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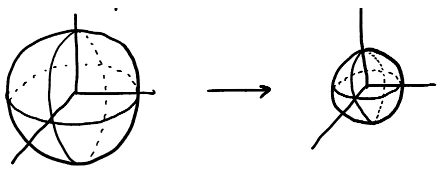
11-3 Examples of Quantum Channels (§8.3)

depolarization

$$\mathcal{E}(\rho) = p\frac{I}{2} + (1-p)\rho \qquad 0 \le p \le 1$$

Bloch sphere



HW11-1 $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\rho = \frac{I + \vec{r}\cdot\vec{\sigma}}{2} = \frac{I}{2} + \frac{r_x}{2}X + \frac{r_y}{2}Y + \frac{r_z}{2}Z = \begin{pmatrix} \frac{1+r_z}{2} & \frac{r_x - ir_y}{2} \\ \frac{r_x + ir_y}{2} & \frac{1-r_z}{2} \end{pmatrix}$$

Show $\mathcal{E}(\rho) = \frac{I}{2}\rho\frac{I}{2} + \frac{X}{2}\rho\frac{X}{2} + \frac{Y}{2}\rho\frac{Y}{2} + \frac{Z}{2}\rho\frac{Z}{2} = \frac{I}{2}.$

Random application of I, X, Y, Z erases all information!

$$\mathcal{E}(\rho) = p\frac{I}{2} + (1-p)\rho = p\left(\frac{I}{2}\rho\frac{I}{2} + \frac{X}{2}\rho\frac{X}{2} + \frac{Y}{2}\rho\frac{Y}{2} + \frac{Z}{2}\rho\frac{Z}{2}\right) + (1-p)\rho$$

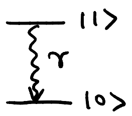
$$= \sum_k E_k \rho E_k^\dagger \qquad E_0 = \frac{\sqrt{4-3p}}{2}I, \quad E_1 = \frac{\sqrt{p}}{2}X, \quad E_2 = \frac{\sqrt{p}}{2}Y, \quad E_3 = \frac{\sqrt{p}}{2}Z$$

amplitude damping

$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix} = |0\rangle\langle 0| + \sqrt{1-\gamma}|1\rangle\langle 1|$$

$$E_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix} = \sqrt{\gamma}|0\rangle\langle 1| \qquad E_1\rho E_1^\dagger = \gamma|0\rangle\langle 1|\rho|1\rangle\langle 0|$$

$$\rho = \begin{pmatrix} 1-p & f \\ f^* & p \end{pmatrix} \to \begin{pmatrix} 1-p+\gamma p & \sqrt{1-\gamma}f \\ \sqrt{1-\gamma}f^* & (1-\gamma)p \end{pmatrix}$$



phase damping

$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix} = |0\rangle\langle 0| + \sqrt{1-\gamma}|1\rangle\langle 1|$$

$$E_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\gamma} \end{pmatrix} = \sqrt{\gamma}|1\rangle\langle 1| \qquad E_1\rho E_1^\dagger = \gamma|1\rangle\langle 1|\rho|1\rangle\langle 1|$$

$$\rho = \begin{pmatrix} 1-p & f \\ f^* & p \end{pmatrix} \to \begin{pmatrix} 1-p & \sqrt{1-\gamma}f \\ \sqrt{1-\gamma}f^* & p \end{pmatrix} \qquad \text{decoherence}$$