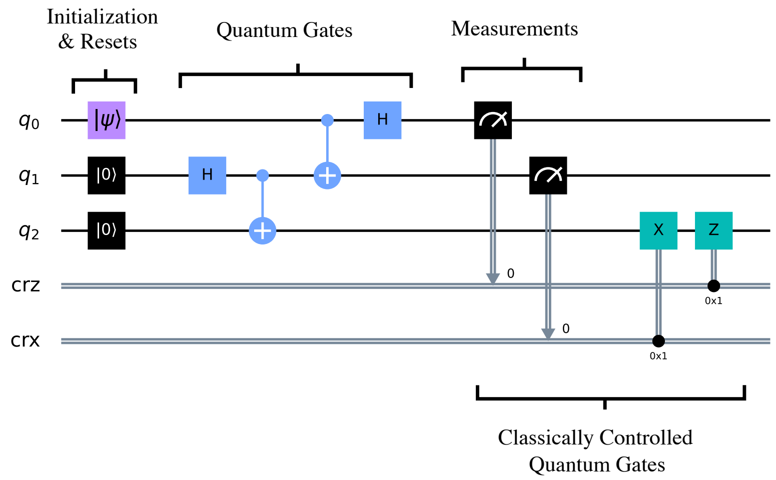
6-1 Quantum Teleportation

§1.3 Quantum computation

quantum circuits



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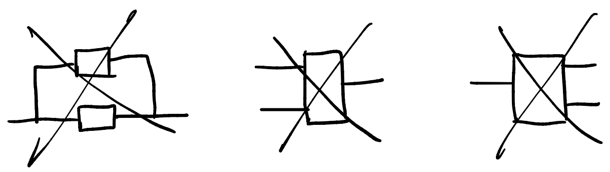
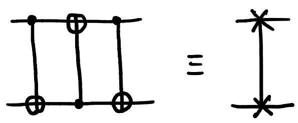
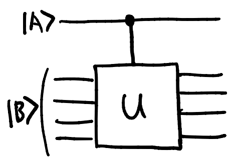


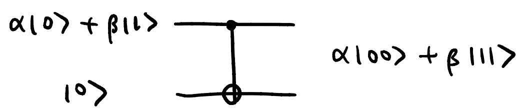
SWAP $\quad |00\rangle \to |00\rangle,\ \ |01\rangle \to |10\rangle,\ \ |10\rangle \to |01\rangle,\ \ |11\rangle \to |11\rangle$



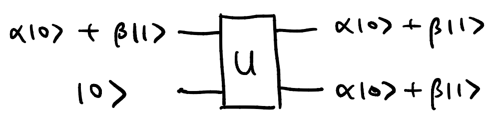
controlled-U



$$|B\rangle \to \begin{cases} |B\rangle & \text{if } \ |A\rangle = |0\rangle \\ U|B\rangle & \text{if } \ |A\rangle = |1\rangle \end{cases}$$



entangled state



???

no-cloning theorem

> You cannot copy an <u>unknown</u> quantum state!

Suppose you can do it with U_{copy} such that

$$U_{\text{copy}}|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$$

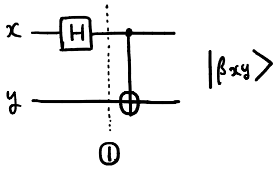
$$U_{\text{copy}}|\phi\rangle|0\rangle = |\phi\rangle|\phi\rangle$$

Then,

$$U_{\text{copy}}(|\psi\rangle + |\phi\rangle)|0\rangle = U_{\text{copy}}|\psi\rangle|0\rangle + U_{\text{copy}}|\phi\rangle|0\rangle = |\psi\rangle|\psi\rangle + |\phi\rangle|\phi\rangle$$

$$\neq (|\psi\rangle + |\phi\rangle)(|\psi\rangle + |\phi\rangle)$$

Bell states



x	y	① (unnormalized)	$\|\beta_{xy}\rangle$
$\|0\rangle$	$\|0\rangle$	$(\|0\rangle + \|1\rangle)\|0\rangle = \|00\rangle + \|10\rangle$	$\frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle)$
$\|0\rangle$	$\|1\rangle$	$(\|0\rangle + \|1\rangle)\|1\rangle = \|01\rangle + \|11\rangle$	$\frac{1}{\sqrt{2}}(\|01\rangle + \|10\rangle)$
$\|1\rangle$	$\|0\rangle$	$(\|0\rangle - \|1\rangle)\|0\rangle = \|00\rangle - \|10\rangle$	$\frac{1}{\sqrt{2}}(\|00\rangle - \|11\rangle)$
$\|1\rangle$	$\|1\rangle$	$(\|0\rangle - \|1\rangle)\|1\rangle = \|01\rangle - \|11\rangle$	$\frac{1}{\sqrt{2}}(\|01\rangle - \|10\rangle)$

x: phase ($x = 0$: phase $= +1$, $x = 1$: phase $= -1$)

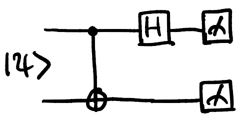
y: parity ($y = 0$: even parity, $y = 1$: odd parity)

$|\beta_{xy}\rangle$ can't be written as $|\psi_1\rangle|\psi_2\rangle$ <u>entangled state</u>

<u>HW6-1</u> Show that any two-qubit state can be written in the basis of Bell states.

(hint) $\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \stackrel{?}{=} c_{00}|\beta_{00}\rangle + c_{01}|\beta_{01}\rangle + c_{10}|\beta_{10}\rangle + c_{11}|\beta_{11}\rangle$

Express $c_{00}, c_{01}, c_{10}, c_{11}$ in terms of $\alpha, \beta, \gamma, \delta$

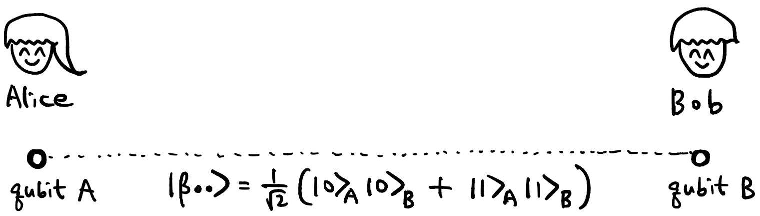


measurement in the basis of Bell states

> Bell states are <u>quantum resources</u>!

superdense coding (§2.3)

<u>objective:</u> to transmit two-bit information by sending only one qubit



Alice alone can transform $|\beta_{00}\rangle$ into any Bell state using local gates!

assumptions of quantum communication

- Classical communication and local quantum operations are cheap.
- Sending qubits and sharing entangled qubits are expensive.

$X_A|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B) = |\beta_{01}\rangle$

$Z_A|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B) = |\beta_{10}\rangle$

$Z_A X_A|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(-|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B) = -|\beta_{11}\rangle$

step 1. Alice applies local gates to her qubit.

step 2. Alice sends her qubit to Bob.

step 3. Bob performs the Bell-state measurement.

Alice never touches Bob's qubit. Classical counterpart?

quantum teleportation

objective: to transmit an unknown qubit state without sending real qubits.

cf) $|\psi\rangle_{AB} = \alpha|0\rangle_A|0\rangle_B + \beta|1\rangle_A|1\rangle_B$

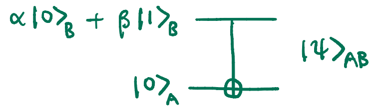
Alice measures qubit A in $|\pm\rangle_A = \frac{1}{\sqrt{2}}(|0\rangle_A \pm |1\rangle_A)$ basis.

$$|\psi\rangle_{AB} = \alpha\frac{1}{\sqrt{2}}(|+\rangle_A + |-\rangle_A)|0\rangle_B + \beta\frac{1}{\sqrt{2}}(|+\rangle_A - |-\rangle_A)|1\rangle_B$$
$$= \frac{1}{\sqrt{2}}|+\rangle_A(\alpha|0\rangle_B + \beta|1\rangle_B) + \frac{1}{\sqrt{2}}|-\rangle_A(\alpha|0\rangle_A - \beta|1\rangle_B)$$

If Alice measures $|+\rangle_A$, Bob has $\alpha|0\rangle_B + \beta|1\rangle_B$

$|-\rangle_A$ $\qquad\qquad$ $\alpha|0\rangle_B - \beta|1\rangle_B \xrightarrow{Z_B} \alpha|0\rangle_B + \beta|1\rangle_B$

Now, they can restore $|\psi\rangle_{AB}$ from Bob's state.



Alice's measurement destroys her qubit, yet they can restore the original state!

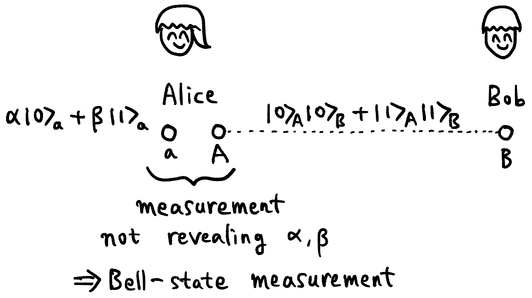
What's going on here?

① $|\psi\rangle_{AB}$ is a two-qubit state, but encodes one-qubit information in α & β.

② Alices's measurement gives NO information on α & β.

$P(|+\rangle_A) = P(|-\rangle_A) = 1/2$ is independent of α, β.

In QM, there exists a trade-off between information gain and state disturbance.



(unnormalized)

$\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle$

$\to \alpha\{(|00\rangle + |11\rangle) + (|00\rangle - |11\rangle)\}|0\rangle$

$+\alpha\{(|01\rangle + |10\rangle) + (|01\rangle - |10\rangle)\}|1\rangle$

$+\beta\{(|01\rangle + |10\rangle) - (|01\rangle - |10\rangle)\}|0\rangle$

$+\beta\{(|00\rangle + |11\rangle) - (|00\rangle - |11\rangle)\}|1\rangle$

$(\alpha|0\rangle_a + \beta|1\rangle_a)\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)_{AB}$

$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{aA}(\alpha|0\rangle + \beta|1\rangle)_B$: case 1. Bob has $\alpha|0\rangle + \beta|1\rangle$ $\quad I = Z^0X^0$

$+\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{aA}(\alpha|1\rangle + \beta|0\rangle)_B$: case 2. Bob needs to perform X $\quad Z^0X^1$

$+\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{aA}(\alpha|0\rangle - \beta|1\rangle)_B$: case 3. Bob needs to perform Z $\quad Z^1X^0$

$+\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{aA}(\alpha|1\rangle - \beta|0\rangle)_B$: case 4. Bob needs to perform ZX $\quad Z^1X^1$

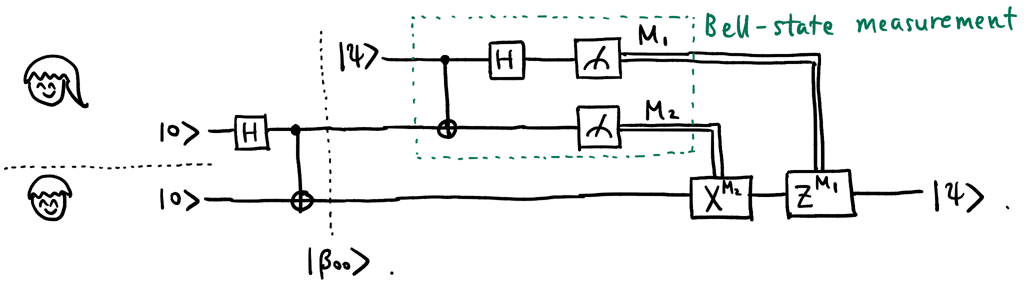
all cases occur with an equal probability for any α, β $\Rightarrow$ no information gain on α, β

protocol ① Alice and Bob share two qubits A and B in state $|\beta_{00}\rangle_{AB}$.

② Alice performs Bell-state measurement on qubits a and A.

③ Alice tells Bob the measurement result by classical communication.

④ Bob performs local operation on qubit B according to the measurement result.



HW6-2 $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$

$$\begin{cases} |\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\ |\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\ |\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\ |\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \end{cases} \qquad \begin{cases} |\eta_{00}\rangle = \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle) \\ |\eta_{01}\rangle = \frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle) \\ |\eta_{10}\rangle = \frac{1}{\sqrt{2}}(|++\rangle - |--\rangle) \\ |\eta_{11}\rangle = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle) \end{cases}$$

Express $|\eta_{ij}\rangle$ in terms of $|\beta_{ij}\rangle$.