Assignment 01

Dep. AI Convergence Engineering

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1. **Q1.** Algebraically prove that if A ⊂ B_1 and C ⊂ B_2 then A∩C = φ if A∩B_1∩B_2 = φ, where B_1∩B_2≠ φ and B_1⊄ B_2, B_2⊄ B_1.

Answer:

Given,

1. A ⊂ B_1
2. C ⊂ B_2
3. A∩B_1∩B_2 = φ
4. B_1∩B_2≠ φ
5. B_1⊄ B_2 and
6. B_2⊄ B_1

Since A ⊂ B_1 and C ⊂ B_2 any element in the intersection of A and C must belong to both B_1 and B_2 . If A∩C is non-empty, then there exists an element x such that x∈A and x∈C.

However, from the given condition A∩B_1∩B_2 = φ, it follows that there can be no element that belongs simultaneously to A, $B_1$and B_2.

Now, let's consider the case where A∩C is not empty. This implies that there exists an element x such that x∈A and x∈C. Since A⊂B_1 and C⊂B_2, it follows that x∈B_1 and x∈B_2.

However, this contradicts the given condition A∩B_1∩B_2= ∅, as we have found an element x that belongs to all three sets A, B_1, and B_2, which is not possible.

Therefore, A∩C must be empty. Thus, algebraically, it is proven that if A∩B_1∩B_1= ∅, where B_1∩ B_2 = ∅ and B_1 ⊈ B_2 and B_2⊈B_1, then A∩C=∅.

2. **Q2.** Assume that P(X_1, X_2) is a proposition. Prepare a set S using P(X_1, X_2) such that φ=S.

Answer:
A proposition is a statement that can be either true or false, but not both.

To construct the set S such that ∅ = S, we need to find values for X_1 and X_2 such that P(X_1, X_2) is always false, resulting in an empty set.

Lets say,

P(X_1, X_2) = X_1+ X_2 < 0 Where X_1 and X_2 are positive number

Then, our set S would be defined as:

S = {(X_1, X_2) | P(X_1, X_2) is true}

However, since there are no values of X_1 and X_1 that satisfies this condition (because the sum of two real positive numbers cannot be less than 0), the set S will be empty.

3. **Q3.** Prove that if A and B are two sets such that, propositions P(x∈A) and Q(x∈B) are tautologies whereas P(x∈B) and Q(x∈A) are contradictions, then A ∩ B = φ.

Given:

Proposition P(x∈A) is a tautology.
Proposition Q(x∈B) is a tautology.
Proposition P(x∈B) is a contradiction.
Proposition Q(x∈A) is a contradiction.
To prove: A∩B = ∅

Let's proceed with the proof:

Since P(x∈A) is a tautology, any element x that belongs to set A satisfies proposition P(x).
Similarly, since Q(x∈B) is a tautology, any element x that belongs to set B satisfies proposition Q(x).

However, if $P(x \in B)$ is a contradiction, there cannot exist an element that belongs to both sets A and B. Similarly, if $Q(x \in A)$ is a contradiction, there cannot exist an element that belongs to both sets A and B.

Therefore, there are no elements that simultaneously belong to both sets A and B, implying that the intersection of sets A and B is empty.

Mathematically, $A \cap B = \varnothing$.

Hence, we have proven that $\varnothing = A \cap B$.