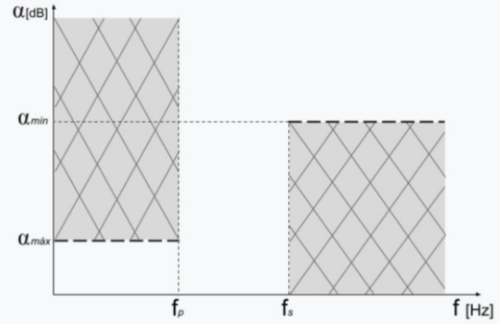
A partir de la siguiente plantilla, sabiendo que:



αmáx [dB]	αmín [dB]	fp [Hz]	fs [Hz]
1	12	1500	3000

1. Obtener la transferencia para máxima planicidad en la banda de paso **utilizando los conceptos de partes de función**. Recordar que: $|T(j\omega)|^2 = T(j\omega) \cdot T(-j\omega) = T(s) \cdot T(-s)|_{s=j\omega}$
2. Obtener el diagrama de polos y ceros, y un bosquejo de la respuesta en frecuencia.
3. Implementar el circuito **normalizado** con estructuras pasivas separadas mediante buffers.
4. Obtenga el circuito que cumpla con la plantilla requerida si dispone de capacitores de 100nf.
5. Proponga una red que se comporte igual a la hallada en **4)** pero con resistores, capacitores y opamps.

Bonus:

- +10 💎 Proponer un planteo alternativo a **1)** usando la ω_{Butter} (**ver Schaumann 6.4**)
- +10 ⚽ Simulación **numérica y circuital.**
- +10 🍺 Presentación en jupyter notebook.

#1

Func atenuación α:

$$\alpha^2 = 1 + \varepsilon^2 \omega^{2n} \longrightarrow \alpha = (1 + \varepsilon^2 \omega^{2n})^{1/2}$$

Expresada en dB: $\alpha_{dB} = 10 \log(1 + \varepsilon^2 \omega^{2n})$

En $\omega = \omega_{p_n} = 1$: $\alpha_{dB}\Big|_{\omega=1} = \alpha_{max_{dB}} = 10 \log(1 + \varepsilon^2)$

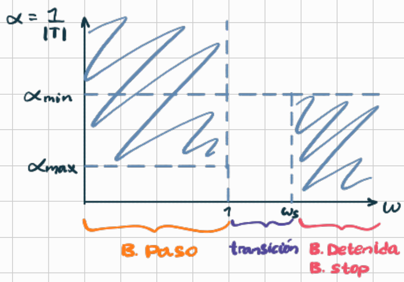
$$\hookrightarrow \varepsilon^2 = 10^{\frac{\alpha_{max\,dB}}{10}} - 1 \quad (1)$$

En $\omega = \omega_{s_n}$: $\alpha_{dB}\Big|_{\omega=\omega_s} = \alpha_{min\,dB} = 10 \log(1 + \varepsilon^2 \omega_{s_n}^{2n}) \quad (2)$

Reemplazando en (1): $\varepsilon^2 = 10^{\frac{\alpha_{max\,dB}}{10}} - 1 \longrightarrow \varepsilon^2 = 0{,}2589$

Reemplazando en (2) e iterando:

- $n=3$: $10 \log(1 + \varepsilon^2 \omega_{s_n}^{2n}) = 12{,}4476 > \alpha_{min_{dB}} = 12 \longleftarrow$ cumple



αmáx [dB]	αmín [dB]	fp [Hz]	fs [Hz]
1	12	1500	3000

$\omega_p = 2\pi f_p = 3000\pi$ rad/seg

$\omega_s = 2\pi f_s = 6000\pi$ rad/seg

✳ Normalizando:

$\omega_{p_n} = 1 \qquad \omega_{s_n} = \frac{\omega_s}{\omega_p} = 2$

/ /

Func. transferencia máxima planicidad para $n=3$:

$$|T_{(s)}|^2 = T_{(s)} \cdot T_{(-s)} = |T_{(j\omega)}|^2\Big|_{\omega=\frac{s}{j}} = \frac{1}{1-\varepsilon^2 s^6} = \frac{-1/\varepsilon^2}{s^6 - 1/\varepsilon^2} = \underbrace{\frac{a}{s^3+bs^2+cs+d}}_{T(s)} \cdot \underbrace{\frac{(-a)}{s^3-bs^2+cs-d}}_{T(-s)}$$

$$a\cdot(-a) = -\frac{1}{\varepsilon^2} \longrightarrow -a^2 = -\frac{1}{\varepsilon^2} \longrightarrow a = \frac{1}{\varepsilon}$$

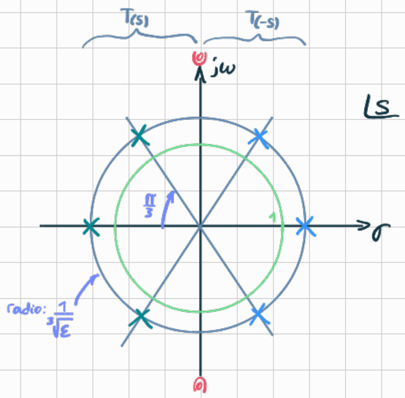
$$\begin{cases} s^6: & 1=1 \\ s^5: & -b+b=0 \\ s^4: & c-b^2+c=0 \longrightarrow 2c=b^2 \\ s^3: & -d+bc-bc+d=0 \\ s^2: & -bd+c^2-bd=0 \longrightarrow c^2=2bd \\ s: & -cd+cd=0 \\ 1: & -d^2=-\frac{1}{\varepsilon^2} \longrightarrow d=\frac{1}{\varepsilon} \end{cases}$$

$$\begin{cases} 2c=b^2 \\ c^2=2bd \\ d=\frac{1}{\varepsilon} \end{cases} \qquad \begin{cases} b=\frac{2}{\sqrt[3]{\varepsilon}} \\ c^2 = 2\sqrt{2}\sqrt{c}\,\frac{1}{\varepsilon} \longrightarrow c^{3/2} = 2\sqrt{2}\,\frac{1}{\varepsilon} \longrightarrow c=\frac{2}{\sqrt[3]{\varepsilon^2}} \\ d=\frac{1}{\varepsilon} \end{cases}$$

$$T_{(s)} = \frac{1/\varepsilon}{s^3 + \frac{2}{\sqrt[3]{\varepsilon}}s^2 + \frac{2}{\sqrt[3]{\varepsilon^2}}s + \frac{1}{\varepsilon}}$$

#2

Diagrama de polos y ceros



Ls

polos: $1-\varepsilon^2 s^6 = 0 \longrightarrow s^6 = \frac{1}{\varepsilon^2}$

$$S_{pk} = \frac{1}{\sqrt[6]{\varepsilon^2}} e^{j\frac{2k\pi}{6}} = \frac{1}{\sqrt[3]{\varepsilon}} e^{j\frac{k\pi}{3}} \quad ; \; k=0,1,2,3,4,5$$

$$S_{p_0} = \frac{1}{\sqrt[3]{\varepsilon}} e^{j0} \qquad S_{p_3} = \frac{1}{\sqrt[3]{\varepsilon}} e^{j\pi}$$

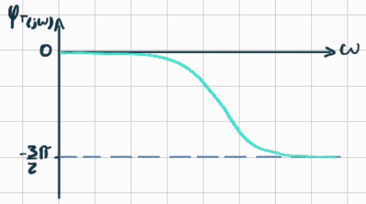
$$S_{p_1} = \frac{1}{\sqrt[3]{\varepsilon}} e^{j\frac{\pi}{3}} \qquad S_{p_4} = \frac{1}{\sqrt[3]{\varepsilon}} e^{j\frac{4}{3}\pi}$$

$$S_{p_2} = \frac{1}{\sqrt[3]{\varepsilon}} e^{j\frac{2}{3}\pi} \qquad S_{p_5} = \frac{1}{\sqrt[3]{\varepsilon}} e^{j\frac{5}{3}\pi}$$

Respuesta en frecuencia:

$$T_{(0)} = \frac{1/\varepsilon}{1/\varepsilon} = 1$$

$$|T| = \frac{\prod_i |V_{z_i,\omega}|}{\prod_j |V_{p_j,\omega}|}$$

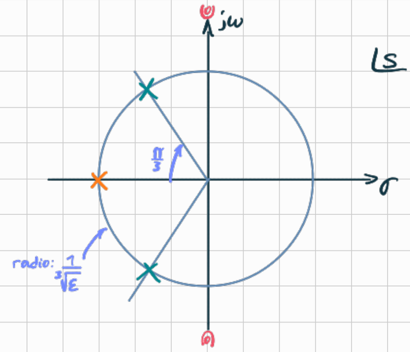


$$\varphi_T = \sum \angle \alpha_{z_i,\omega} - \sum \angle \alpha_{p_j,\omega}$$

#3

Partiendo del sistema de orden 3, se separa en uno de orden 2 y uno de orden 1

$$T_{(s)} = \frac{1/\varepsilon}{s^3 + \frac{2}{\sqrt[3]{\varepsilon}} s^2 + \frac{2}{\sqrt[3]{\varepsilon^2}} s + \frac{1}{\varepsilon}}$$



• Los polos (X) corresponden a la sig. Func. Transferencia:

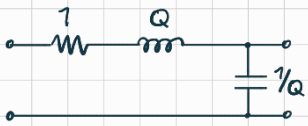
$$T_{2(s)} = \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \qquad Q = \frac{1}{2 \cdot \cos \underbrace{\varphi}_{\frac{\pi}{3}}} = 1 \qquad \omega_0 = \frac{1}{\sqrt[3]{\varepsilon}}$$

$$T_{2(s)} = \frac{1/\sqrt[3]{\varepsilon^2}}{s^2 + s\frac{1}{\sqrt[3]{\varepsilon}} + \frac{1}{\sqrt[3]{\varepsilon^2}}}$$

Normalizando: $\Omega_\omega = \omega_0 = \frac{1}{\sqrt[3]{\varepsilon}}$; $\Omega_{z_2} = R_2$

$$\begin{cases} R_{2n} = \frac{R_2}{\Omega_{z_2}} = 1 \\ L_{1n} = \frac{\Omega_\omega}{\Omega_{z_2}} L_1 = \frac{1}{R_2 \sqrt[3]{\varepsilon}} L_1 \\ C_{2n} = \Omega_\omega \Omega_{z_2} C_2 = \frac{R_2}{\sqrt[3]{\varepsilon}} C_2 \end{cases} \quad \begin{cases} \omega_0^2 = \frac{1}{L_1 C_2} = \frac{1}{\sqrt[3]{\varepsilon^2}} \longrightarrow L_1 = \frac{\sqrt[3]{\varepsilon^2}}{C_2} \\ Q = \sqrt{\frac{L_1}{C_2}} \cdot \frac{1}{R_2} = 1 \longrightarrow C_2 R_2 = \frac{\sqrt[3]{\varepsilon}}{Q} = \sqrt[3]{\varepsilon} \end{cases} \quad \begin{cases} R_{2n} = 1 \\ C_{2n} = \frac{1}{Q} = 1 \\ L_{1n} = Q = 1 \end{cases}$$

$$T_{2(\$)} = \frac{1}{\$^2 + \$ + 1}$$



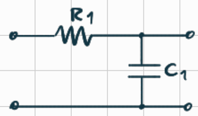
$\Omega_\omega = \frac{1}{\sqrt[3]{\varepsilon}}$

$\Omega_{z_2} = R_2$

• El polo (X) corresponde a la sig. Func. Transferencia:

$$T_{1(s)} = \frac{1/\sqrt[3]{\varepsilon}}{s + \frac{1}{\sqrt[3]{\varepsilon}}}$$

para que $T_{1(0)} = 1$



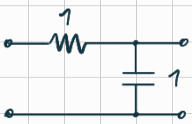
Normalizando: $\Omega_\omega = \omega_0 = \frac{1}{\sqrt[3]{\varepsilon}}$; $\Omega_{z_1} = R_1$

$$T_{2(\$)} = \frac{1}{\$ + 1}$$

$R_{1n} = \frac{R_1}{\Omega_{z_1}} = 1$

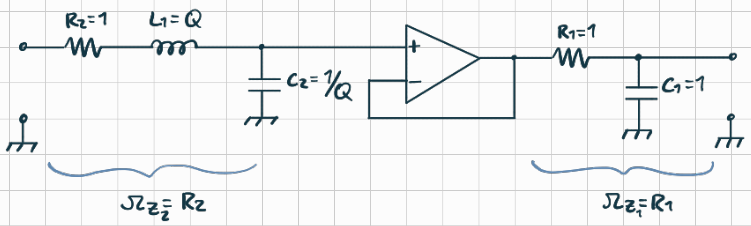
$C_{1n} = \Omega_\omega \Omega_{z_1} C_1 = \frac{R_1}{\sqrt[3]{\varepsilon}} C_1$

$$\begin{cases} \omega_0 = \frac{1}{R_1 C_1} = \frac{1}{\sqrt[3]{\varepsilon}} \\ R_{1n} = 1 \\ C_{1n} = 1 \end{cases}$$

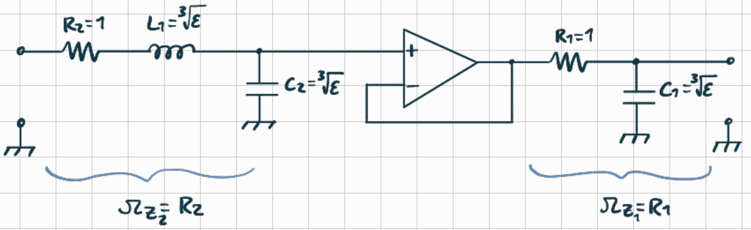


. Quedando $T_{(s)}$ en función de $T_{1(s)}$ y $T_{2(s)}$:

$$T_{(\$)} = \frac{1}{\$^2 + \$ + 1} \cdot \frac{1}{\$ + 1} \qquad \Omega_\omega = \omega_0 = \frac{1}{\sqrt[3]{\varepsilon}}$$



(Filtro Butterworth)



(Filtro Max. Planicidad)

. Renormalizando: $\Omega'_\omega = \omega_B = \omega_p \underbrace{\varepsilon^{-1/3}}_{\Omega\omega}$

#4

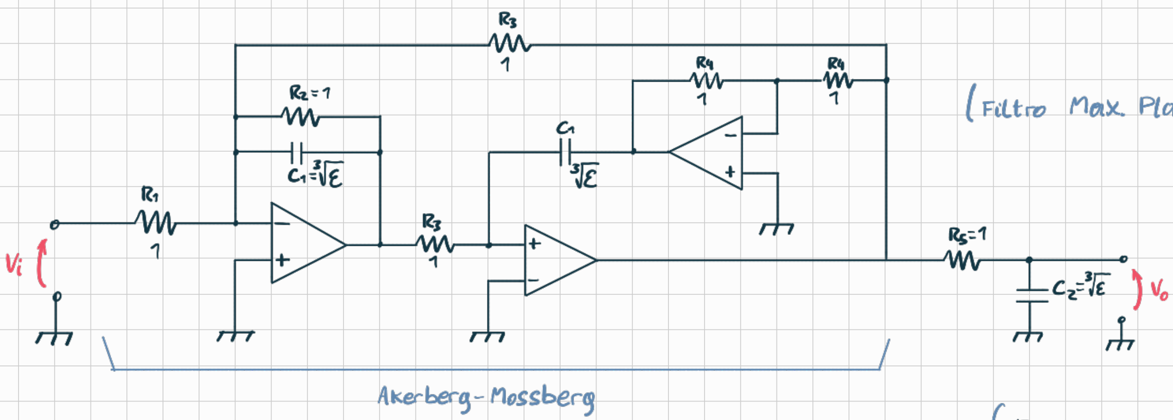
$C_1 = C_2 = 100\,nF$

$$C_{1n} = \Omega'_\omega \Omega_{z_1} C_1 = \frac{\omega_p R_1}{\sqrt[3]{\varepsilon}} C_1 = 1 \longrightarrow R_1 = \frac{\sqrt[3]{\varepsilon}}{\omega_p C_1} \longrightarrow R_1 = 847\,\Omega$$

$$C_{2n} = \Omega'_\omega \Omega_{z_2} C_2 = \frac{\omega_p R_2}{\sqrt[3]{\varepsilon}} C_2 = 1 \longrightarrow R_2 = \frac{\sqrt[3]{\varepsilon}}{\omega_p C_2} \longrightarrow R_2 = 847\,\Omega$$

$$L_{1n} = \frac{\Omega'_\omega}{\Omega_{z_2}} L_1 = \frac{\omega_p}{R_2 \sqrt[3]{\varepsilon}} L_1 = 1 \longrightarrow L_1 = \frac{R_2 \sqrt[3]{\varepsilon}}{\omega_p} \longrightarrow L_1 = 0{,}071746\text{ Hy} = 71{,}746\text{ mHy}$$

#5



(Filtro Max. Planicidad)

$$T_{A\text{-}M(\$)} = k \cdot \frac{1}{\$^2 + \$\frac{1}{Q} + 1} = -\frac{1}{R_{1n}} \frac{1}{\$^2 + \$\frac{1}{R_{2n}} + 1} \quad ; \quad \Omega_z = R_3 \quad ; \quad \Omega_\omega = \frac{1}{R_3 C} = \frac{1}{\sqrt[3]{\varepsilon}}$$

$$\begin{cases} R_{1n} = 1 \\ R_{2n} = 1 \\ R_{3n} = 1 \\ R_{4n} = 1 \\ C_{1n} = \sqrt[3]{\varepsilon} \end{cases}$$

$$T_{(\$)} = \frac{1}{\$^2 + \$ + 1} \cdot \frac{1}{\$ + 1} \qquad \Omega_\omega = \omega_0 = \frac{1}{\sqrt[3]{\varepsilon}}$$