HW 3-1 :

$$-\boxed{X}- = -\boxed{H}-\boxed{Z}-\boxed{H}-$$

$$-\boxed{Z}- = -\boxed{H}-\boxed{X}-\boxed{H}-$$

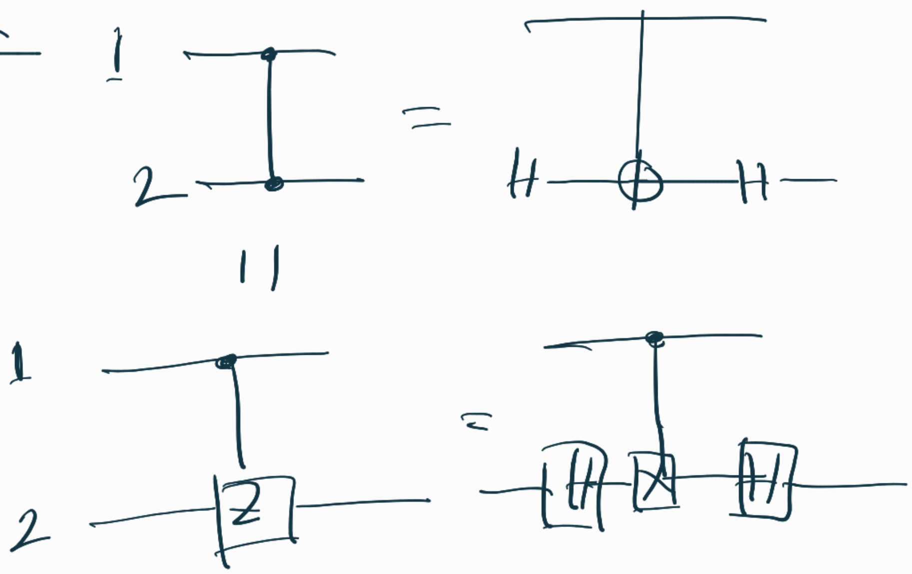
unneccessary stupid but enlightining method.

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{H} \alpha\frac{|0\rangle + |1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\xrightarrow{Z} \alpha\frac{|0\rangle - |1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\xrightarrow{H} \alpha|1\rangle + \beta|0\rangle$$

HW 3-2



$$(I_1 \otimes H)\left(|0\rangle\langle 0| \otimes I_2 + |1\rangle\langle 1| \otimes X_2\right)(I_1 \otimes H_2)$$

$$= \left(|0\rangle\langle 0| \otimes H + |1\rangle\langle 1| \otimes HX\right)\left(I \otimes H\right)$$

$$= |0\rangle\langle 0| \otimes \underbrace{HH}_{I} + |1\rangle\langle 1| \otimes \underbrace{HXH}_{Z}$$

$$= |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Z$$

HW 5-1 Show, $(\hat{n}\cdot\vec{\sigma})^2 = I$

$$(n_x X + n_y Y + n_z Z)(n_x X + n_y Y + n_z Z)$$

$$= n_x^2 X^2 + n_y^2 Y^2 + n_z^2 Z^2$$

$$= (n_x^2 + n_y^2 + n_z^2) I$$

$$= I$$

$|\hat{n}| = 1$

$\hat{n} = (n_x, n_y, n_z)$

$n_x^2 + n_y^2 + n_z^2 = 1$

$X^2 = I$

$Y^2 = I$

$Z^2 = I$

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$(\hat{n}\cdot\vec{\sigma})^2$$

$$= \begin{pmatrix} n_z & n_x - i n_y \end{pmatrix}^2$$

$$\begin{pmatrix} \cdots & \cdots \\ n_x + i n_y & -n_z \end{pmatrix}$$

$$= n_z^2 + n_x^2 + n_y^2$$

$$= I$$

HW 5-2 :

$$\hat{n}_H = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right)$$

$$R_{\hat{n}_H}(\theta) = e^{i \theta/2 \, \hat{n} \cdot \vec{\sigma}}$$

$$\theta = 180^\circ$$

$$= \cos\theta/2 \, I - i \sin\theta/2 \, \hat{n} \cdot \vec{\sigma}$$

$$\hat{n} \cdot \vec{\sigma} \to \frac{X}{\sqrt{2}} + \frac{Z}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = H$$

$$= -iH$$

HW 6.1

$$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$|\beta_{00}\rangle = 1/\sqrt{2}\left(|00\rangle + |11\rangle\right)$$

$$|\beta_{01}\rangle = 1/\sqrt{2}\left(|01\rangle + |10\rangle\right)$$

$$|\beta_{10}\rangle = 1/\sqrt{2}\left(|00\rangle - |11\rangle\right)$$

$$|\beta_{11}\rangle = 1/\sqrt{2}\left(|01\rangle - |10\rangle\right)$$

$$\Rightarrow \alpha\, 1/\sqrt{2}\left(|\beta_{00}\rangle + |\beta_{10}\rangle\right) + \beta\, 1/\sqrt{2}\left(|\beta_{10}\rangle + |\beta_{11}\rangle\right)$$

$$+ \gamma\, 1/\sqrt{2}\left(|\beta_{01}\rangle - |\beta_{11}\rangle\right) + \text{- - - -}$$

HW 6-2:

$$|+\rangle = 1/\sqrt{2}\left(|0\rangle + |1\rangle\right)$$

$|+\rangle = \frac{1}{\sqrt{2}}\left(\quad \right)$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

unnormalized $|\eta_{00}\rangle = |++\rangle + |--\rangle$

$$= \left(|0\rangle + |1\rangle\right)\left(|0\rangle + |1\rangle\right) +$$
$$\left(|0\rangle - |1\rangle\right)\left(|0\rangle - |1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = |\beta_{00}\rangle$$

<u>HW 6-3</u> Alice sends

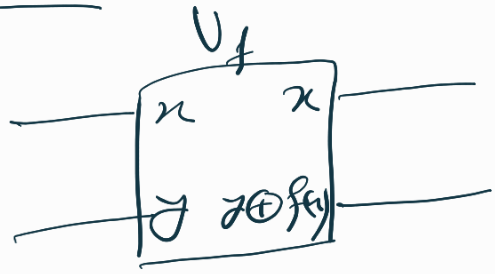
$|+\rangle \quad |1\rangle \quad |-\rangle \quad |+\rangle \quad |+\rangle \quad |1\rangle \quad |0\rangle \quad |0\rangle \quad |-\rangle \quad |+\rangle$

Bob measures

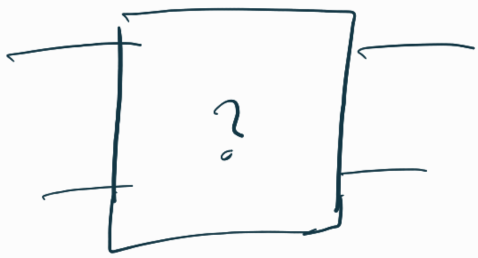
$\rightarrow$ x z z x x z x x x z

$\|+\rangle$	$\|1\rangle$	$\cancel{\|+\rangle}$	$\|+\rangle$	$\|+\rangle$	$\|1\rangle$	$\cancel{\|0\rangle}$	$\cancel{\|0\rangle}$	$\|-\rangle$	$\cancel{\|+\rangle}$
x	z	$\cancel{z}$	x	x	z	$\cancel{x}$	$\cancel{x}$	x	$\cancel{z}$
0	1		0	0	1			1	

HW 7.2



ex) $f(1) = 0$, $f(0) = 1$



x	y	x	$y \oplus f(x)$
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|f(x)\rangle = \|1\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1 \oplus f(x)\rangle = \|0\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|f(x)\rangle = \|0\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|1 \oplus f(x)) = \|1\rangle$

Protocol :

$$\left.\begin{array}{l}|0\rangle \;—\; 0\\ |1\rangle \;—\; 1\end{array}\right\} Z$$

$$\left.\begin{array}{l}|+\rangle \;—\; 0\\ |-\rangle \;—\; 1\end{array}\right\} X$$