Show $(\hat{n}\cdot\vec{\sigma})^2 = I$

$\Rightarrow$ The vector of Pauli matrices $\vec{\sigma}$ can be represented as,

$$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$$

So, $$\hat{n}\,\vec{\sigma} = n_x\sigma_x + n_y\sigma_y + n_z\sigma_z$$

$$\therefore (\hat{n}\,\vec{\sigma})^2 = (n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)^2$$

$$= (n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)(n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)$$

$$= n_x^2\sigma_x^2 + n_y^2\sigma_y^2 + n_z^2\sigma_z^2 + n_x n_y(\sigma_x\sigma_y + \sigma_y\sigma_x)$$
$$+ n_x n_z(\sigma_x\sigma_z + \sigma_z\sigma_x) + n_y n_z(\sigma_y\sigma_z + \sigma_z\sigma_y) \quad \text{---- (i)}$$

Now, $$\sigma_i^2 = I \text{ for } i = x, y, z$$

$$\sigma_i\sigma_j = -\sigma_j\sigma_i \text{ for } i \neq j$$

No simplifying expression (i)

$$(n\cdot\vec{\sigma})^2 = n_x^2 I + n_y^2 I + n_z^2 I + n_x n_y (0) + n_x n_z (0) + n_y n_z (0)$$

$$= I\left(n_x^2 + n_y^2 + n_z^2\right)$$

Since $\hat{n}$ is unit vector $|\hat{n}| = 1$

so, $n_x^2 + n_y^2 + n_z^2 = 1$

$\therefore (\hat{n}\cdot\vec{\sigma})^2 = I$ **Proved**