Assignment 04

Dep. AI Convergence Engineering

Student ID: 2023214019

Name: Md Habibur Rahman

1. **Q1. Prove the chain rule using set algebra.**

Answer:
Formula for conditional probability

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

We can rearrange it to:

$$P(A\cap B) = P(B)P(B|A)$$

We can extend this for three variables:

$$P(A\cap B\cap C) = P(A|B\cap C)\,P(B\cap C) = P(A|B\cap C)\,P(B|C)\,P(C)$$

and in general to n variables:

$$P(A1\cap A2\cap \ldots \cap An) = P(A1|A2\cap \ldots \cap An)\,P(A2|A3\cap \ldots \cap An)\,P(An-1|An)\,P(An)$$

Assignment 05

Dep. AI Convergence Engineering

Student ID: 2023214019

Name: Md Habibur Rahman

2. **Q1. Prove generalized Bayes theorem using set algebra.**

Answer:
Formula for conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{—(1)}$$

We can rearrange it to:

$$P(A \cap B) = P(B)P(B|A) \text{—(2)}$$

Using total probability theorem:

$$P(B) = \sum_{k=1}^{n} P(A_k)P(B|A_k) \text{ —(3)}$$

Putting the values from equations (2) and (3) in equation 1, we get:

$$P(A|B) = \frac{P(B)P(B|A)}{\sum_{k=1}^{n} P(A_k)P(B|A_k)}$$

3. **Q2. Prove that $\lim_{i \to +\infty} P(A_i \cap B_i | C_k) \to 0$ if $i, k \in Z^+$ and k is a positive fixed integer and A_i, B_i are conditionally independent.**

Answer:

Given,

1. A_i *and* B_i are conditionally independent events.
2. k is a positive fixed integer.
3. i and k are both positive integers.

We want to prove that:

$$\lim_{i \to +\infty} P(A_i \cap B_i | C_k) \to 0$$

We can use the definition of conditional probability to express

$$P(A_i \cap B_i | C_k) = \frac{P(A_i \cap B_i \cap C_k)}{P(C_k)}$$

Since A_i *and* B_i are conditionally independent events, we have:

$$P(A_i \cap B_i | c_k) = \frac{P(A_i | C_k) P(B_i | C_k) P(C_k)}{P(C_k)}$$

$$P(A_i \cap B_i | c_k) = P(A_i | C_k) P(B_i | C_k)$$

Since A_i *and* B_i are conditionally independent of C_k

$$P(A_i | C_k) = P(A_i)$$

$$P(B_i | C_k) = P(A_i)$$

Now,

$$P(A_i \cap B_i | C_k) = P(A_i) P(B_i)$$

if P(A_i) $and\ P(B_i)$ approaches zero as i approaches infinity, then $P(A_i \cap B_i | C_k)$ will approach zero.

4. **Q3. Compute the total probability of** $\lim_{i \to k} P(A_i \cap B_i | C_k) \to 0$ **if** $i, k \in Z^+$ **and** $\forall i > 0, P(A_i | C_k) = 2^{-m} P(A_{i+1} | C_k)$ **. Assume** $k <+ \infty$**.**

Answer:

Given,

$$P(A_i | C_k) = 2^{-m} P(A_{i+1} | C_k)$$

= $P(A_i | C_k) = 2^{-(i-1)m} P(A_1 | C_k)$ [recursively express]

Now,

$P(A_i \cap B_i | C_k)$ **=** $P(A_i | C_k) * P(B_i | A_i \cap C_k)$

Given that $P(A_i | C_k)$ tends to 0 as i increases towards k, and $P(B_i | A_i \cap C_k)$ is bounded by 1, $P(A_i \cap B_i | C_k)$ tends to 0 as well.