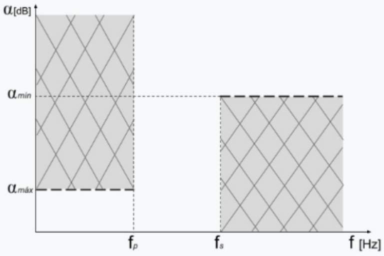
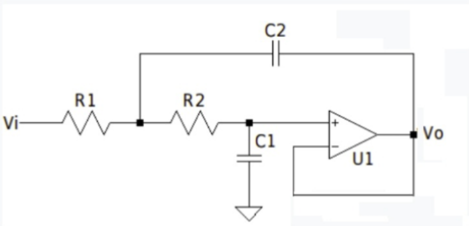
Diseñar un filtro pasabajos normalizado ***Bessel*** para obtener un *Retardo de Grupo* $D(w)$ de 1 segundo.



D[seg]	α_{Max} [dB]	wp [rad/seg]	ws [rad/seg]
1	1 dB	1	2.5

1. Utilizando el metodo de *Storch (pag. 403 del Schaumann)* obtener la T(s) Bessel para **N: 2, 3 y 4** normalizados para D = 1
2. Elegir la T(s) con el minimo orden que cumple con $\alpha_{Max = 1\,dB}$
3. Evaluar el Retardo de Grupo $D(2.5)$. Expresar en forma *porcentual [%]* el error o desviamiento respecto a $D(0)$
4. Sintetizar el circuito NORMALIZADO con estructuras *Sallen-Key con K=1 (real. negativa unitaria)*
5. 

BONUS

- +10 💎 Simulación **numérica en python**
- +10 💎 DESNORMALIZAR los componentes para obtener un $D(1)$ = 200 microseg.
- +10 💎 Simulación Circuital con los valores DESNORMALIZADOS y medir el $D(w)$**(*)**
- +10 🍺 Presentación en jupyter notebook

(*) Para medir el $D(w)$ en LTSpice, click derecho en el y Axis de la phase y seleccionar "Group Delay"

#1 . n = 2:

$$\text{cotgh}(s) = \frac{1}{s} + \frac{s}{3} = \frac{s^2+3}{3s}$$

$$T_2(s) = \frac{1}{\text{senh}(s) + \cosh(s)} = \frac{1}{s^2+3s+3} \longrightarrow T_2(s) = \frac{3}{s^2+3s+3}$$

. n = 3

$$\text{cotgh}(s) = \frac{1}{s} + \frac{1}{\frac{3}{s} + \frac{s}{5}} = \frac{1}{s} + \frac{5s}{s^2+15} = \frac{6s^2+15}{s^3+15s}$$

$$T_3(s) = \frac{15}{s^3+6s^2+15s+15}$$

. n = 4

$$\text{cotgh}(s) = \frac{1}{s} + \frac{1}{\frac{3}{s} + \frac{1}{\frac{5}{s} + \frac{s}{7}}} = \frac{1}{s} + \frac{1}{\frac{3}{s} + \frac{7s}{s^2+35}} = \frac{1}{s} + \frac{s^3+35s}{10s^2+105} = \frac{s^4+45s^2+105}{10s^3+105s}$$

$$T_4(s) = \frac{105}{s^4+10s^3+45s^2+105s+105}$$

#2

$\alpha_{max} = 1$

$$\alpha_{(\omega)} = -20 \log |T_{(\omega)}| \longrightarrow \alpha_{max} = -20 \log |T_{(\omega)}| \Big|_{\omega=\omega_p=1}$$

. $n=2$:

$$T_{2(s)} = \frac{3}{s^2+3s+3} \longrightarrow T_{2(\omega)} = T_{2(s)}\Big|_{s=j\omega} = \frac{3}{3-\omega^2+j3\omega}$$

$$\alpha_{max} = -20 \log \left(\frac{3}{\sqrt{(3-\omega^2)^2+9\omega^2}} \right) \Bigg|_{\omega=\omega_p=1} = 1{,}59 \text{ dB}$$ X la atenuación en la banda de paso es mayor a la pedida

. $n=3$:

$$T_{3(s)} = \frac{15}{s^3+6s^2+15s+15} \longrightarrow T_{3(\omega)} = T_{3(s)}\Big|_{s=j\omega} = \frac{15}{15-6\omega^2+j(15\omega-\omega^3)}$$

$$\alpha_{max} = -20 \log \left(\frac{15}{\sqrt{(15-6\omega^2)^2+(15\omega-\omega^3)^2}} \right) \Bigg|_{\omega=\omega_p=1} = 0{,}9029725095 \text{ dB}$$ ✓ $\alpha_{max} < 1\text{dB}$

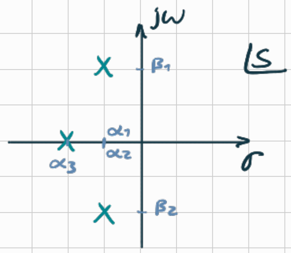
Para cumplir con la plantilla, se deberá utilizar un Bessel de orden 3 $\longrightarrow$ siendo $\alpha_{max} \cong 0{,}9$

#3

$$\varphi = atg\left(\frac{\mathrm{Im}}{\mathrm{Re}}\right) \longrightarrow D_{(\omega)} = -\frac{\partial\left[atg\left(\frac{\mathrm{Im}}{\mathrm{Re}}\right)\right]}{\partial\omega}$$

Se hace uso de la siguiente identidad trigonométrica: $\frac{\partial\{atg(x)\}}{\partial x} = \frac{1}{1+x^2}\, x'$

$$atg\left(\frac{\omega \pm \beta}{\alpha}\right)$$

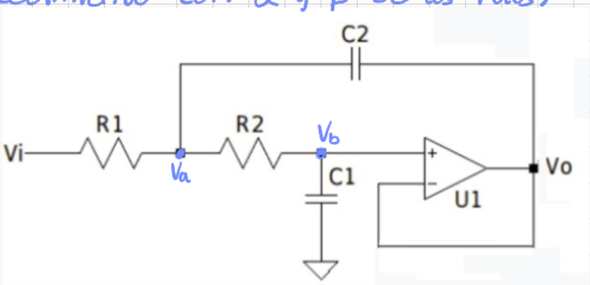


$$D_{(\omega)} = \frac{\alpha_1}{\alpha_1^2+(\omega+\beta_1)^2} + \frac{\alpha_2}{\alpha_2^2+(\omega+\beta_2)^2} + \frac{\alpha_3}{\alpha_3^2+(\omega+\beta_3)^2}$$

$$D_{(\omega)} = \frac{\alpha_1}{\alpha_1^2+(\omega+\beta_1)^2} + \frac{\alpha_1}{\alpha_1^2+(\omega-\beta_1)^2} + \frac{\alpha_3}{\alpha_3^2+\omega^2}$$

$$error\%_{2,5} = \left(D_{(0)} - D_{(2,5)}\right) \cdot 100 \cong 24{,}79175$$ $\longleftarrow$ Calculo realizado en Python

#4 (Procedimiento con α y β de los Polos)



$$\begin{cases} (G_1+G_2+sC_2)V_a - G_1 V_i - G_2 V_b - sC_2 V_o = 0 \\ (G_2 + sC_1) V_b - G_2 V_a = 0 \\ V_b = V_o \end{cases}$$

$$T_{SK(s)} = \frac{\frac{G_1 . G_2}{C_1 C_2}}{s^2 + s \frac{G_1+G_2}{C_2} + \frac{G_1 . G_2}{C_1 C_2}} \longleftarrow \text{Transferencia obtenida con cálculo simbólico}$$

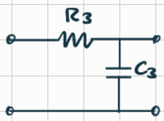
siendo la Transferencia de un Pasa Bajos Bessel de orden 3

$$T_{B3\,(s)} = \frac{15}{s^3 + 6s^2 + 15s + 15} = \underbrace{\frac{\alpha_1^2+\beta_1^2}{s^2 + 2\alpha_1 s + \alpha_1^2 + \beta_1^2}}_{T_1} \cdot \underbrace{\frac{\frac{15}{\alpha_1^2+\beta_1^2}}{s+\alpha_3}}_{T_2}$$

* $T_{1(s)} = T_{SK(s)}$

$$\begin{cases} \frac{G_1 . G_2}{C_1 C_2} = \alpha_1^2+\beta_1^2 \\ \frac{G_1+G_2}{C_2} = 2\alpha_1 \end{cases} ; \; G_1 = G_2 = 1 \begin{cases} \frac{1}{C_1 C_2} = \alpha_1^2+\beta_1^2 \\ \frac{2}{C_2} = 2\alpha_1 \end{cases} \begin{cases} G_1 = 1 \longrightarrow R_1 = 1 \\ G_2 = 1 \longrightarrow R_2 = 1 \\ C_1 = \frac{\alpha_1}{\alpha_1^2+\beta_1^2} \\ C_2 = \frac{1}{\alpha_1} \end{cases}$$

* $$T_{2(s)} = \frac{\frac{15}{\alpha_1^2+\beta_1^2}}{s+\alpha_3}$$



$$T_{(s)} = \frac{\frac{1}{R_3 C_3}}{s + \frac{1}{R_3 C_3}} = \frac{\frac{15}{\alpha_1^2+\beta_1^2}}{s+\alpha_3} \begin{cases} \alpha_3 = \frac{15}{\alpha_1^2+\beta_1^2} \\ R_3 C_3 = \frac{1}{\alpha_3} \end{cases} ; \; R_3 = 1 \begin{cases} R_3 = 1 \\ C_3 = \frac{1}{\alpha_3} = \frac{\alpha_1^2+\beta_1^2}{15} \end{cases}$$

Bonus:

$$D_{(\omega)}\Big|_{\omega=1} = 200\,\mu seg \longrightarrow D_{(0)} \cong D_{(1)} = 200\,\mu seg \longrightarrow \Omega_\omega = \omega_0 = \frac{1}{D_{(0)}} = 5000\ rad/seg$$

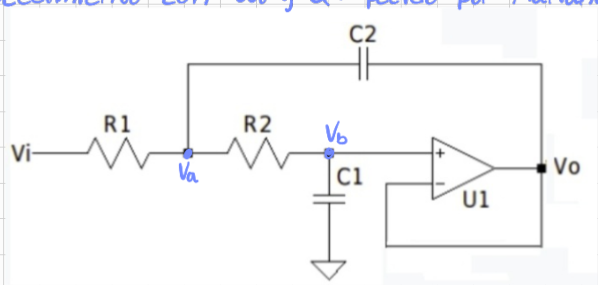
$$C_1 = \frac{C_{1n}}{\Omega_\omega} = \frac{1}{\Omega_\omega} \frac{\alpha_1}{\alpha_1^2+\beta_1^2}$$

$$C_2 = \frac{C_{2n}}{\Omega_\omega} = \frac{1}{\Omega_\omega} \cdot \frac{1}{\alpha_1}$$

$$C_3 = \frac{C_{3n}}{\Omega_\omega} = \frac{1}{\Omega_\omega} \frac{\alpha_1^2+\beta_1^2}{15}$$

$R_1 = R_2 = R_3 = 1$

#4 (Procedimiento con ω_0 y Q ← pedido por Mariano)



$$\begin{cases} (G_1 + G_2 + sC_2)V_a - G_1 V_i - G_2 V_b - sC_2 V_o = 0 \\ (G_2 + sC_1) V_b - G_2 V_a = 0 \\ V_b = V_o \end{cases}$$

$$T_{SK(s)} = \frac{\frac{G_1 . G_2}{C_1 C_2}}{s^2 + s\,\frac{G_1+G_2}{C_2} + \frac{G_1 . G_2}{C_1 C_2}}$$ ← Transferencia obtenida con cálculo simbólico

$$\begin{cases} \omega_{02} = \frac{15}{\omega_0^2} \\ \omega_{02} = \alpha_3 \\ \omega_0^2 = \alpha_1^2 + \beta_1^2 \\ \frac{\omega_0}{Q} = 2\alpha_1 \end{cases} \qquad \begin{cases} \alpha_3 = \frac{15}{\omega_0^2} \end{cases} \qquad \begin{cases} \omega_0^2 = \alpha_1^2 + \beta_1^2 \\ Q = \frac{\sqrt{\alpha_1^2+\beta_1^2}}{2\alpha_1} \end{cases}$$

siendo la Transferencia de un Pasa Bajos Bessel de orden 3

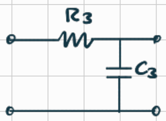
$$T_{B3\,(s)} = \frac{15}{s^3 + 6s^2 + 15s + 15} = \underbrace{\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}}_{T_1} \cdot \underbrace{\frac{\frac{15}{\omega_0^2}}{s + \omega_{02}}}_{T_2}$$

* $T_{1(s)} = T_{SK(s)}$

$$\begin{cases} \frac{G_1 . G_2}{C_1 C_2} = \omega_0^2 \\ \frac{G_1+G_2}{C_2} = \frac{\omega_0}{Q} \end{cases} ;\; G_1 = G_2 = 1 \begin{cases} \frac{1}{C_1 C_2} = \omega_0^2 \\ \frac{2}{C_2} = \frac{\omega_0}{Q} \end{cases}$$

$$\begin{cases} G_1 = 1 \longrightarrow R_1 = 1 \\ G_2 = 1 \longrightarrow R_2 = 1 \\ C_1 = \frac{1}{2Q\omega_0} \\ C_2 = 2\frac{Q}{\omega_0} \end{cases}$$

* $$T_{2(s)} = \frac{\frac{15}{\alpha_1^2+\beta_1^2}}{s + \alpha_3}$$



$$T_{(s)} = \frac{\frac{1}{R_3 C_3}}{s + \frac{1}{R_3 C_3}} = \frac{\frac{15}{\omega_0^2}}{s + \frac{15}{\omega_0^2}} \quad \begin{cases} R_3 C_3 = \frac{\omega_0^2}{15} \end{cases} ;\; R_3 = 1 \begin{cases} R_3 = 1 \\ C_3 = \frac{\omega_0^2}{15} \end{cases}$$

#Bonus:

$$D_{(\omega)}\Big|_{\omega=1} = 200\,\mu seg \longrightarrow D_{(0)} \cong D_{(1)} = 200\,\mu seg \longrightarrow \Omega_\omega = \omega_0 = \frac{1}{D_{(0)}} = 5000\ rad/seg$$

$$C_1 = \frac{C_{1n}}{\Omega_\omega} = \frac{1}{\Omega_\omega}\,\frac{1}{2Q\omega_0}$$

$$C_2 = \frac{C_{2n}}{\Omega_\omega} = \frac{1}{\Omega_\omega}\,.\,2\,\frac{Q}{\omega_0}$$

$$C_3 = \frac{C_{3n}}{\Omega_\omega} = \frac{1}{\Omega_\omega}\,\frac{\omega_0^2}{15}$$

$R_1 = R_2 = R_3 = 1$