION of the wave.

Four different Process for Producing Polarized Light from Unpolarized light

Polarization by Selective Absorption

- * A polaroid polarizes light by selective absorption
- * When unpolarized light is incident on the material, the exiting light is polarized perpendicular to the molecular chains
- * In an ideal polarizer, all light with \vec{E} parallel to the transmission axis is transmitted and all light with \vec{E} perpendicular to the transmission axis is absorbed
- * The intensity I of the (polarized) beam transmitted by an analyzer is given by
$$I = I_{max} \cos^2 \theta$$

I_{max} = intensity of a polarized beam incident on a analyzer

The expression is known as Nyquist's Law

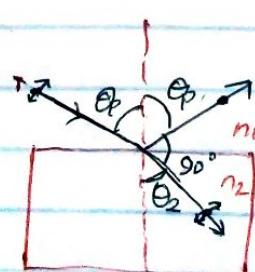
It shows that the intensity of the transmitted beam is maximum when the transmission axes are parallel ($\theta = 0$ or 180°) and zero (Completely absorbed by a analyzer) when the transmission axes are perpendicular to each other.

Polarization by Reflection

- * The polarization of reflected light depends on the angle of incidence
- * If the angle is 0° , the reflected beam is unpolarized. For other angles of incidence, the reflected light is polarized to some extent and for one particular angle of incidence, the reflected light is completely polarized

Suppose the angle of incidence is varied until the angle between the reflected and refracted beam is 90° . At this particular angle of incidence, the reflected beam is completely polarized and the refracted beam is still only partially polarized.

- * The angle of incidence at which this polarization occurs is called the polarizing angle θ_p



$$\theta_p + 90^\circ + \theta_2 = 180$$

$$\theta_2 = 90 - \theta_p$$

Using Snell's law

$$\frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin \theta_p}{\sin \theta_2}$$

$$\text{beas } \sin \theta_2 = \sin (90 - \theta_p) = \cos \theta_p$$

$$n_2 = \frac{\sin \theta_p}{\sin \theta_2}$$

$$n_1 \cos \theta_p$$

$$\boxed{\tan \theta_p = \frac{n_2}{n_1}}$$

This is called Brewster's law and polarizing angle is sometimes called Brewster's angle

Polarisation by double Refraction

Polarisation by separation

Wave Motion

2 Types of waves

Mechanical waves

Electromagnetic waves

- * In the case of mechanical waves, some physical medium is being disturbed
- Electromagnetic waves don't require a material medium for propagation
- Propagation of disturbance
- * All mechanical waves require
 - (i) some source of disturbance
 - (ii) A medium containing elements that can be disturbed
 - (iii) Some physical mechanism in which elements of medium can influence each other
- * Each disturbed element ^{of a string} moves in a direction perpendicular to the direction of propagation
- * A travelling wave that causes the elements to move in a direction perpendicular to the direction of the propagation of the medium is called a transverse wave
- * A travelling wave that causes the elements of the medium to move parallel to the direction of propagation is called a longitudinal wave
- Sound waves are an example of longitudinal waves
- * Some waves exhibit both transverse and longitudinal displacements. Surface water waves are a good example
- * An ideal wave is infinitely long
- * A wave of finite length must necessarily have a mixture of frequencies
- * Highest point of displacement \rightarrow Crest

Lowest Point of displacement \rightarrow Trough

Distance from one crest to another \rightarrow wavelength (λ)

The wavelength is the minimum distance between any two identical points on adjacent waves.

The number of ~~several~~ seconds between arrival of two adjacent crests at a given point in space is the period T of a wave

The frequency f of a periodic wave is the number of ~~waves~~ or ~~crests~~ that pass ~~at~~ a given point in a unit time interval

$$f = \frac{1}{T}$$

The maximum position of an element of the medium relative to its equilibrium position is called the Amplitude A of the wave

$$y(x,t) = A \sin \left[\frac{2\pi}{\lambda} (x - vt) \right] \quad \text{if wave moves right}$$

if it moves left $\approx x+vt$

$$y = A \sin \left[2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \right]$$

$$\text{Wave number } k \equiv \frac{2\pi}{\lambda}$$

$$\text{Angular frequency } \omega \equiv \frac{2\pi}{T} = 2\pi f$$

$$y = A \sin \left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} \right)$$

$$y = A \sin (kx - 2\pi ft)$$

$$y = A \sin (Kx - \omega t)$$

$$V = \frac{\omega}{k} = \frac{2\pi f}{\lambda} \times \frac{A}{2\pi} = \frac{2\pi f A}{2\pi \lambda} = f\lambda$$

$$V = f\lambda$$

also

$$y = A \sin (K\omega t - \omega t + \phi)$$

where $y \neq 0$ at $x=0$ and $t=0$

ϕ = phase constant

Sinusoidal Waves on Strings

$$y = A \sin(Kx - wt)$$

An element on the string moves ^{only} vertically ~~as~~ as ~~x~~ component remains constant

$$\text{The transverse speed } v_y = \frac{dy}{dt} = \frac{A}{x-\text{constant}} \frac{d}{dt} \cos(Kx - wt)$$

Transverse

SOUND WAVES

They are divided into 3 categories

- 1) Audible waves : lie within the range of sensitivity of the human ear
- 2) Infrasonic waves : have frequencies below the audible range. Elephants use infrasonic waves to communicate with one another
- 3) Ultrasonic waves : have frequencies above the audible range. Ultrasonic waves are also used in medical imaging

Pressure Variations in Sound Waves

A piston at the left end can be quickly moved to the right to compress the gas and create the pulse. Before the piston is moved the gas is undisturbed and of uniform density. When the piston is pushed to the right, the gas just in front of it is compressed, the pressure and density in that region are now higher than they were before the piston moved. When the piston comes to rest, the compressed region of the gas continues to move to the right corresponding to a longitudinal pulse travelling down a tube with speed v .

$$\text{If } S(x,t) = S_{\max} \cos(Kx - wt)$$

If $S(x,t)$ is the position of a small element relative to its equilibrium position

$$S(x,t) = S_{\max} \cos(Kx - wt)$$

S_{\max} = maximum ~~displacement~~^{Position} or displacement amplitude

The variation in the gas pressure from equilibrium value

$$\Delta P = \Delta P_{\text{max}} \sin(Kx - vt)$$

The pressure amplitude ΔP_{max} is the maximum change in pressure from the equilibrium value

$$\Delta P = -B \frac{\partial s}{\partial x}$$

$$\Delta P = -B \frac{\partial}{\partial x} [\sin(Kx - vt)] = B s_{\text{max}} K \sin(Kx - vt)$$

$$\Delta P_{\text{max}} = B s_{\text{max}} K$$

- * Pressure wave is 90° out of phase with the displacement wave
- * Pressure variation is maximum when the displacement from equilibrium is zero
- * Displacement from equilibrium is maximum when the pressure variation is zero

Speed of sound waves

- * Speed of all mechanical waves follows an expression of the general form

$$V = \sqrt{\frac{\text{elastic Property}}{\text{Inertial Property}}}$$

$$\text{Speed of sound in gas} ; V = \sqrt{\frac{B}{\rho}} \quad \begin{matrix} \text{bulk modulus} \\ \text{density} \end{matrix}$$

Speed of sound in air depends on Temperature

$$V = 331 \sqrt{1 + \frac{T_c}{273}} \quad 331 \text{ m/s} \leftarrow \text{Speed of sound in air at } 0^\circ\text{C}$$

Relationship b/w pressure amplitude and displacement amplitude for a sound wave

$$\Delta P_{\text{max}} = P v s_{\text{max}}$$

Intensity of Periodic Sound Waves

$$\text{Power} = F \cdot V_a$$

$$(\text{Power})_{\text{avg}} = \frac{1}{2} \rho v w^2 A s^2$$

$$I = \frac{(\text{Power})_{\text{avg}}}{A}$$

$$I = \frac{1}{2} \rho v w^2 s^2$$

$$I = \frac{(D\text{Power})^2}{2\rho v}$$

Wave intensity of a spherical wave

$$I = \frac{\text{Power}}{A} = \frac{\text{Power}}{4\pi r^2} = \frac{1}{2} \rho v w^2$$

Threshold of

$$\text{Threshold of hearing} = 1.00 \times 10^{-12} \text{ W/m}^2$$

$$\text{Threshold of pain} = 1.00 \text{ W/m}^2$$

ΔP_{max} for threshold of hearing and pain

$$\Delta P_{\text{max}} = \sqrt{2\rho v I}$$

$$= \sqrt{2 \times 1.02 \times 343 \times 1.00 \times 10^{-12}}$$

$$= 2.87 \times 10^{-5} \text{ N/m}^2$$

$$S_{\text{max}} = \frac{\Delta P_{\text{max}}}{\rho v w} = \frac{2.87 \times 10^{-5}}{1.02 \times 343 \times (2\pi \times 1000)} \\ \approx 1.11 \times 10^{-11} \text{ m}$$

Sound level in Decibels

$$B = 10 \log \left(\frac{I}{I_0} \right) \quad \text{Where } I_0 = \text{Threshold of hearing}$$

Sound level for threshold of pain = 120 dB

" " " by " hearing = 0 dB

Shock waves

$$\sin \theta = \frac{vt}{st} = \frac{v}{16}$$

$v_s/v = \text{Mach number}$

Conical wavefront produced when $v_s > v$ is a shock wave

Superposition & Standing Waves

- * A particle is zero size, whereas a wave has a characteristic size, its wavelength.
- * Two waves can combine at one point and some medium. particles can be combined to form extended objects, but they must be at different locations.
- * When two waves combine having nearly the same frequency interfere we hear variations in loudness called beats.

Waves in Interference

Waves ~~at~~ combining at the same location are analyzed using the superposition principle

- Superposition Principle: If two or more waves are moving thru a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave function of the individual waves.

- * Waves that obey the principle are called linear waves. Linear waves are generally characterised by having amplitudes much smaller than their wavelengths.
- * Waves that violate the superposition principle are non linear waves and are often characterised by large amplitudes.

- * Two waves can pass in each other without being destroyed or altered.
- * The combination of separate waves in the same region of space produce a resultant wave is called interference.
- * If the displacement caused by two pulses are in same direction, the superposition is referred to as constructive interference.
- * If the displacement caused is in different directions; it known as Destructive interference.

Superposition of Sinusoidal waves

If the two waves are travelling in same direction and have the same frequency, wavelength and amplitude, but differ in phase

Their individual wave function is expressed as

$$y_1 = A \sin(kx - \omega t) \quad y_2 = A \sin(kx - \omega t + \phi)$$

where $k = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$. and ϕ is the phase angle constant.

$$\text{Resultant wave function } y = y_1 + y_2 = A [\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

Using the trigonometric identity

$$\sin a + \sin b = 2 \cos\left(\frac{a-b}{2}\right) \sin\left(\frac{a+b}{2}\right)$$

taking $a = kx - \omega t$ and $b = kx - \omega t + \phi$

$$y = 2A \cos\left(\frac{(kx - \omega t) - (kx - \omega t + \phi)}{2}\right) \sin\left(\frac{(kx - \omega t) + (kx - \omega t + \phi)}{2}\right)$$

$$y = 2A \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

The resultant wave function is also sinusoidal and has same frequency and wavelength as the individual waves

The amplitude is $2A \cos\left(\frac{\phi}{2}\right)$

Constructive interference occurs when $\cos(\phi/2) = \pm 1$ i.e. when ϕ is an even multiple of π .

When ϕ equals π rad or to any odd multiple of π , then $\cos(\phi/2) = \cos(\pi/2) = 0$ and the crest of one wave occur at the same position as the trough of the second wave.

The resultant wave has zero amplitude everywhere

When the phase constant has an arbitrary other than 0 or an integer multiple of π rad, the resultant wave has an amplitude whose value is somewhere between 0 and $2A$.

In a more general case where the waves are same wavelength but different amplitudes, the results are similar with the following exceptions.

(i) In the phase case they do not completely lose the amplitude of d

wave is not trace that of a single wave but rather, is a sum of the amplitudes of the two waves

② When the waves are ~~travel~~ out of phase, they do not completely cancel out. The result is a wave whose amplitude is the different in amplitudes of the individual waves.

Interference of sound waves

* The distance along any path from the sound source to the receiver is called the path length.

* A phase difference may arise between two waves generated by the same source when they travel along paths of unequal lengths.

Standing Waves

$$y = (2A)$$

$$y_1 = A \sin(kx - \omega t) \quad y_2 = A \sin(kx + \omega t)$$

$$\begin{aligned}y = y_1 + y_2 &= A \sin(kx - \omega t) + A \sin(kx + \omega t) \\&= 2A \sin(kx) \cos(\omega t)\end{aligned}$$

Using trigonometric identity

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\text{thus } Y = (2A \sin kx) \cos \omega t$$

wave function of a standing wave

* The amplitude of the simple harmonic motion of an element of the medium has a minimum value of zero when x satisfies the condition $\sin kx = 0$ i.e. when

$$kx = 0, \pi, 2\pi, 3\pi$$

because $k = 2\pi/\lambda$; these values give

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots = \frac{n\lambda}{2} \quad n = 0, 1, 2, 3$$

These points of zero amplitude are called nodes

* The elements of the medium with the greatest possible displacement from equilibrium has an amplitude of ~~ext~~

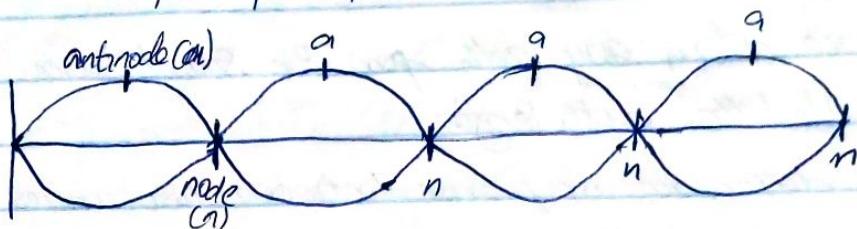
* The position of the medium at which this maximum displacement occurs are called antinodes

The antinodes are located at positions for which the coordinate x satisfies the condition since $x = \pm 1$

$$Kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

Therefore the positions of the antinodes are

$$x = \frac{1}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} = \frac{n\lambda}{4} \quad n=1, 3, 5$$



- * The distance b/w adjacent antinodes is equal $\lambda/2$
- * The distance between adjacent nodes is equal $\lambda/2$
- * The distance between a node and an adjacent antinode is $\lambda/4$

Example

Two waves are travelling in opposite directions produce a standing wave. The individual wave functions are

$$y_1 = 4 \sin(3x - 2t)$$

$$y_2 = 4 \sin(3x + 2t)$$

x and y are in cm and t is seconds

- A) Find a) amplitude of the simple harmonic motion of the medium at $x = 2.3\text{cm}$

$$y = y_1 + y_2 = (2A \sin Kx) \cos \omega t = 8 \sin 3.0x \cos 2.0t$$

$$y_{\max} = 8.0 \sin(3x 2.3\text{ rad})$$

$$y_{\max} = 4.6 \text{ cm}$$

- B) Find the positions of the nodes and antinodes if one end of the string is at $x = 0$

first find the wavelength of the travelling wave

$$K = \frac{2\pi}{\lambda} \quad \lambda = \frac{2\pi}{K}$$

$$\lambda = \frac{2\pi}{3} \text{ cm}$$

$$\text{location of nodes } x = n\lambda = \frac{2\pi}{2} \times \frac{x}{2}$$

$$\Rightarrow x = n \left(\frac{\lambda}{3.0} \right) \text{ cm} \quad n = 0, 1, 2, 3, \dots$$

location of antinode

$$x = n \frac{\lambda}{4} = n \left(\frac{\pi}{6} \right) \text{ cm} \quad n = 1, 3, 5, 7$$

Waves Under boundary Conditions

- * There is a boundary condition for waves on a string fixed at both ends.
- * Because the ends are fixed they must necessarily have zero displacement and are therefore nodes by definition.
- The boundary condition results in the string having a number of discrete natural patterns of oscillations called normal modes, each of which has a characteristic frequency that is easily calculated.
- * The situation in which only certain frequencies of oscillation are allowed is called quantization.
- * There is no quantization without boundary conditions.
- * The section of a standing wave from one node to the next node is called a loop.

for 1P

In the first normal mode; the string is vibrating in one loop
so first harmonic $L = \frac{1}{2}\lambda_1$

In the second mode; the string vibrates in two loops

2nd harmonic $\Rightarrow L = \lambda_2$

The third normal mode; i.e. 3rd harmonic

$$L = \frac{3}{2}\lambda_3$$

so Wavelengths of normal modes ; $\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, \dots$

$$f_n = \frac{V}{\lambda_n} = n \frac{V}{2L} \quad n = 1, 2, 3$$

These natural frequencies are also called quantized frequencies with

the vibrating string fixed at both ends

because $v = \sqrt{\frac{T}{\mu}}$ for waves on string where T is tension
of string and μ is its linear mass density

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

The lowest frequency f_1 which corresponds to $n=1$ is called the fundamental frequency or the fundamental and is given by

$$f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$$

- * The frequency of the remaining normal mode are integer multiple of the fundamental frequency
- * Such frequency that exhibit integer multiple relationship form a harmonic series, and the normal modes are called harmonics
- * The fundamental frequency f_1 is the first harmonic, the frequency $f_2 = 2f_1$ is the second harmonic, and the frequency $f_n = nf_1$ is that of the n th harmonic
- * As the tension on a string increases, the frequency on the normal modes increases in accordance
- * As the length is shortened, the frequency increases
- * The normal mode frequency are inversely proportional to string length

Resonance

If a periodic force is applied to a string with a vibrating blade, the amplitude of the resulting motion of the string is greatest when the frequency of the applied force is equal to one of the natural frequencies of the system.

This phenomenon is known as resonance

The frequencies are also often referred to as resonance frequencies

Standing Waves in Air Columns

for a pipe open at both ends, the ends are displacement antinodes and the harmonic series contains all integer multiples of the fundamental frequency

1st harmonic

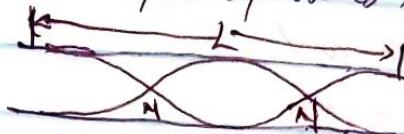


$$\lambda_1 = \frac{L}{2}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{\frac{L}{2}} = \frac{2v}{L}$$

where v = speed of waves in air string

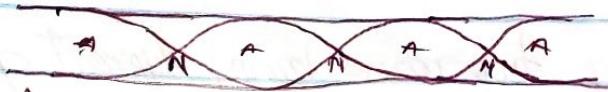
2nd harmonic



$$\lambda_2 = \frac{L}{3}$$

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{\frac{L}{3}} = \frac{3v}{L} = 3f_1$$

3rd harmonic



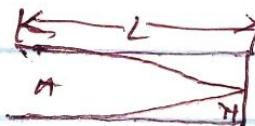
$$\lambda_3 = \frac{2}{3} L$$

$$f_3 = \frac{3v}{\lambda_3} = \frac{3v}{\frac{2}{3}L} = \frac{9v}{2L} = 9f_1$$

In a pipe closed at one end, the open end is a displacement antinode and the closed end is a node

* the harmonic series contains only odd integer multiples of the fundamental frequency

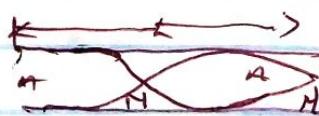
1st harmonic



$$\lambda_1 = 4L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

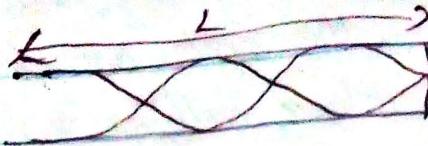
3rd harmonic



$$\lambda_3 = \frac{4}{3} L$$

$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{\frac{4}{3}L} = \frac{9v}{4L} = 9f_1$$

fifth harmonic



$$\lambda_5 = \frac{4L}{5}$$

$$f_5 = \frac{5v}{4L} = \frac{5}{4} f_1$$

So for a pipe open at both ends

$$f_n = n \frac{v}{2L} \quad \text{where } n = 1, 2, 3, \dots, n$$

for a pipe closed at one end

$$f_n = n \frac{v}{4L} \quad n = 1, 3, 5, \dots$$

* Speed of sound increases in warmer air inside a wind instrument (~~Stringed~~ non stringed)

Resultant of two waves having different frequencies but equal amplitude

$$y = \left[2A \cos 2\pi \left(\frac{f_1 - f_2}{2} t \right) \right] \cos 2\pi \left(\frac{f_1 + f_2}{2} t \right)$$

Envelope wave Envelope = $2A \cos 2\pi \left(\frac{f_1 - f_2}{2} t \right)$

$$f = [\lambda R(\nu - \nu_0)]^{1/2}$$

ν = bright ray number

λ = wavelength

R = Radius of curvature

f_p = radius of bright fringe

(LAGOS STATE UNIVERSITY, OJO)

$$f_p^2 = \lambda R (\nu - \nu_0)$$

$$\lambda = \frac{4f^2}{R(\nu - \nu_0)}$$

DEPARTMENT OF PHYSICS

2014/2015 RAIN SEMESTER SESSIONAL EXAMINATION

COURSE TITLE: BASIC OPTICS & SOUND;

TIME: 1HR & 15MINS

COURSE CODE: PHY.102
INSTRUCTION: ANSWER ALL QUESTIONS
OPTION B

QUESTION: A light of wavelength 750nm is used to illuminate the slits which are spaced 2 mm apart in a double slit interference experiment. If the screen is 120 cm from the slits, calculate the distance between the central maximum and the first minimum.

N = bright
 A = angle
 R = Radius
 f = radius
 LAGOS
 DEF

2010/2015 RAIN

COURSE CODE: PHY 102

INSTRUCTION: ANSWER ALL QUESTIONS

1. Sound can travel through?

- A. Only Gases
- B. Only Solids
- C. Only Liquids
- D. In solids, liquid and gases ✓

 Answer-1

 Post-Your-Explanation-1

2. Which of the following voices is likely to have a minimum frequency?

- A. Baby Boy
- B. Baby Gir
- C. A Man ✓
- D. A Women

 Answer-2

 Post-Your-Explanation-2

3. When a wave travels through a medium -----.

- A. energy is
- B. energy is

3. When a wave travels through a medium -----.

- A. energy is transferred at a constant speed ✓
- B. energy is transferred in a periodic manner
- C. particles are transferred from one place to another
- D. none of these

 Answer-3

 Post-Your-Explanation-3

4. Bats detect the obstacles in their path by receiving the reflected -----.

- A. radio waves
- B. ultrasonic waves ✓
- C. electromagnetic waves
- D. infrasonic waves

 Answer-4

 Post-Your-Explanation-4

14. A sound wave is produced when an object

- A. accelerates
- B. decelerates
- C. vibrates ✓
- D. remains stationary

 Answer-14

 Post-Your-Explanation-14

15. The wavelength of a wave is measured in

- A. metres ✓
- B. hertz
- C. seconds
- D. decibels

 Answer-15

 Post-Your-Explanation-15

11. Which of the following will remain unchanged when a sound wave travels in air or in water?

- A. frequency ✓
- B. wavelength
- C. amplitude
- D. speed

 Answer-11

 Post-Your-Explanation-11

12. _____ is an example for mechanical wave.

- A. radio wave
- B. light wave
- C. sound wave ✓
- D. infrared wave

 Answer-12

 Post-Your-Explanation-12

13. Which of the following quantities is transferred during wave propagation?

- A. speed
- B. mass
- C. energy ✓
- D. matter

7. Sound waves do not travel through

- A. vacuum ✓
- B. soild
- C. liquid
- D. gases

 Answer-7

 Post-Your-Explanation-7

8. Sound waves are ----- in nature

- A. longitudinal ✓
- B. transverse
- C. partly longitudinal
- D. sometimes longitudinal and sometimes transverse

 Answer-8

 Post-Your-Explanation-8

19. Which of the following does not describe a sound wave?

- A. transverse wave
- B. longitudinal wave
- C. compression wave
- D. push-pull wave

 Answer-19

 Post-Your-Explanation-19

20. An animal that can hear sound frequencies higher than a human child is

- A. a human adult
- B. a cod fish
- C. a bat
- D. an eagle

 Answer-20

 Post-Your-Explanation-20

9. The amplitude of a wave is _____.

- A. the distance the wave moves in one second
- B. the maximum distance moved by the medium particles on either side of the mean position ✓
- C. the distance the wave moves in one time one wave length period of the wave
- D. the distance equal to one wave length

 Answer-9

 Post-Your-Explanation-9

10. The speed of sound in medium depends upon

- A. frequency
- B. amplitude
- C. wavelength
- D. properties of the medium ✓

 Answer-10

 Post-Your-Explanation-10

5. The minimum distance between the source and the reflector, so that an echo is heard is approximately

- A. 12 m
- B. 15 m
- C. 17 m ✓
- D. 21 m

 Answer-5

← Post-Your-Explanation-5

6. When sound travels through air, the air particles

-----.

- A. do not vibrate
- B. vibrate but not in any fixed direction
- C. vibrate
- D. vibrate along the perpendicular to the direction of wave propagation ✓



HANGOUTS

now

Aswin**Aswin: Okay**

- A. absorbed
- B. transmitted
- C. refracted
- D. reflected ✓

i Answer-16 [Post-Your-Explanation-16](#)**17. Another name for the unit Hertz is**

- A. cycles per second ✓
- B. seconds per cycle
- C. metres per second
- D. decibels

i Answer-17 [Post-Your-Explanation-17](#)**18. Sound travels fastest in**

- A. a vacuum
- B. the sea
- C. the atmosphere
- D. a broom stick ✓

i Answer-18 [Post-Your-Explanation-18](#)

7. Sound waves do not travel through

- A. vacuum ✓
- B. soild
- C. liquid
- D. gases

 Answer-7

 Post-Your-Explanation-7

8. Sound waves are _____ in nature

- A. longitudinal ✓
- B. transverse
- C. partly longitudinal
- D. sometimes longitudinal and sometimes transverse
- and partly transverse

 Answer-8

 Post-Your-Explanation-8

21. Hitting a drum harder makes the sound

- A. higher
- B. lower
- C. louder ✓
- D. softer

 Answer-21

 Post-Your-Explanation-21

22. The part of the ear that responds to sound waves like a microphones diaphragm is the

- A. lobe
- B. eardrum ✓
- C. bones of the middle ear
- fluid in the inner ear

 Answer-22

 Post-Your-Explanation-22

23. Sounds above 20000Hz are called

- A. ultracool
- B. ultrasound ✓
- C. infra-audio
- infrasound