# WhiteMesh: Leveraging White Spaces in Wireless Mesh Networks

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Abstract—Wireless mesh networks were previously thought to be an ideal solution for large-scale Internet connectivity in metropolitan areas. However, in-field trials revealed that the node spacing required for WiFi propagation induced a prohibitive cost model for network carriers to deploy. The digitization of TV channels and new FCC regulations have reapportioned spectrum for data networks with far greater range than WiFi due to lower carrier frequencies. Also, occupancy of frequency varies from populated area to sparse area across both ISM bands and white space bands. In this work, we consider how these white space bands can be leveraged in large-scale wireless mesh network deployments with in-field measured channel capacity. In particular, we present an integer linear programming model to leverage diverse propagation characteristics of white space and WiFi bands to deploy optimal WhiteMesh networks. Since such optimization is known to be NP-hard, we design a heuristic algorithm, Band-based Path Selection (BPS), which we show approaches the performance of the optimal solution with reduced complexity. We additionally compare the performance of BPS against two well-known multi-channel, multi-radio deployment algorithms across a range of scenarios spanning those typical for rural areas to urban areas. In doing so, we achieve between 3 to 6 times the gateway goodput of these existing multichannel, multi-radio algorithms, which are agnostic to diverse propagation characteristics across bands. Moreover, we show that, with similar channel resources and bandwidth, the joint use of WiFi and white space bands can achieve a served user demand of 170% that of mesh networks with only WiFi bands or white space bands, respectively.

# I. INTRODUCTION

About a decade ago, numerous cities solicited proposals from network carriers for exclusive rights to deploy citywide WiFi, spanning hundreds of square miles. While the vast majority of the resulting awarded contracts used a wireless mesh topology, initial field tests revealed that the actual WiFi propagation could not achieve the proposed mesh node spacing. As a result, many network carriers opted to pay millions of dollars in penalties rather than face the exponentially-increasing deployment costs (e.g., Houston [1] and Philadelphia [2]). Thus, while a few mesh networks have been deployed in certain communities [3], [4], wireless mesh networks have largely been unsuccessful in achieving the scale of what was once anticipated [5].

Around the same time, the digital TV transition created more spectrum for use with data networks [6]. These white space bands operate in available channels from 54-806 MHz, having increased propagation characteristics as compared to WiFi [7]. Hence, the FCC has identified rural areas as a key application for white space networks since the reduced population from major metropolitan areas allows a greater service area per backhaul device without saturating wireless capacity. As shown in Google database [8], the number of available white space frequency channels is deferent among cities all over US. And previous work [9] has shown the occupancy of

frequency varies related to population distribution. Naturally, the question arises for these rural communities as well as more dense urban settings: how can the emerging white space bands improve large-scale mesh network deployments? While much work has been done on deploying multihop wireless networks with multiple channels and radios, the differences in propagation have not been exploited in their models [10]–[13], which could be the fundamental issue for the success of mesh networks going forward.

In this paper, we leverage the diversity in propagation of white space and WiFi bands in the planning and deployment of large-scale wireless mesh networks. To do so, we first form an integer linear program to jointly exploit white space and WiFi bands for optimal WhiteMesh topologies. Second, since similar problem formulations have been shown to be NP-hard [13], [14], we design a heuristic algorithm, Bandbased Path Selection (BPS). We then show the algorithm approaches the performance of the optimal solution but with a reduced complexity. We also involve in-field measured data to quantify the frequency occupancy in our algorithms. To assess the performance of our scheme, we compare the performance of BPS against two well-known multi-channel, multi-radio deployment algorithms across a wide range of scenarios, including those typical for rural areas as well as urban settings. Finally, we quantify the degree to which the joint use of both band types can improve the performance of wireless mesh networks.

The main contributions of our work are as follows:

- We analyze the white space bands application in wireless network deployment and develop an optimization framework based on integer linear programming to jointly leverage white space and WiFi bands to serve the greatest user demand in terms of gateway throughput in wireless mesh networks.
- We build an algorithm, Band-based Path Selection (BPS), which considers the diverse propagation and overall interference level of WiFi and white space bands using a twostage approach. In the first stage, we prioritize the bands with the greatest propagation to reduce the overall hop count. In the second stage, we compare the interference level of path choices with similar hop count.
- We perform extensive analysis across offered loads, network sizes, and WiFi/white space band combinations, showing that BPS outperforms existing multi-channel, multi-radio algorithms techniques by 3 and 6 times in terms of the served user demand.
- Given similar channel resources (bandwidth and transmission power), we additionally show that while WiFi-only mesh topologies would largely outperform mesh networks with only white space bands, the joint use of the two types of bands (i.e., WhiteMesh networks) can yield up

to 170% of the served user demand compared to mesh networks with only one type of band.

The remainder of this paper is organized as follows. In Section II, we introduce WhiteMesh network topologies, describe the challenge of diverse frequency band allocation, and formulate the integer linear programming model. In Section III, we develop two heuristic algorithms which consider which bands and multihop paths to select in a WhiteMesh topology. We then validate the performance of the heuristic algorithms versus the upper bound of the optimal solution and compare their performance against two well-known multichannel, multi-radio algorithms in Section IV. Finally, we discuss related work in Section V and conclude in Section VI.

#### II. PROBLEM FORMULATION

In this section, we formulate the problem of how to optimally use WiFi and white space bands in concert when deploying wireless mesh networks. We first describe our system model and illustrate the challenges of such a WhiteMesh architecture. We then discuss how to evaluate WhiteMesh networks and the corresponding goal of both the optimization framework and the heuristic algorithm that we propose in the following section. Finally, we present our integer linear programming model used to address the problem.

#### A. WhiteMesh Network Architecture

Wireless propagation is the behavior of the signal loss characteristics when wireless signals are transmitted from a transmitter to receiver. The strength of the receiving signal depends on both the line-of-sight path (or lack thereof) and multiple other paths that are a result of reflection, diffraction, and scattering from obstacles in the environment [15]. The widely-used Friis equation characterizes the power of the received signal  $P_r$  in terms of the power  $P_t$  and gain  $G_t$  of the transmitting signal, gain of the receiver  $G_r$ , wavelength  $\lambda$  of the carrier frequency, distance R from transmitter to receiver, and path loss exponent n according to [16]:

$$P_r = P_t + G_t + G_r + 10n \log_{10} \left(\frac{\lambda}{4\pi R}\right) \tag{1}$$

Here, the path loss exponent n changes according to the aforementioned environmental factors and ranges from 2 to 5 in typical outdoor settings [17].

A common assumption in works that use many WiFi channels is that the propagation characteristics of one channel is similar to another, since the channel separation is relatively small (e.g., 5 MHz for the 2.4 GHz band). Many works which rely on such an assumption have focused on the allocation of multiple WiFi channels with multiple radios in multihop wireless networks [11]. Here, a frequency band is defined as a group of channels which have similar propagation characteristics. In the application of wireless communication, channel occupancy has to be considered [18]. To quantify the channel occupancy, we inherit the activity level from [19] to quantify the channel occupancy as shown in Eq. 2. The activity level is defined based on a long-term measurement of each band. It is defined as the percentage of sensing samples  $(S_{\theta})$  above an interference threshold  $(\theta)$  over the total samples (S) in a time unit as the activity level (A) of inter-network interference:

$$A = \frac{S_{\theta}}{S_a} \tag{2}$$

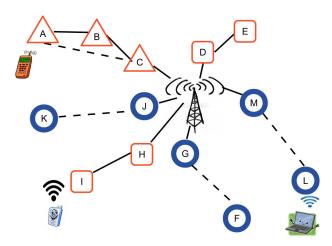


Fig. 1. Example WhiteMesh topology with different mesh-node shapes representing different frequency band choices per link.

In this work, we consider the diverse propagation characteristics for four frequency bands: 450 MHz, 800 MHz, 2.4 GHz, and 5.8 GHz. We refer to the two former frequency bands as white space (WS) bands, whereas we refer to the two latter frequency bands as WiFi bands.

Wireless mesh networks are a particular type of multihop wireless network that are typically considered to have at least two tiers [3]: (i) an access tier, where client traffic is aggregated to and from mesh nodes, and (ii) a multihop backhaul tier for connecting all mesh nodes to the Internet through gateway nodes. In this work, we focus on how to optimally allocate white space and WiFi bands on a finite set of radios per mesh node along the backhaul tier, since we assume that client devices will use WiFi (due to the economies of scale). In each of the WhiteMesh topologies studied in Section IV, a sufficient number of orthogonal WiFi channels remain for the access tier clients connection through additional radios co-located on the mesh nodes.

Due to the broadcast nature of the wireless medium, greater levels of propagation induce higher levels of interference. Thus, in sparsely-populated rural areas, the lower frequencies of the white space bands might be a more appropriate choice for multihop paths to gateways having reduced hop count. However, as the population and demand scales up (e.g., for urban regions), the reduced spatial reuse and greater levels of interference of white space bands might detract from the overall deployment strategy. In such urban areas, select links of greater distance might be the most appropriate choice for white space bands, especially since the number of available channels is often inversely proportional to the population (due to the existence of greater TV channels).

Figure 1 depicts an example where mesh node A could connect to mesh node C through B at 2.4 GHz, or directly connect to C at 450 MHz. If 2.4 GHz were used, link D, E might be able to reuse 2.4 GHz if they are out of the interference range. However, if link A, C used 450 MHz, a lower hop count would result for the path, but lower levels of spatial reuse also result (e.g., for link D, E). While the issues of propagation, interference, and spatial reuse are simple to understand, the joint use of white space and WiFi bands to form optimal WhiteMesh topologies is challenge.

#### B. Model and Problem Formulation

In this work, we have a model similar to prior multi-channel models [10], [11], [13]. However, in these previous models, they fail to distinguish the communication range  $D_c$  and interference range  $D_4$  variation across frequency bands. While these works would attempt to minimize the interference in a multihop network topology by an optimal channel assignment for a given set of radios, we hypothesize that using radios with a greater diversity in propagation could yield overall network performance gains. Therefore, for a given set of radios, we allow the channel choices to come from multiple frequency bands (i.e., multiband channel assignment, which also includes multiple channels in the same band). To simplify the analysis, we assume the channel capacity of each band is equal. In practice, we could easily calculate the proportional ratio of channels in each band according to their bandwidth. We assume that the locations of mesh nodes and gateway nodes are given and all mesh nodes have the same transmit power, channel bandwidth, and antenna gain. Each mesh node operates with a classic protocol model [20].

A mesh network could be represented by a unidirectional graph G=(V,E), where V is the set of mesh nodes, and E is the set of all possible physical links in the network. If the received signal (according to Eq. 1) between two mesh nodes i,j for a given frequency band (from the set of all bands B) is greater than a communication-range threshold, then a data link exists and belongs to the set E with a fixed, non-zero capacity E0 according to the protocol model. Due to the frequency occupancy, the available channel capacity is calculated through Eq. 3.

$$\gamma_r = \gamma * (1 - \bar{A}) \tag{3}$$

The capacity of a clean channel is denoted by  $\gamma$ . With the protocol model, the capacity of a channel with internetwork interference  $\gamma_r$  could be represented as the remaining free time of the channel capacity according to the measured activity level A. Correspondingly, a connectivity graph C is formed for each band in B such that C=(V,L,B). If the received signal for a given band is above an interference-range threshold, then contention occurs between nodes. We extend the conflict matrix in [10] related to different interference per band according to  $F=(E_{i,j},I_{Set},B)$ , where  $E_{i,j}$  represents the link and  $I_{Set}$  includes all the links are physically inside the interference range  $D_r$  when operating on each band b.

Therefore, the problem we model is: to choose the connectivity graph C' which maximizes the metrics according to the constriants of multiband wirless network (defined below). A key challenge is that selecting the optimal channels from the set B leads to a conflict graph F which cannot be known a priori. Previous works have proposed a coloring, cluster-independent set, mixed linear integer methodology for a single band b [10], [13], [21]. However, these works do not address a reduction in hop count or an increase in spatial reuse for a set of diverse bands B.

In network application, the bottleneck of mesh network capacity has been shown to be the gateway's wireless connections [22]. The metric we use to evaluate the proposed algorithm is the traffic arrived at gateway nodes. Networks are operated and maintenaced by vendors. The vendors, such as AT&T, T-Mobile, they chare the custumers based on the

data amount. The traffic arrived at gateway nodes under an random traffic generated area correlated to the incomes of the vendors. We use the performance metric of gateway goodput X, represented the traffic arrived at the gateway nodes, where:

$$X = \sum_{w \in W, v \in V} T(w, v) \tag{4}$$

The goodput of gateway node  $w \in W$  considers all incoming and outgoing wireless traffic from access node  $v \in V$  as T onto the Internet. Obviously, the goodput is also related to the routing, we use a simple routing method to keep the maximum the traffic arrived at the gateway nodes, the exact calculation of gateway goodput is described in Section IV and consider where to put the gateway nodes are outside the scope of this work.

We also use the general metric *network throughput* NT in [10], [13] which directly associated with the interfering number of each link, as 5:

$$NT = \sum_{i=1}^{n} \frac{1}{1 + Interference \ number \ of \ (e \in E)}$$
 (5)

The network throughput is the sum of shared bandwidth of each link in the network, which represents the worst case performance of the wireless network.

## C. Mixed Integer Linear Programming Formulation

To clarify the problem and approach the optimization of the problem, we present a mixed integer, linear programming formulation for optimizing channel assignment in multiband scenario. We assume that the set of available mesh nodes (V), gateways (W), and available bands (B) are pre-known. The communication links and conflict graph are given as parameters. The capacity  $\gamma$  is given as the available channel capacity based on activity level measurement noted in 3.

Sets: V set of nodes B set of bands Parameters:

$$\begin{array}{lll} \gamma_{i,j}^b & (i,j) \in V, b \in B & \text{capacity of link } i,j \text{ on} \\ b & b & b & b \\ I_{ij,lm}^b & (i,j,l,m) \in V, b \in B & \text{Interference of link} \\ W_i & i \in V \text{ binary} & \text{Gateways in network} \\ D_{di} & i \in V & \text{Downlink demand of node i} \\ D_{ui} & i \in V & \text{Uplink demand of node} \\ \end{array}$$

In the variable set, we define a time share represents the percentage of time a single link transmits according to  $\alpha^b_{i,j}$  for link i,j between node i and node j in band b. There are two terms  $uy^b_{i,j,k}$  and  $dy^b_{i,j,k}$  defined as uplink and downlink flows:

## Variables:

$$0 \leq \alpha_{ij}^b \leq 1 \qquad b \in B, (i,j) \in N \qquad \text{Time share of link} \\ 0 \leq uy_{i,j,k}^b \qquad (i,j,k) \in V, b \in B \qquad \text{Uplink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink flow of node} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink} \\ 0 \leq dy_{i,j,k}^b \qquad (i,j,k) \in N, b \in B \qquad \text{Downlink}$$

Our objective could be represented for maximizing the traffic arrived at gateway (X) or the network throughput (NT), defined in Eq. 5.

## **Objective:**

$$Max \sum_{i} \sum_{j} \sum_{k} \sum_{b} (uy_{i,j,k}^{b} + dy_{j,i,k}^{b})$$
 When  $w_{j} = 1$  (6)

$$Max \sum_{i} \sum_{j} \sum_{b} \left( \frac{1}{\sum_{l,m} \alpha_{i,j}^{b} \times I_{ij,lm}} \right) \tag{7}$$

In the ILP, the connectivity, uplink, and downlink constraints are represented as:

## **Connectivity Constraints:**

$$\alpha_{i,j}^b + \alpha_{j,i}^b + \sum_{l} \sum_{m} (\alpha_{l,m}^b \cdot I_{ij,lm}^b) \le 1, i \ne j$$
 (8)

$$\sum_{i} u y_{i,j,k}^b + \sum_{i} d y_{i,j,k}^b \le r_{j,k}^b \cdot \alpha_{j,k}^b \tag{9}$$

## **Uplink Constraints:**

$$\sum_{k} \sum_{b} u y_{i,i,k}^{b} \le D_{ui} \text{ when } w_{k} = 0, i \ne k$$
 (10)

$$uy_{i,j,k}^b = 0w_k = 1 (11)$$

$$\sum_{i} \sum_{b} u y_{i,j,k}^{b} = \sum_{m} \sum_{b} u y_{j,m,k}^{b} \text{ when } w_{k} = 0, i \neq k$$
 (12)

$$uy_{i,j,i}^b = 0 (13)$$

## **Downlink Constraints:**

$$\sum_{j} \sum_{b} dy_{i,j,i}^{b} \le D_{di} \text{ when } w_{i} = 0$$

$$\tag{14}$$

$$dy_{i,j,k}^l = 0 \text{ when } w_k = 1$$
 (15)

$$\sum_{j} \sum_{b} dy_{i,j,k}^{b} = \sum_{m} \sum_{b} dy_{j,m,k}^{b}, \text{ when } w_{k} = 0, i \neq k$$
(16)

$$dy_{i,i,j}^b = 0 (17)$$

In the constraints, (8) represents the summation of the incoming and outgoing wireless time share and the interfering links' wireless time share, which should all be less than 1. Constraint (9) represents the incoming and outgoing wireless traffic, which should be less than the link capacity for link i,j. Uplink constraints (10) and (11) represent that the summation of any wireless flow i,j should be less than the demand of node k. Constraints (12) and (13) are used to restrict the sum of all incoming data flows for a given mesh node k to be equal to the sum of all outgoing flows. Downlink constraints (14) and (15) are similar to (10) and (11) but in the downlink direction. Similarly, constraints (16) and (17) are downlink versions of (12) and (13).

Linear programs which attempt to solve channel assignment and routing in multihop wireless networks have been proved to be NP-hard [23]. Our model jointly considers channel assignment factors and provides the methodology to achieve the optimization solution. When the particular configuration is given, we can choose the objectives, parameters, and relax constraints to find an approaching solution for the network.

#### III. PATH ANALYSIS WITH DIVERSE PROPAGATION

In this section, we discuss the influence of diverse propagation characteristics of the wide range of carrier frequencies of white space and WiFi bands. We then introduce our measurement driven heuristic algorithm for channel assignment in WhiteMesh networks.

## A. Path Interference Induced on the Network

In WhiteMesh networks, multihop paths can be intermixed with WiFi for more spacial reuse and white space bands with less hops. To deal with the tradeoff, we consider analyze the band choices reduce the number of hops along a path and the aggregate level of interference that hop-by-hop path choices have on the network (i.e., Path Interference induced on the Network).

Mesh nodes closer to the gateway generally achieve greater levels of throughput at sufficiently high offered loads. To combat such starvation effects, we treat each flow with equal priority in the network when assigning channels. In the worst case, all nodes along a particular path have equal time shares for contending links (i.e., intra-path interference). We start the channel assignment assuming that h mesh nodes are demanding traffic from each hop of an h-hop path to the gateway. If each link along the path uses orthogonal channels, then each link could be active simultaneously, otherwise they will complete with each other. We note each node along the path had traffic demand  $T_d$ , obviously the bottleneck link along the path would be the one closest to the gateway, and then next. Thus, the total traffic along the path  $h \cdot T_d$  must be less than the bottleneck link's capacity  $\gamma$  estimated from the measurements. In such a scenario, the h-hop mesh node would achieve the minimum served demand, which we define as the network efficiency. In general, the active time per link for an *h*-hop mesh node can be represented by  $1, \frac{h-1}{h}, \frac{h-2}{h} \cdots \frac{1}{h}$ . The summation of all active times for each mesh node along the path is considered the intra-path network cost.

Considering only intra-path interference, using lower carrier frequencies allows a reduction in hop count and increase in the network efficiency of each mesh node along the h-hop path. However, a lower carrier frequency will induce greater interference to other paths to the gateway (i.e., inter-path interference). Fig. 1 depicts such an example where links in different bands are represented by circles for 450 MHz, rectangles for 2.4 GHz, and triangles for the nodes which can choose between the two. Nodes A and C could be connected through two 2.4-GHz links or a single 450-MHz link. With 2.4 GHz, the interfering distance will be less than using 450 MHz. For example, only link D, E will suffer from interference, whereas H, I would not. However, with 450 MHz, link A, C would interfere with links F, G, M, L, and K, J. At each time unit, the number of links interfering with the active links along a path would be the inter-path network cost.

When an h-hop flow is transmitted to a destination node, it prevents activity on a number of links in the same frequency via the protocol model. The active time on a single link can be noted as  $\frac{T}{\gamma_h}$ . An interfering link from the conflict matrix F counts as  $I_h$  per unit time and contributes to the network cost in terms of:  $\frac{hT}{\gamma_1} \cdot I_1 + \frac{(h-1)T}{\gamma_2} \cdot I_2 \cdots \frac{T}{\gamma_h} \cdot I_h$ . Then, the traffic

transmitted in a unit of network cost for the h-hop node is:

$$E_{\eta} = \frac{T}{\sum_{i \in h} \frac{(h-i+1) \cdot T}{\gamma_i} \cdot I_i}$$
 (18)

Using network efficiency, the equation simplifies to:

$$E_{\eta} = \frac{\gamma}{\sum_{i \in h} (h - i + 1) \cdot I_i} \tag{19}$$

The network efficiency is the amount of traffic that could be offered on a path per unit time. With multiple channels from the same band,  $I_i$  will not change due to the common communication range. With multiple bands,  $I_i$  depends on the band choice due to the communication range diversity. This network efficiency jointly considers hop count and interference. We define the Path Interference induced on the Network (PIN) as the denominator of Eq. 19, which represents the sum of all interfering links in the network by a given path. PIN is used to quantify the current state of channel for channel assignment across WiFi and white space bands. To determine when the lower carrier frequency will be better than two or more hops at a higher carrier frequency, we consider the average interference  $\bar{I}$  of a given path at the higher frequency. The problem could be formulated as:

$$\frac{\gamma}{\frac{h(h-1)}{2} \cdot \bar{I} + I_x} \ge \frac{\gamma}{\frac{h(h+1)}{2} \cdot \bar{I}} \tag{20}$$

Here, from Eq. refeq:benefit when  $I_x \leq 2 \cdot h\bar{I}$ , the performacne of a lower-frequency link is better than two higher-frequency hops for the same destination node.  $I_x$  is also a parameter of hop count in Eq. 19. When the hop count is lower which closer to the gateway node, the threshold would be more strict since the interference would have a greater effect on the performance.

## B. Band-based Path Selection (BPS) Algorithm

We design a Band-based Path Selection (BPS) algorithm (described in Alg. 1) which first chooses the mesh node that has the largest physical distance from the gateway nodes. When a path is constructed for the mesh node with the greatest distance, all subsequent mesh nodes along the path are also connected to the gateway. The central concept behind the BPS algorithm is to improve the worst mesh node performance in a path. For large-scale mesh networks, it is impractical to traverse all the paths with different combination of bands from a mesh node to any gateway node. However, based on the discussion in Section III-A, if two paths have the same number of used bands along those paths, then the path with the least hops is likely to have the greatest performance and is chosen. Similarly, if two path have the same path interference, we choose the path which has higher-frequency links for spatial reuse. Thus, the next step of the algorithm is to find the shortest path across band combinations.

Compared to the number of mesh nodes, the amount of channels  $N_B$  in different bands is small. The time complexity of calculating the combination is  $O(2^{N_B})$ . Finding the shortest path in Dijkstra algorithm will cost  $O(N_E^2)$  according to [24], where  $N_E$  is the links in the network, and as a result, the total complexity would be  $O(N_E^2 \cdot 2^{N_B})$ . The algorithm would then calculate the PIN of the candidate path and select the path with the least interference induced on the network for

## Algorithm 1 Band-based Path Selection (BPS)

M: The set of mesh nodes

Input:

```
G: The set of gateway nodes
   C: Communication graph of potential links among all nodes
    I: Interference matrix of all potential links
    B: Available frequency bands
Output:
    CA: Channel Assignment of the Network
   Rank mesh nodes according to physical distance from gateway
 2: Initialize S_{curr} = G, N_{srv} = \emptyset, N_{unsrv} = M, I_{active} = \emptyset
   while N_{srv}=!M do
      Select node with largest distance to gateway
 5:
      Find the adjacency matrix across band combinations A_c
      for all A_i \in A_c do
         Find the shortest path SP_i in mixed adjacency matrix A
 7:
         for all Link l \in SP_i, ordered from gateway to mesh node
 8:
 9.
           Find the least interfering link
10:
           If equally-interfering links, choose higher frequency
11:
           Calculate the path interference of SP_i
         end for
         Store the shortest path SP_i as SP
13:
      end for
14:
      Assign the path in the network
15:
16:
      Update N_{srv}, N_{unsrv}
17:
      Update I_{active} from I
   end while
   Output CA as the locally-optimal solution
```

the source mesh node. After a path is assigned, the algorithm updates the network's channel assignment with served nodes, activated links, and radio information. Then, we iteratively assign channels for all the mesh nodes in the network.

If all the nodes are connected  $(N_E = \binom{n}{2})$  which is  $O(N_V^2)$ ), then the complexity of assigning a channel for a mesh node is  $O(N_E^2 \cdot 2^{N_B})$ . Then, the complexity of assigning a mesh node is  $O(N_V^4 \cdot 2^{N_B})$ . To assign all the nodes in the network, the complexity would be  $O(N_V^5 \cdot 2^{N_B})$ .

## IV. EVALUATION OF WHITEMESH CHANNEL ASSIGNMENT

We now extensively evaluate our proposed heuristic algorithm against the upper bound formed by our integer linear program and versus prior channel assignment strategies. We introduce the topologies and metric calculation used in the analysis and present a set of results based of the linear program and heuristic algorithm.

## A. Experimental Evaluation Setup

A key aspect of WhiteMesh networks is the diversity in propagation from the lowest white space channels (tens to hundreds of MHz) to the highest WiFi channels (multiple GHz). Thus, to evaluate the performance of our algorithms, we consider a wide range of propagation characteristics from four different frequency bands. For white space bands, we consider 450 and 800 MHz. For WiFi bands, we consider 2.4 and 5.8 GHz. With similar transmission power and antenna gain, the highest carrier frequency would have the shortest communication range. Hence, we set a communication-range threshold of -100 dBm, and normalize the communication range with the highest frequency of 5.8 GHz. In particular, the communication range of 450 MHz, 800 MHz, 2.4 GHz, and 5.8 GHz would be normalized to 12.8, 6.2, 2.4, and 1, respectively. The interference range is computed as twice that

of the communication range [25]. We deploy static wireless mesh networks of n mesh nodes along a regular, rectangular grid with a normalized distance of 0.8 between rectangular edges. As a result, we have varying degrees of connectivity in the grid. The gateways are randomly-selected from one out of every six mesh nodes in the network. Unless otherwise, specified in the analysis, all four bands are used in the WhiteMesh topology studied.

The throughput achieved through the gateways is not only critical for network providers (e.g., cellular carriers charge fees for total bandwidth through towers), but also has been used by researchers to evaluate channel assignment [26]. As mentioned previously, the wireless capacity of gateway nodes has been shown to be the bottleneck in mesh networks [27]. Moreover, gateway goodput combines multiple factors, such as mesh node placement, gateway placement, routing, and channel assignment, the latter of which is the focus of our analysis and algorithms. For the purposes of our analysis, we specifically calculate the gateway goodput first introduced in Section II-B in the following way. Mesh nodes that have a one-hop path to the gateway nodes are the first ones served. Where there are nodes with the same hop count, the least interfering mesh nodes are chosen for channel assignment. Then, the demand of multihop mesh nodes are served until there is no remaining demand to be satisfied through any path.

We vary the average population distribution of the target area, assuming 10% of the residents will use our service. An individual would have a 100KB/s traffic demand on average. Then, we randomly assign the user distribution across the area and run the analysis of each case 20 times. We relax our ILP model to keep the link capacity constraints, given the demand of the mesh nodes as a parameter to achieve the maximum throughput at the gateways. The ILP Bound assumes the mesh nodes could connect to all the gateways if there is a path that exists to each gateway. However, we restrict a mesh node to only connect to one gateway in the network, and this represents the main difference from the ILP Bound to our heuristic algorithm.

## B. Experimental Analysis of WhiteMesh Backhaul

Typically, clients and mesh nodes have diverse traffic patterns with the download direction dominating the total traffic demand (e.g., consider service agreements for cellular data or Internet connectivity). Hence, to simplify the analysis and scale the ILP Bound to larger network sizes, we remove the uplink constraints while maintaining the downlink constraints. We then randomly assign demand per mesh node with a maximum offered load as specified in Fig. 2 and Table I. We repeat the process 20 times, averaging the results of each of the algorithms for the given network configuration.

In the first experiment, we consider different network sizes according to the number of mesh nodes in the aforementioned regular grid. In particular, we expect that as the network size grows, so too does the number of gateways, producing a greater total gateway goodput. Fig. 2(a) shows the total gateway goodput when the population distribution is  $600 \ ppl/km^2$  for the ILP formulation and the heuristic algorithms: (i) Common Channel Assignment (CCA) from [28], (ii) Breadth First Search Channel Assignment (BFS-CA) from [29], (iv) our algorithm BPS (Section III-B). In CCA [28], two nodes will assign a channel for each other when there is a common

free channel for both of the nodes. In BFS-CA [10], a node will search all the available one-hop connections then choose the one has the largest capacity for a new assignment. These two methods assume the one-hop connections are equal when there is no assignment on the channel. In BPS, we both consider and leverage propagation differences of diverse bands.

In Fig. 2(a), we observe dips in the curves for all algorithms, representing the randomly generated demand. The ILP Bound shows what could be expected, that an increasing number of gateways produce an increase in total gateway goodput. However, we observe that the other algorithms are not able to achieve such behavior for various reasons. CCA increases the average hop count the most, meaning that it was unable to find shorter routes to new gateways. BFS-CA fails to increase the reach of the gateway by attempting to optimize the first hop from the gateway. Conversely, BPS alleviates the strain on these first-hop, bottleneck links, achieving up to 63% of the ILP Bound. The key difference of BPS to the ILP is that BPS only considers one path to a gateway node for each mesh node, whereas the ILP allows multiple paths to the gateways.

Next, we consider a different form of scalability in our analysis. Namely, we increase the average population distribution from 150 to 1,500 per km<sup>2</sup>, while maintaining a 30node regular grid topology. Fig. 2(b) shows that all of the algorithms are able to achieve comparable gateway goodput at 150  $ppl/km^2$ . However, as the population distribution increases, the ILP and BPS diverge greatly from the remaining algorithms. Similar to Fig. 2(a), the wireless capacity around a gateway is quickly saturated if the algorithm is not focused on preserving that resource. Nonetheless, we do observe a leveling off at approximately 60 Mbps offered load due to a similar effect of the saturation of wireless capacity around the gateway node. For BPS, this saturation point at 60 Mbps has a gateway goodput from 1.5 to 3.2 times that of the remaining three algorithms, whereas the ILP Bound has between 2.7 and 4.18 times the gateway goodput. BPS has a performance of 75% of the ILP Bound, on average.

WhiteMesh networks will be deployed across a vast array of environments, from rural to urban areas. Each of these areas will have varying amounts of user demand in proportion to the population densities. However, since a greater number of TV stations exist in urban areas, the available white space bands are often inversely related to the population density. To capture these varying degrees of demand and white space availability we consider three likely scenarios and one final scenario for comparison purposes: (i) two WiFi bands (2.4 and 5.8 GHz) with two white space channels (450 and 800 MHz), (ii) two WiFi bands (2.4 and 5.8 GHz) without any white space channels, and (iv) two white space bands (450 and 800 MHz) with no WiFi bands (for comparison).

In Fig. 2(c), we consider the performance of BPS in the four aforementioned scenarios of varying white space availability with varying offered load from 1 to 10 Mbps, representing different population densities. A regular, 30-node grid is again used. Immediately, we observe that the WiFi-only scenario has greater gateway goodput than the white-space-only scenario. This is due to the lack of spatial reuse achieved by white spaces. Interestingly, however, the joint use of both white space and WiFi bands has significant gains over the single type of band scenarios (37% greater than WiFi and 85% over

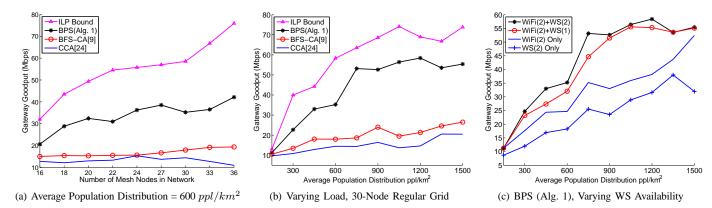


Fig. 2. Performance in terms of gateway goodput for various offered loads, network sizes, and configurations of WiFi or white space (WS) channels.

Bands/ Algorithms	WiFi Only	WS Only	WS & WiFi	WS & WiFi	WS & WiFi	WS & WiFi	WS & Multi-WiFi	WS & Multi-WiFi	Multi-WS & WiFi	Multi-WS & WiFi	Multi-WS & Multi-WiFi
WS (MHz)		450,800	450	800	450	800	450	800	450,800	450,800	450,800
WiFi (GHz)	2.4, 5		2.4	2.4	5	5	2.4, 5	2.4, 5	2.4	5	2.4, 5
CCA [28]	3.1	7.3	8.2	8.1	8.3	7.8	8.7	9.3	9.0	11.9	14.4
BFS-CA [29]	8.9	6.2	7.9	9.0	13.6	13.8	14.9	13.8	14.9	14.3	18.6
BPS (Alg. 1)	22.2	18.2	28.4	25.0	30.9	25.8	32.0	33.5	34.5	30.9	35.2

TABLE I Throughput achieved through Gateway nodes (MBPS) for various combinations of WiFi and Average Population Distribution =  $600 \ ppl/km^2$ , Network Size = 30 mesh nodes).

white space, on average). This can be explained in part because the joint use of WiFi and white space has more total bands to use. However, we will see in Table I that even with the same number of available bands (2), the combination of the two can achieve significant gains over scenarios with only WiFi or white space.

Table I describes the achieved gateway goodput for various combinations of WiFi and white space bands with a maximum offered load of 4 Mbps and a regular 30-node grid. The second reason for the gains in Fig. 2(c) with WiFi and white space is completely isolated in this scenario. Consider the case where we use a white space band of 450 MHz and a WiFi band of 5.8 GHz. With BPS and the same bandwidth, transmission power, and antenna gain, we achieve 30.9 Mbps of gateway goodput through jointly use of these bands, versus 22.2 Mbps with WiFi only or 18.2 Mbps for white space only with gains of 40% and 70%, respectively. If we have one channel in a white space band and one channel in a WiFi band, then we could use the advantage of WiFi for spatial reuse and white spaces to reduce the hop count. These two points become more critical at different points in the WhiteMesh topology. For the sake of completeness, we present many other scenarios which could be interesting for WhiteMesh deployments.

## V. RELATED WORK

Wireless mesh network deployment is to design wireless architecture for offering Internet service in an target area. There are significant challenges in wireless mesh network deployment, such as user priorities, user behaviors, long term throughput estimation, selfish clients, interference and energy efficiency, etc. [30] These challenges are distinguished under the topics of channel assignment, cognitive radio, protocol design, etc. [30], [31] Previous works have recognize the impact of interference in wireless mesh network deployment is the key issue [10], [32], [33]. To overcome the challenges, a lot of works have been done to optimize the deployment in increasing throughput, minimize resource, reducing interference,

etc. [13], [32], [34] Many works have studied the network deployment problem in multihop wireless networks [14], [30], [35], [36]. In addition, multiple radios have been used to improve the routing in multi-channel scenarios [28], [32]. Both static and dynamic network deployments have been discussed in previous works under the 802.11 WiFi scenario [29], [34], [37]. However, all of the aforementioned works have not considered propagation differences of the diverse frequency bands of white space and WiFi, which we show are critical improving the performance of mesh networks. Frequency agility in multiband scenario brings more traffic capacity to wireless network deployment as well as more complexity of resolving the interference issues.

Previous work [9] involve the inter-network interference in multiband scenario, but did not offer the solution of intranetwork interference. As a new designed wireless network, intra-network interference is more important for performance estimation. Previous work focus on WiFi wireless networks proposed several methods to reduce the interference targeting on multiple metrics. [10], [27], [38] focus on reducing the gateway mesh nodes. [32], [34] try to reduce the overall interference in the worst case of traffic independent scenario. [33], [39] improve the performance in throughput. However, these works fails to involve the traffic demands of clients in their solutions. [12], [27] consider the QoS requirements in the WiFi network design. Our work also consider the traffic demands from the client as part of our network design to satisfy both customers and vendors.

The wireless network deployment problem has been proved as a NP-hard problem [11]. Several works introduce relaxed linear program formulation to find the optimization of multihop wireless networks [10], [32], [40]. Also, game theory methods is another option to solve the problem [25], [41]. Social network analysis is also popular in wireless network design [42]. In this work, we model the problem to a linear program to approach the optimal solution and generate heuristic algorithms to find a practical solution for the problem.

To be used effectively, white space bands must ensure that available TV bands exist but no interference exists between microphones and other devices [43]. White space bands availability has to be known in prior of network deployment. TV channels freed by FCC are fairly static in their channel assignment, databases have been used to account for white space channel availability (e.g., Microsoft's White Space Database [44]). In fact, Google has even visualized the licensed white space channels in US cities with an API for research and commercial use [8]. In contrast, we study the performance of mesh networks with a varying number of available white space channels at varying population densities, assuming such white space databases and mechanisms are in place. As FCC release these bands for research, many methods have been proposed to employ these frequency bands. [43] introduce WiFi like white space link implementation on USRP and link protocols. [19] discuss the point to point communication in multiband scenario. In [40], white space band application is discussed in cognitive radio network for reducing maintenance cost. In [45], the white space is proposed to increase the data rates through spectrum allocation. In our work, we focus on maximizing the throughput of the wireless network.

## VI. CONCLUSION

In this paper, we exploited the joint use of WiFi and white space bands for improving the served user demand of wireless mesh networks. To do so, we used an integer programming model to find optimal WhiteMesh topologies. We then constructed a heuristic algorithms, Band-based Path Selection, to achieve similar performance with reduced complexity. Through extensive analysis across varying offered loads, network sizes, and white space channel availability. we show that our algorithms can achieve 3 to 6 times the served user demand versus previous multi-channel, multi-radio solutions, since we leverage diverse propagation characteristics offered by WiFi and white space bands. Moreover, we quantify the degree to which the joint use of these bands can improve the served user demand. Our BPS algorithm shows that WhiteMesh topologies can achieve up to 170% of the gateway goodput of similar WiFi- or white-space-only configurations.

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