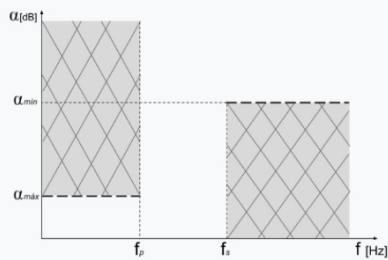


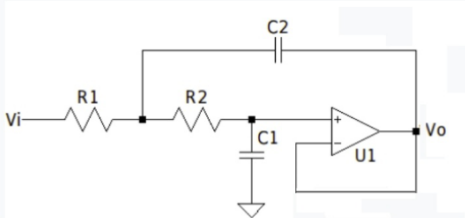
Diseñar un filtro pasabajos normalizado **Bessel** para obtener un Retardo de Grupo $D(w)$ de 1 segundo.



D[seg] α_{Max} [dB] ω_p [rad/seg] ω_s [rad/seg]

1 1 dB 1 2.5

- Utilizando el metodo de Storch (pag. 403 del Schaumann) obtener la T(s) Bessel para N: 2, 3 y 4 normalizados para D = 1
- Elegir la T(s) con el minimo orden que cumple con $\alpha_{Max} = 1$ dB
- Evaluar el Retardo de Grupo $D(2.5)$. Expresar en forma porcentual [%] el error o desviamiento respecto a $D(0)$
- Sintetizar el circuito NORMALIZADO con estructuras Sallen-Key con K=1 (real. negativa unitaria)



5.

BONUS

- +10 Simulación numérica en python
- +10 DESNORMALIZAR los componentes para obtener un $D(1) = 200$ microseg.
- +10 Simulación Circuitual con los valores DESNORMALIZADOS y medir el $D(w)^{(*)}$
- +10 Presentación en jupyter notebook

(*) Para medir el $D(w)$ en LTSpice, click derecho en el y Axis de la phase y seleccionar "Group Delay"

#1 . n = 2:

$$\coth(s) = \frac{1}{s} + \frac{s}{3} = \frac{s^2 + 3}{3s}$$

$$T_2(s) = \frac{1}{\sinh(s) + \cosh(s)} = \frac{1}{s^2 + 3s + 3} \rightarrow T_2(s) = \frac{3}{s^2 + 3s + 3}$$

. n = 3

$$\coth(s) = \frac{1}{s} + \frac{1}{\frac{3+s}{s}} = \frac{1}{s} + \frac{s}{s^2 + 3} = \frac{6s^2 + 15}{s^3 + 3s}$$

$$T_3(s) = \frac{15}{s^3 + 6s^2 + 15s + 15}$$

. n = 4

$$\coth(s) = \frac{1}{s} + \frac{1}{\frac{3}{s} + \frac{1}{\frac{5}{s} + \frac{s}{7}}} = \frac{1}{s} + \frac{1}{\frac{3}{s} + \frac{7s}{5s^2 + 35}} = \frac{1}{s} + \frac{s^3 + 35s}{10s^2 + 105} = \frac{s^4 + 45s^2 + 105}{10s^3 + 105s}$$

$$T_4(s) = \frac{105}{s^4 + 10s^3 + 45s^2 + 105s + 105}$$

#2

$$\alpha_{\max} = 1$$

$$\alpha(\omega) = -20 \log |T(\omega)| \rightarrow \alpha_{\max} = -20 \log |T(\omega)| \Big|_{\omega=\omega_p=1}$$

. n = 2:

$$T_2(s) = \frac{3}{s^2 + 3s + 3} \rightarrow T_2(\omega) = T_2(s) \Big|_{s=j\omega} = \frac{3}{3 - \omega^2 + j3\omega}$$

$$\alpha_{\max} = -20 \log \left(\frac{3}{\sqrt{(3 - \omega^2)^2 + 9\omega^2}} \right) \Big|_{\omega=\omega_p=1} = 1,59 \text{ dB} \quad \times \text{ la atenuaci3n en la banda de paso es mayor a la pedida}$$

. n = 3:

$$T_3(s) = \frac{15}{s^3 + 6s^2 + 15s + 15} \rightarrow T_3(\omega) = T_3(s) \Big|_{s=j\omega} = \frac{15}{15 - 6\omega^2 + j(15\omega - \omega^3)}$$

$$\alpha_{\max} = -20 \log \left(\frac{15}{\sqrt{(15 - 6\omega^2)^2 + (15\omega - \omega^3)^2}} \right) \Big|_{\omega=\omega_p=1} = 0,9029725095 \text{ dB} \quad \checkmark \alpha_{\max} < 1 \text{ dB}$$

Para cumplir con la plantilla, se deber3 utilizar un Bessel de orden 3 \rightarrow siendo $\alpha_{\max} \approx 0,9$

#3

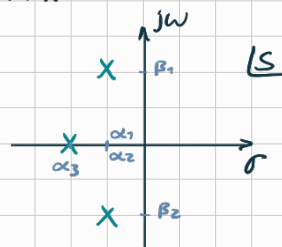
$$\varphi = \alpha \operatorname{tg} \left(\frac{\operatorname{Im}}{\operatorname{Re}} \right) \rightarrow D(\omega) = - \frac{2 \left[\operatorname{atg} \left(\frac{\operatorname{Im}}{\operatorname{Re}} \right) \right]}{2\omega}$$

Se hace uso de la siguiente identidad trigonom3trica: $\frac{2 \{ \operatorname{atg}(x) \}}{2x} = \frac{1}{1+x^2} x'$

$$\operatorname{atg} \left(\frac{\omega \pm \beta}{\alpha} \right)$$

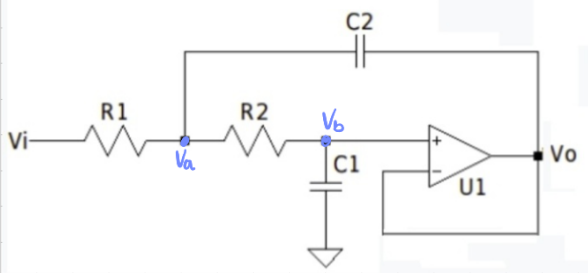
$$D(\omega) = \frac{\alpha_1}{\alpha_1^2 + (\omega + \beta_1)^2} + \frac{\alpha_2}{\alpha_2^2 + (\omega + \beta_2)^2} + \frac{\alpha_3}{\alpha_3^2 + (\omega + \beta_3)^2}$$

$$D(\omega) = \frac{\alpha_1}{\alpha_1^2 + (\omega + \beta_1)^2} + \frac{\alpha_1}{\alpha_1^2 + (\omega - \beta_1)^2} + \frac{\alpha_3}{\alpha_3^2 + \omega^2}$$



$$\text{error}\%_{2,5} = (D(0) - D(2,5)) \cdot 100 \approx 24,79175 \leftarrow \text{C3lculo realizado en Python}$$

#4 (Procedimiento con α y β de los Polos)



$$\begin{cases} (G_1 + G_2 + sC_2)V_a - G_1V_i - G_2V_b - sC_2V_o = 0 \\ (G_2 + sC_1)V_b - G_2V_a = 0 \\ V_b = V_o \end{cases}$$

$$T_{SK}(s) = \frac{\frac{G_1 \cdot G_2}{C_1 C_2}}{s^2 + s \frac{G_1 + G_2}{C_2} + \frac{G_1 \cdot G_2}{C_1 C_2}}$$

← Transferencia obtenida con cálculo simbólico

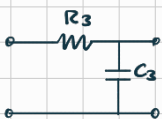
Siendo la Transferencia de un Pasa Bajas Bessel de orden 3

$$T_{B3}(s) = \frac{15}{s^3 + 6s^2 + 15s + 15} = \underbrace{\frac{\alpha_1^2 + \beta_1^2}{s^2 + 2\alpha_1 s + \alpha_1^2 + \beta_1^2}}_{T_1} \cdot \underbrace{\frac{15}{s + \alpha_3}}_{T_2}$$

$$* T_1(s) = T_{SK}(s)$$

$$\begin{cases} \frac{G_1 \cdot G_2}{C_1 C_2} = \alpha_1^2 + \beta_1^2 \\ \frac{G_1 + G_2}{C_2} = 2\alpha_1 \end{cases}, G_1 = G_2 = 1 \left\{ \begin{array}{l} \frac{1}{C_1 C_2} = \alpha_1^2 + \beta_1^2 \\ \frac{2}{C_2} = 2\alpha_1 \end{array} \right. \left\{ \begin{array}{l} G_1 = 1 \rightarrow R_1 = 1 \\ G_2 = 1 \rightarrow R_2 = 1 \\ C_1 = \frac{\alpha_1}{\alpha_1^2 + \beta_1^2} \\ C_2 = \frac{1}{\alpha_1} \end{array} \right.$$

$$* T_2(s) = \frac{15}{\alpha_1^2 + \beta_1^2} \cdot \frac{1}{s + \alpha_3}$$



$$T(s) = \frac{1}{R_3 C_3} \cdot \frac{15}{s + \frac{1}{R_3 C_3}} = \frac{15}{s + \alpha_3} \left\{ \begin{array}{l} \alpha_3 = \frac{15}{R_3 C_3} \\ R_3 C_3 = \frac{1}{\alpha_3} \end{array} \right. j, R_3 = 1 \left\{ \begin{array}{l} R_3 = 1 \\ C_3 = \frac{1}{\alpha_3} = \frac{\alpha_1^2 + \beta_1^2}{15} \end{array} \right.$$

Bonus:

$$D(\omega) \Big|_{\omega=1} = 200 \mu\text{seg} \rightarrow \Omega_{\omega} = \omega_0 = \frac{1}{D(1)} = 5000 \text{ rad/seg}$$

$$C_1 = \frac{C_{1n}}{\Omega_{\omega}} = \frac{1}{\Omega_{\omega}} \cdot \frac{\alpha_1}{\alpha_1^2 + \beta_1^2}$$

$$C_2 = \frac{C_{2n}}{\Omega_{\omega}} = \frac{1}{\Omega_{\omega}} \cdot \frac{1}{\alpha_1}$$

$$C_3 = \frac{C_{3n}}{\Omega_{\omega}} = \frac{1}{\Omega_{\omega}} \cdot \frac{\alpha_1^2 + \beta_1^2}{15}$$

$$R_1 = R_2 = R_3 = 1$$

$$D_n(1) = 0,9963$$

$$D_n(0) = 1$$

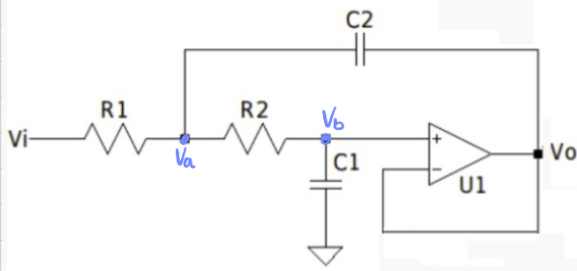
$$D_{(1)} = 200 \mu\text{s}$$

$$D_{(0)} =$$

$$D_{(0)} = \frac{1 \cdot 200 \mu\text{seg}}{0,9963}$$

$$\Omega_{\omega} = \frac{1}{D_{(0)}} = 4981,95 \text{ rad/seg}$$

#4 (Procedimiento con ω_0 y Q ← pedido por Mariano)



$$\begin{cases} (G_1 + G_2 + sC_2)V_a - G_1V_i - G_2V_b - sC_2V_o = 0 \\ (G_2 + sC_1)V_b - G_2V_a = 0 \\ V_b = V_o \end{cases}$$

$$T_{SK}(s) = \frac{\frac{G_1 \cdot G_2}{C_1 C_2}}{s^2 + s \frac{G_1 + G_2}{C_2} + \frac{G_1 \cdot G_2}{C_1 C_2}}$$

← Transferencia obtenida con cálculo simbólico

Siendo la Transferencia de un Pasa Bajas Bessel de orden 3

$$T_{B3}(s) = \frac{15}{s^3 + 6s^2 + 15s + 15} = \underbrace{\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}}_{T_1} \cdot \underbrace{\frac{\frac{15}{\omega_0^2}}{s + \omega_0}}_{T_2}$$

$$\begin{cases} \omega_{02} = \frac{15}{\omega_0^2} \\ \omega_{02} = \alpha_3 \end{cases} \Rightarrow \alpha_3 = \frac{15}{\omega_0^2}$$

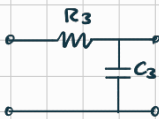
$$\begin{cases} \omega_0^2 = \alpha_1^2 + \beta_1^2 \\ \frac{\omega_0}{Q} = 2\alpha_1 \end{cases} \Rightarrow \begin{cases} \omega_0^2 = \alpha_1^2 + \beta_1^2 \\ Q = \frac{\sqrt{\alpha_1^2 + \beta_1^2}}{2\alpha_1} \end{cases}$$

* $T_1(s) = T_{SK}(s)$

$$\begin{cases} \frac{G_1 \cdot G_2}{C_1 C_2} = \omega_0^2 \\ \frac{G_1 + G_2}{C_2} = \frac{\omega_0}{Q} \end{cases}, G_1 = G_2 = 1 \Rightarrow \begin{cases} \frac{1}{C_1 C_2} = \omega_0^2 \\ \frac{2}{C_2} = \frac{\omega_0}{Q} \end{cases}$$

$$\begin{cases} G_1 = 1 \rightarrow R_1 = 1 \\ G_2 = 1 \rightarrow R_2 = 1 \\ C_1 = \frac{1}{2Q\omega_0} \\ C_2 = 2 \frac{Q}{\omega_0} \end{cases}$$

* $T_2(s) = \frac{\frac{15}{\alpha_1^2 + \beta_1^2}}{s + \alpha_3}$



$$T(s) = \frac{\frac{1}{R_3 C_3}}{s + \frac{1}{R_3 C_3}} = \frac{\frac{15}{\omega_0^2}}{s + \frac{15}{\omega_0^2}} \Rightarrow R_3 C_3 = \frac{\omega_0^2}{15}, R_3 = 1 \Rightarrow \begin{cases} R_3 = 1 \\ C_3 = \frac{\omega_0^2}{15} \end{cases}$$