3-1 Introduction to Quantum Mechanics

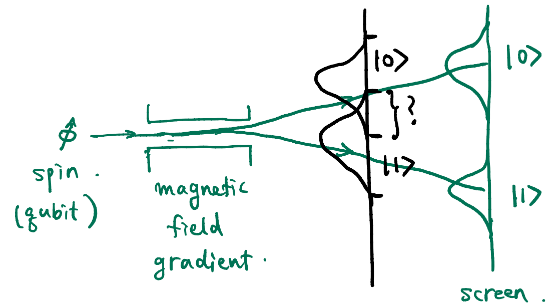
3 bits vs 3 qubits

Postulate 3: called a projective measurement

general measurement

(combination of Postulates 1~4)

ex) Stern – Gerlach experiment.



$\{M_m\}$: set of measurement operators s.t.

$$\sum_m E_m \equiv \sum_m M_m^\dagger M_m = I$$

$$\sum_m M_m M_m^\dagger \neq I$$

$E_m \equiv M_m^\dagger M_m$: called the POVM element

(Positive-Operator Valued Measure)

$\langle\phi|E_m|\phi\rangle \geq 0$ for any $|\phi\rangle$

$\because \langle\phi|M_m^\dagger M_m|\phi\rangle = \langle\phi'|\phi'\rangle \geq 0 \qquad |\phi'\rangle \equiv M_m|\phi\rangle$

measurement outcome: $P(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle$

$$|\psi\rangle \to \frac{M_m|\psi\rangle}{\sqrt{P(m)}}$$

ex) You have a state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. $\quad(\alpha, \beta \in \mathbb{C})$

Measurement operators: $M_1 = \sqrt{p}|0\rangle\langle 0|$, $M_2 = \sqrt{p}|1\rangle\langle 1|$, $M_3 = \sqrt{1-p}\, I$

Your measurement outcome is 1 or 2 or 3.

$P(1) = ?$, State after measuring 1: ?

$P(2) = ?$, State after measuring 2: ?

$P(3) = ?$, State after measuring 3: ?

How? 1. apply a unitary operator U_{SA} s.t. $U_{SA}|\psi\rangle_S|0\rangle_A = \sum_m (M_m|\psi\rangle_s)|m\rangle_A$

S: system, A: "ancilla"

2. measure the ancilla in $\{|m\rangle_A\}$ projective measurement

$$P_m = I_S \otimes |m\rangle_A\langle m|$$

Is U_{SA} unitary? you can see U_{SA} preserves the norm

$$
\begin{aligned}
{}_S\langle\phi|_A\langle 0|U_{SA}^\dagger U_{SA}|\psi\rangle_S|0\rangle_A &= \sum_{m,m'} \Big({}_S\langle\phi|M_m^\dagger \otimes {}_A\langle m|\Big)\Big(M_{m'}|\psi\rangle_S \otimes |m'\rangle_A\Big) \\
&= \sum_{m,m'} {}_S\langle\phi|M_m^\dagger M_{m'}|\psi\rangle_S\, {}_A\langle m|m'\rangle_A \\
&= \sum_m {}_S\langle\phi|M_m^\dagger M_m|\psi\rangle_S = {}_S\langle\phi|\psi\rangle_S
\end{aligned}
$$

$P(m) = {}_S\langle\psi|M_m^\dagger M_m|\psi\rangle_S$

expectation values

You have a state $|\psi\rangle$ and want to get the expectation value of an observable A.

$$A = \sum_m a_m P_m \qquad \sum_m P_m = I$$

$$\langle A \rangle = \sum_m P(a_m) a_m = \sum_m a_m \langle\psi|P_m|\psi\rangle = \langle\psi| \sum_m a_m P_m |\psi\rangle$$

$$\boxed{\langle A \rangle = \langle\psi|A|\psi\rangle}$$