# Assignment 02

Dep. AI Convergence Engineering

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1. **Q1. Prove that if $A_i \subset$ R ( R is set of real numbers) and S = {$A_i$ : i ≥ 0 } then $\forall A_i \in$ S, $\cap_{i \in I} A_i \neq \varphi$ if $A_i \subset A_k$, $i > k$.**

**Answer:**

- Let's denote $\cap_{i \in I} A_i$ = X, where is the intersection of all sets in S.
- Suppose $A_i \subset A_k$, for some $i > k$
- Then, X ⊂ $A_k$ because X is the intersection of all $A_i$ and $A_i \subset A_k$.
- Since X ⊂ $A_k$ X is a subset of each $A_i$ for $i > k$
- By the definition of intersection, X contains all elements that are common to each set $A_i$ for $i > k$.
- Since X is a subset of each $A_i$ for $i > k$, X itself contains these common elements.
- Thus, is not empty, as it contains at least one element that is common to all sets $A_i$ for $i > k$

Therefor, if $A_i \subset$ R and S = {$A_i$ : i ≥ 0 }, then for all $A_i$ in not empty if $A_i \subset A_k$ and $i > k$.

Example:
Explanation of Notation:

- ∀ represents "for all" or "for every".
- $A_i$ denotes a set in the sequence of sets.
- S represents the set containing all sets $A_i$
- denotes the intersection of sets $A_i$ where $i$ belongs to the index set $I$.

Let's consider the sequence of sets:

$A_0$ = {1,2,3}

$A_1$ = {2,3}

$A_2$ = {3}

Here $A_0 \supset A_1 \supset A_2$, as each subsequent set is a subset of the previous one.

Proof:

We want to show that for any set $A_i$ in the sequence, the intersection $\cap_{i\in I} A_i$ is non-empty. Consider $A_2$ from our example. Since $A_0 \supset A_1 \supset A_2$, we can take $I$ = {0,1,2}.

$\cap_{i\in I} A_i = A_0 \cap A_1 \cap A_2$

= {1,2,3} ∩ {2,3} ∩ {3}

= {3}

As we can see, the intersection is not empty ({3} in this case).
Hence, for any sequence of sets $A_i \subset$ R such that $A_i \subset A_k$ for some $i > k$, for any $A_i$ in S, the intersection $\cap_{i\in I} A_i$ is non-empty.

2. **Q2. Prove that a surjective function does not have a preimage in the traditional sense.**

### Answer:

A surjective function is one for which every element in the codomain has at least one corresponding element in the domain. In other words, every element in the codomain is mapped to by at least one element in the domain.

To prove that a surjective function does not necessarily have a unique preimage for each element in the codomain, let's consider an example:

Consider the function *f*:R→R defined by *f*(x) = $x^2$.

This function is surjective because for any y in the codomain (R), there exists an x in the domain (R) such that *f*(x) = y. This is because every real number has a square root in the real numbers.

However, if we take y = 4, for instance, there are two preimages: x = 2 and x = −2. So, $f^{-1}(4)$ = {2, −2}. This shows that a surjective function does not have a unique preimage for every element in the codomain.

Therefore, a surjective function does not necessarily have a unique preimage in the traditional sense.

3. **Q3. Prove that if E ⊂ A*B and *f* : E→R such that *f*((a,b)) = a+b then *f* may not have an inverse. Here R is a set of real numbers.**

## Answer:

To prove that the function *f*:E→R defined by *f*((a,b))=a+b may not have an inverse, we'll show that *f* is not injective (one-to-one).

Suppose we have two distinct elements $(a_1, b_1)$ and $(a_2, b_2)$ in E such that *f*($(a_1, b_1)$) = *f*($(a_2, b_2)$).
According to the given function definition:

*f*($(a_1, b_1)$) = $a_1 + b_1$

*f*($(a_2, b_2)$) = $a_2 + b_2$

If *f* has an inverse, then by definition, $f^{-1}(a_1 + b_1) = (a_1, b_1)$ and $f^{-1}(a_2 + b_2) = (a_2, b_2)$.
However, if $a_1 + b_1 = a_2 + b_2$ , it doesn't necessarily imply that $(a_1, b_1) = (a_2, b_2)$.
For example, consider $a_1$= 1, $b_1$= 2, $a_2$= 3, and $b_2$= 0.
Then:

*f*((1,2)) = 1+2 = 3
*f*((3,0)) = 3+0 = 3

Even though *f*((1,2)) = *f*((3,0)), (1,2) is not equal to (3,0).

Hence, *f* is not injective, and thus, it may not have an inverse.

This demonstrates that the function f((a,b))=a+b defined on E may not have an inverse due to its lack of injectivity.