# Assignment 03

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1. **Q1. Algebraically prove that if P(A ⊂ Ω) = 1, then P(Ω\A) = 0 . Consider discrete probability for simplicity.**

**Answer:**

- Recall that for any event B, the probability of the complement of B (denoted as $B^c$) is P( $B^c$) = 1 - P(B).
- Given that P(A ⊂ Ω) = 1, it means that event A covers the entire sample space Ω. In other words, A is certain to occur whenever an outcome is observed.
- Now, consider the complement of A, denoted as $A^c$ which represents all the outcomes not in A.
- Since A covers the entire sample space, $A^c$ must be an empty set, and its probability is P($A^c$) = 0
- So, P(Ω\A) = P(Ω ∩ $A^c$) = P( $A^c$) = 0, as required

**Example:**
Suppose we have a fair six-sided die, with outcomes Ω = {1,2,3,4,5,6}.

Let A be the event that the outcome is an even number, i.e., A={2,4,6}.

Now, let's calculate the probabilities:
1. P(A): Since there are 3 outcomes in event A and 6 possible outcomes in total, P(A) = 3/6 = 1/2.
2. P(Ω): Since the sample space contains all possible outcomes, P(Ω) = 1.
3. P(Ω∖A): This represents the probability of outcomes not in A, i.e., the probability of rolling an odd number. There are 3 odd numbers in the sample space, so P(Ω∖A) = 3/6 = 1/2.

As we can see, in this example, P(A) = 1 since event A covers the entire sample space (all outcomes are even numbers). Consequently, P(Ω∖A) = 0, representing the probability of outcomes not in A (odd numbers).

2. **Q2. Prove that if $A_1 \subset A_2$ ….. $\subset A_n \subset \Omega$ is a finite sequence such that P($A_i$) > P($A_{i+k>0}$) then P cannot be a discrete uniform probability.**

**Answer:**

- Assume P is a discrete uniform probability. This means that each outcome in Ω is equally likely, so P($A_i$) = $\frac{|A_i|}{|\Omega|}$ where $|A_i|$ denotes the number of elements in $A_i$ and $|\Omega|$ denotes the total number of elements in Ω.
- Since $A_1 \subset A_2$ ….. $\subset A_n \subset \Omega$, we have $|A_1| < |A_2| <$ ….. $|A_n| = |\Omega|$
- Now, according to the given condition, P($A_i$) > P($A_{i+k>0}$).
- Let's consider i=1 for simplicity. It implies P($A_1$) > P($A_{k+1>0}$)
- Since P is a discrete uniform probability, P($A_1$) = $\frac{|A_1|}{|\Omega|}$ and P($A_{k+1}$) = $\frac{|A_{k+1}|}{|\Omega|}$
- But $|A_1| < |A_{k+1}|$ ( as $A_1 \subset A_{k+1}$), which contradicts the assumption that P($A_1$) > P($A_{k+1}$ ). Hence, our initial assumption that P is a discrete uniform probability must be false.
- Therefore, if $A_1 \subset A_2$ ….. $\subset A_n \subset \Omega$ is a finite sequence such that P($A_i$) > P($A_{i+k>0}$) for some k > 0, then P cannot be a discrete uniform probability.

3. **Q3. Prove that if Ω is continuous and $A_1 \subset \Omega$, $A_2 \subset \Omega$ such that P($A_1$) > 0, P($A_2$) > 0 then P($A_1 \cap A_2$) ≥ 0 if $A_1 \cap A_2 \neq \varphi$**

**Answer:**

- Given that P($A_1$) > 0, P($A_2$) > 0 it means that both $A_1$ and $A_2$ have non-zero probabilities under the probability measure P.
- Since $A_1 \cap A_2$ is the intersection of two sets, if $A_1 \cap A_2 \neq \varphi$ it implies that there are elements that belong to both $A_1$ and $A_2$.

By the properties of probability measures:

1. P($A_1 \cup A_2$) = P($A_1$) + P($A_2$) - P($A_1 \cap A_2$)
    - Given that P($A_1 \cup A_2$) ≤ 1 for any probability measure, and since both P($A_1$) and P($A_2$) are greater than zero, P($A_1$) + P($A_2$) > 0
2. P($A_1 \cup A_2$) ≥ 0

Combining these two inequalities:

- P($A_1$) + P($A_2$) - P($A_1$∩ $A_2$) ≥ 0
- Since both P($A_1$) and P($A_2$) are greater than zero, and P($A_1$) + P($A_2$) is positive, we have P($A_1$) + P($A_2$) ≥ P($A_1$∩ $A_2$)

Rearranging the terms, we get:

- P($A_1$∩ $A_2$) ≥ 0

Therefore, if Ω is continuous and A1⊂ Ω, A2⊂ Ω such that P(A1) > 0, P(A2) > 0 then P(A1∩ A2) ≥ 0 if A1∩ A2≠ φ