3-2 Postulates of quantum mechanics

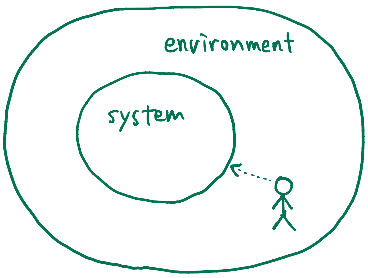
§2.2 Postulates of quantum mechanics

Postulate 1 (state space)

A quantum state $|\psi\rangle$ completely describes a closed quantum system.

$|\psi\rangle$ is a unit vector in a Hilber space.

$\langle\psi|\psi\rangle = 1$



- closed system: No exchange of any form—energy, particles, etc.—with the environment
- open system, otherwise

how do we handle open systems?

Postulate 2 (dynamics)

The time evolution of a closed quantum system is described by a unitary transformation.

closed $\Rightarrow H$ (Hermian operator called "Hamiltonian") is conserved and satisfies

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle \qquad \text{Schrödinger equation}$$

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

$$U(t) \equiv e^{-iHt} \qquad \text{time-evolution operator, unitary}$$

Postulate 3 (measurement)

An observable is a Hermitian operator $A = A^\dagger = \sum_m a_m |m\rangle\langle m|$.

$a_m \in \mathbb{R}$, $\{|m\rangle\}$: orthonormal basis

Let $P_m \equiv |m\rangle\langle m|$ projector ($P_m^2 = P_m$)

$A = \sum_m a_m P_m \qquad P_m P_n = 0$ for $m \neq n$, $\sum_m P_m = I$

A measurement of A goes as follows:

If $|\psi\rangle$ is the state before the measurement,

the measurement outcome is a_m with probability $P(a_m) = \langle\psi|P_m|\psi\rangle$.

After measuring a_m, the state becomes $\frac{P_m|\psi\rangle}{\||P_m\psi\rangle\|}$ irreversibly.

non-unitary. how?

ex) measurement of $Z = |0\rangle\langle 0| - |1\rangle\langle 1|$

Postulate 4 (composite systems)

Suppose system A is in state $|\psi\rangle_A$ in Hilbert space $\mathcal{H}_A$.

Suppose system B is in state $|\psi\rangle_B$ in Hilbert space $\mathcal{H}_B$.

Then, system $A + B$ is in state $|\psi\rangle_A \otimes |\psi\rangle_B$ in Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$.