

All equations

$$\rightarrow |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \langle 0| = (1, 0), \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle 1| = (0, 1)$$

$$\rightarrow |\psi\rangle = \alpha |0\rangle + \beta |1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\rightarrow \langle \psi| = \alpha^* \langle 0| + \beta^* \langle 1| = \begin{pmatrix} \alpha & \beta \end{pmatrix}^\dagger = (\alpha^* \quad \beta^*)$$

$$\rightarrow \langle \psi | \psi \rangle = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha^* \alpha + \beta^* \beta = |\alpha|^2 + |\beta|^2 = \text{norm}$$

$$\rightarrow |\psi\rangle \otimes |\phi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \otimes \begin{pmatrix} \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \alpha\gamma & \alpha\delta \\ \beta\gamma & \beta\delta \end{pmatrix} \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix}$$

$$= (\alpha |0\rangle + \beta |1\rangle) \otimes (\gamma |0\rangle + \delta |1\rangle)$$

$$= \alpha\gamma |00\rangle + \alpha\delta |01\rangle + \beta\gamma |10\rangle + \beta\delta |11\rangle$$

$$\rightarrow |0\rangle \langle 0| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = |1\rangle \langle 1|$$

$$\rightarrow |1\rangle \langle 0|0\rangle = |1\rangle (1) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = |1\rangle (1) = (1) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow (A \otimes B) |\psi\rangle \otimes |\phi\rangle = A|\psi\rangle \otimes B|\phi\rangle$$

\Rightarrow Diagonalization process

① find Eigenvalues and vector

$$\textcircled{2} D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

③ $X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$ Eigenvalue for λ_1, λ_2

$$\textcircled{4} X^{-1} = \frac{1}{\det A} \text{Adj } A$$

$$\textcircled{5} XDX^{-1} = A \rightarrow \underline{\text{Proved}}$$

\Rightarrow

