Assignment 08

Dep. AI Convergence Engineering

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1. **Q1. Compare geometric and Poisson distributions by using numerical computations and graphs.**

Answer:
Geometric Distribution:
The geometric distribution represents the number of trials needed to get the first success in a series of Bernoulli trials (where each trial is independent and has a constant probability of success q).

The probability mass function (PMF) for a geometric distribution is given by:

$$P(X = n) = (1 - q)^{n-1}q$$

where q is the probability of success on each trial and n is the trial number on which the first success occurs.

Poisson Distribution:
The Poisson distribution represents the number of events occurring in a fixed interval of time or space, with these events happening at a constant rate λ and independently of the time since the last event.

The probability mass function (PMF) for a Poisson distribution is given by:

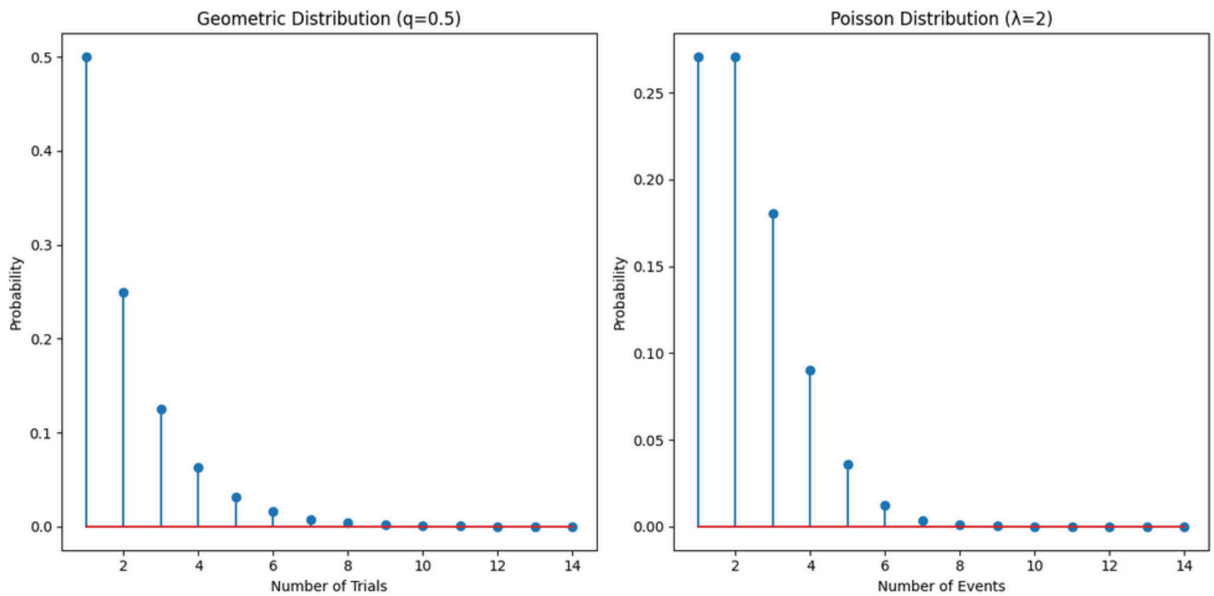
$$P(X = n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

where λ is the average number of events in the interval and n is the number of occurrences.

Comparison Using Numerical Computations and Graphs:
Let's perform numerical computations and plot the graphs for both distributions.

1. Parameters:
    a. For the geometric distribution, choose q=0.5.
    b. For the Poisson distribution, choose λ=2.
2. Computation and Plotting:

We'll use Python to compute and plot these distributions.



2. **Check whether convergence distribution of X_n (i.e. Concept 3) holds or not if f is a real valued constant function. Prove your result.**

Answer:

Concept 3 states that a sequence of random variables X_n converges in distribution to a random variable X if for any bounded continuous function f: $lim_{n \to \infty} E[f(X_n)] = E[f(X)]$

If f is a constant function, say $f(X) = c$, where c is a constant, then:

$$E[f(X_n)] = E[c] = c$$

and

$$E[f(X)] = E[c]$$

Since the expectation of a constant function is the constant itself, we have:

$$f: lim_{n \to \infty} E[f(X_n)] = c = E[f(X)]$$

Thus, the convergence distribution (Concept 3) holds for a real-valued constant function f.

3. **Q3. Start from probabilistic convergence (Concept 1) and derive to Almost-Sure-Convergence (Concept 2) by using limiting value** $\epsilon \to 0$.

Answer:
Concept 1 (Probabilistic Convergence):
A sequence of random variables X_n converges in probability to a random variable X if for every $\epsilon > 0$:

$$lim_{n \to \infty} P(|X_n - X| \geq \epsilon) = 0$$

Concept 2 (Almost-Sure Convergence):
A sequence of random variables X_n converges almost surely to a random variable X if:

$$P(lim_{n \to \infty} X_n = X) = 1$$

To derive almost-sure convergence from probabilistic convergence as $\epsilon \to 0$.

For almost-sure convergence, we need:

$$P(|X_n - X| \geq \epsilon \, for \, infinitely \, many \, n) = 0$$

By the Borel-Cantelli lemma, if $\sum_{n=1}^{\infty} P(|X_n - X| \geq \epsilon) < \infty$ for every $\epsilon > 0$ then,

$$P(|X_n - X| \geq \epsilon \, infinitely \, often) = 0$$

Given probabilistic convergence:

$$lim_{n \to \infty} P(|X_n - X| \geq \epsilon) = 0$$

this implies that for large n, $P(|X_n - X| \geq \epsilon) = 0$ becomes arbitrarily small.

Thus, if $P(|X_n - X| \geq \epsilon)$ decreases rapidly enough so that the series $\sum_{n=1}^{\infty} P(|X_n - X| \geq \epsilon)$ converges for every $\epsilon > 0$, then by the Borel-Cantelli lemma, we get almost-sure convergence:

$$P(|X_n - X| \geq \epsilon \, for \, infinitely \, many \, n) = 0$$

Hence, starting from probabilistic convergence and considering the limit as $\epsilon \to 0$., we derive almost-sure convergence.