Because $V_{ab} = I_1R_1 = I_2R_2$, it follows that

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \quad \text{(two resistors in parallel)} \tag{26.4}$$

Thus the currents carried by two resistors in parallel are *inversely proportional* to their resistances. More current goes through the path of least resistance.

PROBLEM-SOLVING STRATEGY 26.1 Resistors in Series and Parallel

IDENTIFY *the relevant concepts:* As in Fig. 26.1, many resistor networks are made up of resistors in series, in parallel, or a combination thereof. Such networks can be replaced by a single equivalent resistor. The logic is similar to that of Problem-Solving Strategy 24.1 for networks of capacitors.

SET UP *the problem* using the following steps:

1. Make a drawing of the resistor network.
2. Identify groups of resistors connected in series or parallel.
3. Identify the target variables. They could include the equivalent resistance of the network, the potential difference across each resistor, or the current through each resistor.

EXECUTE *the solution* as follows:

1. Use Eq. (26.1) or (26.2), respectively, to find the equivalent resistance for series or parallel combinations.
2. If the network is more complex, try reducing it to series and parallel combinations. For example, in Fig. 26.1c we first replace the parallel combination of R_2 and R_3 with its equivalent resistance; this then forms a series combination with R_1. In Fig. 26.1d, the combination of R_2 and R_3 in series forms a parallel combination with R_1.
3. Keep in mind that the total potential difference across resistors connected in series is the sum of the individual potential differences. The potential difference across resistors connected in parallel is the same for every resistor and equals the potential difference across the combination.
4. The current through resistors connected in series is the same through every resistor and equals the current through the combination. The total current through resistors connected in parallel is the sum of the currents through the individual resistors.

EVALUATE *your answer:* Check whether your results are consistent. The equivalent resistance of resistors connected in series should be greater than that of any individual resistor; that of resistors in parallel should be less than that of any individual resistor.

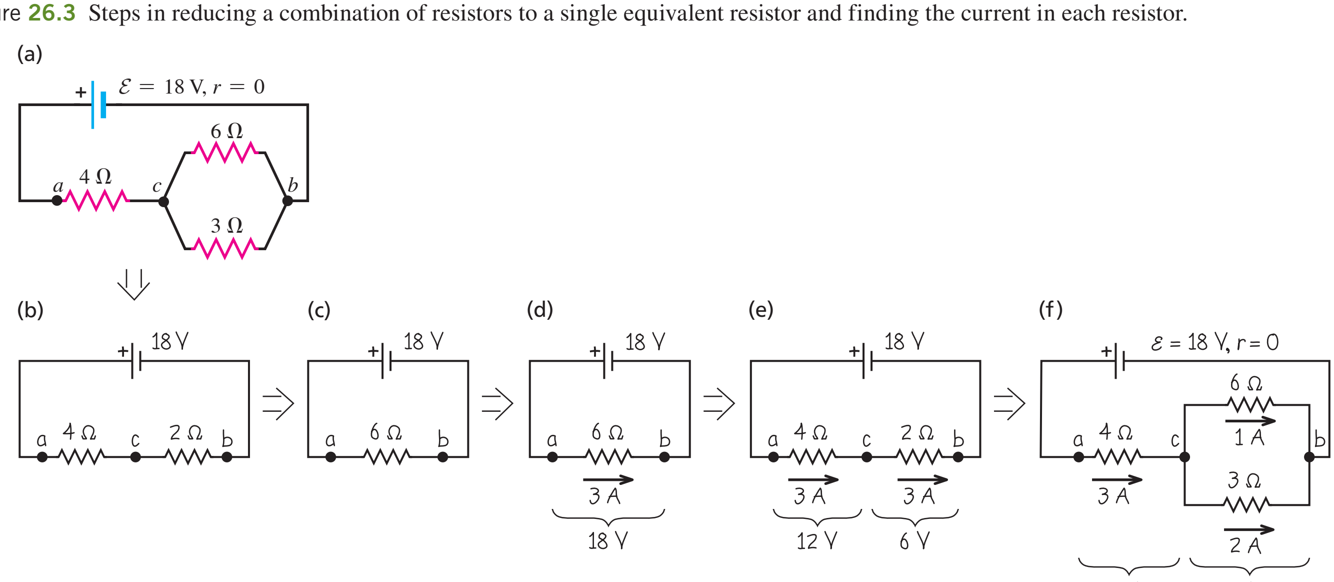
EXAMPLE 26.1 Equivalent resistance

WITH VARIATION PROBLEMS

Find the equivalent resistance of the network in **Fig. 26.3a** and the current in each resistor. The source of emf has negligible internal resistance.

IDENTIFY and SET UP This network of three resistors is a *combination* of series and parallel resistances, as in Fig. 26.1c. We determine the equivalent resistance of the parallel 6 Ω and 3 Ω resistors, and then that of their series combination with the 4 Ω resistor: This is the equivalent resistance R_{eq} of the network as a whole. We then find the current in the emf, which is the same as that in the 4 Ω resistor. The potential difference is the same across each of the parallel 6 Ω and 3 Ω resistors; we use this to determine how the current is divided between these.

Figure 26.3 Steps in reducing a combination of resistors to a single equivalent resistor and finding the current in each resistor.



Continued

EXECUTE Figures 26.3b and 26.3c show successive steps in reducing the network to a single equivalent resistance R_{eq}. From Eq. (26.2), the 6 Ω and 3 Ω resistors in parallel in Fig. 26.3a are equivalent to the single 2 Ω resistor in Fig. 26.3b:

$$\frac{1}{R_{6\,\Omega+3\,\Omega}} = \frac{1}{6\,\Omega} + \frac{1}{3\,\Omega} = \frac{1}{2\,\Omega}$$

[Equation (26.3) gives the same result.] From Eq. (26.1) the series combination of this 2 Ω resistor with the 4 Ω resistor is equivalent to the single 6 Ω resistor in Fig. 26.3c.

We reverse these steps to find the current in each resistor of the original network. In the circuit shown in Fig. 26.3d (identical to Fig. 26.3c), the current is $I = V_{ab}/R = (18\text{ V})/(6\,\Omega) = 3\text{ A}$. So the current in the 4 Ω and 2 Ω resistors in Fig. 26.3e (identical to Fig. 26.3b) is also 3 A. The potential difference V_{cb} across the 2 Ω resistor is therefore $V_{cb} = IR = (3\text{ A})(2\,\Omega) = 6\text{ V}$. This potential difference must also be 6 V in Fig. 26.3f (identical to Fig. 26.3a). From $I = V_{cb}/R$, the currents in the 6 Ω and 3 Ω resistors in Fig. 26.3f are, respectively, $(6\text{ V})/(6\,\Omega) = 1\text{ A}$ and $(6\text{ V})/(3\,\Omega) = 2\text{ A}$.

EVALUATE Note that for the two resistors in parallel between points *c* and *b* in Fig. 26.3f, there is twice as much current through the 3 Ω resistor as through the 6 Ω resistor; more current goes through the path of least resistance, in accordance with Eq. (26.4). Note also that the total current through these two resistors is 3 A, the same as it is through the 4 Ω resistor between points *a* and *c*.

KEYCONCEPT When resistors are combined in *series,* the current is the same through each resistor but the potential differences across the resistors may not be the same. The equivalent resistance of a series combination equals the sum of the individual resistances. When resistors are combined in *parallel,* the potential difference across each resistor is the same but the currents through the resistors may not be the same. The reciprocal of the equivalent resistance of a parallel combination equals the sum of the reciprocals of the individual resistances.

EXAMPLE 26.2 Series versus parallel combinations

WITH VARIATION PROBLEMS

Two identical incandescent light bulbs, each with resistance $R = 2\,\Omega$, are connected to a source with $\mathcal{E} = 8\text{ V}$ and negligible internal resistance. Find the current through each bulb, the potential difference across each bulb, and the power delivered to each bulb and to the entire network if the bulbs are connected (a) in series and (b) in parallel. (c) Suppose one of the bulbs burns out; that is, its filament breaks and current can no longer flow through it. What happens to the other bulb in the series case? In the parallel case?

IDENTIFY and SET UP The light bulbs are just resistors in simple series and parallel connections (**Figs. 26.4a** and 26.4b). Once we find the current I through each bulb, we can find the power delivered to each bulb by using Eq. (25.18), $P = I^2R = V^2/R$.

EXECUTE (a) From Eq. (26.1) the equivalent resistance of the two bulbs between points *a* and *c* in Fig. 26.4a is $R_{eq} = 2R = 2(2\,\Omega) = 4\,\Omega$. In series, the current is the same through each bulb:

$$I = \frac{V_{ac}}{R_{eq}} = \frac{8\text{ V}}{4\,\Omega} = 2\text{ A}$$

Since the bulbs have the same resistance, the potential difference is the same across each bulb:

$$V_{ab} = V_{bc} = IR = (2\text{ A})(2\,\Omega) = 4\text{ V}$$

From Eq. (25.18), the power delivered to each bulb is

$$P = I^2R = (2\text{ A})^2(2\,\Omega) = 8\text{ W} \qquad \text{or}$$

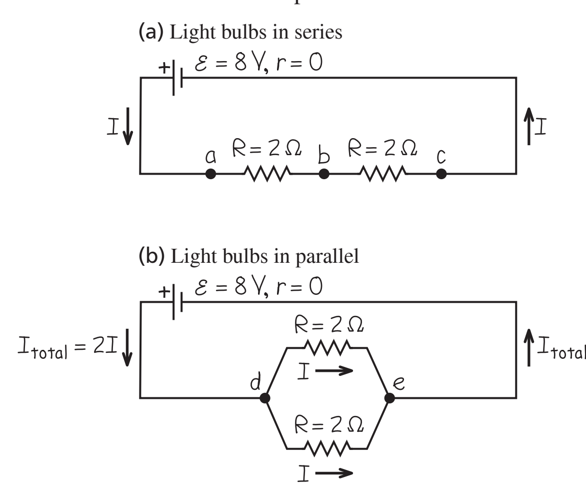
$$P = \frac{V_{ab}^2}{R} = \frac{V_{bc}^2}{R} = \frac{(4\text{ V})^2}{2\,\Omega} = 8\text{ W}$$

The total power delivered to both bulbs is $P_{tot} = 2P = 16\text{ W}$.

(b) If the bulbs are in parallel, as in Fig. 26.4b, the potential difference V_{de} across each bulb is the same and equal to 8 V, the terminal voltage of the source. Hence the current through each light bulb is

$$I = \frac{V_{de}}{R} = \frac{8\text{ V}}{2\,\Omega} = 4\text{ A}$$

Figure **26.4** Our sketches for this problem.



and the power delivered to each bulb is

$$P = I^2R = (4\text{ A})^2(2\,\Omega) = 32\text{ W} \qquad \text{or}$$

$$P = \frac{V_{de}^2}{R} = \frac{(8\text{ V})^2}{2\,\Omega} = 32\text{ W}$$

Both the potential difference across each bulb and the current through each bulb are twice as great as in the series case. Hence the power delivered to each bulb is *four* times greater, and each bulb is brighter.

The total power delivered to the parallel network is $P_{total} = 2P = 64\text{ W}$, four times greater than in the series case. The increased power compared to the series case isn't obtained "for free"; energy is extracted from the source four times more rapidly in the parallel case than in the series case. If the source is a battery, it will be used up four times as fast.

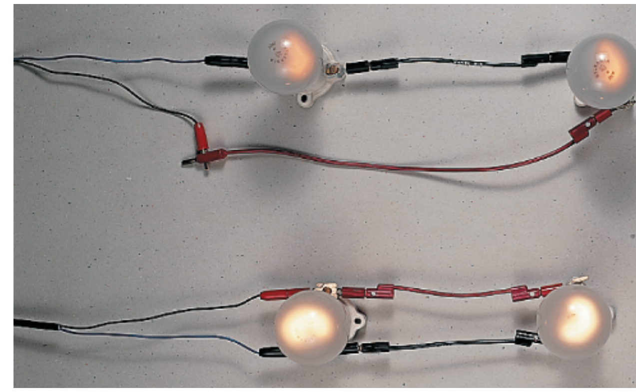
(c) In the series case the same current flows through both bulbs. If one bulb burns out, there will be no current in the circuit, and neither bulb will glow.

In the parallel case the potential difference across either bulb is unchanged if a bulb burns out. The current through the functional bulb and the power delivered to it are unchanged.

EVALUATE Our calculation isn't completely accurate, because the resistance $R = V/I$ of real light bulbs depends on the potential difference V across the bulb. That's because the filament resistance increases with increasing operating temperature and therefore with increasing V. But bulbs connected in series across a source do in fact glow less brightly than when connected in parallel across the same source (**Fig. 26.5**).

KEYCONCEPT Two or more resistors connected in parallel to a source each have more current through them and each draw more power than if they were connected in series to the same source.

Figure **26.5** When connected to the same source, two incandescent light bulbs in series (shown at top) draw less power and glow less brightly than when they are in parallel (shown at bottom).



TEST YOUR UNDERSTANDING OF SECTION 26.1 Suppose all three of the resistors shown in Fig. 26.1 have the same resistance, so $R_1 = R_2 = R_3 = R$. Rank the four arrangements shown in parts (a)–(d) of Fig. 26.1 in order of their equivalent resistance, from highest to lowest.

ANSWER

❙ **(a), (c), (d), (b)** Here's why: The three resistors in Fig. 26.1a are in series, so $R_{eq} = R + R + R = 3R$. In Fig. 26.1b the three resistors are in parallel, so $1/R_{eq} = 1/R + 1/R + 1/R = 3/R$ and $R_{eq} = R/3$. In Fig. 26.1c the second and third resistors are in parallel, so their equivalent resistance R_{23} is given by $1/R_{23} = 1/R + 1/R = 2/R$; hence $R_{23} = R/2$. This combination is in series with the first resistor, so the three resistors together have equivalent resistance $R_{eq} = R + R/2 = 3R/2$. In Fig. 26.1d the second and third resistors are in series, so their equivalent resistance is $R_{23} = R + R = 2R$. This combination is in parallel with the first resistor, so the equivalent resistance of the three-resistor combination is given by $1/R_{eq} = 1/R + 1/2R = 3/2R$. Hence $R_{eq} = 2R/3$.

26.2 KIRCHHOFF'S RULES

Many practical resistor networks cannot be reduced to simple series-parallel combinations. **Figure 26.6a** shows a dc power supply with emf $\mathcal{E}_1$ charging a battery with a smaller emf $\mathcal{E}_2$ and feeding current to a light bulb with resistance R. Figure 26.6b is a "bridge" circuit, used in many different types of measurement and control systems. (Problem 26.74 describes one important application of a "bridge" circuit.) To analyze these networks, we'll use the techniques developed by the German physicist Gustav Robert Kirchhoff (1824–1887).

First, here are two terms that we'll use often. A **junction** in a circuit is a point where three or more conductors meet. A **loop** is any closed conducting path. In Fig. 26.6a points a and b are junctions, but points c and d are not; in Fig. 26.6b points a, b, c, and d are junctions, but points e and f are not. The blue lines in Figs. 26.6a and 26.6b show some possible loops in these circuits.

Kirchhoff's rules are the following two statements:

Kirchhoff's junction rule (valid at any junction): The sum of the currents into any junction ...

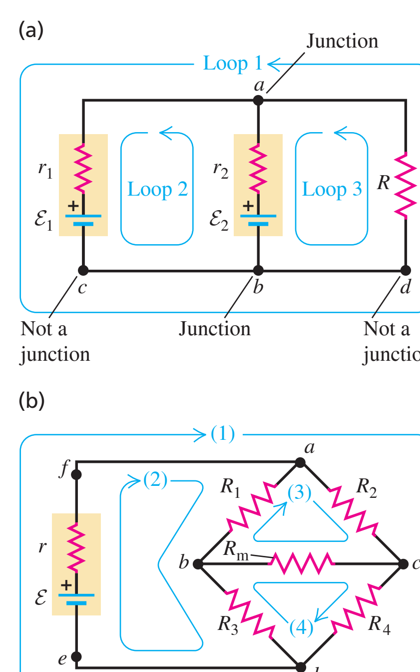
$$\sum I = 0 \quad \text{... equals zero.} \tag{26.5}$$

Kirchhoff's loop rule (valid for any closed loop): The sum of the potential differences around any loop ...

$$\sum V = 0 \quad \text{... equals zero.} \tag{26.6}$$

Note that the potential differences V in Eq. (26.6) include those associated with all circuit elements in the loop, including emfs and resistors.

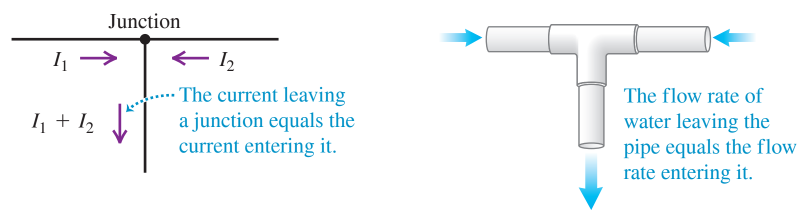
Figure **26.6** Two networks that cannot be reduced to simple series-parallel combinations of resistors.



CAUTION **Current need not split equally at a junction** As a rule, at a junction where the current in a wire splits to follow two or more separate paths, the current does *not* split equally among the different paths. More current flows along a path that has less resistance.

Figure 26.7 Kirchhoff's junction rule states that as much current flows into a junction as flows out of it.

(a) Kirchhoff's junction rule (b) Water-pipe analogy



The junction rule is based on *conservation of electric charge*. No charge can accumulate at a junction, so the total charge entering the junction per unit time must equal the total charge leaving per unit time (**Fig. 26.7a**). Charge per unit time is current, so if we consider the currents entering a junction to be positive and those leaving to be negative, the algebraic sum of currents into a junction must be zero. It's like a T branch in a water pipe (Fig. 26.7b); if you have a total of 1 liter per minute coming in the two pipes, you can't have 3 liters per minute going out the third pipe. We used the junction rule (without saying so) in Section 26.1 in the derivation of Eq. (26.2) for resistors in parallel.

The loop rule is a statement that the electrostatic force is *conservative*. Suppose we go around a loop, measuring potential differences across successive circuit elements as we go. When we return to the starting point, we must find that the *algebraic sum* of these differences is zero; otherwise, we could not say that the potential at this point has a definite value.

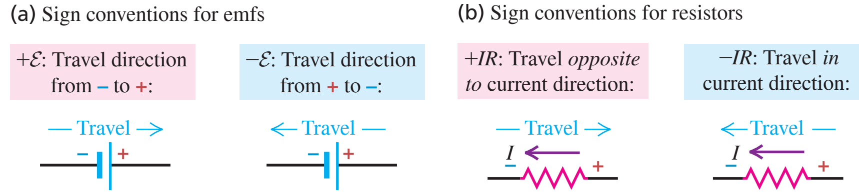
Sign Conventions for the Loop Rule

In applying the loop rule, we need some sign conventions. Problem-Solving Strategy 26.2 describes in detail how to use these, but here's a quick overview. We first assume a direction for the current in each branch of the circuit and mark it on a diagram of the circuit. Then, starting at any point in the circuit, we imagine traveling around a loop, adding emfs and IR terms as we come to them. When we travel through a source in the direction from $-$ to $+$, the emf is considered to be *positive;* when we travel from $+$ to $-$, the emf is considered to be *negative* (**Fig. 26.8a**). When we travel through a resistor in the *same* direction as the assumed current, the IR term is *negative* because the current goes in the direction of decreasing potential. When we travel through a resistor in the direction *opposite* to the assumed current, the IR term is *positive* because this represents a rise of potential (Fig. 26.8b).

Kirchhoff's two rules are all we need to solve a wide variety of network problems. Usually, some of the emfs, currents, and resistances are known, and others are unknown. We must always obtain from Kirchhoff's rules a number of independent equations equal to the number of unknowns so that we can solve the equations simultaneously. Often the hardest part of the solution is keeping track of algebraic signs!

Figure 26.8 Use these sign conventions when you apply Kirchhoff's loop rule. In each part of the figure "Travel" is the direction that we imagine going around the loop, which is not necessarily the direction of the current.

(a) Sign conventions for emfs (b) Sign conventions for resistors



PROBLEM-SOLVING STRATEGY 26.2 Kirchhoff's Rules

IDENTIFY *the relevant concepts:* Kirchhoff's rules are useful for analyzing any electric circuit.

SET UP *the problem* using the following steps:

1. Draw a circuit diagram, leaving room to label all quantities, known and unknown. Indicate an assumed direction for each unknown current and emf. (Kirchhoff's rules will yield the magnitudes *and directions* of unknown currents and emfs. If the actual direction of a quantity is opposite to your assumption, the resulting quantity will have a negative sign.)
2. As you label currents, it's helpful to use Kirchhoff's junction rule, as in **Fig. 26.9**, so as to express the currents in terms of as few quantities as possible.
3. Identify the target variables.

EXECUTE *the solution* as follows:

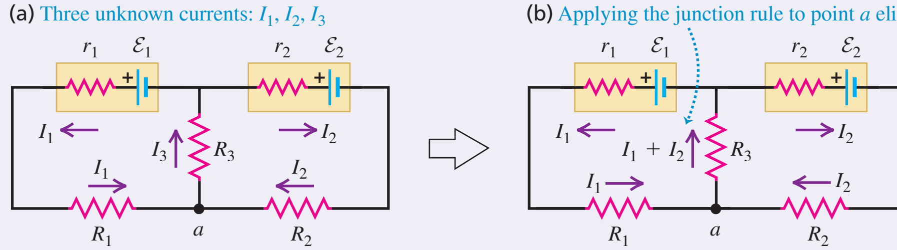
1. Choose any loop in the network and choose a direction (clockwise or counterclockwise) to travel around the loop as you apply Kirchhoff's loop rule. The direction need not be the same as any assumed current direction.
2. Travel around the loop in the chosen direction, adding potential differences algebraically as you cross them. Use the sign conventions of Fig. 26.8.
3. Equate the sum obtained in step 2 to zero in accordance with the loop rule.
4. If you need more independent equations, choose another loop and repeat steps 1–3; continue until you have as many independent equations as unknowns or until every circuit element has been included in at least one loop.
5. Solve the equations simultaneously to determine the unknowns.
6. You can use the loop-rule bookkeeping system to find the potential V_{ab} of any point a with respect to any other point b. Start at b and add the potential changes you encounter in going from b to a; use the same sign rules as in step 2. The algebraic sum of these changes is $V_{ab} = V_a - V_b$.

EVALUATE *your answer:* Check all the steps in your algebra. Apply steps 1 and 2 to a loop you have not yet considered; if the sum of potential drops isn't zero, you've made an error somewhere.

Figure **26.9** Applying the junction rule to point a reduces the number of unknown currents from three to two.

(a) Three unknown currents: I_1, I_2, I_3

(b) Applying the junction rule to point a eliminates I_3.



EXAMPLE 26.3 A single-loop circuit

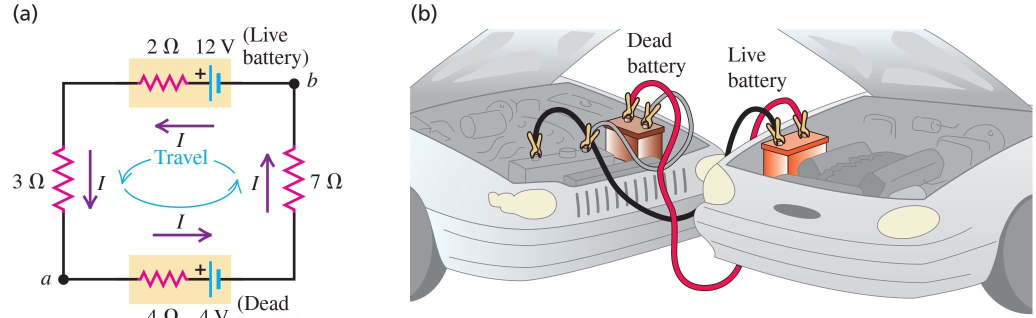
WITH VARIATION PROBLEMS

The circuit shown in **Fig. 26.10a** contains two batteries, each with an emf and an internal resistance, and two resistors. Find (a) the current in the circuit, (b) the potential difference V_{ab}, and (c) the power output of the emf of each battery.

IDENTIFY and SET UP There are no junctions in this single-loop circuit, so we don't need Kirchhoff's junction rule. To apply Kirchhoff's loop rule, we first assume a direction for the current; let's assume a counterclockwise direction as shown in Fig. 26.10a.

Figure **26.10** (a) In this example we travel around the loop in the same direction as the assumed current, so all the IR terms are negative. The potential decreases as we travel from + to − through the bottom emf but increases as we travel from − to + through the top emf. (b) A real-life example of a circuit of this kind.

(a) (b)



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EXECUTE (a) Starting at a and traveling counterclockwise with the current, we add potential increases and decreases and equate the sum to zero as in Eq. (26.6):

$$-I(4\ \Omega) - 4\ \text{V} - I(7\ \Omega) + 12\ \text{V} - I(2\ \Omega) - I(3\ \Omega) = 0$$

Collecting like terms and solving for I, we find

$$8\ \text{V} = I(16\ \Omega) \qquad \text{and} \qquad I = 0.5\ \text{A}$$

The positive result for I shows that our assumed current direction is correct.

(b) To find V_{ab}, the potential at a with respect to b, we start at b and add potential changes as we go toward a. There are two paths from b to a; taking the lower one, we find

$$V_{ab} = (0.5\ \text{A})(7\ \Omega) + 4\ \text{V} + (0.5\ \text{A})(4\ \Omega) = 9.5\ \text{V}$$

Point a is at 9.5 V higher potential than b. All the terms in this sum, including the IR terms, are positive because each represents an *increase* in potential as we go from b to a. For the upper path,

$$V_{ab} = 12\ \text{V} - (0.5\ \text{A})(2\ \Omega) - (0.5\ \text{A})(3\ \Omega) = 9.5\ \text{V}$$

Here the IR terms are negative because our path goes in the direction of the current, with potential decreases through the resistors. The results for V_{ab} are the same for both paths, as they must be in order for the total potential change around the loop to be zero.

(c) The power outputs of the emf of the 12 V and 4 V batteries are

$$P_{12\text{V}} = \mathcal{E}I = (12\ \text{V})(0.5\ \text{A}) = 6\ \text{W}$$
$$P_{4\text{V}} = \mathcal{E}I = (-4\ \text{V})(0.5\ \text{A}) = -2\ \text{W}$$

The negative sign in $\mathcal{E}$ for the 4 V battery appears because the current actually runs from the higher-potential side of the battery to the lower-potential side. The negative value of P means that we are *storing* energy in that battery; the 12 V battery is *recharging* it (if it is in fact rechargeable; otherwise, we're destroying it).

EVALUATE By applying the expression $P = I^2R$ to each of the four resistors in Fig. 26.10a, you can show that the total power dissipated in all four resistors is 4 W. Of the 6 W provided by the emf of the 12 V battery, 2 W goes into storing energy in the 4 V battery and 4 W is dissipated in the resistances.

The circuit shown in Fig. 26.10a is much like that used when a fully charged 12 V storage battery (in a car with its engine running) "jump-starts" a car with a run-down battery (Fig. 26.10b). The run-down battery is slightly recharged in the process. The 3 Ω and 7 Ω resistors in Fig. 26.10a represent the resistances of the jumper cables and of the conducting path through the car with the run-down battery. (The values of the resistances in actual automobiles and jumper cables are considerably lower, and the emf of a run-down car battery isn't much less than 12 V.)

KEYCONCEPT In any loop of a circuit, Kirchhoff's loop rule applies: The sum of the potential changes as you travel around the loop must be zero. If the circuit has only a single loop, the current has the same value everywhere in the circuit.

EXAMPLE 26.4 Charging a battery

WITH VARIATION PROBLEMS

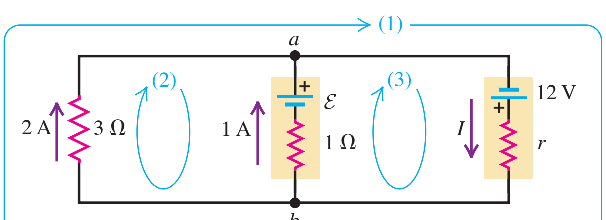
In the circuit shown in **Fig. 26.11**, a 12 V power supply with unknown internal resistance r is connected to a run-down rechargeable battery with unknown emf $\mathcal{E}$ and internal resistance 1 Ω and to an indicator light bulb of resistance 3 Ω carrying a current of 2 A. The current through the run-down battery is 1 A in the direction shown. Find r, $\mathcal{E}$, and the current I through the power supply.

IDENTIFY and SET UP This circuit has more than one loop, so we must apply both the junction and loop rules. We assume the direction of the current through the 12 V power supply, and the polarity of the run-down battery, to be as shown in Fig. 26.11. There are three target variables, so we need three equations.

EXECUTE We apply the junction rule, Eq. (26.5), to point a:

$$-I + 1\ \text{A} + 2\ \text{A} = 0 \qquad \text{so} \qquad I = 3\ \text{A}$$

Figure **26.11** In this circuit a power supply charges a run-down battery and lights a bulb. An assumption has been made about the polarity of the emf $\mathcal{E}$ of the battery. Is this assumption correct?



To determine r, we apply the loop rule, Eq. (26.6), to the large, outer loop (1):

$$12\ \text{V} - (3\ \text{A})r - (2\ \text{A})(3\ \Omega) = 0 \qquad \text{so} \qquad r = 2\ \Omega$$

To determine $\mathcal{E}$, we apply the loop rule to the left-hand loop (2):

$$-\mathcal{E} + (1\ \text{A})(1\ \Omega) - (2\ \text{A})(3\ \Omega) = 0 \qquad \text{so} \qquad \mathcal{E} = -5\ \text{V}$$

The negative value for $\mathcal{E}$ shows that the actual polarity of this emf is opposite to that shown in Fig. 26.11. As in Example 26.3, the battery is being recharged.

EVALUATE Try applying the junction rule at point b instead of point a, and try applying the loop rule counterclockwise rather than clockwise around loop (1). You'll get the same results for I and r. We can check our result for $\mathcal{E}$ by using loop (3):

$$12\ \text{V} - (3\ \text{A})(2\ \Omega) - (1\ \text{A})(1\ \Omega) + \mathcal{E} = 0$$

which again gives us $\mathcal{E} = -5$ V.

As an additional check, we note that $V_{ba} = V_b - V_a$ equals the voltage across the 3 Ω resistance, which is $(2\ \text{A})(3\ \Omega) = 6$ V. Going from a to b by the right-hand branch, we encounter potential differences $+12\ \text{V} - (3\ \text{A})(2\ \Omega) = +6$ V, and going by the middle branch, we find $-(-5\ \text{V}) + (1\ \text{A})(1\ \Omega) = +6$ V. The three ways of getting V_{ba} give the same results.

KEYCONCEPT In any circuit that has more than one loop, Kirchhoff's junction rule applies: At each junction, the sum of the currents into the junction must be zero.

EXAMPLE 26.5 Power in a battery-charging circuit

WITH VARIATION PROBLEMS

In the circuit of Example 26.4 (shown in Fig. 26.11), find the power delivered by the 12 V power supply and by the battery being recharged, and find the power dissipated in each resistor.

IDENTIFY and SET UP We use the results of Section 25.5, in which we found that the power delivered *from* an emf to a circuit is $\mathcal{E}I$ and the power delivered *to* a resistor from a circuit is $V_{ab}I = I^2R$. We know the values of all relevant quantities from Example 26.4.

EXECUTE The power output from the emf of the power supply is

$$P_{\text{supply}} = \mathcal{E}_{\text{supply}}I_{\text{supply}} = (12\text{ V})(3\text{ A}) = 36\text{ W}$$

The power dissipated in the power supply's internal resistance r is

$$P_{r-\text{supply}} = I_{\text{supply}}{}^2 r_{\text{supply}} = (3\text{ A})^2(2\ \Omega) = 18\text{ W}$$

so the power supply's *net* power output is $P_{\text{net}} = 36\text{ W} - 18\text{ W} = 18\text{ W}$. Alternatively, from Example 26.4 the terminal voltage of the battery is $V_{ba} = 6\text{ V}$, so the net power output is

$$P_{\text{net}} = V_{ba}I_{\text{supply}} = (6\text{ V})(3\text{ A}) = 18\text{ W}$$

The power output of the emf $\mathcal{E}$ of the battery being charged is

$$P_{\text{emf}} = \mathcal{E}I_{\text{battery}} = (-5\text{ V})(1\text{ A}) = -5\text{ W}$$

This is negative because the 1 A current runs through the battery from the higher-potential side to the lower-potential side. (As we mentioned in Example 26.4, the polarity assumed for this battery in Fig. 26.11 was wrong.) We are storing energy in the battery as we charge it. Additional power is dissipated in the battery's internal resistance; this power is

$$P_{r-\text{battery}} = I_{\text{battery}}{}^2 r_{\text{battery}} = (1\text{ A})^2(1\ \Omega) = 1\text{ W}$$

The total power input to the battery is thus $1\text{ W} + |-5\text{ W}| = 6\text{ W}$. Of this, 5 W represents useful energy stored in the battery; the remainder is wasted in its internal resistance. The power dissipated in the light bulb is

$$P_{\text{bulb}} = I_{\text{bulb}}{}^2 R_{\text{bulb}} = (2\text{ A})^2(3\ \Omega) = 12\text{ W}$$

EVALUATE As a check, note that all of the power from the supply is accounted for. Of the 18 W of net power from the power supply, 5 W goes to recharge the battery, 1 W is dissipated in the battery's internal resistance, and 12 W is dissipated in the light bulb.

KEYCONCEPT In any circuit that contains a source (such as a battery or power supply) and resistors, the sum of the power dissipated in each resistor must equal the power supplied by the source.

EXAMPLE 26.6 A complex network

WITH VARIATION PROBLEMS

Figure 26.12 shows a "bridge" circuit of the type described at the beginning of this section (see Fig. 26.6b). Find the current in each resistor and the equivalent resistance of the network of five resistors.

IDENTIFY and SET UP This network is neither a series combination nor a parallel combination. Hence we must use Kirchhoff's rules to find the values of the target variables. There are five unknown currents, but by applying the junction rule to junctions a and b, we can represent them in terms of three unknown currents I_1, I_2, and I_3, as shown in Fig. 26.12.

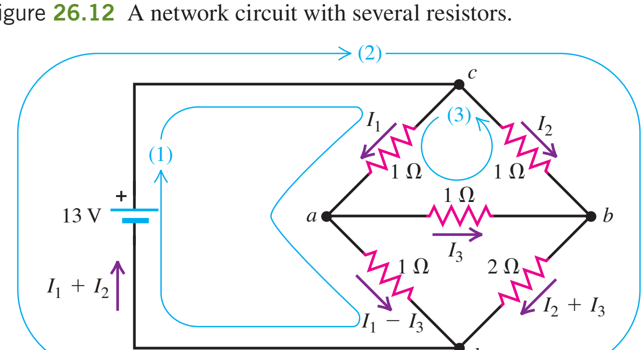
EXECUTE We apply the loop rule to the three loops shown:

$$13\text{ V} - I_1(1\ \Omega) - (I_1 - I_3)(1\ \Omega) = 0 \qquad (1)$$

$$-I_2(1\ \Omega) - (I_2 + I_3)(2\ \Omega) + 13\text{ V} = 0 \qquad (2)$$

$$-I_1(1\ \Omega) - I_3(1\ \Omega) + I_2(1\ \Omega) = 0 \qquad (3)$$

Figure **26.12** A network circuit with several resistors.



One way to solve these simultaneous equations is to solve Eq. (3) for I_2, obtaining $I_2 = I_1 + I_3$, and then substitute this expression into Eq. (2) to eliminate I_2. We then have

$$13\text{ V} = I_1(2\ \Omega) - I_3(1\ \Omega) \qquad (1')$$

$$13\text{ V} = I_1(3\ \Omega) + I_3(5\ \Omega) \qquad (2')$$

Now we can eliminate I_3 by multiplying Eq. (1′) by 5 and adding the two equations. We obtain

$$78\text{ V} = I_1(13\ \Omega) \qquad I_1 = 6\text{ A}$$

We substitute this result into Eq. (1′) to obtain $I_3 = -1\text{ A}$, and from Eq. (3) we find $I_2 = 5\text{ A}$. The negative value of I_3 tells us that its direction is opposite to the direction we assumed.

The total current through the network is $I_1 + I_2 = 11\text{ A}$, and the potential drop across it is equal to the battery emf, 13 V. The equivalent resistance of the network is therefore

$$R_{\text{eq}} = \frac{13\text{ V}}{11\text{ A}} = 1.2\ \Omega$$

EVALUATE You can check our results for I_1, I_2, and I_3 by substituting them back into Eqs. (1)–(3). What do you find?

KEYCONCEPT Some circuits have combinations of resistors that are neither in series nor in parallel. To analyze any such circuit, use Kirchhoff's loop rule and junction rule.

EXAMPLE 26.7 A potential difference in a complex network

WITH VARIATION PROBLEMS

In the circuit of Example 26.6 (Fig. 26.12), find the potential difference V_{ab}.

IDENTIFY and SET UP Our target variable $V_{ab} = V_a - V_b$ is the potential at point a with respect to point b. To find it, we start at point b and follow a path to point a, adding potential rises and drops as we go. We can follow any of several paths from b to a; the result must be the same for all such paths, which gives us a way to check our result.

EXECUTE The simplest path is through the center $1\ \Omega$ resistor. In Example 26.6 we found $I_3 = -1$ A, showing that the actual current direction through this resistor is from right to left. Thus, as we go from b to a, there is a *drop* of potential with magnitude $|I_3|R = (1\text{ A})(1\ \Omega) = 1$ V. Hence $V_{ab} = -1$ V, and the potential at a is 1 V less than at point b.

EVALUATE To check our result, let's try a path from b to a that goes through the lower two resistors. The currents through these are

$$I_2 + I_3 = 5\text{ A} + (-1\text{ A}) = 4\text{ A} \quad \text{and}$$
$$I_1 - I_3 = 6\text{ A} - (-1\text{ A}) = 7\text{ A}$$

and so

$$V_{ab} = -(4\text{ A})(2\ \Omega) + (7\text{ A})(1\ \Omega) = -1\text{ V}$$

You can confirm this result by using some other paths from b to a.

KEYCONCEPT A useful way to restate Kirchhoff's loop rule is that for any path in a circuit from point a to point b, the potential difference V_{ab} must be the same.

TEST YOUR UNDERSTANDING OF SECTION 26.2 Subtract Eq. (1) from Eq. (2) in Example 26.6. To which loop in Fig. 26.12 does this equation correspond? Would this equation have simplified the solution of Example 26.6?

ANSWER

loop *cbdac*, no Equation (2) minus Eq. (1) gives $-I_2(1\ \Omega) - (I_2 + I_3)(2\ \Omega) + (I_1 - I_3)(1\ \Omega) + I_1(1\ \Omega) = 0$. We can obtain this equation by applying the loop rule around the path from c to b to d to a to c in Fig. 26.12. This isn't an independent equation, so it would not have helped with the solution of Example 26.6.

26.3 ELECTRICAL MEASURING INSTRUMENTS

Figure **26.13** Both this voltmeter (left) and ammeter (right) are d'Arsonval galvanometers. The difference has to do with their internal connections (see Fig. 26.15).

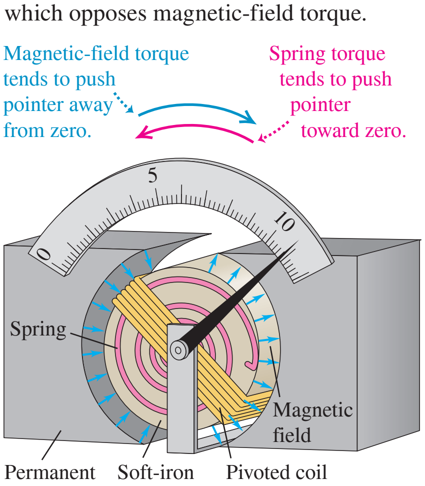
We've been talking about potential difference, current, and resistance for two chapters, so it's about time we said something about how to *measure* these quantities. Many common devices, including car instrument panels, battery chargers, and inexpensive electrical instruments, measure potential difference (voltage), current, or resistance with a **d'Arsonval galvanometer** (**Fig. 26.13**). In the following discussion we'll often call it just a *meter.* A pivoted coil of fine wire is placed in the magnetic field of a permanent magnet (**Fig. 26.14**). Attached to the coil is a spring, similar to the hairspring on the balance wheel of a watch. In the equilibrium position, with no current in the coil, the pointer is at zero. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to the current. (We'll discuss this magnetic interaction in detail in Chapter 27.) As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement.

Thus the angular deflection of the coil and pointer is directly proportional to the coil current, and the device can be calibrated to measure current. The maximum deflection, typically 90° or so, is called *full-scale deflection*. The essential electrical characteristics of the meter are the current I_{fs} required for full-scale deflection (typically on the order of 10 μA to 10 mA) and the resistance R_c of the coil (typically on the order of 10 Ω to 1000 Ω).

The meter deflection is proportional to the *current* in the coil. If the coil obeys Ohm's law, the current is proportional to the *potential difference* between the terminals of the coil, and the deflection is also proportional to this potential difference. For example, consider a meter whose coil has a resistance $R_c = 20.0\ \Omega$ and that deflects full scale when the current in its coil is $I_{fs} = 1.00$ mA. The corresponding potential difference for full-scale deflection is

$$V = I_{fs}R_c = (1.00 \times 10^{-3}\text{ A})(20.0\ \Omega) = 0.0200\text{ V}$$

Figure **26.14** A d'Arsonval galvanometer, showing a pivoted coil with attached pointer, a permanent magnet supplying a magnetic field that is uniform in magnitude, and a spring to provide restoring torque, which opposes magnetic-field torque.



Ammeters

A current-measuring instrument is usually called an **ammeter** (or milliammeter, microammeter, and so forth, depending on the range). *An ammeter always measures the current passing through it.* An *ideal* ammeter, discussed in Section 25.4, would have *zero* resistance, so including it in a branch of a circuit would not affect the current in that branch.

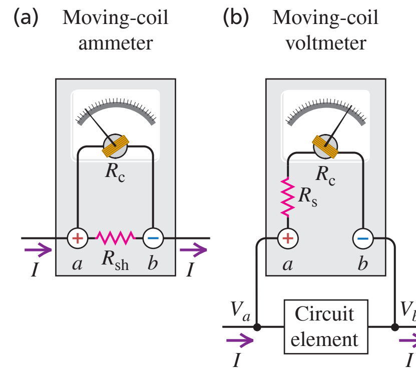
Real ammeters always have a finite resistance, but it is always desirable for an ammeter to have as little resistance as possible.

We can adapt any meter to measure currents that are larger than its full-scale reading by connecting a resistor in parallel with it (**Fig. 26.15a**) so that some of the current bypasses the meter coil. The parallel resistor is called a **shunt resistor** or simply a *shunt,* denoted as R_{sh}.

Suppose we want to make a meter with full-scale current I_{fs} and coil resistance R_c into an ammeter with full-scale reading I_a. To determine the shunt resistance R_{sh} needed, note that at full-scale deflection the total current through the parallel combination is I_a, the current through the coil of the meter is I_{fs}, and the current through the shunt is the difference $I_a - I_{fs}$. The potential difference V_{ab} is the same for both paths, so

$$I_{fs}R_c = (I_a - I_{fs})R_{sh} \qquad \text{(for an ammeter)} \tag{26.7}$$

Figure **26.15** Using the same meter to measure (a) current and (b) voltage.



EXAMPLE 26.8 Designing an ammeter

What shunt resistance is required to make the 1.00 mA, 20.0 Ω meter described above into an ammeter with a range of 0 to 50.0 mA?

IDENTIFY and SET UP Since the meter is being used as an ammeter, its internal connections are as shown in Fig. 26.15a. Our target variable is the shunt resistance R_{sh}, which we'll find from Eq. (26.7). The ammeter must handle a maximum current $I_a = 50.0 \times 10^{-3}$ A. The coil resistance is $R_c = 20.0\ \Omega$, and the meter shows full-scale deflection when the current through the coil is $I_{fs} = 1.00 \times 10^{-3}$ A.

EXECUTE Solving Eq. (26.7) for R_{sh}, we find

$$R_{sh} = \frac{I_{fs}R_c}{I_a - I_{fs}} = \frac{(1.00 \times 10^{-3}\ \text{A})(20.0\ \Omega)}{(50.0 \times 10^{-3}\ \text{A}) - (1.00 \times 10^{-3}\ \text{A})}$$
$$= 0.408\ \Omega$$

EVALUATE It's useful to consider the equivalent resistance R_{eq} of the ammeter as a whole. From Eq. (26.2),

$$R_{eq} = \left(\frac{1}{R_c} + \frac{1}{R_{sh}}\right)^{-1} = \left(\frac{1}{20.0\ \Omega} + \frac{1}{0.408\ \Omega}\right)^{-1}$$
$$= 0.400\ \Omega$$

The shunt resistance is so small in comparison to the coil resistance that the equivalent resistance is very nearly equal to the shunt resistance. The result is an ammeter with a low equivalent resistance and the desired 0–50.0 mA range. At full-scale deflection, $I = I_a = 50.0$ mA, the current through the galvanometer is 1.00 mA, the current through the shunt resistor is 49.0 mA, and $V_{ab} = 0.0200$ V. If the current I is *less* than 50.0 mA, the coil current and the deflection are proportionally less.

KEYCONCEPT An ammeter measures the current that passes through it. It should have a very small equivalent resistance in order to minimize its effect on the circuit being measured. This can be accomplished using a small shunt resistance in parallel with the meter.

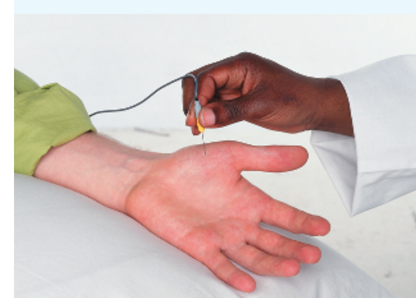
Voltmeters

This same basic meter may also be used to measure potential difference or *voltage.* A voltage-measuring device is called a **voltmeter.** A voltmeter always measures the potential difference between two points, and its terminals must be connected to these points. (Example 25.6 in Section 25.4 described what can happen if a voltmeter is connected incorrectly.) As we discussed in Section 25.4, an ideal voltmeter would have *infinite* resistance, so connecting it between two points in a circuit would not alter any of the currents. Real voltmeters always have finite resistance, but a voltmeter should have large enough resistance that connecting it in a circuit does not change the other currents appreciably.

For the meter of Example 26.8, the voltage across the meter coil at full-scale deflection is only $I_{fs}R_c = (1.00 \times 10^{-3}\ \text{A})(20.0\ \Omega) = 0.0200$ V. We can extend this range by connecting a resistor R_s in *series* with the coil (Fig. 26.15b). Then only a fraction of the total potential difference appears across the coil itself, and the remainder appears across R_s. For a voltmeter with full-scale reading V_V, we need a series resistor R_s in Fig. 26.15b such that

$$V_V = I_{fs}(R_c + R_s) \qquad \text{(for a voltmeter)} \tag{26.8}$$

BIO APPLICATION
Electromyography A fine needle containing two electrodes is being inserted into a muscle in this patient's hand. By using a sensitive voltmeter to measure the potential difference between these electrodes, a physician can probe the muscle's electrical activity. This is an important technique for diagnosing neurological and neuromuscular diseases.



EXAMPLE 26.9 Designing a voltmeter

What series resistance is required to make the 1.00 mA, 20.0 Ω meter described above into a voltmeter with a range of 0 to 10.0 V?

IDENTIFY and SET UP Since this meter is being used as a voltmeter, its internal connections are as shown in Fig. 26.15b. The maximum allowable voltage across the voltmeter is $V_V = 10.0$ V. We want this to occur when the current through the coil is $I_{fs} = 1.00 \times 10^{-3}$ A. Our target variable is the series resistance R_s, which we find from Eq. (26.8).

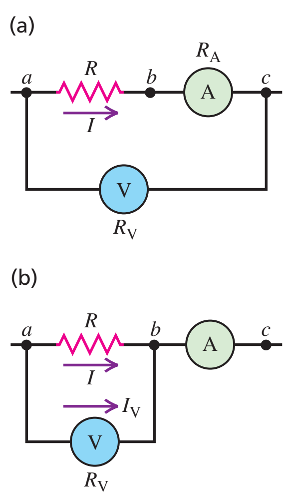
EXECUTE From Eq. (26.8),

$$R_s = \frac{V_V}{I_{fs}} - R_c = \frac{10.0 \text{ V}}{0.00100 \text{ A}} - 20.0\ \Omega = 9980\ \Omega$$

EVALUATE At full-scale deflection, $V_{ab} = 10.0$ V, the voltage across the meter is 0.0200 V, the voltage across R_s is 9.98 V, and the current through the voltmeter is 0.00100 A. Most of the voltage appears across the series resistor. The meter's equivalent resistance is a desirably high $R_{eq} = 20.0\ \Omega + 9980\ \Omega = 10{,}000\ \Omega$. Such a meter is called a "1000 ohms-per-volt" meter, referring to the ratio of resistance to full-scale deflection. In normal operation the current through the circuit element being measured (I in Fig. 26.15b) is much greater than 0.00100 A, and the resistance between points a and b in the circuit is much less than 10,000 Ω. The voltmeter draws off only a small fraction of the current and thus disturbs the circuit being measured only slightly.

KEYCONCEPT A voltmeter measures the potential difference (voltage) between its terminals. It should have a very large equivalent resistance in order to minimize the amount of current it draws from the circuit being measured. This can be accomplished using a large shunt resistance in series with the meter.

Figure 26.16 Ammeter–voltmeter method for measuring resistance.



Ammeters and Voltmeters in Combination

A voltmeter and an ammeter can be used together to measure *resistance* and *power*. The resistance R of a resistor equals the potential difference V_{ab} between its terminals divided by the current I; that is, $R = V_{ab}/I$. The power input P to any circuit element is the product of the potential difference across it and the current through it: $P = V_{ab}I$. In principle, the most straightforward way to measure R or P is to measure V_{ab} and I simultaneously.

With practical ammeters and voltmeters this isn't quite as simple as it seems. In **Fig. 26.16a**, ammeter A reads the current I in the resistor R. Voltmeter V, however, reads the *sum* of the potential difference V_{ab} across the resistor and the potential difference V_{bc} across the ammeter. If we transfer the voltmeter terminal from c to b, as in Fig. 26.16b, then the voltmeter reads the potential difference V_{ab} correctly, but the ammeter now reads the *sum* of the current I in the resistor and the current I_V in the voltmeter. Either way, we have to correct the reading of one instrument or the other unless the corrections are small enough to be negligible.

EXAMPLE 26.10 Measuring resistance I

The voltmeter in the circuit of Fig. 26.16a reads 12.0 V and the ammeter reads 0.100 A. The meter resistances are $R_V = 10{,}000\ \Omega$ (for the voltmeter) and $R_A = 2.00\ \Omega$ (for the ammeter). What are the resistance R and the power dissipated in the resistor?

IDENTIFY and SET UP The ammeter reads the current $I = 0.100$ A through the resistor, and the voltmeter reads the potential difference between a and c. If the ammeter were *ideal* (that is, if $R_A = 0$), there would be zero potential difference between b and c, the voltmeter reading $V = 12.0$ V would be equal to the potential difference V_{ab} across the resistor, and the resistance would be equal to $R = V/I = (12.0 \text{ V})/(0.100 \text{ A}) = 120\ \Omega$. The ammeter is *not* ideal, however (its resistance is $R_A = 2.00\ \Omega$), so the voltmeter reading V is actually the sum of the potential differences V_{bc} (across the ammeter) and V_{ab} (across the resistor). We use Ohm's law to find the voltage V_{bc} from the known current and ammeter resistance. Then we solve for V_{ab} and R. Given these, we are able to calculate the power P into the resistor.

EXECUTE From Ohm's law, $V_{bc} = IR_A = (0.100 \text{ A})(2.00\ \Omega) = 0.200$ V and $V_{ab} = IR$. The sum of these is $V = 12.0$ V, so the potential difference across the resistor is $V_{ab} = V - V_{bc} = (12.0 \text{ V}) - (0.200 \text{ V}) = 11.8$ V. Hence the resistance is

$$R = \frac{V_{ab}}{I} = \frac{11.8 \text{ V}}{0.100 \text{ A}} = 118\ \Omega$$

The power dissipated in this resistor is

$$P = V_{ab}I = (11.8 \text{ V})(0.100 \text{ A}) = 1.18 \text{ W}$$

EVALUATE You can confirm this result for the power by using the alternative formula $P = I^2R$. Do you get the same answer?

KEYCONCEPT To find the resistance of a resistor and the power that the resistor dissipates requires measuring both the current through the resistor and the potential difference across the resistor. To do this, put an ammeter in series with the resistance and put a voltmeter across the resistor and ammeter.

EXAMPLE 26.11 Measuring resistance II

Suppose the meters of Example 26.10 are connected to a different resistor as shown in Fig. 26.16b, and the readings obtained on the meters are the same as in Example 26.10. What is the value of this new resistance R, and what is the power dissipated in the resistor?

IDENTIFY and SET UP In Example 26.10 the ammeter read the actual current through the resistor, but the voltmeter reading was not the same as the potential difference across the resistor. Now the situation is reversed: The voltmeter reading $V = 12.0\ \text{V}$ shows the actual potential difference V_{ab} across the resistor, but the ammeter reading $I_A = 0.100\ \text{A}$ is *not* equal to the current I through the resistor. Applying the junction rule at b in Fig. 26.16b shows that $I_A = I + I_V$, where I_V is the current through the voltmeter. We find I_V from the given values of V and the voltmeter resistance R_V, and we use this value to find the resistor current I. We then determine the resistance R from I and the voltmeter reading, and calculate the power as in Example 26.10.

EXECUTE We have $I_V = V/R_V = (12.0\ \text{V})/(10{,}000\ \Omega) = 1.20\ \text{mA}$. The actual current I in the resistor is $I = I_A - I_V = 0.100\ \text{A} - 0.0012\ \text{A} = 0.0988\ \text{A}$, and the resistance is

$$R = \frac{V_{ab}}{I} = \frac{12.0\ \text{V}}{0.0988\ \text{A}} = 121\ \Omega$$

The power dissipated in the resistor is

$$P = V_{ab}I = (12.0\ \text{V})(0.0988\ \text{A}) = 1.19\ \text{W}$$

EVALUATE Had the meters been ideal, our results would have been $R = 12.0\ \text{V}/0.100\ \text{A} = 120\ \Omega$ and $P = VI = (12.0\ \text{V})(0.100\ \text{A}) = 1.2\ \text{W}$ both here and in Example 26.10. The actual (correct) results are not too different in either case. That's because the ammeter and voltmeter are nearly ideal: Compared with the resistance R under test, the ammeter resistance R_A is very small and the voltmeter resistance R_V is very large. Under these conditions, treating the meters as ideal yields pretty good results; accurate work requires calculations as in these two examples.

KEYCONCEPT An alternative way to find the resistance of a resistor and the power that the resistor dissipates is to put a voltmeter across the resistor and put an ammeter in the circuit downstream of the resistor–voltmeter combination.

Ohmmeters

An alternative method for measuring resistance is to use a d'Arsonval meter in an arrangement called an **ohmmeter.** It consists of a meter, a resistor, and a source (often a flashlight battery) connected in series (**Fig. 26.17**). The resistance R to be measured is connected between terminals x and y.

The series resistance R_s is variable; it is adjusted so that when terminals x and y are short-circuited (that is, when $R = 0$), the meter deflects full scale. When nothing is connected to terminals x and y, so that the circuit between x and y is *open* (that is, when $R \rightarrow \infty$), there is no current and hence no deflection. For any intermediate value of R the meter deflection depends on the value of R, and the meter scale can be calibrated to read the resistance R directly. Larger currents correspond to smaller resistances, so this scale reads backward compared to the scale showing the current.

In situations in which high precision is required, instruments containing d'Arsonval meters have been supplanted by electronic instruments with direct digital readouts. Digital voltmeters can be made with extremely high internal resistance, of the order of $100\ \text{M}\Omega$. **Figure 26.18** shows a digital *multimeter,* an instrument that can measure voltage, current, or resistance over a wide range.

Figure 26.17 Ohmmeter circuit. The resistor R_s has a variable resistance, as is indicated by the arrow through the resistor symbol. To use the ohmmeter, first connect x directly to y and adjust R_s until the meter reads zero. Then connect x and y across the resistor R and read the scale.

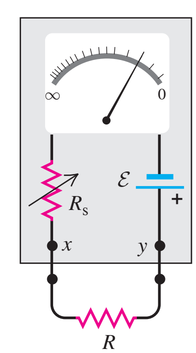
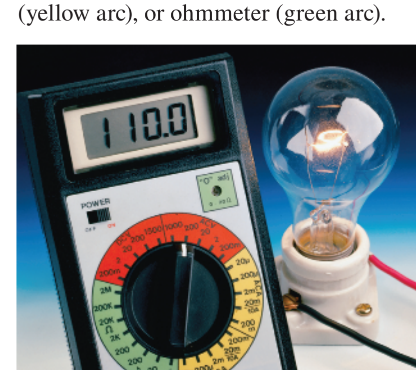


Figure 26.18 This digital multimeter can be used as a voltmeter (red arc), ammeter (yellow arc), or ohmmeter (green arc).

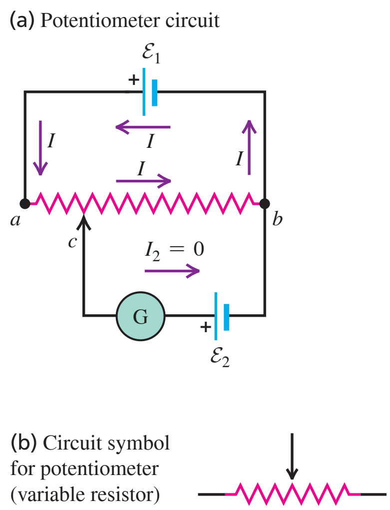


The Potentiometer

The *potentiometer* is an instrument that can be used to measure the emf of a source without drawing any current from the source; it also has a number of other useful applications. Essentially, it balances an unknown potential difference against an adjustable, measurable potential difference.

Figure 26.19a shows the principle of the potentiometer. A resistance wire ab of total resistance R_{ab} is permanently connected to the terminals of a source of known emf $\mathcal{E}_1$. A sliding contact c is connected through the galvanometer G to a second source whose emf $\mathcal{E}_2$ is to be measured. As contact c is moved along the resistance wire, the resistance R_{cb} between points c and b varies; if the resistance wire is uniform, R_{cb} is proportional to the length of wire between c and b. To determine the value of $\mathcal{E}_2$, contact c

Figure 26.19 A potentiometer.



is moved until a position is found at which the galvanometer shows no deflection; this corresponds to zero current passing through $\mathcal{E}_2$. With $I_2 = 0$, Kirchhoff's loop rule gives

$$\mathcal{E}_2 = IR_{cb}$$

With $I_2 = 0$, the current I produced by the emf $\mathcal{E}_1$ has the same value no matter what the value of the emf $\mathcal{E}_2$. We calibrate the device by replacing $\mathcal{E}_2$ by a source of known emf; then to find any unknown emf $\mathcal{E}_2$, we measure the length of wire cb for which $I_2 = 0$. Note: For this to work, V_{ab} must be greater than $\mathcal{E}_2$.

The term *potentiometer* is also used for any variable resistor, usually having a circular resistance element and a sliding contact controlled by a rotating shaft and knob. Figure. 26.19b shows the circuit symbol for a potentiometer.

TEST YOUR UNDERSTANDING OF SECTION 26.3 You want to measure the current through and the potential difference across the 2 Ω resistor shown in Fig. 26.12 (Example 26.6 in Section 26.2). (a) How should you connect an ammeter and a voltmeter to do this? (i) Both ammeter and voltmeter in series with the 2 Ω resistor; (ii) ammeter in series with the 2 Ω resistor and voltmeter connected between points b and d; (iii) ammeter connected between points b and d and voltmeter in series with the 2 Ω resistor; (iv) both ammeter and voltmeter connected between points b and d. (b) What resistances should these meters have? (i) Both ammeter and voltmeter resistances should be much greater than 2 Ω; (ii) ammeter resistance should be much greater than 2 Ω and voltmeter resistance should be much less than 2 Ω; (iii) ammeter resistance should be much less than 2 Ω and voltmeter resistance should be much greater than 2 Ω; (iv) both ammeter and voltmeter resistances should be much less than 2 Ω.

ANSWER

(a) **(ii)**, (b) **(iii)** An ammeter must always be placed in series with the circuit element of interest, and a voltmeter must always be placed in parallel. Ideally the ammeter would have zero resistance and the voltmeter would have infinite resistance so that their presence would have no effect on either the resistor current or the voltage. Neither of these idealizations is possible, but the ammeter resistance should be much less than 2 Ω and the voltmeter resistance should be much greater than 2 Ω.

26.4 *R-C* CIRCUITS

In the circuits we have analyzed up to this point, we have assumed that all the emfs and resistances are *constant* (time independent) so that all the potentials, currents, and powers are also independent of time. But in the simple act of charging or discharging a capacitor we find a situation in which the currents, voltages, and powers *do* change with time.

Many devices incorporate circuits in which a capacitor is alternately charged and discharged. These include flashing traffic lights, automobile turn signals, and electronic flash units. Understanding what happens in such circuits is thus of great practical importance.

Charging a Capacitor

Figure 26.20 shows a simple circuit for charging a capacitor. A circuit such as this that has a resistor and a capacitor in series is called an ***R-C* circuit.** We idealize the battery (or power supply) to have a constant emf $\mathcal{E}$ and zero internal resistance $(r = 0)$, and we ignore the resistance of all the connecting conductors.

We begin with the capacitor initially uncharged (Fig. 26.20a); then at some initial time $t = 0$ we close the switch, completing the circuit and permitting current around the loop to begin charging the capacitor (Fig. 26.20b). For all practical purposes, the current begins at the same instant in every conducting part of the circuit, and at each instant the current is the same in every part.

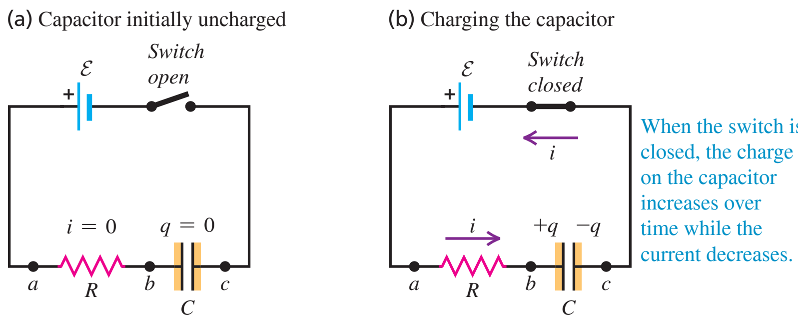


Figure **26.20** Charging a capacitor. (a) Just before the switch is closed, the charge q is zero. (b) When the switch closes (at $t = 0$), the current jumps from zero to $\mathcal{E}/R$. As time passes, q approaches Q_f and the current i approaches zero.

Because the capacitor in Fig. 26.20 is initially uncharged, the potential difference v_{bc} across it is zero at $t = 0$. At this time, from Kirchhoff's loop law, the voltage v_{ab} across the resistor R is equal to the battery emf $\mathcal{E}$. The initial $(t = 0)$ current through the resistor, which we'll call I_0, is given by Ohm's law: $I_0 = v_{ab}/R = \mathcal{E}/R$.

As the capacitor charges, its voltage v_{bc} increases and the potential difference v_{ab} across the resistor decreases, corresponding to a decrease in current. The sum of these two voltages is constant and equal to $\mathcal{E}$. After a long time the capacitor is fully charged, the current decreases to zero, and v_{ab} across the resistor becomes zero. Then the entire battery emf $\mathcal{E}$ appears across the capacitor and $v_{bc} = \mathcal{E}$.

Let q represent the charge on the capacitor and i the current in the circuit at some time t after the switch has been closed. We choose the positive direction for the current to correspond to positive charge flowing onto the left-hand capacitor plate, as in Fig. 26.20b. The instantaneous potential differences v_{ab} and v_{bc} are

$$v_{ab} = iR \qquad v_{bc} = \frac{q}{C}$$

Using these in Kirchhoff's loop rule, we find

$$\mathcal{E} - iR - \frac{q}{C} = 0 \tag{26.9}$$

The potential drops by an amount iR as we travel from a to b and by q/C as we travel from b to c. Solving Eq. (26.9) for i, we find

$$i = \frac{\mathcal{E}}{R} - \frac{q}{RC} \tag{26.10}$$

At time $t = 0$, when the switch is first closed, the capacitor is uncharged, and so $q = 0$. Substituting $q = 0$ into Eq. (26.10), we find that the *initial* current I_0 is given by $I_0 = \mathcal{E}/R$, as we have already noted. If the capacitor were not in the circuit, the last term in Eq. (26.10) would not be present; then the current would be *constant* and equal to $\mathcal{E}/R$.

As the charge q increases, the term q/RC becomes larger and the capacitor charge approaches its final value, which we'll call Q_f. The current decreases and eventually becomes zero. When $i = 0$, Eq. (26.10) gives

$$\frac{\mathcal{E}}{R} = \frac{Q_f}{RC} \qquad Q_f = C\mathcal{E} \tag{26.11}$$

Note that the final charge Q_f does not depend on R.

Figure 26.21 shows the current and capacitor charge as functions of time. At the instant the switch is closed $(t = 0)$, the current jumps from zero to its initial value $I_0 = \mathcal{E}/R$; after that, it gradually approaches zero. The capacitor charge starts at zero and gradually approaches the final value given by Eq. (26.11), $Q_f = C\mathcal{E}$.

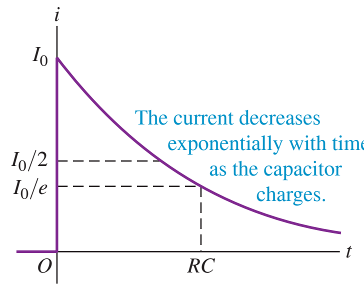
We can derive general expressions for charge q and current i as functions of time. With our choice of the positive direction for current (Fig. 26.20b), i equals the rate at which positive charge arrives at the left-hand (positive) plate of the capacitor, so $i = dq/dt$. Making this substitution in Eq. (26.10), we have

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\mathcal{E})$$

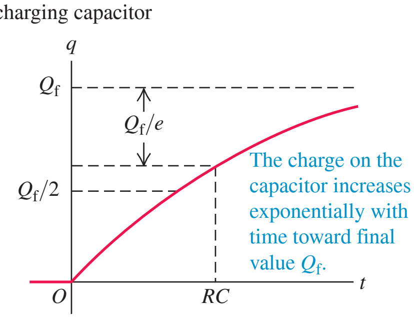
CAUTION Lowercase means time-varying Up to this point we have been working with constant potential differences (voltages), currents, and charges, and we have used *capital* letters V, I, and Q, respectively, to denote these quantities. To distinguish between quantities that vary with time and those that are constant, we'll use *lowercase* letters v, i, and q for time-varying voltages, currents, and charges, respectively. We suggest that you follow this same convention in your own work.

Figure **26.21** Current i and capacitor charge q as functions of time for the circuit of Fig. 26.20. The initial current is I_0 and the initial capacitor charge is zero. The current asymptotically approaches zero, and the capacitor charge asymptotically approaches a final value of Q_f.

(a) Graph of current versus time for a charging capacitor



(b) Graph of capacitor charge versus time for a charging capacitor



We can rearrange this to

$$\frac{dq}{q - C\mathcal{E}} = -\frac{dt}{RC}$$

and then integrate both sides. We change the integration variables to q' and t' so that we can use q and t for the upper limits. The lower limits are $q' = 0$ and $t' = 0$:

$$\int_0^q \frac{dq'}{q' - C\mathcal{E}} = -\int_0^t \frac{dt'}{RC}$$

When we carry out the integration, we get

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

Exponentiating both sides (that is, taking the inverse logarithm) and solving for q, we find

$$\frac{q - C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC}$$

***R-C* circuit, charging capacitor:**

Capacitor charge · Capacitance · Final capacitor charge $= C\mathcal{E}$

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right) \qquad (26.12)$$

Battery emf · Time since switch closed · Resistance

The instantaneous current i is just the time derivative of Eq. (26.12):

***R-C* circuit, charging capacitor:**

Current · Battery emf · Time since switch closed

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \qquad (26.13)$$

Initial current $= \mathcal{E}/R$

Rate of change of capacitor charge · Resistance · Capacitance

The charge and current are both *exponential* functions of time. Figure 26.21a is a graph of Eq. (26.13) and Fig. 26.21b is a graph of Eq. (26.12).

Time Constant

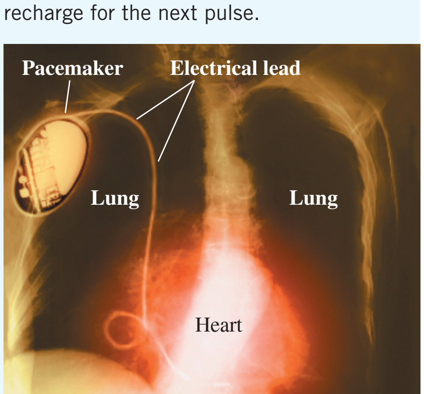
After a time equal to RC, the current in the *R-C* circuit has decreased to $1/e$ (about 0.368) of its initial value. At this time, the capacitor charge has reached $(1 - 1/e) = 0.632$ of its final value $Q_f = C\mathcal{E}$. The product RC is therefore a measure of how quickly the capacitor charges. We call RC the **time constant,** or the **relaxation time,** of the circuit, denoted by τ:

$$\tau = RC \qquad \text{(time constant for } R\text{-}C \text{ circuit)} \qquad (26.14)$$

When τ is small, the capacitor charges quickly; when it is larger, the charging takes more time. If the resistance is small, it's easier for current to flow, and the capacitor charges more quickly. If R is in ohms and C in farads, τ is in seconds.

In Fig. 26.21a the horizontal axis is an *asymptote* for the curve. Strictly speaking, i never becomes exactly zero. But the longer we wait, the closer it gets. After a time equal to $10RC$, the current has decreased to 0.000045 of its initial value. Similarly, the curve in Fig. 26.21b approaches the horizontal dashed line labeled Q_f as an asymptote. The charge q never attains exactly this value, but after a time equal to $10RC$, the difference between q and Q_f is only 0.000045 of Q_f. We invite you to verify that the product RC has units of time.

BIO APPLICATION Pacemakers and Capacitors This x-ray image shows a pacemaker implanted in a patient with a malfunctioning sinoatrial node, the part of the heart that generates the electrical signal to trigger heartbeats. The pacemaker circuit contains a battery, a capacitor, and a computer-controlled switch. To maintain regular beating, once per second the switch discharges the capacitor and sends an electrical pulse along the lead to the heart. The switch then flips to allow the capacitor to recharge for the next pulse.



Discharging a Capacitor

Now suppose that after the capacitor in Fig. 26.21b has acquired a charge Q_0, we remove the battery from our *R-C* circuit and connect points *a* and *c* to an open switch (**Fig. 26.22a**). We then close the switch and at the same instant reset our stopwatch to $t = 0$; at that time, $q = Q_0$. The capacitor then *discharges* through the resistor, and its charge eventually decreases to zero.

Again let *i* and *q* represent the time-varying current and charge at some instant after the connection is made. In Fig. 26.22b we make the same choice of the positive direction for current as in Fig. 26.20b. Then Kirchhoff's loop rule gives Eq. (26.10) but with $\mathcal{E} = 0$; that is,

$$i = \frac{dq}{dt} = -\frac{q}{RC} \tag{26.15}$$

The current *i* is now negative; this is because positive charge *q* is leaving the left-hand capacitor plate in Fig. 26.22b, so the current is in the direction opposite to that shown. At time $t = 0$, when $q = Q_0$, the initial current is $I_0 = -Q_0/RC$.

To find *q* as a function of time, we rearrange Eq. (26.15), again change the variables to q' and t', and integrate. This time the limits for q' are Q_0 to *q*:

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC}\int_0^t dt'$$

$$\ln\frac{q}{Q_0} = -\frac{t}{RC}$$

***R-C* circuit, discharging capacitor:**

Capacitor charge; Initial capacitor charge; Capacitance; Time since switch closed; Resistance

$$q = Q_0 e^{-t/RC} \tag{26.16}$$

The instantaneous current *i* is the derivative of this with respect to time:

***R-C* circuit, discharging capacitor:**

Current; Initial capacitor charge; Capacitance; Time since switch closed; Rate of change of capacitor charge; Resistance; Initial current $= -Q_0/RC$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \tag{26.17}$$

We graph the current and the charge in **Fig. 26.23**; both quantities approach zero exponentially with time. Comparing these results with Eqs. (26.12) and (26.13), we note that the expressions for the current are identical, apart from the sign of I_0. The capacitor charge approaches zero asymptotically in Eq. (26.16), while the *difference* between *q* and *Q* approaches zero asymptotically in Eq. (26.12).

Energy considerations give us additional insight into the behavior of an *R-C* circuit. While the capacitor is charging, the instantaneous rate at which the battery delivers energy to the circuit is $P = \mathcal{E}i$. The instantaneous rate at which electrical energy is dissipated in the resistor is i^2R, and the rate at which energy is stored in the capacitor is $iv_{bc} = iq/C$. Multiplying Eq. (26.9) by *i*, we find

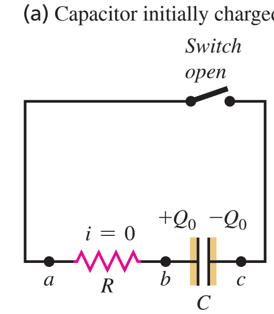
$$\mathcal{E}i = i^2R + \frac{iq}{C} \tag{26.18}$$

This means that of the power $\mathcal{E}i$ supplied by the battery, part (i^2R) is dissipated in the resistor and part (iq/C) is stored in the capacitor.

The *total* energy supplied by the battery during charging of the capacitor equals the battery emf $\mathcal{E}$ multiplied by the total charge Q_f, or $\mathcal{E}Q_f$. The total energy stored in the capacitor, from Eq. (24.9), is $Q_f\mathcal{E}/2$. Thus, of the energy supplied by the battery, *exactly half*

Figure **26.22** Discharging a capacitor. (a) Before the switch is closed at time $t = 0$, the capacitor charge is Q_0 and the current is zero. (b) At time *t* after the switch is closed, the capacitor charge is *q* and the current is *i*. The actual current direction is opposite to the direction shown; *i* is negative. After a long time, *q* and *i* both approach zero.

(a) Capacitor initially charged



(b) Discharging the capacitor

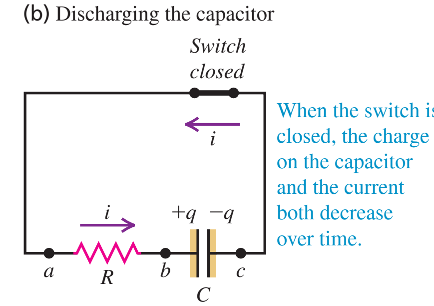
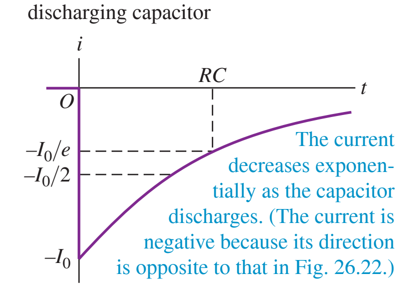
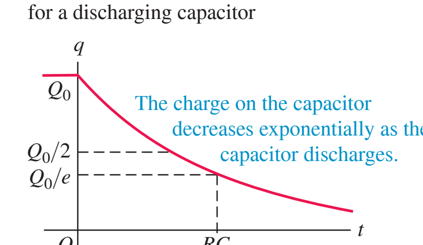


Figure **26.23** Current *i* and capacitor charge *q* as functions of time for the circuit of Fig. 26.22. The initial current is I_0 and the initial capacitor charge is Q_0. Both *i* and *q* asymptotically approach zero.

(a) Graph of current versus time for a discharging capacitor



(b) Graph of capacitor charge versus time for a discharging capacitor



is stored in the capacitor, and the other half is dissipated in the resistor. This half-and-half division of energy doesn't depend on C, R, or $\mathcal{E}$. You can verify this result by taking the integral over time of each of the power quantities in Eq. (26.18).

EXAMPLE 26.12 Charging a capacitor

WITH VARIATION PROBLEMS

A 10 MΩ resistor is connected in series with a 1.0 μF capacitor and a battery with emf 12.0 V. Before the switch is closed at time $t = 0$, the capacitor is uncharged. (a) What is the time constant? (b) What fraction of the final charge Q_f is on the capacitor at $t = 46$ s? (c) What fraction of the initial current I_0 is still flowing at $t = 46$ s?

IDENTIFY and SET UP This is the situation shown in Fig. 26.20, with $R = 10\ \text{M}\Omega$, $C = 1.0\ \mu\text{F}$, and $\mathcal{E} = 12.0$ V. The charge q and current i vary with time as shown in Fig. 26.21. Our target variables are (a) the time constant τ, (b) the ratio q/Q_f at $t = 46$ s, and (c) the ratio i/I_0 at $t = 46$ s. Equation (26.14) gives τ. For a capacitor being charged, Eq. (26.12) gives q and Eq. (26.13) gives i.

EXECUTE (a) From Eq. (26.14),

$$\tau = RC = (10 \times 10^6\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 10\ \text{s}$$

(b) From Eq. (26.12),

$$\frac{q}{Q_f} = 1 - e^{-t/RC} = 1 - e^{-(46\ \text{s})/(10\ \text{s})} = 0.99$$

(c) From Eq. (26.13),

$$\frac{i}{I_0} = e^{-t/RC} = e^{-(46\ \text{s})/(10\ \text{s})} = 0.010$$

EVALUATE After 4.6 time constants the capacitor is 99% charged and the charging current has decreased to 1.0% of its initial value. The circuit would charge more rapidly if we reduced the time constant by using a smaller resistance.

KEYCONCEPT When a capacitor is charging in a circuit with a resistor and a source of emf, the capacitor charge q and the current i both vary with time: q approaches its final value asymptotically and i approaches zero asymptotically. The time constant for both quantities is the product RC of the resistance and the capacitance of the circuit elements.

EXAMPLE 26.13 Discharging a capacitor

WITH VARIATION PROBLEMS

The resistor and capacitor of Example 26.12 are reconnected as shown in Fig. 26.22. The capacitor has an initial charge of 5.0 μC and is discharged by closing the switch at $t = 0$. (a) At what time will the charge be 0.50 μC? (b) What is the current at this time?

IDENTIFY and SET UP Now the capacitor is being discharged, so q and i vary with time as in Fig. 26.23, with $Q_0 = 5.0 \times 10^{-6}$ C. Again we have $RC = \tau = 10$ s. Our target variables are (a) the value of t at which $q = 0.50\ \mu$C and (b) the value of i at this time. We first solve Eq. (26.16) for t, and then solve Eq. (26.17) for i.

EXECUTE (a) Solving Eq. (26.16) for the time t gives

$$t = -RC \ln\frac{q}{Q_0} = -(10\ \text{s})\ln\frac{0.50\ \mu\text{C}}{5.0\ \mu\text{C}}$$
$$= 23\ \text{s} = 2.3\tau$$

(b) From Eq. (26.17), with $Q_0 = 5.0\ \mu\text{C} = 5.0 \times 10^{-6}$ C,

$$i = -\frac{Q_0}{RC}e^{-t/RC} = -\frac{5.0 \times 10^{-6}\ \text{C}}{10\ \text{s}}e^{-2.3} = -5.0 \times 10^{-8}\ \text{A}$$

EVALUATE The current in part (b) is negative because i has the opposite sign when the capacitor is discharging than when it is charging. Note that we could have avoided evaluating $e^{-t/RC}$ by noticing that at the time in question, $q = 0.10Q_0$; from Eq. (26.16) this means that $e^{-t/RC} = 0.10$.

KEYCONCEPT When a capacitor is discharging in a circuit with a resistor, the capacitor charge q and the current i both vary with time, and both approach zero asymptotically. The time constant for both quantities is the product RC of the resistance and the capacitance of the circuit elements.

TEST YOUR UNDERSTANDING OF SECTION 26.4 The energy stored in a capacitor is equal to $q^2/2C$. When a capacitor is discharged, what fraction of the initial energy remains after an elapsed time of one time constant? (i) $1/e$; (ii) $1/e^2$; (iii) $1 - 1/e$; (iv) $(1 - 1/e)^2$; (v) answer depends on how much energy was stored initially.

ANSWER

(ii) After one time constant, $t = RC$ and the initial charge Q_0 has decreased to $Q_0e^{-t/RC} = Q_0e^{-RC/RC} = Q_0e^{-1} = Q_0/e$. Hence the stored energy has decreased from $Q_0^2/2C$ to $(Q_0/e)^2/2C = Q_0^2/2Ce^2$, a fraction $1/e^2 = 0.135$ of its initial value. This result doesn't depend on the initial value of the energy.

26.5 POWER DISTRIBUTION SYSTEMS

We conclude this chapter with a brief discussion of practical household and automotive electric-power distribution systems. Automobiles use direct-current (dc) systems, while nearly all household, commercial, and industrial systems use alternating current (ac) because of the ease of stepping voltage up and down with transformers. Most of the same basic wiring concepts apply to both. We'll talk about alternating-current circuits in greater detail in Chapter 31.

The various lamps, motors, and other appliances to be operated are always connected in *parallel* to the power source (the wires from the power company for houses, or from the battery and alternator for a car). If appliances were connected in series, shutting one appliance off would shut them all off (see Example 26.2 in Section 26.1). **Figure 26.24** shows the basic idea of house wiring. The cable bringing the mains electricity into a house contains two wires. One of these is called, in technical jargon, the "line" or the "phase," and is commonly referred to as "*live*" (a term which we will use later), "active" or "hot", and the other called is "*neutral*". The neutral wire is connected to *earth*, or *earthed*, at the local transformer substation, but in some countries it can be re-earthed at the entrance panel. *Earth* is provided by an electrode driven into the ground (which is usually a good conductor), either locally (adjacent to the house or in its foundations), or at the local substation. As a general rule, all metallic services in a building (such as water pipes and gas pipes) are connected (or *bonded*) to its earthing installation.

In most of the world, household voltage is nominally between 220 V and 240 V. In particular, it is 230 V in the United Kingdom and most of continental Europe, as well as in Australia, New Zealand, and India; whereas in China, Russia, and a large proportion of South American countries it is 220 V. Most African countries also use either 220 V or 230 V voltage. A smaller number of countries use lower voltage, between 100 V and 127 V—for example, the United States and Canada use 120 V, Japan 100 V and Mexico 127 V. Finally, a few countries have double voltage—for example in Brazil both 220 V and 127 V is supplied. (For alternating current, which varies sinusoidally with time, these numbers represent the *root-mean-square* voltage, which is $1/\sqrt{2}$ times the peak voltage. We'll discuss this further in Section 31.1.) The amount of current I drawn by a given device is determined by its power input P, given by Eq. (25.17): $P = VI$. Hence $I = P/V$. For example, for 230 V supply, the current in a 100 W light bulb is

$$I = \frac{P}{V} = \frac{100\text{ W}}{230\text{ V}} = 0.43\text{ A}$$

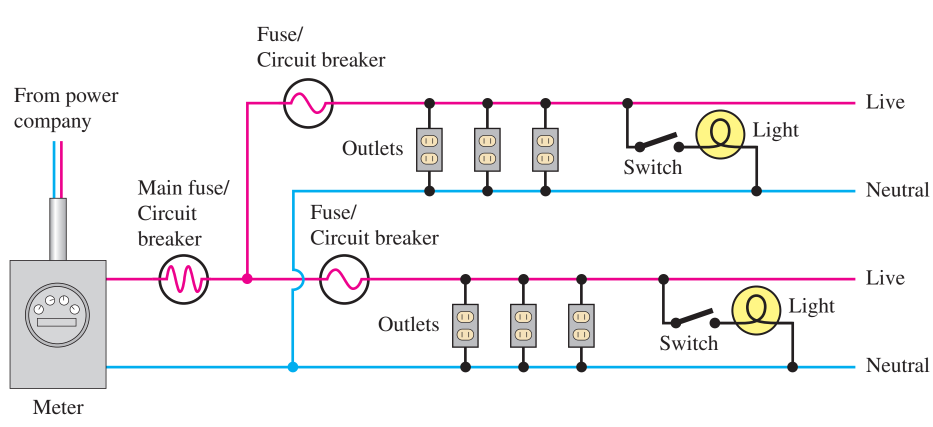


Figure **26.24** Schematic diagram of part of a house wiring system. Only two circuits are shown; an actual system might have four to thirty circuits. Lamps and appliances may be plugged into the outlets. In some countries (e.g. in the UK), lighting circuits are separate from the socket outlet circuits. The earthing wires, which normally carry no current, are not shown. The neutral wire can be re-earthed at the entry.

The power input to this bulb is actually determined by its resistance R. Using Eq. (25.18), which states that $P = VI = I^2R = V^2/R$ for a resistor, we get the resistance of this bulb at operating temperature:

$$R = \frac{V}{I} = \frac{230\text{ V}}{0.43\text{ A}} = 529\ \Omega \qquad \text{or} \qquad R = \frac{V^2}{P} = \frac{(230\text{ V})^2}{100\text{ W}} = 529\ \Omega$$

Similarly, a 950 W toaster draws a current of $(950\text{ W})/(230\text{ V}) = 4.1\text{ A}$ and has a resistance, at operating temperature, of $55.7\ \Omega$. Because of the temperature dependence of resistivity, the resistances of these devices are considerably less when they are cold. If you measure the resistance of a 100 W light bulb with an ohmmeter (whose small current causes very little temperature rise), you'll probably get a value of about $36\ \Omega$. When a light bulb is turned on, this low resistance causes an initial surge of current until the filament heats up. That's why a light bulb that's ready to burn out nearly always does so just when you turn it on.

Circuit Overloads and Short Circuits

The maximum current available from an individual circuit is limited by the resistance of the wires. As we discussed in Section 25.5, the I^2R power loss in the wires causes them to become hot, and in extreme cases this can cause a fire or melt the wires. Wires with cross-sectional area 1.5 mm^2, which can carry currents of up to 14.5 A, are usually used for fixed lighting. Ordinary outlet wiring usually uses 2.5 mm^2-size wires, which can carry a maximum current of 20 A safely (without overheating). Larger-section wires of the same length have lower resistance [see Eq. (25.10)]. Hence wires with cross sections 4 mm^2 and 6 mm^2 are used for high-current appliances such as cookers and storage heaters, and 16 mm^2 or larger are used for the main power lines entering a house.

Protection against overloading and overheating of circuits is provided by fuses or circuit breakers. A *fuse* contains a link of lead–tin alloy with a very low melting temperature; the link melts and breaks the circuit when its rated current is exceeded (**Fig. 26.25a**). A *circuit breaker* is an electromechanical device that performs the same function, using an electromagnet or a bimetallic strip to "trip" the breaker and interrupt the circuit when the current exceeds a specified value (**Fig. 26.25b**). Circuit breakers have the advantage that they can be reset after they are tripped, while a blown fuse must be replaced.

Figure 26.25 (a) Excess current will melt the thin wire of lead–tin alloy that runs along the length of a fuse, inside the transparent housing. (b) The switch on this US circuit breaker will flip if the maximum allowable current is exceeded.

(a)

(b)

CAUTION **Fuses** If your system has fuses and you plug too many high-current appliances into the same outlet, the fuse blows. Do not replace the fuse with one that has a higher rating; if you do, you risk overheating the wires and starting a fire. The only safe solution is to distribute the appliances among several circuits. Modern kitchens often have three or four separate circuits.

Contact between the live and neutral wires causes a *short circuit*. Such a situation, which can be caused by faulty insulation or by a variety of mechanical malfunctions, provides a very low-resistance current path, permitting a very large current that would quickly melt the wires and ignite their insulation if the current were not interrupted by a fuse or circuit breaker (see Example 25.10 in Section 25.5). An equally dangerous situation is a broken wire that interrupts the current path, creating an *open circuit*. This is hazardous because of the sparking that can occur at the point of intermittent contact.

In approved wiring practice, a fuse or breaker is placed *only* in the live side of the supply, never in the neutral side. Otherwise, if a short circuit should develop because of faulty insulation or other malfunction, even if the fuse should blow, the live side would pose a shock hazard if you were to touch the live conductor (still connected to the supply) and a grounded object such as a water pipe. For similar reasons the wall switch for a light fixture should always be in the live side of the supply, never the neutral side.

Further protection against shock hazard is provided by a third conductor called the *earth wire* (or ground wire), included in all present-day wiring. Depending on the plug and socket type used in your country, the earth conductor corresponds to either the third pin of the plug or to the socket's male pin. The earth wire is connected to earth and is therefore at the earth potential. It normally carries no current, but it connects the metal case or frame of the device to earth. If a conductor on the live side of the supply accidentally contacts the frame or case, the earth conductor provides a current path, and the fuse blows. Without the earth wire, the frame could become "live"—that is, at a potential 230 V with respect to earth. Then if you touched it and a water pipe (or even a damp floor) at the same time, you could get a dangerous shock (**Fig. 26.26**). Modern

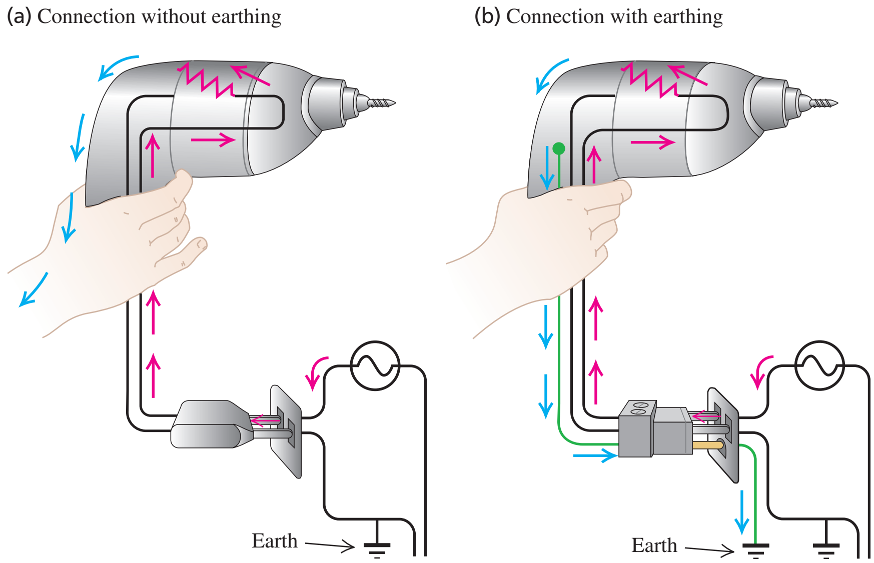


Figure **26.26** (a) If a malfunctioning electric drill is connected to a wall socket via a plug without earthing, a person may receive a shock. (b) When the drill malfunctions when connected via a plug which provides earthing, a person touching it receives no shock, because electric charge flows through the earth wire (shown in green) into the earth rather than into the person's body. If the earth current is appreciable, the fuse blows.

installations usually contain an additional protection, a special kind of circuit breaker called a *residual current device* (RCD) or *ground-fault interrupter* (GFI or GFCI). This device senses the difference in current between the live and neutral conductors (which is normally zero) and trips when some very small value, typically 30 mA, is exceeded.

Household and Automotive Wiring

In some countries, household wiring systems use elaborations of the system described above. For example, most modern domestic supply in the U.S. has *three* conductors (plus earth): one neutral and *two live* ones at 120 V with respect to the neutral, but with opposite polarity, giving a voltage between them of 240 V. With this three-wire supply, 120 V lamps and appliances can be connected between neutral and either live conductor, and high-power devices requiring 240 V, such as electric ranges and tumble dryers, are connected between the two live conductors. In some countries in Europe, the domestic supply is *three-phase*: it has four conductors (plus earth): the neutral one and three *lines* or *phases* (the live wires). The voltage between any single phase and the neutral is 230 V, but between any two phases it is 400 V. Again, it allows high-power appliances such as heat pumps or cookers to be supplied from 400 V, whereas the rest of of the circuits in the house use 230 V.

All of the above discussion can be applied directly to automobile wiring. The voltage is about 13 V (direct current); the power is supplied by the battery and by the alternator, which charges the battery when the engine is running. The neutral side of each circuit is connected to the body and frame of the vehicle. For this low voltage a separate earth conductor is not required for safety. The fuse or circuit breaker arrangement is the same in principle as in household wiring. Because of the lower voltage (less energy per charge), more current (a greater number of charges per second) is required for the same power; a 100 W headlight bulb requires a current of about $(100\text{ W})/(13\text{ V}) = 8\text{ A}$.

Although we spoke of *power* in the above discussion, what we buy from the power company is *energy*. Power is energy transferred per unit time, so energy is average power multiplied by time. The usual unit of energy sold by the power company is the kilowatt-hour (kWh):

$$1\text{ kWh} = (10^3\text{ W})(3600\text{ s}) = 3.6 \times 10^6\text{ W}\cdot\text{s} = 3.6 \times 10^6\text{ J}$$

In Europe, one kilowatt-hour typically costs between 10 and 30 euro cents, depending on the country and quantity of energy purchased. To operate a 2000 W (2 kW) electric heater continuously for 1 hour requires 2.0 kWh of energy; at 18 euro cents per kilowatt-hour, the energy cost is 36 euro cents. The cost of operating any lamp or appliance for a specified time can be calculated in the same way if the power rating is known. However, many electric heating or cooling devices (such as oil heaters or air-conditioning units) or cooking utensils (e.g. grills or waffle makers) cycle on and off to maintain a constant temperature, so the average power may be less than the power rating marked on the device.

EXAMPLE 26.14 A kitchen circuit

An 850 W toaster, a 2400 W electric kettle, and a 1450 W coffee machine are plugged into the same 16 A, 230 V circuit. (a) What current is drawn by each device, and what is the resistance of each device? (b) Will this combination trip the circuit breaker?

IDENTIFY and SET UP When plugged into the same circuit, the three devices are connected in parallel, so the voltage across each appliance is $V = 230$ V. We find the current I drawn by each device from the relationship $P = VI$, where P is the power input of the device. To find the resistance R of each device we use the relationship $P = V^2/R$.

EXECUTE (a) To simplify the calculation of current and resistance, we note that $I = P/V$ and $R = V^2/P$. Hence

$$I_{\text{toaster}} = \frac{850\text{ W}}{230\text{ V}} = 3.7\text{ A} \qquad R_{\text{toaster}} = \frac{(230\text{ V})^2}{850\text{ W}} = 62.2\ \Omega$$

$$I_{\text{kettle}} = \frac{2400\text{ W}}{230\text{ V}} = 10.4\text{ A} \qquad R_{\text{kettle}} = \frac{(230\text{ V})^2}{2400\text{ W}} = 22.0\ \Omega$$

$$I_{\text{coffee machine}} = \frac{1450\text{ W}}{230\text{ V}} = 6.3\text{ A} \quad R_{\text{coffee machine}} = \frac{(230\text{ V})^2}{1450\text{ W}} = 36.5\ \Omega$$

For constant voltage the device with the *least* resistance (in this case the electric kettle) draws the most current and receives the most power.

(b) The total current through the circuit is the sum of the currents drawn by the three devices:

$$\begin{aligned} I &= I_{\text{toaster}} + I_{\text{kettle}} + I_{\text{coffee machine}} \\ &= 3.7\text{ A} + 10.4\text{ A} + 6.3\text{ A} = 20.4\text{ A} \end{aligned}$$

This exceeds the 16 A rating of the circuit, and the circuit breaker will indeed trip.

EVALUATE We could also find the total current by using $I = P/V$ and the total power P delivered to all three devices:

$$\begin{aligned} I &= \frac{P_{\text{toaster}} + P_{\text{kettle}} + P_{\text{coffee machine}}}{V} \\ &= \frac{850\text{ W} + 2400\text{ W} + 1450\text{ W}}{230\text{ V}} = 20.4\text{ A} \end{aligned}$$

A third way to determine I is to use $I = V/R_{\text{eq}}$, where R_{eq} is the equivalent resistance of the three devices in parallel:

$$I = \frac{V}{R_{\text{eq}}} = (230\text{ V})\left(\frac{1}{62.2\ \Omega} + \frac{1}{22.0\ \Omega} + \frac{1}{36.5\ \Omega}\right) = 20.4\text{ A}$$

Appliances with such current demands are common, so modern kitchens have more than one 16 A circuit. To keep currents safely below 16 A, the electric kettle and coffee machine should be plugged into different circuits.

KEYCONCEPT The voltage across any device plugged into a household wiring system has the same value. The smaller the resistance of the device, the more current it draws and the more power it receives.

TEST YOUR UNDERSTANDING OF SECTION 26.5 To prevent the circuit breaker in Example 26.14 from blowing, a home electrician replaces the circuit breaker with one rated at 32 A. Is this a reasonable thing to do?

ANSWER

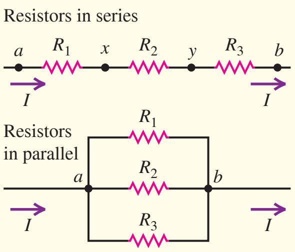
no This is a very dangerous thing to do. The circuit breaker will allow currents up to 32 A, much higher than the rated value of the wiring (20 A). The amount of power $P = I^2R$ dissipated in a section of wire can therefore be up to four times the rated value, so the wires could get hot and start a fire. (This assumes the resistance R remains unchanged. In fact, R increases with temperature, so the dissipated power can be even greater, and more dangerous, than we have estimated.)

CHAPTER 26 SUMMARY

Resistors in series and parallel: When several resistors $R_1, R_2, R_3, \ldots$ are connected in series, the equivalent resistance R_{eq} is the sum of the individual resistances. The same *current* flows through all the resistors in a series connection. When several resistors are connected in parallel, the reciprocal of equivalent resistance R_{eq} is the sum of the reciprocals of the individual resistances. All resistors in a parallel connection have the same *potential difference* between their terminals. (See Examples 26.1 and 26.2.)

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \quad (26.1)$$
(resistors in series)

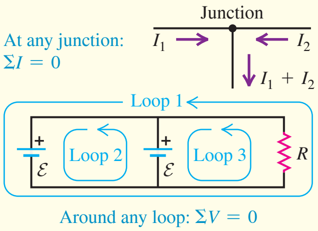
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \quad (26.2)$$
(resistors in parallel)



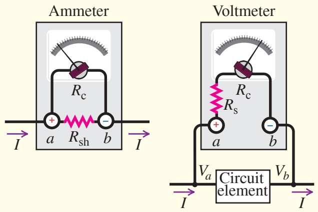
Kirchhoff's rules: Kirchhoff's junction rule is based on conservation of charge. It states that the algebraic sum of the currents into any junction must be zero. Kirchhoff's loop rule is based on conservation of energy and the conservative nature of electrostatic fields. It states that the algebraic sum of potential differences around any loop must be zero. Careful use of consistent sign rules is essential in applying Kirchhoff's rules. (See Examples 26.3–26.7.)

$$\sum I = 0 \quad \text{(junction rule)} \quad (26.5)$$

$$\sum V = 0 \quad \text{(loop rule)} \quad (26.6)$$



Electrical measuring instruments: In a d'Arsonval galvanometer, the deflection is proportional to the current in the coil. For a larger current range, a shunt resistor is added, so some of the current bypasses the meter coil. Such an instrument is called an ammeter. If the coil and any additional series resistance included obey Ohm's law, the meter can also be calibrated to read potential difference or voltage. The instrument is then called a voltmeter. A good ammeter has very low resistance; a good voltmeter has very high resistance. (See Examples 26.8–26.11.)



***R-C* circuits:** When a capacitor is charged by a battery in series with a resistor, the current and capacitor charge are not constant. The charge approaches its final value asymptotically and the current approaches zero asymptotically. The charge and current in the circuit are given by Eqs. (26.12) and (26.13). After a time $\tau = RC$, the charge has approached within $1/e$ of its final value. This time is called the time constant or relaxation time of the circuit. When the capacitor discharges, the charge and current are given as functions of time by Eqs. (26.16) and (26.17). The time constant is the same for charging and discharging. (See Examples 26.12 and 26.13.)

Capacitor charging:

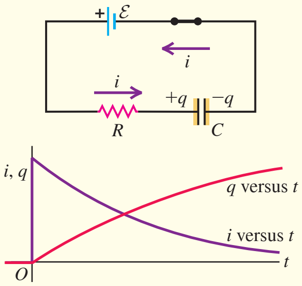
$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right) \quad (26.12)$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \quad (26.13)$$

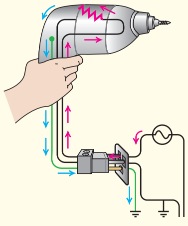
Capacitor discharging:

$$q = Q_0 e^{-t/RC} \quad (26.16)$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \quad (26.17)$$



Household wiring: In household wiring systems, the various electrical devices are connected in parallel across the supply. A mains electricity cable contains a pair of conductors, one "live" and the other "neutral." An additional "earth" wire is included for safety. The maximum permissible current in a circuit is determined by the size of the wires and the maximum temperature they can tolerate. Protection against excessive current and the resulting fire hazard is provided by fuses or circuit breakers. (See Example 26.14.)



Chapter 26 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLES 26.1 and 26.2 (Section 26.1) before attempting these problems.

VP26.2.1 You have three resistors: $R_1 = 1.00\,\Omega$, $R_2 = 2.00\,\Omega$, and $R_3 = 4.00\,\Omega$. Find the equivalent resistance for the combinations shown in (a) Fig. 26.1a, (b) Fig. 26.1b, (c) Fig. 26.1c, and (d) Fig. 26.1d.

VP26.2.2 You have three resistors, $R_1 = 4.00\,\Omega$, $R_2 = 5.00\,\Omega$, and $R_3 = 6.00\,\Omega$, connected as shown in Fig. 26.1d. The terminals of a battery with emf 24.0 V and negligible internal resistance are connected to points a and b. Find the current through (a) the battery, (b) R_1, (c) R_2, and (d) R_3.

VP26.2.3 You have three resistors, $R_1 = 7.00\,\Omega$, $R_2 = 8.00\,\Omega$, and $R_3 = 9.00\,\Omega$, connected as shown in Fig. 26.1b. The terminals of a battery with emf 12.0 V and negligible internal resistance are connected to points a and b. Find (a) the power output of the battery and the power input to (b) R_1, (c) R_2, and (d) R_3.

VP26.2.4 You have three resistors, $R_1 = 5.00\,\Omega$, $R_2 = 6.00\,\Omega$, and $R_3 = 7.00\,\Omega$, connected as shown in Fig. 26.1c. The terminals of a battery with emf 9.00 V and negligible internal resistance are connected to points a and b. Find (a) the power output of the battery and the power input to (b) R_1, (c) R_2, and (d) R_3.

Be sure to review EXAMPLES 26.3, 26.4, 26.5, 26.6, and 26.7 (Section 26.2) before attempting these problems.

VP26.7.1 In the circuit shown in Fig. 26.10a, you reverse the 4 V battery so that its positive terminal is on the right side instead of the left. Find (a) the current in the circuit, (b) the potential difference V_{ab}, and (c) the power output of the emf of each battery.

VP26.7.2 Figure 26.6a shows two batteries, one with emf $\mathcal{E}_1$ and one with emf $\mathcal{E}_2$, connected to a resistance R. Current I_1 flows through emf $\mathcal{E}_1$ from point c toward point a, and current I_2 flows through emf $\mathcal{E}_2$ from point b toward point a. If $\mathcal{E}_1 = 8.00$ V, $\mathcal{E}_2 = 9.00$ V, $R = 5.00\,\Omega$, $I_1 = 0.200$ A, and $I_2 = 1.35$ A, find the values of (a) the potential difference V_{ab}, (b) the internal resistance r_1, and (c) the internal resistance r_2.

VP26.7.3 In the circuit shown in Figure 26.12, you replace the 2 Ω resistor between points b and d with a 3 Ω resistor. Find the power dissipated in (a) the 1 Ω resistor between points c and a, (b) the 1 Ω resistor between points c and b, (c) the 1 Ω resistor between points a and b, (d) the 1 Ω resistor between points a and d, and (e) the 3 Ω resistor between points b and d. Give each answer to three significant figures.

VP26.7.4 In the circuit shown in Fig. 26.9a, let $r_1 = r_2 = 0$ (that is, the internal resistances are very small) and let $R_1 = R_2 = R_3 = R$. Find the values of (a) I_1, (b) I_2, and (c) I_3.

Be sure to review EXAMPLES 26.12 and 26.13 (Section 26.4) before attempting these problems.

VP26.13.1 You connect a 10.0 MΩ resistor in series with a 3.20 μF capacitor and a battery with emf 9.00 V. Before you close the switch at $t = 0$ to complete the circuit, the capacitor is uncharged. Find (a) the final capacitor charge, (b) the initial current, (c) the time constant, (d) the fraction of the final charge on the capacitor at $t = 18.0$ s, and (e) the fraction of the initial current present at $t = 18.0$ s.

VP26.13.2 A 2.20 μF capacitor initially has charge 4.20 μC. You connect it in series with a 4.00 MΩ resistor and an open switch. If $t = 0$ is the time when you close the switch, (a) at what time will the capacitor charge be 1.20 μC and (b) what will be the current in the circuit at this time?

VP26.13.3 You connect an unknown resistor in series with an 8.00 μF capacitor and an open switch. Before you close the switch at $t = 0$, the capacitor has charge 5.50 μC; at $t = 17.0$ s, the charge has decreased to 1.10 μC. Find (a) the resistance of the resistor, (b) the current in the circuit just after you close the switch, and (c) the current in the circuit at $t = 17.0$ s.

VP26.13.4 You connect an initially uncharged 6.40 μF capacitor in series with a 5.00 MΩ resistor and a battery with emf 12.0 V. After letting the capacitor charge for 51.0 s, you disconnect it from this circuit and connect it in series to an open switch and a 6.00 MΩ resistor. Find the charge on the capacitor (a) when you disconnect it from the first circuit and (b) 70.0 s after you close the switch in the second circuit.

BRIDGING PROBLEM Two Capacitors and Two Resistors

A 2.40 μF capacitor and a 3.60 μF capacitor are connected in series. (a) A charge of 5.20 mC is placed on each capacitor. What is the energy stored in the capacitors? (b) A 655 Ω resistor is connected to the terminals of the capacitor combination, and a voltmeter with resistance $4.58 \times 10^4\ \Omega$ is connected across the resistor (**Fig. 26.27**). What is the rate of change of the energy stored in the capacitors just after the connection is made? (c) How long after the connection is made has the energy stored in the capacitors decreased to $1/e$ of its initial value? (d) At the instant calculated in part (c), what is the rate of change of the energy stored in the capacitors?

SOLUTION GUIDE

IDENTIFY and SET UP

1. The two capacitors act as a single equivalent capacitor (see Section 24.2), and the resistor and voltmeter act as a single equivalent resistor. Select equations that will allow you to calculate the values of these equivalent circuit elements.
2. In part (a) you'll need to use Eq. (24.9), which gives the energy stored in a capacitor.
3. For parts (b), (c), and (d), you'll need to use Eq. (24.9) as well as Eqs. (26.16) and (26.17), which give the capacitor charge and current as functions of time. (*Hint:* The rate at which energy is lost by the capacitors equals the rate at which energy is dissipated in the resistances.)

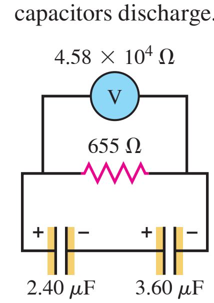
EXECUTE

4. Find the stored energy at $t = 0$.
5. Find the rate of change of the stored energy at $t = 0$.
6. Find the value of t at which the stored energy has $1/e$ of the value you found in step 4.
7. Find the rate of change of the stored energy at the time you found in step 6.

EVALUATE

8. Check your results from steps 5 and 7 by calculating the rate of change in a different way. (*Hint:* The rate of change of the stored energy U is dU/dt.)

Figure **26.27** When the connection is made, the charged capacitors discharge.



PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

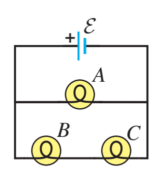
Q26.1 In which 230 V light bulb does the filament have greater resistance: a 60 W bulb or a 100 W bulb? If the two bulbs are connected to a 230 V supply in series, through which bulb will there be the greater voltage drop? What if they are connected in parallel? Explain your reasoning.

Q26.2 Two 120 V light bulbs, one 25 W and one 200 W, were connected in series across a 230 V supply. It seemed like a good idea at the time, but one bulb burned out almost immediately. Which one burned out, and why?

Q26.3 You connect a number of identical light bulbs to a flashlight battery. (a) What happens to the brightness of each bulb as more and more bulbs are added to the circuit if you connect them (i) in series and (ii) in parallel? (b) Will the battery last longer if the bulbs are in series or in parallel? Explain your reasoning.

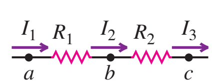
Q26.4 In the circuit shown in **Fig. Q26.4**, three identical light bulbs are connected to a flashlight battery. How do the brightnesses of the bulbs compare? Which light bulb has the greatest current passing through it? Which light bulb has the greatest potential difference between its terminals? What happens if bulb A is unscrewed? Bulb B? Bulb C? Explain your reasoning.

Figure **Q26.4**



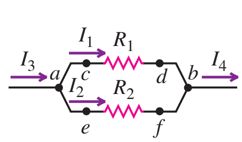
Q26.5 If two resistors R_1 and R_2 ($R_2 > R_1$) are connected in series as shown in **Fig. Q26.5**, which of the following must be true? In each case justify your answer. (a) $I_1 = I_2 = I_3$. (b) The current is greater in R_1 than in R_2. (c) The electrical power consumption is the same for both resistors. (d) The electrical power consumption is greater in R_2 than in R_1. (e) The potential drop is the same across both resistors. (f) The potential at point a is the same as at point c. (g) The potential at point b is lower than at point c. (h) The potential at point c is lower than at point b.

Figure **Q26.5**



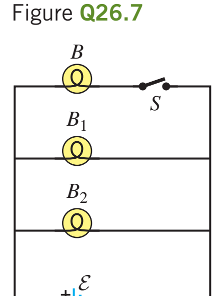
Q26.6 If two resistors R_1 and R_2 ($R_2 > R_1$) are connected in parallel as shown in **Fig. Q26.6**, which of the following must be true? In each case justify your answer. (a) $I_1 = I_2$. (b) $I_3 = I_4$. (c) The current is greater in R_1 than in R_2. (d) The rate of electrical energy consumption is the same for both resistors. (e) The rate of electrical energy consumption is greater in R_2 than in R_1. (f) $V_{cd} = V_{ef} = V_{ab}$. (g) Point c is at higher potential than point d. (h) Point f is at higher potential than point e. (i) Point c is at higher potential than point e.

Figure **Q26.6**



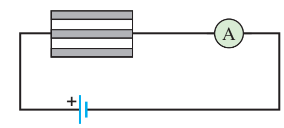
Q26.7 A battery with no internal resistance is connected across identical light bulbs as shown in **Fig. Q26.7**. When you close the switch S, will the brightness of bulbs B_1 and B_2 change? If so, how will it change? Explain.

Figure **Q26.7**



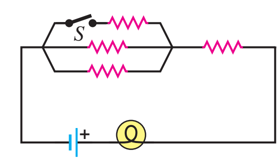
Q26.8 A resistor consists of three identical metal strips connected as shown in **Fig. Q26.8**. If one of the strips is cut out, does the ammeter reading increase, decrease, or stay the same? Why?

Figure **Q26.8**



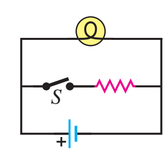
Q26.9 A light bulb is connected in the circuit shown in **Fig. Q26.9**. If we close the switch S, does the bulb's brightness increase, decrease, or remain the same? Explain why.

Figure **Q26.9**



Q26.10 A real battery, having nonnegligible internal resistance, is connected across a light bulb as shown in **Fig. Q26.10**. When the switch S is closed, what happens to the brightness of the bulb? Why?

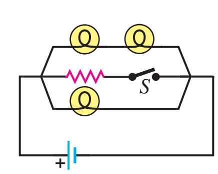
Figure **Q26.10**



Q26.11 If the battery in Discussion Question Q26.10 is ideal with no internal resistance, what will happen to the brightness of the bulb when S is closed? Why?

Q26.12 Consider the circuit shown in **Fig. Q26.12**. What happens to the brightnesses of the bulbs when the switch S is closed if the battery (a) has no internal resistance and (b) has nonnegligible internal resistance? Explain why.

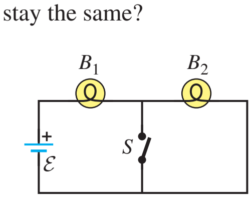
Figure **Q26.12**



Q26.13 Is it possible to connect resistors together in a way that cannot be reduced to some combination of series and parallel combinations? If so, give examples. If not, state why not.

Q26.14 The battery in the circuit shown in **Fig. Q26.14** has no internal resistance. After you close the switch S, will the brightness of bulb B_1 increase, decrease, or stay the same?

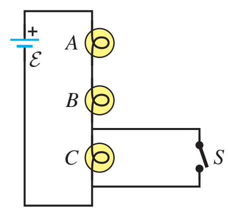
Figure **Q26.14**



Q26.15 In a two-cell flashlight, the batteries are usually connected in series. Why not connect them in parallel? What possible advantage could there be in connecting several identical batteries in parallel?

Q26.16 Identical light bulbs A, B, and C are connected as shown in **Fig. Q26.16**. When the switch S is closed, bulb C goes out. Explain why. What happens to the brightness of bulbs A and B? Explain.

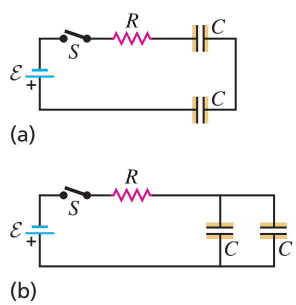
Figure **Q26.16**



Q26.17 The emf of a flashlight battery is roughly constant with time, but its internal resistance increases with age and use. What sort of meter should be used to test the freshness of a battery?

Q26.18 Will the capacitors in the circuits shown in **Fig. Q26.18** charge at the same rate when the switch S is closed? If not, in which circuit will the capacitors charge more rapidly? Explain.

Figure **Q26.18**



Q26.19 Verify that the time constant RC has units of time.

Q26.20 For very large resistances it is easy to construct R-C circuits that have time constants of several seconds or minutes. How might this fact be used to measure very large resistances, those that are too large to measure by more conventional means?

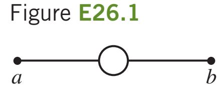
Q26.21 When a capacitor, battery, and resistor are connected in series, does the resistor affect the maximum charge stored on the capacitor? Why or why not? What purpose does the resistor serve?

EXERCISES

Section 26.1 Resistors in Series and Parallel

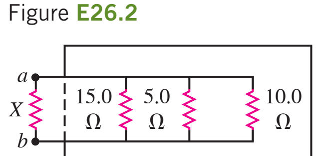
26.1 •• A uniform wire of resistance R is cut into three equal lengths. One of these is formed into a circle and connected between the other two (**Fig. E26.1**). What is the resistance between the opposite ends a and b?

Figure **E26.1**



26.2 •• A machine part has a resistor X protruding from an opening in the side. This resistor is connected to three other resistors, as shown in **Fig. E26.2**. An ohmmeter connected across a and b reads 2.00 Ω. What is the resistance of X?

Figure **E26.2**

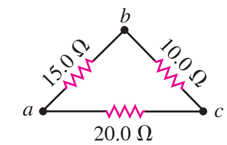


26.3 • A resistor with $R_1 = 25.0\ \Omega$ is connected to a battery that has negligible internal resistance and electrical energy is dissipated by R_1 at a rate of 36.0 W. If a second resistor with $R_2 = 15.0\ \Omega$ is connected in series with R_1, what is the total rate at which electrical energy is dissipated by the two resistors?

26.4 • A 46 Ω resistor and a 20 Ω resistor are connected in parallel, and the combination is connected across a 230 V dc supply. (a) What is the resistance of the parallel combination? (b) What is the total current through the parallel combination? (c) What is the current through each resistor?

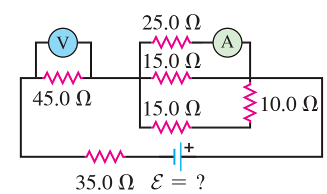
26.5 • A triangular array of resistors is shown in **Fig. E26.5**. What current will this array draw from a 30.0 V battery having negligible internal resistance if we connect it across (a) ab; (b) bc; (c) ac? (d) If the battery has an internal resistance of 5.00 Ω, what current will the array draw if the battery is connected across bc?

Figure **E26.5**



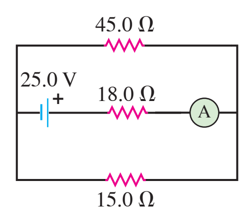
26.6 •• For the circuit shown in **Fig. E26.6** both meters are idealized, the battery has no appreciable internal resistance, and the ammeter reads 1.15 A. (a) What does the voltmeter read? (b) What is the emf $\mathcal{E}$ of the battery?

Figure **E26.6**



26.7 •• For the circuit shown in **Fig. E26.7** find the reading of the idealized ammeter if the battery has an internal resistance of 3.86 Ω.

Figure **E26.7**



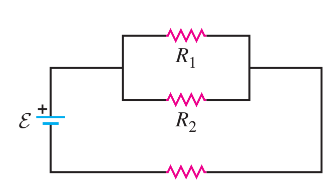
26.8 • Two resistors, R_1 and R_2, are connected in parallel to a power supply that has voltage V and negligible internal resistance. $R_2 = 8.00\ \Omega$ and the resistance of R_1 is not known. For several values of V, you measure the current I flowing through the voltage source. You plot the data as I versus V and find that they lie close to a straight line that has slope $0.208\ \Omega^{-1}$. What is the resistance of R_1?

26.9 • Six identical resistors, each with resistance R, are connected to an emf $\mathcal{E}$. (a) In terms of $\mathcal{E}$ and R, what is the current I through each of the resistors if they are connected in parallel? (b) In series? (c) For which network of resistors, series or parallel, is the power consumed in each resistor greater?

26.10 •• Power Rating of a Resistor. The *power rating* of a resistor is the maximum power the resistor can safely dissipate without too great a rise in temperature and hence damage to the resistor. (a) If the power rating of a 15 kΩ resistor is 5.0 W, what is the maximum allowable potential difference across the terminals of the resistor? (b) A 10.0 kΩ resistor is to be connected across a 230 V potential difference. What power rating is required? (c) A 100.0 Ω and a 150.0 Ω resistor, both rated at 3.00 W, are connected in series across a variable potential difference. What is the greatest this potential difference can be without overheating either resistor, and what is the rate of heat generated in each resistor under these conditions?

26.11 • In **Fig. E26.11**, $R_1 = 6.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 7.00\ \Omega$. The battery has negligible internal resistance. The current I_2 through R_2 is 4.00 A. (a) What are the currents I_1 and I_3? (b) What is the emf of the battery?

Figure **E26.11**



26.12 •• In Fig. E26.11 the battery has emf 35.0 V and negligible internal resistance. $R_1 = 5.00\ \Omega$. The current through R_1 is 1.50 A, and the current through $R_3 = 4.50$ A. What are the resistances R_2 and R_3?