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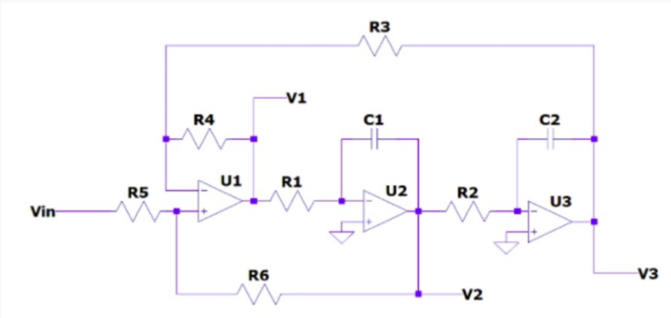
Diseñar un filtro pasabajos ***Chebyshev*** para obtener una atenuación de 48 dB para frecuencias mayores a 900 Hz, con una atenuación máxima de 0.4 dB desde continua hasta 300 Hz. (Ancho de Banda reducido para canal Sub Lows)

α_{Max} [dB]	$\alpha_{mín}$ [dB]	fp [Hz]	fs [Hz]
0.4	48	300	900

1. Determinar el orden del filtro y el parámetro ε.
2. Obtener la expresion completa de la Transferencia NORMALIZADA de T(s)
3. Obtener el diagrama de polos y ceros y graficar a mano alzada en forma cualitativa la respuesta de modulo y fase.
4. Sintetizar el circuito NORMALIZADO utilizando estructuras Kerwin−Huelsman−Newcomb (KHN, también conocido como Variable de Estado) saliendo desde la salida **V3** como indica el siguiente circuito de referencia:



BONUS

- +10 💎 Simulación **numérica en python**
- +10 💎 Simulación Circuital con los valores **DESNORMALIZADOS (cumpliendo la plantilla pedida)**
- +10 🍺 Presentación en jupyter notebook

#1

$$\varepsilon^2 = 10^{\alpha_{max}/10} - 1 = 0{,}0964782 \longrightarrow \varepsilon = 0{,}310609$$

Normalizando:

$$\left.\begin{aligned} \omega_p &= 2\pi f_p = 1884{,}96 \ \text{rad/seg} \\ \omega_s &= 2\pi f_s = 5654{,}87 \ \text{rad/seg} \end{aligned}\right\} \quad \omega_{p_n} = 1 \qquad \omega_{s_n} = \frac{\omega_s}{\omega_p} = 3$$

Obtención de n a partir de iterar $\alpha_{dB}\Big|_{\omega=\omega_{sn}=3}$ hasta que $\alpha_{dB} \geq \alpha_{min}$

$$\alpha_{dB} = 10 \log\left(1 + \varepsilon^2 \cosh^2(n \operatorname{arccos}(\omega_{sn}))\right)$$

- $n = 4$: $\alpha_{dB} = 45{,}07 < 48$ X
- $n = 5$: $\alpha_{dB} = 60{,}38 > \underbrace{48}_{\alpha_{min}}$ ✓

∴ el filtro Chebyshev que cumple con la plantilla será de orden 5 y $\varepsilon \cong 0{,}310609$

#2 $$|T_{C5(j\omega)}|^2 = \frac{1}{1+\varepsilon^2\, C_{5(\omega)}^2}$$

$$\left.\begin{cases} C_n = 2\omega\, C_{n-1} - C_{n-2} \\ C_0 = 1 \\ C_1 = \omega \end{cases}\right\} C_n^2(\omega)$$

$$C_2 = 2\omega(\omega) - 1 = 2\omega^2 - 1$$

$$C_3 = 2\omega(2\omega^2-1) - \omega = 4\omega^3 - 3\omega$$

$$C_4 = 2\omega(4\omega^3 - 3\omega) - (2\omega^2 - 1) = 8\omega^4 - 8\omega^2 + 1$$

$$C_5 = 2\omega(8\omega^4 - 8\omega^2 + 1) - (4\omega^3 - 3\omega) = 16\omega^5 - 20\omega^3 + 5\omega$$

$$|T_{C5(j\omega)}|^2 = \frac{1}{1+\varepsilon^2(16\omega^5 - 20\omega^3 + 5\omega)^2} = \frac{1}{1+\varepsilon^2(256\,\omega^{10} + 400\,\omega^6 + 25\omega^2 - 640\,\omega^8 + 160\,\omega^6 - 200\,\omega^4)}$$

$$|T_{C5(j\omega)}|^2 = \frac{1}{256\,\varepsilon^2\,\omega^{10} - 640\varepsilon^2\omega^8 + 560\,\varepsilon^2\,\omega^6 - 200\,\varepsilon^2\,\omega^4 + 25\,\varepsilon^2\,\omega^2 + 1}$$

$$|T_{C5(s)}|^2\Big|_{\omega=\frac{s}{j}} = \frac{1}{-256\,\varepsilon^2\,s^{10} - 640\varepsilon^2\,s^8 - 560\varepsilon^2\,s^6 - 200\,\varepsilon^2\,s^4 - 25\,\varepsilon^2\,s^2 + 1}$$

$$|T_{C5}(s)|^2 = \frac{-\frac{1}{256\,\varepsilon^2}}{s^{10} + \frac{5}{2}\,s^8 + \frac{35}{16}\,s^6 + \frac{25}{32}\,s^4 + \frac{25}{256}\,s^2 - \frac{1}{256\,\varepsilon^2}}$$

Se calculan las raíces del polinomio del denominador:

~~0.11932228+1.01949544j~~, ~~0.11932228-1.01949544j~~,
(3) -0.11932228+1.01949544j, -0.11932228-1.01949544j,
~~0.31238979+0.63008283j~~, ~~0.31238979-0.63008283j~~,
(2) -0.31238979+0.63008283j, -0.31238979-0.63008283j,
(1) -0.38613502+0.j , ~~0.38613502+0.j~~]

2 pares de polos comp. conj. y un polo Real
No se tienen en cuenta aquellos que estén
en el semiplano derecho

(1) $$T_{1(s)} = \frac{1}{s+0{,}386}$$

(2) $\alpha \pm j\beta \rightarrow \alpha_2 = 0{,}312 \quad \beta_2 = 0{,}63$
$$T_{2(s)} = \frac{1}{s^2 + 2\alpha_2 s + \alpha_2^2 + \beta_2^2} \longrightarrow T_{2(s)} = \frac{1}{s^2 + 0{,}625\,s + 0{,}495}$$

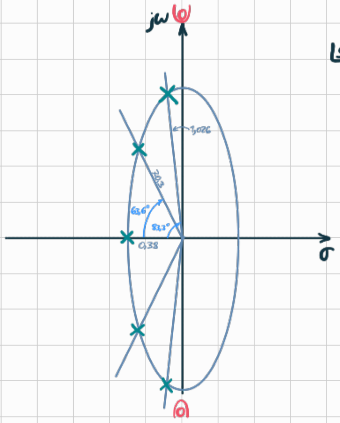
(3) $\alpha \pm j\beta \rightarrow \alpha_3 = 0{,}119 \quad \beta_3 = 1{,}019$
$$T_{3(s)} = \frac{1}{s^2 + 2\alpha_3 s + \alpha_3^2 + \beta_3^2} \longrightarrow T_{3(s)} = \frac{1}{s^2 + 0{,}239\,s + 1{,}054}$$

$$T_{(s)} = T_{1(s)} \cdot T_{2(s)} \cdot T_{3(s)} = \frac{1}{s+0{,}386} \cdot \frac{1}{s^2 + 0{,}625\,s + 0{,}495} \cdot \frac{\overbrace{(-0{,}2012)}^{-\sqrt{\frac{1}{256\,\varepsilon^2}}}}{s^2 + 0{,}239\,s + 1{,}054}$$

$$T_{(s)} = -\frac{0{,}2012}{s^5 + 1{,}2496\,s^4 + 2{,}0307\,s^3 + 1{,}4317\,s^2 + 0{,}8209\,s + 0{,}2012}$$

#3

Diagrama de Polos y Ceros:



Ls Polos:

```
[-0.11932228+1.01949544j, -0.11932228-1.01949544j,
 -0.31238979+0.63008283j, -0.31238979-0.63008283j,
 -0.38613502+0.j          ])
```

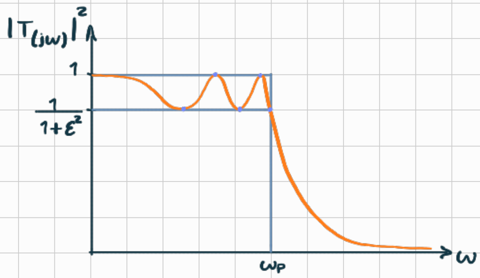
* Ángulos:

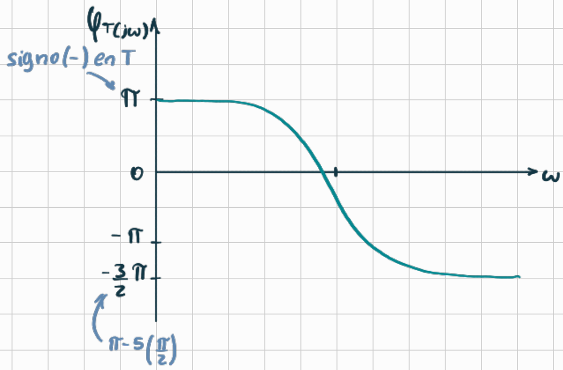
```
[  96.67555691,  -96.67555691,  116.37182893, -116.37182893,
  180.         ])
```

* Módulos:

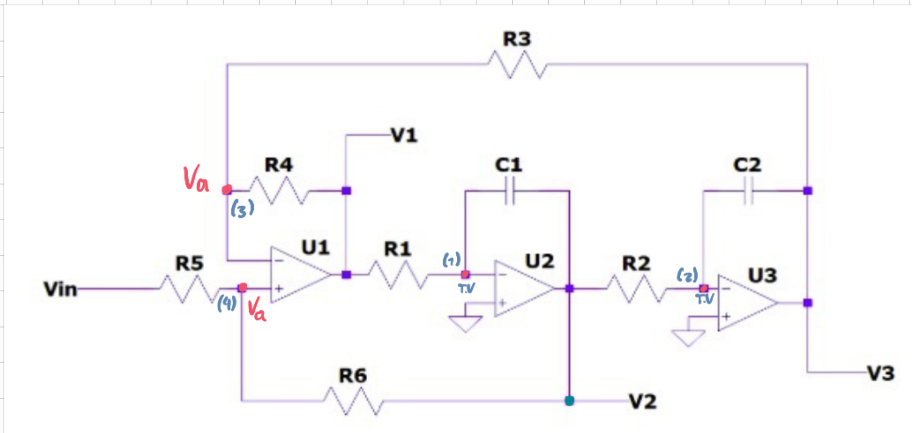
```
[1.02645446, 1.02645446, 0.70327218, 0.70327218, 0.38613502]
```

Diagrama aproximado de módulo y fase:





#4



$$
\left.\begin{aligned}
&(1)\quad V_1 G_1 + V_2\, s C_1 = 0\\
&(2)\quad V_2 G_2 + V_3\, s C_2 = 0\\
&(3)\quad V_a\left(G_3 + G_4\right) - V_1 G_4 - V_3 G_3 = 0\\
&(4)\quad V_a\left(G_5 + G_6\right) - V_{in} G_5 - V_2 G_6 = 0
\end{aligned}\right\}
$$

* Se obtiene $T_{(s)}$ por medio de cálculo simbólico:

$$
T_{(s)} = \frac{V_3}{V_{in}} = \frac{G_1 G_2 G_5 (G_3 + G_4)}{s^2 G_4 C_1 C_2 (G_5 + G_6) + s\, G_1 G_6 C_2 (G_3 + G_4) + G_1 G_2 G_3 (G_5 + G_6)}
$$

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$$T_{(s)} = \frac{G_1G_2G_5(G_3+G_4)}{G_4C_1C_2(G_5+G_6)} \frac{1}{s^2 + s\,\frac{G_1G_6(G_3+G_4)}{G_4C_1(G_5+G_6)} + \frac{G_1G_2G_3}{G_4C_1C_2}}$$ ← Chequeo de unidades OK

$$T_{(s)} = \frac{G_5(G_3+G_4)}{G_3(G_5+G_6)} \frac{\frac{G_1G_2G_3}{G_4C_1C_2}}{s^2 + s\,\frac{G_1G_6(G_3+G_4)}{G_4C_1(G_5+G_6)} + \frac{G_1G_2G_3}{G_4C_1C_2}} = K\,\frac{\omega_o^2}{s^2 + s\,\frac{\omega_o}{Q} + \omega_o^2}$$

$$\left.\begin{aligned} \omega_o &= \sqrt{\frac{G_1G_2G_3}{G_4C_1C_2}} \\ \frac{\omega_o}{Q} &= \frac{G_1G_6(G_3+G_4)}{G_4C_1(G_5+G_6)} \end{aligned}\right\} Q = \frac{G_4C_1(G_5+G_6)}{G_1G_6(G_3+G_4)}\sqrt{\frac{G_1G_2G_3}{G_4C_1C_2}} \quad ; \quad K = \frac{G_5(G_3+G_4)}{G_3(G_5+G_6)}$$

Partiendo de la func. transferencia obtenida para el filtro Chebyshev

$$T_{cs(s)} = T_{1(s)} \cdot T_{2(s)} \cdot T_{3(s)} = \underbrace{\frac{1}{s+0{,}386}}_{T_{1(s)}} \cdot \underbrace{\frac{1}{s^2+0{,}625\,s+0{,}495}}_{T_{2(s)}} \cdot \underbrace{\frac{0{,}2012}{s^2+0{,}239\,s+1{,}054}}_{T_{3(s)}}$$

*2) $T_{(s)} = T_{2(s)}$

$$T_{(s)} = K\,\frac{\omega_o^2}{s^2 + s\,\frac{\omega_o}{Q} + \omega_o^2} = \frac{1}{0{,}495}\,\frac{0{,}495}{s^2 + s\,0{,}625 + 0{,}495}$$

$$\begin{cases} K = \frac{G_5(G_3+G_4)}{G_3(G_5+G_6)} = \frac{1}{0{,}495} \\ \omega_o^2 = \frac{G_1G_2G_3}{G_4C_1C_2} = 0{,}495 \\ \frac{\omega_o}{Q} = \frac{G_1G_6(G_3+G_4)}{G_4C_1(G_5+G_6)} = 0{,}625 \end{cases}$$

Como hay 8 incógnitas y solo 3 datos, establezco relaciones p/ poder resolver el sist.

$$\begin{cases} C_1 = C_2 = C \\ G_3 + G_4 = G_5 + G_6 \\ G_1 = G_4 \\ G_5 = G_6 = 1 \end{cases} \qquad \begin{cases} K = \frac{1}{G_3} = \frac{1}{0{,}495} \\ \omega_o^2 = \frac{G_2G_3}{C^2} = 0{,}495 \\ \frac{\omega_o}{Q} = \frac{1}{C} = 0{,}625 \end{cases}$$

Resolución por cálculo simbólico:

$C = 1{,}6$

$G_2 = 2{,}56$

$G_3 = 0{,}495$

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$$\begin{cases} C_1 = C_2 = 1{,}6 \\ G_1 = G_4 = 1{,}505 \\ G_2 = 2{,}56 \\ G_3 = 0{,}495 \\ G_5 = G_6 = 1 \end{cases} \qquad \begin{cases} C_1 = C_2 = 1{,}6 \\ R_1 = R_4 = 0{,}664 \\ R_2 = 0{,}391 \\ R_3 = 2{,}02 \\ R_5 = R_6 = 1 \end{cases}$$

*3) $T_{(s)} = T_{2(s)}$

$$T_{(s)} = K \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} = 0{,}1909 \frac{1{,}054}{s^2 + s\,0{,}239 + 1{,}054}$$

$$\begin{cases} K = \dfrac{G_5(G_3+G_4)}{G_3(G_5+G_6)} = 0{,}1909 \\ \omega_0^2 = \dfrac{G_1 G_2 G_3}{G_4 C_1 C_2} = 1{,}054 \\ \dfrac{\omega_0}{Q} = \dfrac{G_1 G_6 (G_3+G_4)}{G_4 C_1 (G_5+G_6)} = 0{,}239 \end{cases}$$

De forma similar a la resolución anterior, establezco relaciones p/ poder resolver el sist.

$$\begin{cases} C_1 = C_2 = 1 \\ G_5 = G_3 \\ G_1 = G_4 = G_3 \end{cases} \qquad \begin{cases} K = \dfrac{2G_3}{G_3+G_6} = 0{,}1909 \\ \omega_0^2 = G_2 G_3 = 1{,}054 \\ \dfrac{\omega_0}{Q} = G_6 \dfrac{2G_3}{G_3+G_6} = 0{,}239 \end{cases}$$

Resolución por cálculo simbólico:

$G_2 = 7{,}978$

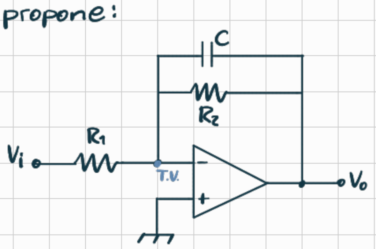
$G_3 = 0{,}132$

$G_6 = 1{,}252$

$$\begin{cases} C_1 = C_2 = 1 \\ G_1 = G_4 = G_3 = G_5 = 0{,}132 \\ G_2 = 7{,}978 \\ G_6 = 1{,}252 \end{cases} \qquad \begin{cases} C_1 = C_2 = 1 \\ R_1 = R_3 = R_4 = R_5 = 7{,}576 \\ R_2 = 0{,}125 \\ R_6 = 0{,}799 \end{cases}$$

* 1) $T_{1(s)} = \dfrac{-1}{s+0{,}386}$

Se propone:



$V_i\, G_1 + V_o\,(G_2 + sC) = 0$

$T_{(s)} = \dfrac{V_o}{V_i} = -\dfrac{G_1}{sC+G_2} = -\dfrac{\frac{1}{R_1 C}}{s+\frac{1}{R_2 C}}$

$T_{1(s)} = \dfrac{-1}{s+0{,}386} = -\dfrac{\frac{1}{R_1 C}}{s+\frac{1}{R_2 C}}$

$\begin{cases} \dfrac{1}{R_1 C} = 1 \\ \dfrac{1}{R_2 C} = 0{,}386 \end{cases}$; $R_1 = 1$ $\begin{cases} R_1 = 1 \\ R_2 = 2{,}59 \\ C = 1 \end{cases}$

Bonus (Desnormalización en frecuencia)

$L = \dfrac{L_n}{\Omega_\omega}$ $C = \dfrac{C_n}{\Omega_\omega}$ $R = R_n$; siendo $\Omega_\omega = \omega_p = 2\pi . f_p = 2\pi . 300 = 1884{,}96$ rad/seg

* Los valores desnormalizados fueron reemplazados directamente en la simulación circuital.