Let $\hat{n}_H = \left(1/\sqrt{2}, 0, 1/\sqrt{2}\right)$. Show $R_{\hat{n}_H}(180°) = iH$

$\Rightarrow$ General rotation

$$R_{\hat{n}}(\theta) = e^{-i\theta/2(\hat{n}\,\vec{\sigma})}$$

Lets compute

$$\hat{n}\,\vec{\sigma} = \left(1/\sqrt{2}, 0, 1/\sqrt{2}\right)\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\end{pmatrix}$$

$$= 1/\sqrt{2}\,\sigma_x + 1/\sqrt{2}\,\sigma_z$$

$\theta = 180°$

$\sigma = \left(1/\sqrt{2}, 0\ 1/\sqrt{2}\right)$

$180° = \pi$

$$R_{\hat{n}}(\pi) = e^{-i\,\pi/2\left(1/\sqrt{2}\,\sigma_x + 1/\sqrt{2}\,\sigma_z\right)}$$

$$= e^{-i\,\pi/2\;1/\sqrt{2}\,\sigma_x} \cdot e^{-i\,\pi/2\;1/\sqrt{2}\,\sigma_z}$$

as we know

$$e^{-i\theta/2\,\hat{n}\,\vec{\sigma}} = \cos\theta/2\, I - i\sin\theta/2\,\hat{n}\,\vec{\sigma}$$

We can write,

$$\left(\cos\left(-\pi/4\right) I - i\sin\left(-\pi/4\right)\sigma_x\right)\left(\cos\left(-\pi/4\right) I - i\sin\left(-\pi/4\right)\sigma_z\right)$$

$$= \left(\frac{\sqrt{2}}{2} I - i\frac{\sqrt{2}}{2}\sigma_x\right)\left(\frac{\sqrt{2}}{2} I - i\frac{\sqrt{2}}{2}\sigma_z\right)$$

$$= \frac{1}{2}(I + i\sigma_x)(I - i\sigma_z)$$

$$= \frac{1}{2}(I - i\sigma_y - i\sigma_y \quad i^2\sigma_x\sigma_z)$$

$$= \frac{1}{2}(I - 2i\sigma_y - i^2\sigma_y)$$

$$= \frac{1}{2}(I + 2i\sigma_y)$$

$$= \frac{1}{2} I + i\sigma_y$$

$$= i\sigma_y$$

$$= i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = iH$$

$\therefore R_{\hat{n}}(\pi) = -iH$ <u>Proved</u>