HW 6-2

Sol^n: Given,

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right)$$

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

and

$$|\eta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|++\rangle + |--\rangle\right)$$

$$|\eta_{01}\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle + |-+\rangle\right)$$

$$|\eta_{10}\rangle = \frac{1}{\sqrt{2}}\left(|++\rangle - |--\rangle\right)$$

$$|\eta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right)$$

We can see $|\eta_{00}\rangle$ and $|\eta_{10}\rangle$ are linear combinations of $|\beta_{00}\rangle$ and $|\beta_{11}\rangle$, while $|\eta_{01}\rangle$ and $|\eta_{11}\rangle$ are linear combinations of $|\beta_{01}\rangle$ and $|\beta_{10}\rangle$.

So, expressing each η_{ij} in terms of β_{ij}',

$$|\eta_{00}\rangle = \tfrac{1}{\sqrt{2}}\left(|++\rangle + |--\rangle\right)$$

$$= \tfrac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) + \tfrac{1}{\sqrt{2}}\left(|11\rangle + |00\rangle\right)$$

$$= 2|\beta_{00}\rangle$$

$$|\eta_{01}\rangle = \tfrac{1}{\sqrt{2}}\left(|+-\rangle + |-+\rangle\right)$$

$$= \tfrac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right) + \tfrac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)$$

$$= 2|\beta_{01}\rangle$$

$$|\eta_{10}\rangle = 2|\beta_{10}\rangle$$

$$|\eta_{11}\rangle = 2|\beta_{11}\rangle$$

Solved