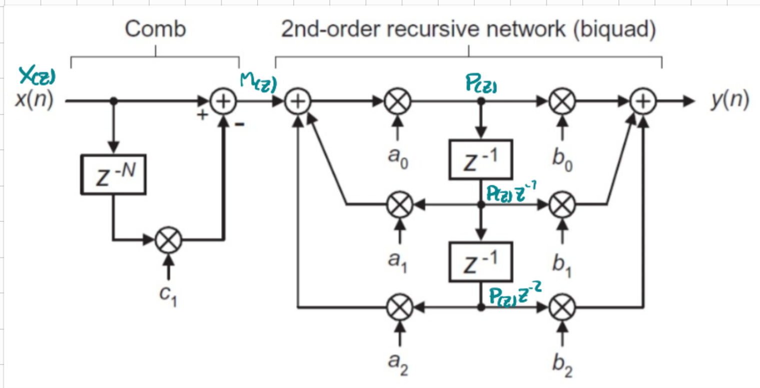
#4.a



- $M(z) = X(z) - X(z)\, c_1 z^{-N} = (1 - c_1 z^{-N})\, X(z)$

- $P(z) = a_0 \left(M(z) + a_1 P(z) z^{-1} + a_2 P(z) z^{-2} \right)$

$$P(z)\left(1 - a_0 a_1 z^{-1} - a_0 a_2 z^{-2}\right) = a_0 M(z) \longrightarrow P(z) = \frac{a_0 (1 - c_1 z^{-N})\, X(z)}{(1 - a_0 a_1 z^{-1} - a_0 a_2 z^{-2})}$$

- $Y(z) = b_0 P(z) + b_1 P(z) z^{-1} + b_2 P(z) z^{-2}$

$$Y(z) = P(z)\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right) = \frac{a_0 (1 - c_1 z^{-N})(b_0 + b_1 z^{-1} + b_2 z^{-2})}{(1 - a_0 a_1 z^{-1} - a_0 a_2 z^{-2})}\, X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = (1 - c_1 z^{-N})\, \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{\frac{1}{a_0} - a_1 z^{-1} - a_2 z^{-2}}$$

#4.b (Filtro de media móvil)

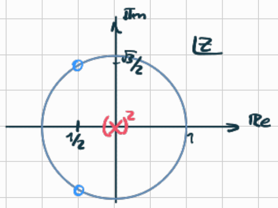
$a_0 = 1$, $a_1 = 1$, $b_0 = \frac{1}{N}$, $c_1 = 1$, $N = (3, 4 \text{ y } 5) \longrightarrow a_2 = b_1 = b_2 = 0$

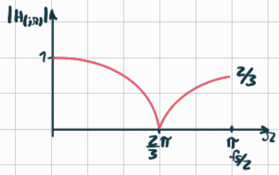
$$H(z) = (1 - z^{-N})\, \frac{\frac{1}{N}}{1 - z^{-1}} = \frac{1}{N}\, \frac{1 - z^{-N}}{1 - z^{-1}}$$

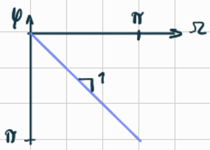
- $N = 3$: $H(z) = \frac{1}{3}\, \frac{1 - z^{-3}}{1 - z^{-1}} = \frac{1}{3}\, \frac{z^3 - 1}{z^2 (z - 1)} = \frac{1}{3}\, \frac{\cancel{(z-1)}(z^2 + z + 1)}{z^2 \cancel{(z-1)}} = \frac{1}{3}\, \frac{z^2 + z + 1}{z^2}$

$$H(j\Omega) = H(z)\Big|_{z = e^{j\Omega}} = \frac{1}{3}\, \frac{e^{j2\Omega} + e^{j\Omega} + 1}{e^{j2\Omega}} = \frac{1}{3}\left(e^{j0} + e^{-j\Omega} + e^{-j2\Omega}\right) = \frac{1}{3} e^{-j\Omega}\left(e^{j\Omega} + e^{j0} + e^{-j\Omega}\right)$$

$$H(j\Omega) = \frac{1}{3}\left(1 + 2\cos(\Omega)\right) e^{-j\Omega}$$

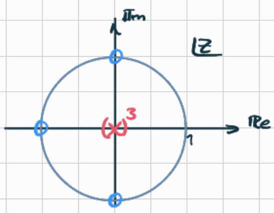


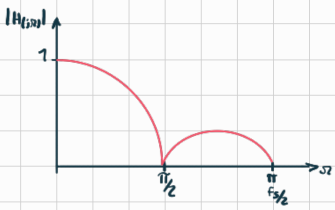


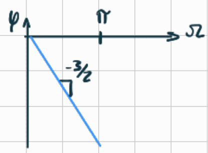


- $N = 4$: $H(z) = \frac{1}{4}\, \frac{1 - z^{-4}}{1 - z^{-1}} = \frac{1}{4}\, \frac{z^4 - 1}{z^3 (z - 1)} = \frac{1}{4}\, \frac{\cancel{(z-1)}(z+1)(z^2+1)}{z^3 \cancel{(z-1)}} = \frac{1}{4}\, \frac{z^3 + z^2 + z + 1}{z^3}$

$$H(j\Omega) = \frac{1}{4}\left(e^{j0} + e^{-j\Omega} + e^{-j2\Omega} + e^{-j3\Omega}\right) = \frac{1}{4} e^{-j\frac{3}{2}\Omega}\left(e^{j\frac{3}{2}\Omega} + e^{j\frac{1}{2}\Omega} + e^{-j\frac{1}{2}\Omega} + e^{-j\frac{3}{2}\Omega}\right) = \frac{1}{2}\left[\cos\left(\tfrac{1}{2}\Omega\right) + \cos\left(\tfrac{3}{2}\Omega\right)\right] e^{-j\frac{3}{2}\Omega}$$



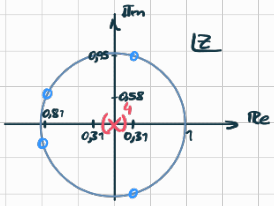


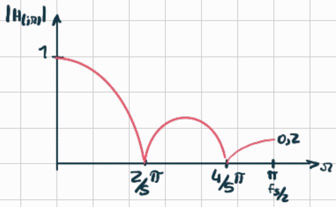


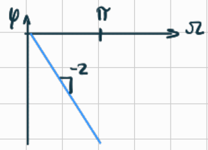
/ /

• $N=5$: $H_{(z)} = \frac{1}{5}\frac{1-z^{-5}}{1-z^{-1}} = \frac{1}{5}\frac{z^5-1}{z^4(z-1)} = \frac{1}{5}\frac{\cancel{(z-1)}(z^4+z^3+z^2+z+1)}{z^4\cancel{(z-1)}} = \frac{1}{5}\frac{\left(z^2+\frac{1-\sqrt{5}}{2}z+1\right)\left(z^2+\frac{1+\sqrt{5}}{2}z+1\right)}{z^4}$

$$H_{(j\Omega)} = \frac{1}{5}\left(e^{j0}+e^{-j\Omega}+e^{-j2\Omega}+e^{-j3\Omega}+e^{-j4\Omega}\right) = \frac{1}{5}e^{-j2\Omega}\left(e^{j2\Omega}+e^{j\Omega}+e^{j0}+e^{-j\Omega}+e^{-j2\Omega}\right) = \frac{1}{5}\left[1+2\cos(2\Omega)+2\cos(\Omega)\right]e^{-j2\Omega}$$







* Es un filtro FIR, todos sus polos se encuentran en 0

* Esta topología tiene como ventaja frente al filtro de media movil convencional, su fácil escalabilidad ya que solo depende del retardo en la primer etapa. Además los requerimientos de memoria parecen ser menores a grandes escalas.

* Sí, se podría implementar $h_{6(k)} = (1,1,1,1,1,1,1)$, solo habría que aumentar el $N=6$

$$h_{(k)} = (1,1,1,1,1,1,1) \xrightarrow{Z} H_{(z)} = 1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}+z^{-6} = \frac{z^6+z^5+z^4+z^3+z^2+z+1}{z^6}$$

#4.c (Filtro diferenciador)

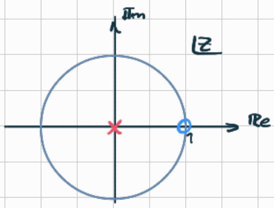
* Diferenciador de primer orden:

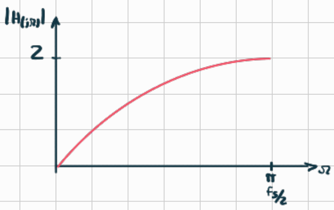
$$y_{[n]} = x_{[n]} - x_{[n-1]} \xrightarrow{Z} Y_{(z)} = X_{(z)} - z^{-1}X_{(z)} \longrightarrow H_{(z)} = \frac{Y_{(z)}}{X_{(z)}} = 1-z^{-1}$$

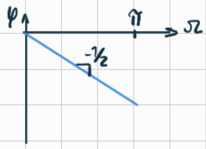
Para que el filtro sea equivalente a un diferenciador de primer orden:

$$H_{(z)} = \frac{Y_{(z)}}{X_{(z)}} = (1-C_1 z^{-N})\frac{b_0+b_1z^{-1}+b_2z^{-2}}{\frac{1}{a_0}-a_1z^{-1}-a_2z^{-2}} \quad \left\{\begin{array}{l} C_1=b_2=a_1=a_2=0 \\ b_0=b_1=a_0=1 \end{array}\right. \quad \left\{ H_{(z)} = 1-z^{-1} = \frac{z-1}{z}\right.$$

$$H_{(j\Omega)} = e^{j0}-e^{-j\Omega} = e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}}-e^{-j\frac{\Omega}{2}}\right) = 2\,\text{sen}\left(\frac{\Omega}{2}\right)e^{-j\frac{\Omega}{2}}$$





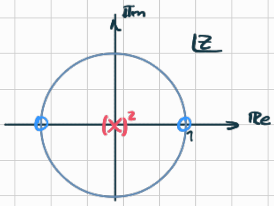


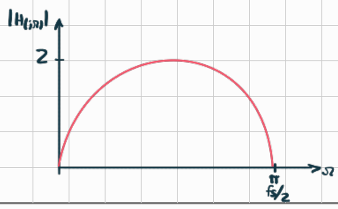
* Diferenciador de segundo orden:

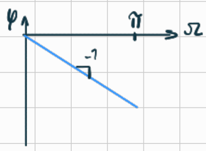
$$h_{[n]} = (1,0,-1) \xrightarrow{Z} H_{(z)} = 1-z^{-2} = \frac{z^2-1}{z^2} \longrightarrow y_{[n]} = x_{[n]} - x_{[n-2]}$$

$$H_{(z)} = \frac{Y_{(z)}}{X_{(z)}} = (1-C_1 z^{-N})\frac{b_0+b_1z^{-1}+b_2z^{-2}}{\frac{1}{a_0}-a_1z^{-1}-a_2z^{-2}} \quad \left\{\begin{array}{l} b_0=-1\ ;\ b_2=a_0=1 \\ C_1=b_1=a_1=a_2=0 \end{array}\right. \quad \left\{ H_{(z)} = 1-z^{-2} = \frac{z^2-1}{z^2}\right.$$

$$H_{(j\Omega)} = e^{j0}-e^{-j2\Omega} = e^{-j\Omega}\left(e^{j\Omega}-e^{-j\Omega}\right) = 2\,\text{sen}(\Omega)\,e^{-j\Omega}$$



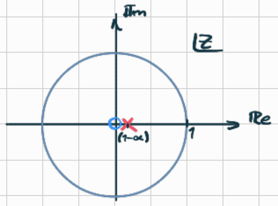




#4.d (Integrador con pérdidas)

$a_0 = 1$; $a_1 = 1-\alpha$; $b_0 = \alpha$; $\alpha = 0{,}9$

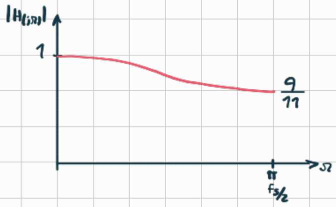
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\alpha}{1-(1-\alpha)z^{-1}} = \frac{\alpha z}{z-(1-\alpha)}$$

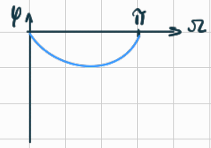


$$H(j\Omega) = \frac{\alpha}{1-(1-\alpha)e^{-j\Omega}} = \frac{\alpha}{1-(1-\alpha)(\cos(\Omega)-j\,\text{sen}(\Omega))}$$

$$|H(j\Omega)| = \frac{\alpha}{\sqrt{[1-(1-\alpha)\cos(\Omega)]^2 + (1-\alpha)^2\,\text{sen}^2(\Omega)}}$$

$$\varphi(\Omega) = -\text{arctg}\left(\frac{(1-\alpha)\,\text{sen}(\Omega)}{1-(1-\alpha)\cos(\Omega)}\right)$$

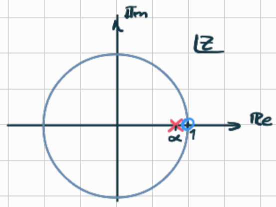




#4.e (Filtro elimina continua / DC Blocker)

$a_0 = 1$, $a_1 = \alpha$, $b_0 = 1$, $b_1 = -1$, $\alpha = 0{,}9$

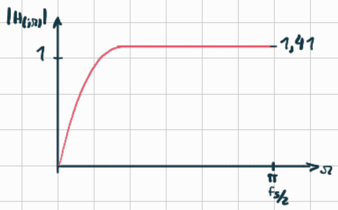
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-z^{-1}}{1-\alpha z^{-1}} = \frac{z-1}{z-\alpha}$$

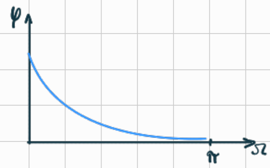


$$H(j\Omega) = \frac{e^{j\Omega}-1}{e^{j\Omega}-\alpha} = \frac{\cos(\Omega)-1+j\,\text{sen}(\Omega)}{\cos(\Omega)-\alpha+j\,\text{sen}(\Omega)}$$

$$|H(j\Omega)| = \sqrt{\frac{(\cos(\Omega)-1)^2+\text{sen}^2(\Omega)}{(\cos(\Omega)-\alpha)^2+\text{sen}^2(\Omega)}} = \sqrt{\frac{\cos^2(\Omega)-2\cos(\Omega)+1+\text{sen}^2(\Omega)}{\cos^2(\Omega)-2\alpha\cos(\Omega)+\alpha^2+\text{sen}^2(\Omega)}} = \sqrt{\frac{2-2\cos(\Omega)}{1+\alpha^2-2\alpha\cos(\Omega)}} = \frac{|2\,\text{sen}(\frac{\Omega}{2})|}{\sqrt{1+\alpha^2-2\alpha\cos(\Omega)}}$$

$$\varphi(\Omega) = \text{arctg}\left(\frac{\text{sen}(\Omega)}{\cos(\Omega)-1}\right) - \text{arctg}\left(\frac{\text{sen}(\Omega)}{\cos(\Omega)-\alpha}\right)$$





$$|H(j\Omega)|\Big|_{\Omega=0{,}1\pi} = \overset{3dB}{\frac{\sqrt{2}}{2}}\,|H(j\Omega)|\Big|_{\Omega=\pi}$$

$$\frac{2\,\text{sen}(0{,}05\pi)}{\sqrt{1+\alpha^2-2\alpha\cos(0{,}1\pi)}} = \frac{\sqrt{2}}{2}\,\frac{2}{\sqrt{\alpha^2+2\alpha+1}} = \frac{\sqrt{2}}{\alpha+1}$$

$$\frac{1+\alpha^2-2\alpha\cos(0{,}1\pi)}{\alpha^2+2\alpha+1} = 2\,\text{sen}^2(0{,}05\pi)$$

$$1+\alpha^2-2\alpha\cos(0{,}1\pi)-2\alpha^2\,\text{sen}^2(0{,}05\pi)-4\alpha\,\text{sen}^2(0{,}05\pi)-2\,\text{sen}^2(0{,}05\pi)=0$$

$$\alpha^2\left(1-2\,\text{sen}^2(0{,}05\pi)\right)-\alpha\left(2\cos(0{,}1\pi)+4\,\text{sen}^2(0{,}05\pi)\right)+1-2\,\text{sen}^2(0{,}05\pi)=0 \longrightarrow \alpha = 1{,}37 \times$$

$$\searrow \alpha = 0{,}7265$$

#4.f (Filtro ecualizador de fase de primer orden)

$a_0 = 1$; $a_1 = -R$; $b_0 = R$; $b_1 = 1$ / $R = \frac{-D}{D+2}$

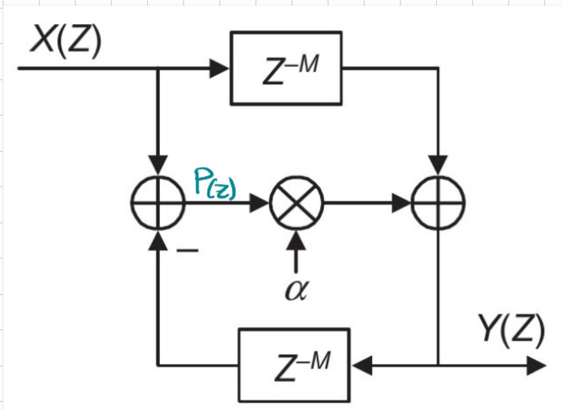
$$H_{(z)} = \frac{Y_{(z)}}{X_{(z)}} = (1 - c_1 z^{-N}) \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{\frac{1}{a_0} - a_1 z^{-1} - a_2 z^{-2}} = \frac{R + z^{-1}}{1 + R z^{-1}} = R \frac{z + 1/R}{z + R}$$

$$H_{(j\Omega)} = R \frac{e^{j\Omega} + 1/R}{e^{j\Omega} + R} = R \frac{\cos(\Omega) + j\,\mathrm{sen}(\Omega) + 1/R}{\cos(\Omega) + j\,\mathrm{sen}(\Omega) + R} = R \frac{1/R + \cos(\Omega) + j\,\mathrm{sen}(\Omega)}{R + \cos(\Omega) + j\,\mathrm{sen}(\Omega)}$$

$$\varphi_{(j\Omega)} = \mathrm{arctg}\left(\frac{\mathrm{sen}(\Omega)}{1/R + \cos(\Omega)}\right) - \mathrm{arctg}\left(\frac{\mathrm{sen}(\Omega)}{R + \cos(\Omega)}\right)$$

$$D_{(j\Omega)} = -\frac{d\varphi_{(j\Omega)}}{d\Omega}$$

#2



$$\begin{cases} P_{(z)} = X_{(z)} - Y_{(z)} z^{-M} \\ Y_{(z)} = X_{(z)} z^{-M} + \alpha P_{(z)} \end{cases}$$

$$Y_{(z)} = X_{(z)} z^{-M} + \alpha X_{(z)} - \alpha Y_{(z)} z^{-M}$$

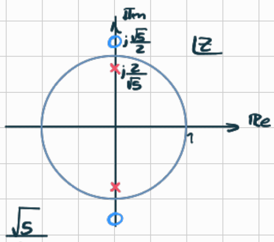
$$Y_{(z)}(1 + \alpha z^{-M}) = X_{(z)}(\alpha + z^{-M})$$

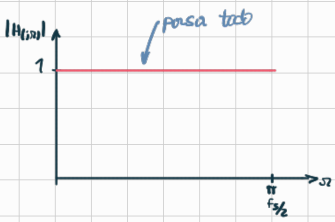
$$H_{(z)} = \frac{Y_{(z)}}{X_{(z)}} = \frac{\alpha + z^{-M}}{1 + \alpha z^{-M}} = \frac{\alpha z^M + 1}{z^M + \alpha} = \alpha \frac{z^M + 1/\alpha}{z^M + \alpha}$$

* $M = 2$, $\alpha = 0{,}8 = \frac{4}{5}$

$$H_{(z)} = \frac{Y_{(z)}}{X_{(z)}} = \frac{4}{5} \frac{z^2 + 5/4}{z^2 + 4/5}$$

polos: $z_p = \pm j \frac{2}{\sqrt{5}}$ ceros: $z_0 = \pm j \frac{\sqrt{5}}{2}$



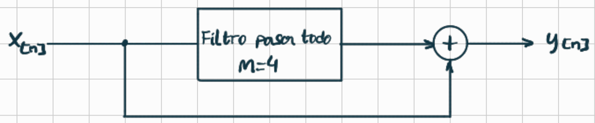


* $$H_{(j\Omega)} = \frac{4}{5} \frac{e^{j2\Omega} + 5/4}{e^{j2\Omega} + 4/5} = \frac{4}{5} \frac{5/4 + \cos(2\Omega) + j\,\mathrm{sen}(2\Omega)}{4/5 + \cos(2\Omega) + j\,\mathrm{sen}(2\Omega)}$$

$$|H_{(j\Omega)}| = \frac{4}{5}\sqrt{\frac{(5/4 + \cos(2\Omega))^2 + \mathrm{sen}^2(2\Omega)}{(4/5 + \cos(2\Omega))^2 + \mathrm{sen}^2(2\Omega)}} = \sqrt{\left(\frac{4}{5}\right)^2 \frac{(5/4)^2 + 1 + 5/2 \cos(2\Omega)}{(4/5)^2 + 1 + 8/5 \cos(2\Omega)}} = \sqrt{\frac{41 + 40\cos(2\Omega)}{41 + 40\cos(2\Omega)}} = 1$$

$$\varphi_{(j\Omega)} = \mathrm{arctg}\left(\frac{\mathrm{sen}(2\Omega)}{5/4 + \cos(2\Omega)}\right) - \mathrm{arctg}\left(\frac{\mathrm{sen}(2\Omega)}{4/5 + \cos(2\Omega)}\right)$$

* Se anula senoidal 125Hz y su armónica de 375Hz



$$H_{PT(z)} = \alpha \frac{z^4 + 1/\alpha}{z^4 + \alpha}$$

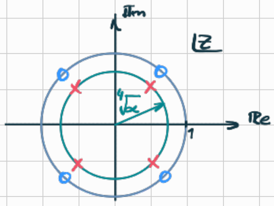
$$H_{T(z)} = 1 + H_{PT(z)} = \frac{z^4 + \alpha + \alpha z^4 + 1}{z^4 + \alpha} = (1+\alpha)\frac{z^4+1}{z^4+\alpha}$$

- ceros: $z_0 = \sqrt[4]{-1} = \sqrt[4]{1}\, e^{\frac{j(\pi + 2k\pi)}{4}}$, $k = 0,1,2,3$

$$z_{0_0} = 1e^{j\frac{\pi}{4}} \qquad z_{0_1} = 1e^{j\frac{3}{4}\pi} \qquad z_{0_2} = 1e^{j\frac{5}{4}\pi} \qquad z_{0_3} = 1e^{j\frac{7}{4}\pi}$$

- polos: $z_P = \sqrt[4]{-\alpha} = \sqrt[4]{\alpha}\, e^{\frac{j(\pi + 2k\pi)}{4}}$, $k = 0,1,2,3$

$$z_{P_0} = \sqrt[4]{\alpha}\, e^{j\frac{\pi}{4}} \qquad z_{P_1} = \sqrt[4]{\alpha}\, e^{j\frac{3}{4}\pi} \qquad z_{P_2} = \sqrt[4]{\alpha}\, e^{j\frac{5}{4}\pi} \qquad z_{P_3} = \sqrt[4]{\alpha}\, e^{j\frac{7}{4}\pi}$$



Para anular $f_1 = 125$ Hz y $f_3 = 375$ Hz:

$$\frac{f_s}{2} \equiv \pi \rightarrow f_s = \frac{2\pi}{\pi/4} f_1 = 8 f_1 = 1\,\text{kHz}$$

$$\rightarrow f_s = \frac{2\pi}{3\pi/4} f_3 = \frac{8}{3} f_3 = 1\,\text{kHz}$$

Si $f_s = 1$ kHz, se anulan ambas señales