12-1 Quantum State Tomography

You are given multiple copies of ρ.

How do you determine ρ?

$\rho = \frac{I + \vec{r}\cdot\vec{\sigma}}{2} = \frac{I}{2} + \frac{1}{2}(r_x X + r_y Y + r_z Z)$

$XY = iZ = -YX, \quad YZ = iX = -ZY, \quad ZX = iY = -XZ$

$X^2 = Y^2 = Z^2 = I$

$\mathrm{Tr}(X) = \mathrm{Tr}(Y) = \mathrm{Tr}(Z) = 0$

$\mathrm{Tr}(\rho) = 1$

$\langle X \rangle = \mathrm{Tr}(X\rho) = \frac{1}{2}\mathrm{Tr}(X) + \frac{r_x}{2}\mathrm{Tr}(X^2) + \frac{r_y}{2}\mathrm{Tr}(XY) + \frac{r_z}{2}\mathrm{Tr}(XZ) = r_x$

$\langle Y \rangle = r_y, \quad \langle Z \rangle = r_z$

$\therefore \rho = \frac{1}{2}\langle I \rangle I + \frac{1}{2}\langle X \rangle X + \frac{1}{2}\langle Y \rangle Y + \frac{1}{2}\langle Z \rangle Z$

two-qubit states

You need to measure all combinations of $\sigma_0 \equiv I, \ \sigma_1 \equiv X, \ \sigma_2 \equiv Y, \ \sigma_3 \equiv Z$

$$\rho = \sum_{i,j=0}^{3} \frac{1}{2^2} \left\langle \sigma_i \otimes \sigma_j \right\rangle \sigma_i \otimes \sigma_j$$

$$= \frac{1}{4}\langle I \otimes I \rangle I \otimes I + \frac{1}{4}\langle I \otimes X \rangle I \otimes X + \frac{1}{4}\langle I \otimes Y \rangle I \otimes Y + \frac{1}{4}\langle I \otimes Z \rangle I \otimes Z$$

$$+ \frac{1}{4}\langle X \otimes I \rangle X \otimes I + \frac{1}{4}\langle X \otimes X \rangle X \otimes X + \cdots$$